Heavy-flavor baryon physics: QCD and CPV

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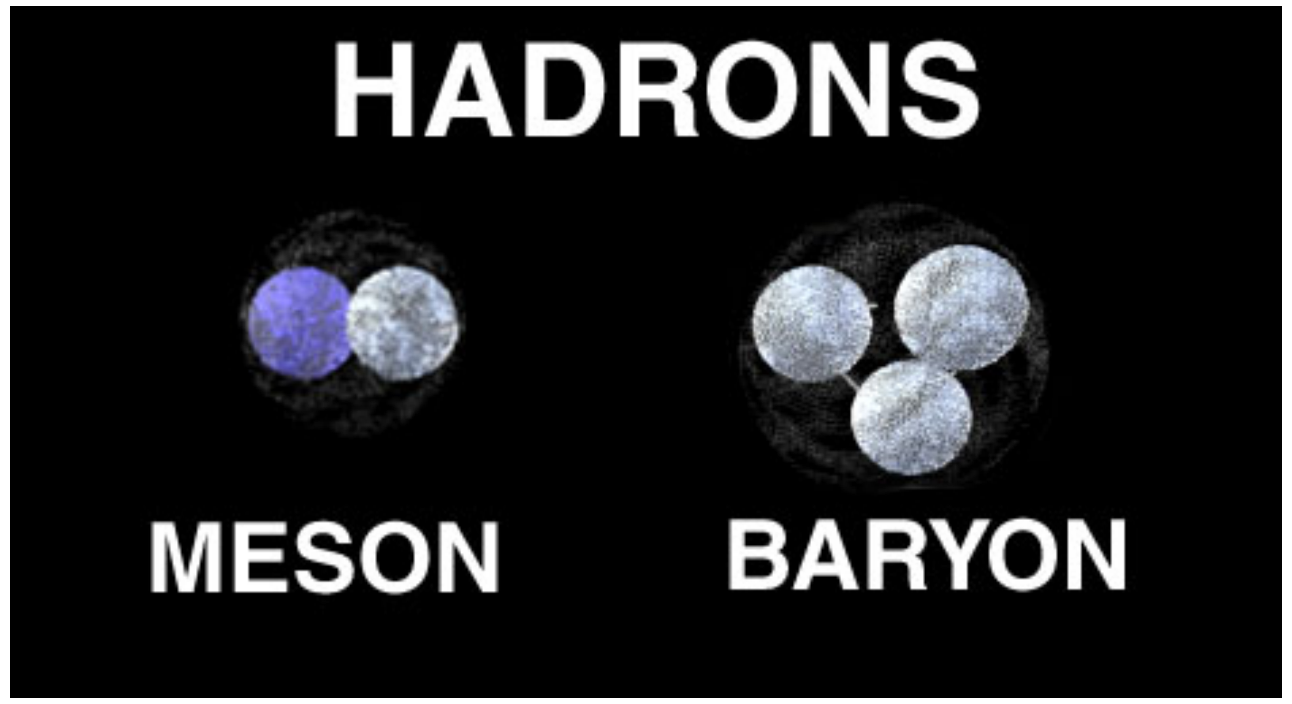
Outline

- Why baryon physics?
- QCD dynamics of non-leptonic decays
- CP violation of $\Lambda_b \to p\pi$
- Summary

Heavy flavor physics

- Heavy flavor physics has achieved a great progress in the heavy meson systems during the past two decades.
- It established the KM mechanism for the CP violation in B meson decays.
- However, the studies on heavy-flavor baryons are limited.



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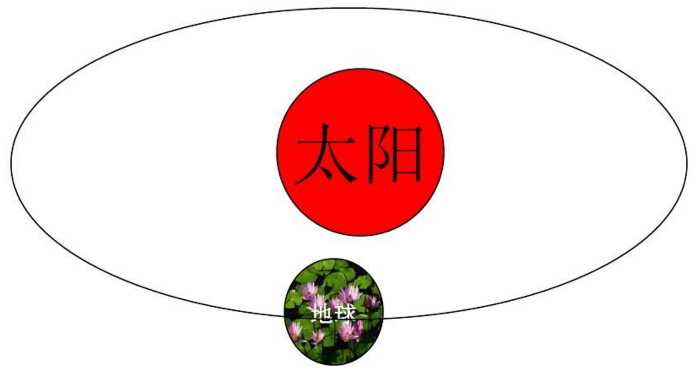

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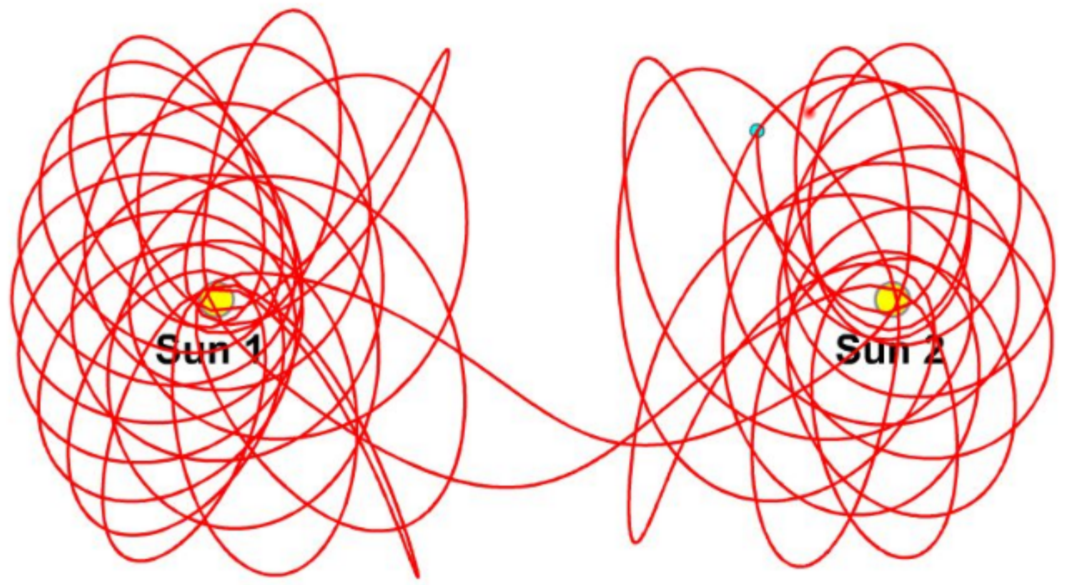
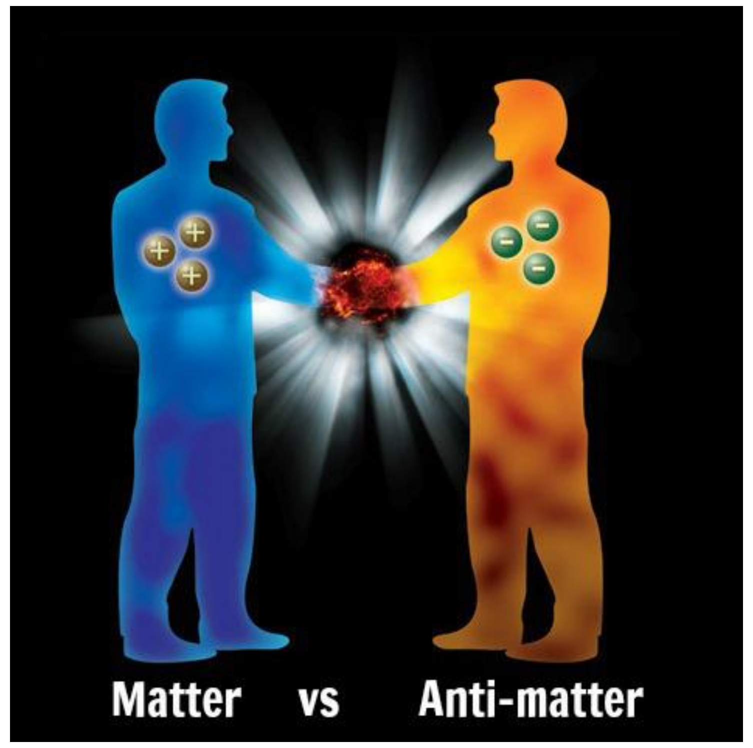


It is a non-trivial extension

More is different

CP violation in baryons

- Sakharov conditions for **Baryogenesis**:
  1) **baryon** number violation
  2) C and **CP violation**
  3) out of thermal equilibrium
- **CPV: SM < BAU. => new source of CPV, NP**
- CPV well established in K, B and D mesons,
  but CPV never established in any baryon
- **Comparison between precise prediction and measurement is helpful to test the SM and search for NP**



Opportunities

- **LHCb is a baryon factory !!** **Large Production:** $\frac{f_{\Lambda_b}}{f_{u,d}} \sim 0.5 \longrightarrow \frac{N_{\Lambda_b}}{N_{B^{0(-)}}} \sim 0.5$

- Precision of baryon CPV measurements has reached to the order of **1%** [LHCb, PLB2018]

$$A_{CP}(\Lambda_b^0 \to p\pi^-) = (-3.5 \pm 1.7 \pm 2.0)\,\%\,, \quad A_{CP}(\Lambda_b^0 \to pK^-) = (-2.0 \pm 1.3 \pm 1.0)\,\%$$

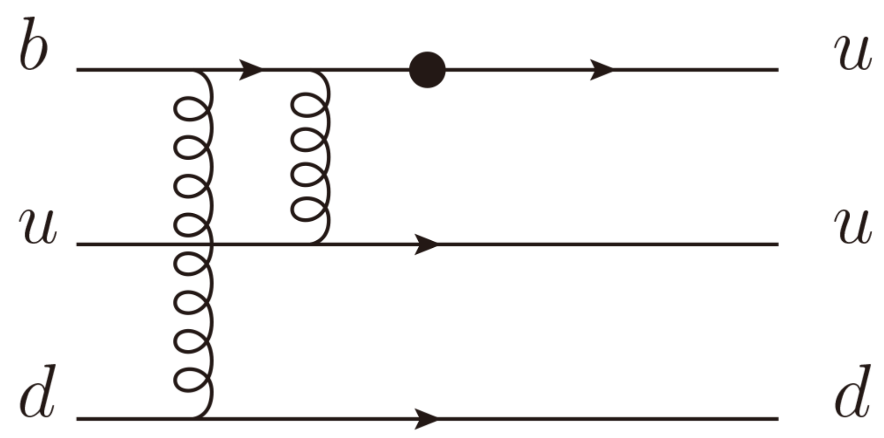
- CPV in some B-meson decays are as large as **10%**:

$$A_{CP}(\overline{B}^0 \to \pi^+\pi^-) = -\,(32 \pm 4)\,\%\,, \quad A_{CP}(\overline{B}_s^0 \to K^+\pi^-) = +\,(21.3 \pm 1.7)\,\%$$

- **It can be expected that CPV in b-baryons might be observed soon !!**

Challenges

1. QCD dynamics for non-leptonic decays



- One more energetic quark, one more hard gluon.
  Counting rule of power expansion is violated by α_s .
- Why the CPV of $\Lambda_b \to p\pi, pK$ are so small?

2. Non-perturbative inputs

- Theoretical uncertainties are dominated by the non-perturbative input parameters, such as the light-cone distribution amplitudes (LCDA).

3. Observables

- T-odd triple products $(\vec{p}_1 \times \vec{p}_2) \cdot \vec{p}_3$, 3σ signal in $\Lambda_b \to p\pi\pi\pi$[LHCb2017].
  Defined by kinematics, but unclear relation to the decay amplitudes.
  No way for theoretical explanations and predictions.

See J-P Wang's talk

Theoretical challenges

- **QCD studies on baryons are limited**

- Generalized factorization [Hsiao, Geng, 2015; Liu, Geng, 2021]:
  lost of non-factorizable contributions, such as W-exchange diagrams.

- QCDF [Zhu, Ke, Wei, 2016, 2018]: based on diquark picture, No W-exchange diagrams.

- PQCD [Lu, Wang, Zou, Ali, Kramer, 2009]: only considering the leading twists of LCDAs.

- **Currently, no complete QCD-inspired method for non-leptonic b-baryon decays**

	EXP	GF	PQCD	QCDF
$Br(\Lambda_b \to p\pi)[\times 10^{-6}]$	4.3 ± 0.8	4.2+-0.7	$4.66^{+2.22}_{-1.81}$	4.11~4.57
$Br(\Lambda_b \to pK)[\times 10^{-6}]$	5.1 ± 0.9	4.8+-0.7	$1.82^{+0.97}_{-1.07}$	1.70~3.15
$A_{CP}(\Lambda_b \to p\pi)[\%]$	-2.5 ± 2.9	-3.9+-0.2	-32^{+49}_{-1}	-3.74~-3.08
$A_{CP}(\Lambda_b \to pK)[\%]$	-2.5 ± 2.2	5.8+-0.2	-3^{+25}_{-4}	8.1~11.4

Theoretical opportunities

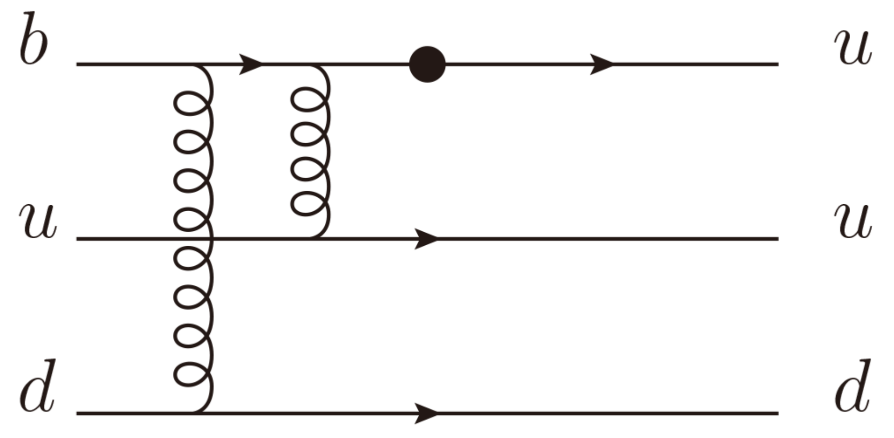
- **Baryons are very different from mesons!!**

- **Factorization:** Heavy-to-light form factor is factorizable at leading power in SCET.
  No end-point singularity! [Wei Wang, 1112.0237] Taking $\Lambda_b \to \Lambda$ as an example,

$$\xi_\Lambda = f_{\Lambda_b}\Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes f_\Lambda \Phi_\Lambda(y_i)$$

- However, the **leading-power result is one order of magnitude smaller** than the total one
  - Leading power: $\xi_\Lambda(0) = -0.012$ [W.Wang, 2011]
  - Total form factor: $\xi_\Lambda(0) = 0.18$ [Y.L.Shen, Y.M.Wang, 2016]

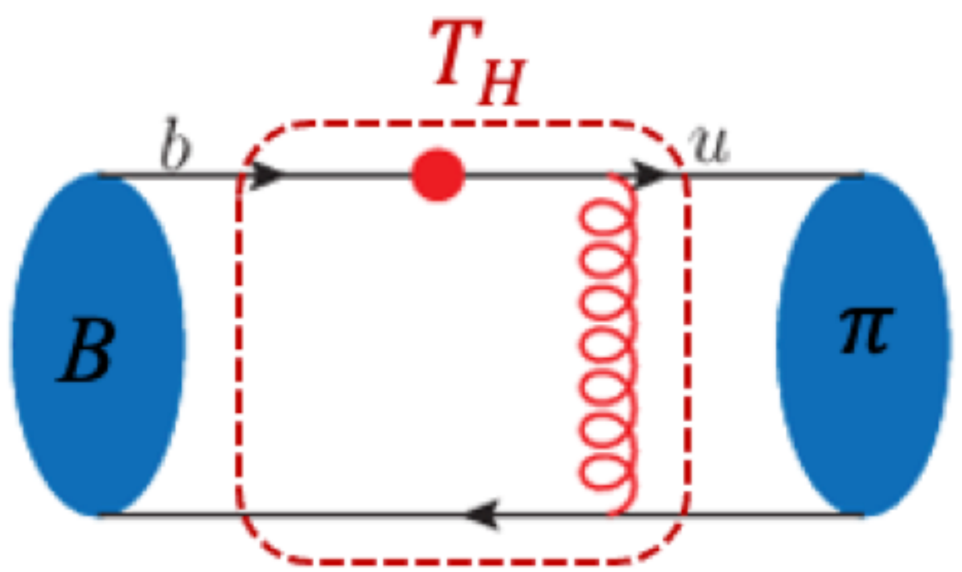
- Two hard gluons suppressed by α_s^2 at the leading power.
  Compared to the soft contributions in the power corrections.



PQCD approach

- PQCD successfully predicted CPV in B meson decays [Keum, H.n.Li, Sanda, 2000; C.D.Lu, Ukai, M.Z.Yang, 2000].

直接CP破坏(%)	GFA	QCDF	2000 PQCD	2004 exp.
$B \to \pi^+\pi^-$	-5 ± 3	-6 ± 12	$+30 \pm 20$	$+32 \pm 4$
$B \to K^+\pi^-$	$+10 \pm 3$	$+5 \pm 9$	-17 ± 5	-8.3 ± 0.4



- under collinear factorization:
  - Endpoint singularity: propagator $\sim 1/x_1x_2Q^2 \to \infty$ when $x_{1,2} \to 0{,}1$

$$M(Q^2) = \int_0^1 dx_1 dx_2\, \phi_B(x_2,\mu^2) * T_H\left(x_1, x_2, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1,\mu^2)$$

- PQCD approach (based on k_T factorization): retain transverse momentum of parton k_T,
  - propagator $\sim 1/(x_1x_2Q^2 + k_T^2)$

$$M(Q^2) = \int_0^1 dx_1 dx_2 \int d\boldsymbol{k}_{1T} d\boldsymbol{k}_{2T} \phi_B(x_2, \boldsymbol{k}_{2T}, \mu^2) * T_H\left(x_1, x_2, \boldsymbol{k}_{2T}, \boldsymbol{k}_{1T}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) * \phi_\pi(x_1, \boldsymbol{k}_{1T}, \mu^2)$$

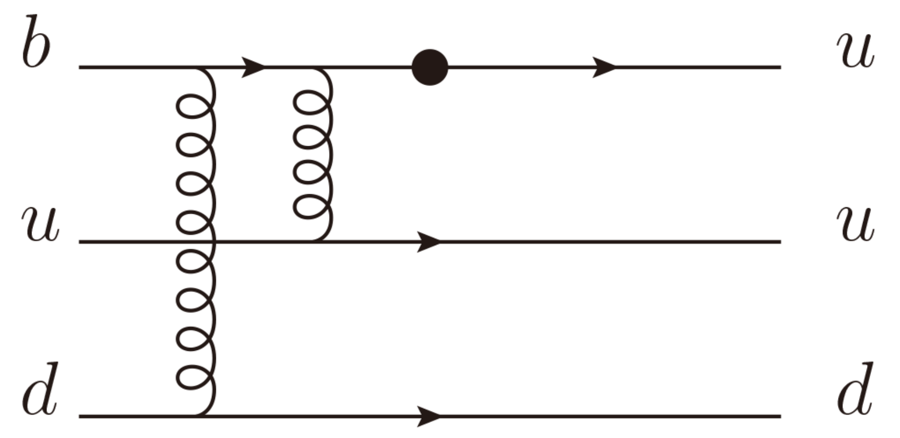
$\Lambda_b \to p$ form factors in PQCD

- In 2009, the form factors are two orders of magnitude smaller than LatticeQCD/experiments, considering only the leading twist of LCDAs of baryons. [C.D.Lu, Y.M.Wang, et al, 2009]
- In 2022, when consider contributions of high-twist LCDAs, they are consistent with LatticeQCD. [J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2022]

	Lattice/exp	PQCD(2009)	PQCD(2022)
$f_1^{\Lambda_b \to p}(0)$	0.22 ± 0.08	0.002 ± 0.001	0.27 ± 0.12

	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3^{+-}	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3^{-+}	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	$\boxed{0.25}$	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

2022, $\Lambda_b \to p$ form factor



J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, 2022

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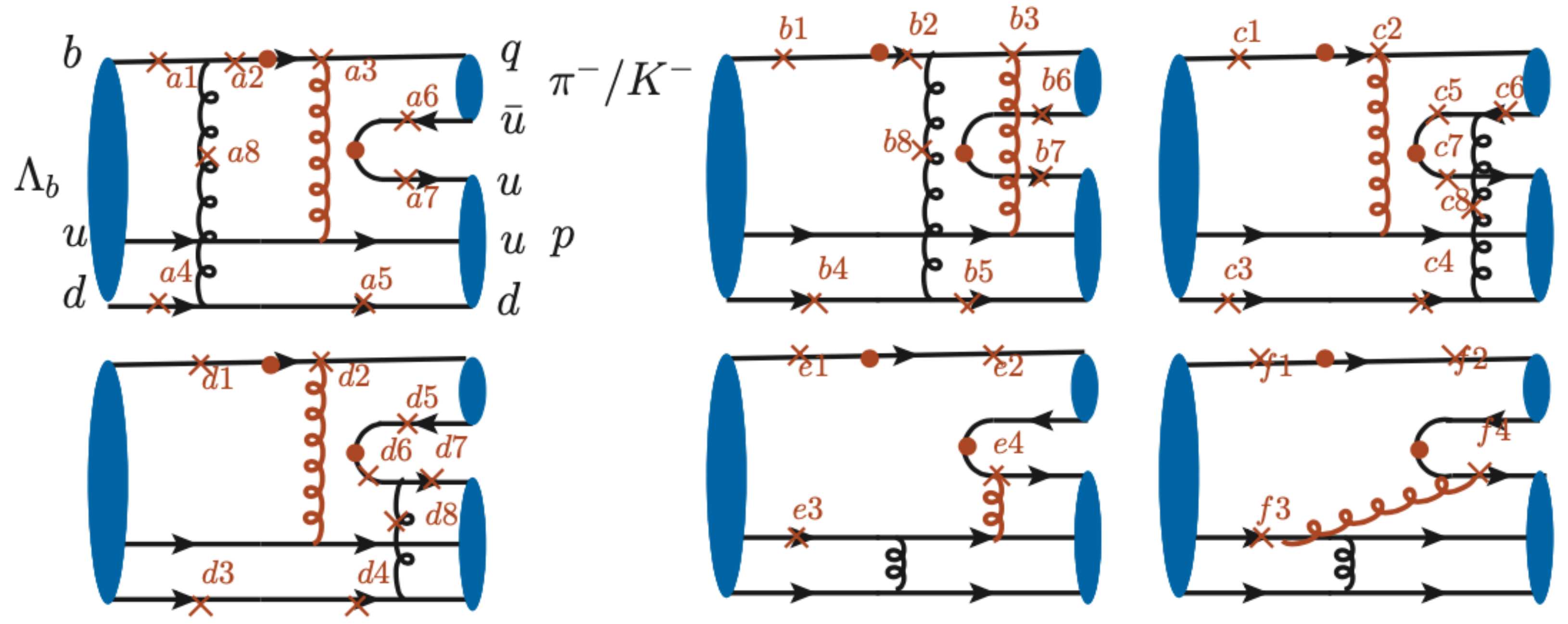
2023, $\Lambda_b \to p\pi$ non-leptonic decay

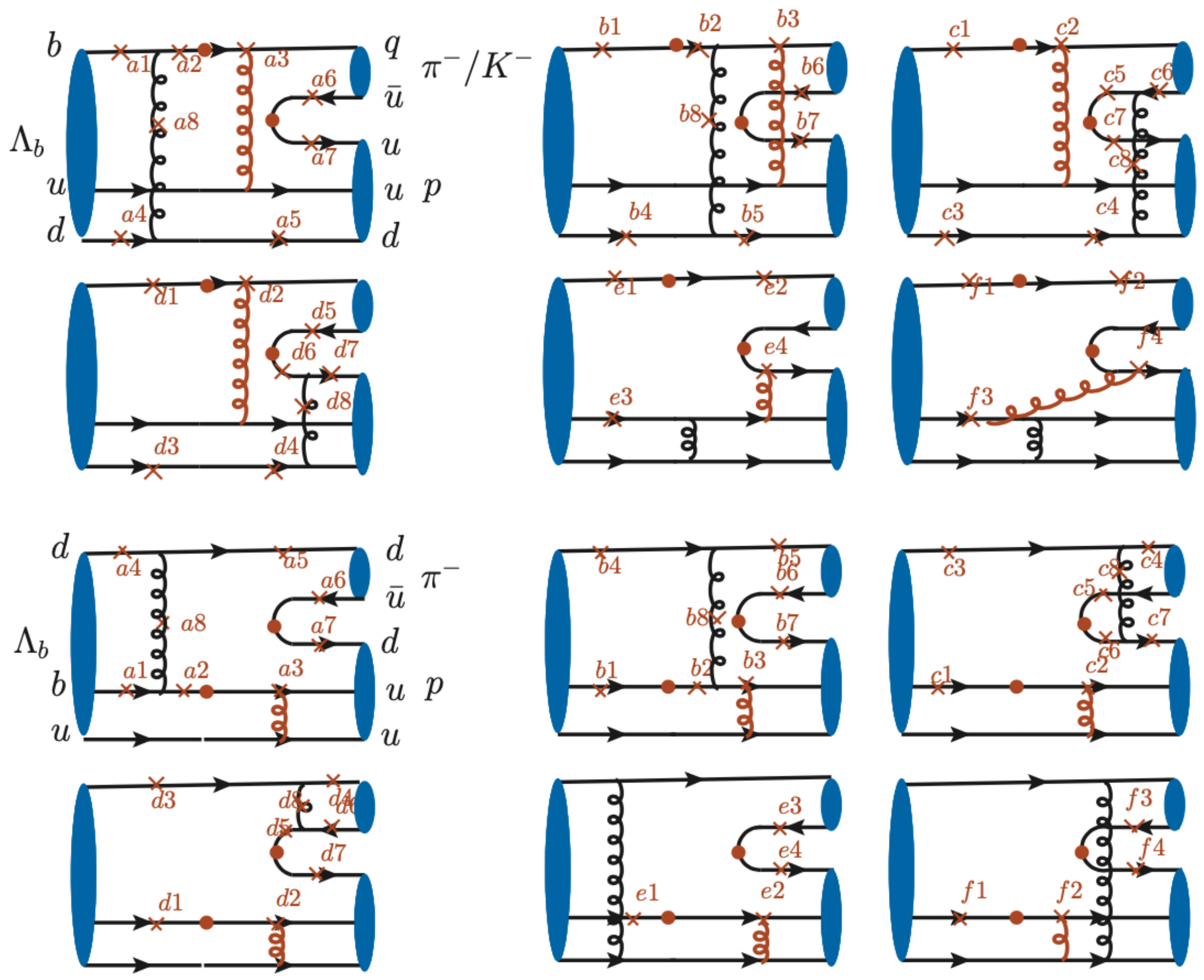


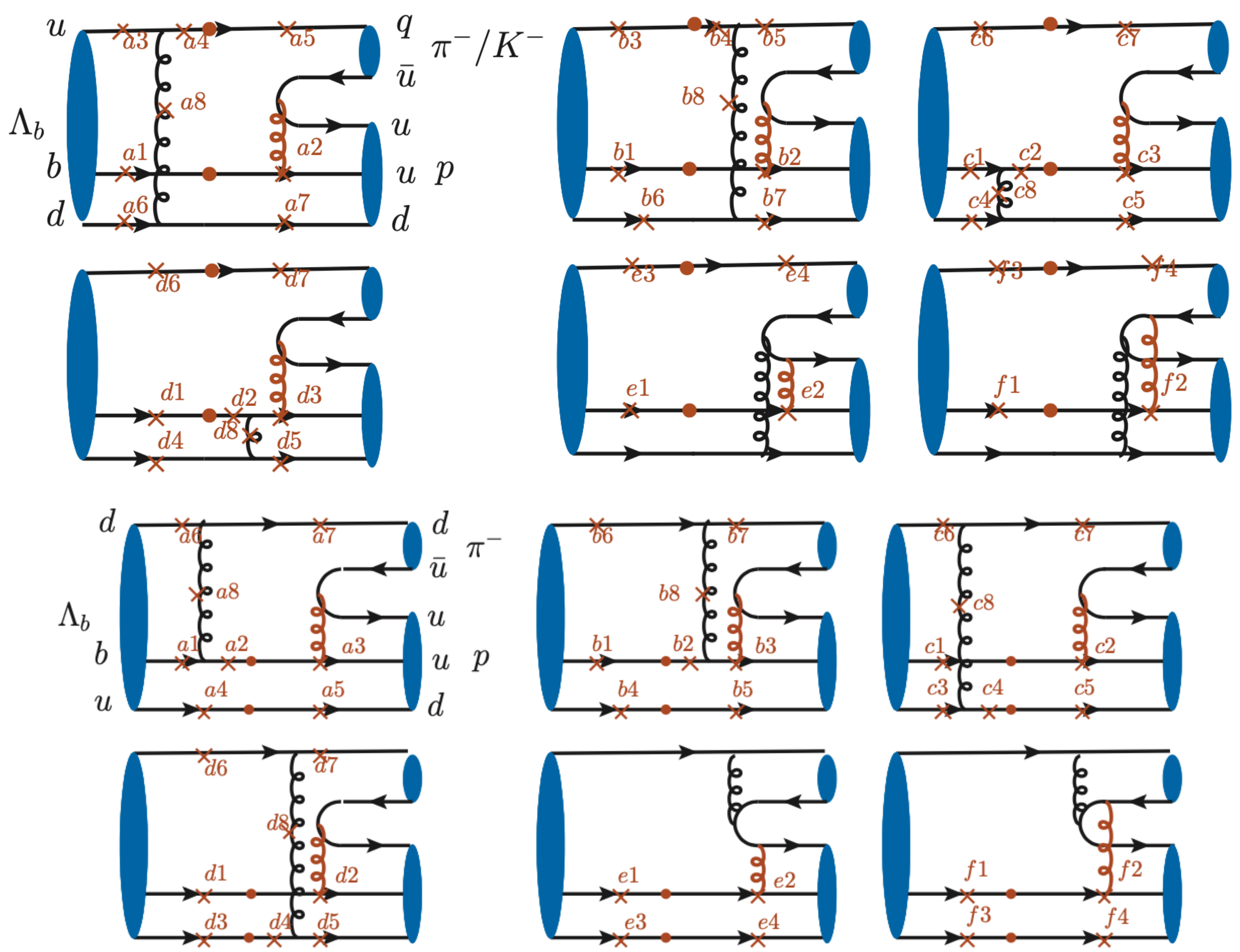
J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, in preparation

Non-leptonic decays

- $\Lambda_b \to \Lambda_c \pi, \Lambda_c K, \Lambda J/\Psi, \Lambda\phi$ are recently studied by [C.Q.Zhang, J.M.Li, M.K.Jia, Zhou Rui, 2022]
- **It can be expected that PQCD can predict CPV of b-baryons**

J.J.Han, Y.Li, H.n.Li, Y.L.Shen, Z.J.Xiao, FSY, in preparation

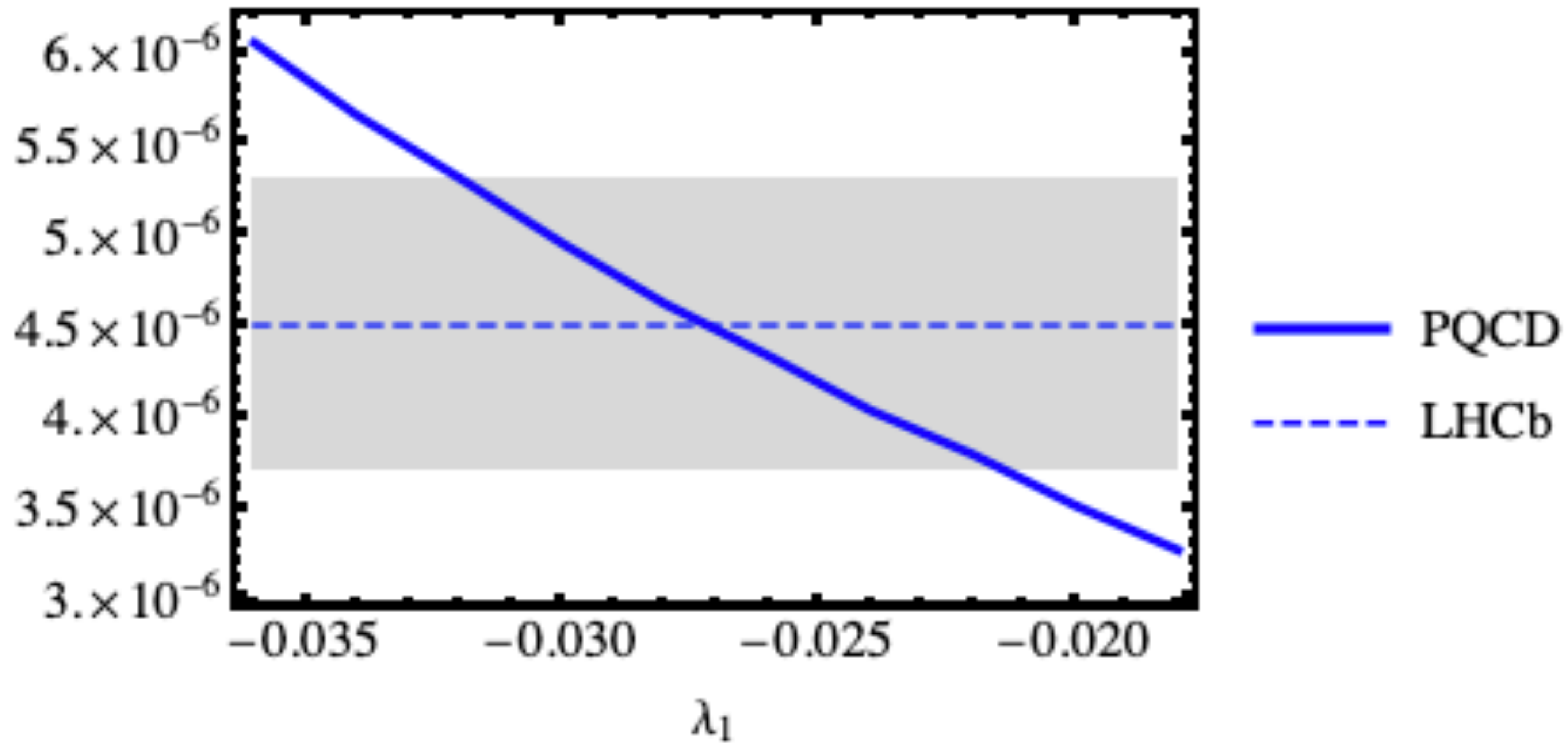




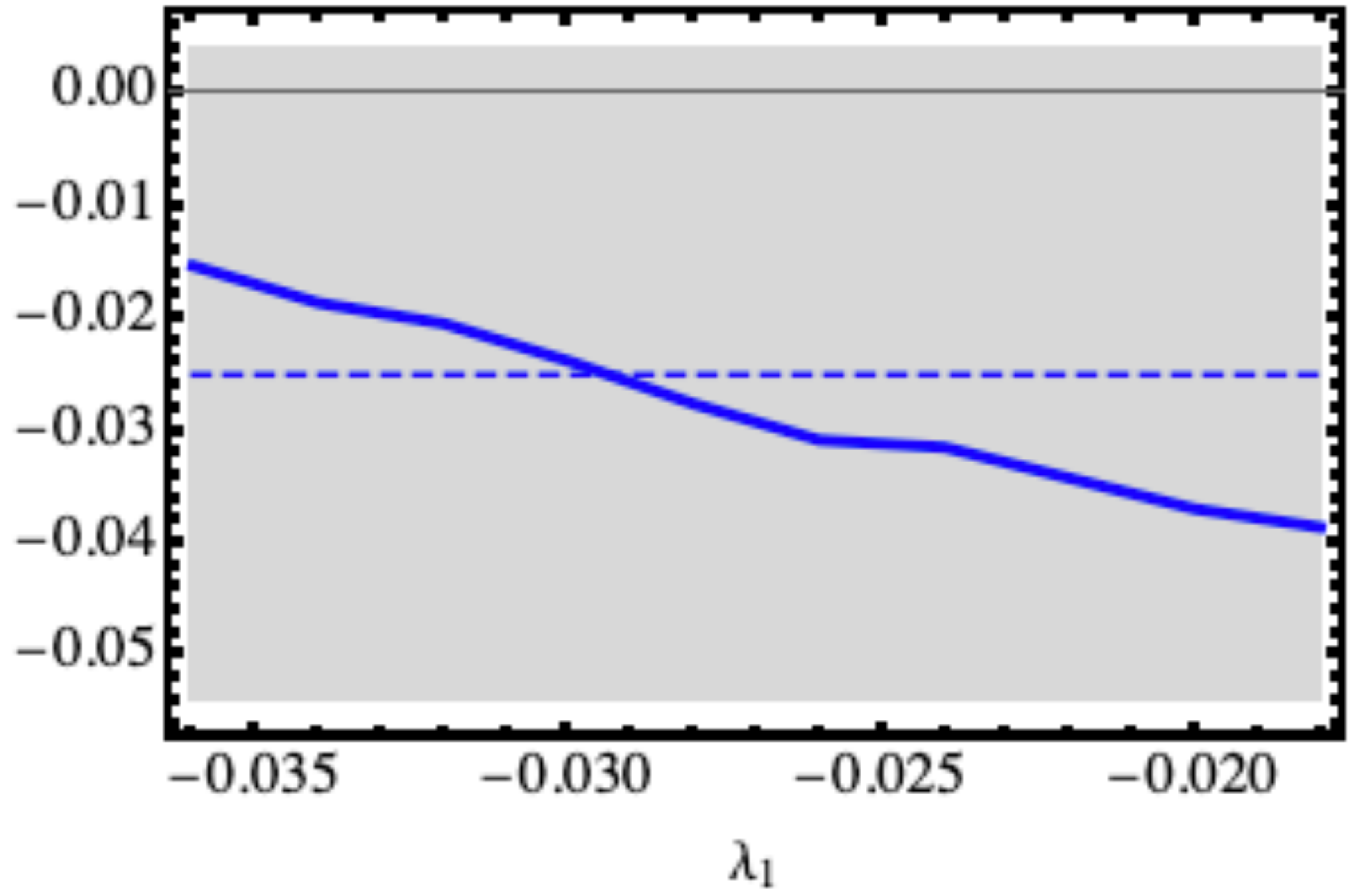
There are 200 Feynman diagrams for $\Lambda_b \to p\pi$, and 120 diagrams for $\Lambda_b \to pK$.

Branching fractions and CPV

$BR(\Lambda_b^0 \to p\pi^-)$



$A_{CP}(\Lambda_b^0 \to p\pi^-)$

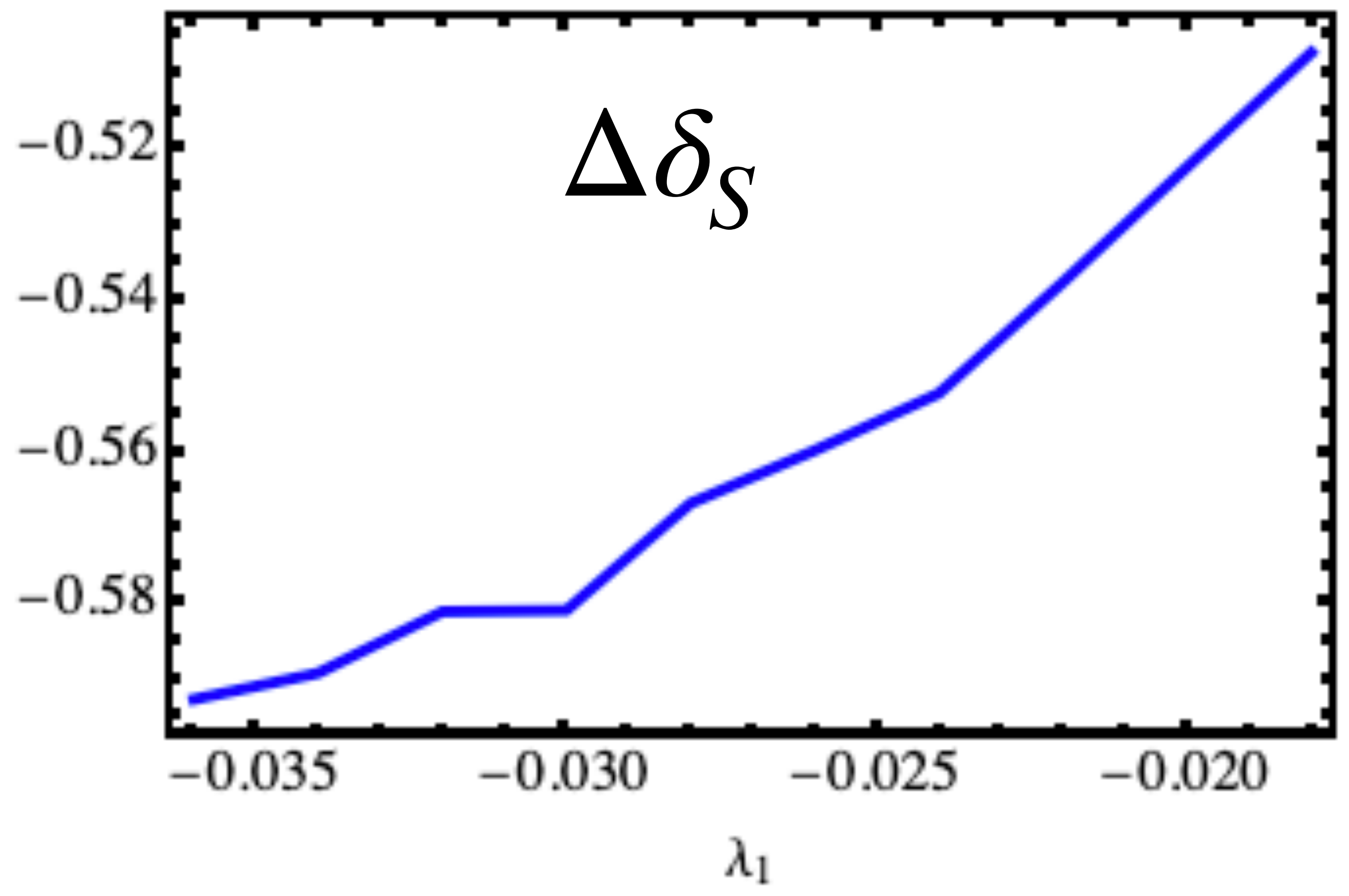


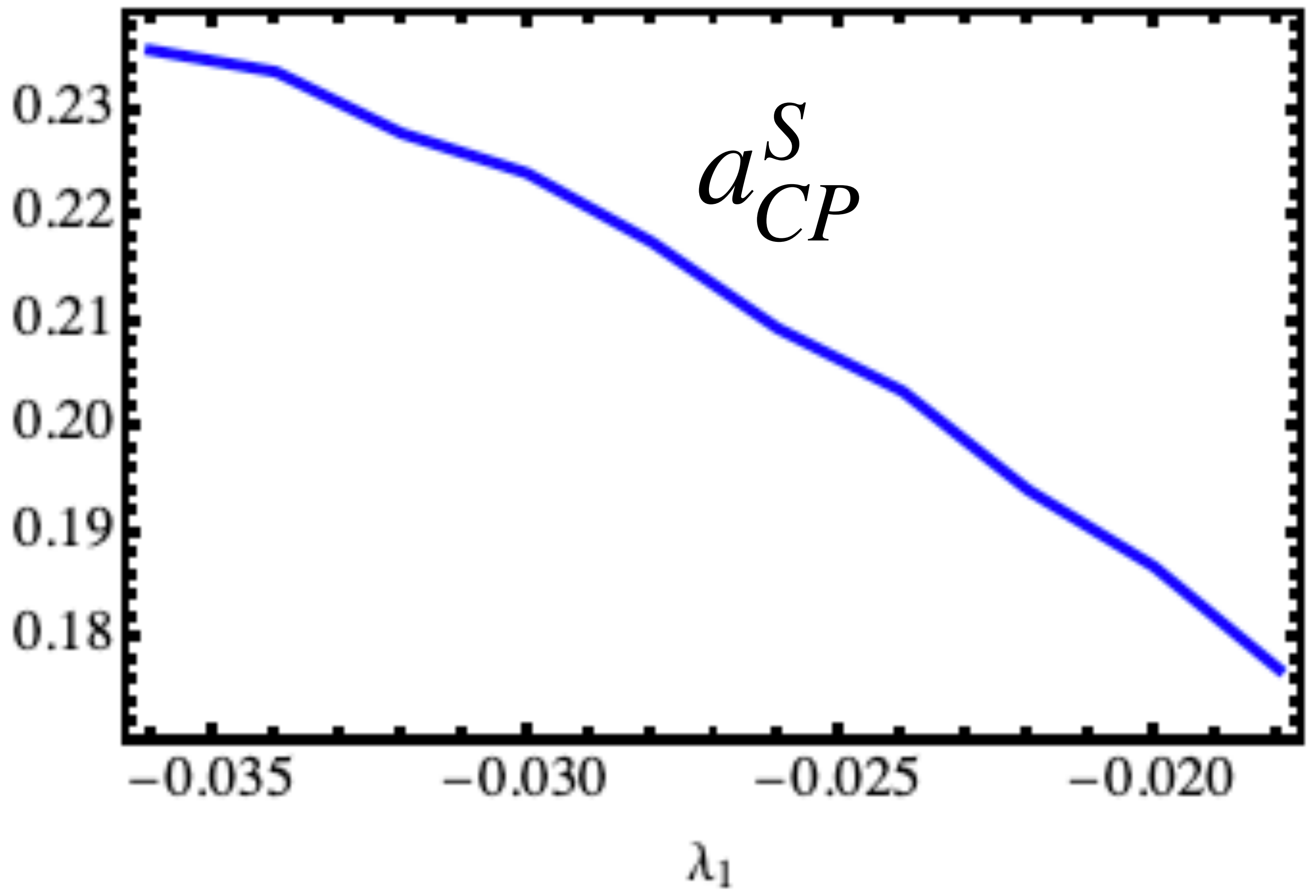
- λ_1 is one parameter in the proton LCDA. Within the allowed region of λ_1, both the branching fraction and CPV of $\Lambda_b \to p\pi$ can be understood.
- Why is CPV of $\Lambda_b \to p\pi$ so small, compared to B meson decays?

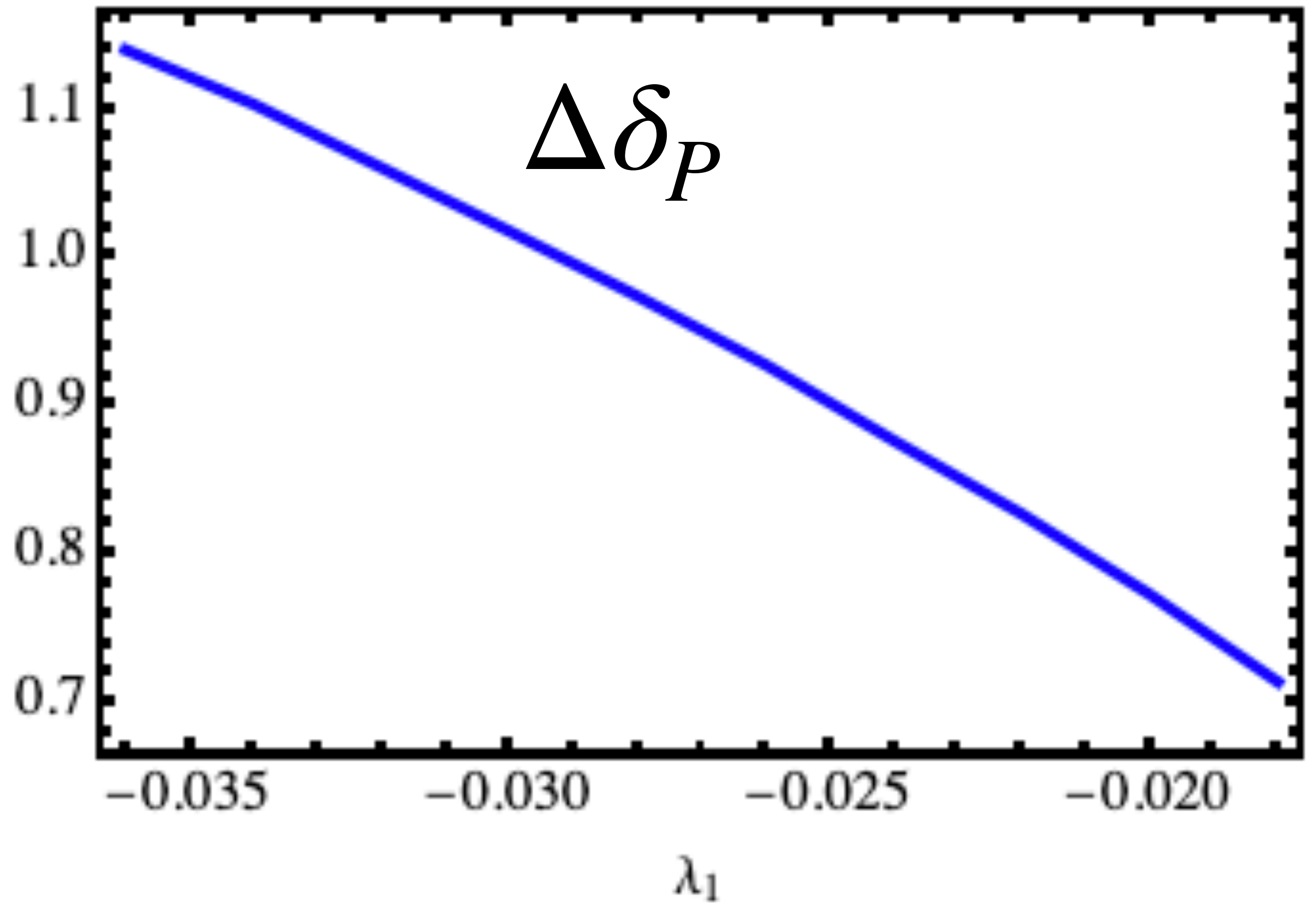
CPV are cancelled by S- and P-wave amplitudes

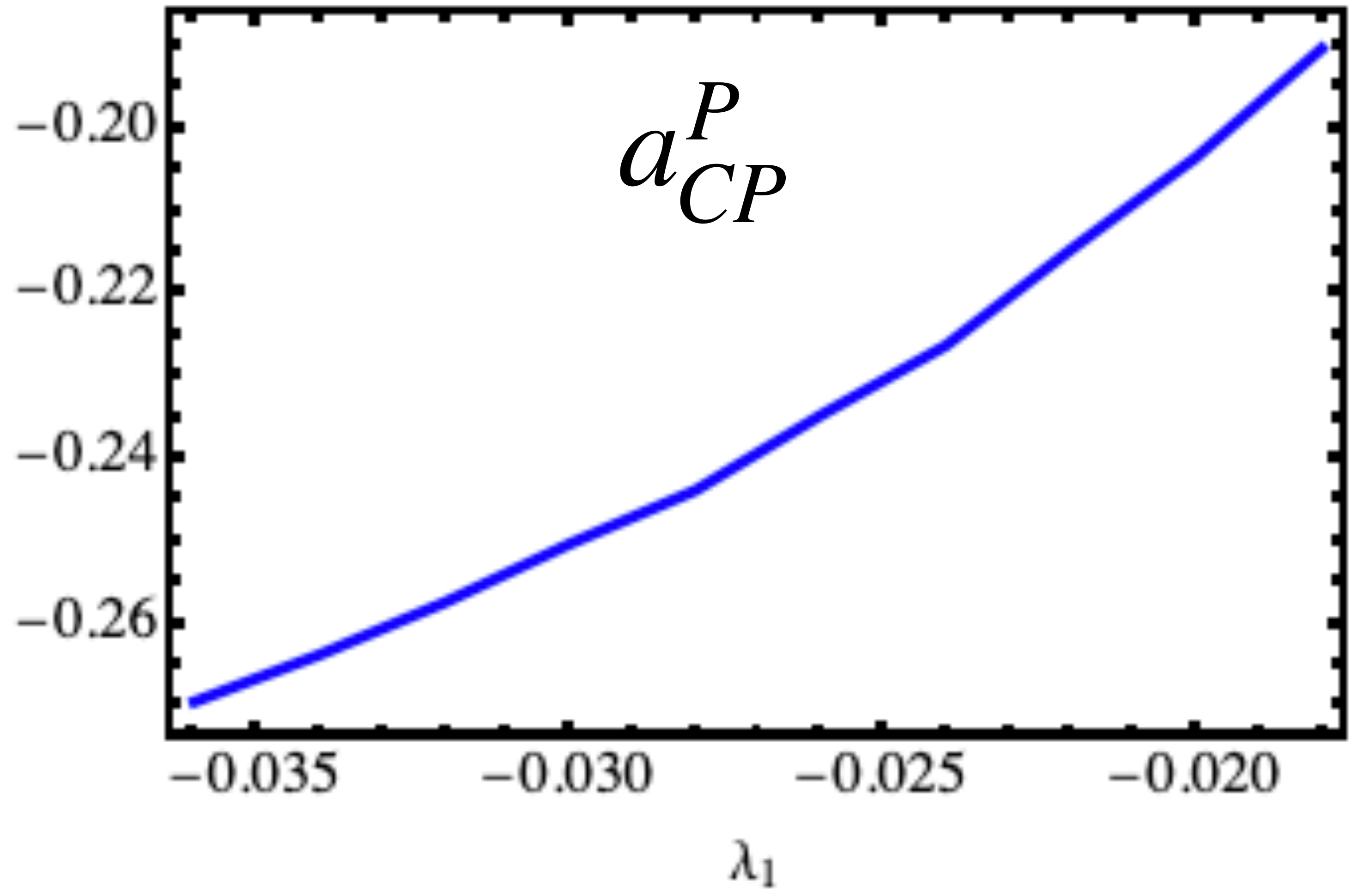
$$\mathscr{M} = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

- CPV of each of S- and P-wave contributions are as large as 20%.
- Strong phases of S- and P-wave amplitudes are opposite in sign.
- Large cancellation between S-wave and P-wave amplitudes



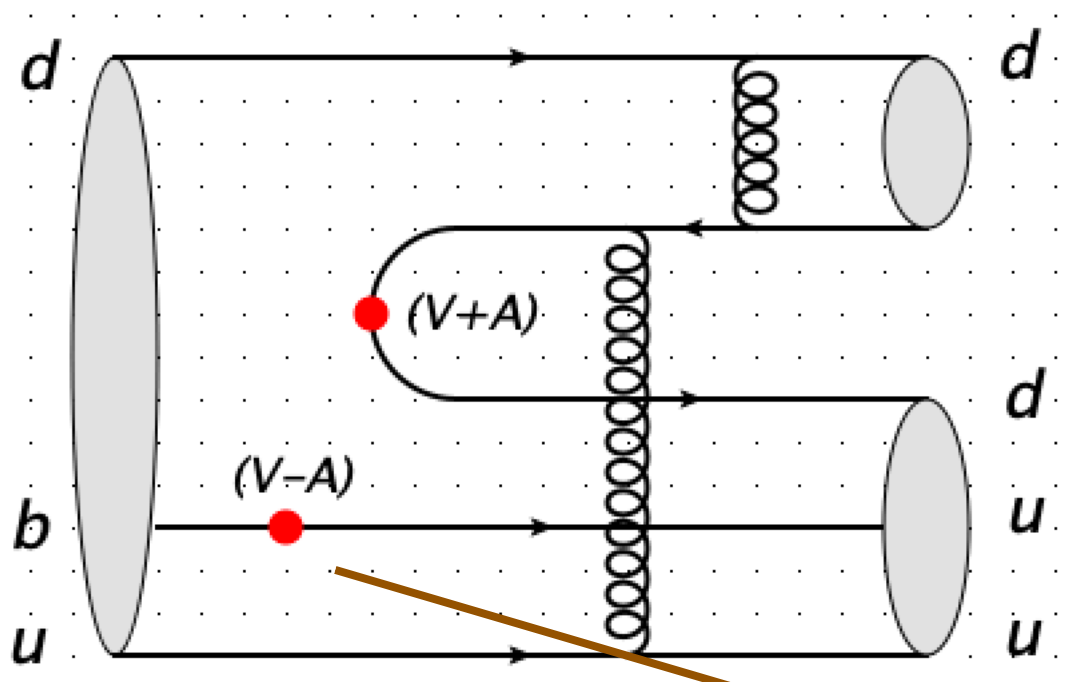


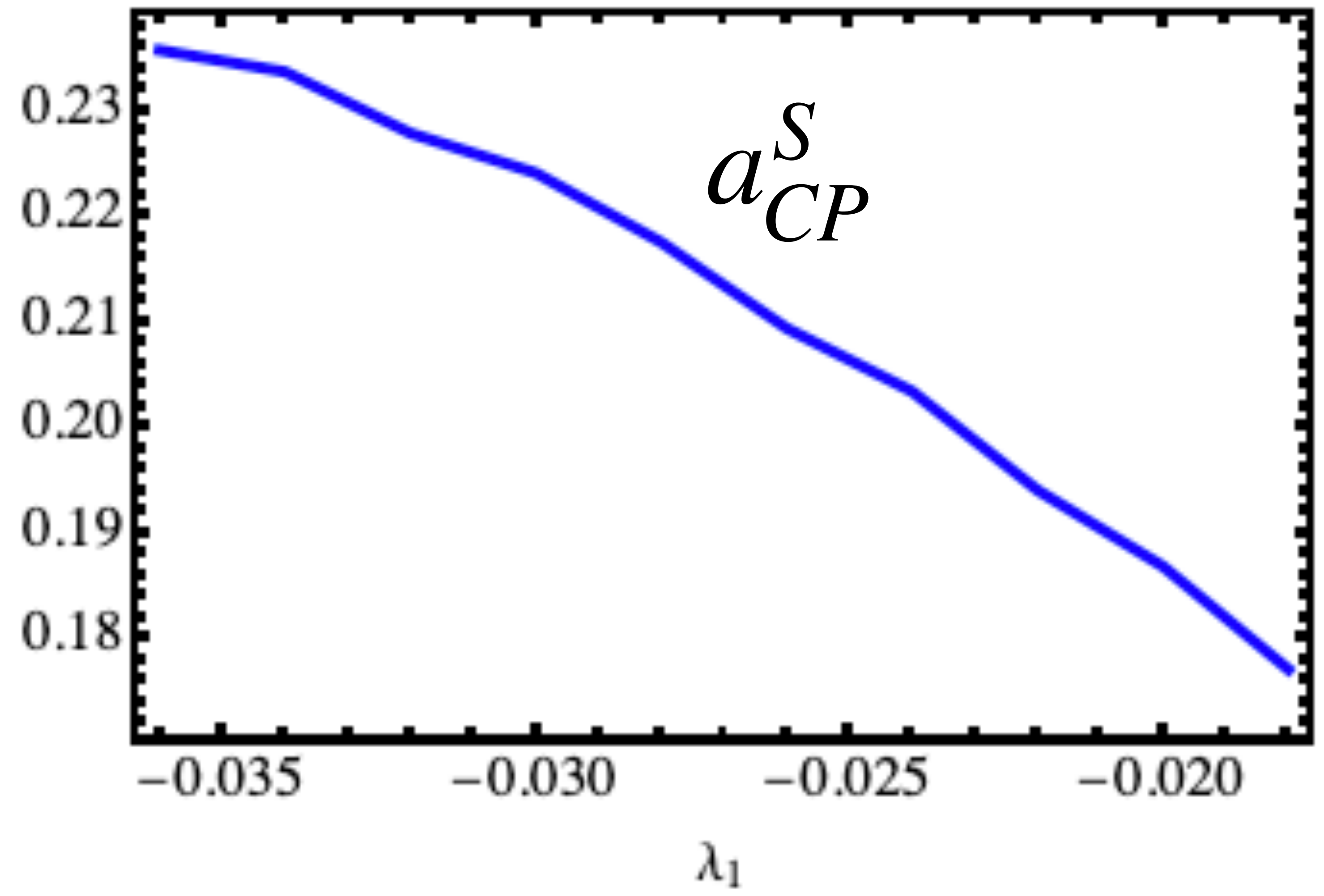




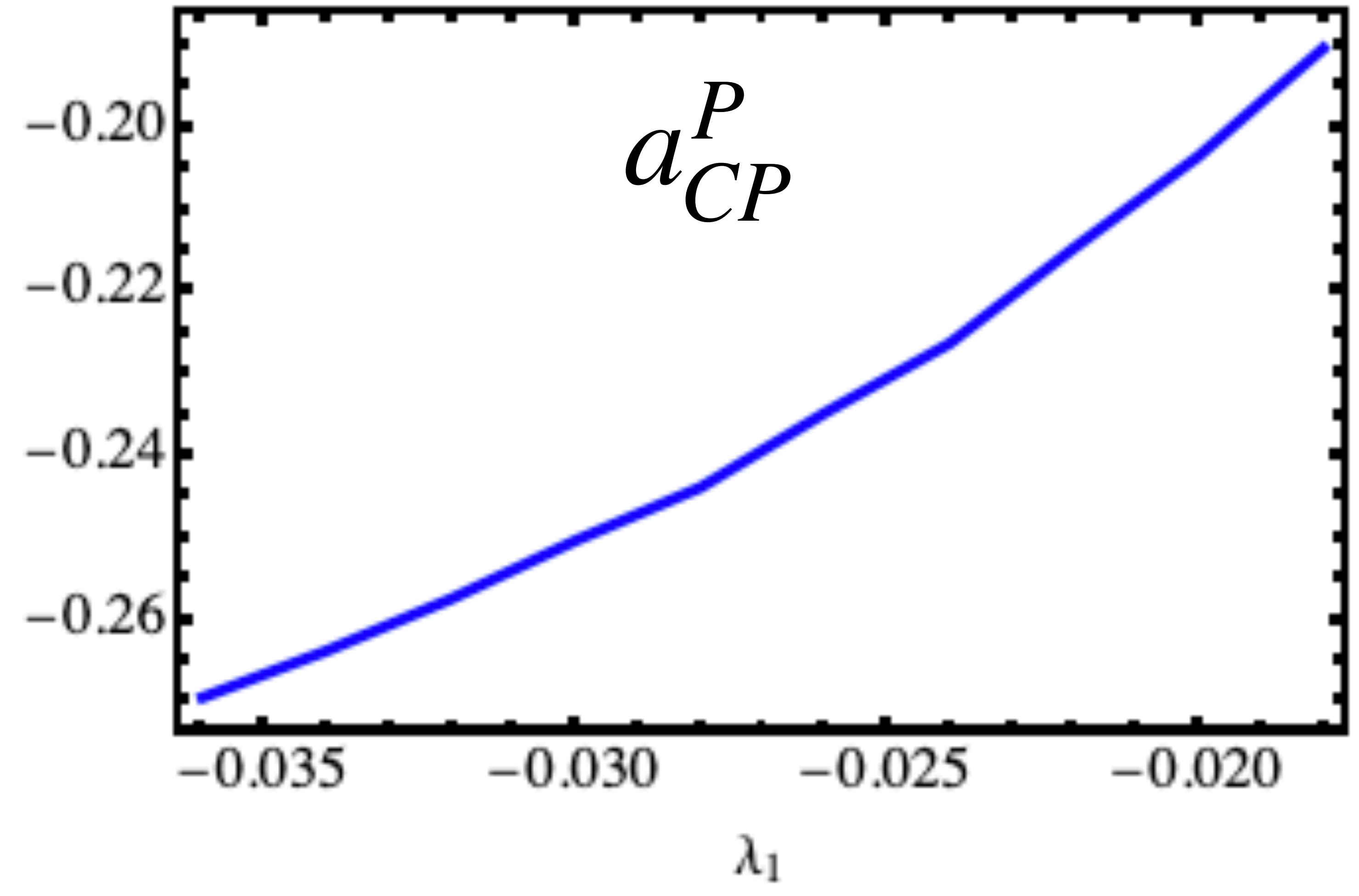
CPV are cancelled by S- and P-wave amplitudes

$$\mathcal{M} = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$





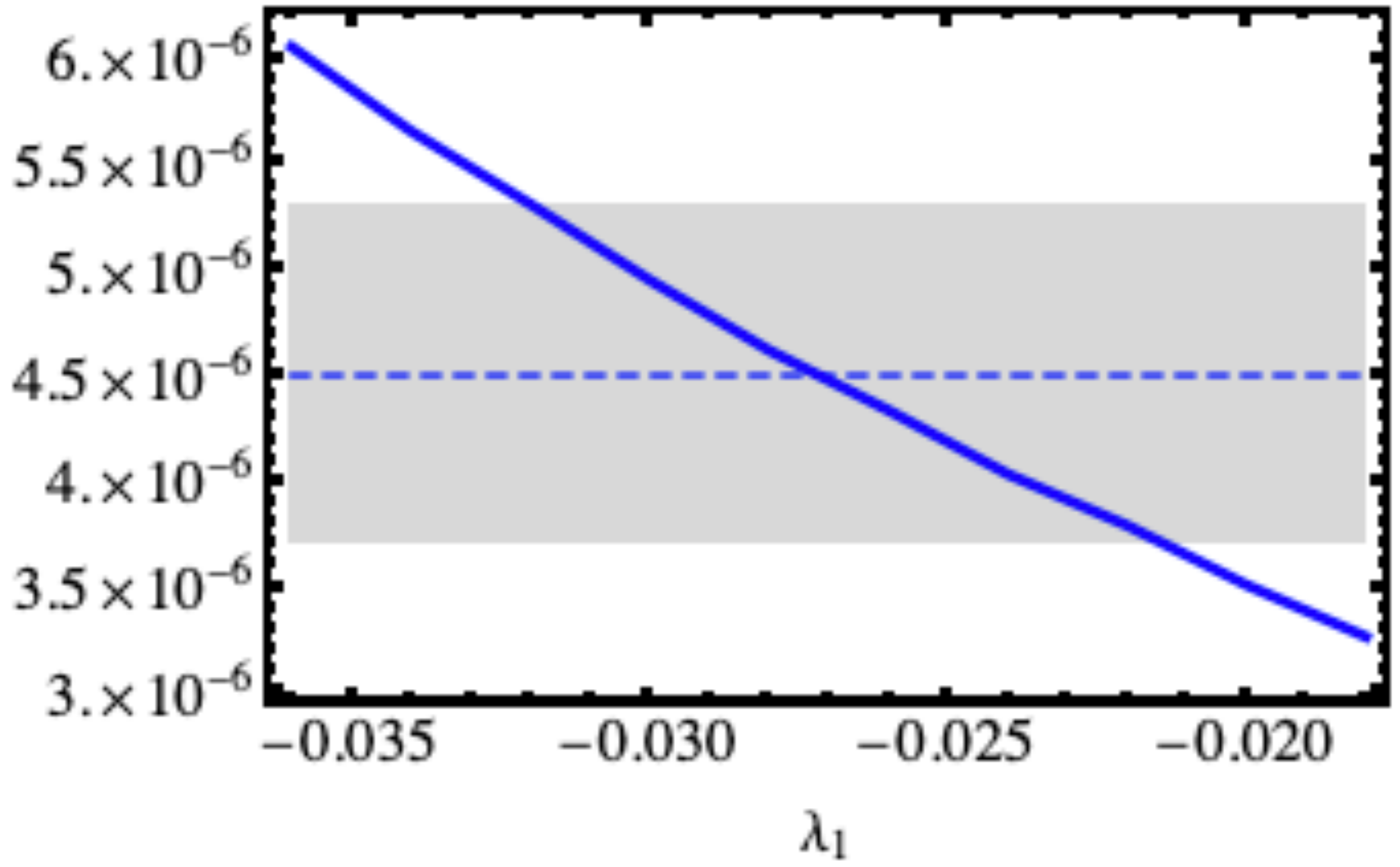
$$\gamma_5\gamma^\mu(1+\gamma_5)(\not{p} - \not{p}' + \not{k}'_2)\gamma^\nu(\not{p} - \not{p}' - \not{k}'_3)\gamma^\rho\gamma_5\not{q}\gamma_\rho\gamma_5(-\not{p} + m_b)\gamma_\nu\not{p}'\gamma_\mu(1-\gamma_5)$$



- Minus sign comes from V-A current in penguin diagram.
- Non-factorizable contributions, benefitted by PQCD.

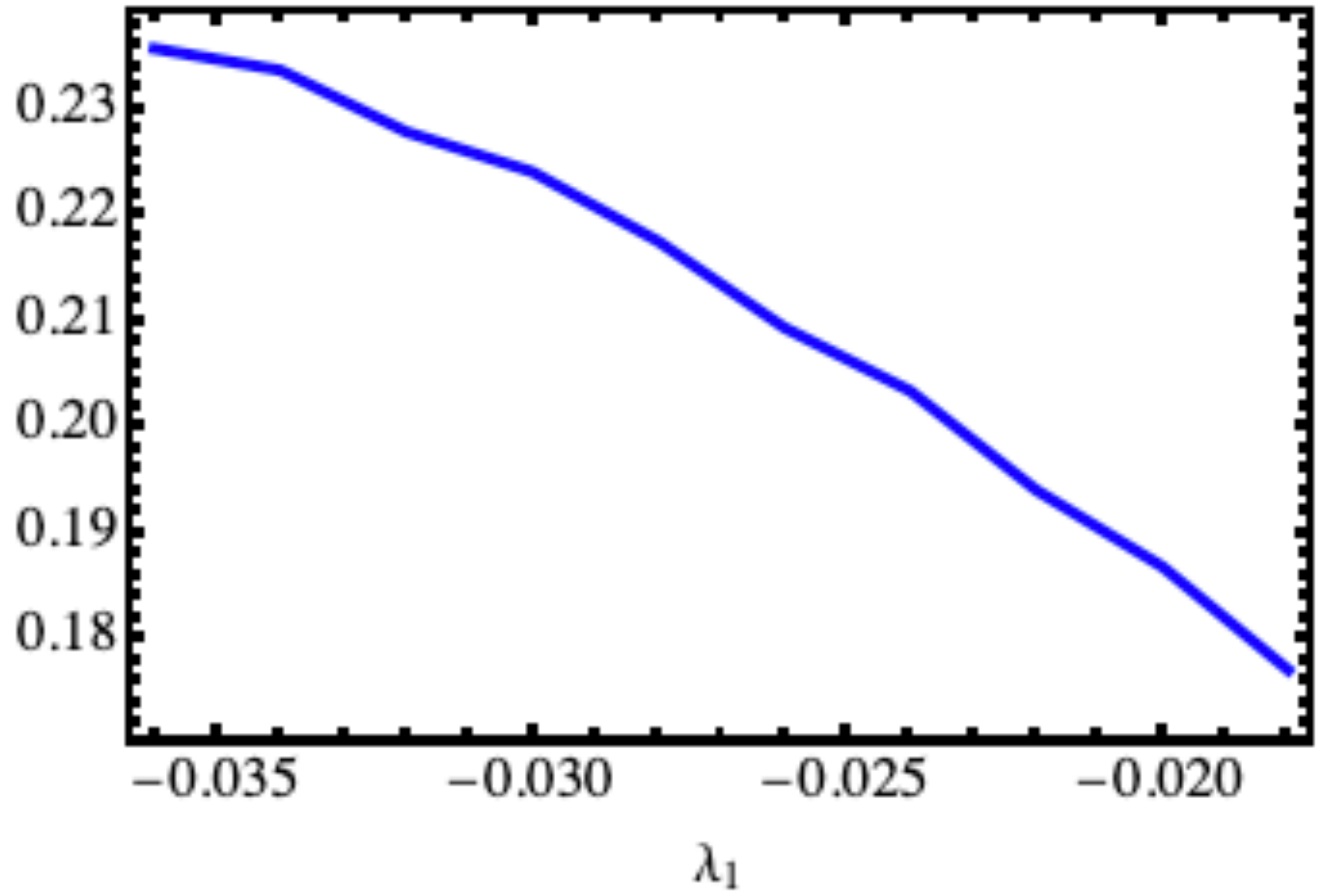
Branching fractions and CPV

$BR(\Lambda_b^0 \to p\pi^-)$



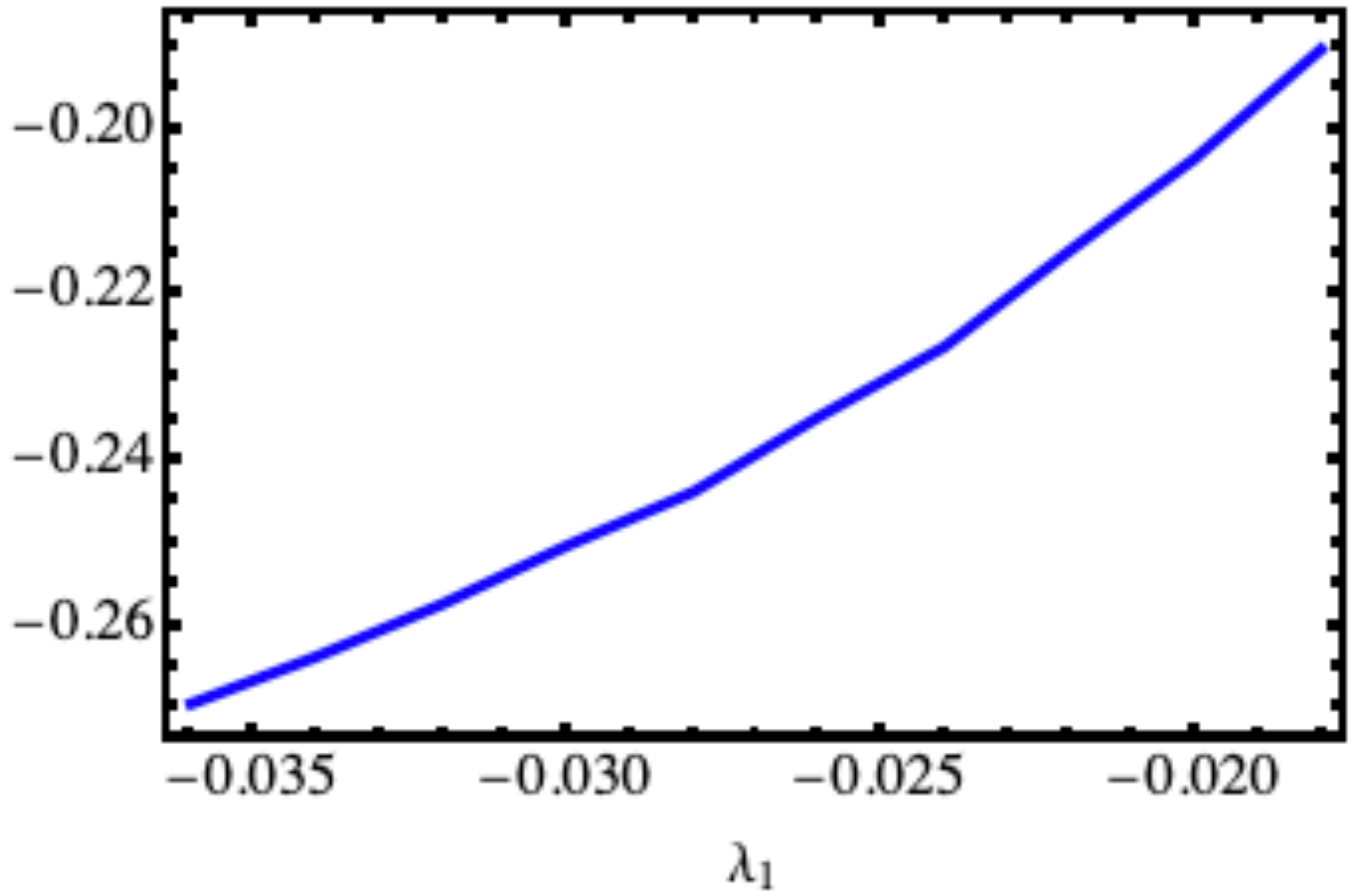
30%

a_{CP}^S



10%

a_{CP}^P

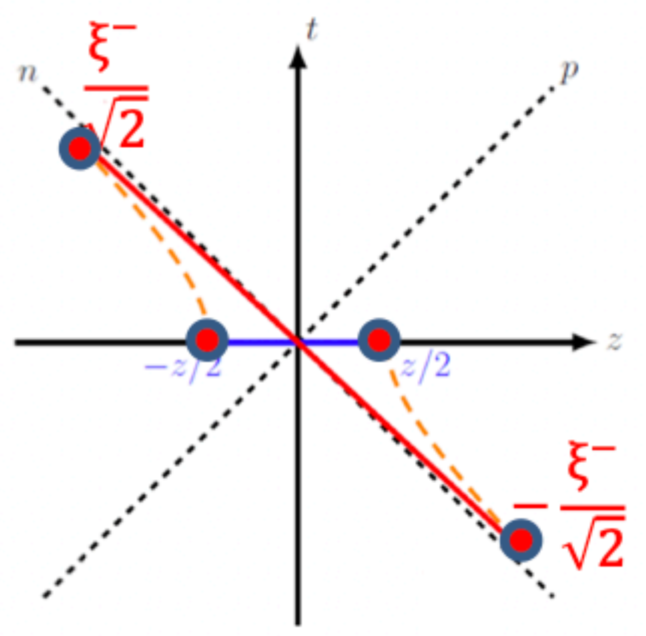


15%

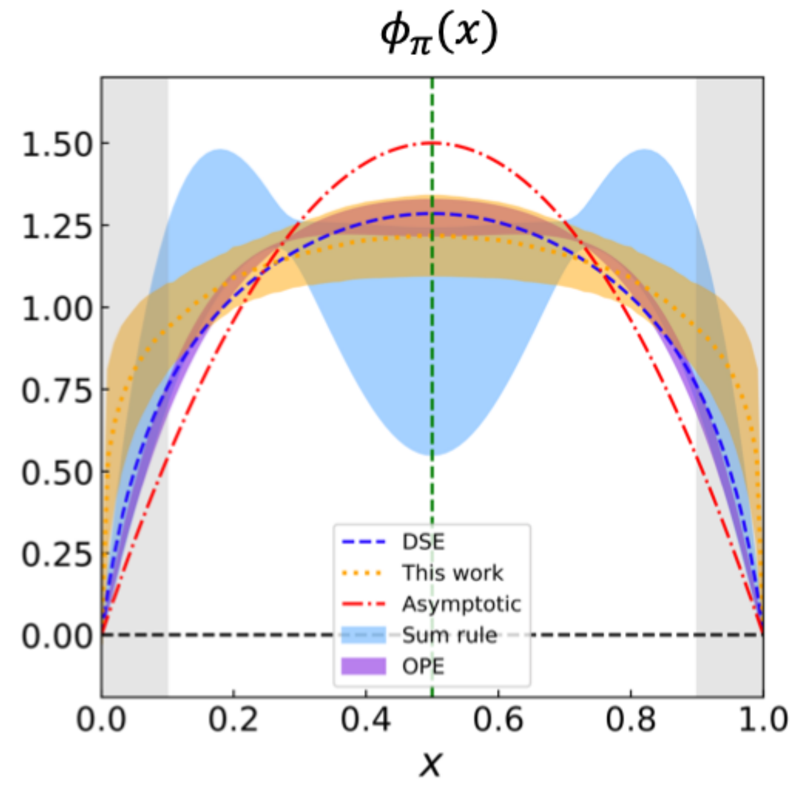
- Branching fractions are more sensitive to the parameters of LCDA, compared to CPV.
- Some of theoretical uncertainties are cancelled in CPV by the ratio.

Prospects: LCDA

- **Theoretical uncertainties are dominated by the baryon LCDAs.**
- Limited knowledge for nucleons. VERY very limited for all the others, especially for HIGH TWISTs.
- Experiments: $eN \to eN$ and $ee \to p\bar{p}, \Lambda\bar{\Lambda}$ by PQCD or light-cone sum rules
- Non-perturbative methods:
  - LaMET and Lattice QCD
  - Inverse Problem

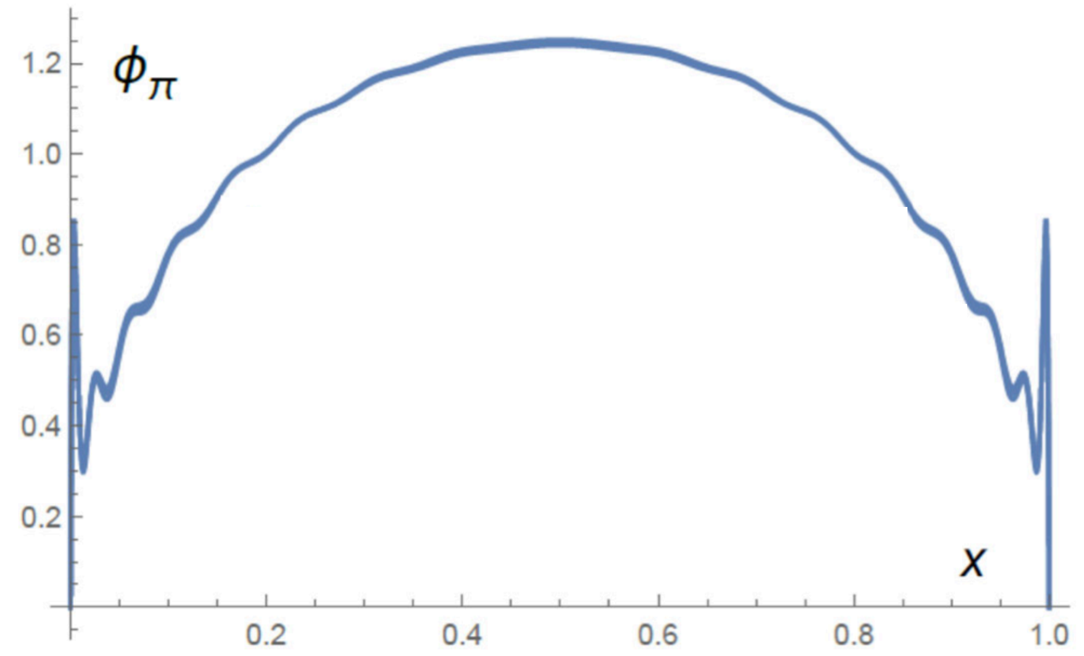


Z.F.Deng, C.Han, W.Wang, J.Zeng, J.L.Zhang, 2304.09004



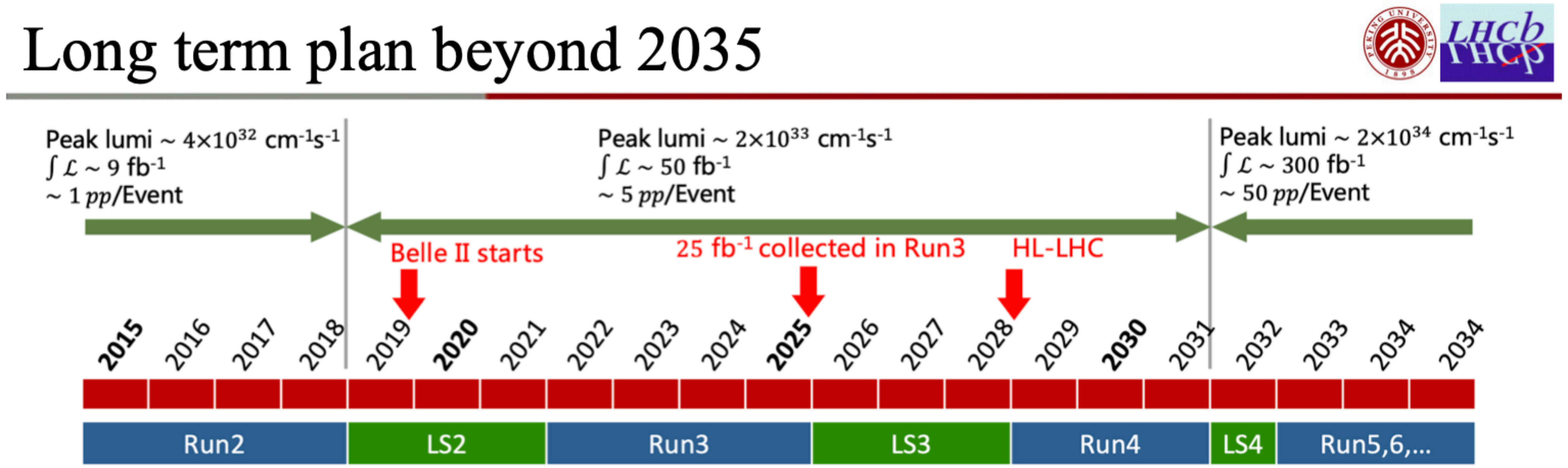
Hua, et al, 2021

$$(a_2^\pi, a_4^\pi, a_6^\pi, a_8^\pi, a_{10}^\pi, a_{12}^\pi, \cdots, a_{32}^\pi, a_{34}^\pi)|_{\mu=2\,\mathrm{GeV}}$$
$$= (0.1775^{+0.0036}_{-0.0040}, 0.0957^{+0.0011}_{-0.0012}, 0.0762^{+0.0006}_{-0.0003}, 0.0688^{+0.0016}_{-0.0012}, 0.0643^{+0.0021}_{-0.0017}, 0.0603^{+0.0024}_{-0.0019},$$
$$\cdots, 0.0089^{+0.0004}_{-0.0006}, 0.0028^{+0.0001}_{-0.0003}),$$



Summary and outlook

- Baryon physics is an opportunity of heavy flavor physics at the current stage.
- LHCb Run3 begins collecting more data.
- We are ready to predict CPV of heavy-flavor baryon decays.



Thank you very much!

Observables

$$\mathscr{M} = i\bar{u}_p(f_1 + f_2\gamma_5)u_{\Lambda_b}$$

$$f_1 = |f_1^T|e^{i\phi_1^T}e^{i\delta_1^T} + |f_1^P|e^{i\phi_1^P}e^{i\delta_1^P} \qquad f_2 = |f_2^T|e^{i\phi_2^T}e^{i\delta_2^T} + |f_2^P|e^{i\phi_2^P}e^{i\delta_2^P}$$

$$A_{CP}^{dir}(\Lambda_b \to pM) \equiv \frac{\mathcal{B}r(\Lambda_b \to pM) - \mathcal{B}r(\bar{\Lambda}_b \to \bar{p}\bar{M})}{\mathcal{B}r(\Lambda_b \to pM) + \mathcal{B}r(\bar{\Lambda}_b \to \bar{p}\bar{M})}$$

$$A_{CP}^{dir} = \frac{-2A|f_1^T|^2 r_1 sin\Delta\phi_1 sin\Delta\delta_1 - 2B|f_2^T|^2 r_2 sin\Delta\phi_2 sin\Delta\delta_2}{A|f_1^T|^2(1 + r_1^2 + 2r_1 cos\Delta\phi_1 cos\Delta\delta_1) + B|f_2^T|^2(1 + r_2^2 + 2r_2 cos\Delta\phi_2 cos\Delta\delta_2)}$$

$$A = \frac{(M_{\Lambda_b} + M_p)^2 - M_M^2}{M_{\Lambda_b}^2} \qquad B = \frac{(M_{\Lambda_b} - M_p)^2 - M_M^2}{M_{\Lambda_b}^2}$$

$$A_{CP}^{dir}(f_1) = \frac{-2r_1 sin\Delta\phi_1 sin\Delta\delta_1}{(1 + r_1^2 + 2r_1 cos\Delta\phi_1 cos\Delta\delta_1)} \qquad A_{CP}^{dir}(f_2) = \frac{-2r_2 sin\Delta\phi_2 sin\Delta\delta_2}{(1 + r_2^2 + 2r_2 cos\Delta\phi_2 cos\Delta\delta_2)}$$

Light-Cone Distribution Amplitudes: Λ_b

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{1}{8\sqrt{2}N_c}\left\{f^{(1)}_{\Lambda_b}(\mu)[M_1(x_2,x_3)\gamma_5 C^T]_{\gamma\beta} + f^{(2)}_{\Lambda_b}(\mu)[M_2(x_2,x_3)\gamma_5 C^T]_{\gamma\beta}\right\}[\Lambda_b(p)]_\alpha$$

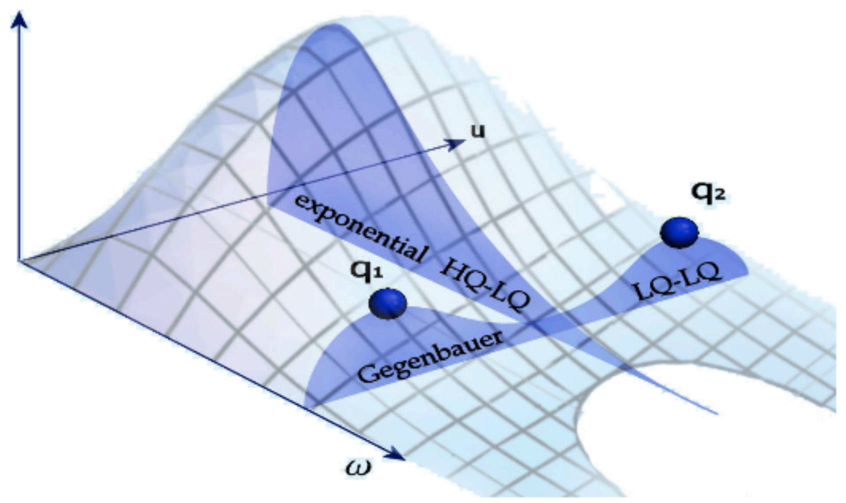
$$M_1(x_2,x_3) = \frac{\bar{\not n}\not n}{4}\psi_3^{+-}(x_2,x_3) + \frac{\not n\bar{\not n}}{4}\psi_3^{-+}(x_2,x_3),$$

$$M_2(x_2,x_3) = \frac{\not n}{\sqrt{2}}\psi_2(x_2,x_3) + \frac{\bar{\not n}}{\sqrt{2}}\psi_4(x_2,x_3),$$

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{f'_{\Lambda_b}}{8\sqrt{2}N_c}[(\not p + m_{\Lambda_b})\gamma_5 C]_{\beta\gamma}[\Lambda_b(p)]_\alpha \psi(x_i,\mu),$$

$$\psi(x_i) = N x_1 x_2 x_3\, exp\left(-\frac{m^2_{\Lambda_b}}{2\beta^2 x_1} - \frac{m_l^2}{2\beta^2 x_2} - \frac{m_l^2}{2\beta^2 x_3}\right),$$

Light-Cone Distribution Amplitudes: Λ_b



Model-I: Gegenbauer-1

$$\psi_2(x_2,x_3) = m_{\Lambda_b}^4 x_2 x_3 \left[\frac{1}{\epsilon_0^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_0} + a_2 C_2^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right)\frac{1}{\epsilon_1^4} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_1}\right]$$

$$\psi_3^{+-}(x_2,x_3) = \frac{2m_{\Lambda_b}^3 x_2}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3},$$

$$\psi_3^{-+}(x_2,x_3) = \frac{2m_{\Lambda_b}^3 x_3}{\epsilon_3^3} e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_3},$$

$$\psi_4(x_2,x_3) = \frac{5}{\mathcal{N}} m_{\Lambda_b}^2 \int_{m_{\Lambda_b}(x_2+x_3)/2}^{s_0} ds e^{-s/\tau}(s - m_{\Lambda_b}(x_2+x_3)/2)^3,$$

with the Gegenbauer moment $a_2 = 0.333^{0.250}_{-0.333}$, the Gegenbauer polynomial $C_2^{3/2}(x) = 3(5x^2-1)/2$, the parameters $\epsilon_0 = 200^{+130}_{-60}$ MeV, $\epsilon_1 = 650^{+650}_{-300}$ MeV and $\epsilon_3 = 230 \pm 60$

Ball, Braun, Gardi, 0804.2424, PLB 2008

Model-II: Gegenbauer-2

$$\psi_2(x_2,x_3) = m_{\Lambda_b}^4 x_2 x_3 \frac{a_2^{(2)}}{{\epsilon_2^{(2)}}^4} C_2^{3/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}/(x_2+x_3)/\epsilon_2^{(2)}},$$

$$\psi_3^{+-}(x_2,x_3) = m_{\Lambda_b}^3 (x_2+x_3)\left[\frac{a_2^{(3)}}{{\epsilon_2^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(3)}} + \frac{b_3^{(3)}}{{\eta_3^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\eta_3^{(3)}}\right]$$

$$\psi_3^{-+}(x_2,x_3) = m_{\Lambda_b}^3 (x_2+x_3)\left[\frac{a_2^{(3)}}{{\epsilon_2^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(3)}} - \frac{b_3^{(3)}}{{\eta_3^{(3)}}^3} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\eta_3^{(3)}}\right]$$

$$\psi_4(x2,x3) = m_{\Lambda_b}^2 \frac{a_2^{(4)}}{{\epsilon_2^{(4)}}^2} C_2^{1/2}\left(\frac{x_2-x_3}{x_2+x_3}\right) e^{-m_{\Lambda_b}(x_2+x_3)/\epsilon_2^{(4)}},$$

$a_2^{(2)} = 0.391 \pm 0.279$, $a_2^{(3)} = -0.161^{+0.108}_{-0.207}$, $a_2^{(4)} = -0.541^{+0.173}_{-0.09}$, $b_3^{(3)} = -0.24^{+0.24}_{-0.147}$, $\epsilon_2^{(2)} = 0.551^{+\infty}_{-0.356}$ GeV, $\epsilon_2^{(3)} = 0.055^{+0.01}_{-0.02}$ GeV, $\epsilon_2^{(4)} = 0.262^{+0.116}_{-0.132}$ GeV and $\eta_3^{(3)} = 0.633 \pm 0.099$ GeV.

Ali, Hambrock, Parkhomenko, W.Wang, 2012

Light-Cone Distribution Amplitudes: Λ_b

Model-III: Exponential

$$\begin{aligned}
\psi_2(x_2,x_3) &= \frac{x_2 x_3}{\omega_0^4} m_{\Lambda_b}^4 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0}, \\
\psi_3^{+-}(x_2,x_3) &= \frac{2x_2}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0}, \\
\psi_3^{-+}(x_2,x_3) &= \frac{2x_3}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0}, \\
\psi_4(x_2,x_3) &= \frac{1}{\omega_0^2} m_{\Lambda_b}^2 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},
\end{aligned}$$

$\omega_0 = 0.4\ \mathrm{GeV}$

Model-IV: Free Parton

$$\begin{aligned}
\psi_2(x_2,x_3) &= \frac{15 x_2 x_3 m_{\Lambda_b}^4 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})}{4\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}) \\
\psi_3^{+-}(x_2,x_3) &= \frac{15 x_2 m_{\Lambda_b}^3 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^2}{4\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}), \\
\psi_3^{-+}(x_2,x_3) &= \frac{15 x_3 m_{\Lambda_b}^3 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^2}{4\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}), \\
\psi_4(x_2,x_3) &= \frac{5 m_{\Lambda_b}^2 (2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b})^3}{8\bar{\Lambda}^5} \Theta(2\bar{\Lambda} - x_2 m_{\Lambda_b} - x_3 m_{\Lambda_b}),
\end{aligned}$$

$\bar{\Lambda} \equiv (m_{\Lambda_b} - m_b)/2 \approx 0.8\ \mathrm{GeV}$

Bell, Feldmann, Y.M.Wang, Yip, 1308.6114, JHEP2013

Light-Cone Distribution Amplitudes: proton

$$\langle 0|\, \varepsilon^{ijk} u^{i'}_\alpha(a_1 z)\, [a_1 z, a_0 z]_{i',i}\, u^{j'}_\beta(a_2 z)\, [a_2 z, a_0 z]_{j',j}\, d^{k'}_\gamma(a_3 z)\, [a_3 z, a_0 z]_{k',k}\, |P(P,\lambda)\rangle$$

$$\begin{aligned}
&4\,\langle 0|\, \varepsilon^{ijk} u^i_\alpha(a_1 z) u^j_\beta(a_2 z) d^k_\gamma(a_3 z)\,|P\rangle = \\
&= S_1 M C_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + S_2 M C_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + P_1 M\,(\gamma_5 C)_{\alpha\beta}\, N^+_\gamma + P_2 M\,(\gamma_5 C)_{\alpha\beta}\, N^-_\gamma \\
&\quad + V_1\,(\not{p} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + V_2\,(\not{p} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + \frac{V_3}{2} M\,(\gamma_\perp C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma \\
&\quad + \frac{V_4}{2} M\,(\gamma_\perp C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma + V_5 \frac{M^2}{2pz}\,(\not{z} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma + \frac{M^2}{2pz} V_6\,(\not{z} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma \\
&\quad + A_1\,(\not{p}\gamma_5 C)_{\alpha\beta}\, N^+_\gamma + A_2\,(\not{p}\gamma_5 C)_{\alpha\beta}\, N^-_\gamma + \frac{A_3}{2} M\,(\gamma_\perp \gamma_5 C)_{\alpha\beta}\left(\gamma^\perp N^+\right)_\gamma \\
&\quad + \frac{A_4}{2} M\,(\gamma_\perp \gamma_5 C)_{\alpha\beta}\left(\gamma^\perp N^-\right)_\gamma + A_5 \frac{M^2}{2pz}\,(\not{z}\gamma_5 C)_{\alpha\beta}\, N^+_\gamma + \frac{M^2}{2pz} A_6\,(\not{z}\gamma_5 C)_{\alpha\beta}\, N^-_\gamma \\
&\quad + T_1\,(i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma + T_2\,(i\sigma_{\perp p} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma + T_3 \frac{M}{pz}\,(i\sigma_{pz} C)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma \\
&\quad + T_4 \frac{M}{pz}\,(i\sigma_{zp} C)_{\alpha\beta}\left(\gamma_5 N^-\right)_\gamma + T_5 \frac{M^2}{2pz}\,(i\sigma_{\perp z} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^+\right)_\gamma + \frac{M^2}{2pz} T_6\,(i\sigma_{\perp z} C)_{\alpha\beta}\left(\gamma^\perp \gamma_5 N^-\right)_\gamma \\
&\quad + M\frac{T_7}{2}\,(\sigma_{\perp\perp'} C)_{\alpha\beta}\left(\sigma^{\perp\perp'} \gamma_5 N^+\right)_\gamma + M\frac{T_8}{2}\,(\sigma_{\perp\perp'} C)_{\alpha\beta}\left(\sigma^{\perp\perp'} \gamma_5 N^-\right)_\gamma\,, \qquad (2.9)
\end{aligned}$$

Braun, Fries, Mahnke, Stein, hep-ph/0007279, NPB 2000

Light-Cone Distribution Amplitudes: proton

- Twist-3 LCDAs

$$
\begin{aligned}
V_1(x_i) =& 120x_1x_2x_3[\phi_3^0 + \phi_3^+(1-3x_3)],\\
A_1(x_i) =& 120x_1x_2x_3(x_2-x_1)\phi_3^-,\\
T_1(x_i) =& 120x_1x_2x_3[\phi_3^0 + \frac{1}{2}(\phi_3^- - \phi_3^+)(1-3x_3)].
\end{aligned}
$$

- Twist-4 LCDAs

$$
\begin{aligned}
V_2(x_i) =& 24x_1x_2[\phi_4^0 + \phi_4^+(1-5x_3)],\\
V_3(x_i) =& 12x_3[\psi_4^0(1-x_3) + \psi_4^-(x_1^2+x_2^2-x_3(1-x_3)) + \psi_4^+(1-x_3-10x_1x_2)],\\
A_2(x_i) =& 24x_1x_2(x_2-x_1)\phi_4^-,\\
A_3(x_i) =& 12x_3(x_2-x_1)[(psi_4^0 + \psi_4^+) + \psi_4^-(1-2x_3)],\\
T_2(x_i) =& 24x_1x_2[\xi_4^0 + \xi_4^+(1-5x_3)],\\
T_3(x_i) =& 6x_3[(\xi_4^0 + \phi_4^0 + \psi_4^0)(1-x_3) + (\xi_4^- + \phi_4^- - \psi_4^-)(x_1^2+x_2^2-x_3(1-x_3))\\
&+ (\xi_4^+ + \phi_4^+ + \psi_4^+)(1-x_3-10x_1x_2)],\\
T_7(x_i) =& 6x_3[(-\xi_4^0 + \phi_4^0 + \psi_4^0)(1-x_3) + (-\xi_4^- + \phi_4^- - \psi_4^-)(x_1^2+x_2^2-x_3(1-x_3))\\
&+ (-\xi_4^+ + \phi_4^+ + \psi_4^+)(1-x_3-10x_1x_2)],\\
S_1(x_i) =& 6x_3(x_2-x_1)[(\xi_4^0 + \phi_4^0 + \psi_4^0 + \xi_4^+ + \phi_4^+ + \psi_4^+) + (\xi_4^- + \phi_4^- - \psi_4^-)(1-2x_3)],\\
P_1(x_i) =& 6x_3(x_2-x_1)[(\xi_4^0 - \phi_4^0 - \psi_4^0 + \xi_4^+ - \phi_4^+ - \psi_4^+) + (\xi_4^- - \phi_4^- + \psi_4^-)(1-2x_3)].
\end{aligned}
$$

- Twist-5 LCDAs

$$
\begin{aligned}
V_4(x_i) =& 3[\psi_5^0(1-x_3) + \psi_5^-(2x_1x_2 - x_3(1-x_3)) + \psi_5^+(1-x_3-2(x_1^2+x_2^2))],\\
V_5(x_i) =& 6x_3[\phi_5^0 + \phi_5^+(1-2x_3)],\\
A_4(x_i) =& 3(x_2-x_1)[-\psi_5^0 + \psi_5^- x_3 + \psi_5^+(1-2x_3)],\\
A_5(x_i) =& 6x_3(x_2-x_1)\phi_5^-,\\
T_4(x_i) =& \frac{3}{2}[(\xi_5^0 + \psi_5^0 + \phi_5^0)(1-x_3) + (\xi_5^- + \phi_5^- - \psi_5^-)(2x_1x_2 - x_3(1-x_3))\\
&+ (\xi_5^+ + \phi_5^+ + \psi_5^+)(1-x_3-2(x_1^2+x_2^2))],\\
T_5(x_i) =& 6x_3[\xi_5^0 + \xi_5^+(1-2x_3)],\\
T_8(x_i) =& \frac{3}{2}[(\psi_5^0 + \phi_5^0 - \xi_5^0)(1-x_3) + (\phi_5^- - \phi_5^- - \xi_5^-)(2x_1x_2 - x_3(1-x_3))\\
&+ (\phi_5^+ + \phi_5^+ - \xi_5^+)(\mu)(1-x_3-2(x_1^2+x_2^2))],\\
S_2(x_i) =& \frac{3}{2}(x_2-x_1)[-(\psi_5^0 + \phi_5^0 + \xi_5^0) + (\xi_5^- + \phi_5^- - \psi_5^0)x_3 + (\xi_5^+ + \phi_5^+ + \psi_5^0)(1-2x_3)],\\
P_2(x_i) =& \frac{3}{2}(x_2-x_1)[(\psi_5^0 + \phi_5^0 - \xi_5^0) + (\xi_5^- - \phi_5^- + \psi_5^0)x_3 + (\xi_5^+ - \phi_5^+ - \psi_5^0)(1-2x_3)].
\end{aligned}
$$

- Twist-6 LCDAs

$$
\begin{aligned}
V_6(x_i) =& 2[\phi_6^0 + \phi_6^+(1-3x_3)],\\
A_6(x_i) =& 2(x_2-x_1)\phi_6^-,\\
T_6(x_i) =& 2[\phi_6^0 + \frac{1}{2}(\phi_6^- - \phi_6^+)(1-3x_3)],
\end{aligned}
$$

- LCDAs V_i, A_i, T_i, S_i, P_i are functions of parameters $\phi_i^{\pm,0}, \psi_i^{\pm,0}, \xi_i^{\pm,0}$ Braun, 2001

$$V_1(x_i) = 120x_1x_2x_3[\phi_3^0 + \phi_3^+(1-3x_3)],$$
$$A_1(x_i) = 120x_1x_2x_3(x_2-x_1)\phi_3^-,$$
$$T_1(x_i) = 120x_1x_2x_3[\phi_3^0 + \frac{1}{2}(\phi_3^- - \phi_3^+)(1-3x_3)].$$

- The parameters $\phi_i^{\pm,0}, \psi_i^{\pm,0}, \xi_i^{\pm,0}$ depend on 8 parameters

$$\phi_3^0 = \phi_6^0 = f_N, \qquad \phi_4^0 = \phi_5^0 = \frac{1}{2}(\lambda_1 + f_N),$$
$$\xi_4^0 = \xi_5^0 = \frac{1}{6}\lambda_2, \qquad \psi_4^0 = \psi_5^0 = \frac{1}{2}(f_N - \lambda_1).$$

$$\psi_4^+ = -\frac{1}{4}\Big(\lambda_1(-2 + 5f_1^d + 5f_1^u) + f_N(2 + 5A_1^u - 5V_1^d)\Big),$$
$$\xi_4^- = \frac{5}{16}\lambda_2(4 - 15f_2^d),$$
$$\xi_4^+ = \frac{1}{16}\lambda_2(4 - 15f_2^d),$$

$$\phi_4^- = \frac{5}{4}\Big(\lambda_1(1 - 2f_1^d - 4f_1^u) + f_N(2A_1^u - 1)\Big),$$
$$\phi_4^+ = \frac{1}{4}\Big(\lambda_1(3 - 10f_1^d) - f_N(10V_1^d - 3)\Big),$$
$$\psi_4^- = -\frac{5}{4}\Big(\lambda_1(2 - 7f_1^d + f_1^u) + f_N(A_1^u + 3V_1^d - 2)\Big),$$

$$\phi_5^- = \frac{5}{3}\Big(\lambda_1(f_1^d - f_1^u) + f_N(2A_1^u - 1)\Big),$$
$$\phi_5^+ = -\frac{5}{6}\Big(\lambda_1(4f_1^d - 1) + f_N(3 + 4V_1^d)\Big),$$
$$\psi_5^- = \frac{5}{3}\Big(\lambda_1(f_1^d - f_1^u) + f_N(2 - A_1^u - 3V_1^d)\Big),$$
$$\psi_5^+ = \frac{5}{3}\Big(\lambda_1(-1 + f_1^u) + f_N(1 + A_1^u + V_1^d)\Big),$$
$$\xi_5^- = -\frac{5}{4}\lambda_2 f_2^d,$$
$$\xi_5^+ = \frac{5}{12}\lambda_2(2 - 3f_2^d),$$

$$\phi_6^- = \frac{1}{2}\Big(\lambda_1(1 - 4f_1^d - 2f_1^u) + f_N(1 + 4A_1^u)\Big),$$
$$\phi_6^+ = \Big(\lambda_1(1 - 2f_1^d) + f_N(4V_1^d - 1)\Big).$$

	$f_N(GeV^2)$	$\lambda_1(GeV^2)$	$\lambda_2(GeV^2)$	V_1^d	A_1^u	f_1^d	f_2^d	f_1^u
QCDSR(2001) 8	$(5.3 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.1 \pm 1.9) \times 10^{-2}$	0.23 ± 0.03	0.38 ± 0.15	0.6 ± 0.2	0.15 ± 0.06	0.22 ± 0.15
QCDSR(2006) 9	$(5.0 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.4 \pm 1.9) \times 10^{-2}$	0.23 ± 0.03	0.38 ± 0.15	0.4 ± 0.05	0.22 ± 0.05	0.07 ± 0.05
LCSR(2006) 9	$(5.0 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.4 \pm 1.9) \times 10^{-2}$	0.3	0.13	0.33	0.25	0.09

Light-Cone Distribution Amplitudes: proton

Table 2: Parameters in the proton LCDAs in units of 10^{-2} GeV^2 [73]. The accuracy of those parameters without uncertainties is of order of 50%.

	ϕ_i^0	ϕ_i^-	ϕ_i^+	ψ_i^0	ψ_i^-	ψ_i^+	ξ_i^0	ξ_i^-	ξ_i^+
twist-3 $(i = 3)$	0.53 ± 0.05	2.11	0.57						
twist-4 $(i = 4)$	-1.08 ± 0.47	3.22	2.12	1.61 ± 0.47	-6.13	0.99	0.85 ± 0.31	2.79	0.56
twist-5 $(i = 5)$	-1.08 ± 0.47	-2.01	1.42	$1.61 \pm .047$	-0.98	-0.99	0.85 ± 0.31	-0.95	0.46
twist-6 $(i = 6)$	0.53 ± 0.05	3.09	-0.25						

Parameters of LCDAs of proton

Model	Method	$f_N \cdot 10^3$ Gev^2	$\lambda_1 \cdot 10^3$ Gev^2	$\lambda_2 \cdot 10^3$ Gev^2	A_1^u	V_1^d	f_1^u	f_1^d	f_2^d	Ref.
	QCDSR	5.0(5)	-27(9)	54(19)						
ASY		-	-	-	0	1/3	1/10	3/10	4/15	
CZ	QCDSR	5.3(5)	-	-	0.47	0.22	-	-	-	[1]
KS	QCDSR	5.1(3)	-	-	0.34	0.24	-	-	-	[2]
COZ	QCDSR	5.0(3)	-	-	0.39	0.23	-	-	-	[3]
SB	QCDSR	-	-	-	0.38	0.24	-	-	-	[4]
BK	PQCD	6.64	-	-	0.08	0.31	-	-	-	[5]
BLW	QCDSR	-	-	-	0.38(15)	0.23(3)	0.07(5)	0.40(20)	0.22(5)	[6]
BLW	LCSR（LO）	-	-	-	0.13	0.30	0.09	0.33	0.25	[6]
ABO1	LCSR（NLO）	-	-	-	0.11	0.30	0.11	0.27	-	[7]
ABO2	LCSR（NLO）				0.11	0.30	0.11	0.29	-	[7]
LAT09	LATTICE	3.23 (63)	-35.57 (65)	70.02 (13)	0.19 (2)	0.20 (1)	-	-	-	[8]
LAT14	LATTICE	3.07 (36)	-38.77 (18)	77.64 (37)	0.07 (4)	0.31 (2)	-	-	-	[9]
LAT19	LATTICE	3.54（6)	-44.9 (42)	93.4 (48)	0.30 (32)	0.192 (22)	-	-	-	[10]

thanks to K.S.Huang

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