

Angular distribution of E_c four body decays

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TDLI & SJTU

PRD 107, 074024 (2023)



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- Introduction
- Theoretical formwork
- Angular distribution
- numerical results

- Introduction



PART 01

Introduction

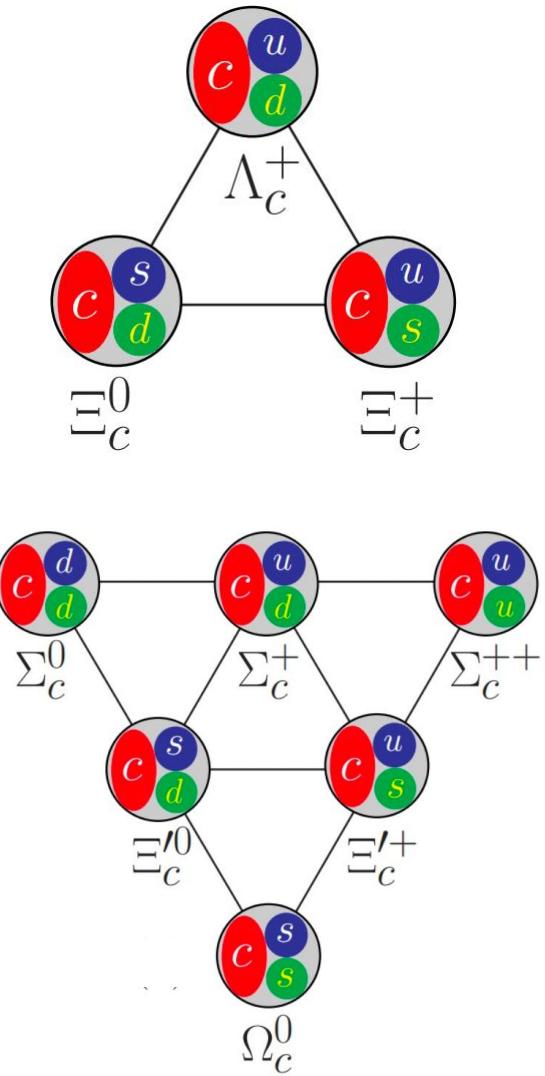
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Introduction

Charmed baryon physics

- There are various experimental facilities focus on charmed baryon physics in the world.
- The theoretical analysis is still incomplete.
- Charmed baryon physics are hopeful to become a new platform for study NP.





Introduction



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Charmed baryon physics in experiment

$$R(\Lambda_c) = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$

First observation of $\Lambda_c \rightarrow n\pi^+$

First observation of $\Lambda_c \rightarrow pK^-e^+\nu$

First observation of $\Lambda_c \rightarrow p\eta'$

First search for the weak radiative decays $\Lambda_c^+ \rightarrow \Sigma^+\gamma$ and $\Xi_c^0 \rightarrow \Xi^0\gamma$

$$\text{PRD 107 (2023) 3, 032001}$$

Search for the weak radiative decay $\Lambda_c^+ \rightarrow \Sigma^+\gamma$ at BESIII

$$\text{PRD 107 (2023) 5, 052002}$$

$$\text{PRL 128 (2022) 191803}$$

$$\text{PRL 128 (2022) 142001}$$

$$\text{PRD 106 (2022) 11, 112010}$$

$$\text{JHEP 03 (2022) 090}$$

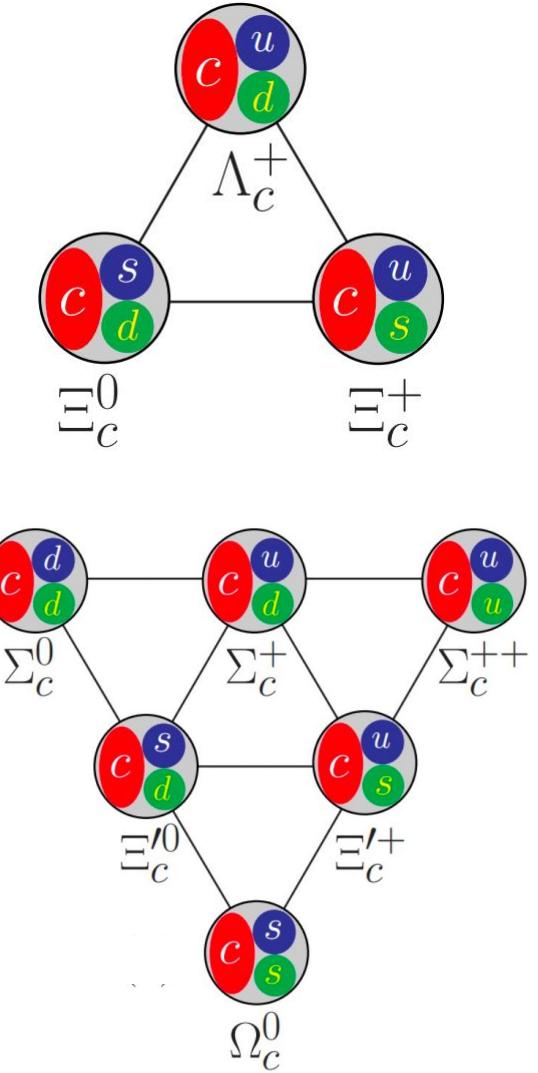
$$\mathcal{R}_{SM}(\Lambda_c) = 0.324 \pm 0.004.$$



Introduction

Charmed baryon physics in theory

- SU(3) analysis: [[PLB 823 \(2021\) 136765](#)], [[JHEP 03 \(2022\) 143](#)], [[JHEP 12 \(2022\) 003](#)],
[[JHEP 09 \(2022\) 035](#)], [[JHEP 02 \(2023\) 235](#)].
- $\Lambda_c \rightarrow \Lambda$ Form Factors in Lattice QCD [Turk.J.Phys. 45 \(2021\) 4](#)
- First lattice QCD calculation of semileptonic decays of charmed-strange baryons Ξ_c^* [Chin.Phys.C 46 \(2022\) 1, 011002](#)
- QCDSR : [[PRD 104 \(2021\) 5, 054030](#)], [[PRD 106 \(2022\) 9, 9](#)], [[2103.09436](#)],
- Weak radiative decay $\Lambda_c^+ \rightarrow \Sigma^+ \gamma$ using light-cone sum rules
[EPJ.C 83 \(2023\) 3, 224](#)





Introduction

SU(3) analysis

Channel	experiment data(%)	SU(3) symmetry(%)
$\Lambda_c \rightarrow \Lambda e^+ \nu_e$	$3.6 \pm 0.4[1]$	3.6 ± 0.4
$\Lambda_c \rightarrow \Lambda \mu^+ \nu_\mu$	$3.5 \pm 0.5[1]$	3.5 ± 0.5
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	$7 \pm 4[1]$	12.7 ± 1.35
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	$1.54 \pm 0.35[2, 3]$	4.10 ± 0.46
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	$1.27 \pm 0.44[3]$	3.98 ± 0.57

Not consist with SU(3) symmetry

[1].PDG

[2].S.Acharya et al. [ALICE], Phys. Rev. Lett. 127, no.27, 272001 (2021)

[3].Y. B. Li et al. [Belle], Phys. Rev. Lett. 127, no.12, 121803 (2021)

X.G.He, F.Huang, W.Wang and
Z.P.Xing, Phys. Lett. B 823, 136765 (2021)



input data

{ 2 σ standard deviation
6 σ standard deviation
5 σ standard deviation



Introduction



Ξ_c anomaly

Channel	experiment data(%)	SU(3) symmetry(%)	Lattice(%)	QCD sum rules(%)	light-cone sum rules(%)
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	$1.04 \pm 0.24[1]$	$4.10 \pm 0.46[2]$	$2.38 \pm 0.44[3]$	$3.4 \pm 0.7[4]$	$2.81_{-0.15}^{+0.17}[5]$
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	$1.01 \pm 0.25[1]$	$3.98 \pm 0.57[2]$	$2.29 \pm 0.42[3]$	-	$2.72_{-0.15}^{+0.17}[5]$

Table 3
Experimental and fit data of anti-triplet charmed baryons decays.

globe fit

Not consist with SU(3)
symmetry

[1].PDG

[2].X.G.He, F.Huang, W.Wang and Z.P.Xing, Phys. Lett. B 823, 136765 (2021)

[3].Q.A.Zhang, et al.Chin. Phys. C 46, no.1, 011002 (2022)

[4].Z.X.Zhao, [arXiv:2103.09436 [hep-ph]]

[5].H.H.Duan, Y.L.Liu and M.Q.Huang, Phys. Rev. D 106, no.9, 9 (2022)

Channel	Branching ratio (%)	
	Experimental data	Fit data
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.60 ± 0.40	1.94 ± 0.18
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	1.87 ± 0.176
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5	6.53 ± 0.60
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35	2.17 ± 0.20
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	2.09 ± 0.19
$\chi^2/d.o.f = 14.3$	$f_1 = 1.05 \pm 0.30$	$f'_1 = 0.11 \pm 0.95$



Introduction



SU(3) symmetry breaking from the quark mass

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \sim m_s \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = m_s \times \boxed{\omega}.$$

SU(3) symmetry breaking

$$\begin{aligned}
 H_{\lambda, \lambda_w} = & a_1^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} (H_3)^k \epsilon_{ikm} (T_8)_j^m + a_2^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H_3)^k \epsilon_{ikm} (T_8)_j^m \omega_n^j \\
 & + a_3^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H_3)^k \epsilon_{kjm} (T_8)_i^m \omega_n^j + a_4^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H_3)^k \epsilon_{jim} (T_8)_k^m \omega_n^j \\
 & + a_5^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} (H_3)^k \epsilon_{inm} (T_8)_j^m \omega_k^n.
 \end{aligned}$$

Amplitude:

$\Lambda_c^+ \rightarrow \Lambda^0 l^+ \nu$	$-\sqrt{\frac{2}{3}}(a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}) V_{cs}^*$	$\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$	$-(a_1^{\lambda, \lambda_w} + a_2'^{\lambda, \lambda_w} - a_4'^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}) V_{cs}^*$
$\Lambda_c^+ \rightarrow n l^+ \nu$	$a_1 V_{cd}^*$	$\Xi_c^0 \rightarrow \Sigma^- \ell^+ \nu_\ell$	$(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4'^{\lambda, \lambda_w}) V_{cd}^*$
$\Xi_c^+ \rightarrow \Sigma^0 \ell^+ \nu_\ell$	$\frac{(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4'^{\lambda, \lambda_w}) V_{cd}^*}{\sqrt{2}}$	$\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$	$(a_1^{\lambda, \lambda_w} + a_2'^{\lambda, \lambda_w} - a_4'^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}) V_{cs}^*$
$\Xi_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$	$-\frac{(a_1^{\lambda, \lambda_w} + 2a_2'^{\lambda, \lambda_w} - a_3^{\lambda, \lambda_w} - a_4'^{\lambda, \lambda_w}) V_{cd}^*}{\sqrt{6}}$		



Introduction

global fit

Helicity amplitude:

$$a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w} = f_1(q^2) \times \bar{u}(\lambda) \gamma^\mu u(\lambda_i) \epsilon_\mu^*(\lambda_w) - f'_1(q^2) \times \bar{u}(\lambda) \gamma^\mu \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w),$$

$$a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} = \delta f_1(q^2) \times \bar{u}(\lambda) \gamma^\mu u(\lambda_i) \epsilon_\mu^*(\lambda_w) - \delta f'_1(q^2) \times \bar{u}(\lambda) \gamma^\mu \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w),$$

$$a_3^{\lambda, \lambda_w} = \Delta f_1(q^2) \times \bar{u}(\lambda) \gamma^\mu u(\lambda_i) \epsilon_\mu^*(\lambda_w) - \Delta f'_1(q^2) \times \bar{u}(\lambda) \gamma^\mu \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w),$$

$$f_i(q^2) = \frac{f_i}{1 - \frac{q^2}{m_p^2}},$$

Pole model

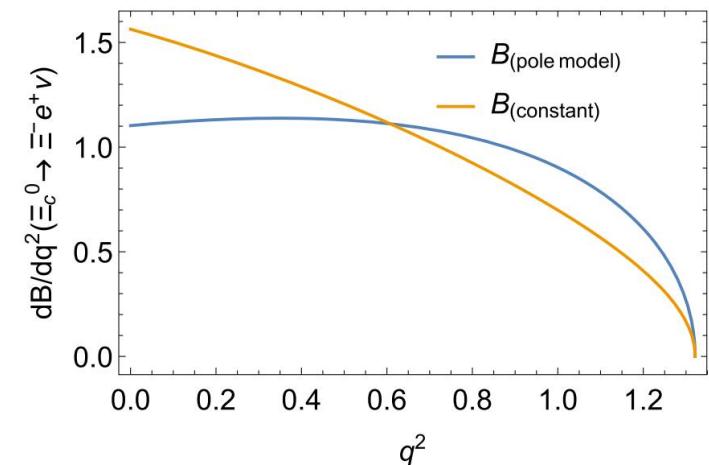


$$f_i(q^2) = f_i$$

Constant

	Experimental data	Fit data (pole model)	Fit data (constant).
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.6 ± 0.4	3.61 ± 0.32	3.62 ± 0.32
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	3.48 ± 0.30	3.45 ± 0.30
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5	3.89 ± 0.73	3.92 ± 0.73
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35	1.29 ± 0.24	1.31 ± 0.24
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	1.24 ± 0.23	1.24 ± 0.23
Fit parameter	$f_1 = 1.01 \pm 0.87, \delta f_1 = -0.51 \pm 0.92$		$\chi^2/d.o.f = 1.6$
(Pole model)		$f'_1 = 0.60 \pm 0.49, \delta f'_1 = -0.23 \pm 0.41$	
Fit parameter	$f_1 = 0.86 \pm 0.92, \delta f_1 = -0.25 \pm 0.88$		$\chi^2/d.o.f = 1.9$
(Constant)		$f'_1 = 0.85 \pm 0.36, \delta f'_1 = -0.43 \pm 0.50$	

The differential branching ratio with two different treatments of form factors





Introduction

$\Xi_c^{0/+} - \Xi_c'^{0/+}$ mixing effect

Symmetry breaking matrix: $\omega = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Amplitude:

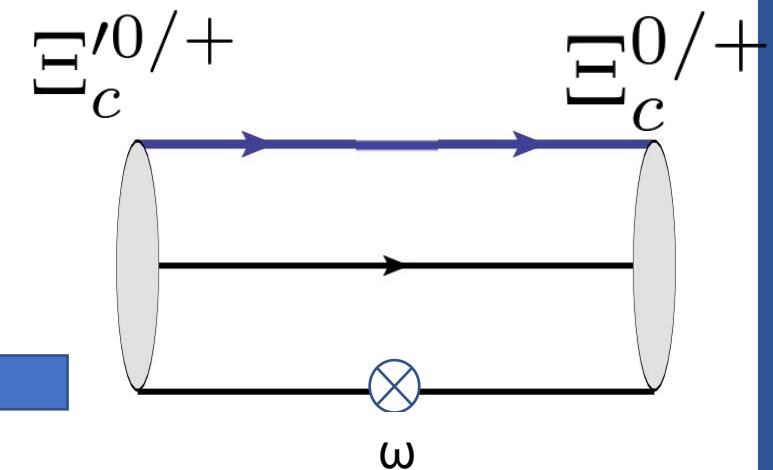
$$H_{\lambda, \lambda_w}(T_{c\bar{3}}\omega \rightarrow T_{c6}) = d^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} \omega_j^k (T_{c6})_{\{ki\}}.$$

The anti-triplet and sextet and mixing by the SU(3) symmetry breaking effect ω .



$$\Xi_c^{0/+mass} = \cos \theta \times \Xi_c^{0/+} + \sin \theta \times \Xi_c'^{0/+}$$

$$T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{+'}}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^{0'}}{\sqrt{2}} \\ \frac{\Xi_c^{+'}}{\sqrt{2}} & \frac{\Xi_c^{0'}}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}$$





Introduction

$\Xi_c^{0/+} - \Xi_c'^{0/+}$ mixing effect

Redefine the helicity amplitude:

$$a_4'^{\lambda, \lambda_w} = a_4^{\lambda, \lambda_w} + c_1^{\lambda, \lambda_w} \theta / \sqrt{2} \quad a_2'^{\lambda, \lambda_w} = a_2^{\lambda, \lambda_w} + \sqrt{2} c_1^{\lambda, \lambda_w} \theta$$

The θ and c_1 can be absorbed into a_2 and a_4

The SU(3) symmetry breaking term a2 and a4 can also be explained by the mixing effect

SU(3) amplitude

$$\begin{aligned} \Lambda_c^+ &\rightarrow \Lambda^0 l^+ \nu & -\sqrt{\frac{2}{3}}(a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}) V_{cs}^* \\ \Lambda_c^+ &\rightarrow n l^+ \nu & a_1 V_{cd}^* \\ \Xi_c^+ &\rightarrow \Sigma^0 l^+ \nu_\ell & \frac{(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} - \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}} \theta) V_{cd}^*}{\sqrt{2}} \\ \Xi_c^+ &\rightarrow \Lambda^0 l^+ \nu_\ell & -\frac{(a_1^{\lambda, \lambda_w} + 2a_2^{\lambda, \lambda_w} - a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + \frac{3c_1^{\lambda, \lambda_w}}{\sqrt{2}} \theta) V_{cd}^*}{\sqrt{6}} \\ \Xi_c^+ &\rightarrow \Xi^0 l^+ \nu_\ell & -(a_1^{\lambda, \lambda_w} + a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w} + \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}} \theta) V_{cs}^* \\ \Xi_c^0 &\rightarrow \Sigma^- l^+ \nu_\ell & (a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} - \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}} \theta) V_{cd}^* \\ \Xi_c^0 &\rightarrow \Xi^- l^+ \nu_\ell & (a_1^{\lambda, \lambda_w} + a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w} + \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}} \theta) V_{cs}^* \end{aligned}$$

$\Xi_c^{0/+} - \Xi_c^{\prime 0/+}$ mixing effect

Lattice[1]

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

QCD sum rules[2]

$$\theta_c = 5.5^\circ \pm 1.8^\circ$$

HQET[3]

$$|\theta_c| = 8.12^\circ \pm 0.80^\circ$$

LFQM[4]

$$\theta_c = 16.27^\circ \pm 2.30^\circ$$

Mass spectrum[5]

$$|\theta_c| = 0.137\pi$$

- [1].H.Liu, L.Liu, P.Sun, W.Sun, J.X.Tan, W.Wang, Y.B.Yang and Q.A.Zhang,[arXiv:2303.17865 [hep-lat]]
- [2].T.M.Aliev, A.Ozpineci and V.Zamiralov,Phys. Rev. D 83, 016008 (2011)
- [3].Y.Matsui, Nucl. Phys. A \textbf{1008}, 122139 (2021)
- [4].H.W.Ke and X.Q.Li, Phys. Rev. D 105, no.9, 9 (2022)
- [5].C.Q.Geng, X.N.Jin and C.W.Liu,Phys. Lett. B 838, 137736 (2023)

$\Xi_c - \Xi'_c$ mixing From Lattice QCD

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²Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou, 730000, China

³University of Chinese Academy of Sciences, Beijing 100049, China

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⁵Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China

⁶CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

⁷School of Fundamental Physics and Mathematical Sciences,

Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China

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In heavy quark limit, the lowest-lying charmed baryons with two light quarks can form an SU(3) triplet and sextet. The Ξ_c in the SU(3) triplet and Ξ'_c in the sextet have the same J^{PC} quantum number and can mix due to the finite charm quark mass and the fact the strange quark is heavier than the up/down quark. We explore the $\Xi_c - \Xi'_c$ mixing by calculating the two-point correlation functions of the Ξ_c and Ξ'_c baryons from lattice QCD. Based on the lattice data, we adopt two independent methods to determine the mixing angle between Ξ_c and Ξ'_c . After making the chiral and continuum extrapolation, it is found that the mixing angle θ is $1.2^\circ \pm 0.1^\circ$, which seems insufficient to account for the large SU(3) symmetry breaking effects found in weak decays of charmed baryons.

$\Xi_c^0/+ - \Xi_c'^0/+$ mixing effect

Lattice[1]

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

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$\Xi_c - \Xi'_c$ mixing From Lattice QCD

Mixing Angle of Hadrons in QCD: A New View

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^bInstitute of Nuclear Physics, M. V. Lomonosov MSU, Moscow, Russia

October 30, 2018

Abstract

A new method for calculation of the mixing angle between the hadrons within QCD sum rules is proposed. In this method, the mixing is expressed in terms of quark and gluon degrees of freedom. As an application, the detailed calculation of the mixing angle between heavy cascade baryons Ξ_Q and Ξ'_Q , $Q = c, b$ is presented and it is found that the mixing angle between Ξ_b (Ξ_c) and Ξ'_b (Ξ'_c) is given by $\theta_b = 6.4^\circ \pm 1.8^\circ$ ($\theta_c = 5.5^\circ \pm 1.8^\circ$).

$\Xi_c - \Xi'_c$ mixing effect



$\Xi_c^0/+ - \Xi_c'^0/+$ mixing effect

Lattice[1]

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

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- [3].Y.Matsui, Nucl. Phys. A 1008, 122139 (2021)
- [4].H.W.Ke and X.Q.Li, Phys. Rev. D 105, no.9, 9 (2022)
- [5].C.Q.Geng, X.N.Jin and C.W.Liu,Phys. Lett. B 838, 137736 (2023)

$\Xi_c - \Xi'_c$ mixing From Lattice QCD

Mixing Angle of Hadrons in QCD: A New View

Mixing Angle of $\Xi_Q - \Xi'_Q$ in Heavy Quark Effective Theory

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Abstract

The Heavy Quark Effective Theory provides a systematic method to estimate a mixing angle of hadron states in a heavy quark, such as the charm quark (c) and the bottom quark (b). By using this method, the mixing angle of the baryons $\Xi_Q - \Xi'_Q$ can be estimated. It is found that the mixing angle between Ξ_Q and Ξ'_Q is given by $\theta_b = 4.51^\circ \pm 0.79^\circ$ for $Q = b$ case and $\theta_c = 8.12^\circ \pm 0.80^\circ$ for $Q = c$ case.

$\Xi_c^{0/+} - \Xi_c'^{0/+}$ mixing effect

Lattice[1]

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

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$\Xi_c - \Xi'_c$ mixing From Lattice QCD

Mixing Angle of Hadrons in QCD: A New View

Mixing Angle of $\Xi_Q - \Xi'_Q$ in Heavy Quark Effective Theory

Revisiting the transition $\Xi_{cc}^+ \rightarrow \Xi_c^{(\prime)+}$ to understand the data from LHCb

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Abstract

The LHCb collaboration newly measured the decay rate of doubly charmed baryon $\Xi_{cc}^{++} \rightarrow \Xi^+\pi^+$ and a ratio of its branching fraction with respect to that of the decay $\Xi_{cc}^{++} \rightarrow \Xi^+\pi^+$ is reported as $1.41 \pm 0.17 \pm 0.10$. This result conflicts with the theoretical predictions made by several groups. In our previous work, following the prescription given in early literature where the us diquark in Ξ_c^+ is assumed to be a scalar whereas in $\Xi_c'^+$ is a vector i.e. the spin-flavor structure of Ξ_c^+ is $[us]_0 c$ and that of $\Xi_c'^+$ is $[us]_1 c$, we studied the case of $\Xi_{cc}^{++} \rightarrow \Xi^{(\prime)+}$ with the light front quark model. Numerically we obtained $\Gamma(\Xi_{cc}^{++} \rightarrow \Xi^+\pi^+)/\Gamma(\Xi_{cc}^{++} \rightarrow \Xi^+\pi^+) = 0.56 \pm 0.18$ which is about half of the data. While abandoning the presupposition, we suppose the spin-flavor structure of us in Ξ_c^+ may be a mixture of scalar and vector, namely the spin-flavor function of Ξ_c^+ could be $\cos\theta [us]_0[c] + \sin\theta [us]_1[c]$. An alternative combination $-\sin\theta [us]_0[c] + \cos\theta [us]_1[c]$ would correspond to $\Xi_c'^+$. Introducing the mixing mechanism the ratio $\Gamma(\Xi_{cc}^{++} \rightarrow \Xi^+\pi^+)/\Gamma(\Xi_{cc}^{++} \rightarrow \Xi^+\pi^+)$ depends on the mixing angle θ . With the mixing scenario, the theoretical prediction on the ratio between the transition rate of $\Xi_{cc}^+ \rightarrow \Xi_c'^+$ and that of $\Xi_{cc}^+ \rightarrow \Xi_c^+$ can coincide with the data as long as $\theta = 16.27^\circ \pm 2.30^\circ$ or $85.54^\circ \pm 2.30^\circ$ is set. Definitely, more precise measurements on other decay portals of Ξ_{cc}^+ are badly needed for testing the mixing mechanism and further determining the mixing angle.

PACS numbers: 13.30.-a, 12.39.Ki, 14.20.Lq

• $E_c - E'_c$ mixing effect



$E_c^{0/+} - E_c'^{0/+}$ mixing effect

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$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

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Mass spectrum [5]

$$|\theta_c| = 0.137\pi$$

- [1].H.Liu, L.Liu, P.Sun, W.Sun, J.X.Tan, W.Wang, Y.B.Yang and Q.A.Zhang,[arXiv:2303.17865 [hep-lat]]
 - [2].T.M.Aliev, A.Ozpineci and V.Zamiralov,Phys. Rev. D 83, 016008 (2011)
 - [3].Y.Matsui, Nucl. Phys. A \textbf{1008}, 122139 (2021)
 - [4].H.W.Ke and X.Q.Li, Phys. Rev. D 105, no.9, 9 (2022)
 - [5].C.Q.Geng, X.N.Jin and C.W.Liu,Phys. Lett. B 838, 137736 (2023)

$\Xi_c - \Xi'_c$ mixing From Lattice QCD

Mixing Angle of Hadrons in QCD: A New View

Mixing Angle of $\Xi_Q - \Xi'_Q$ in Heavy Quark Effective Theory

Revisiting the transition $\Xi_{cc}^+ \rightarrow \Xi_c^{(\prime)+}$ to understand the data from LHCb

Resolving puzzle in $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$ with $\Xi_c - \Xi'_c$ mixing

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Hangzhou Institute for Advanced Study,

SAS, Hangzhou 310024, China

Abstract

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PART 02

- Theoretical framework
-



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Theoretical formwork

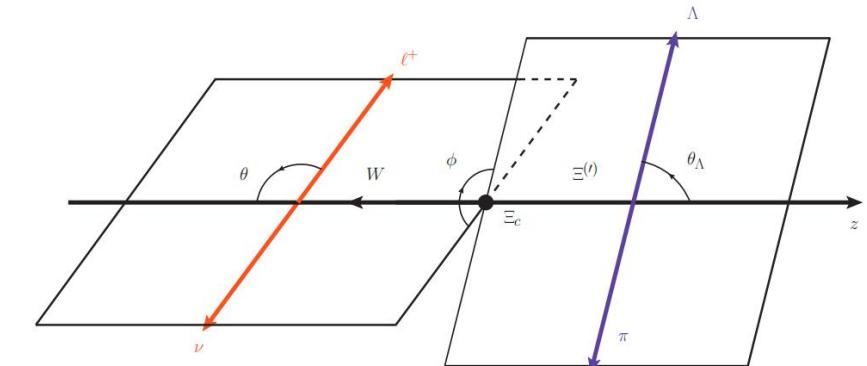
$\Xi_c \rightarrow l\nu\Lambda\pi$ four body decays

$$\Xi_c \rightarrow \Xi \ell^+ \nu \rightarrow \Lambda \pi \ell^+ \nu$$

Resonance: $\Xi^0: \frac{1}{2}^+$, $\Xi^0(1530): \frac{3}{2}^+$, $\Xi^0(1620): \frac{1}{2}$, $\Xi^0(1690): \frac{1}{2}$

$3: \Xi$ $6: \Xi'$

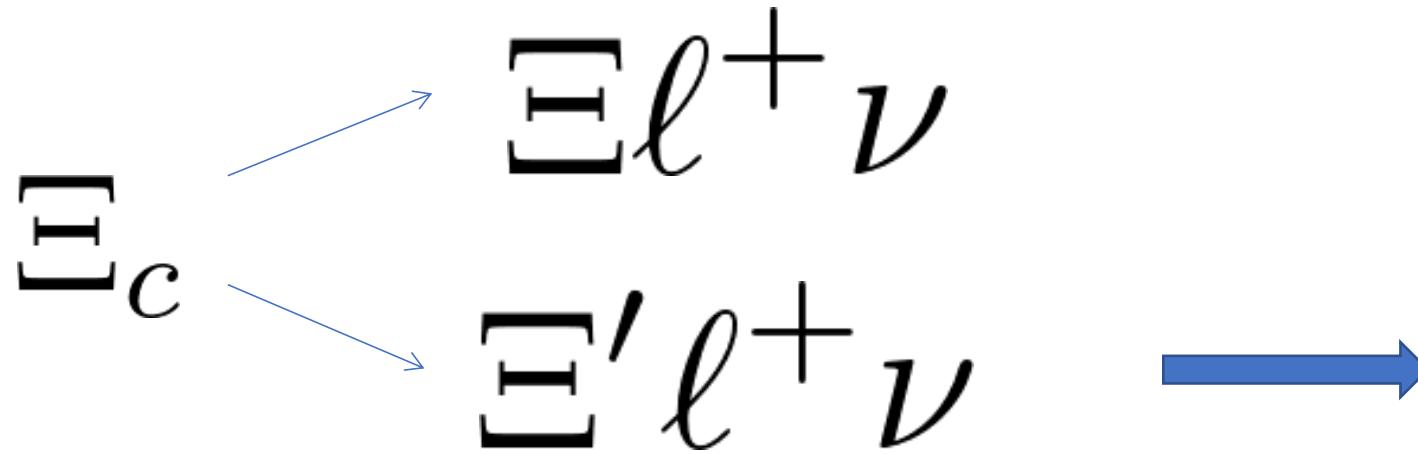
kinematics



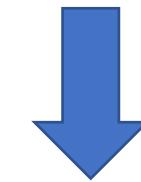
Angular distribution

$\Xi_c^{0/+} - \Xi_c'^{0/+}$ mixing effect

$$\Xi_c^{0+p} = (\Xi_c^{0+})_{flavor} \cos \theta + (\Xi_c'^{0+})_{flavor} \sin \theta.$$



Mixing

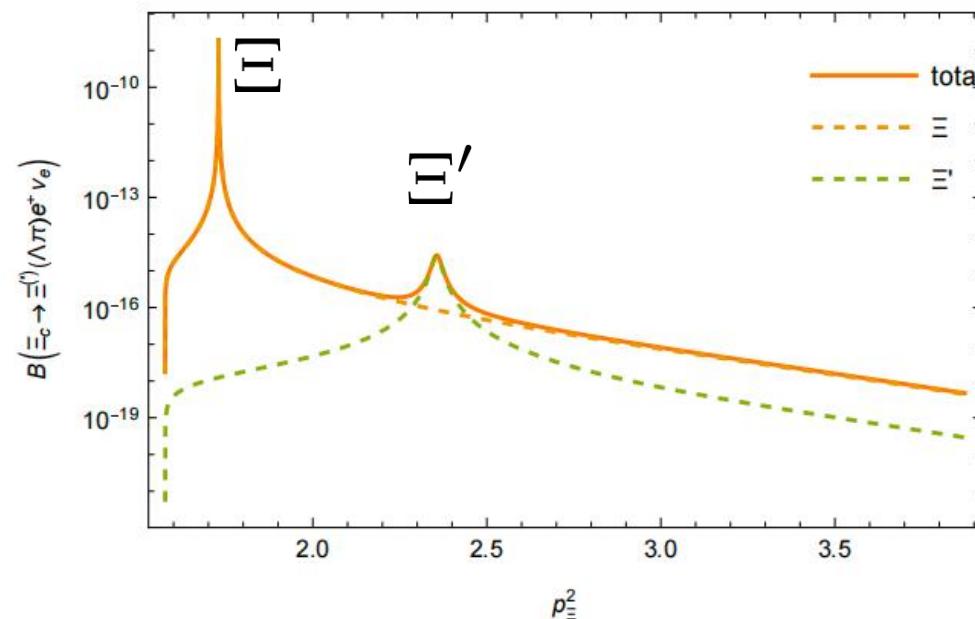


SU(3) forbidden process

Angular distribution

$\Xi_c^{0/+} - \Xi_c'^{0/+}$ mixing effect

$\Xi_c \rightarrow \Xi^{(\prime)} (\Lambda\pi)\ell\nu$



preliminary analysis

$$\Gamma(\Xi_c \rightarrow \Xi(\Lambda\pi)\ell\nu) \propto Br(\Xi \rightarrow \Lambda\pi)$$



99.8%

$$\Gamma(\Xi_c \rightarrow \Xi'(\Lambda\pi)\ell\nu) \propto Br(\Xi' \rightarrow \Lambda\pi)$$



5.02×10^{-13}

[1] R. M. Wang, M. Z. Yang, H. B. Li and X. D. Cheng, Phys. Rev. D 100, no.7, 076008 (2019)

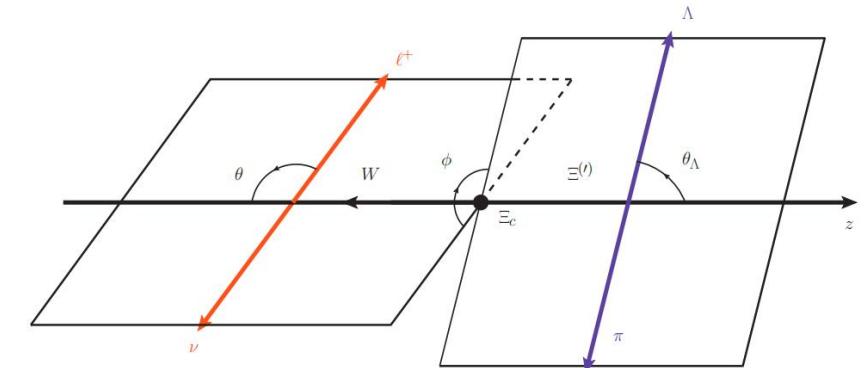
Amplitude

$$\mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)}(\Lambda\pi)\ell^+\nu) = \sum_{J_{\Xi^{(\prime)}}} \sum_{s_{\Xi^{(\prime)}}} i\mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)}\ell^+\nu) \\ \times \frac{i}{p_{\Xi^{(\prime)}}^2 - m_{\Xi^{(\prime)}}^2 + im_{\Xi^{(\prime)}}\Gamma_{\Xi^{(\prime)}}} i\mathcal{M}(\Xi^{(\prime)} \rightarrow \Lambda\pi), \quad (2)$$

Breit-Wigner form

Sub-processes amplitude

kinematics



Amplitude

Hamiltonian

$$\mathcal{H}_{c \rightarrow s} = \frac{G_F}{\sqrt{2}} [V_{cs}^* \bar{s} \gamma^\mu (1 - \gamma_5) c \bar{\nu} \gamma_\mu (1 - \gamma_5) \ell] + h.c.. \quad (3)$$

$$\begin{aligned} i\mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)} \ell^+ \nu) &= \sum_{s_w} \frac{G_F}{\sqrt{2}} V_{cs}^* \bar{u}_\nu \gamma_\rho (1 - \gamma_5) \nu_\ell \epsilon^\rho(s_w) \\ &\quad \times \langle \Xi^{(\prime)} | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c \rangle \epsilon_\mu^*(s_w) \\ &= \sum_{s_w} \frac{G_F}{\sqrt{2}} V_{cs}^* L_{s_\ell}^{s_w}(\phi, \theta) \times h_{s_w, s_{\Xi}}^{s_{\Xi_c}} \quad (4) \end{aligned}$$

Wigner Function

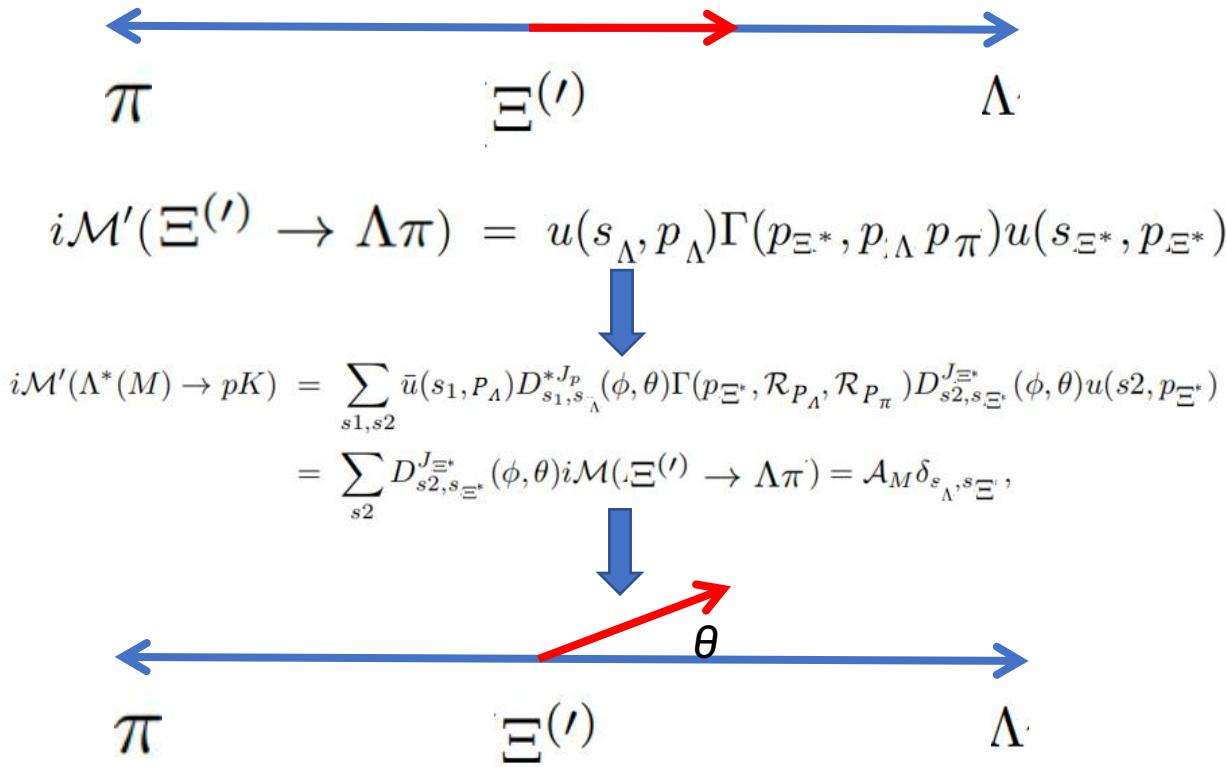
$$i\mathcal{M}(\Xi^{(\prime)} \rightarrow \Lambda \pi) = \mathcal{A}^{(\prime)} \times D_{s_{\Xi^{(\prime)}}, s_\Lambda}^{J_\Xi}(\phi_\Xi, \theta_\Xi),$$

$$A = \sqrt{\Gamma(\Xi \rightarrow \Lambda \pi) 8\pi m_\Xi^2 / |p_\Lambda|}$$

$$A' = \sqrt{\Gamma(\Xi' \rightarrow \Lambda \pi) 16\pi m_{\Xi^{(\prime)}}^2 / |p_\Lambda|}.$$

Amplitude

Special process



Wigner Function

$$i\mathcal{M}(\Xi^{(\prime)} \rightarrow \Lambda\pi) = \mathcal{A}^{(\prime)} \times D^{J_\Xi}_{s_{\Xi^{(\prime)}}, s_\Lambda}(\phi_\Xi, \theta_\Xi),$$

$$A = \sqrt{\Gamma(\Xi \rightarrow \Lambda\pi) 8\pi m_\Xi^2 / |p_\Lambda|}$$

$$A' = \sqrt{\Gamma(\Xi' \rightarrow \Lambda\pi) 16\pi m_{\Xi'}^2 / |p_\Lambda|}.$$

Angular distribution

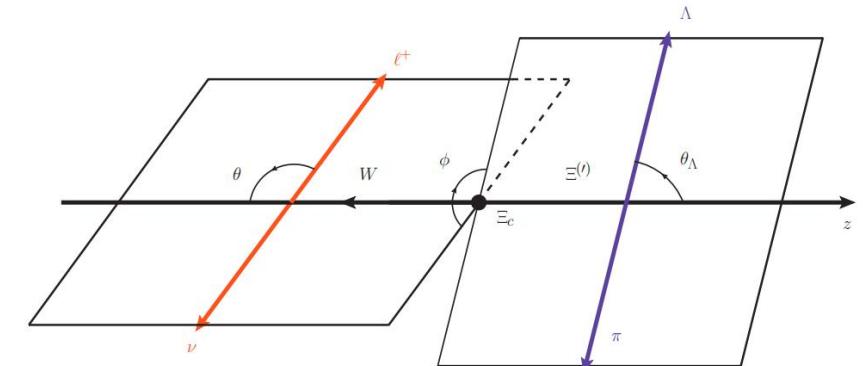
decay width

$$d\Gamma = d\Pi_4 \times \frac{(2\pi)^4}{2m_{\Xi_c}} |\mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)} (\Lambda\pi)\ell^+\nu)|^2,$$

$$\begin{aligned} \mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)} (\Lambda\pi)\ell^+\nu) &= \sum_{J_{\Xi^{(\prime)}}} \sum_{s_{\Xi^{(\prime)}}, s_w} \frac{G_F}{\sqrt{2}} V_{cs}^* \\ &\quad \times H_{s_{\Xi_c}, s_{\Xi^{(\prime)}}}^{J_{\Xi^{(\prime)}}} L_{s_\ell}^{s_w}(\phi, \theta) D_{s_{\Xi^{(\prime)}}, s_\Lambda}^{J_\Xi}(\phi_\Xi, \theta_\Xi), \end{aligned}$$

$$\begin{aligned} H_{s_{\Xi_c}, s_{\Xi^{(\prime)}}}^{J_{\Xi^{(\prime)}}} &= \frac{iA^{(\prime)}}{p_{\Xi^{(\prime)}}^2 - m_{\Xi^{(\prime)}}^2 + im_{\Xi^{(\prime)}}\Gamma_{\Xi^{(\prime)}}} h_{s_w, s_{\Xi}}^{s_{\Xi_c}} \\ &= L_{\Xi^{(\prime)}} h_{s_w, s_{\Xi}}^{s_{\Xi_c}}. \end{aligned}$$

kinematics



PART 03

- Angular distribution

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Angular distribution

$$\begin{aligned} \frac{d\Gamma}{d \cos \theta d \cos \theta_{\Xi} d\phi dp_{\Xi}^2 dq^2} = & \mathcal{P} \left(L_{11} + L_{12} \cos \theta_{\Lambda} + L_{13} \cos 2\theta_{\Lambda} + (L_{21} + L_{22} \cos \theta_{\Lambda}) \cos 2\phi \right. \\ & + (L_{31} + L_{32} \cos \theta_{\Lambda} + L_{33} \cos 2\theta_{\Lambda}) \cos \theta \\ & + (L_{41} + L_{42} \cos 2\phi + L_{43} \cos \theta_{\Lambda} + L_{44} \cos 2\theta_{\Lambda} + L_{45} \cos 2\theta_{\Lambda} \cos 2\phi) \cos 2\theta \\ & + (L_{51} \sin \theta_{\Lambda} + L_{52} \sin 2\theta_{\Lambda}) \sin \theta \cos \phi + (L_{61} \sin \theta_{\Lambda} + L_{62} \sin 2\theta_{\Lambda}) \sin 2\theta \cos \phi \\ & + (L_{71} \sin \theta_{\Lambda} + L_{72} \sin 2\theta_{\Lambda}) \sin \theta \sin \phi + (L_{81} \sin \theta_{\Lambda} + L_{82} \sin 2\theta_{\Lambda}) \sin 2\theta \sin \phi \\ & \left. + (L_{91} + L_{92} \cos 2\theta_{\Lambda}) \sin 2\phi + (L_{101} + L_{102} \cos 2\theta_{\Lambda}) \sin 2\phi \cos 2\theta \right), \end{aligned}$$

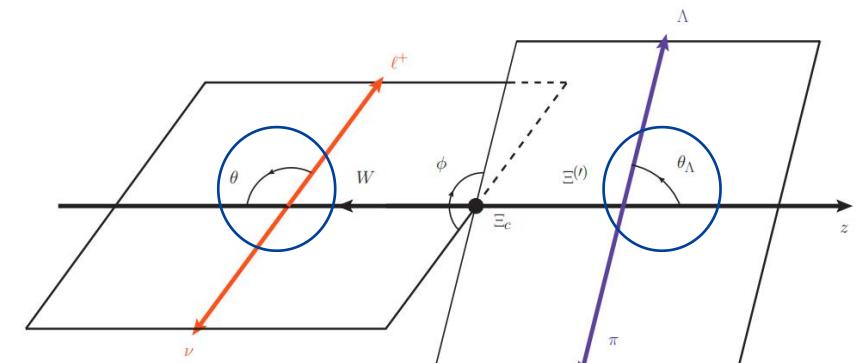
$$\mathcal{P} = \frac{G_F^2 |V_{cs}|^2}{2} \frac{(1 - \hat{m}_{\ell}^2) \sqrt{\lambda(m_{\Xi_c}, \sqrt{p_{\Xi}^2}, \sqrt{q^2}) \lambda(\sqrt{p_{\Xi}^2}, m_{\Lambda}, m_{\pi})}}{(2\pi)^6 512 m_{\Xi_c}^3 p_{\Xi}^2 \sqrt{p_{\Xi}^2}},$$

Observables

$$\frac{d\Gamma}{dp_{\Xi}^2} = \frac{8\pi}{9} \mathcal{P} \left(9L_{11} - 3L_{13} - 3L_{41} + L_{44} \right).$$

$$\begin{aligned} \frac{dA_{FB}}{dp_{\Xi}^2} &= \frac{\left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_{\lambda} \frac{d\Gamma}{dp_{\Xi}^2 d \cos \theta_{\Lambda}}}{\int_{-1}^1 d \cos \theta_{\lambda} \frac{d\Gamma}{dp_{\Xi}^2 d \cos \theta_{\Lambda}}} \\ &= \frac{3}{2} \frac{3L_{12} - L_{33}}{9L_{11} - 3L_{13} - 3L_{41} + L_{44}} \\ &= \frac{4}{3} \frac{\sum_{s_{\Xi_c}, s_{\Xi}} \mathcal{R}_e(H_{s_{\Xi_c}, s_{\Xi}}^{\frac{1}{2}}, H_{s_{\Xi_c}, s_{\Xi}}^{\frac{3}{2}*})}{\sum_{s_{\Xi_c}, s_{\Xi'}} (2|H_{s_{\Xi_c}, s_{\Xi}}^{\frac{1}{2}}|^2 + |H_{s_{\Xi_c}, s_{\Xi'}}^{\frac{3}{2}}|^2)}. \end{aligned}$$

kinematics



$$A_{FB}^{\theta}$$

Interference of vector
and axis-vector current

$$A_{FB}^{\theta_{\Lambda}}$$

Interference of resonance
with different spin

Observables

previous work: $\Lambda_b \rightarrow \Lambda^*(pK^-)J/\psi(\rightarrow \ell^+\ell^-)$ [PRD 106 \(2022\) 11, 114041](#)

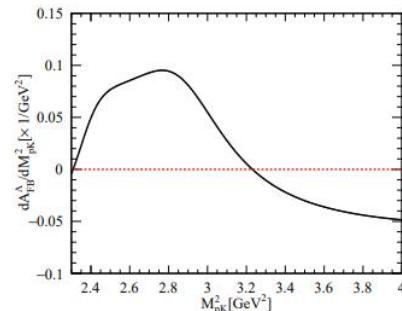


FIG. 4: The dA_F^Λ/dM_{pK}^2 of process $\Lambda_b \rightarrow \Lambda^*(pK^-)J/\psi(\ell^+\ell^-)$ for $\ell = \mu$.

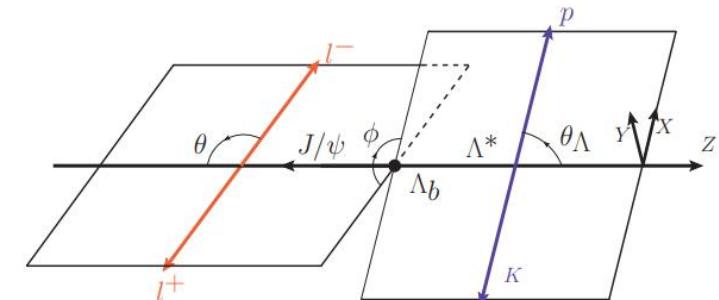
It can be seen from Fig.4 that there are two cross point s_0^1 and s_0^2 :

$$s_0^1 = 2.307 \text{ GeV}^2, \quad s_0^2 = 3.231 \text{ GeV}^2.$$

The two points are very close to the invariant mass square of $\Lambda_{1520,1800}^*$: $m_{\Lambda_{1520}^*}^2 = 2.308 \text{ GeV}^2, m_{\Lambda_{1800}^*}^2 = 3.240 \text{ GeV}^2$.

Thus the s_0^1 and s_0^2 should be close to the mass square of $\Lambda_{1520,1800}^*$. It will be a new method for precisely measuring resonant mass in experiments. Besides, one can find that the A_F^Λ is positive in the region $M_{pK}^2 = [s_0^1, s_0^2]$ and negative

kinematics

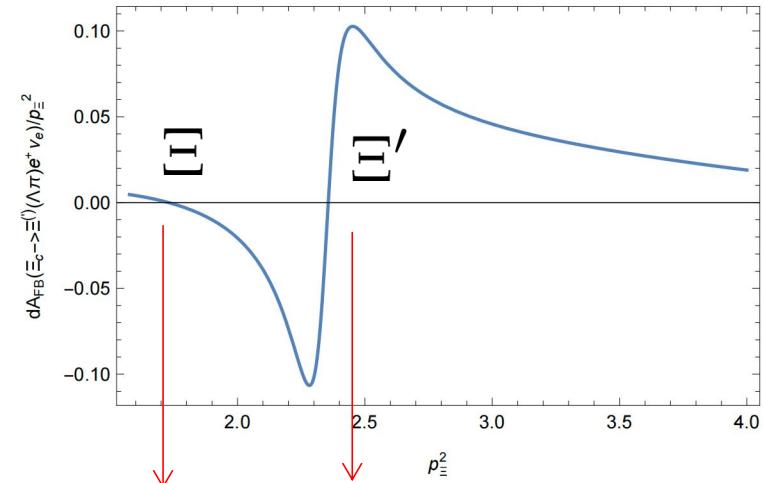


Can we search the resonance Ξ' and θ by forward-backward assymetry?

Forward-backward Asymmetry

$$\begin{aligned} & \sum_{s_{\Xi_c}, s_{\Xi}} \mathcal{R}_e(H_{s_{\Xi_c}, s_{\Xi}}^{\frac{1}{2}}, H_{s_{\Xi_c}, s_{\Xi}}^{\frac{3}{2}*}) \\ &= \frac{(p_{\Xi}^2 - m_{\Xi}^2)(p_{\Xi}^2 - m_{\Xi'}^2) - \Gamma_{\Xi} m_{\Xi} \Gamma_{\Xi'} m_{\Xi'}}{((p_{\Xi}^2 - m_{\Xi}^2)^2 + \Gamma_{\Xi}^2 m_{\Xi}^2)((p_{\Xi'}^2 - m_{\Xi'}^2)^2 + \Gamma_{\Xi'}^2 m_{\Xi'}^2)} \\ &\times (\cos \theta_c h_{s_w, s_{\Xi}}^{3, s_{\Xi_c}} + \sin \theta_c h_{s_w, s_{\Xi}}^{6, s_{\Xi_c}}) \sin \theta_c h_{s_w, s_{\Xi}}^{6, s_{\Xi_c}}. \quad (11) \end{aligned}$$

$$\begin{aligned} s_1 &= \frac{1}{2}(m_{\Xi}^2 + m_{\Xi'}^2 - \sqrt{(m_{\Xi}^2 - m_{\Xi'}^2)^2 - 4\Gamma_{\Xi} m_{\Xi} \Gamma_{\Xi'} m_{\Xi'}}) \\ &= \boxed{m_{\Xi}^2} - \frac{\Gamma_{\Xi} m_{\Xi} \Gamma_{\Xi'} m_{\Xi'}}{m_{\Xi'}^2 - m_{\Xi}^2} + O(\Gamma_{\Xi'})^2, \\ s_2 &= \frac{1}{2}(m_{\Xi}^2 + m_{\Xi'}^2 + \sqrt{(m_{\Xi}^2 - m_{\Xi'}^2)^2 - 4\Gamma_{\Xi} m_{\Xi} \Gamma_{\Xi'} m_{\Xi'}}) \\ &= \boxed{m_{\Xi'}^2} + \frac{\Gamma_{\Xi} m_{\Xi} \Gamma_{\Xi'} m_{\Xi'}}{m_{\Xi'}^2 - m_{\Xi}^2} + O(\Gamma_{\Xi'})^2. \quad (14) \end{aligned}$$



two zero point

$$\begin{aligned} \Gamma_{\Xi} &= 2.26 \times 10^{-15} \text{GeV} \\ \Gamma_{\Xi'} &= 0.0093 \text{GeV} \end{aligned}$$

PART 05

- Numerical results
-



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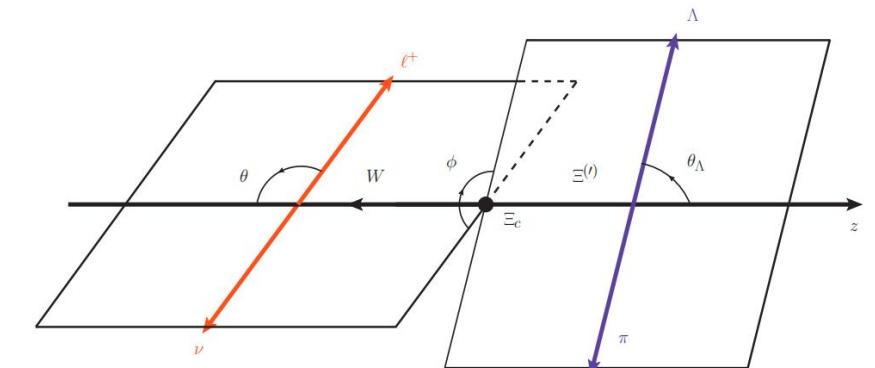
decay width

$$d\Gamma = d\Pi_4 \times \frac{(2\pi)^4}{2m_{\Xi_c}} |\mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)} (\Lambda\pi)\ell^+\nu)|^2,$$

$$\begin{aligned} \mathcal{M}(\Xi_c \rightarrow \Xi^{(\prime)} (\Lambda\pi)\ell^+\nu) &= \sum_{J_{\Xi^{(\prime)}}} \sum_{s_{\Xi^{(\prime)}}, s_w} \frac{G_F}{\sqrt{2}} V_{cs}^* \\ &\quad \times H_{s_{\Xi_c}, s_{\Xi^{(\prime)}}}^{J_{\Xi^{(\prime)}}} L_{s_\ell}^{s_w}(\phi, \theta) D_{s_{\Xi^{(\prime)}}, s_\Lambda}^{J_\Xi}(\phi_\Xi, \theta_\Xi), \end{aligned}$$

$$\begin{aligned} H_{s_{\Xi_c}, s_{\Xi^{(\prime)}}}^{J_{\Xi^{(\prime)}}} &= \frac{iA^{(\prime)}}{p_{\Xi^{(\prime)}}^2 - m_{\Xi^{(\prime)}}^2 + im_{\Xi^{(\prime)}}\Gamma_{\Xi^{(\prime)}}} h_{s_w, s_{\Xi}}^{s_{\Xi_c}} \\ &= L_{\Xi^{(\prime)}} h_{s_w, s_{\Xi}}^{s_{\Xi_c}}. \end{aligned}$$

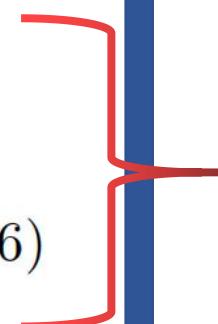
kinematics



Hadron matrix element

$$h_{s_w, s_{\Xi}^{(\prime)}}^{s_{\Xi_c}} = \langle \Xi^{(\prime)} | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c^p \rangle \epsilon_\mu^*(s_w)$$

$$\begin{aligned} \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c^p \rangle &= \cos \theta \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c \rangle \\ &\quad + \sin \theta \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi'_c \rangle \\ \langle \Xi' | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c^p \rangle &= \sin \theta \langle \Xi' | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi'_c \rangle. \end{aligned} \quad (16)$$



Three hadronic matrix element

Numerical results

$$\begin{aligned} \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c \rangle = \\ \times \left(\bar{u}(p_\Xi, s_\Xi) [f_1 \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Xi_c}} f_2 + \frac{q^\mu}{m_{\Xi_c}} f_3] u(p_{\Xi_c}, s_{\Xi_c}) \right. \\ \left. - \bar{u}(p_\Xi, s_\Xi) [g_1 \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Xi_c}} g_2 + \frac{q^\mu}{m_{\Xi_c}} g_3] \gamma_5 u(p_{\Xi_c}, s_{\Xi_c}) \right), \end{aligned}$$

$$\begin{aligned} \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi'_c \rangle = \\ \times \left(\bar{u}(p_\Xi, s_\Xi) [f'_1 \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Xi_c}} f'_2 + \frac{q^\mu}{m_{\Xi_c}} f'_3] u(p_{\Xi_c}, s_{\Xi_c}) \right. \\ \left. - \bar{u}(p_\Xi, s_\Xi) [g'_1 \gamma^\mu + \frac{i\sigma^{\mu\nu} q_\nu}{m_{\Xi_c}} g'_2 + \frac{q^\mu}{m_{\Xi_c}} g'_3] \gamma_5 u(p_{\Xi_c}, s_{\Xi_c}) \right), \end{aligned}$$

$$\begin{aligned} \langle \Xi' | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi'_c \rangle = & \left(\bar{u}_\rho(p_\Xi, s_{\Xi'}) \left[(F_1 \gamma^\mu + \frac{p_{\Xi_c}^\mu}{m_{\Xi_c}} F_2 \right. \right. \\ & \left. \left. + \frac{p_{\Xi'}^\mu}{m_{\Xi'}} F_3 \right) \frac{p_{\Xi_c}^\rho}{m_{\Xi_c}} + g^{\mu\rho} F_4 \right] \gamma_5 u(p_{\Xi_c}, s_{\Xi_c}) \right. \\ & - \bar{u}_\rho(p_\Xi, s_{\Xi'}) \left[(G_1 \gamma^\mu + \frac{p_{\Xi_c}^\mu}{m_{\Xi_c}} G_2 + \frac{p_{\Xi'}^\mu}{m_{\Xi'}} G_3) \frac{p_{\Xi_c}^\rho}{m_{\Xi_c}} \right. \\ & \left. \left. + g^{\mu\rho} G_4 \right] u(p_{\Xi_c}, s_{\Xi_c}) \right). \end{aligned} \quad (17)$$



Lattice Calculation

Eur. Phys. J. C (2022) 82:11



Light-cone Sum rule Calculaion

arXiv:1107.5925v2



Light-front quark model

Eur. Phys. J. C (2020) 80:1066

Mixing angle

Lattice[1]

$$\theta = (1.200 \pm 0.090 \pm 0.020)^\circ$$

QCD sum rules[2]

$$\theta_c = 5.5^\circ \pm 1.8^\circ$$

HQET[3]

$$|\theta_c| = 8.12^\circ \pm 0.80^\circ$$

LFQM[4]

$$\theta_c = 16.27^\circ \pm 2.30^\circ$$

Mass spectrum[5]

$$|\theta_c| = 0.137\pi$$

- [1].H.Liu, L.Liu, P.Sun, W.Sun, J.X.Tan, W.Wang, Y.B.Yang and Q.A.Zhang,[arXiv:2303.17865 [hep-lat]]
- [2].T.M.Aliev, A.Ozpineci and V.Zamiralov,Phys. Rev. D 83, 016008 (2011)
- [3].Y.Matsui, Nucl. Phys. A 1008, 122139 (2021)
- [4].H.W.Ke and X.Q.Li, Phys. Rev. D 105, no.9, 9 (2022)
- [5].C.Q.Geng, X.N.Jin and C.W.Liu,Phys. Lett. B 838, 137736 (2023)

The matrix element depends on the mixing angle

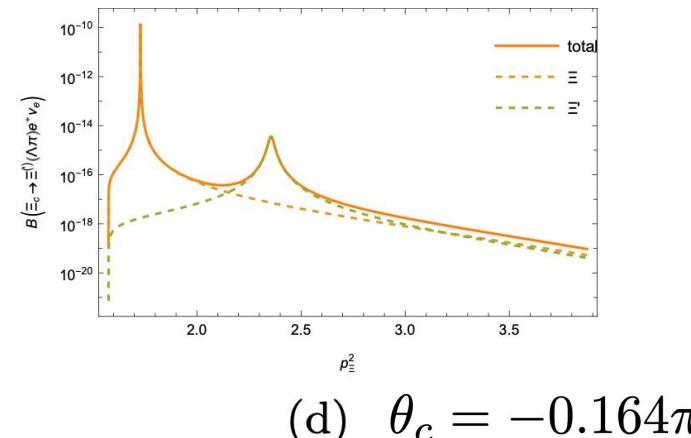
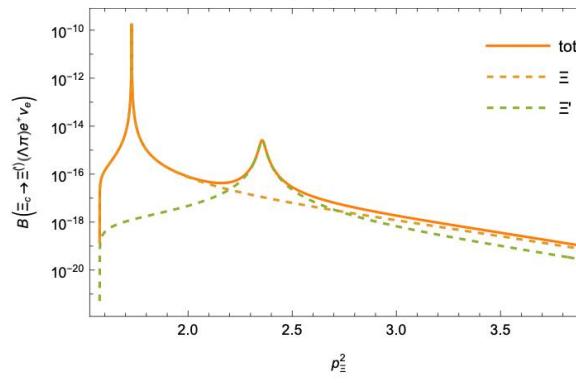
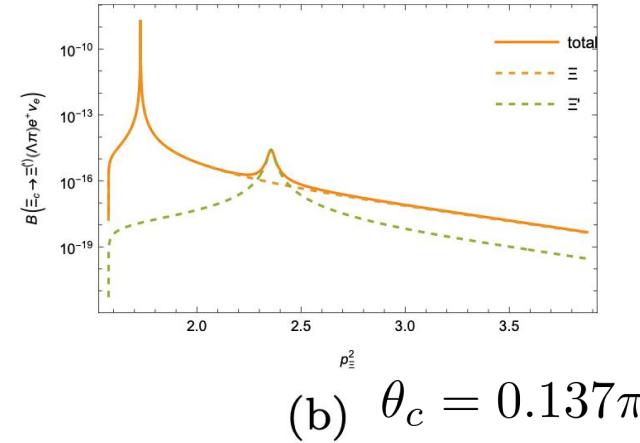
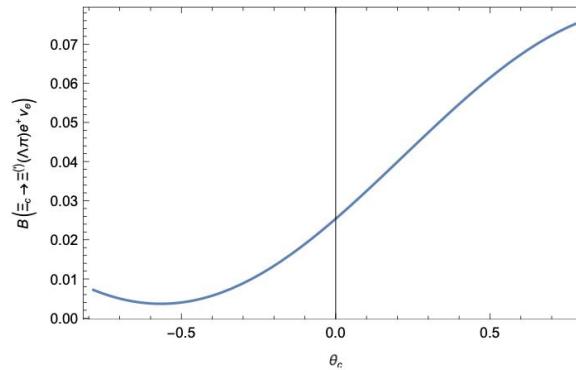
$$\begin{aligned} \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c^p \rangle &= \cos \theta \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c \rangle \\ &\quad + \sin \theta \langle \Xi | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi'_c \rangle \\ \langle \Xi' | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi_c^p \rangle &= \sin \theta \langle \Xi' | \bar{s} \gamma^\mu (1 - \gamma_5) c | \Xi'_c \rangle. \end{aligned} \quad (16)$$

Comparing with Belle data:
 $\mathcal{B}(\Xi_c^p \rightarrow \Xi e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.03 \pm 0.38)\%$

$$\theta_c = -0.164\pi$$

Y. B. Li et al. [Belle], Phys. Rev. Lett. 127, no.12, 121803 (2021)

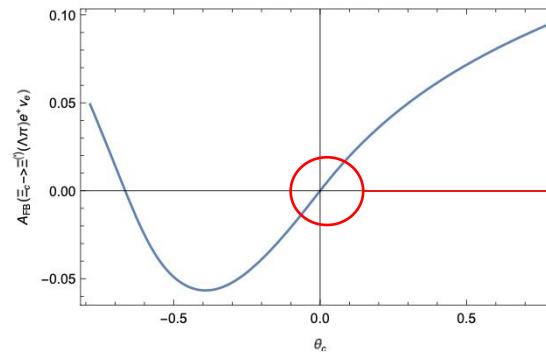
Decay width



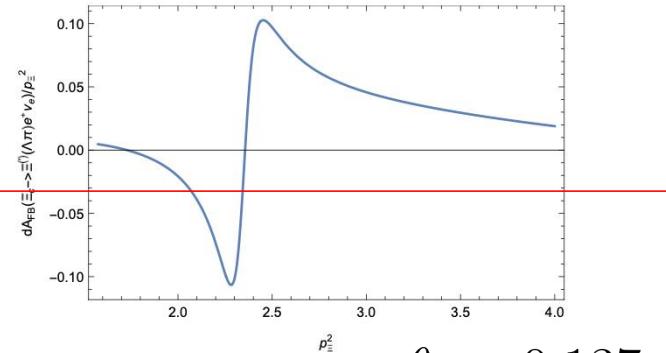
Observables	$\theta_c = 0.137\pi$	$\theta_c = -0.137\pi$	$\theta_c = -0.164\pi$
$\Gamma(\Xi^-)(\text{GeV})$	2.476×10^{-13}	2.198×10^{-14}	1.678×10^{-14}
$\Gamma(\Xi'^-)(\text{GeV})$	4.975×10^{-28}	4.975×10^{-28}	4.975×10^{-28}
$\Gamma(\Xi^{(\prime)-})(\text{GeV})$	2.476×10^{-13}	2.198×10^{-14}	1.678×10^{-14}
$\mathcal{B}(\Xi^{(\prime)-})(\%)$	5.68	0.504	0.385

- Model dependence
- Does not change significantly with angle

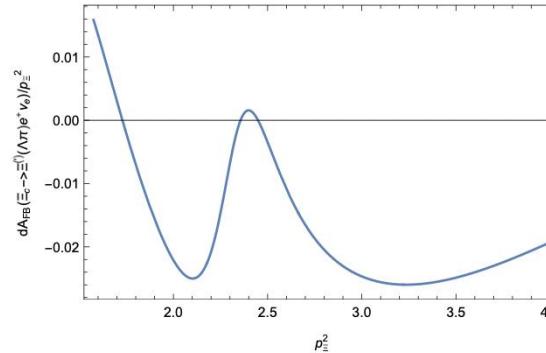
Forward-backward Asymmetry



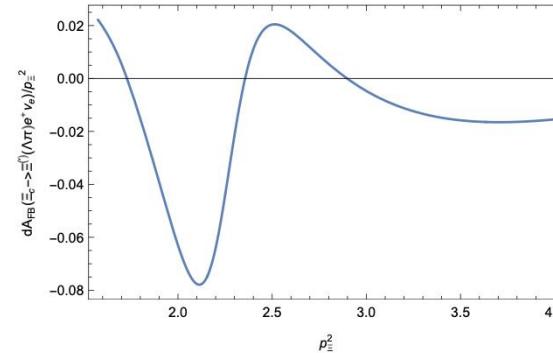
(a)



$\theta_c = 0.137\pi$



(c) $\theta_c = -0.137\pi$



(d) $\theta_c = -0.164\pi$

Observables	$\theta_c = 0.137\pi$	$\theta_c = -0.137\pi$	$\theta_c = -0.164\pi$
$\Gamma(\Xi^-)(\text{GeV})$	2.476×10^{-13}	2.198×10^{-14}	1.678×10^{-14}
$\Gamma(\Xi'^-)(\text{GeV})$	4.975×10^{-28}	4.975×10^{-28}	4.975×10^{-28}
$\Gamma(\Xi^{(\prime)-})(\text{GeV})$	2.476×10^{-13}	2.198×10^{-14}	1.678×10^{-14}
$\mathcal{B}(\Xi^{(\prime)-})(\%)$	5.68	0.504	0.385
$A_{FB}(\text{GeV}^2)$	0.0647	-0.0557	-0.0466

disappeared when mixing angle is zero

$$\begin{aligned}
 & \sum_{s_{\Xi_c}, s_{\Xi}} \mathcal{R}_e(H_{s_{\Xi_c}, s_{\Xi}}^{\frac{1}{2}} H_{s_{\Xi_c}, s_{\Xi}}^{\frac{3}{2}*}) \\
 &= \frac{(p_{\Xi}^2 - m_{\Xi}^2)(p_{\Xi}^2 - m_{\Xi'}) - \Gamma_{\Xi} m_{\Xi} \Gamma_{\Xi'} m_{\Xi'}}{((p_{\Xi}^2 - m_{\Xi}^2)^2 + \Gamma_{\Xi}^2 m_{\Xi}^2)((p_{\Xi}^2 - m_{\Xi'}^2)^2 + \Gamma_{\Xi'}^2 m_{\Xi'}^2)} \\
 & \quad \times (\cos \theta_c h_{s_w, s_{\Xi}}^{3, s_{\Xi_c}} + \sin \theta_c h_{s_w, s_{\Xi}}^{6, s_{\Xi_c}}) \sin \theta_c h_{s_w, s'_{\Xi}}^{6, s_{\Xi_c}}. \quad (11)
 \end{aligned}$$

- The puzzle in charm baryon decays can be solved by the $\Xi_c - \Xi'_c$ mixing effect.
- The mixing effect can be observed in angular distribution of $\Xi_c \rightarrow \Xi^{(\prime)} (\Lambda\pi)\ell\nu$ process
- The forward-backward asymmetry is good observables for searching the mixing effect.

Thanks!

