

QCD two-loop results of $t\bar{W}$ production at hadron colliders

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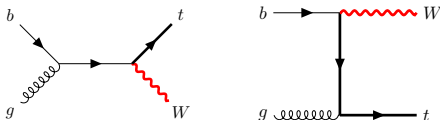
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Motivation

The top quark is the **heaviest** elementary particle in the Standard Model.

Three major modes for single top productions, s-channel, t-channel and **tW production**.

tW production can be used to probe the **CKM matrix element V_{tb}** .



The uncertainty of the measured cross section is about 10 %. [CMS, 2022].

To match experiments, the theoretical predictions must include higher-order corrections.

Motivation

NLO correction [S. Zhu 2002, Q.-H. Cao 2008] with top and W decay [Campbell, Tramontano 2005]

Approximate higher order corrections [N. Kidonakis 2006, 2010, 2017, 2021]

Effect of the parton shower [Frixione, Laenen, Motylinski, Webber, White 2008, E. Re 2011, Ježo, Lindert, Nason, Oleari, Pozzorini 2016]

NNLL soft-gluon resummation [Li, Li, Shao, Wang 2019]

To match experiments, the complete NNLO QCD corrections are important.

Factorization formula

The N -jettiness subtraction is based on the soft-collinear effective theory (SCET).

$$\frac{d\sigma}{d\tau_N} \propto \int H \otimes B_1 \otimes B_2 \otimes S \otimes \left(\prod_{n=1}^N J_n \right). \quad (1)$$

NNLO Beam functions B_i [Stewart, Tackmann, Waalewijn 2010, Berger, Marcantonini, Stewart, Tackmann, Waalewijn 2011, Gaunt, Stahlhofen, Tackmann, 2014]

NNLO Jet function J [Becher, Bell 2006, Becher, Neubert 2011]

NNLO Soft function S [Li, Wang 2016, 2018]

The missing part is **NNLO hard function**, which demands **one-loop squared amplitudes** and the interference between **two-loop** and tree-level amplitudes.

Kinematics and notations

Two massive particles

$$g(k_1) + b(k_2) \rightarrow W(k_3) + t(k_4),$$
$$k_1^2 = k_2^2 = 0, \quad k_3^2 = m_W^2, \quad k_4^2 = (k_1 + k_2 - k_3)^2 = m_t^2. \quad (2)$$

The polarization summation

$$\sum_i \epsilon_i^{*\mu}(k_3) \epsilon_i^\nu(k_3) = -g^{\mu\nu} + \frac{k_3^\mu k_3^\nu}{m_W^2}$$
$$\sum_i \epsilon_i^\mu(k_1) \epsilon_i^{*\nu}(k_1) = -g^{\mu\nu} + \frac{k_1^\mu n^\nu + k_1^\nu n^\mu}{k_1 \cdot n} \text{ (can be neglected here)}. \quad (3)$$

Kinematics and notations

The **anticommuting γ_5 scheme** is implemented.

The tW amplitude can be written as

$$\mathcal{M} = \mathcal{M}^{(0)} + \frac{\alpha_s}{4\pi} \mathcal{M}^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \mathcal{M}^{(2)} + \dots \quad (4)$$

We don't consider the decay of top quark and W boson here, focus on **amplitude squared**,

$$|\mathcal{M}^{(1)}|^2, \quad |\mathcal{M}^{(0)*} \mathcal{M}^{(2)}|. \quad (5)$$

All the Lorentz indices are contracted.

Color structures

According to color structures, we have

$$\begin{aligned} & \mathcal{M}^{(2)} \mathcal{M}^{(0)*} + \mathcal{M}^{(0)} \mathcal{M}^{(2)*} \\ &= N_c^4 A + N_c^2 B + C + \frac{1}{N_c^2} D + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l) \\ & \quad + n_h (N_c^3 E_h + N_c F_h + \frac{1}{N_c} G_h), \end{aligned} \tag{6}$$

$n_l = 5$ ($n_h = 1$) is the number of light (heavy) quark flavors. N_c is the color factor.

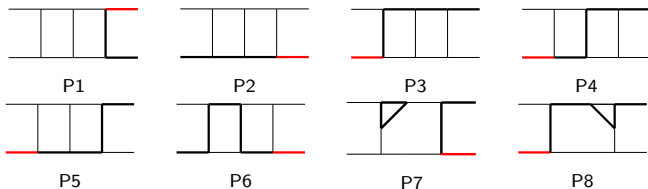
Leading color and light-fermion part

$$\mathcal{A}_{\text{L.C.}}^{(2)} \equiv N_c^4 A, \quad \mathcal{A}_{n_l}^{(2)} \equiv n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l), \tag{7}$$

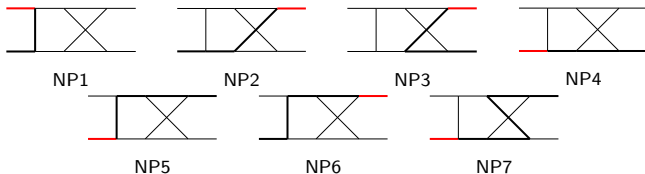
Topologies of Master Integrals

After IBP (integration-by-parts) reduction of package FIRE [Smirnov, Chuharev 2019],

$\mathcal{A}^{(2)} = \sum_{\text{spins}} |\mathcal{M}^{(0)*} \mathcal{M}^{(2)}|$ can be reduced to several families of master integrals.



Red lines are W boson, thick lines are top quarks.

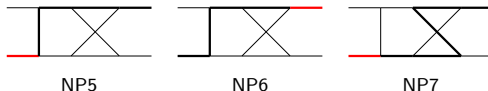


The difficulty of Reduction

There are about $\mathcal{O}(10^3)$ master integrals in our calculations.

Due to the internal massive propagators and multi-scale

1. the reduction of non-planar diagrams are quite difficult.
2. the number of master integrals is large.

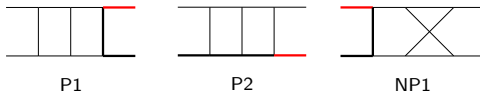


For example, the NP7 family has 90 master integrals.

If directly use package FIRE, we cannot complete reduction even in two week. And the reduction result file can be large ($\mathcal{O}(10^4)$ MB).

Master Integrals Calculations

Analytical results of P1 and P2 and NP1 families have been obtained [Chen, Wang 2021, Long, Zhang, Ma, Jiang, Han, Li, Wang 2021, Wang, Wang 2023] by canonical differential equations.

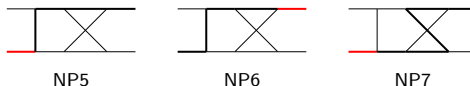


With these analytical results, we obtain the results of $\mathcal{A}_{\text{L.C.}}^{(2)}$ and $\mathcal{A}_{n_1}^{(2)}$.

$$\mathcal{A}_{\text{L.C.}+n_1}^{(2)} \equiv N_c^4 A + n_l (N_c^3 E_l + N_c F_l + \frac{1}{N_c} G_l). \quad (8)$$

For other integral families, analytical calculations are quite difficult because of

1. Rationalization of multiple square roots
2. Several elliptic curves



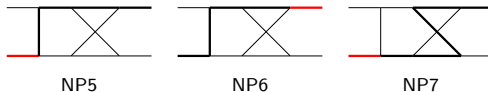
The Difficulty of Master Integrals Calculations

The analytical structures are complicated, we use numerical calculations.

Package AMFlow is powerful.

It is not efficient to directly use AMFlow to get grids in full phase space.

Numerical calculation of master integrals in NP7 in one phase space point need several hours.



Our Improvements in Reduction

We set $m_t^2 = 1, m_w^2 = 3/14$.

We use the FIRE function "ImproveMasters" [Smirnov, Smirnov 2020] to select good integral basis.

The reductions can be down. The reduction of NP7 family still need more than one week.

Construct Differential Equations

Construct the differential equations of master integrals of s and t .

One kinematic point (s_0, t_0) is calculated by AMFlow [Liu, Ma 2022] as boundary condition.

Numerically solve the differential equations by the AMFlow function "DESolver".

By differential equation, the numerical calculations are much more efficient.

Master Integrals Check

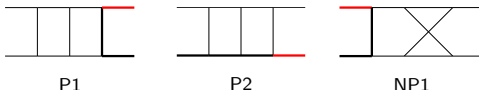
We can obtain numerical results at phase space point (s_i, t_i) in two ways:

1. Direct evaluation of AMFlow
2. Solving the differential equation (more efficient)

Numerical agreements between these two methods reach high precision.

Cross check with FIESTA [Smirnov, Shapurov, Vysotsky, 2022]

Cross check with our analytical results in P1, P2 and NP1 families.



IR divergences

After renormalization, we subtract UV divergence .

IR divergences can be subtracted with a factor \mathbf{Z} .

$$\mathcal{M}_{\text{fin}} = \mathbf{Z}^{-1} \mathcal{M}_{\text{ren}} . \quad (9)$$

In the framework of SCET, \mathbf{Z} can be investigated through the [anomalous-dimensions](#) of the effective operators.

$$\mathbf{Z} = 1 + \frac{\alpha_s}{4\pi} \mathbf{Z}^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \mathbf{Z}^{(2)} + \mathcal{O}(\alpha_s^3). \quad (10)$$

Hard Function

$|\mathcal{M}_{\text{fin}}|^2$ is hard function

$$\begin{aligned} H &= |\mathcal{M}_{\text{fin}}|^2 = H^{(0)} + \frac{\alpha_s}{4\pi} H^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 H^{(2)} + \mathcal{O}(\alpha_s^3), \\ H^{(2)} &= \mathcal{M}_{\text{fin}}^{(2)} \mathcal{M}_{\text{fin}}^{(0)*} + \mathcal{M}_{\text{fin}}^{(0)} \mathcal{M}_{\text{fin}}^{(2)*} + \left|\mathcal{M}_{\text{fin}}^{(1)}\right|^2 \end{aligned} \quad (11)$$

According to **color structures**,

$$\begin{aligned} H^{(2)} &= N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) \\ &\quad + n_h \left(N_c^3 H_{Eh} + N_c H_{Fh} + \frac{1}{N_c} H_{Gh} \right). \end{aligned} \quad (12)$$

Leading color and light fermion parts of hard function

$$H_{\text{L.C.}+n_l}^{(2)} \equiv N_c^4 H_A + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right). \quad (13)$$

Numerical results

$\beta_t = \sqrt{1 - m_t^2/E_t^2}$ measures the **velocity** of top quark.

θ is the **angle** between gluon and top quark.

The full phase space spans over $0 \leq \beta_t < 1$ and $-1 \leq \cos \theta \leq 1$.

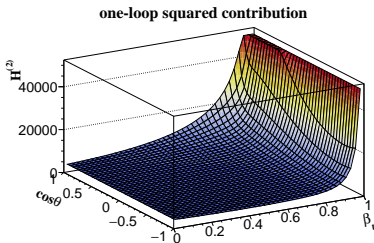
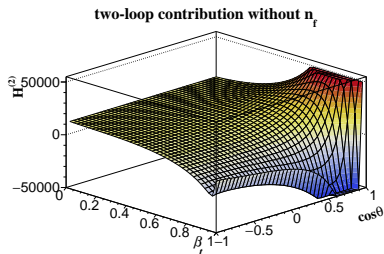
We generated a grid of 80×42 **phase space points**.

The divergence of ϵ^{-n} has been **canceled in high precision**.

Numerical results

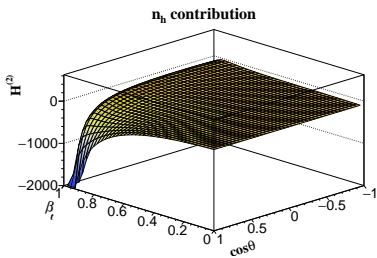
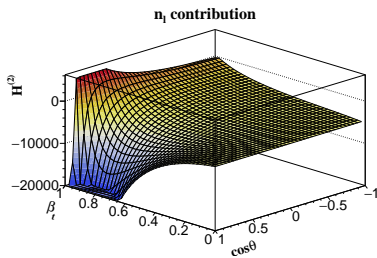
$$H^{(2)} = N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) + n_h \left(N_c^3 H_{Eh} + N_c H_{Fh} + \frac{1}{N_c} H_{Gh} \right), \quad (14)$$

$$N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D \left\{ \begin{array}{l} \text{two-loop contribution without } n_f \\ \text{one-loop squared contribution} \end{array} \right. \quad (15)$$



Numerical results

$$\begin{aligned}
 H^{(2)} = & N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D + n_l \left(N_c^3 H_{El} + N_c H_{Fl} + \frac{1}{N_c} H_{Gl} \right) \\
 & + n_h \left(N_c^3 H_{Eh} + N_c H_{Fh} + \frac{1}{N_c} H_{Gh} \right).
 \end{aligned} \tag{16}$$

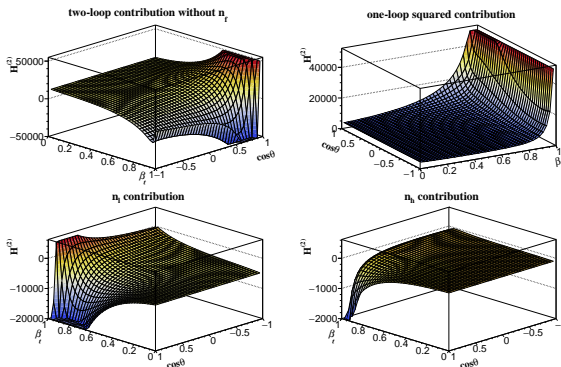


Numerical results

The two-loop without quark loops and the one-loop squared corrections are positive.

The quark-loop corrections are negative.

They vary dramatically toward the boundary of $\beta_t = 1$ or $\cos\theta = 1$.



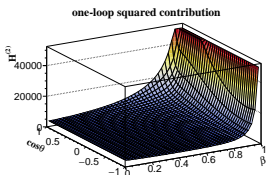
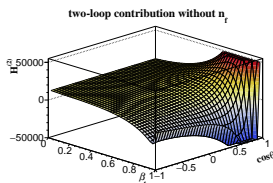
Divergence Behavior

LO squared amplitude contains $1/(1 - \beta_t \cos \theta)$.

The higher-order corrections develop additional **collinear divergences** in the form of $\ln(1 \pm \beta_t \cos \theta)$ and $\ln(1 - \beta_t^2)$.

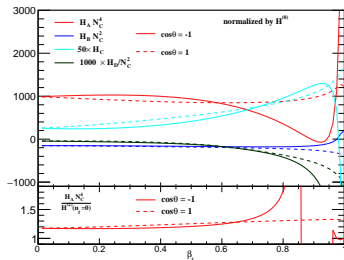
These logarithms can be predicted with the method in boosted top production [Ferroglia, Marzani, Pecjak, Yang 2014].

The **suppression** of the parton distribution functions near the region of $\beta_t \rightarrow 1$ **dominates over these large logarithms**.



Leading Color Contribution

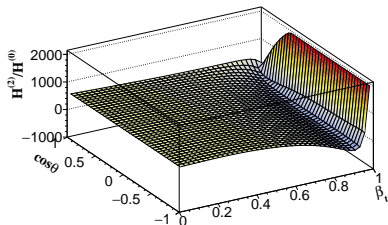
$$N_c^4 H_A + N_c^2 H_B + H_C + \frac{1}{N_c^2} H_D \quad (17)$$



One phase space point

$$\left. \frac{H^{(2)}}{H^{(0)}} \right|_{\beta_t=0.4, \cos \theta=-1, n_f=0} = \underbrace{997}_{H_A N_c^4} \underbrace{-162}_{H_B N_c^2} \underbrace{-6.60}_{H_C} \underbrace{-0.098}_{H_D/N_c^2}, \quad (18)$$

Full NNLO Hard Function



The function is flat over the large region of $\beta_t < 0.8$ as a consequence of **strong cancellation** among different contributions.

After phase space integration and convolution with PDFs, NNLO hard function provides a correction of about **3% to the LO cross section**.

Summary and Outlook

We present the calculation of **complete two-loop amplitudes** for tW production.

We estimate the **leading color approximation** of NNLO hard function.

After **phase space integration and convolution with the PDFs**, NNLO hard function **increases the LO cross section by about 3%**.

Next target is **complete NNLO QCD corrections** for tW production.