

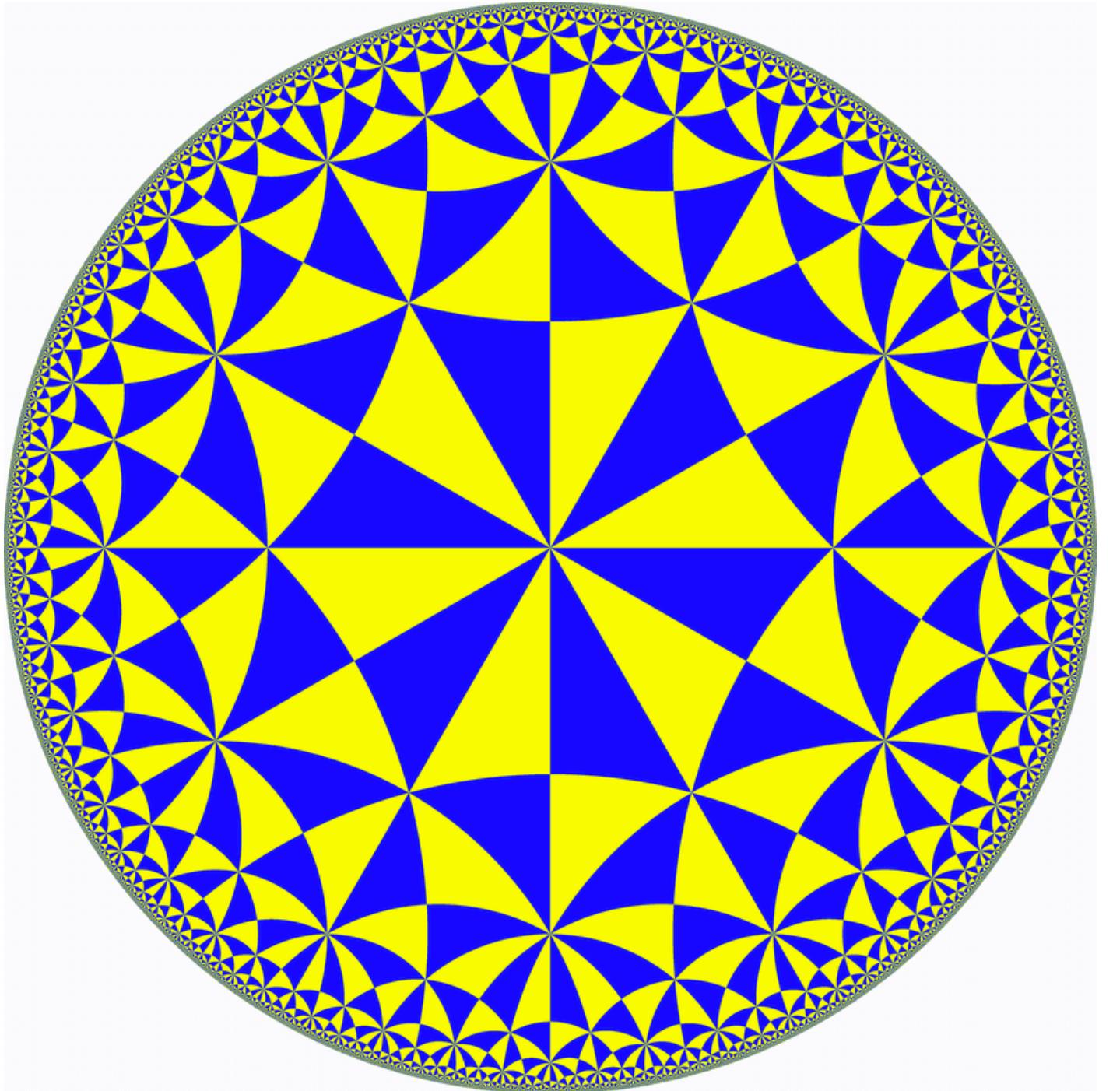
Conformal symmetry and collider physics

朱华星
浙江大学

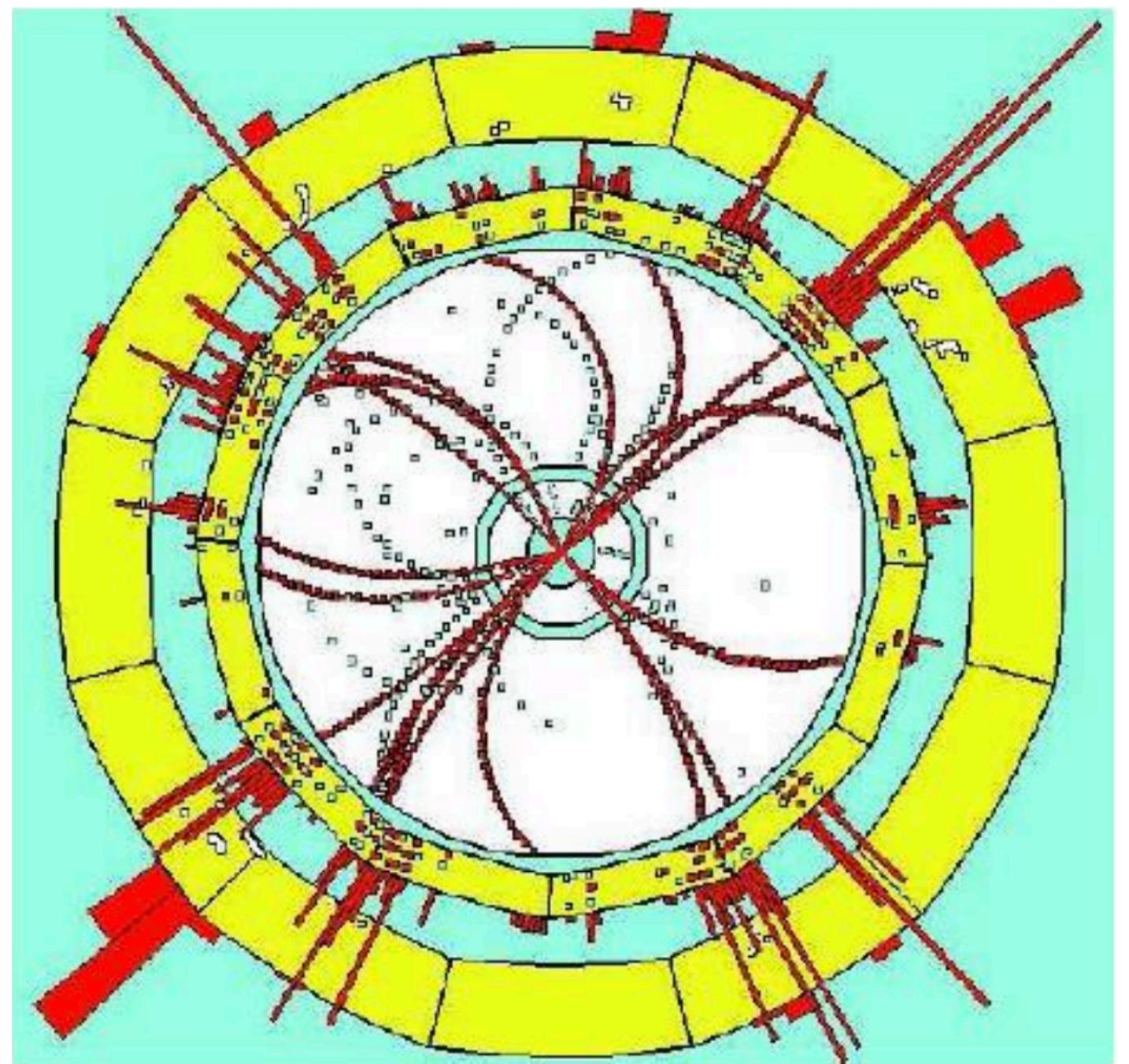
2301.03616 合作者：陈豪，周稀楠

第五届重味物理与量子色动力学研讨会
2023年4月21日至23日，武汉

Conformal field theory



Collider Physics

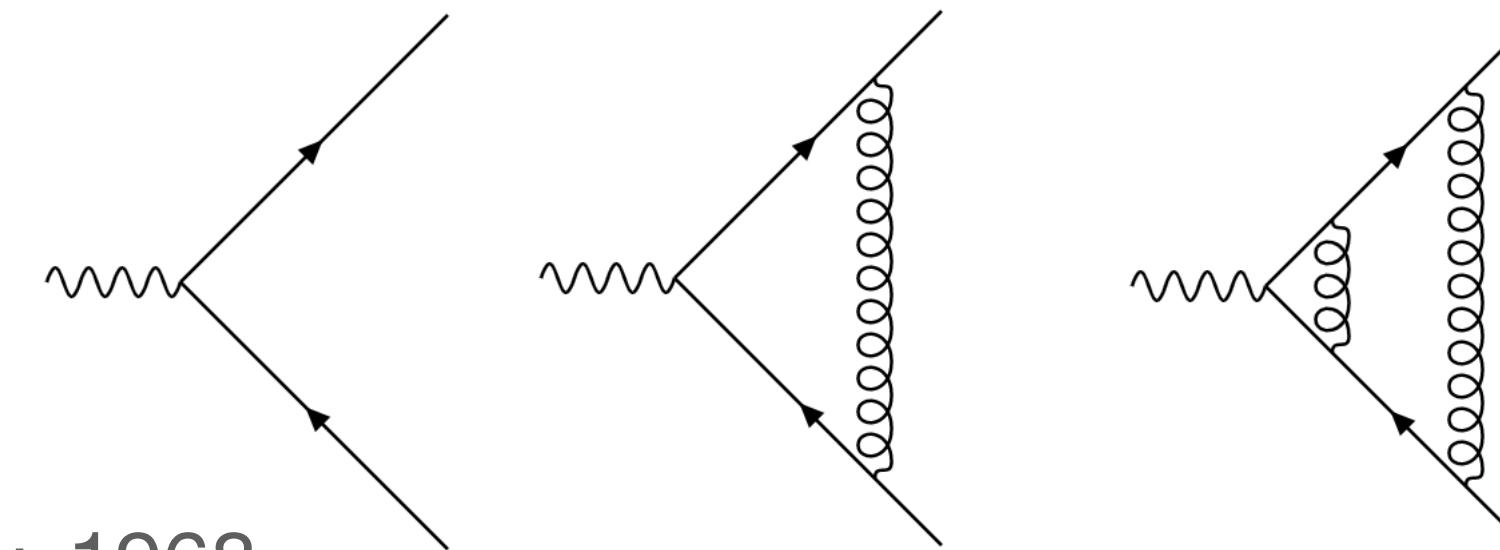


conformal partial wave expansion
Infinite spin summation
Analyticity in spin

Sudakov logarithms

Sudakov logarithms

form factors:



Sudakov, 1956; Jackiw, 1968

W/Z/H pT:

$$\frac{1}{\sigma_0} \frac{d\sigma}{dq_T^2} = \alpha_s \frac{\log q_T^2}{q_T^2} - \alpha_s^2 \frac{\log^3 q_T^2}{q_T^2} + \alpha_s^3 \frac{\log^5 q_T^2}{q_T^2} + \dots$$

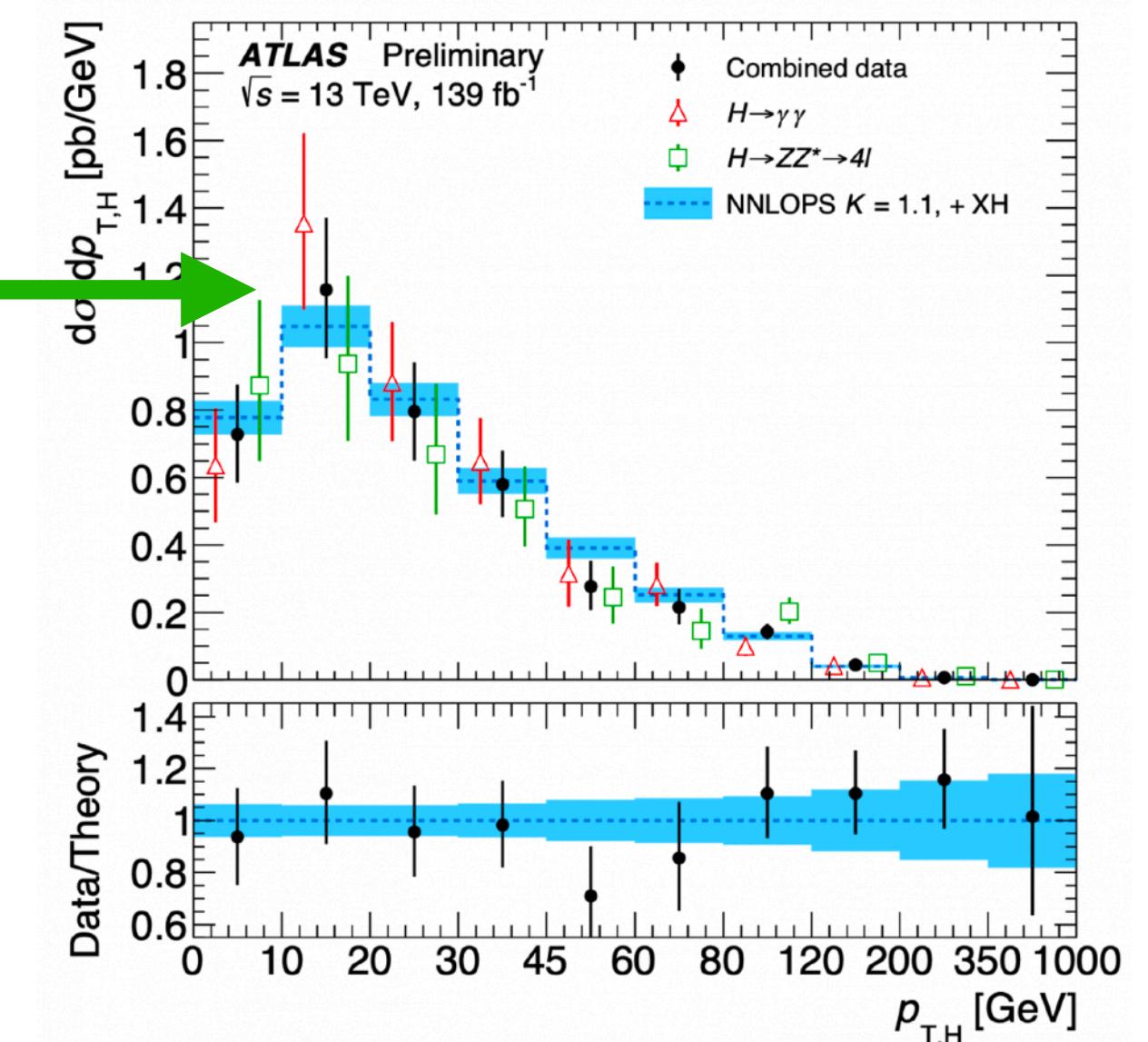
Dokshitzer, Dirkonov, Troian, 1978

Collins, Soper, Sterman, 1985

$$1 - \frac{a}{2\pi} \log^2 Q^2 + \frac{a^2}{8\pi^2} \log^4 Q^2 - \frac{a^3}{48\pi^3} \log^6 Q^2 + \dots$$

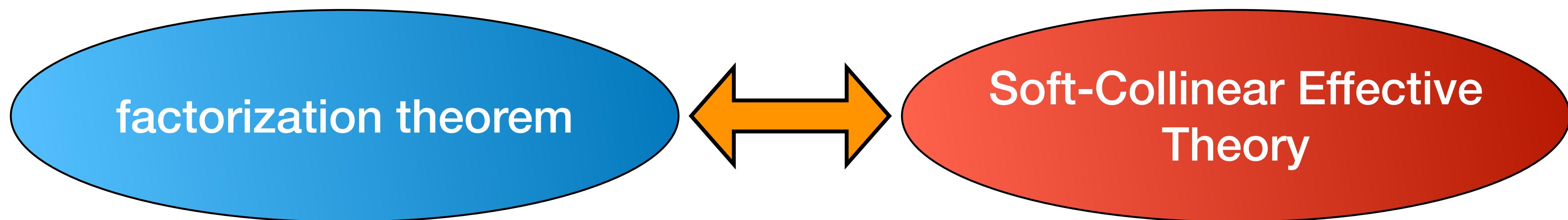
$$= \exp \left[-\frac{a}{2\pi} \log^2 Q^2 \right]$$

Sudakov peak

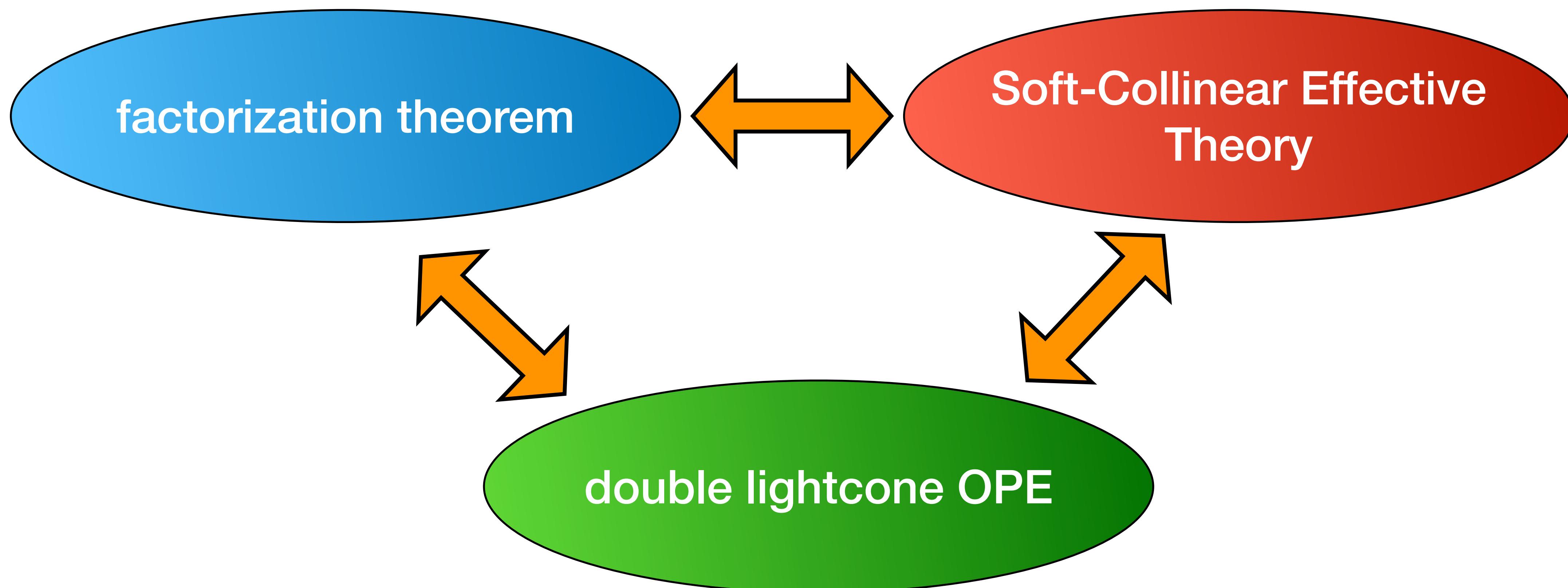


Origin: soft and collinear D.O.F. of gauge theory

A new method for Sudakov resummation



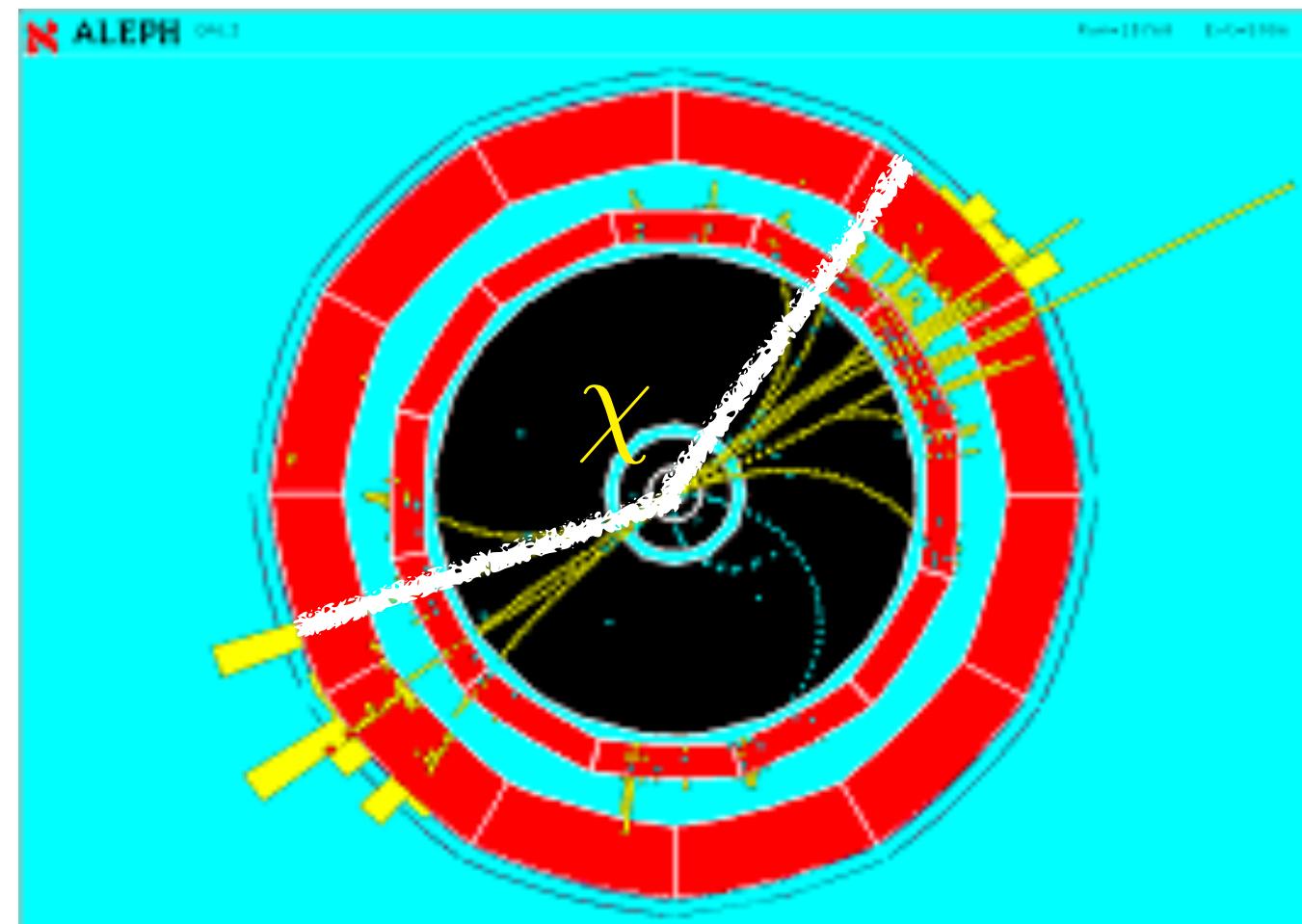
A new method for Sudakov resummation



Physical observable organized by irreducible representation of conformal group
Simplifying calculation for power corrections

Energy-Energy Correlator and Sudakov logarithms

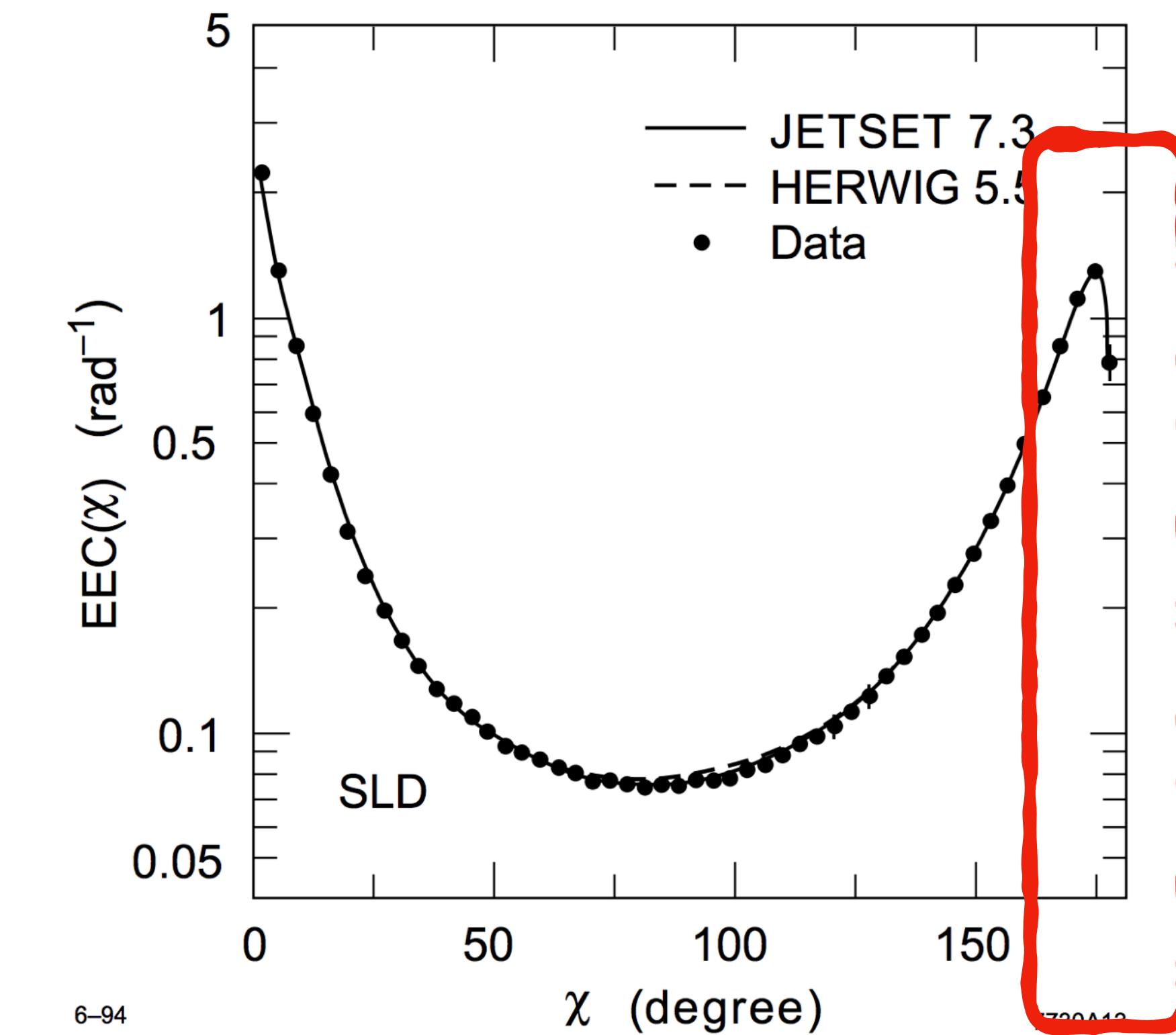
Basham, Brown, Love, Ellis, 1978



$$\text{EEC}(\chi) = \frac{1}{\sigma} \sum_{i,j} \int d\sigma_{e^+e^- \rightarrow i+j+X} \frac{E_i E_j}{Q^2} \delta(\cos \chi - \frac{\vec{p}_i \cdot \vec{p}_j}{|\vec{p}_i||\vec{p}_j|})$$

back-to-back limit: $\chi \rightarrow \pi$ $y = \frac{1 + \cos \chi}{2}$

$$\text{EEC}(y) \sim \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \left(c_{n,m} \frac{\log^m y}{y} + d_{n,m} \log^m y \right)$$



$$\text{EEC}(y) \Big|_{y \rightarrow 0} \sim H(Q, \mu) S(b, \mu, \nu) J_q(b, \mu, \nu) J_q(b, \mu, \nu)$$

TMD soft function
First moment of TMD fragmentation function

Collins, Soper, 1983
Moult, Zhu, 2018

Position space definition of EEC

EEC as Lorentzian Wightman correlation function of null-integrated opeartors

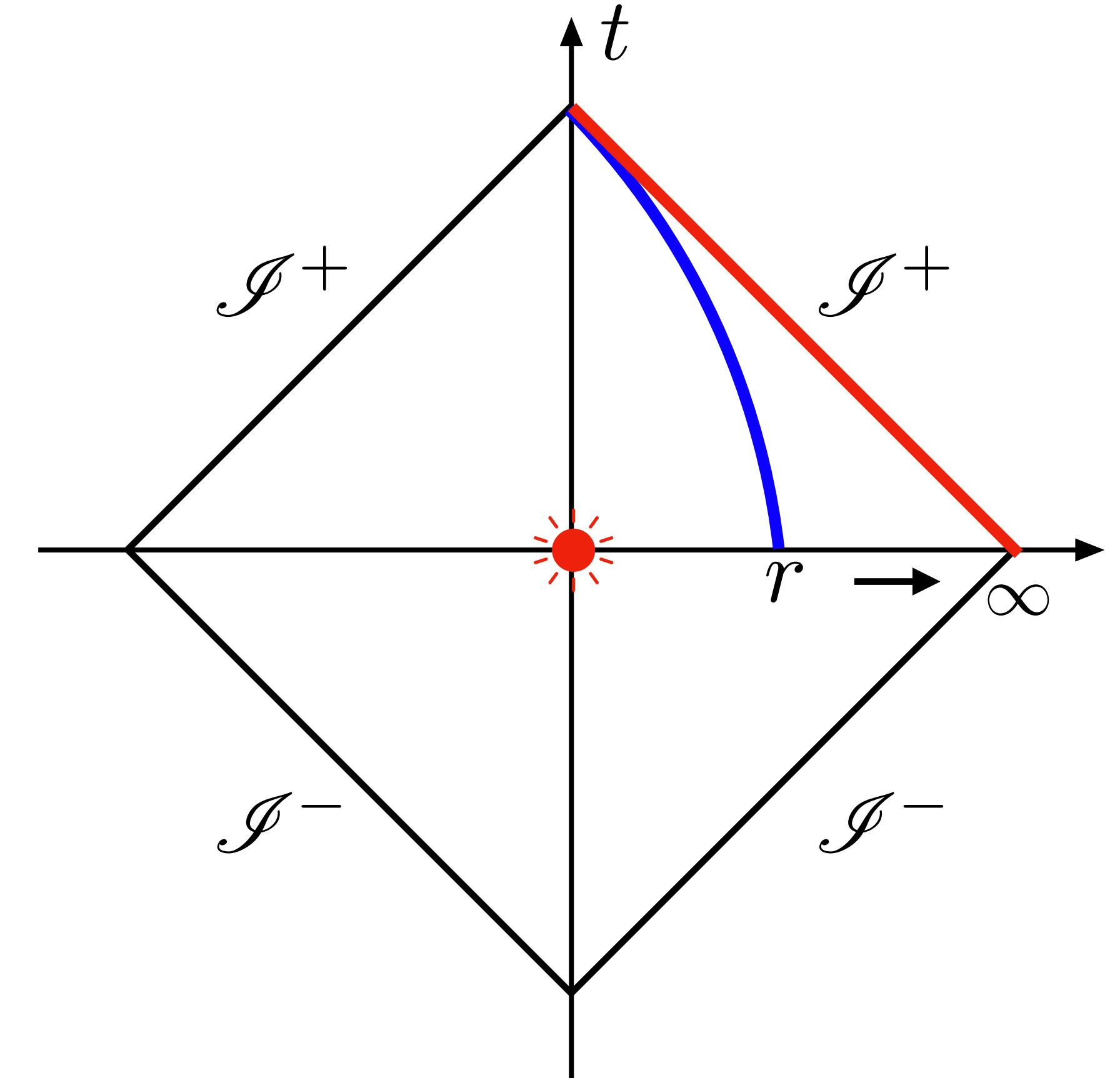
Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 2013

$$\text{EEC}(y) = \frac{8\pi^2}{q^2\sigma_0} \int d^4x e^{iq \cdot x_{13}} \langle J^\mu(x_1) \mathcal{E}(n_2) \mathcal{E}(n_4) J_\mu^\dagger(x_3) \rangle$$

Tkachov, 1995; Hofman, Maldacena 2008; Bauer, Fleming, Lee, Sterman, 2008

$$\mathcal{E}(n_i) = \int_{-\infty}^{\infty} \frac{d n_i \cdot x_i}{16} \lim_{\bar{n}_i x_i \rightarrow \infty} (\bar{n}_i \cdot x_i)^2 T_{\mu\nu}(x_i) \bar{n}_i^\mu \bar{n}_i^\nu$$

$$\mathcal{E}(n_i)|p\rangle = \omega|p\rangle, \quad p^\mu = \omega n_i^\mu$$



Penrose diagram for Minkowski spacetime

Double lightcone dominance

Korchemsky 2019; Chen, Zhou, HXZ, 2023

$$\int d^4x_{13} e^{iq \cdot x_{13}} + \text{Light transformation}$$

q: virtual photon momentum

$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle$$

$$x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$$

Double lightcone dominance

Korchemsky 2019; Chen, Zhou, HXZ, 2023

$$\int d^4x_{13} e^{iq \cdot x_{13}} + \text{Light transformation}$$

q: virtual photon momentum

$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle$$

$$x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$$

Where is the back-to-back limit in position space?

Double lightcone dominance

Korchemsky 2019; Chen, Zhou, HXZ, 2023

$$\int d^4x_{13} e^{iq \cdot x_{13}} + \text{Light transformation}$$

q: virtual photon momentum

$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle$$

$$x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$$

Where is the back-to-back limit in position space?

Choose a frame where detectors
are exactly back-to-back $n_2 = \bar{n}_4$

$$y \sim \frac{q_\perp^2}{q^2} \rightarrow 0$$

Double lightcone dominance

Korchemsky 2019; Chen, Zhou, HXZ, 2023

$$\int d^4x_{13} e^{iq \cdot x_{13}} + \text{Light transformation}$$

q: virtual photon momentum

$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle$$
$$x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$$

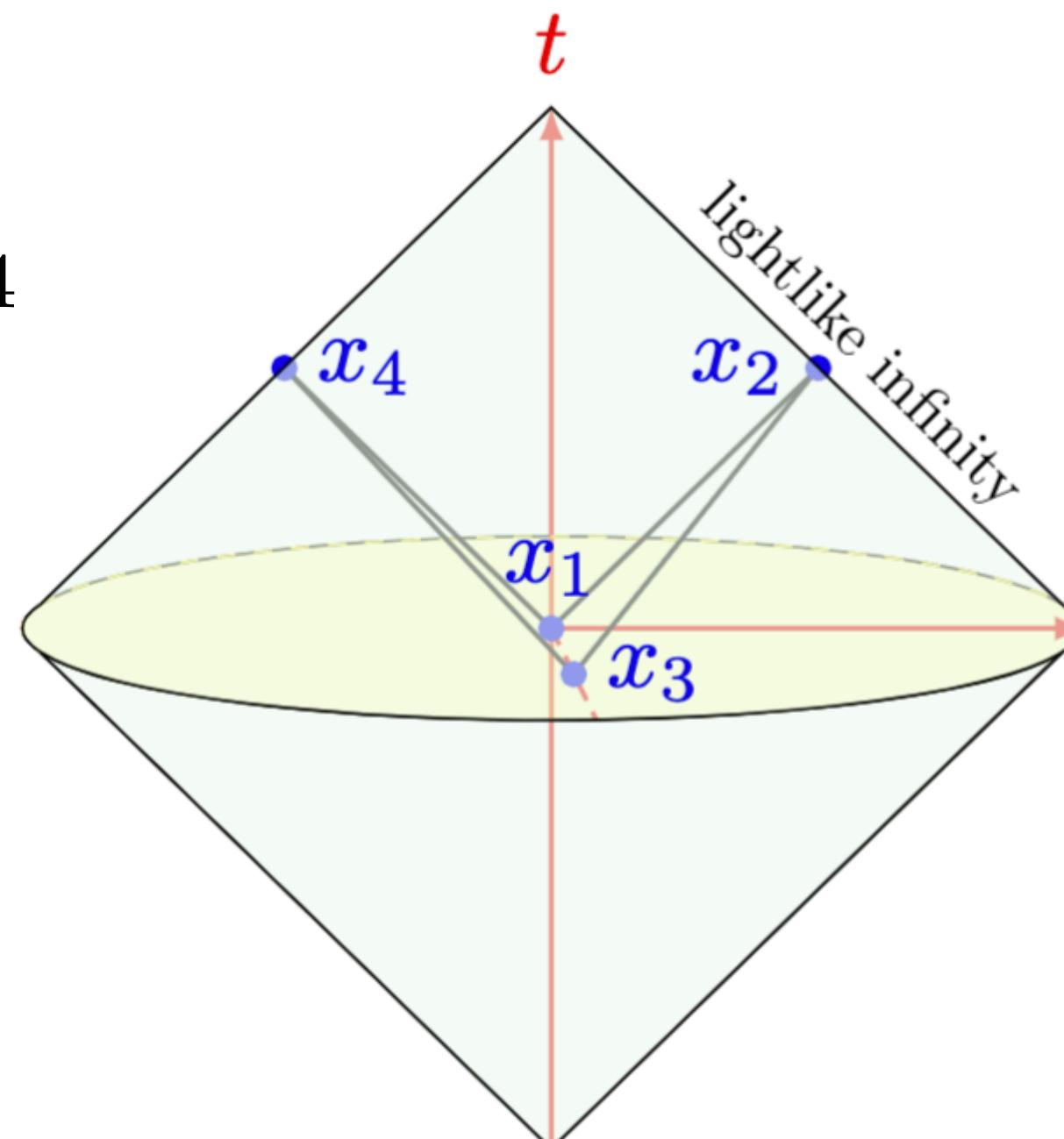
Where is the back-to-back limit in position space?

Choose a frame where detectors
are exactly back-to-back $n_2 = \bar{n}_4$

$$y \sim \frac{q_\perp^2}{q^2} \rightarrow 0$$

↳ $y \sim \frac{x_{13}^+ x_{13}^-}{x_{13}^2} \rightarrow 0$

$$x_{12}^2, x_{23}^2 \ll x_{13}^2$$



Double lightcone dominance

Korchemsky 2019; Chen, Zhou, HXZ, 2023

$$\int d^4x_{13} e^{iq \cdot x_{13}} + \text{Light transformation}$$

q: virtual photon momentum

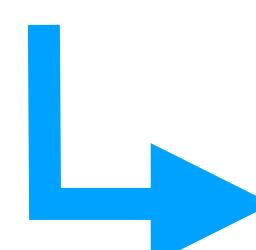
$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3) \rangle$$

$$x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$$

Where is the back-to-back limit in position space?

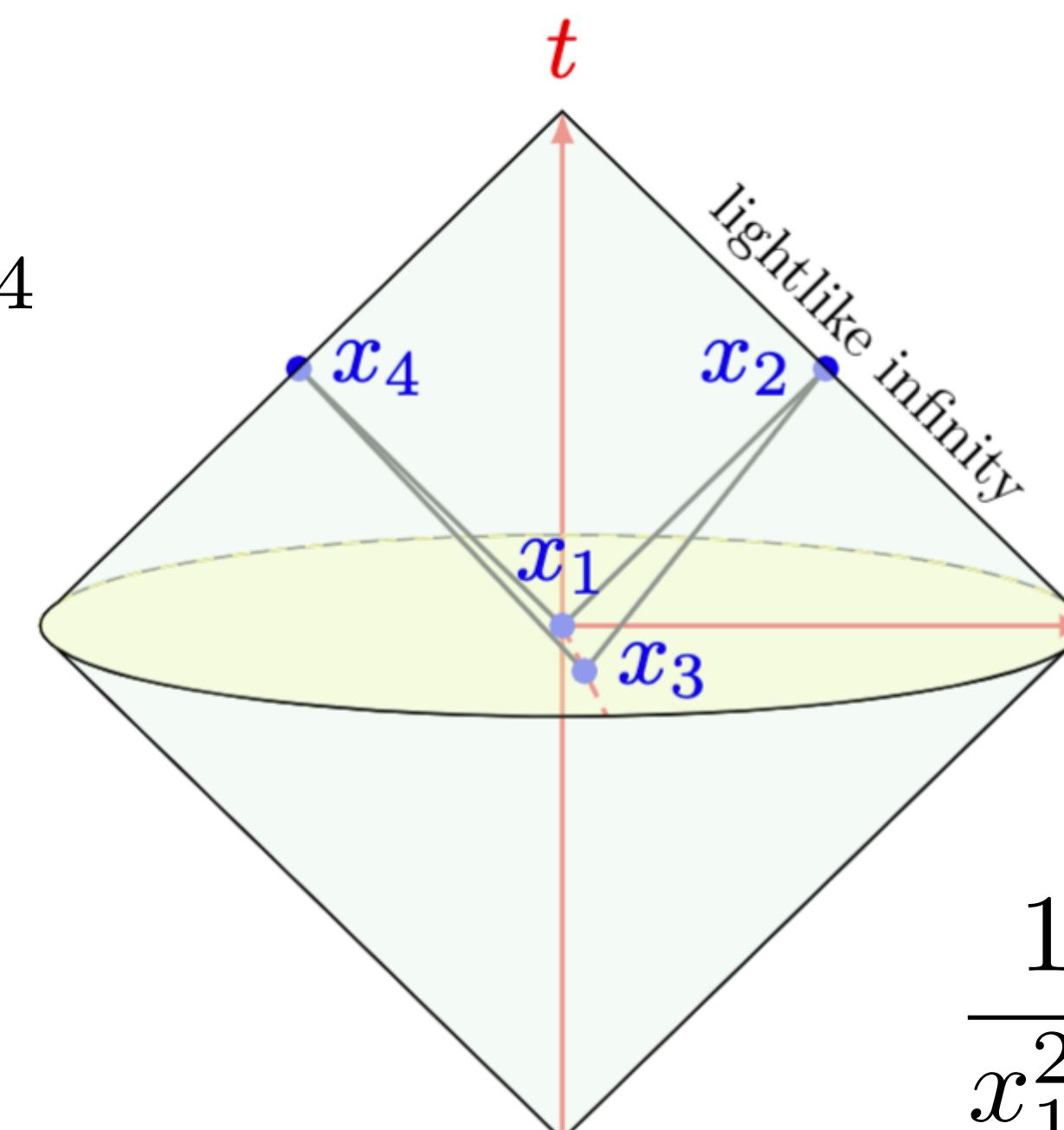
Choose a frame where detectors
are exactly back-to-back $n_2 = \bar{n}_4$

$$y \sim \frac{q_\perp^2}{q^2} \rightarrow 0$$



$$y \sim \frac{x_{13}^+ x_{13}^-}{x_{13}^2} \rightarrow 0$$

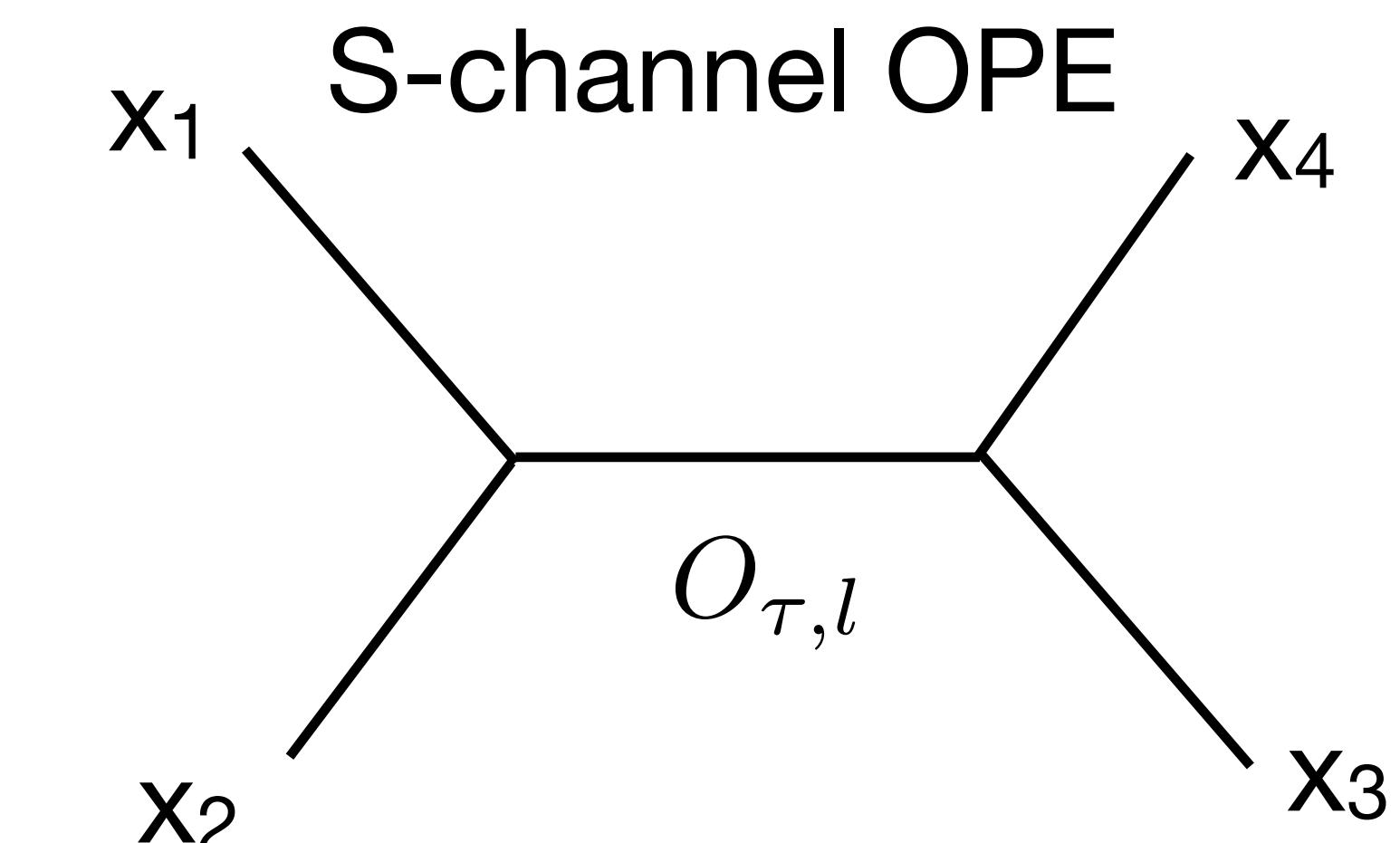
$$x_{12}^2, x_{23}^2 \ll x_{13}^2$$



dominated by

$$\frac{1}{x_{12}^2}$$

$$\frac{1}{x_{23}^2}$$



lightcone OPE
twist expansion

$$\tau = \Delta - l$$

requires sum over infinite operators
with degenerate twist to reproduce

Concrete example: N=4 SYM

$$L = \text{tr} \left\{ -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\theta_I}{8\pi^2} F_{\mu\nu} \bar{F}^{\mu\nu} - i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda_a - D_\mu X^i D^\mu X^i + g C_i^{ab} \lambda_a [X^i, \lambda_b] + g \bar{C}_{iab} \bar{\lambda}^a [X^i, \bar{\lambda}^b] + \frac{g^2}{2} [X^i, X^j]^2 \right\},$$

A close cousin of QCD
Contains gauge interactions similar to QCD
Leads to technical simplification

$$\langle \mathcal{O}(x_1) \mathcal{T}(x_2) \mathcal{T}(x_4) \mathcal{O}(x_3) \rangle_{\text{dyn}} = \frac{1}{(2\pi)^4} \frac{x_{13}^4 x_{24}^4}{(x_{12}^2 x_{34}^2)^4} \mathcal{F}(u, v)$$

conformal cross ratio

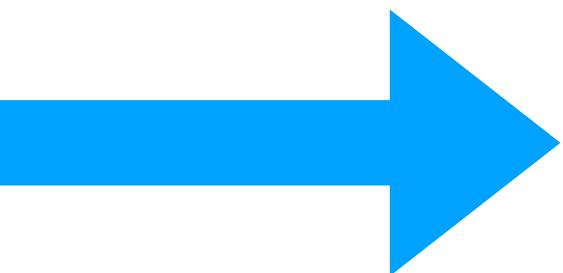
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

In Minkowski signature z and \bar{z} are independent real valuable

Double lightcone expansion

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

Sudakov limit in EEC



singularities in $u \rightarrow 0, v \rightarrow 0$
 $z \rightarrow 0, \bar{z} \rightarrow 1$

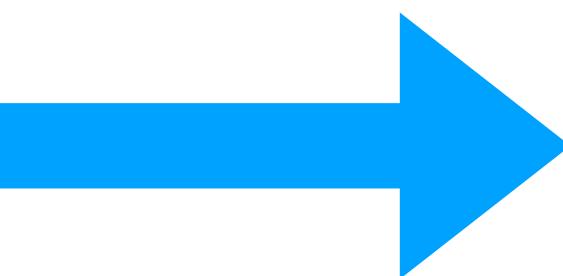
collinear $u \rightarrow 0$ twist expansion. Power corrections computed by including higher twist operator

soft $v \rightarrow 0$ Large spin expansion. Power corrections computed by retaining 1/spin suppression terms

Double lightcone expansion

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

Sudakov limit in EEC



singularities in $u \rightarrow 0, v \rightarrow 0$
 $z \rightarrow 0, \bar{z} \rightarrow 1$

collinear

$$u \rightarrow 0$$

twist expansion. Power corrections computed by including higher twist operator

soft

$$v \rightarrow 0$$

Large spin expansion. Power corrections computed by retaining 1/spin suppression terms

example:

$$\gamma_{2,\ell}^{(1)} = \log J_{6,\ell}^2 + 2\gamma_E + \frac{1}{3J_{6,\ell}^2} + \mathcal{O}(J_{6,\ell}^{-4})$$

cusp anomalous dim.

normal anomalous dim.

large spin suppression

Implication of crossing symmetry

$$\langle \mathcal{O}(x_1)\mathcal{T}(x_2)\mathcal{T}(x_4)\mathcal{O}(x_3) \rangle = \langle \mathcal{O}(x_3)\mathcal{T}(x_2)\mathcal{T}(x_4)\mathcal{O}(x_1) \rangle$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

$$u \leftrightarrow v$$

For a large part, twist suppression is determined by large spin suppression

Implication of crossing symmetry

$$\langle \mathcal{O}(x_1)\mathcal{T}(x_2)\mathcal{T}(x_4)\mathcal{O}(x_3) \rangle = \langle \mathcal{O}(x_3)\mathcal{T}(x_2)\mathcal{T}(x_4)\mathcal{O}(x_1) \rangle$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \quad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$$

$u \leftrightarrow v$

For a large part, twist suppression is determined by large spin suppression

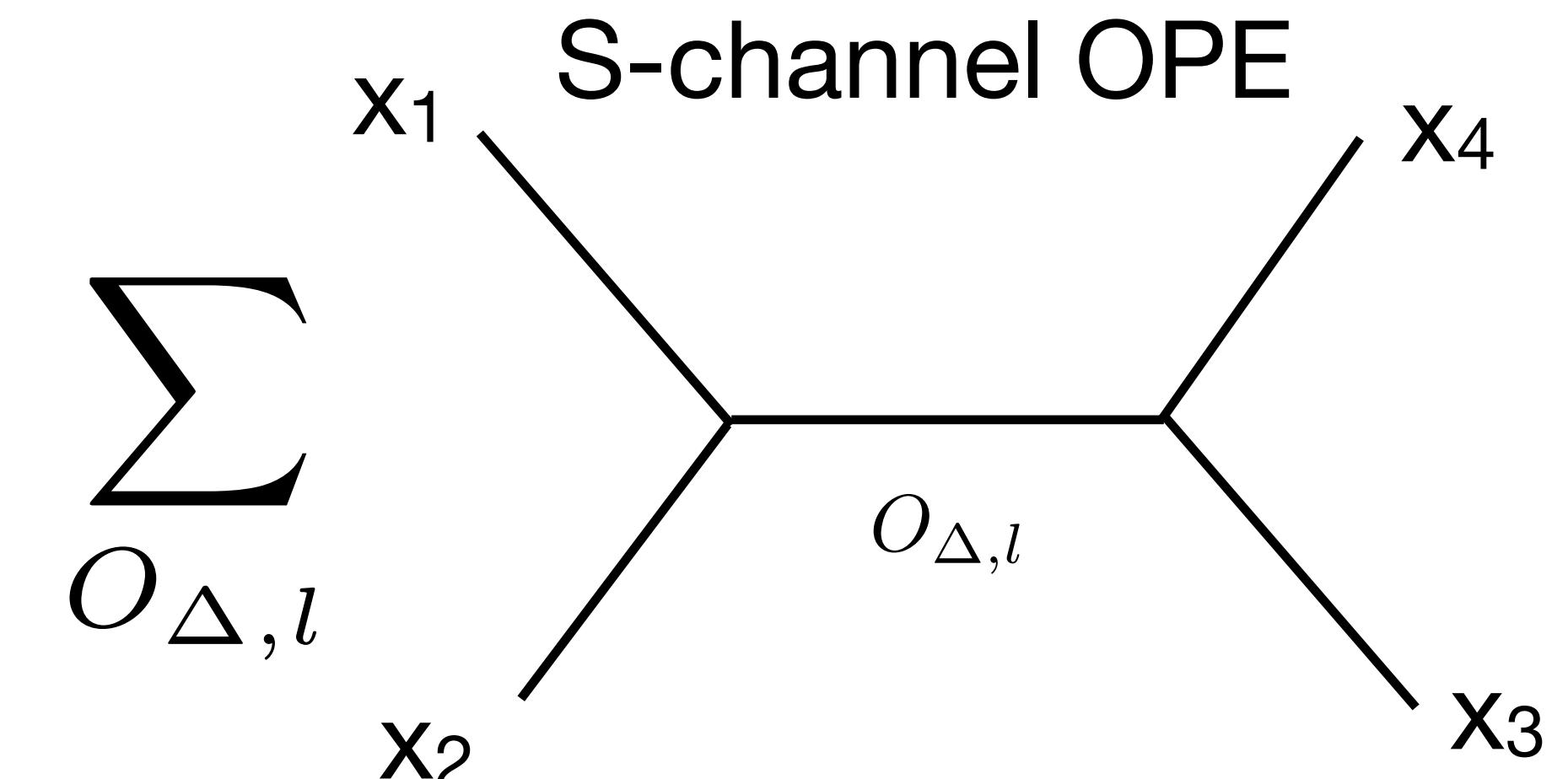
Leading Power	Next-to-Leading Power
one loop = $\overbrace{\left[-\frac{1}{4} \log u \log v + 0 \cdot \log(uv) + \dots \right]}^{\text{Leading Power}} - \overbrace{\left[\frac{1}{4}(u+v) \log u \log v + \frac{1}{2}(u \log u + v \log v) + \dots \right]}^{\text{Next-to-Leading Power}} + \dots,$	
two loop = $\left[\frac{1}{16} \log^2 u \log^2 v + 0 \cdot \log u \log v \log(uv) + \dots \right] + \overbrace{\left[\frac{1}{8}(u+v) \log^2 u \log^2 v \right.}$	
	$\left. + \frac{3}{16} \log u \log v (u \log u + v \log v) + \frac{1}{8} \log u \log v (v \log u + u \log v) + \dots \right] + \dots,$
three loop = $\left[-\frac{1}{96} \log^3 u \log^3 v + 0 \cdot \log^2 u \log^2 v \log(uv) + \dots \right] - \overbrace{\left[\frac{1}{48}(u+v) \log^3 u \log^3 v \right]}$	

For leading and sub-leading log, we are left with large spin corrections

Conformal block expansion

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

eigenfunction of conformal Casimir



$$\mathcal{C}_\tau G_{\Delta, l}(z, \bar{z}) = J_{\tau, l}^2 G_{\Delta, l}(z, \bar{z})$$

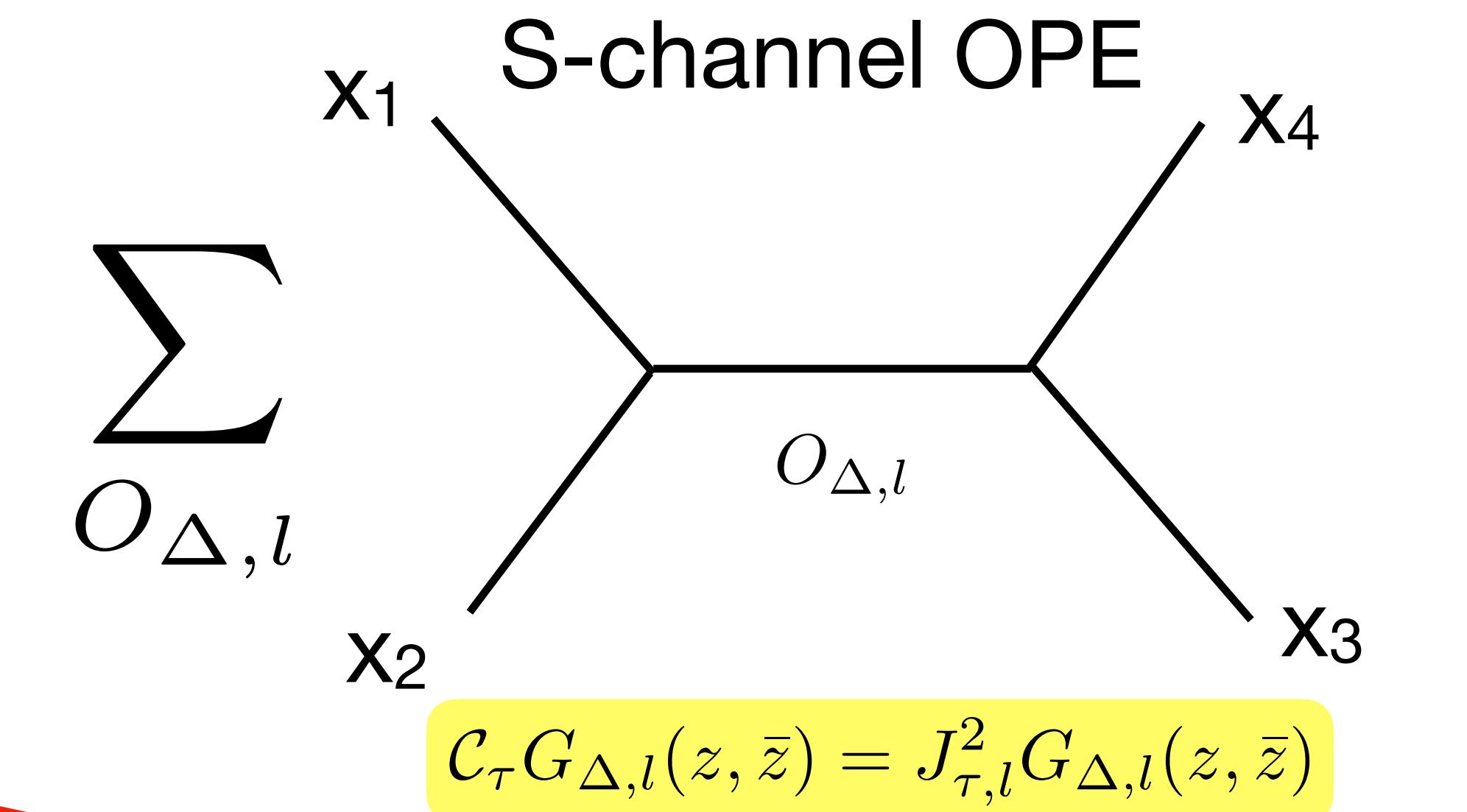
Conformal block expansion

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

eigenfunction of conformal Casimir

Dolan, Osborn, 2001

$$G_{\Delta, \ell}(u, v) = \frac{z\bar{z}}{\bar{z} - z} [k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})]$$



$$k_\beta(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x)$$

Conformal block expansion

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

eigenfunction of conformal Casimir

Dolan, Osborn, 2001

$$G_{\Delta, \ell}(u, v) = \frac{z\bar{z}}{\bar{z} - z} [k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})]$$

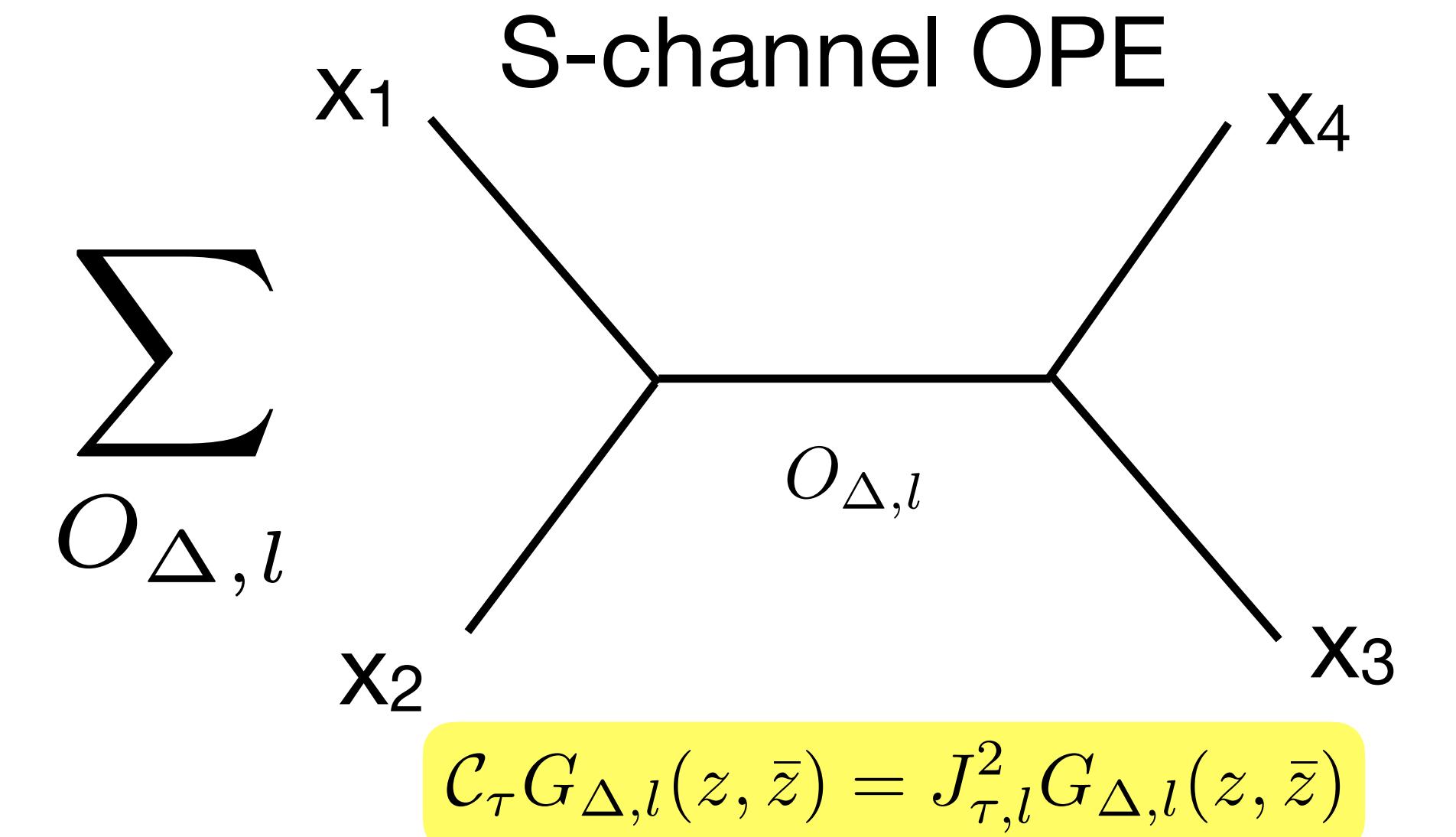
Leading twist expansion $u \rightarrow 0$ ($z \rightarrow 0$): $L_z = \log z$

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a_{2, \ell}^{(0)} \left\{ \frac{\left(\gamma_{2, \ell}^{(1)}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma_{2, \ell}^{(1)}\right)^{n-1} L_z^{n-1}}{2^{n-1} (n-1)!} \times \left[\frac{a_{2, \ell}^{(1)}}{a_{2, \ell}^{(0)}} + (n-1) \frac{\gamma_{2, \ell}^{(2)}}{\gamma_{2, \ell}^{(1)}} + \frac{\gamma_{2, \ell}^{(1)} \partial_\ell}{2} \right] \right\} k_{2\ell+6}(\bar{z}) + \dots$$

Leading in u but including infinite powers in v (fixed by conformal symmetry)

Resumming large logarithms in v requires sum over infinite spin

Resumming Power corrections in v requires systematic expansion over large spin



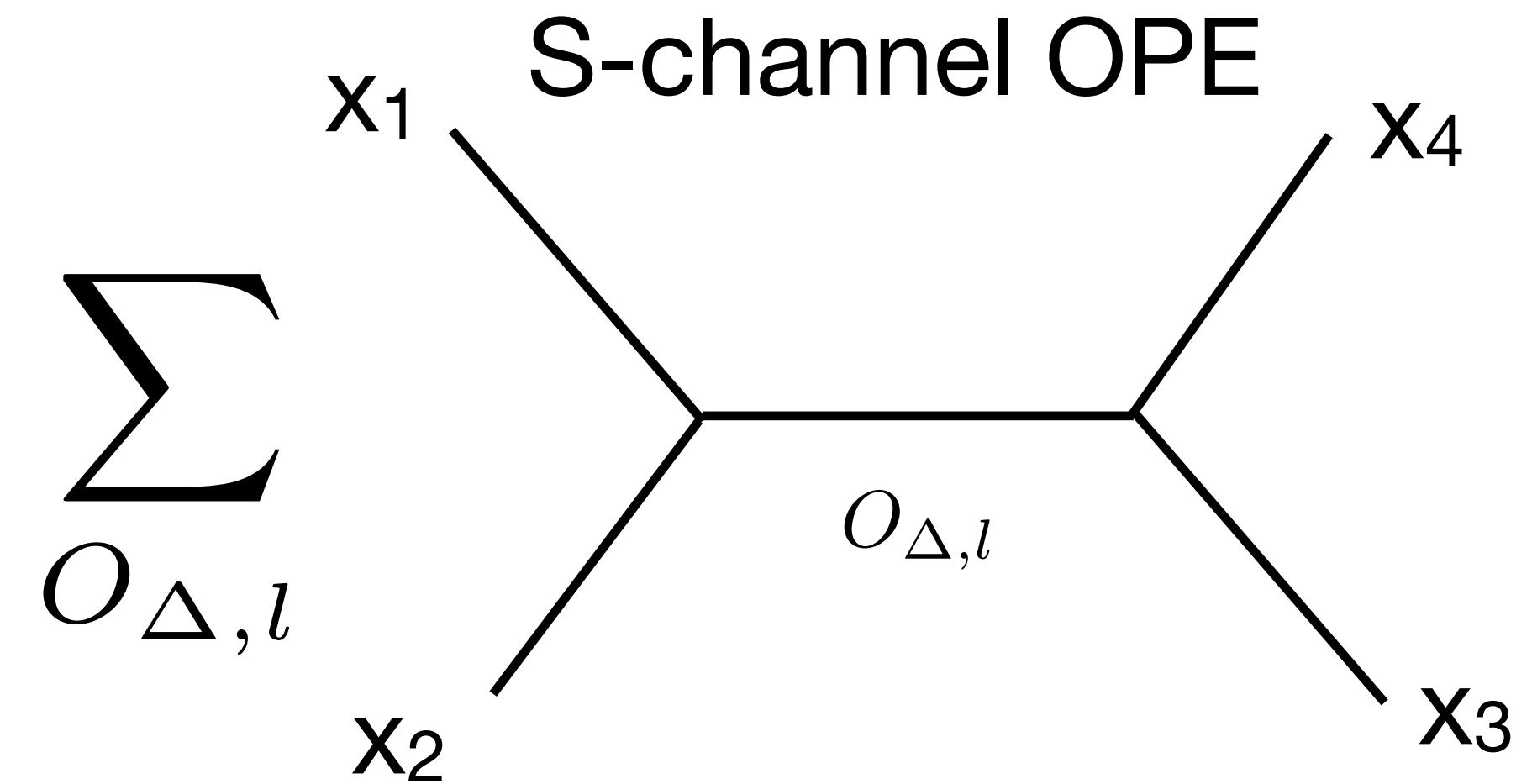
$$k_\beta(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x)$$

Twist conformal block

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016



Leading twist expansion $u \rightarrow 0$ ($z \rightarrow 0$): $L_z = \log z$

LL

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a_{2, \ell}^{(0)} \left\{ \frac{\left(\gamma_{2, \ell}^{(1)}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma_{2, \ell}^{(1)}\right)^{n-1} L_z^{n-1}}{2^{n-1} (n-1)!} \times \left[\frac{a_{2, \ell}^{(1)}}{a_{2, \ell}^{(0)}} + (n-1) \frac{\gamma_{2, \ell}^{(2)}}{\gamma_{2, \ell}^{(1)}} + \frac{\gamma_{2, \ell}^{(1)} \partial_\ell}{2} \right] \right\} k_{2\ell+6}(\bar{z}) + \dots$$

$k_\beta(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x)$

↓

Twist conformal block

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

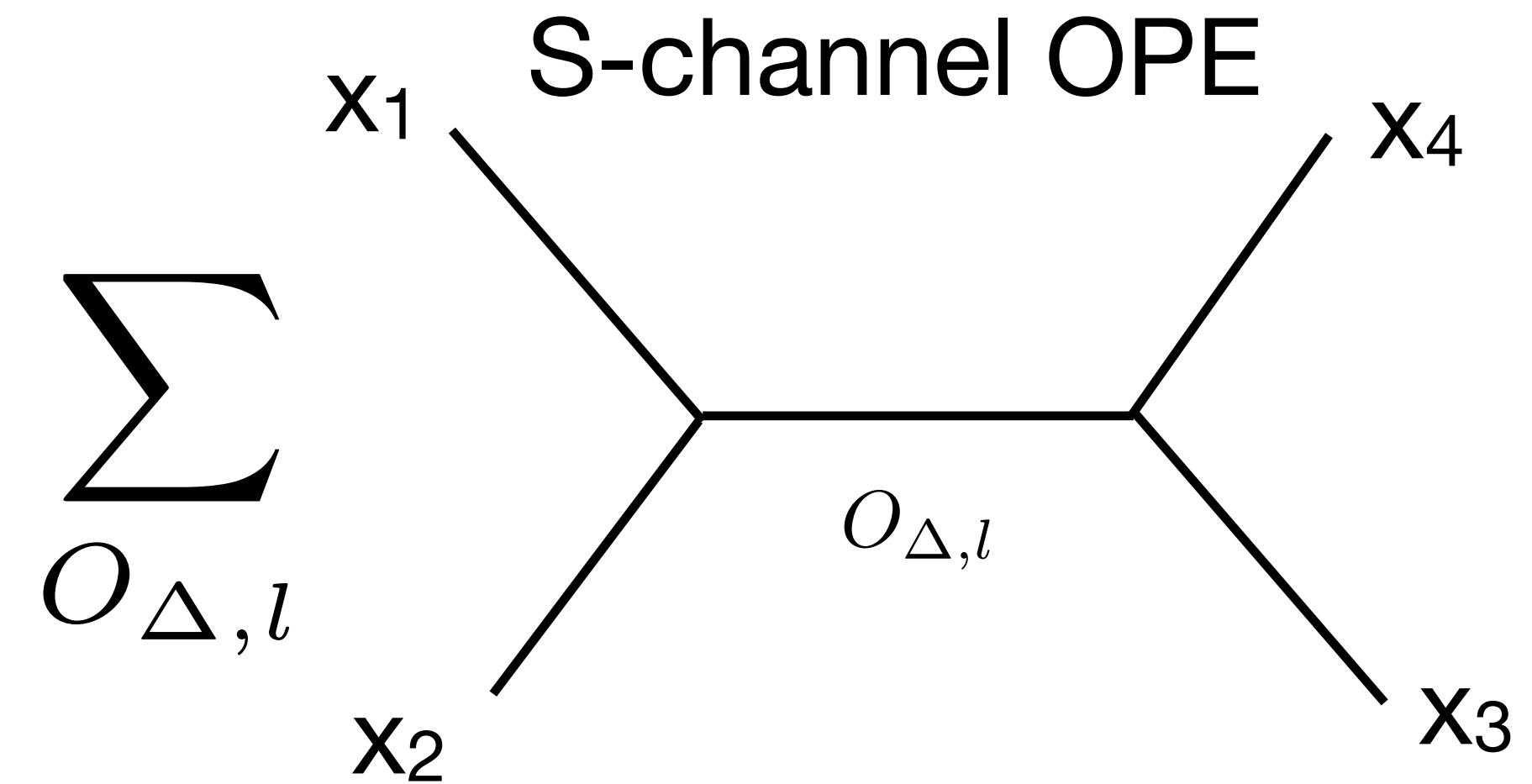
$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

Leading twist expansion $u \rightarrow 0$ ($z \rightarrow 0$): $L_z = \log z$

LL

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a_{2, \ell}^{(0)} \left\{ \frac{\left(\gamma_{2, \ell}^{(1)}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma_{2, \ell}^{(1)}\right)^{n-1} L_z^{n-1}}{2^{n-1} (n-1)!} \times \left[\frac{a_{2, \ell}^{(1)}}{a_{2, \ell}^{(0)}} + (n-1) \frac{\gamma_{2, \ell}^{(2)}}{\gamma_{2, \ell}^{(1)}} + \frac{\gamma_{2, \ell}^{(1)} \partial_{\ell}}{2} \right] \right\} k_{2\ell+6}(\bar{z}) + \dots$$



$$k_{\beta}(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x)$$



Twist conformal block

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

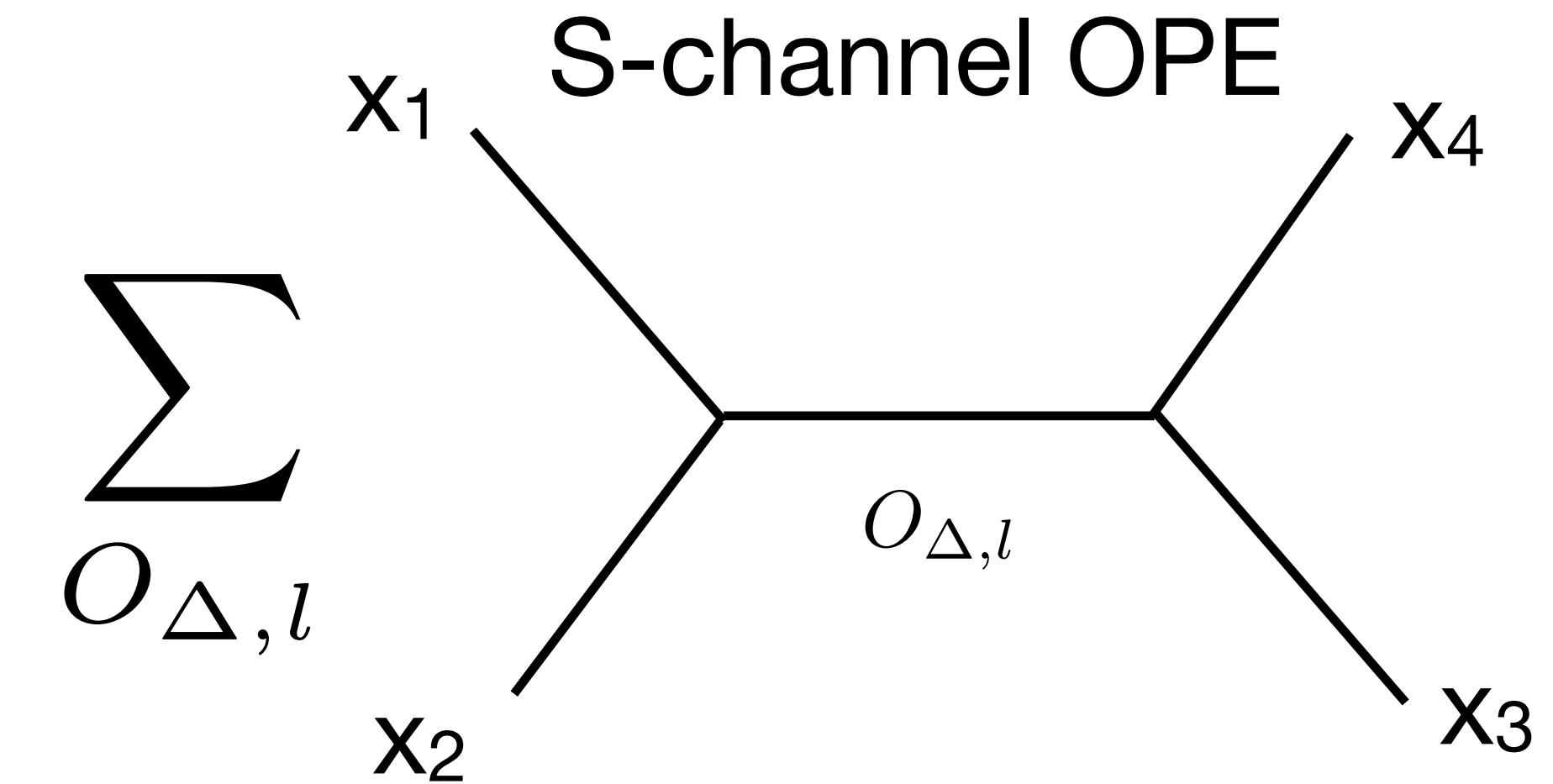
$$\tilde{\tau}_0 = \tau + 4$$

Leading twist expansion $u \rightarrow 0$ ($z \rightarrow 0$): $L_z = \log z$

$$\mathcal{F}^{(n)} = z^3 \sum_{\text{even } \ell} a_{2, \ell}^{(0)} \left\{ \frac{\left(\gamma_{2, \ell}^{(1)}\right)^n}{2^n n!} L_z^n + \frac{\left(\gamma_{2, \ell}^{(1)}\right)^{n-1} L_z^{n-1}}{2^{n-1} (n-1)!} \times \left[\frac{a_{2, \ell}^{(1)}}{a_{2, \ell}^{(0)}} + (n-1) \frac{\gamma_{2, \ell}^{(2)}}{\gamma_{2, \ell}^{(1)}} + \frac{\gamma_{2, \ell}^{(1)} \partial_{\ell}}{2} \right] \right\} k_{2\ell+6}(\bar{z}) + \dots$$

LL

$$= \frac{L_z^n}{n!} \sum_i \binom{n}{i} \left[\frac{\gamma_E^{n-i}}{2^i} H_2^{(0,i)} + \frac{n-i}{3} \frac{\gamma_E^{n-1-i}}{2^{i+1}} H_2^{(1,i)} \right] + \dots$$



$$k_{\beta}(x) = x^{\beta/2} {}_2F_1(\beta/2, \beta/2, \beta; x)$$

↓

$$a_{2, \ell}^{(0)} = \frac{\Gamma(\ell+3)^2}{\Gamma(2\ell+5)},$$

$$\gamma_{2, \ell}^{(1)} = \log J_{6, \ell}^2 + 2\gamma_E + \frac{1}{3J_{6, \ell}^2} + \mathcal{O}(J_{6, \ell}^{-4})$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta \text{ even}} \sum_{\ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad \begin{aligned} J_{\tau, \ell}^2 &= (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1) \\ \tilde{\tau}_0 &= \tau + 4 \end{aligned}$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$

$$H_{\tau_0}^{(0,0)}(z, \bar{z})$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z} \log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$

$$H_{\tau_0}^{(-1,0)}(z, \bar{z}) \xleftarrow{\mathcal{C}_{\tilde{\tau}_0}} H_{\tau_0}^{(0,0)}(z, \bar{z})$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z} \log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

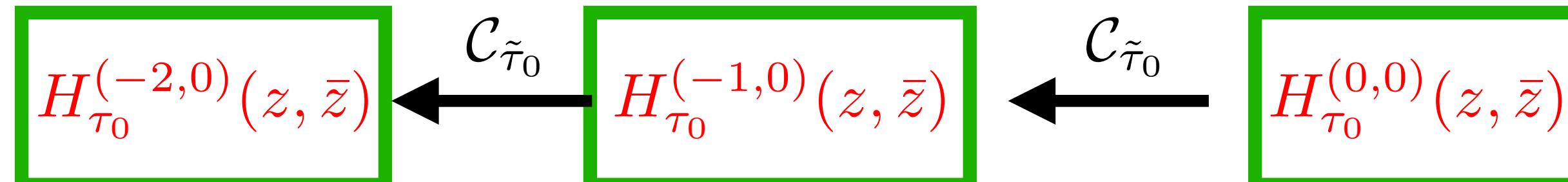
$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$



$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z} \log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

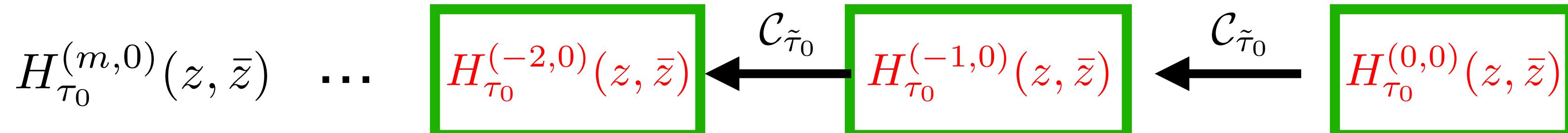
$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$



$$\begin{aligned} \bar{H}_2^{(m,0)}(\bar{z}) &= \frac{1}{2} \epsilon^{m-1} \Gamma(1-m)^2 + \frac{1}{6} m (2m^2 - 6m + 1) \epsilon^m \Gamma(-m)^2 \\ &+ \frac{1}{180} (m-1)m(m+1) (20m^3 - 54m^2 - 35m + 36) \epsilon^{m+1} \Gamma(-m-1)^2 + \dots \end{aligned}$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z} \log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

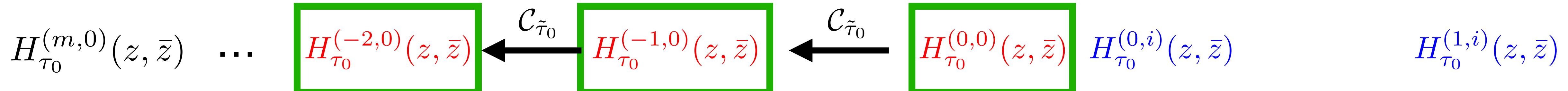
$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z}) \quad J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$



$$\begin{aligned} \bar{H}_2^{(m,0)}(\bar{z}) &= \frac{1}{2} \epsilon^{m-1} \Gamma(1-m)^2 + \frac{1}{6} m (2m^2 - 6m + 1) \epsilon^m \Gamma(-m)^2 \\ &+ \frac{1}{180} (m-1)m(m+1) (20m^3 - 54m^2 - 35m + 36) \epsilon^{m+1} \Gamma(-m-1)^2 + \dots \end{aligned}$$

$$\frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z} \log(1-\bar{z}) = \frac{1}{2\epsilon} + \text{regular terms}$$

$$1 - \bar{z} = \epsilon$$

Analyticity in spin

$$\mathcal{F}(u, v) = \sum_{\Delta} \sum_{\text{even } \ell} a_{\tau, \ell} G_{\Delta+4, \ell}(u, v)$$

twist conformal block $H_{\tau=2} = \sum_{\text{even } \ell} a_{\tau, \ell}^{(0)} G_{\ell+6, \ell}(u, v)$

Alday, 2016

$$H_{\tau_0}^{(m,i)}(z, \bar{z}) = \sum_{\ell} a_{\tau_0, \ell}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, \ell}^2}{J_{\tilde{\tau}_0, \ell}^{2m}} G_{\Delta+4, \ell}(z, \bar{z})$$

$$J_{\tau, \ell}^2 = (\ell + \frac{\tau}{2})(\ell + \frac{\tau}{2} - 1)$$

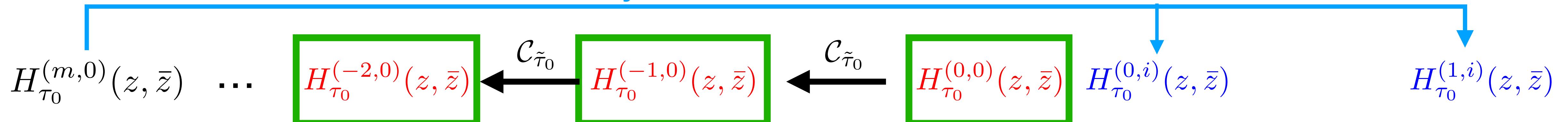
$$\tilde{\tau}_0 = \tau + 4$$

$$\mathcal{C}_{\tilde{\tau}_0} G_{\Delta+4, \ell}(z, \bar{z}) = J_{\tilde{\tau}_0, \ell}^2 G_{\Delta+4, \ell}(z, \bar{z})$$

$$\mathcal{C}_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)} = \sum_l a_{\tau_0, l}^{(0)} \frac{\log^i J_{\tilde{\tau}_0, l}^2}{J_{\tilde{\tau}_0, l}^{2(m-1)}} G_{\Delta+4, l}$$

$$C_{\tilde{\tau}_0} H_{\tau_0}^{(m,i)}(z, \bar{z}) = H_{\tau_0}^{(m-1,i)}(z, \bar{z})$$

analytic continuation in m



$$\begin{aligned} \bar{H}_2^{(m,0)}(\bar{z}) &= \frac{1}{2} \epsilon^{m-1} \Gamma(1-m)^2 + \frac{1}{6} m (2m^2 - 6m + 1) \epsilon^m \Gamma(-m)^2 \\ &\quad + \frac{1}{180} (m-1)m(m+1) (20m^3 - 54m^2 - 35m + 36) \epsilon^{m+1} \Gamma(-m-1)^2 + \dots \end{aligned}$$

$$\begin{aligned} \frac{\bar{z}^2(2-\bar{z})}{2(1-\bar{z})} + \bar{z} \log(1-\bar{z}) &= \frac{1}{2\epsilon} + \text{regular terms} \\ 1 - \bar{z} &= \epsilon \end{aligned}$$

Alday, 2016
Henriksson, Lukowski, 2017

Explicit example for a toy model: N=4 SYM

$$\begin{aligned} \mathcal{F}^{(n)}(z, \bar{z}) = z^3 & \left\{ \frac{1}{n!} \log^n z \left[\frac{1}{\epsilon} \left(\frac{(-1)^n}{2^{n+1}} \log^n \epsilon + \dots \right) + \left(\frac{(-1)^{n+1}}{2^{n+1}} \log^n \epsilon + \frac{(-1)^n n}{3 \times 2^n} \log^{n-1} \epsilon + \dots \right) + \dots \right] \right. \\ & + \frac{\log^{n-1} z}{(n-1)!} \left[\frac{1}{\epsilon} \left(\frac{(-1)^n}{2^{n+1}} (n+1) \zeta_2 \log^{n-1} \epsilon - \frac{(-1)^n}{2^{n+1}} 3(n-1) \zeta_3 \log^{n-2} \epsilon + \dots \right) \right. \\ & \quad \left. \left. + \left(\frac{(-1)^n}{2^{n+1} n} \log^n \epsilon + \frac{(-1)^{n+1}}{2^{n+1}} (n+1) \zeta_2 \log^{n-1} \epsilon + \dots \right) \right] + \dots \right\} + \mathcal{O}(z^4) \end{aligned}$$

Agree with fixed-order expansion (up to terms not enhanced by large spin)

$$\text{one loop} = \left[-\frac{1}{4} \log u \log v + 0 \cdot \log(uv) + \dots \right] - \left[\frac{1}{4} (u+v) \log u \log v + \frac{1}{2} (u \log u + v \log v) + \dots \right] + \dots,$$

$$\begin{aligned} \text{two loop} = & \left[\frac{1}{16} \log^2 u \log^2 v + 0 \cdot \log u \log v \log(uv) + \dots \right] + \left[\frac{1}{8} (u+v) \log^2 u \log^2 v \right. \\ & \quad \left. + \frac{3}{16} \log u \log v (u \log u + v \log v) + \frac{1}{8} \log u \log v (v \log u + u \log v) + \dots \right] + \dots, \end{aligned}$$

$$\text{three loop} = \left[-\frac{1}{96} \log^3 u \log^3 v + 0 \cdot \log^2 u \log^2 v \log(uv) + \dots \right] - \left[\frac{1}{48} (u+v) \log^3 u \log^3 v \right]$$

Resummation for EEC beyond leading power

$$\text{EEC}(y) \sim \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \left(c_{n,m} \frac{\log^m y}{y} + d_{n,m} \log^m y \right)$$

NLL: $m \geq 2n - 2$

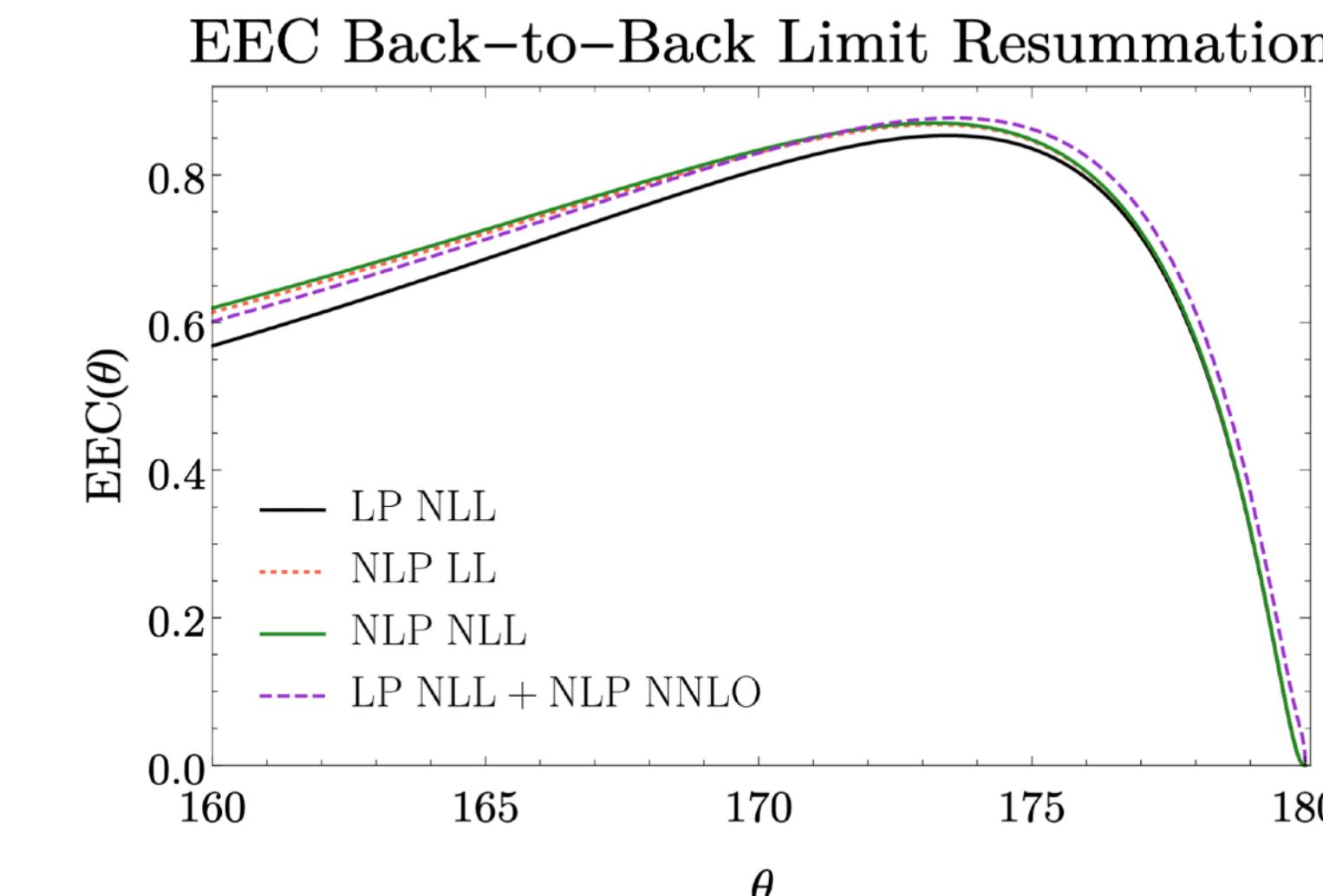
	Power Corrections		Perturbative Corrections	
	twist	large spin	LL	NLL
LP	2	$\mathcal{O}(\ell^0)$	$a_{2,\ell}^{(0)}, \gamma_{2,\ell}^{(1)}$	$a_{2,\ell}^{(1)}, \gamma_{2,\ell}^{(2)}$
NLP	2	$\mathcal{O}(\ell^{-2})$	$a_{2,\ell}^{(0)}, \gamma_{2,\ell}^{(1)}$	$a_{2,\ell}^{(1)}, \gamma_{2,\ell}^{(2)}$
	4	$\mathcal{O}(\ell^0)$	$a_{4,\ell}^{(0)}, \gamma_{4,\ell}^{(1)}$	$a_{4,\ell}^{(1)}, \gamma_{4,\ell}^{(2)}$

$$\text{EEC}(y) = -\frac{aL_y e^{-\frac{aL_y^2}{2}}}{4y} - \frac{1}{4} \left[\sqrt{\frac{\pi}{2}} \sqrt{a} \operatorname{erf} \left(\sqrt{\frac{a}{2}} L_y \right) + aL_y e^{-\frac{aL_y^2}{2}} \right] + \frac{a}{48} (7aL_y^2 - 4)e^{-\frac{aL_y^2}{2}} + \frac{a}{12} + \dots$$

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$$

Discrepancy with NLP-LL prediction of Moult, Vita, Yan, 2019

$$\text{EEC}^{(2)} = -\frac{\sqrt{2a\pi}}{2} e^{-x^2} \operatorname{erfi} \left[\sqrt{\frac{\Gamma_{\text{cusp}}}{2}} \log y \right]$$



Summary

- We have proposed a new method to resum Sudakov logarithms based on double lightcone OPE
 - Power corrections organized by conformal symmetry
 - Kinematical power corrections -> descendant operator contribution + perturbation expansion at large spin limit
 - Dynamical power corrections -> higher twist primary operator
- Towards QCD (work in progress):
 - External operator spin can not be neglected
 - Running coupling effects (only appear at NLL and beyond)
 - Generalization to more observables