

Semileptonic $\Lambda_b \rightarrow \Lambda_c$ decays beyond leading order

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Motivation

- Understanding strong interaction dynamics in heavy baryon system:
 - Factorization properties
 - Higher order α_s effects
 - Renormalization properties of the (higher-twist) Λ_b LCDAs
- Test *Standard Model* and explore *New Physics*:
 - R_{Λ_c} is an independent observable for testing LFU, relative to $R_{D^{(*)}}$,
 - $\Lambda_b \rightarrow \Lambda_c \ell \nu$ will contribute to the extraction of V_{cb} .
 - A_{FB} and abundant spin/angular distributions in $\Lambda_b \rightarrow \Lambda_c (\rightarrow \Lambda \pi/p K_S) \ell \nu$.
- In BGL expansion [Boyd, Grinstein, Lebed, 1997], $\Lambda_b \rightarrow \Lambda_c$ form factors can provide constraints on $B_q^{(*)} \rightarrow D_q^{(*)}$ form factors and vice versa.

Status of $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$

- Progress in experiment

- DELPHI 2004, PDG[2020](Updated):

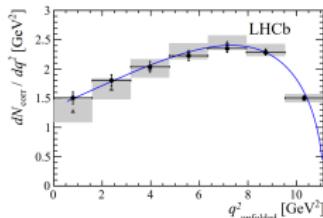
$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell \bar{\nu}) = (6.2^{+0.14}_{-0.13})\%,$$

- LHCb, 2022:

$$\begin{aligned}\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \tau \bar{\nu}) &= [1.50 \pm 0.16 \pm 0.25 \pm 0.23]\%, \\ \Rightarrow R_{\Lambda_c} &= 0.242 \pm 0.026 \pm 0.040 \pm 0.059\end{aligned}$$

- The recent development in theoretical side

- LHCb, 2017:



Method	R_{Λ_c}	Ref.
LQCD	0.333 ± 0.013	[Detmold, Lehner, Meinel, 2015]
LCSR	$0.274^{+0.009}_{-0.005}$ or $0.239^{+0.070}_{-0.021}$ 0.268 ± 0.015	[Duan, Liu, Huang, 2022] [Miao, Deng, Huang, Gao, Shen, 2022]
QCDSR	0.31 ± 0.11	[Azizi, Süngeü, 2018]
HQET	0.331 ± 0.010 and 0.324 ± 0.004	[Bernlochner, Ligeti, Robinson, Sutcliffe, 2018]
LFQM	0.30 ± 0.09 0.28	[Li, Liu, Yu, 2021] [Zhu, Wei, Ke, 2019]

Parameterization of $\Lambda_b \rightarrow \Lambda_c$ form factors

- Traditional parameterizations [Manohar and Wise \oplus many others]:

$$\langle \Lambda_c(p') | \bar{c} \gamma_\mu b | \Lambda_b(p) \rangle = \bar{u}_{\Lambda_c(p')} \left[F_1 \gamma_\mu + i F_2 \sigma_{\mu\nu} \frac{q^\nu}{m_{\Lambda_b}} + F_3 \frac{q_\mu}{m_{\Lambda_b}} \right] u_{\Lambda_b(p)},$$

$$\langle \Lambda_c(p') | \bar{c} \gamma_\mu \gamma_5 b | \Lambda_b(p) \rangle = \bar{u}_{\Lambda_c(p')} \left[G_1 \gamma_\mu + i G_2 \sigma_{\mu\nu} \frac{q^\nu}{m_{\Lambda_b}} + G_3 \frac{q_\mu}{m_{\Lambda_b}} \right] \gamma_5 u_{\Lambda_b(p)}.$$

- Conveniently, in helicity basis [Feldmann and Yip, 2011]

$$\begin{aligned} \langle \Lambda_c(p') | \bar{c} \gamma_\mu b | \Lambda_b(p) \rangle = & \bar{u}_{\Lambda_c}(p') \left[f_+(q^2) \frac{m_{\Lambda_b} + m_{\Lambda_c}}{s_+} \left(p_\mu + p'_\mu - \frac{m_{\Lambda_b}^2 - m_{\Lambda_c}^2}{q^2} q_\mu \right) \right. \\ & \left. + f_0(q^2) \frac{m_{\Lambda_b} - m_{\Lambda_c}}{q^2} q_\mu + f_\perp(q^2) \left(\gamma_\mu - \frac{2m_{\Lambda_c}}{s_+} p_\mu - \frac{2m_{\Lambda_b}}{s_+} p'_\mu \right) \right] u_{\Lambda_b}(p), \end{aligned}$$

with the boundary conditions $f_0(0) = f_+(0)$, $g_0(0) = g_+(0)$, $g_+(q_{\max}^2) = g_\perp(q_{\max}^2)$.

- In the heavy quark limit, they related to Isgur-Wise function $\zeta(w)$

$$F_1 = G_1 = \zeta, \quad F_{2,3} = G_{2,3} = 0, \quad f_0 = f_+ = f_\perp = g_0 = g_+ = g_\perp = \zeta.$$

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Framework

- LCSR and the method of regions [Wang, Shen, 2015]:
Systematic power expansion, factorization and resummation of large logs.
- Starting point: correlation function [Feldmann, Yip, 2011; Wang, Shen, 2015]

$$\Pi_{\mu,a}(p, q) = i \int d^4x e^{iq \cdot x} \langle 0 | T\{j_{\Lambda_c}(x), j_{\mu,a}(0)\} | \Lambda_b(p) \rangle,$$

where $j_{\mu,a} \equiv \bar{c}\Gamma_{\mu,a}b$ and the leading power Λ_c current

$$j_{\Lambda_c} \equiv \epsilon_{ijk} (u_i^T C \gamma_5 \not{d}_j) c_k.$$

- Hadronic dispersion relations:

$$\begin{aligned} \Pi_{\mu,\nu}(p, q) &= \frac{f_{\Lambda_c}(\mu)(n \cdot p')}{m_{\Lambda_c}^2/n \cdot p' - \bar{n} \cdot p'} \not{\not{d}} \left[f_{\perp}(q^2) \gamma_{\perp\mu} + \frac{f_0(q^2) - f_+(q^2)}{2(1 - n \cdot p'/m_{\Lambda_b})} n_\mu + \frac{f_0(q^2) + f_+(q^2)}{2} \bar{n}_\mu \right] u_{\Lambda_b}(p) \\ &+ \int_{\omega_s}^{+\infty} d\omega \frac{1}{\omega + \omega_c - \bar{n} \cdot p'} \times \not{\not{d}} \left[\rho_{V,\perp}^h(\omega, n \cdot p') \gamma_{\perp\mu} + \rho_{V,n}^h(\omega', n \cdot p') n_\mu + \rho_{V,\bar{n}}^h(\omega, n \cdot p') \bar{n}_\mu \right] u_{\Lambda_b}(p). \end{aligned}$$

Factorization of the correlation functions

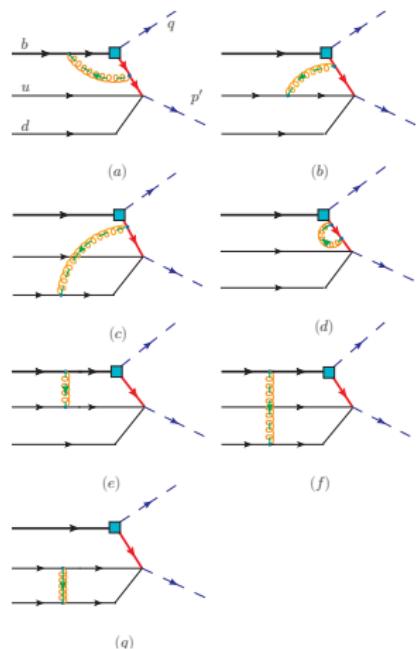
- QCD factorization of the correlation functions [Wang, Shen, 2015]

$$\begin{aligned}\Pi_{\mu, V(A)} &= (I, \gamma_5) \frac{\not{p}}{2} [\Pi_{\perp, V(A)} \gamma_{\perp\mu} + \Pi_{\bar{n}, V(A)} \bar{n}_\mu + \Pi_{n, V(A)} n_\mu] u_{\Lambda_b}(v), \\ \Pi_{\perp, V(A)} &= f_{\Lambda_b}^{(2)}(\mu) C_{\perp, V(A)}(n \cdot p', \mu) \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{1}{\omega_1 + \omega_2 + \omega_c - \bar{n} \cdot p' - i\epsilon} \\ &\quad \times J\left(\frac{\mu^2}{\bar{n} \cdot p' \omega_i}, \frac{\omega_i}{\bar{n} \cdot p'}\right) \psi_4(\omega_1, \omega_2, \mu)\end{aligned}$$

- Hard functions the same as the matching coefficients of the weak currents from QCD onto SCET_I
 - Already known at one loop [Bauer et al, 2000; Beneke et al, 2004]
 - and at two loops [Bonciani et al, 2008; Aastrihan et al, 2008; Beneke et al, 2008; Bell et al, 2008, 2010]
- Two equivalent strategies for computing the jet function:
 - Apply the SCET_I Feynman rules.
 - Extract the hard-collinear contributions of QCD diagrams with the methods of regions.
Algebraically simpler → this talk!
- Factorization-scale independence of the correlation functions:
 - Explicit RGE of $\psi_4(\omega_1, \omega_2, \mu)$ unknown.

Factorization of the correlation functions

- NLO diagrams



- Hard functions:

$$C_{\perp, V(A)}(n \cdot p', \mu) = 1 - \frac{\alpha_s(\mu)}{4\pi} \left[2 \ln^2 \frac{\mu}{n \cdot p'} + 5 \ln \frac{\mu}{m_b} - 2 \text{Li}_2 \left(1 - \frac{1}{r} \right) - \ln^2 r + \frac{3r-2}{1-r} \ln r + \frac{\pi^2}{12} + 6 \right]$$

All hard functions obey the same RGEs.

- Jet function:

$$J \left(\frac{\mu^2}{\bar{n} \cdot p'}, \frac{\omega_i}{\bar{n} \cdot p'} \right) = 1 + \frac{\alpha_s C_F}{4\pi} \left\{ 2 \left[\text{Li}_2 \left(\frac{1}{r_2+1} \right) - 2 \text{Li}_2 \left(\frac{r_3+1}{r_2+r_3+1} \right) \right] - \frac{r_2 \ln r_2}{2(r_3+1)^2(r_2+r_3+1)} \left[r_2^2 [r_3(r_3+2)+3] + r_2(r_3+1)[r_3(r_3+6)+11] + 4(r_3-1)(r_3+1)^2 \right] + (r_2+1) \ln(r_2+1) \left(\frac{r_2+3}{2} - \frac{2}{r_3} \right) + [\ln R + \ln(r_3+1)] \left[-\frac{6r_2}{r_2+r_3+1} - \frac{1}{2} - 4 \ln(r_2+r_3+1) \right] + \frac{\ln(r_2+r_3+1)}{(r_3+1)^2} \left[\frac{2(r_2+1)}{r_3} + (r_2^2 + 4r_2r_3 + 6r_2 - r_3^2 + 3) \right] + 2 \ln^2(r_2+r_3+1) - \ln^2(r_2+1) + 2 \ln(r_2+1) [\ln R + \ln(r_3+1)] + [\ln R + \ln(r_3+1)]^2 - \frac{r_2}{r_3+1} - \frac{8r_2}{r_2+r_3+1} + \frac{\pi^2}{6} - \frac{r_2+1}{2} \right\}.$$

Universal jet function for all the form factors.

NLL resummation of $\Lambda_b \rightarrow \Lambda_c$ form factors

- Resummation improved form factors:

$$\begin{aligned}
& f_{\Lambda_c}(\nu)(n \cdot p') e^{-m_{\Lambda_c}^2 / (n \cdot p' \omega_M)} \left\{ f_\perp(q^2), g_\perp(q^2) \right\} \\
&= f_{\Lambda_b}^{(2)}(\mu) [U_1(n \cdot p', \mu_h, \mu) C_{\perp, V(A)}(n \cdot p', \mu_h)] \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \psi_{4,\text{eff}}(\omega', \mu, \nu), \\
& f_{\Lambda_c}(\nu)(n \cdot p') e^{-m_{\Lambda_c}^2 / (n \cdot p' \omega_M)} \left\{ f_0(q^2), g_0(q^2) \right\} \\
&= f_{\Lambda_b}^{(2)}(\mu) [U_1(n \cdot p', \mu_h, \mu) C_{n, V(A)}(n \cdot p', \mu_h)] \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \psi_{4,\text{eff}}(\omega', \mu, \nu) \\
&+ f_{\Lambda_b}^{(2)}(\mu) \left(1 - \frac{n \cdot p'}{m_{\Lambda_b}} \right) U_1(n \cdot p', \mu_h, \mu) C_{n, V(A)}(n \cdot p', \mu_h) \int_0^{\omega_s} d\omega' e^{-\omega'/\omega_M} \psi_{4,\text{eff}}^{(0)}(\omega', \mu).
\end{aligned}$$

NLL resummation for the twist-4 Λ_b -DA not included (unknown 2-loop RGE)

- "Effective" DA $\psi_{4,\text{eff}}(\omega', \mu, \nu)$ absorbing the hard-collinear QCD corrections

$$\begin{aligned}
\psi_{4,\text{eff}}(\omega', \mu) = & (\omega' - \omega_c) \phi_4(\omega' - \omega_c) \theta(\omega' - \omega_c) + \frac{\alpha_s C_F}{4\pi} \left\{ \phi_4(0) \theta(\omega') \rho^{(1)}(\omega') + \phi_4(\omega') \theta(\omega') \rho^{(2)}(\omega') \right. \\
& + \phi_4(\omega' - \omega_c) \theta(\omega' - \omega_c) \rho^{(3)}(\omega') + \phi'_4(\omega') \theta(\omega') \rho^{(4)}(\omega') + \phi'_4(\omega' - \omega_c) \theta(\omega' - \omega_c) \rho^{(5)}(\omega') \\
& \left. + \int_0^\infty d\omega \phi_4(\omega) \rho^{(6)}(\omega, \omega') + \int_0^\infty d\omega \phi'_4(\omega) \rho^{(7)}(\omega, \omega') \right\}
\end{aligned}$$

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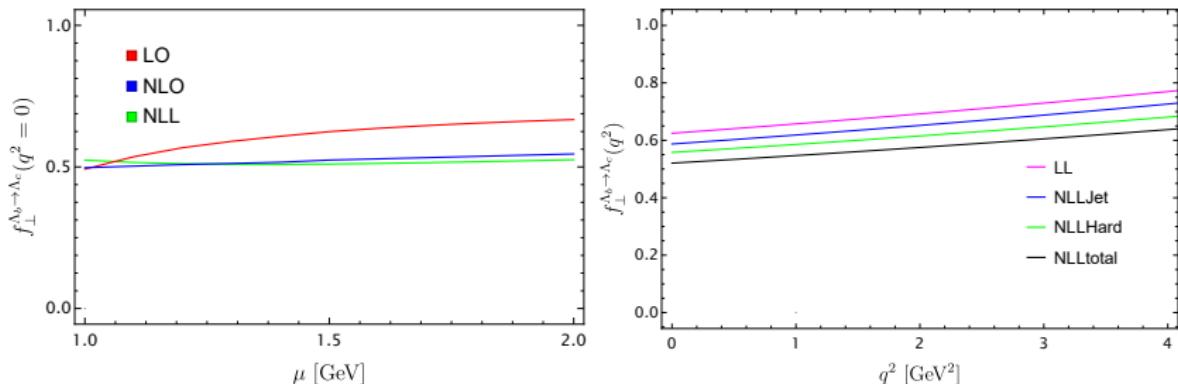
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Scale dependence and higher order effects

- Factorization scale dependence and radiative correction:

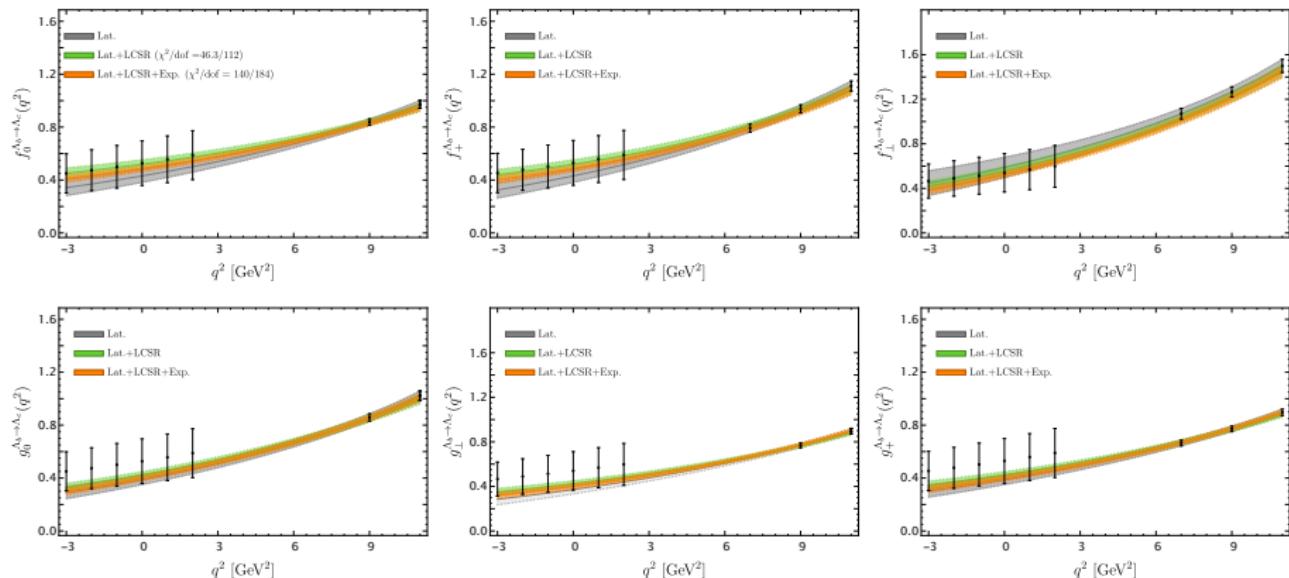


- $f_{\Lambda_b}^{(2)}(\mu)\psi_4(\mu)$ almost no scale dependence.
- NLO correction dominated by the hard contributions.
- Radiative effect can induce about 15% reduction of the form factors.

Results of $\Lambda_b \rightarrow \Lambda_c$ form factors

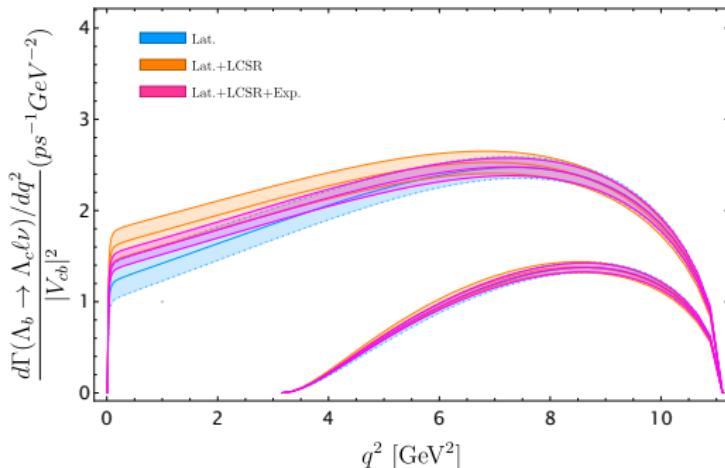
Joint fitting with LQCD[Detmold,Lehner,Meinel,2015] and $B_q^{(*)} \rightarrow D_q^{(*)}$ (LCSR and LQCD) [Cui,Huang,Wang,Zhao,2023] and/or Exp. data in BGL(or BCL) expansion:

$$f_i(t) = \frac{1}{B_i(t)\phi_i(t; t_0)} \sum_{n=0}^2 a_n^i z(t; t_0)^n \quad \sum_{i,n=0}^2 (a_n^i)^2 \leq 1$$



Decay width

- Differential decay width: $\frac{d\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \nu)/dq^2}{|V_{cb}|^2}$



- Total decay width in unit of ps^{-1}

Channel	LCSRs+LQCD	LQCD	Exp.(BR)
$\Gamma(\Lambda_b \rightarrow \Lambda_c \tau \nu)/ V_{cb} ^2$	7.20 ± 0.30	7.15 ± 0.31	$(1.50 \pm 0.38)\%$
$\Gamma(\Lambda_b \rightarrow \Lambda_c \mu \nu)/ V_{cb} ^2$	22.42 ± 1.05	21.5 ± 1.4	$(6.2^{+0.14}_{-0.13})\%$
$\Gamma(\Lambda_b \rightarrow \Lambda_c e \nu)/ V_{cb} ^2$	22.49 ± 1.06	21.5 ± 1.4	$(6.2^{+0.14}_{-0.13})\%$

- V_{cb} from [PDG 2022]

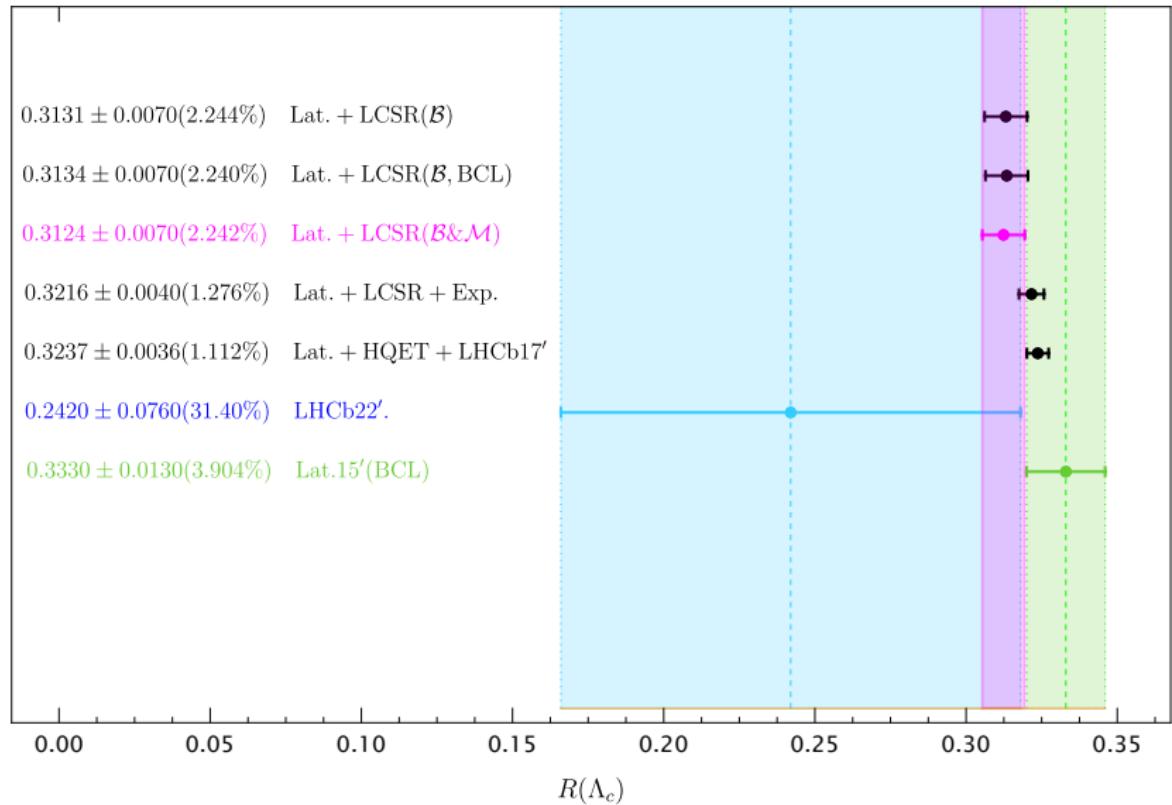
$$|V_{cb}^{\text{excl}}| = (39.4 \pm 0.8) \times 10^{-3}$$

$$|V_{cb}^{\text{incl}}| = (42.2 \pm 0.8) \times 10^{-3}$$

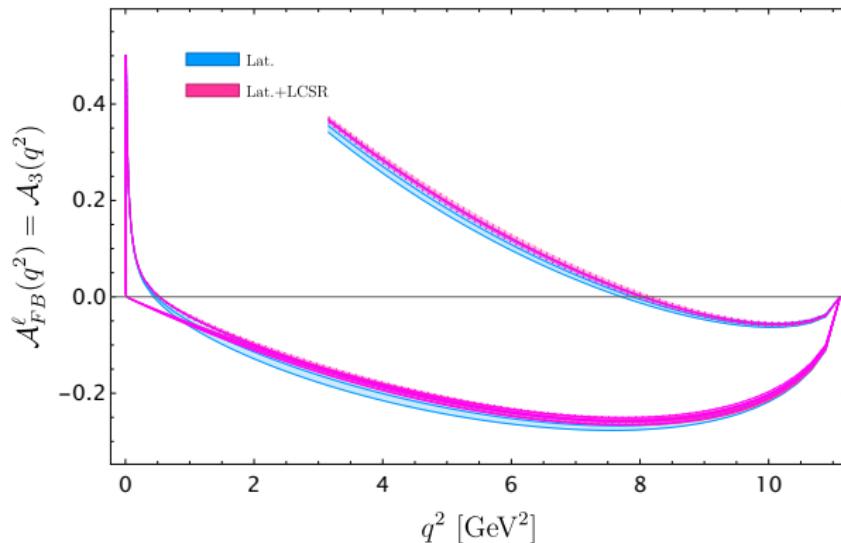
- Our fitting

Scheme	$V_{cb}(10^{-3})$	Relative uncertainty
$B_q^{(*)} \rightarrow D_q^{(*)}$ [Cui,Huang,Wang,Zhao,2023]	39.63 ± 0.63	1.59%
$\Lambda_b \rightarrow \Lambda_c$	40.10 ± 3.50	8.73%
$\Lambda_b \rightarrow \Lambda_c \otimes B_q^{(*)} \rightarrow D_q^{(*)}$	39.79 ± 0.62	1.56%
$\Lambda_b \rightarrow \Lambda_c$ [(Exp. Err)/2]	40.07 ± 1.91	4.77%
$\Lambda_b \rightarrow \Lambda_c \otimes B_q^{(*)} \rightarrow D_q^{(*)}$ [(Exp. Err)/2]	39.80 ± 0.60	1.51%

R_{Λ_c} in SM



Forward-backward asymmetry



$$\langle A_{FB}^{\tau} \rangle = +(0.0725 \pm 0.0062)$$

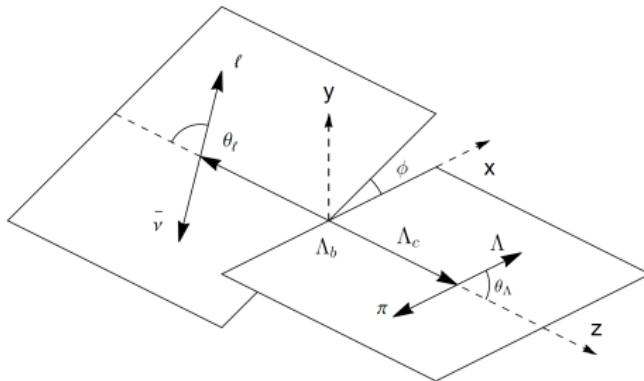
$$\langle A_{FB}^{\mu} \rangle = -(0.1912 \pm 0.0056)$$

$$\langle A_{FB}^e \rangle = -(0.1970 \pm 0.0056)$$

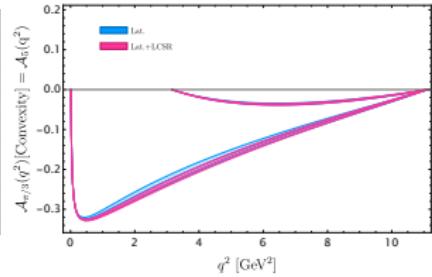
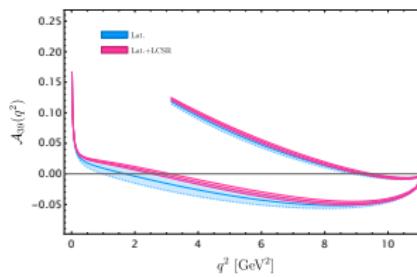
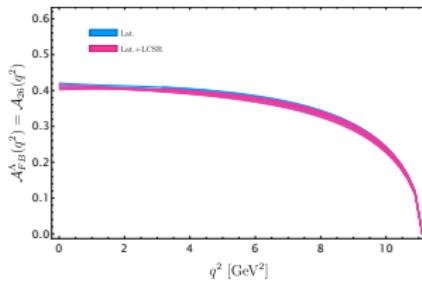
$$\Delta A_{FB} = \langle A_{FB}^{\mu} \rangle - \langle A_{FB}^e \rangle = (5.86 \pm 0.02) \times 10^{-3}$$

Angular distributions in $\Lambda_b \rightarrow \Lambda_c (\rightarrow \Lambda\pi) \ell\nu$

- Schematic diagram defining various angles



- Four-body angular distributions



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Conclusions and outlook

• Conclusions

- For the first time, the $\Lambda_b \rightarrow \Lambda_c$ form factors at small q^2 are calculated to $O(\alpha_s)$ at leading power of Λ/m_b based on LCSR and the method of regions.
- Radiative effect can induce about 15% reduction of the form factors.
- With our small- q^2 constraint and joint fitting with LQCD at large- q^2 , we get the most exact $\Lambda_b \rightarrow \Lambda_c$ form factors in SM.
- We obtained the most precise SM predictions for $\Gamma(\Lambda_b \rightarrow \Lambda_c \ell \nu)/V_{cb}$, R_{Λ_c} , A_{FB} and three(four)-body angular distributions so far.

• Outlook

- The subleading power corrections to $\Lambda_b \rightarrow \Lambda_c$ form factors may play important role, indicated by the research on $B \rightarrow D^{(*)}$.
- Renormalization properties of the Λ_b LCDA, RGE.

Thanks

Thank you for your attention!

Back up: Inputs

Parameter	value/interval	unit	prior	source/comments
quark masses				
$\overline{m}_b(\overline{m}_b)$	4.193 ± 0.035	GeV	Gaussian @ 68%	[Beneke,2014]
$\overline{m}_c(\overline{m}_c)$	1.288 ± 0.02	GeV	Gaussian @ 68%	[Dehnadi,2015]
m_b^{Pole}	4.8 ± 0.1	GeV	Gaussian @ 68%	
hadron masses				
m_{Λ_b}	5619.60	MeV	—	[PDG,2020]
m_{Λ_c}	2286.46	MeV	—	
vacuum condensate densities				
$\langle \frac{\alpha_s}{\pi} G^2 \rangle$	$0.012^{+0.006}_{-0.012}$	GeV ⁴	Uniform @ 100%	[Duplancic,2008]
parameters of the Λ_b DAs				
$f_{\Lambda_b}^{(2)}(1\text{GeV})$	$(3.0 \pm 0.5) \times 10^{-2}$	GeV ³	Gaussian @ 68%	[Groote,1997]
sum rule parameters and scales				
μ	[1.0,2.0]	GeV	Uniform @ 100%	[Wang,2015]
ν	[1.0,2.0]	GeV	Uniform @ 100%	
μ_h	$[\overline{m}_b/2, 2\overline{m}_b]$	GeV	Uniform @ 100%	
M_{2pt}^2	2.5 ± 0.5	GeV ²	Uniform@ 100%	[Khodjamirian,2011]
M_{SR}^2	5.0 ± 1.0	GeV ²	Uniform@ 100%	
s_0	[7.0,8.0]	GeV ²	Uniform@ 100%	[Duplancic,2008]

- Light-cone distribution amplitudes of the Λ_b -baryon(three-parameter model):

$$\phi_4(\omega; \mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\beta(\beta+1)}{\alpha(\alpha+1)} \frac{\mathcal{N}}{\omega_0^2} e^{-\omega/\omega_0} U(\beta-\alpha, 2-\alpha, \omega/\omega_0).$$

- Analogy to the mesonic counterparts[Beneke,Braun et al,2018]
- three parameters ω_0 , α and β are assumed to determine $\phi_4(\omega)$ at the scale μ_0 , which satisfy

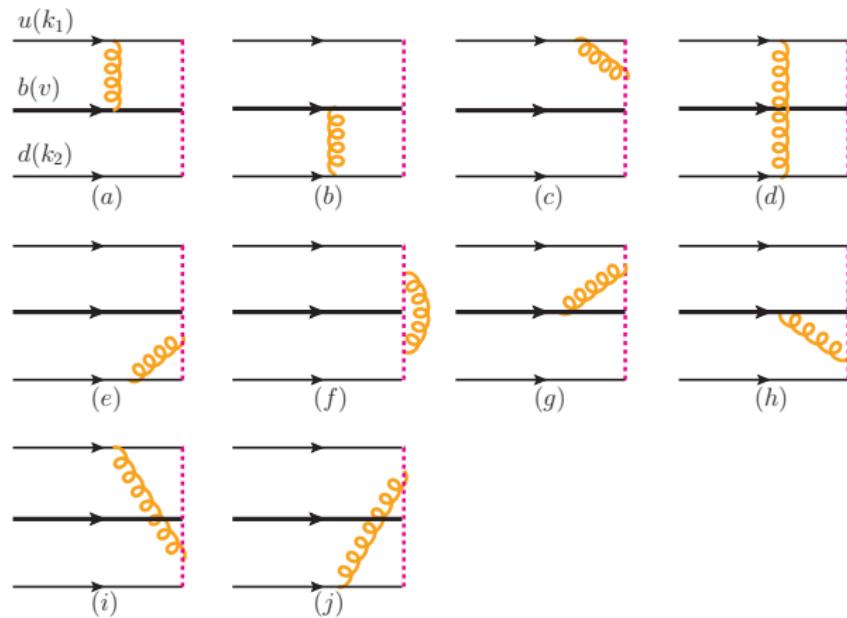
$$\lambda_B(\mu) = \frac{\alpha - 1}{\beta - 1} \omega_0, \hat{\sigma}_1(\mu) = \psi(\beta - 1) - \psi(\alpha - 1) + \ln \frac{\alpha - 1}{\beta - 1},$$

$$\hat{\sigma}_2(\mu) = \hat{\sigma}_1^2(\mu) + \psi'(\alpha - 1) - \psi'(\beta - 1) + \frac{\pi^2}{6}.$$

- In the numerical calculation, We use the following (α, β) tuples together with ω_0 by apply the identities

	ω_0	α	β	λ_B	$\hat{\sigma}_1$	$\hat{\sigma}_2$	\mathcal{N}
ϕ_4	0.28(10)GeV	1.2583	1.2583	0.28(10)GeV	0	$\pi^2/6$	1
	0.329(118)GeV	1.29771	1.35034	0.28(10)GeV	0.40	5.00	1
	0.264(94)GeV	1.1361	1.12827	0.28(10)GeV	-0.40	-5.00	1

- Renormalization of Λ_b LCDA:



- Explicit RGE of $\psi_4(\omega_1, \omega_2, \mu)$ unknown.
 - Extract the UV divergence of the amplitude from the 10 diagrams.
(radiative correction to the Λ_b -baryon DA at one loop)

Factorization-scale dependence

- Factorization-scale dependence:

$$\frac{d}{d \ln \mu} \Pi_{\perp, V(A)} = \frac{\alpha_s(\mu)}{4\pi} C_F \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{1}{\omega_1 + \omega_2 + \omega_c - \bar{n} \cdot p' - i0} \times \left\{ \frac{12\omega_c}{\omega_1 + \omega_2 + \omega_c - \bar{n} \cdot p'} \right. \\ \left[-\frac{12r_2}{1+r_2+r_3} - 1 - 4\ln \frac{(1+r_2+r_3)^2}{(1+r_2)(1+r_3)} + 4\ln \frac{\mu^2}{n \cdot p' (\omega - \bar{n} \cdot p')} \right] - \left[4 \cdot \ln \frac{\mu}{n \cdot p'} + 5 \right] \left. \right\} \left[f_{\Lambda_b}^{(2)}(\mu) \psi_4(\omega_1, \omega_2, \mu) \right] \\ + \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{1}{\omega_1 + \omega_2 + \omega_c - \bar{n} \cdot p' - i0} \frac{d}{d \ln \mu} \left[f_{\Lambda_b}^{(2)}(\mu) \psi_4(\omega_1, \omega_2, \mu) \right] + \mathcal{O}(\alpha_s^2)$$

the three terms in blue arise from the RG running of the charm-quark mass, the jet function J and the hard function $C_{\perp(\bar{n}), V(A)}$ respectively.

- Renormalization of $\psi_4(\omega_1, \omega_2, \mu)$ yields:

$$\int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{1}{\omega_1 + \omega_2 + \omega_c - \bar{n} \cdot p' - i0} \frac{d}{d \ln \mu} \left[f_{\Lambda_b}^{(2)}(\mu) \psi_4(\omega_1, \omega_2, \mu) \right] \\ = -\frac{\alpha_s(\mu)}{4\pi} \frac{4}{3} \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \frac{1}{\omega_1 + \omega_2 + \omega_c - \bar{n} \cdot p' - i0} \\ \times \left[4 \ln \frac{\mu}{\omega + \omega_c - \bar{n} \cdot p'} - 4 \ln \frac{1+r_2+r_3}{1+r_2} - 5 \right] \left[f_{\Lambda_b}^{(2)}(\mu) \psi_4(\omega_1, \omega_2, \mu) \right]$$

- Implying

$$\frac{d}{d \ln \mu} \left[\frac{\Pi_{\perp, V(A)}(n \cdot p', \bar{n} \cdot p', \mu)}{f_{\Lambda_c}(\mu)} \right] = 0.$$

Unitarity bound

The unitarity bound for the BGL form factor \tilde{F}_i expressed entirely in terms of the conformal variable z reads

$$\frac{1}{2\pi i} \sum_i \oint_C \frac{dz}{z} |\phi_i(z) B_i(z) \tilde{F}_i(z)|^2 \leq 1,$$

where Blaschke factors $B_i(z)$ takes into account the subthreshold B_c resonant pole at $t = t_p = m_{B_c^{(*)}}^2$ and reads

$$B(t; t_p) = \prod_p \frac{z(t; t_0) - z(t_p; t_0)}{1 - z(t; t_0)z(t_p; t_0)} = \prod_p z(t; t_p).$$

The weight function $\phi_i(t; t_0)$ known as an *outer function* can be expressed in a general form for given BGL form factors \tilde{F}_i as

$$\begin{aligned} \phi_i(t; t_0) &= \sqrt{\frac{n_I}{K\pi\chi}} \left(\frac{t_{bd} - t}{t_{bd} - t_0} \right)^{\frac{1}{4}} (\sqrt{t_{bd} - t} + \sqrt{t_{bd} - t_0}) (t_{bd} - t)^{\frac{a}{4}} \\ &\times \left(\sqrt{t_{bd} - t} + \sqrt{t_{bd} - t_-} \right)^{\frac{b}{2}} (\sqrt{t_{bd} - t} + \sqrt{t_{bd}})^{-(c+3)}, \end{aligned}$$

where n_I is an isospin Clebsch-Gordan factor, which is 2 and 1 for $B \rightarrow D^{(*)}$ and $\Lambda_b \rightarrow \Lambda_c$.

Weak/Strong unitary bounds

- We will express form factors in term of BGL form factors

$$F_i(w) = \frac{1}{B_i(z)\phi_i(z)} \sum_{n=0}^N a_n^i z^n.$$

- The weak unitary bound which $\Lambda_b \rightarrow \Lambda_c$ form factors satisfy reads:

$$\sum_{n=0}^N (a_n^{f_0})^2 \leq 1, \sum_{n=0}^N (a_n^{f\perp})^2 + (a_n^{f+})^2 \leq 1; \quad \sum_{n=0}^N (a_n^{g_0})^2 \leq 1, \sum_{n=0}^N (a_n^{g\perp})^2 + (a_n^{g+})^2 \leq 1.$$

- The strong unitary bound including all the $\Lambda_b \rightarrow \Lambda_c$ and $B_{(s)} \rightarrow D_{(s)}^{(*)}$ are

$$J^P = 1^- : \quad \sum_{n=0}^N (a_n^{f\perp})^2 + (a_n^{f+})^2 + \sum_{n=i}^7 \sum_{n=0}^N (b_n^{V_i})^2 + \sum_{n=i}^7 \sum_{n=0}^N (b_n^{V_i^s})^2 \leq 1,$$

$$J^P = 1^+ : \quad \sum_{n=0}^N (a_n^{g\perp})^2 + (a_n^{g+})^2 + \sum_{n=i}^7 \sum_{n=0}^N (b_n^{A_i})^2 + \sum_{n=i}^7 \sum_{n=0}^N (b_n^{A_i^s})^2 \leq 1,$$

$$J^P = 0^+ : \quad \sum_{n=0}^N (a_n^{f_0})^2 + \sum_{n=i}^3 \sum_{n=0}^N (b_n^{S_i})^2 + \sum_{n=i}^3 \sum_{n=0}^N (b_n^{S_i^s})^2 \leq 1,$$

$$J^P = 0^- : \quad \sum_{n=0}^N (a_n^{g_0})^2 + \sum_{n=i}^3 \sum_{n=0}^N (b_n^{P_i})^2 + \sum_{n=i}^3 \sum_{n=0}^N (b_n^{P_i^s})^2 \leq 1.$$

Angular distributions

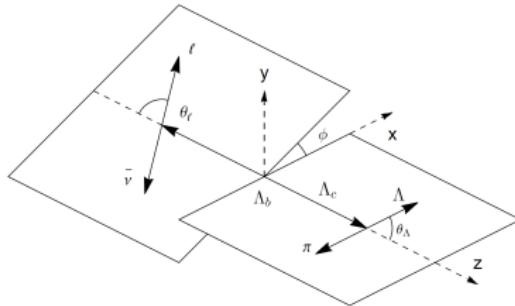
- Three-body angular distributions for $\Lambda_b \rightarrow \Lambda_c \ell \nu$

$$\frac{d^2 \Gamma_{\lambda_c}^{\lambda_\ell}}{dq^2 d \cos \theta} = a_{\lambda_c}^{\lambda_\ell}(q^2) + b_{\lambda_c}^{\lambda_\ell}(q^2) \cos \theta + c_{\lambda_c}^{\lambda_\ell}(q^2) \cos^2 \theta.$$

- Four-body angular distributions for $\Lambda_b \rightarrow \Lambda_c (\rightarrow \Lambda \pi) \ell \nu$

$$\begin{aligned} \frac{d^4 \Gamma^{\lambda_\ell}}{dq^2 d \cos \theta_\ell d \cos \theta_\Lambda d \phi} = & J_1^{\lambda_\ell} + J_2^{\lambda_\ell} \cos \theta_\Lambda + (J_3^{\lambda_\ell} + J_4^{\lambda_\ell} \cos \theta_\Lambda) \cos \theta_\ell \\ & + (J_5^{\lambda_\ell} + J_6^{\lambda_\ell} \cos \theta_\Lambda) \cos^2 \theta_\ell + (J_7^{\lambda_\ell} \sin \theta_\Lambda \cos \phi + J_8^{\lambda_\ell} \sin \theta_\Lambda \sin \phi) \sin \theta_\ell \\ & + (J_9^{\lambda_\ell} \sin \theta_\Lambda \cos \phi + J_{10}^{\lambda_\ell} \sin \theta_\Lambda \sin \phi) \sin \theta \cos \theta_\ell. \end{aligned}$$

- Where a, b, c and J_i are relevant helicity amplitudes, details can be found in [Bećirević and Jaffredo, 2022].



Angular distributions

- Forward-backward asymmetry

$$\mathcal{A}_{FB}^\ell = \frac{\left[\int_0^1 - \int_{-1}^0 \right] \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell}{d\Gamma/dq^2} = \frac{J_3}{2(J_1 + J_5/3)} \equiv \mathcal{A}_3$$

$$\mathcal{A}_{FB}^\Lambda = \frac{\left[\int_0^1 - \int_{-1}^0 \right] \frac{d^2\Gamma}{dq^2 d\cos\theta_\Lambda} d\cos\theta_\Lambda}{d\Gamma/dq^2} = \frac{J_2 + J_6/3}{2(J_1 + J_5/3)} \equiv \mathcal{A}_{26}$$

- Convexity for θ distribution

$$\mathcal{A}_{\pi/3}^\ell = \frac{\left[\int_{1/2}^1 - \int_{-1/2}^{1/2} + \int_{-1}^{-1/2} \right] \frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} d\cos\theta_\ell}{d\Gamma/dq^2} = \frac{J_5}{4(J_1 + J_5/3)} \equiv \mathcal{A}_5$$

- Convexity for ϕ distribution

$$A_{39}(q^2) = \frac{\left[\int_0^{2\pi/3} - \int_{2\pi/3}^{4\pi/3} + \int_{4\pi/3}^{2\pi} \right] \hat{\mathcal{A}}(q^2, \phi) d\phi}{d\Gamma/dq^2} = \frac{\sqrt{3}}{3\pi} \frac{J_9}{J_1 + J_5/3} + \frac{J_3/6}{J_1 + J_5/3}$$