

Precision Predictions for Top-quark Width

李海涛



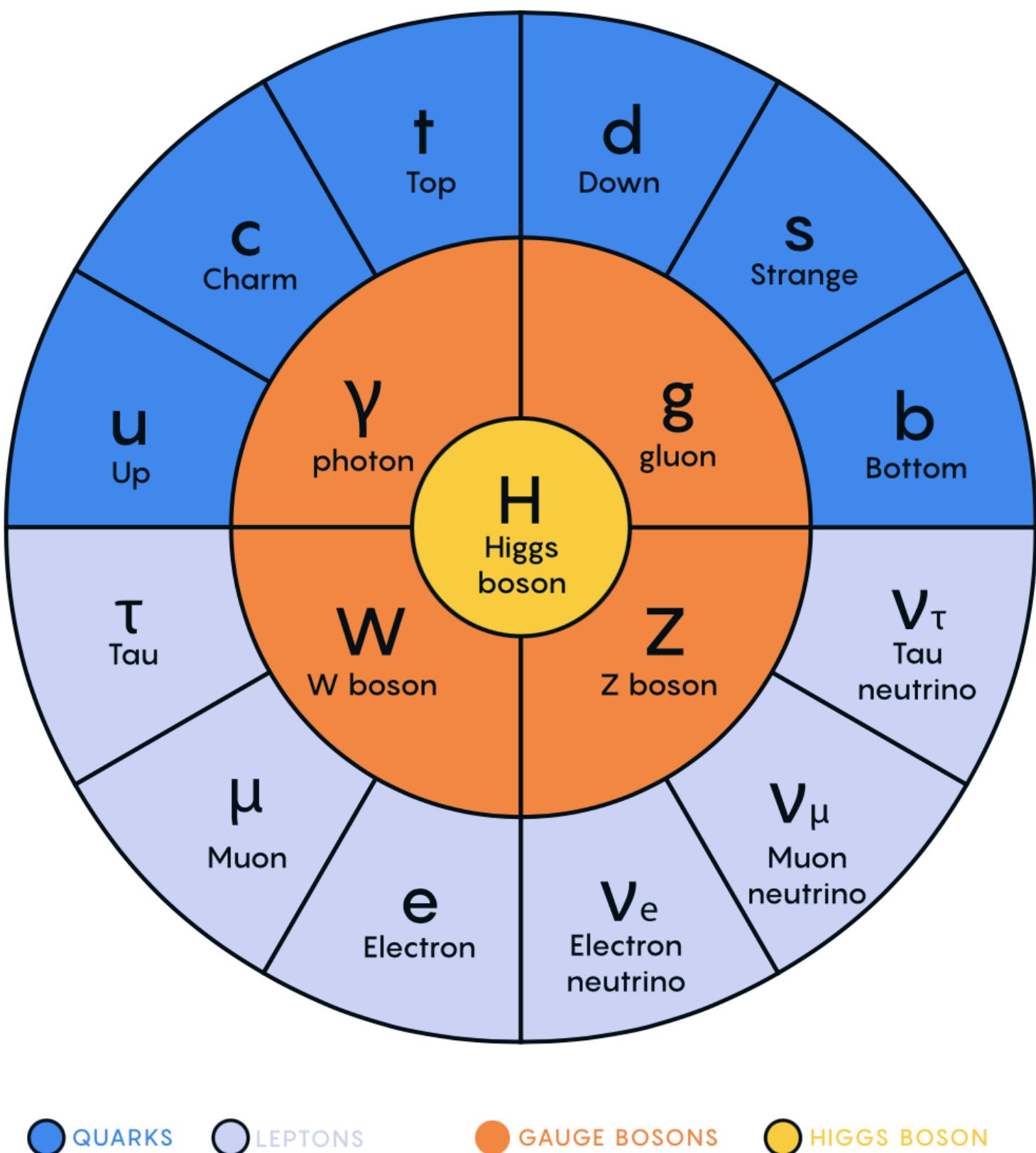
In collaboration with 陈龙斌, 王健, 王烨凡

arXiv:2212.06341

第五届重味物理与量子色动力学研讨会, 华中科技大学

2023年4月22日

Introduction

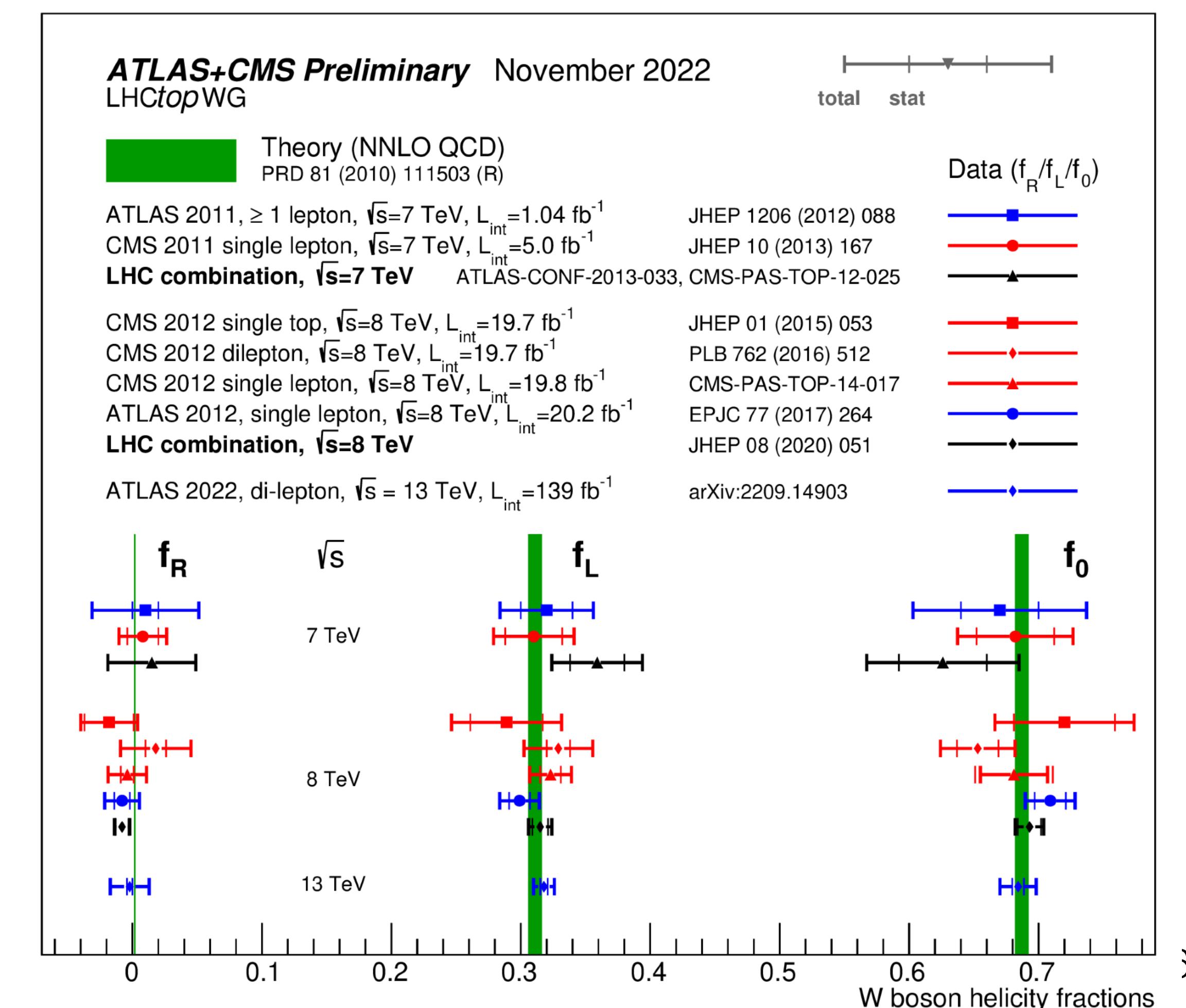
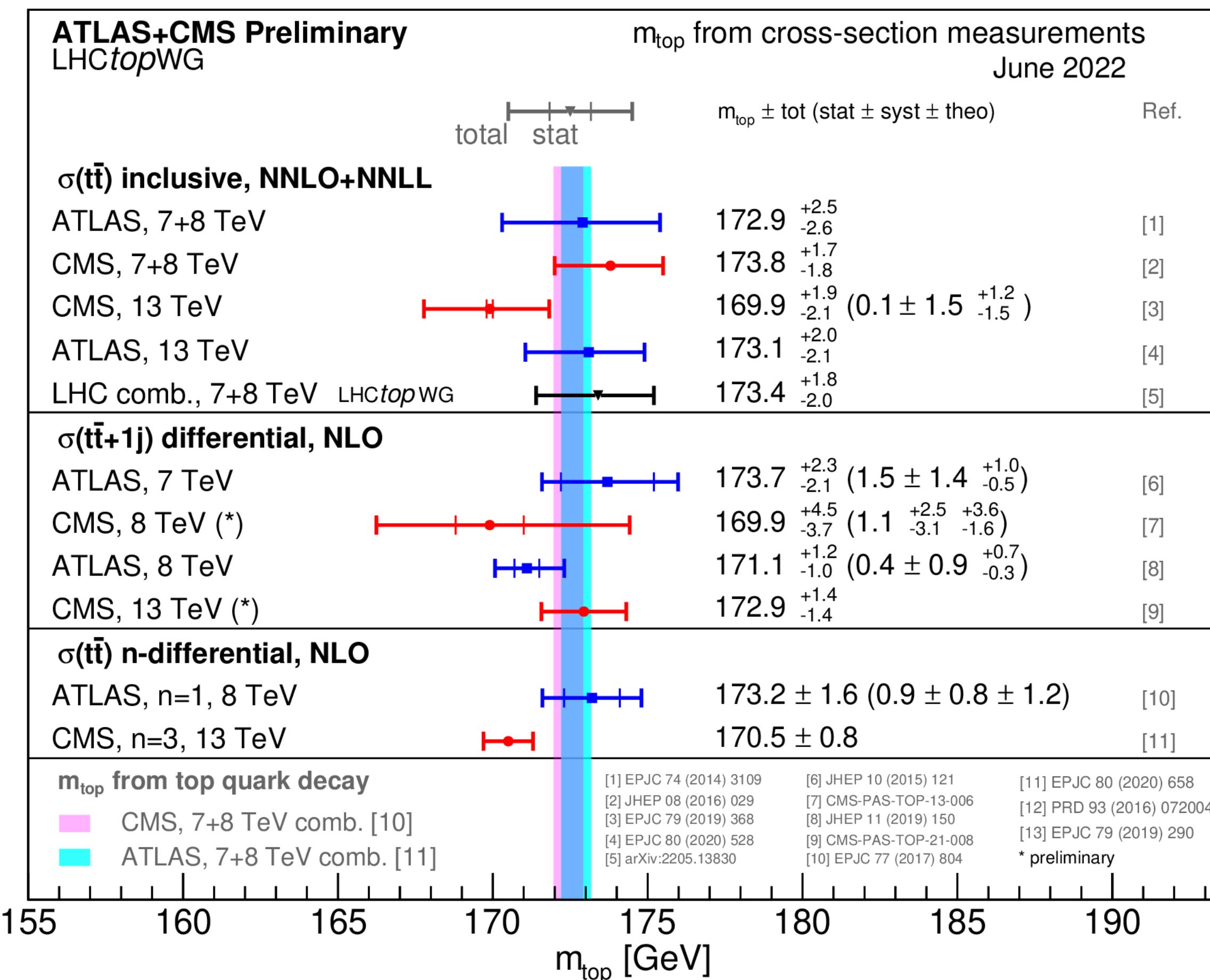


- Top quark is the heaviest elementary particle in the Standard Model.

- Top quark provides the strongest coupling to the SM Higgs boson and opens doors to new physics.

Introduction

Top quark intrinsic properties, Mass, Width, Spin 1/2, e-charge 2/3



Introduction

The top-quark decays almost exclusively to Wb , ie $\Gamma_t = \Gamma(t \rightarrow Wb)$.

At LHC, indirect techniques are precise but model dependent.

The most precise measurement is $\Gamma = 1.36 \pm 0.2^{+0.14}_{-11}$ GeV

$$\mathcal{R} = \frac{\mathcal{B}(t \rightarrow Wb)}{\mathcal{B}(t \rightarrow Wq)} = \mathcal{B}(t \rightarrow Wb)$$

Measuring \mathcal{R} with top pair event

$$\Gamma_t = \frac{\sigma_{t\text{-ch.}}}{\mathcal{B}(t \rightarrow Wb)} \cdot \frac{\Gamma(t \rightarrow Wb)}{\sigma_{t\text{-ch.}}^{\text{theor.}}}$$

CMS, 2014

Direct techniques are less precise but model independent.

Direct result by ATLAS is $\Gamma_t = 1.9 \pm 0.5$ GeV ATLAS, 2019

Using events away from the resonance peak $\Gamma_t = 1.28 \pm 0.3$ GeV

Baskakov, Boos, Dudko, 2018
Herwig, Jazo, Nachman, 2019

In the future e^+e^- -collider, Γ_t can be measured with an uncertainty of 30 MeV Martinez, Miquel 2003

Introduction

NLO QCD corrections

Bigi, Dokshitzer, Khoze, Kuhn,Zerwas, 1986

Jezabek, Kuhn 1989, Czarnecki 1990, Li, Oakes, Yuan 1991

NLO EW corrections

Denner, Sack 1991, Eilam, Mendel, Migneron, Soni 1991

Asymptotic expansions of NNLO QCD corrections near $m_W \rightarrow 0$ and $m_W \rightarrow m_t$

Czarnecki, Melnikov 1999, Chetyrkin, Harlander, Seidensticker, Steinhauser 1999,
Blokland, Czarnecki, Slusarczyk, Tkachov 2004, 2005

Numerical results of full NNLO QCD corrections Gao, Li, Zhu 2013, Brucherseifer, Caola, Melnikov 2013

Polarized top decay up to NNLO QCD

Czarnecki, Groote, Körner and Piclum, 2018

PMC scale settings up to NNLO QCD

Meng, Wang, Sun, Luo, Shen, Wu, 2022

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Analytical results of NNLO QCD corrections Chen, HTL, Wang, Wang, 2022

Optical Theory

Unitarity implies the S -matrix

$$S^\dagger S = 1, \text{ with } S = 1 + iT \quad T \text{ is the transfer matrix}$$

$$-i(T - T^\dagger) = TT^\dagger$$

Take the matrix element of this equation

$$\langle f | T | i \rangle = (2\pi)^4 \delta^{(4)}(p_i - p_f) \mathcal{M}(i \rightarrow f) \quad \left\langle \mathbf{p}_1 \mathbf{p}_2 \left| T^\dagger T \right| \mathbf{k}_1 \mathbf{k}_2 \right\rangle = \sum_n \int d\Phi_n \langle \mathbf{p}_1 \mathbf{p}_2 | T^\dagger | \{\mathbf{q}_n\} \rangle \langle \{\mathbf{q}_n\} | T | \mathbf{k}_1 \mathbf{k}_2 \rangle$$

The generalized optical theorem is

$$-i(\mathcal{M}(i \rightarrow f) - \mathcal{M}^*(i \rightarrow f)) = \sum_X \int d\Phi_X \mathcal{M}(i \rightarrow X) \mathcal{M}^*(f \rightarrow X)$$

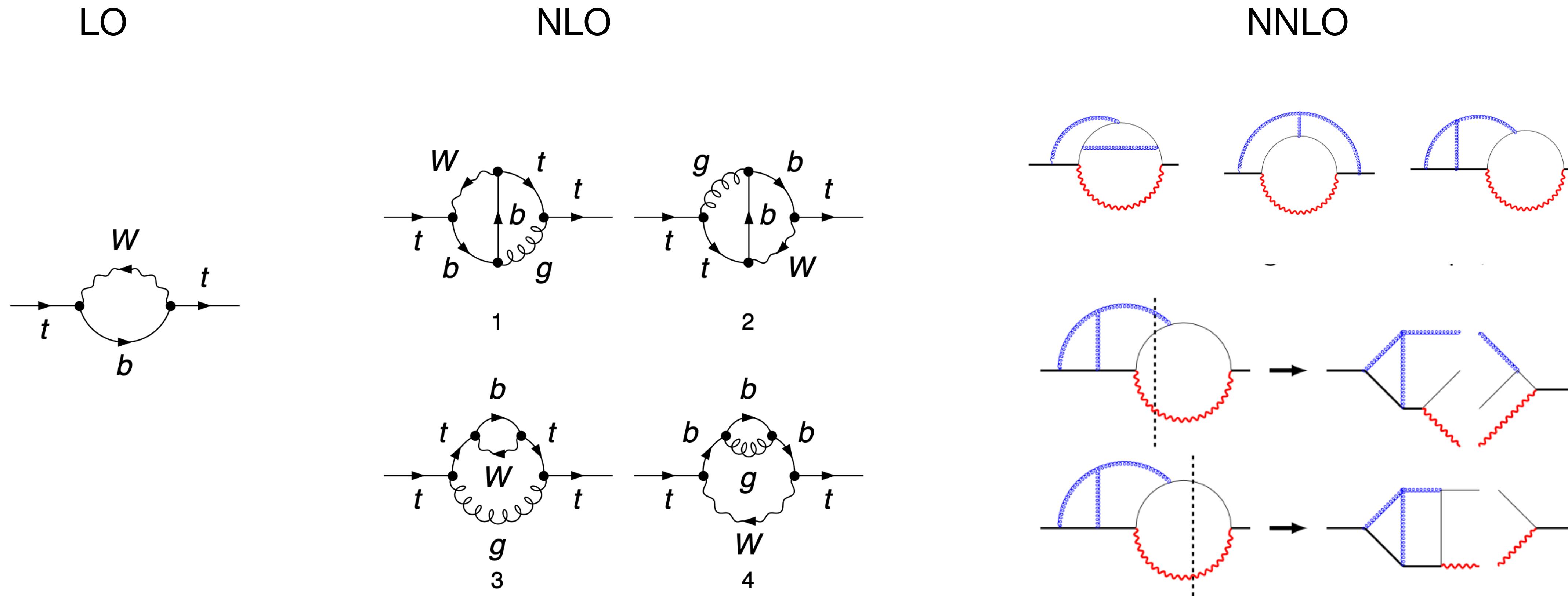
For top quark decay width $|i\rangle = |f\rangle = |A\rangle$

$$\text{Im } \mathcal{M}(A \rightarrow A) = m_A \sum_X \Gamma(A \rightarrow X) = m_A \Gamma_{\text{tot}}$$

Optical Theory

Top decaying into massless b quark and on-shell W boson

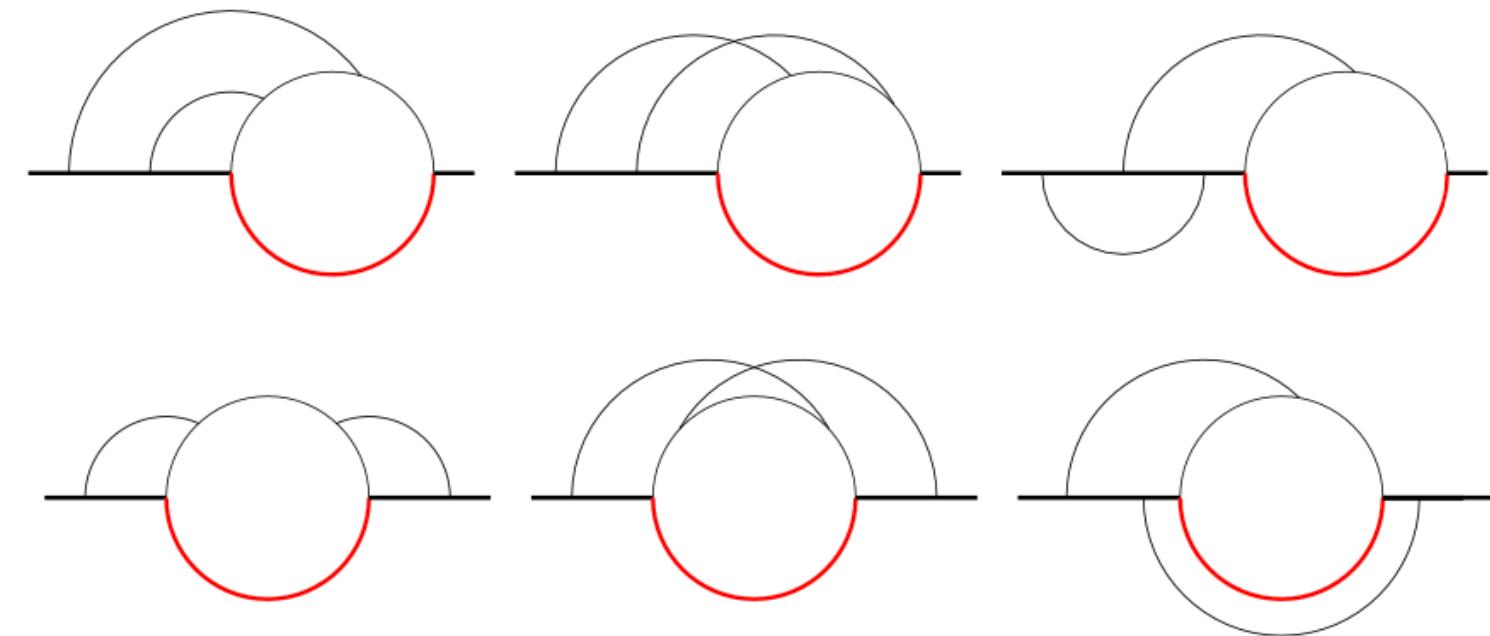
$$\Gamma_t = \frac{\text{Im}\Sigma}{m_t}$$



Amplitude and Master integrals

The amplitudes generated by FeynArts and FeynCalc

Scalar integrals reduced to master integrals using FIRE 6



Canonical differential equations constructed for the cut MIs by choosing a proper basis $\mathbf{I}(w, \epsilon)$ [Henn, 2013](#)

$$d\mathbf{I}(w, \epsilon) = \epsilon d \left[\sum_{i=1}^4 R_i \log(l_i) \right] \mathbf{I}(w, \epsilon)$$

$$\text{Boundary conditions: } w = \frac{m_W^2}{m_t^2} = 0$$

The analytical results of some master integrals in $w = 0$ can be found in literatures

[Blokland, Czarnecki, Slusarczyk, Tkachov 2005,](#)
[Ritbergen, Stuart 2000.](#)

Others reconstructed by PSLQ algorithm with AMFlow [Liu, Ma 2022](#)

Top decay width

Top decaying into massless b quark and on-shell W boson

$$\Gamma(t \rightarrow Wb) = \Gamma_0 \left[X_0 + \frac{\alpha_s}{\pi} X_1 + \left(\frac{\alpha_s}{\pi} \right)^2 X_2 \right],$$

$$X_2 = C_F(T_R n_l X_l + T_R n_h X_h + C_F X_F + C_A X_A)$$

NNLO contributions written as HPLs

$$X_l = -\frac{X_0}{3} [H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3)] + \dots$$

$$X_F = \frac{1}{12} X_0 [-6 (2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3)H_1(w)) - \pi^2 H_{1,0}(w)] + (X_0 + 4w) \left(-\frac{1}{6} \pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w) \right) + \dots$$

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NNLO contributions written as HPLs

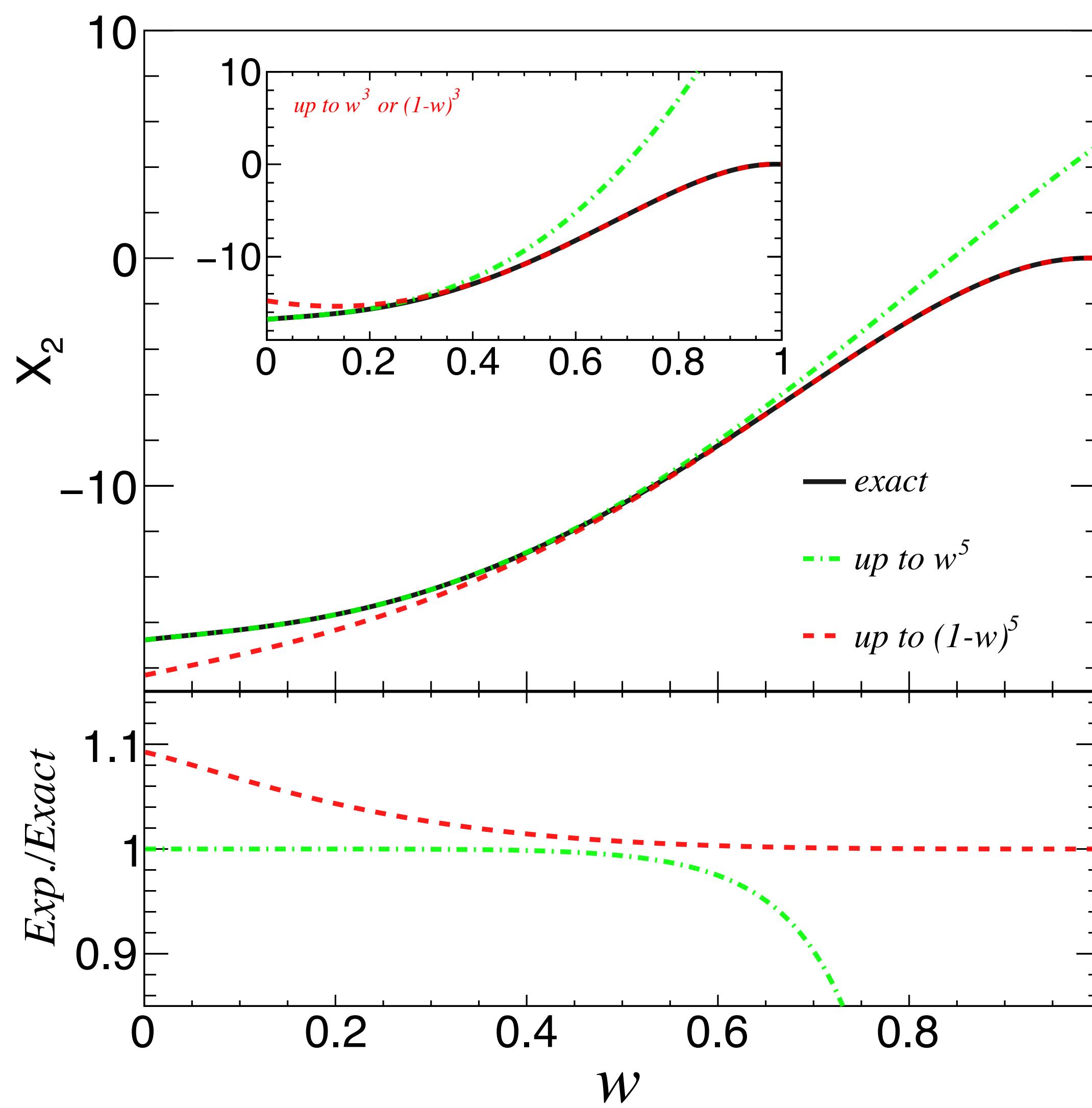
	width (GeV)	delta/LO width
LO	1.48642	—
NLO	1.35897	-8.58%
NNLO	1.32825	-2.07%

$$X_l = -\frac{X_0}{3} [H_{0,1,0}(w) - H_{0,0,1}(w) - 2H_{0,1,1}(w) + 2H_{1,1,0}(w) - \pi^2 H_1(w) - 3\zeta(3)] + \dots$$

Consistent with Gao, Li, Zhu 2013

$$X_F = \frac{1}{12} X_0 [-6 (2H_{0,1,0,1}(w) + 6H_{1,0,0,1}(w) - 3H_{1,0,1,0}(w) - 12\zeta(3)H_1(w)) - \pi^2 H_{1,0}(w)] + (X_0 + 4w) \left(-\frac{1}{6} \pi^2 H_{0,-1}(w) - 2H_{0,-1,0,1}(w) \right) + \dots$$

Top decay width



Though the decay width at $w=0$ and $w=1$ is finite, it exhibits logarithmic structures at these two boundaries

$$X_l = \ln(w) \left(-\frac{1}{3}(2w+1)(w-1)^2(H_{0,1}(w) + 2H_{1,1}(w)) - \frac{1}{18} (38w^3 - 93w^2 + 18w + 37) H_1(w) + \frac{1}{36} w (-106w^2 + 25w + 86) \right) + \dots,$$

$$X_h = \frac{1}{54} \ln(1-w) \left(-18 (2w^3 - 3w^2 - 12w + 1) H_{0,0}(w) + \frac{6 (19w^4 + 32w^3 - 18w^2 - 8w + 23)}{w-1} H_0(w) - 265w^3 - 168w^2 + 498w - \frac{9}{w} - 344 \right) + \dots,$$

All the coefficients can be expanded in series of w or $1 - w$

The result expanded in $w = 0$ and $w = 1$ coincides with
Blokland, Czarnecki, Slusarczyk, Tkachov 2004 2005

Top decay width

Corrections by keeping b quark mass

$$\Gamma_0 = \frac{G_F m_t^3}{8\sqrt{2}\pi} |V_{tb}|^2 \lambda^{1/2} \left[1, \frac{m_b}{m_t}, \frac{m_W}{m_t} \right] \\ \times \left[\left[1 - \frac{m_b^2}{m_t^2} \right]^2 + \left[1 + \frac{m_b^2}{m_t^2} \right] \frac{m_W^2}{m_t^2} - 2 \frac{m_W^4}{m_t^4} \right].$$

0.27% decrease compare to LO width

same order with $\frac{m_b^2}{m_t^2}$

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0.27% decrease compare to LO width

same order with $\frac{m_b^2}{m_t^2}$

$$\Gamma = \Gamma_0 \left\{ 1 + \frac{C_F \alpha_s}{2\pi} \left[2 \left[\frac{(1-\beta_W^2)(2\beta_W^2-1)(\beta_W^2-2)}{\beta_W^4(3-2\beta_W^2)} \right] \ln(1-\beta_W^2) - \frac{9-4\beta_W^2}{3-2\beta_W^2} \ln \beta_W^2 \right. \right. \\ \left. \left. + 2 \text{Li}_2 \beta_W^2 - 2 \text{Li}_2(1-\beta_W^2) - \frac{6\beta_W^4-3\beta_W^2-8}{2\beta_W^2(3-2\beta_W^2)} - \pi^2 \right] \right\}.$$

0.126% increase compared to LO width

Li, Oakes, Yuan, 1991

Top decay width

For off-shell W effects, we could integrate over the

$$\Gamma(t \rightarrow W^* b) = \frac{1}{\pi} \int_0^{m^2} dq^2 \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \left(\Gamma_t \Big|_{m_W^2 \rightarrow q^2} \right) \quad \text{Taking the limits } \Gamma_W \rightarrow 0, \frac{m_W \Gamma_W}{(q^2 - m_W^2)^2 + m_W^2 \Gamma_W^2} \rightarrow \pi \delta(q^2 - m_W^2)$$

$$\Gamma(t \rightarrow W^* b) = \Gamma_0 \left(\tilde{X}_0 + \frac{\alpha_s}{\pi} \tilde{X}_1 + \left(\frac{\alpha_s}{\pi} \right)^2 \tilde{X}_2 \right), \quad X_i \text{ is expressed in terms of GPLs}$$

For example $\tilde{X}_0 = 2rw(2w - 1) - \frac{1}{2} [(2(r - i)w - i)((r - i)w + i)^2 G_{w+irw}(1) + (2(r + i)w + i)((r + i)w - i)^2 G_{w-irw}(1)]$

this effect can be included independently
with QCD corrections

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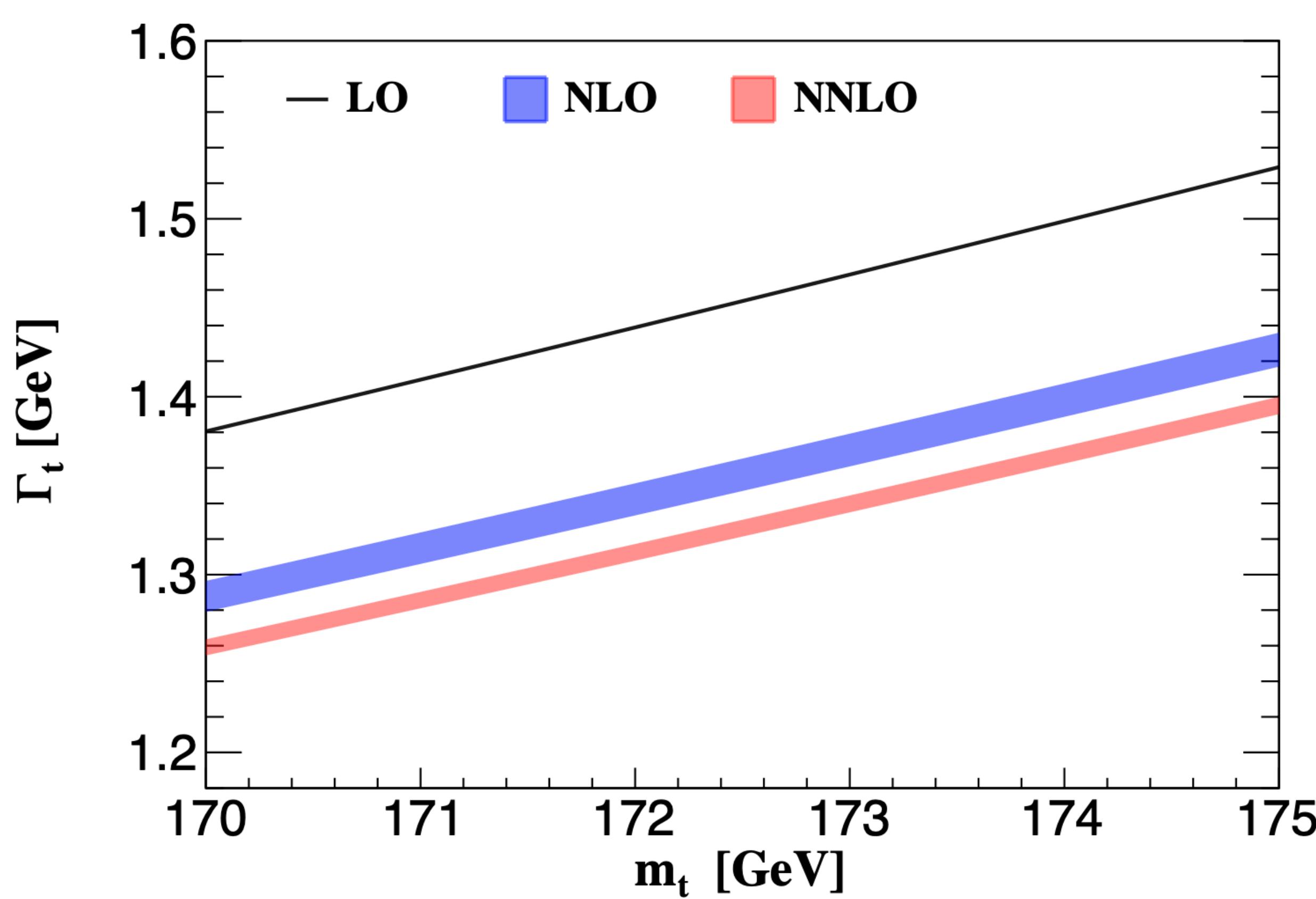
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	LO	NLO	NNLO
delta/LO width	-1.54%	0.13%	0.03%

Top decay width

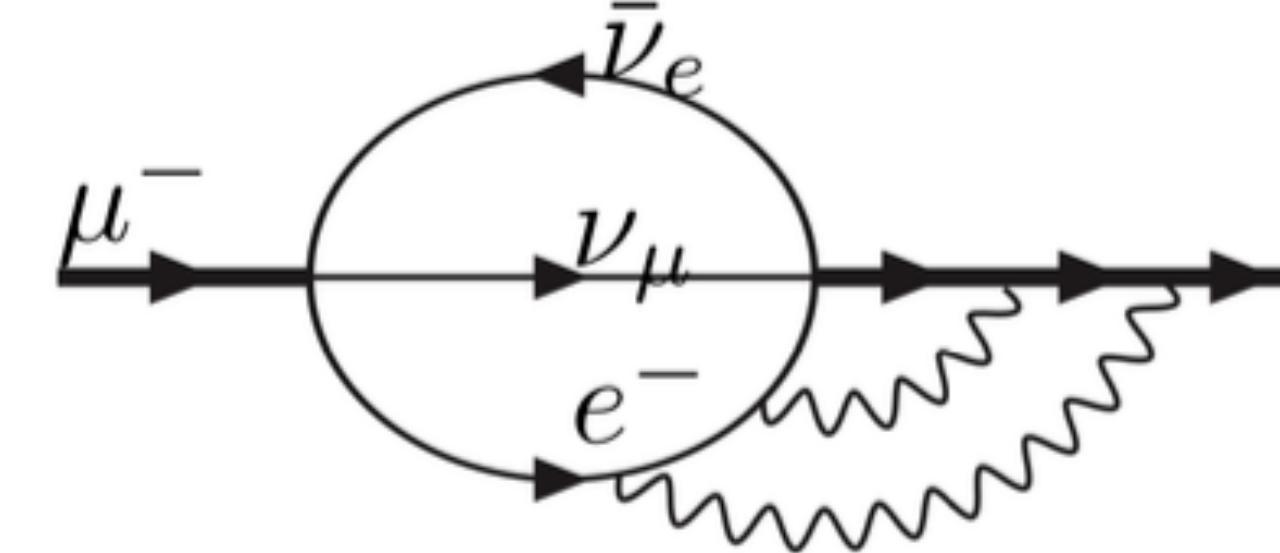
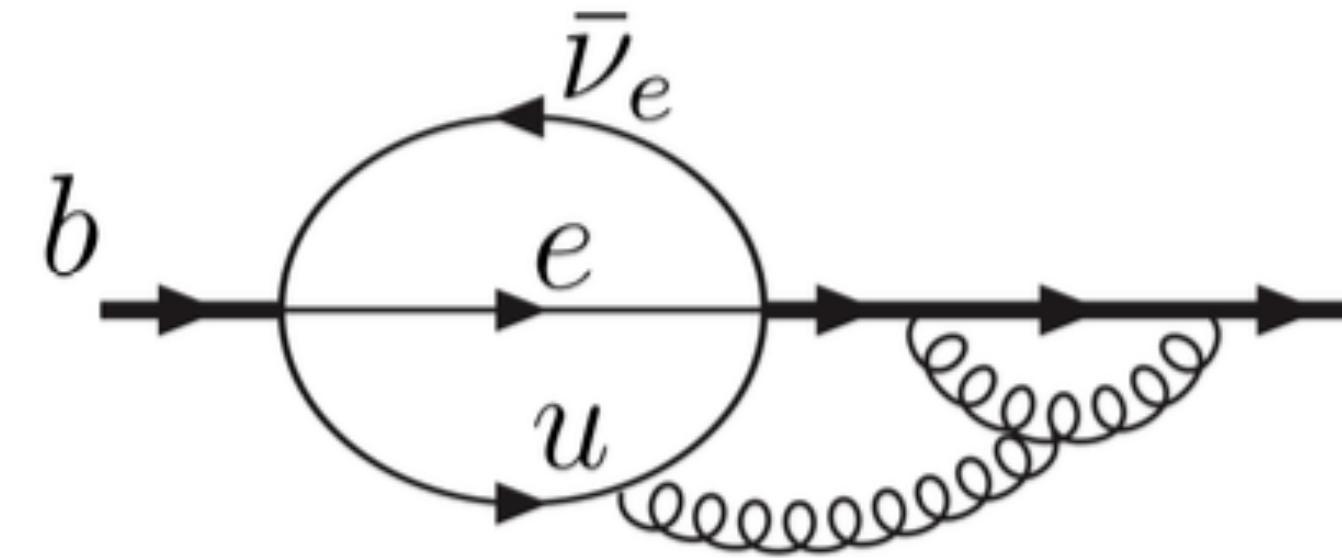
QCD renormalization scale $\mu \in [m_t/2, 2m_t]$, the variation is about $\pm 0.8\%$ and $\pm 0.4\%$ at NLO and NNLO.



- On-shell renormalization scheme adopted
- This scale uncertainty has been reduced dramatically after including NNLO QCD corrections.
- MSbar scheme, $\Gamma_t^{\text{NLO}} = 1.309$ GeV, $\Gamma_t^{\text{NNLO}} = 1.332$ GeV. QCD corrections are -3.79% and 0.09% at NLO and NNLO.
- Assuming power like growth for QCD corrections, NNNLO corrections would be of around 0.4%

Top decay width

Cross-check and other applications



w can be understood as q^2/m^2

with q^2 the invariant mass of lepton sector, m^2 the mass of the parent particle

Integrating over w from 0 to 1, we reproduce NNLO QCD corrections in semileptonic decay $\Gamma(b \rightarrow X e \bar{\nu})$

Ritbergen 1999

Integrating X_F over w , we obtain the analytic two-loop QED correction to the muon lifetime $\Gamma(\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e)$.

Ritbergen, Stuart 1999

Mathematica Package

<https://github.com/haitaoli1/TopWidth>

main · 1 branch · 0 tags

Go to file Add file · Code

haitaoli1 typo corrected · ce94fe8 on Feb 10 · 30 commits

LICENSE.md · license added · 4 months ago
README.md · typo corrected · 2 months ago
TopWidth.m · arXiv information added · 4 months ago
example.nb · typo corrected · 2 months ago

README.md

TopWidth

Mathematica Package to calculate the top decay width with NNLO corrections in QCD and NLO corrections in EW.

Requirement

The HPL package is required to generate the numerics of the harmonic polylogarithm, which can be downloaded from <https://krone.physik.uzh.ch/data/HPL/>.

HPL is supposed to be initialized through "`<<HPL``". If not please set the path
"`$HPLPath="the\path\of\the\installation".`"

Download

Download the package through
git clone <https://github.com/haitaoli1/TopWidth.git>
go to the directory "TopWidth", run the example notebook "example.nb".

About

Mathematica Package to calculate the top decay width with NNLO corrections in QCD and NLO corrections in EW.

Readme · LGPL-3.0 license · 1 star · 1 watching · 0 forks

Releases

No releases published · Create a new release

Packages

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Languages

Mathematica 100.0%

`<< TopWidth``

(***** TopWidth-1.0 *****)

Authors: Long-Bin Chen, Hai Tao Li, Jian Wang, YeFan Wang

`TopWidth[QCDorder, mbCorr, WwidthCorr, EWcorr, mu]` is provided for top width calculations

Please cite the paper for reference: arXiv:2212.06341

***** HPL 2.0 *****

Author: Daniel Maitre, University of Zurich

Rules for minimal set loaded for weights: 2, 3, 4, 5, 6.

Rules for minimal set for + - weights loaded for weights: 2, 3, 4, 5, 6.

Table of MZVs loaded up to weight 6

Table of values at I loaded up to weight 6

`$HPLFunctions` gives a list of the functions of the package.

`$HPLOptions` gives a list of the options of the package.

More info in hep-ph/0507152, hep-ph/0703052 and at

<http://krone.physik.unizh.ch/~maitreda/HPL/>

In[5]:= `SetParameters[$\frac{17269}{100}, \frac{478}{100}, 80377/1000, 2085/1000, 911876/10000, 11663788 \times 10^{-12}$]`

In[6]:= `TopWidth[2, 1 (* with mb effects *), 1 (* with Iw effects *), 1 (* with NLO EW effects *), $\frac{17269}{100}$ (*scale*)]`

Out[6]= 1.33051

Summary

- We provide the full analytical result of top-quark width at NNLO.
- The b mass effect is included up to NLO
- The off-shell W boson contribution is calculated analytically up to NNLO.
- The analytical result can be used to perform both fast and exact evaluations.
- The missing contributions is supposed to be less than 1%.

	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\text{EW}}^{(i)}$	$\delta_{\text{QCD}}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	—	—	1.459
NLO	0.126	0.132	1.683	-8.575	1.361
NNLO	*	0.030	*	-2.070	1.331

謝謝！