Parton shower generator at small x

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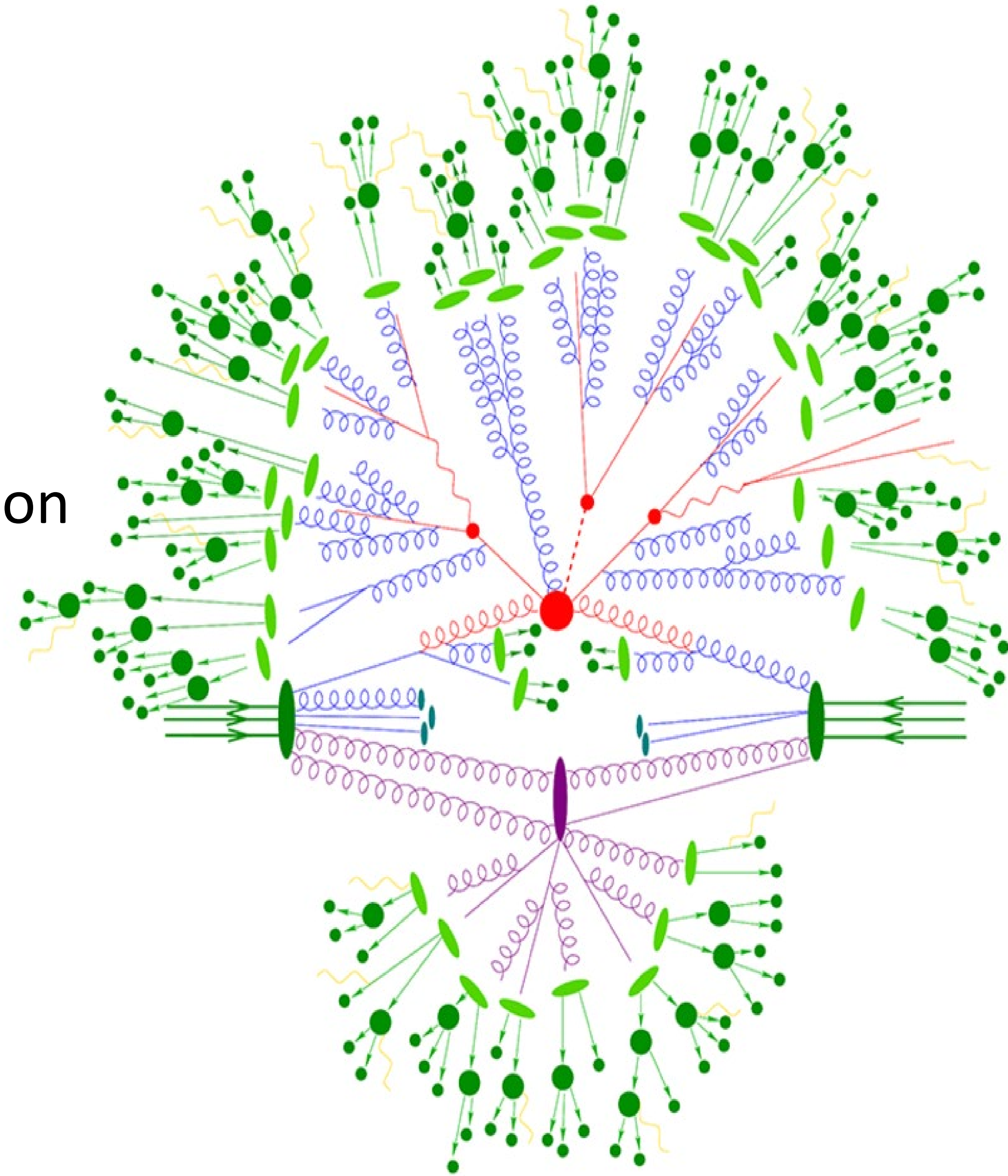
Outline:

- Background
- Forward evolution & backward evolution
- Joint small x and kt resummation
- Summary



Why parton shower generator

- Describe fully exclusive hadronic state
- Coherent branching & recoiled effect
- Keep four momentum conservation in each branching
- Impact studies for future experiments

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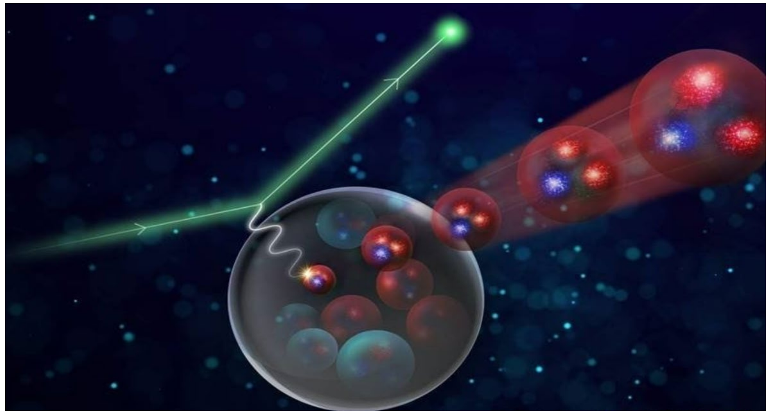
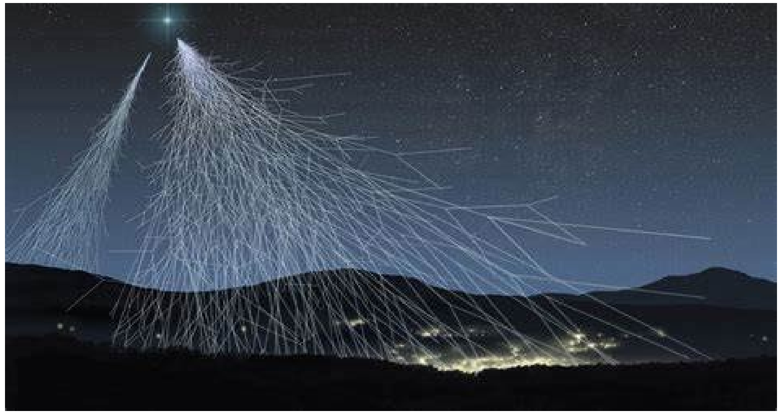
Why small *x* parton shower generator

- Saturation effect is absent in all existing generators

- Aim at developing a PS algorithm to be used:



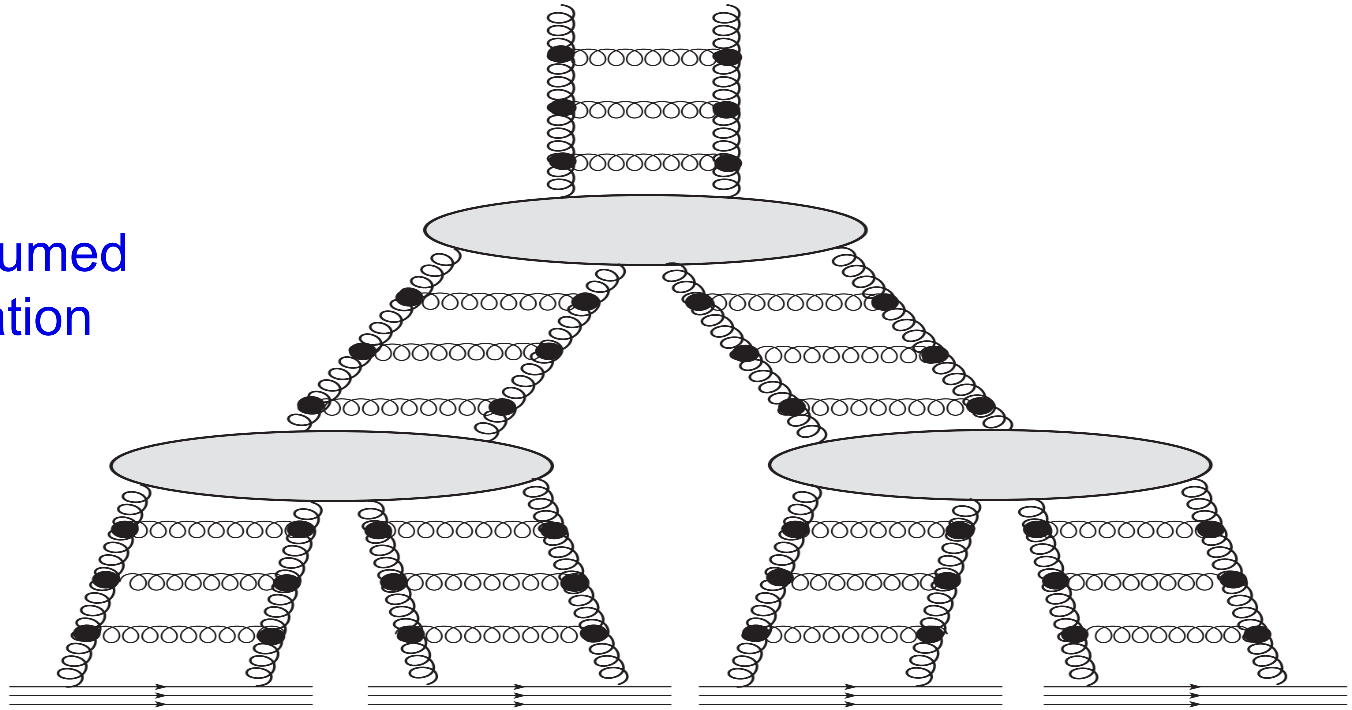
- Phenomology in eA collisions @EIC
- Forward physics in pA collisions @LHC
- Cosmic ray event generator



Small x evolution equations: $\ln\frac{S}{Q^2}$ or $\ln\frac{1}{x}$

- **BK equation** 2 →1, 3 →1, 4 →1 … gluon fusion & $\ln\frac{1}{x}$ Balitsky, 1996; Kovchegov, 1997
- **GLR equation** 2 →1 gluon fusion & $\ln\frac{1}{x}$ Gribov-Levin-Ryskin,1983

“fan” diagram resumed
by the GLR equation



Folded and unfolded GLR equation

- The standard GLR equation(unfolded one)

$$\frac{\partial N(\eta,k_\perp)}{\partial\eta}=\frac{\bar{\alpha}_s}{\pi}\left[\int\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)-\int\frac{\mathrm{d}^2l_\perp}{(k_\perp-l_\perp)^2}\frac{k_\perp^2}{2l_\perp^2}N(\eta,k_\perp)\right]-\bar{\alpha}_sN^2(\eta,k_\perp)$$

- Resolved and unresolved branching:

$$\int\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)\approx\int_\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp+l_\perp)+\int_0^\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}N(\eta,k_\perp)$$

- Folded GLR equation by resmming virtual correction

$$\frac{\partial}{\partial\eta}\frac{N(\eta,k_\perp)}{\Delta(\eta,k_\perp)}=\frac{\bar{\alpha}_s}{\pi}\int_\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}\frac{N(\eta,l_\perp+k_\perp)}{\Delta(\eta,k_\perp)}$$

- Non-Sudakov form factor

$$\Delta(\eta,k_\perp)=\exp\left\{-\bar{\alpha}_s\int_{\eta_0}^{\eta}d\eta'\left[\ln\frac{k_\perp^2}{\mu^2}+N(\eta',k_\perp)\right]\right\}$$

Shi-Wei-ZJ, 2022

GLR v.s. DGLAP

- Determine the next x

$$\mathcal{R} = \exp\left[-\bar{\alpha}_s \int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta' \left(\ln \frac{k_\perp^2}{\mu^2} + N(\eta', k_\perp)\right)\right]$$

  - PS based on DGLAP type dynamics: evolution variable Q

- The generated event has to be re-weighted

$$\mathcal{W}(\eta_i, \eta_{i+1}; k_{\perp,i}) = \frac{\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \ln(P_\perp^2/\mu^2)}{\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \left[\ln(k_{\perp,i}^2/\mu^2) + N(\eta, k_{\perp,i})\right]}$$

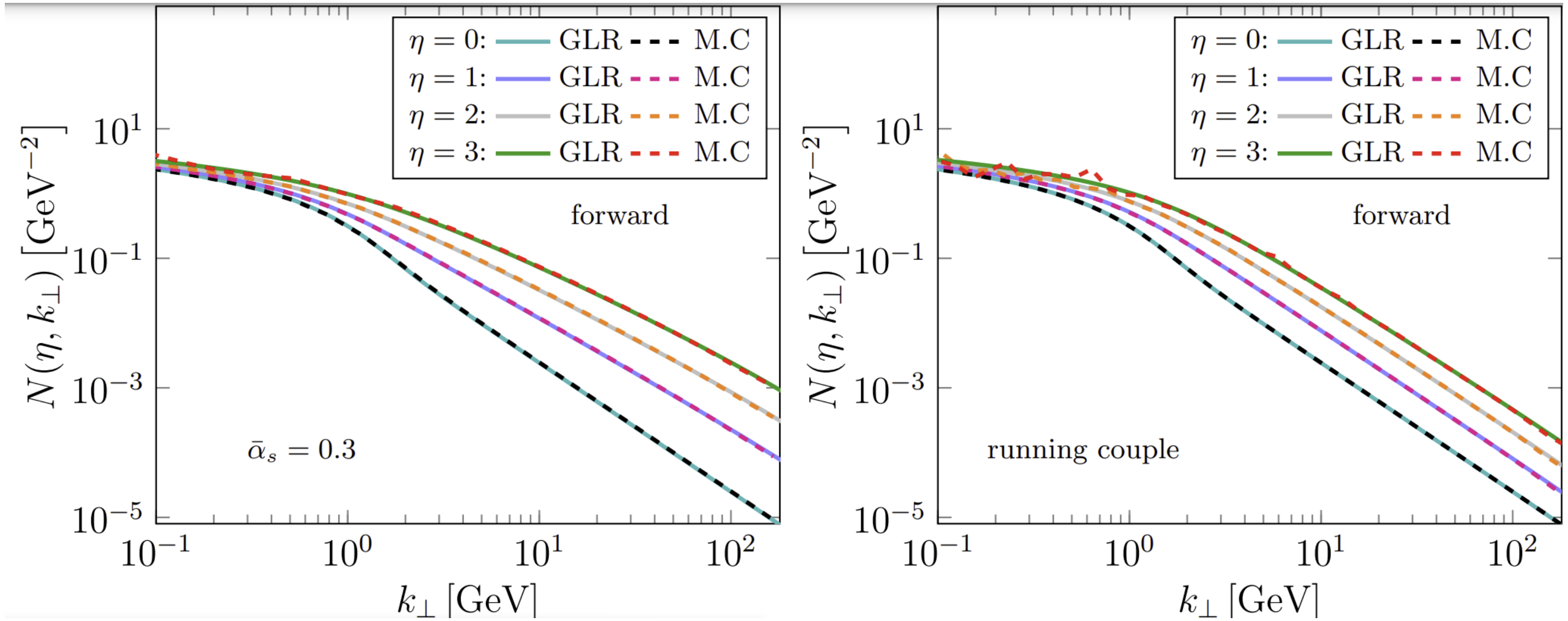
  - Unitary is preserved under DGLAP evolution

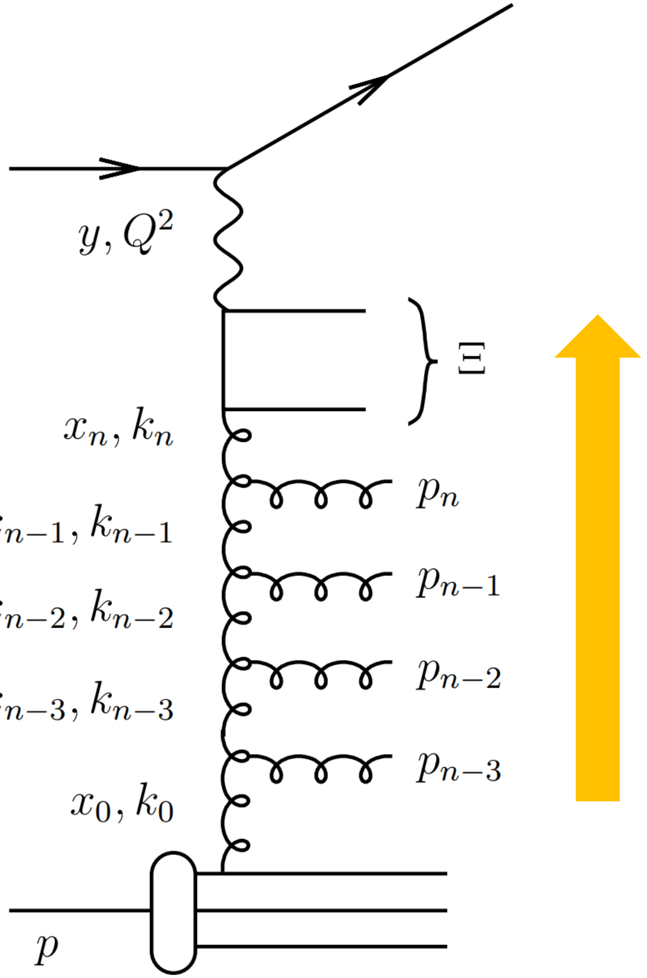
Forward evolution

- MV model as the initial condition at x_0=0.01:

$$N(\eta=0,k_\perp)=\int\frac{d^2r_\perp}{2\pi}e^{-ik_\perp\cdot r_\perp}\frac{1}{r_\perp^2}\left(1-\exp[-\frac{1}{4}Q_{s0}^2r_\perp^2\ln(e+\frac{1}{\Lambda r_\perp})]\right)$$

- Test the algorithm



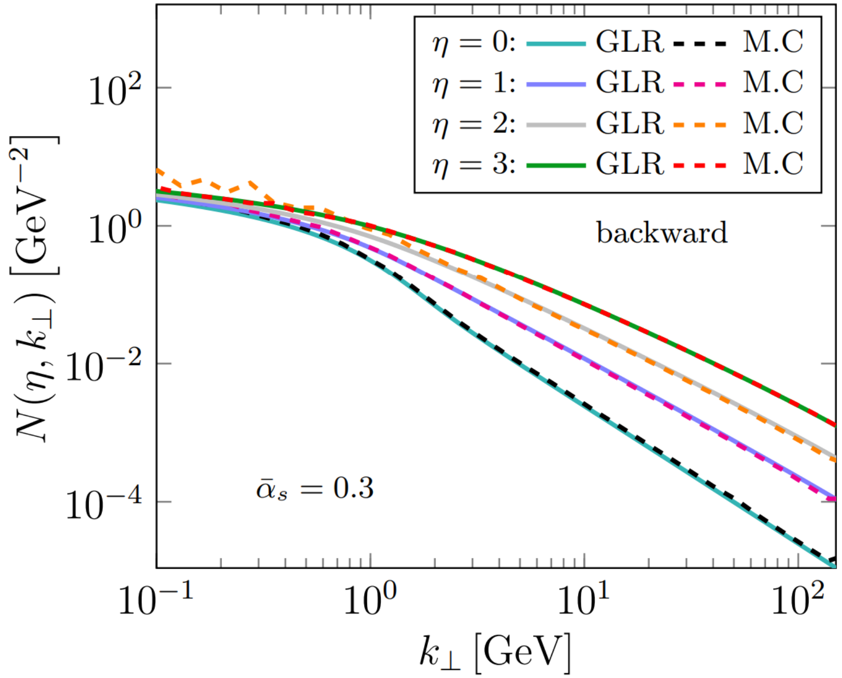


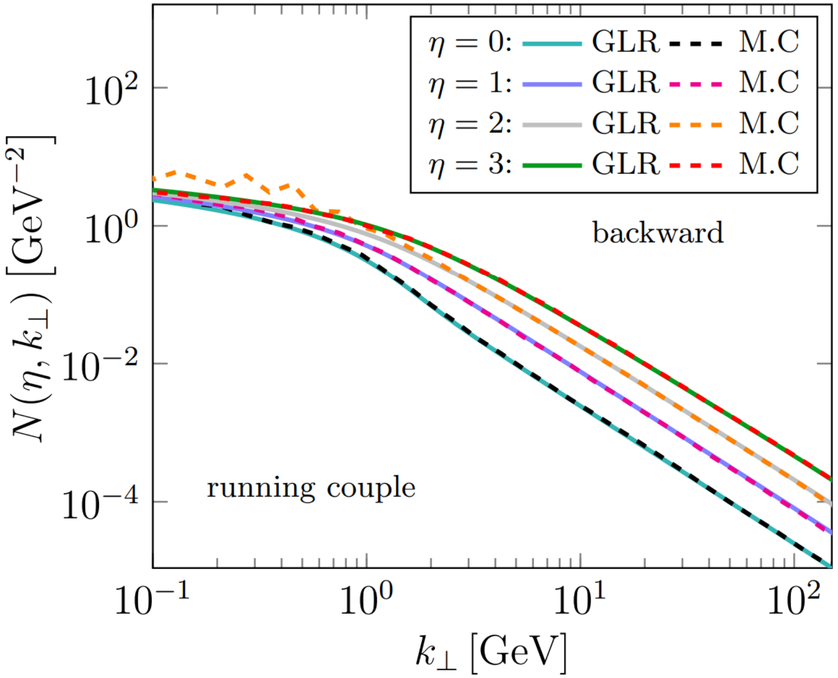
Backward evolution

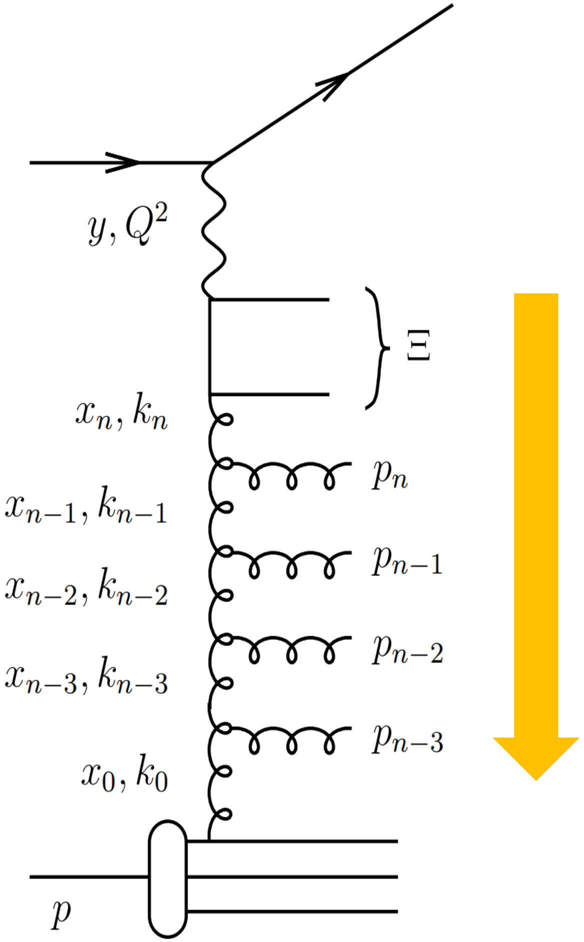
- Non-Sudakov form factor:

$$\frac{\Delta(\eta_{i+1}, k_{\perp,i+1})N(\eta_i, k_{\perp,i+1})}{\Delta(\eta_i, k_{\perp,i+1})N(\eta_{i+1}, k_{\perp,i+1})} = \exp\left[-\frac{\bar{\alpha}_s}{\pi}\int_{\eta_i}^{\eta_{i+1}} \mathrm{d}\eta \int_{\mu} \frac{\mathrm{d}^2 l_\perp}{l_\perp^2} \frac{N(\eta, k_{\perp,i+1} + l_\perp)}{N(\eta, k_{\perp,i+1})}\right]$$

- $l_\perp$ **is also sampled in a different way!**

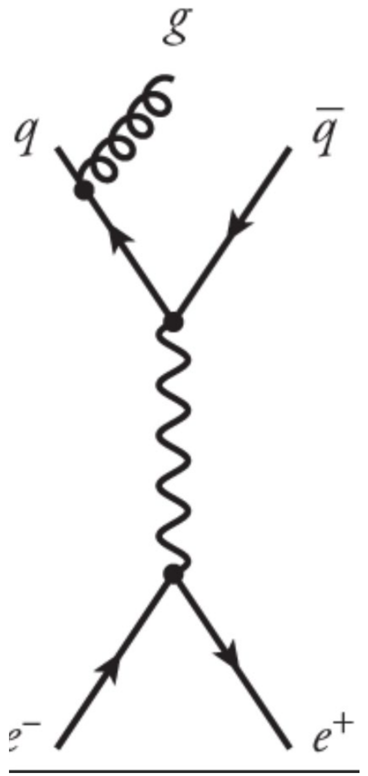






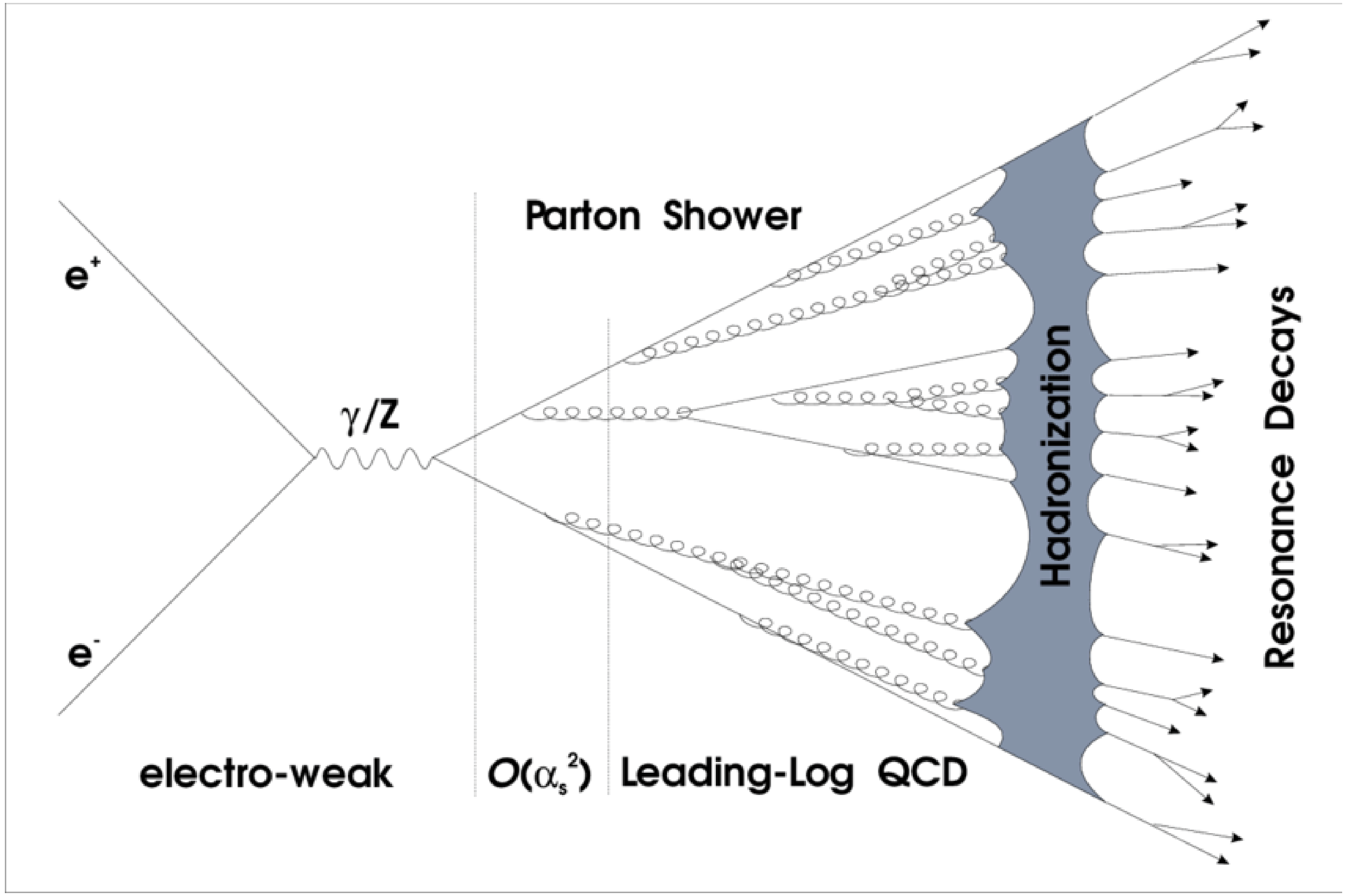
Coherent branching

- The Chudakov effect



➢ Wide angle radiation suppressed!

- Angular ordering



Coherent branching in the GLR evolution

- Kinematic constrained GLR equation

$$\frac{\partial}{\partial\eta}\frac{N(x,k_\perp)}{\Delta(\eta,k_\perp)}=\frac{\bar{\alpha}_s}{\pi}\int_\mu\frac{\mathrm{d}^2l_\perp}{l_\perp^2}\frac{N\left(\eta+\ln\left[\frac{k_\perp^2}{k_\perp^2+l_\perp^2}\right],l_\perp+k_\perp\right)}{\Delta(\eta,k_\perp)}$$

Shi-Wei-ZJ, 2022

- Weight factor for forward evolution:

$$\mathcal{W}_{kc}(\eta_i,\eta_{i+1};k_\perp)=\frac{(\eta_{i+1}-\eta_i)\int_\mu^{\min\left[P_\perp,\sqrt{(k_\perp-l_\perp)^2\frac{1-z}{z}}\right]}\frac{\mathrm{d}^2l_\perp}{l_\perp^2}e^{-\bar{\alpha}_s\int_{\eta_{i+1}}^{\eta_{i+1}+\ln\frac{(k_\perp-l_\perp)^2}{(k_\perp-l_\perp)^2+l_\perp^2}}\mathrm{d}\eta\left[\ln\frac{k_\perp^2}{\mu^2}+N(\eta,k_\perp)\right]}}{(\eta_{i+1}-\eta_i)\ln\frac{k_\perp^2}{\mu^2}+\int_{\eta_i}^{\eta_{i+1}}d\eta N(\eta,k_\perp)}$$

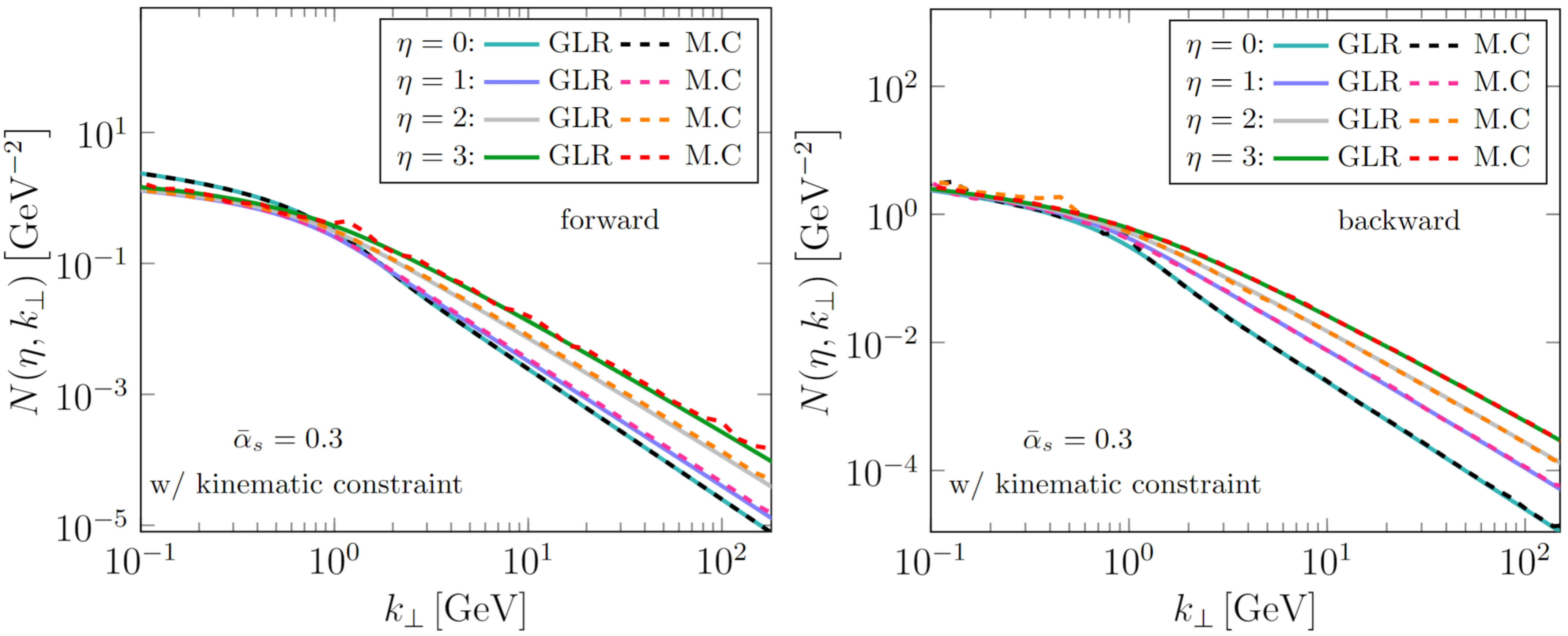
- Weight factor for backward evolution:

$$\mathcal{W}(\eta_{i+1},\eta_i;k_{\perp,i+1})=\frac{(\eta_{i+1}-\eta_i)\ln\frac{k_{\perp,i}^2}{\mu^2}+\int_{\eta_i}^{\eta_{i+1}}d\eta N(\eta,k_{\perp,i})}{(\eta_{i+1}-\eta_i)\ln\frac{P_\perp^2}{\mu^2}}\frac{N(\eta_i,k_{\perp,i})}{N\left(\eta_i+\ln\left[\frac{k_{\perp,i+1}^2}{k_{\perp,i+1}^2+l_\perp^2}\right],k_{\perp,i}\right)}$$

- Non-Sudakov factor for backward evolution

$$\frac{\Delta(\eta_{i+1},k_{\perp,i+1})N(\eta_i,k_{\perp,i+1})}{\Delta(\eta_i,k_{\perp,i+1})N(\eta_{i+1},k_{\perp,i+1})}=\exp\left[-\frac{\bar{\alpha}_s}{\pi}\int_{\eta_i}^{\eta_{i+1}}\mathrm{d}\eta\int_\mu^{\min\left[P_\perp,\sqrt{\frac{1-z}{z}k_{\perp,i+1}^2}\right]}\frac{\mathrm{d}^2l_\perp}{l_\perp^2}\frac{N\left(\eta+\ln\left[\frac{k_{\perp,i+1}^2}{k_{\perp,i+1}^2+l_\perp^2}\right],k_{\perp,i+1}+l_\perp\right)}{N(\eta,k_{\perp,i+1})}\right]$$

Coherent branching: test against the numerical results



Shi-Wei-ZJ, 2022

Joint small x and kt resummation

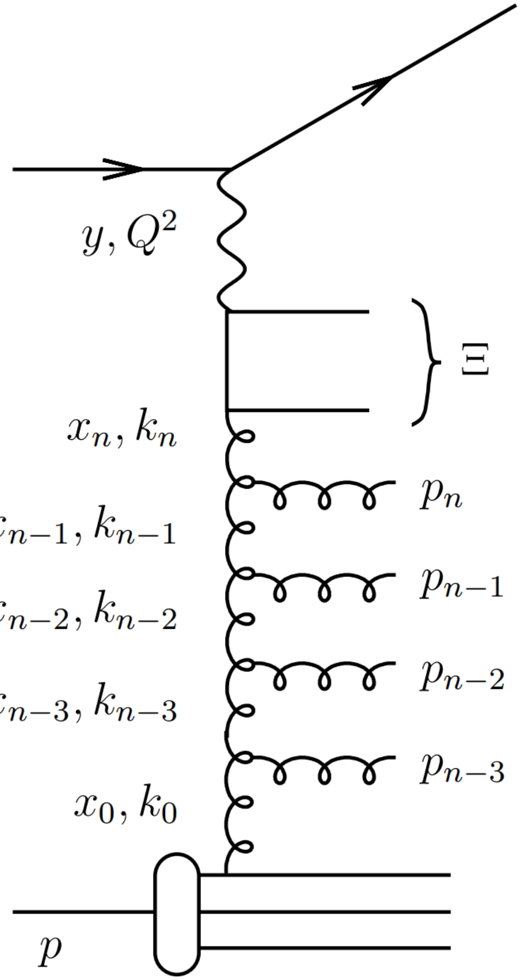
- When $S >> Q^2 >> k_T^2$, resume two type large logs

$$\ln \frac{S}{Q^2} \qquad \ln^2 \frac{Q^2}{k_\perp^2}$$

◆ Starting point:

$$xG(x, l_\perp, x\zeta) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3 P^+} e^{-ixP^+ y^- + i l_\perp \cdot y_\perp} \langle P | F_\mu^+(y^-, y_\perp) \mathcal{L}_{\tilde{n}}^\dagger (y^-, y_\perp) \mathcal{L}_{\tilde{n}}(0, 0_\perp) F^{\mu+}(0) | P \rangle$$

Collins, Soper, 81; Collins, Soper, Sterman, 85



Gloun---->gloun splitting kernel; $\mathcal{P}_{gg}(z) = 2C_A \frac{(z^2 - z + 1)^2}{z(1-z)}$

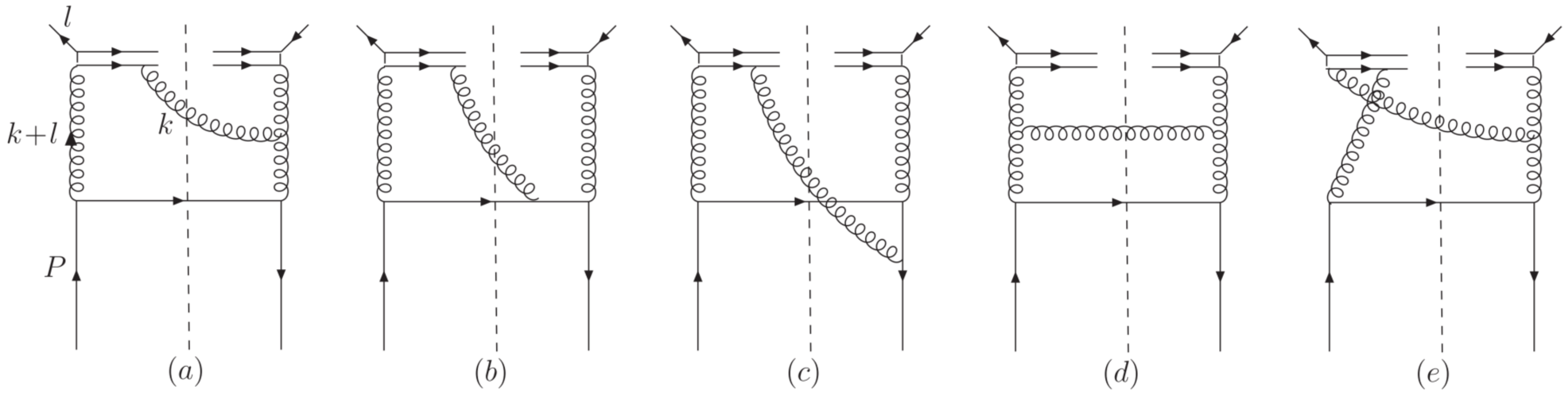
When z(or x) --->0, large logarithm $\ln \frac{1}{x}$ summed by BFKL

When z--->1, light cone divergence, introduce ζ to regularize

large logarithm $\ln \frac{x^2 \varsigma^2}{k_T^2}$ summed by CS

Small x gluon TMD in dilute limit

- Sample real diagrams



- The resulting gluon TMD indeed simultaneously satisfies the both

$$\int_0^\infty \frac{dk^+}{k^+} = \int_{l^+}^\infty \frac{dk^+}{k^+} + \int_0^{l^+} \frac{dk^+}{k^+}$$

2016, ZJ

BFKL equation:

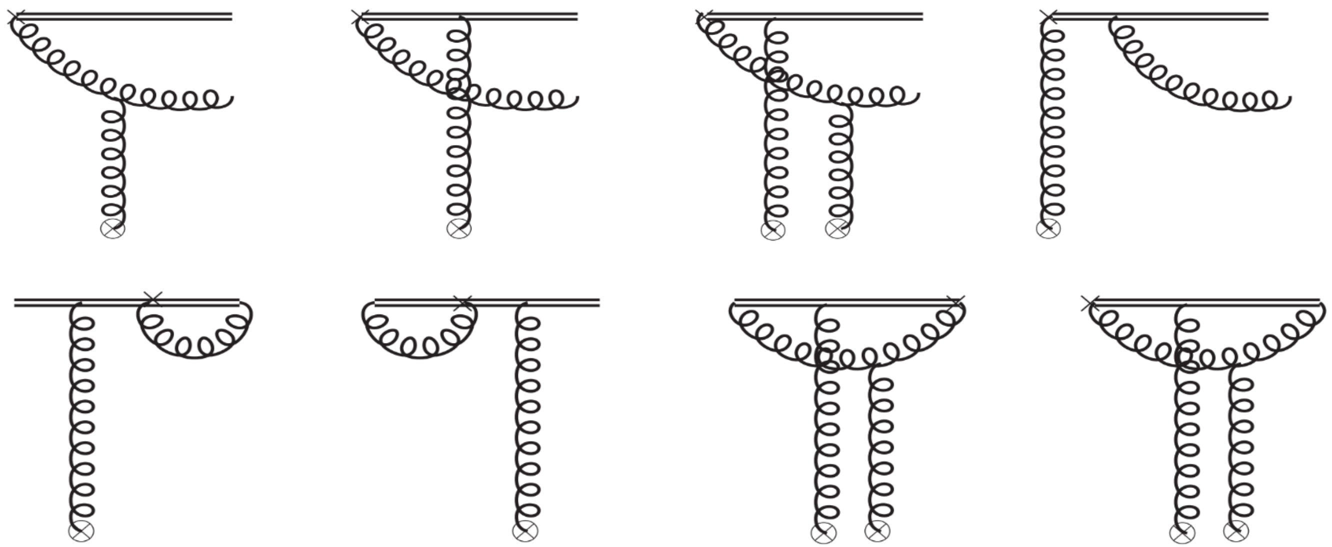
$$\frac{\partial\left[xG(x,l_\perp,x\zeta)\right]}{\partial \ln(1/x)} = \frac{\alpha_s N_c}{\pi^2}\int \frac{d^2k_\perp}{k_\perp^2}\left\{xG(x,k_\perp+l_\perp,x\zeta) - \frac{l_\perp^2}{2(l_\perp+k_\perp)^2} xG(x,l_\perp,x\zeta)\right\}$$

CS equation:

$$\frac{\partial\left[G(x,b_\perp,x\zeta)\right]}{\partial \ln\zeta} = -\frac{\alpha_s N_c}{\pi}\ln\left[\frac{x^2\zeta^2 b_\perp^2}{4} e^{2\gamma_E - \frac{1}{2}}\right] G(x,b_\perp,x\zeta)$$

Small x TMDs in CGC at NLO(double log)

Sample diagrams **(Collins-2011 scheme)**



Xiao, Yuan and ZJ, 2017.

The basic nonperturbative ingredient:

$$U(x_\perp) = \langle \mathcal{P} e^{-ig\int_{-\infty}^{+\infty} dx^- A^+(x^-,\, x_\perp)} \rangle$$

Multiple gluon rescattering is treated in an equal way.

Collinear approach v.s. CGC

- Collinear factorization:

$$\tilde{f}_g^{(sub.)}(x, r_\perp, \zeta_c{}') = e^{-S^g_{pert}(Q,r_\perp)} \sum_i C_{g/i}(\mu_r/\mu) \otimes f_i(x, \mu)$$

Sudakov factor

Hard coefficient

Colliner PDF

- CGC **(Colliner divergence absent)**

$$xG^{(1)}(x, k_\perp, \zeta) = -\frac{2}{\alpha_S}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{ik_\perp \cdot r_\perp} \mathcal{H}^{WW}(\alpha_s(Q)) e^{-S_{sud}(Q^2, r_\perp^2)} \mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp)$$

Hard coefficient

Sudakov factor

Two point function

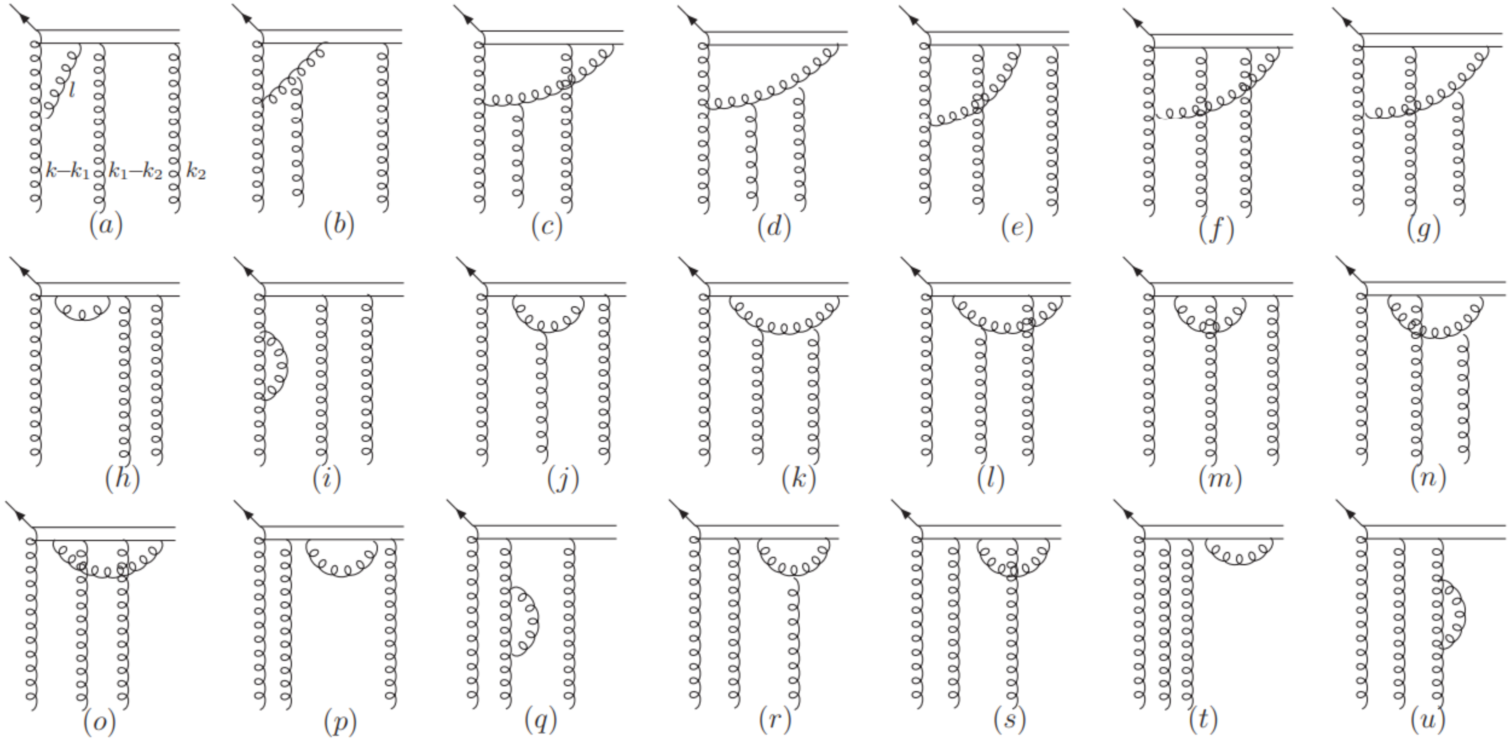
Two step evolution: x_0 → x kt → Q

Sudakov Single log at small x

The anomalous dimension at small x?

$$\frac{\mathrm{d}\,\ln G(x, b_\perp, \mu^2, \zeta_c^2)}{\mathrm{d}\,\ln\mu} = \gamma_G\left(g(\mu), \zeta_c^2/\mu^2\right)$$

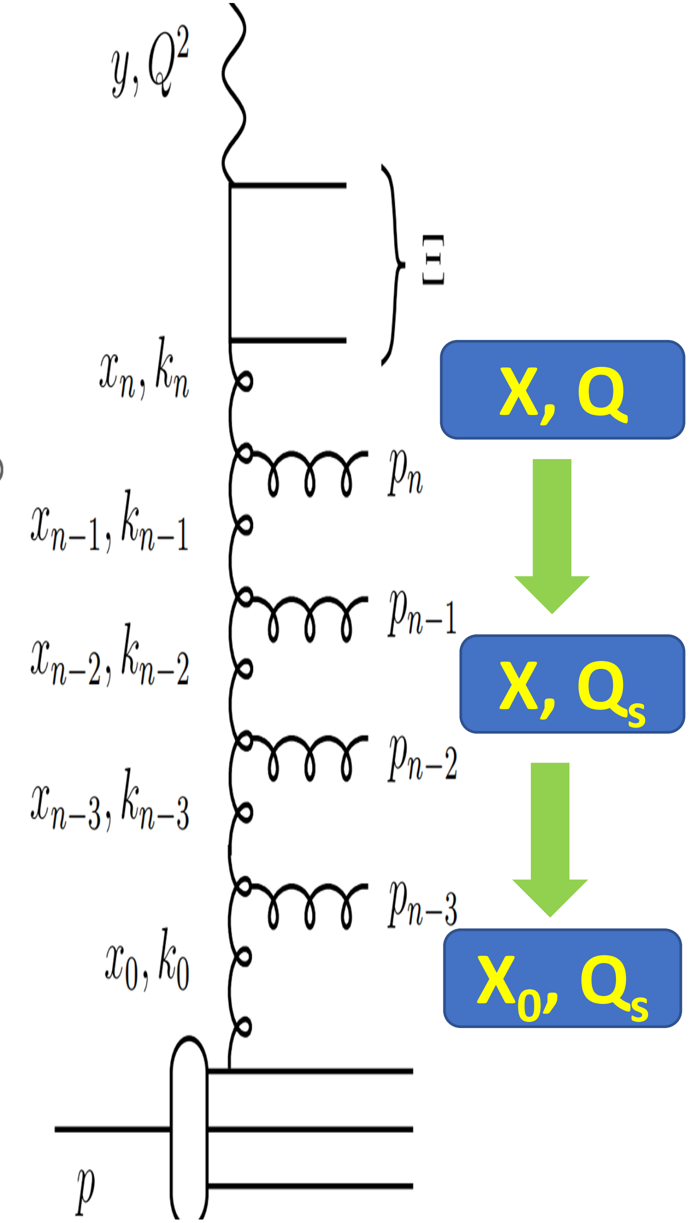
Compute UV part first; employ the Eikonal approximation next!



2019, ZJ

- Single log is not affected by saturation effect.

Monte Carlo implementation of the joint resummation



- Combing Collins-Soper equation

$$\frac{\partial N(\mu^2,\zeta^2,x,k_\perp)}{\partial \ln \zeta^2} = \frac{2\alpha_s N_c}{\pi^2}\int_0^\zeta \frac{d^2 l_\perp}{l_\perp^2}\left[N(\mu^2,\zeta^2,x,k_\perp+l_\perp) - N(\mu^2,\zeta^2,x,k_\perp)\right]$$

with renormalization group equation

$$\frac{\partial N(\mu^2,\zeta^2,x,k_\perp)}{\partial \ln \mu^2} = \frac{\alpha_s N_c}{\pi}\left[\frac{\beta_0}{6} - \ln\frac{\zeta^2}{\mu^2}\right] N(\mu^2,\zeta^2,x,k_\perp)$$

- **Folded CS+RG:** $N(Q^2,x,k_\perp) \equiv N(\mu^2 = Q^2, \zeta^2 = Q^2, x, k_\perp)$

$$\frac{\partial}{\partial \ln Q^2}\frac{N(Q^2,x,k_\perp)}{\Delta_s(Q^2,k_\perp)} = \frac{2\bar{\alpha}_s}{\pi}\int_{Q_0}^{Q}\frac{d^2 l_\perp}{l_\perp^2}\frac{N(Q^2,x,k_\perp+l_\perp)}{\Delta_s(Q^2,k_\perp)}$$

Shi-Wei-ZJ, in preparation

Sudakov form factor

$$\Delta_s(Q^2,k_\perp) = \exp\left[-\bar{\alpha}_s\int_{Q_0^2}^{Q^2}\frac{dt}{t}\left(2\ln\frac{t}{Q_0^2} - \frac{\beta_0}{6}\right)\right]$$

Monte Carlo implementation of kt resummation

- The modified Sudakov factor in the backward evolution:

$$\frac{\Delta_s(Q_n^2,k_{\perp,n})N(Q_{n-1}^2,x_n,k_{\perp,n})}{\Delta_s(Q_{n-1}^2,k_{\perp,n})N(Q_n^2,x_n,k_{\perp,n})}=\exp\left[-\int_{Q_{n-1}^2}^{Q_n^2}\frac{dt}{t}\int_{Q_0}^{t}\frac{d^2l_\perp}{l_\perp^2}\frac{2\bar{\alpha}_s(l_\perp^2)}{\pi}\frac{N(t,x_n,k_{\perp,n}+l_\perp)}{N(t,x_n,k_{\perp,n})}\right]=\mathcal{R}$$

- Sample lt and reconstruct z

Shi-Wei-ZJ, in preparation

$$\int\frac{d^2l_\perp}{l_\perp^2}\frac{\bar{\alpha}_s(l_\perp^2)}{\pi}N(Q_{n-1}^2,x_n,k_{\perp,n}+l_\perp)\qquad l_{\perp,n}^2\approx Q_{n-1}^2(1-z_n)$$

- Weight factor

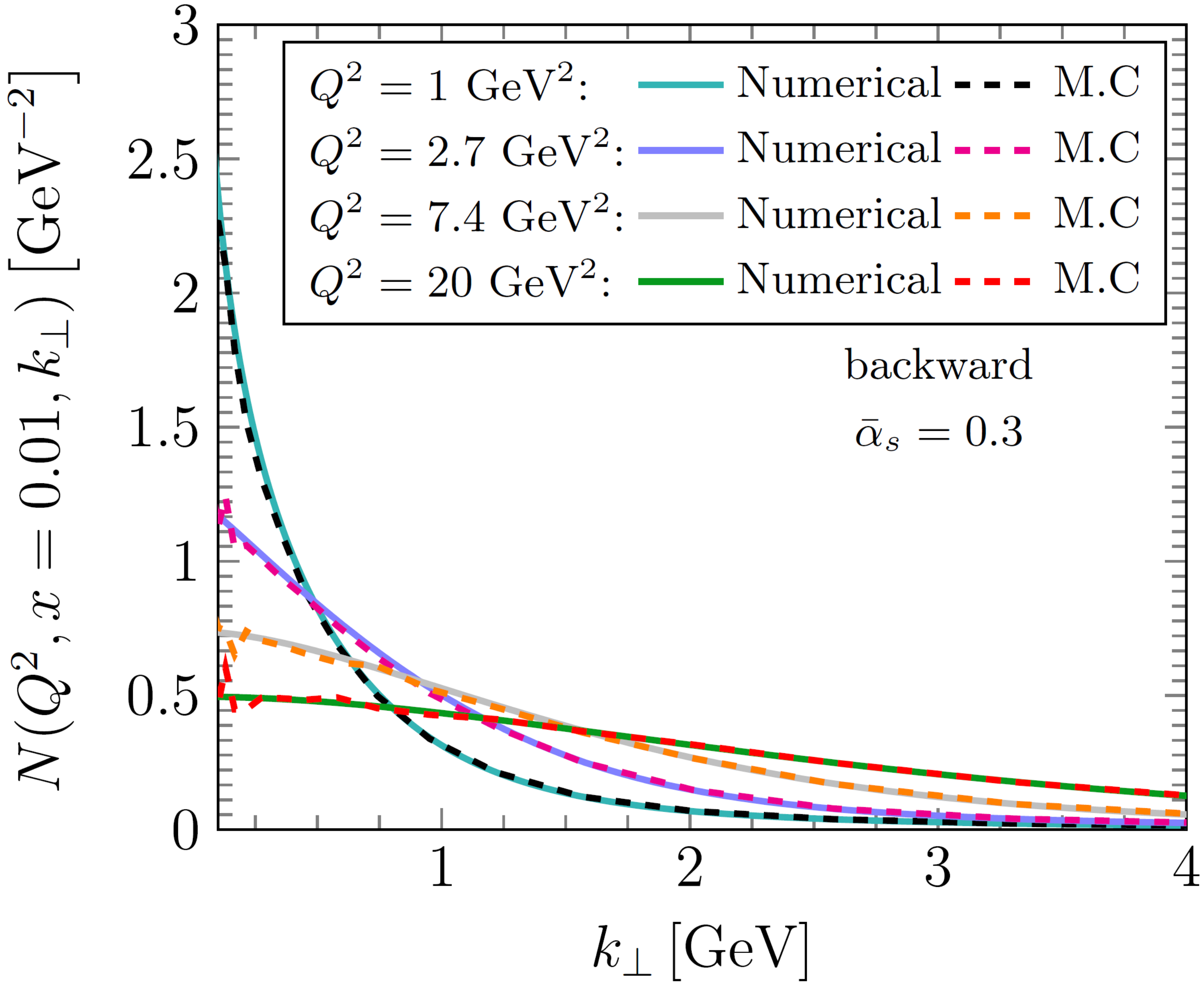
$$\mathcal{W}=\frac{\int_{Q_{n-1}^2}^{Q_n^2}\frac{dt}{t}\ln\frac{t^2}{Q_0^2}}{\int_{Q_{n-1}^2}^{Q_n^2}\frac{dt}{t}\left[\ln\frac{t^2}{Q_0^2}-\frac{\beta_0}{12}\right]}$$

- Unitary is preserved only within the double leading log approximation

- Terminate kt PS and turn on small x PS

$$Q_{n-1}^2>2k_{\perp,n}^2\text{ and }Q_{n-1}^2>2l_{\perp,n}^2.$$

Numerical V.S. MC:



Initial condition:

$$N(Q_0^2 = 1\ \mathrm{GeV}^2, x = 0.01, k_\perp) = \int \frac{d^2 r_\perp}{2\pi} e^{ik_\perp \cdot r_\perp} \frac{1}{r_\perp^2} \left[1 - e^{-\frac{Q_s^2 r_\perp^2}{4} \log(\frac{1}{r_\perp \Lambda_{\mathrm{MV}}} + e)}\right]$$

Summary and Outlook

- The first PS algorithm incorporating saturation effect
- The implementation of the joint resummation is sketched

- Recoiled effect; linear polarization effect; multiple gluon fusion effect
- Perform hadronization using PYTHIA

Thank you for your attention!

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