QCD calculations of $B_{d,s} \to \pi, K$ form factors beyond leading power

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[Cui, BY., Huang, YK., Shen, YL. et al. J. High Energ. Phys. 2023, 140 (2023).]

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April 22, 2023

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Calculational tools for $B \to P$ form factors

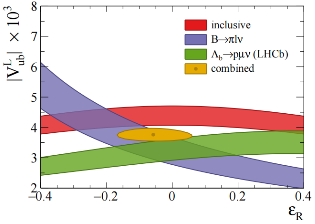
- ▽ QCD/SCET Factorization
  - Factorization formulae for semeleptonic B-meson decays [BBNS, BPRS, and many others]

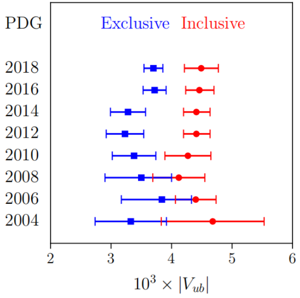
$$F_i^{B\to M}(E) = C_i(E)\xi_a(E) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 dv\, C_i^{(B1)}(E,\tau) \otimes_\tau J_a(\tau; v, \omega)\phi_B^+(\omega)\phi_M(v).$$

  - Perturbative calculations of the hard matching coefficients. [Bauer, Fleming, Pirjol, Stewart, 2001]
  - Perturbative calculations of the hard-collinear matching coefficients. [Becher, Hill, 2004; Hill, Becher, Lee, Neubert, 2004; Beneke, Yang, 2006]
- ▽ Transverse-Momentum Dependent(TMD) Factorization [Botts, Sterman, 1989; Li, Sterman, 1992]
- ▽ Lattice QCD Technique
  - First-principles calculations numerically at small hadronic recoil.
- ▽ Light-Cone Sum Rules in QCD/SCET(LCSR)
  - Light-cone QCD sum rules with the light-meson LCDA. [Ball, Braun, Khodjamirian, etc]
  - Light-cone QCD sum rules with the B-meson LCDA. [Khodjamirian, Lü, Shen, Wang, etc]
  - Light-cone SCET sum rules with B-meson LCDA. [[Feldmann, Lü, Shen, Wang, etc]
  - Light-cone QCD sum rules with the chiral current for the light meson.[Huang, Li, Wu, etc]

Why subleading power corrections

- Understanding the general properties of power expansion in EFTs (HQET, SCET, NRQCD).
- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of the (higher-twist) B-meson DAs.
  - Interplay of different QCD techniques.
- Precision determination of the CKM matrix element $|V_{ub}|$.
  - Power corrections, QED corrections.
  - Long-standing tension between the exclusive and inclusive $|V_{ub}|$.





B-meson Light-Cone Sum Rules

- The form factors for $B \to P$ are defined as [Beneke,Feldmann,2001]:

$$\langle P(p)|\bar{q}\,\gamma_\mu\, b|\bar{B}(P)\rangle = 2 f^{+}_{B\,P}(q^2)\, p_\mu + \left[f^{+}_{B\,P}(q^2) + f^{-}_{B\,P}(q^2)\right] q_\mu ,$$

$$\langle P(p)|\bar{q}\,\sigma_{\mu\nu} q^\nu\, b|\bar{B}(P)\rangle = i\,\frac{f^{T}_{B\,P}(q^2)}{m_B + m_P}\,\left[q^2 (2p+q)_\mu - (m_B^2 - m_P^2)\, q_\mu\right].$$

- Vacuum-to-B-meson correlation function defined as:

$$\Pi_\mu(n\cdot p, \bar{n}\cdot p) = \int d^4x\, e^{ip\cdot x} \langle 0|T\left\{\bar{d}(x)\, \not{\bar{n}}\gamma_5\, q(x),\; \bar{q}(0)\,\Gamma_\mu\, b(0)\right\}|B(P)\rangle$$
$$= \begin{cases} \Pi(n\cdot p, \bar{n}\cdot p)\, n_\mu + \tilde{\Pi}(n\cdot p, \bar{n}\cdot p)\, \bar{n}_\mu , & \Gamma_\mu = \gamma_\mu \\ i\Pi_T(n\cdot p, \bar{n}\cdot p)\, \left[n\cdot q\, \bar{n}_\mu - \bar{n}\cdot q\, n_\mu\right], & \Gamma_\mu = \sigma_{\mu\nu} q^\nu \end{cases} .$$

- Power counting: $\quad n\cdot p \sim \mathcal{O}(m_b), \qquad \bar{n}\cdot p \sim \mathcal{O}(\Lambda)$
- The hadronic representation and dispersion relation:

$$\Pi_{\mu,\mathrm{had}}(p,q) = \frac{-i\langle 0|\bar{d}\,\not{\bar{n}}\gamma_5\, q|P(p)\rangle\langle P(p)|\bar{q}\,\Gamma^\mu\, b|B(P)\rangle}{m_P^2 - p^2} + \text{continuum},$$
$$= \frac{f_P}{m_P^2/n\cdot p - \bar{n}\cdot p}\langle P(p)|\bar{q}\,\Gamma^\mu\, b|B(P)\rangle + \frac{1}{\pi}\int_{\omega_s}^{\infty} \frac{d\omega'}{\omega' - \bar{n}\cdot p - i0} \mathrm{Im}'_{\omega}\Pi_\mu(n\cdot p, \omega')$$

B-meson Light-Cone Sum Rules

- LCSR for $B \to P$ form factors:

$$f^+_{B\,P}(q^2) = \frac{1}{f_P\, n\cdot p}\, e^{\frac{m_p^2}{(n\cdot p\,\omega_M)}} \int_0^{\omega_s} \frac{d\omega'}{\pi}\, e^{\frac{\omega'}{\omega_M}}\, \mathrm{Im}_{\omega'} \left[\tilde{\Pi}(n\cdot p, \omega') + \frac{n\cdot p - m_B}{m_B} \Pi(n\cdot p, \omega')\right],$$

$$f^0_{B\,P}(q^2) = \frac{1}{f_P\, m_B}\, e^{\frac{m_p^2}{(n\cdot p\,\omega_M)}} \int_0^{\omega_s} \frac{d\omega'}{\pi}\, e^{\frac{\omega'}{\omega_M}}\, \mathrm{Im}_{\omega'} \left[\tilde{\Pi}(n\cdot p, \omega') - \frac{n\cdot p - m_B}{m_B} \Pi(n\cdot p, \omega')\right],$$

$$f^T_{B\,P}(q^2) = \frac{2(m_B + m_P)}{f_P\, m_B\, n\cdot p}\, e^{\frac{m_p^2}{(n\cdot p\,\omega_M)}} \int_0^{\omega_s} \frac{d\omega'}{\pi}\, e^{\frac{\omega'}{\omega_M}}\, \mathrm{Im}_{\omega'} \Pi_T(n\cdot p, \omega').$$

- In HQET limit:

$$\sigma_{\mu\nu} q^\nu h_v = \frac{i}{4}\left[\bar{n}\cdot q\, n_\mu - n\cdot q \bar{n}_\mu\right] (\not{\bar{n}} - \not{n})\, h_v,$$

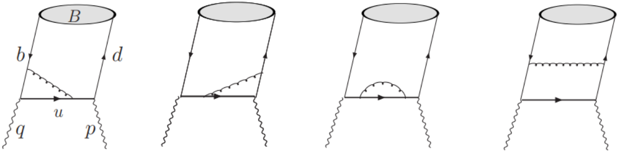
$$\to \qquad \Pi_T = -\frac{i}{2}\left(\tilde{\Pi} - \Pi\right).$$

- Large recoil symmetry with LP contribution at LO [Beneke,Feldmann,2001]:

$$f^+_{B\,P} = \frac{m_B}{n\cdot p} f^0_{B\,P} = \frac{m_B}{m_B + m_P} f^T_{B\,P}.$$

Leading power at NLL

- Leading power contribution for $B \to \pi$ form factors at NLL[Wang,Shen,2015]:



(a) (b) (c) (d)

- Light-quark(s-quark) mass effect for $B \to K$ form factors[Lü,Shen,Wang,Wei,2018].
- Spectator-quark mass effect for $B_s \to K$ form factors have been computed in this work.

$$
\begin{aligned}
&\left\{\tilde{\phi}^{+}_{B,\mathrm{eff}}(\omega',\mu),\, \hat{\phi}^{+}_{B,\mathrm{eff}}(\omega',\mu)\right\} \\
&= \frac{\alpha_s C_F}{4\pi}\Bigg[\left\{r,-1\right\}\int_{\omega'}^{\infty} d\omega\, \frac{\phi_B^+(\omega,\mu)}{\omega} - (m_q + 2m_{q'})\int_{\omega'}^{\infty} d\omega \ln\left(\frac{\omega-\omega'}{\omega'}\right)\frac{d}{d\omega}\frac{\phi_B^+(\omega,\mu)}{\omega} \\
&\quad - 2m_{q'}\int_0^{\infty}\frac{d\omega}{\omega}\left[\theta(\omega-\omega')\left(\ln\frac{\mu^2}{n\cdot p\,\omega'}+\frac{5}{2}\right)+\ln\left(\frac{\omega'-\omega}{\omega'}\right)\theta(\omega'-\omega)\right]\frac{d}{d\omega}\phi_B^+(\omega)\Bigg].
\end{aligned}
$$

- The s-quark mass effect in B_s-meson LCDAs are necessary.

NLP corrections to $B \to P$ form factors

△ Subleading power heavy-to-light current $\bar{q}\gamma^\mu \frac{i\vec{\not{D}}}{2m_b} h_v$.

- Subleading power correction from b-**quark field expansion** in HQET as [Mannel,Moreno,Pivovarov, 2020]:

$$b(x) = e^{-im_b v\cdot x} \left[h_v + \frac{i\vec{\not{D}}}{2m_b} h_v + \cdots \right].$$

- Large recoil symmetry breaking effect:

$$\not{v}\vec{\not{D}} h_v = -\vec{\not{D}} h_v \qquad \to \qquad \Pi_T^{\mathrm{HQE}}(q^2) = \frac{i}{2}[\tilde{\Pi}^{\mathrm{HQE}}(q^2) - \Pi^{\mathrm{HQE}}(q^2)].$$

- Equations of operators derived from the QCD equation of motion [Kawamura,Kodaira,Qiao,Tanaka,2001]:

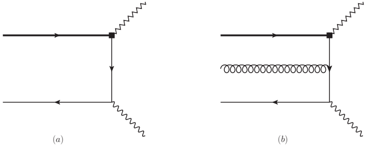
$$\begin{aligned}
\bar{q}(x)\,\Gamma D_\rho\, h_v(0) =& \partial_\rho[\bar{q}(x)\,\Gamma h_v(0)] + i\int_0^1 du(1-u)[\bar{q}(x) x^\lambda g_s G_{\lambda\rho}\Gamma\, h_v(0)] \\
& - \frac{\partial}{\partial x_\rho}[\bar{q}(x)\,\Gamma\, h_v(0)], \\
v_\rho \frac{\partial}{\partial x_\rho}[\bar{q}(x)\,\Gamma\, h_v(0)] =& i\int_0^1 du(1-u)[\bar{q}(x)\, x^\lambda g_s G_{\lambda\rho} v^\rho \Gamma\, h_v(0)] + \partial_\rho[\bar{q}(x)\, v^\rho \Gamma\, h_v(0)], \\
\frac{\partial}{\partial x_\rho}[\bar{q}(x)\,\gamma_\rho \Gamma\, h_v(0)] =& -i\int_0^1 du\, u[\bar{q}(x)\, x^\lambda g_s G_{\lambda\rho}\gamma^\rho \Gamma\, h_v(0)] + i m_q \bar{q}(x)\Gamma h_v(0)\,.
\end{aligned}$$

NLP corrections to $B \to P$ form factors

- Subleading power correction from the **hard-collinear quark propagator**:

$$\underbrace{\frac{\not p}{n\cdot p(\bar n\cdot p-\bar n\cdot k)}}_{\mathrm{LP}}+\underbrace{\frac{-\not k}{(p-k)^2}+\frac{\not p\bar n\cdot p n\cdot k}{(p-k)^4}}_{\mathrm{NLP}}+\underbrace{\frac{m_q}{(p-k)^2}+\frac{\not p\, m_q^2}{(p-k)^4}}_{\mathrm{light-quark\ mass}}.$$

- The two-particle and three-particle higher-twist B-meson LCDAs corrections are given by [Lü,Shen,Wang,Wei,2018].



  - Definition of two-particle B-meson [Beneke,Feldmann,2001;Beneke,Braun,Ji,Wei,2018]:

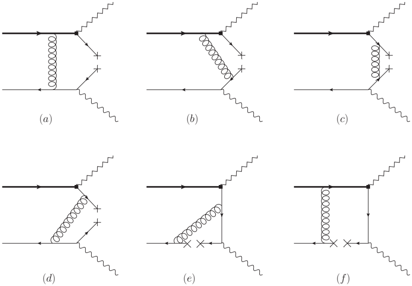
$$\langle 0|\bar q(x)\,\Gamma[x,0]\,h_v(0)|\bar B(v)\rangle = -\frac{i\tilde f_B(\mu)m_B}{4}\,\mathrm{Tr}\Big[\frac{1+\not v}{2}\Big\{2\Big(\Phi_B^+(v\cdot x)+x^2G_B^+(v\cdot x)\Big)$$
$$-\frac{\not x}{v\cdot x}\Big[\Big(\Phi_B^+(v\cdot x)-\Phi_B^-(v\cdot x)\Big)+x^2\Big(G_B^+(v\cdot x)-G_B^-(v\cdot x)\Big)\Big]\Big\}\gamma_5\Gamma\Big].$$

  - The light-cone expansion of the quark propagator in the background gluon field,

$$\langle 0|\mathrm{T}\{\bar q(x),q(0)\}|0\rangle \subset i g_s\int_0^\infty \frac{dk^4}{2\pi^4}\,e^{ik\cdot x}\int_0^1 du\left[\frac{u x_\mu\gamma_\nu}{k^2-m_q^2}-\frac{(\not k+m_q)\sigma_{\mu\nu}}{2(k^2-m_q^2)^2}\right]G^{\mu\nu}(ux)\,.$$

NLP corrections to $B \to P$ form factors

- Four-particle B-meson LCDAs in factorization limit



- The contributions from diagram $a - c$ are power suppressed.
- A similar calculation for $B \to \gamma \ell \nu$ form factors [Beneke,Braun,Ji,Wei,2018].
- A complete parameterization of four-particle B-meson LCDA is necessary.

B-meson LCDAs and RGE

- Two-particle B-meson LCDAs:
  - Shape parameters of $\phi_B^+(\omega,\mu)$:

$$\hat{\sigma}_n(\mu) = \int_0^\infty d\omega\, \frac{\lambda_B}{\omega} \ln^n \frac{\lambda_B e^{-\gamma_E}}{\omega} \phi_B^+(\omega,\ \mu).$$

  - The equations of motion for the light-quark field yield[Kawamura,Kodaira,Qiao,Tanaka,2001]:

$$\phi_B^-(\omega) = \underbrace{\int_\omega^\infty \frac{\phi_B^+(\omega')}{\omega'} d\omega'}_{\text{W-W approximation}} - \underbrace{\int_\omega^\infty \frac{d\omega'}{\omega'} \frac{d}{d\omega'} \int_0^\omega d\omega_1 \int_{\omega'-\omega_1}^\infty \frac{d\omega_2}{\omega_2} \frac{d}{d\omega_2} \phi_3(\omega_1,\omega_2)}_{\text{three-particle LCDAs correction}} .$$

  - Twist-five LCDA[Beneke,Braun,Ji,Wei,2018]:

$$\begin{aligned} g_B^-(\omega) =& \frac{1}{4} \int_\omega^\infty dx \Big\{ (x-\omega) \left[\phi_B^+(\omega) - \phi_B^-(\omega) \right] - 2(\overline{\Lambda} - x) \phi_B^-(\omega) \Big\} \\ & - \frac{1}{2} \int_0^\omega d\omega_1 \int_{\omega-\omega_1}^\infty d\omega_2\, \frac{1}{\omega_2} \left(1 - \frac{\omega-\omega_1}{\omega_2} \right) \psi_5(\omega_1,\omega_2). \end{aligned}$$

- Three-particle higher-twist B-meson LCDAs:
  - Three-particle B-meson LCDAs[Kawamura,Kodaira,Qiao,Tanaka,2001; Braun,Ji,Manashov,2017]:

$$
\begin{aligned}
&\langle 0|\bar{q}_\alpha(z_1\bar{n}) g_s G_{\mu\nu}(z_2\bar{n}) h_{\nu\beta}(0)|\bar{B}(v)\rangle \\
&= \frac{\tilde{f}_B(\mu) m_B}{4}\Big[(1+\not{v})\Big\{(v_\mu\gamma_\nu - v_\nu\gamma_\mu)\left[\Psi_A(z_1,z_2)-\Psi_V(z_1,z_2)\right] - i\sigma_{\mu\nu}\Psi_V(z_1,z_2) \\
&- (\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu) X_A(z_1,z_2) + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\left[W(z_1,z_2)+Y_A(z_1,z_2)\right] \\
&+ i\epsilon_{\mu\nu\alpha\beta}\bar{n}^\alpha v^\beta\gamma_5\tilde{X}_A(z_1,z_2) - i\epsilon_{\mu\nu\alpha\beta}\bar{n}^\alpha\gamma^\beta\gamma_5\tilde{Y}_A(z_1,z_2) \\
&- (\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu)\not{\bar{n}}\, W(z_1,z_2) + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\not{\bar{n}}\, Z(z_1,z_2)\Big\}\gamma_5\Big]_{\beta\alpha}.
\end{aligned}
$$

  - Those eight invariant functions are related to the B-meson higher-twist LCDAs:

$$
\begin{aligned}
\Phi_3 &= \Psi_A - \Psi_V, & \Phi_4 &= \Psi_A + \Psi_V. \\
\Psi_4 &= \Psi_A + X_V, & \tilde{\Psi}_4 &= \Phi_A - \tilde{X}_A. \\
\Psi_5 &= -\Psi_A + X_A - 2Y_A, & \Phi_5 &= \Psi_A + \Psi_V + 2Y_A - 2\tilde{Y}_A + 2W. \\
\tilde{\Psi}_5 &= -\Psi_V - \tilde{X}_A + 2\tilde{Y}_A, & \Phi_6 &= \Psi_A - \Psi_V + 2Y_A + 2\tilde{Y}_A + 2W - 4Z.
\end{aligned}
$$

  - Asymptotic behavior at small quark and gluon momenta [Braun,Filyanov,1990]:

$$
\phi(\omega_1,\omega_2) \sim \omega_1^{2j_1-1}\omega_2^{2j_2-1}, \quad \phi \in \{\phi_3, \phi_4, \psi_4 \cdots\}.
$$

  - Normalization conditions:

$$
\Psi_V(z=0) = \frac{1}{3}\lambda_H^2, \qquad \Psi_A(z=0) = \frac{1}{3}\lambda_E^2.
$$

B-meson LCDAs and RGE

- The general model of B-meson LCDAs:

$$
\begin{aligned}
\phi_B^+(\omega) &= \omega f(\omega), \\
\phi_B^-(\omega) &= \int_\omega^\infty d\rho\, f(\rho) + \frac{1}{6}\varkappa(\lambda_E^2 - \lambda_H^2)\left[\omega^2 f'(\omega) + 4\omega f(\omega) - 2\int_\omega^\infty d\rho\, f(\rho)\right], \\
g_-(\omega) &= \frac{1}{4}\int_\omega^\infty dx\, \left\{(x-\omega)\left[\phi_B^+(\omega) - \phi_B^-(\omega)\right] - 2(\bar{\Lambda}_q - x)\phi_B^-(\omega)\right\} \\
&\quad -\frac{1}{2}\int_0^\omega d\omega_1 \int_{\omega-\omega_1}^\infty d\omega_2\, \frac{1}{\omega_2}(1 - \frac{\omega-\omega_1}{\omega_2})\psi_5(\omega_1,\omega_2), \\
\phi_3(\omega_1,\omega_2) &= -\frac{1}{2}\varkappa(\lambda_E^2 - \lambda_H^2)\omega_1\omega_2^2 f'(\omega_1+\omega_2), \\
\phi_4(\omega_1,\omega_2) &= \frac{1}{2}\varkappa(\lambda_E^2 + \lambda_H^2)\omega_2^2 f(\omega_1+\omega_2), \\
\psi_4(\omega_1,\omega_2) &= \varkappa\lambda_E^2\omega_1\omega_2 f(\omega_1+\omega_2), \\
\phi_5(\omega_1,\omega_2) &= \varkappa(\lambda_E^2 + \lambda_H^2)\omega_1 \int_{\omega_1+\omega_2}^\infty d\omega\, f(\omega), \\
\psi_5(\omega_1,\omega_2) &= \varkappa\lambda_E^2\omega_2 \int_{\omega_1+\omega_2}^\infty d\omega\, f(\omega), \\
\tilde{\psi}_5(\omega_1,\omega_2) &= \varkappa\lambda_H^2\omega_2 \int_{\omega_1+\omega_2}^\infty d\omega\, f(\omega), \\
\phi_6(\omega_1,\omega_2) &= \varkappa(\lambda_E^2 - \lambda_H^2) \int_{\omega_1+\omega_2}^\infty d\omega \int_\omega^\infty d\omega'\, f(\omega').
\end{aligned}
$$

B-meson LCDAs and RGE

- Exponential model:

$$f(\omega) = \frac{1}{\omega_0^2} e^{-\frac{\omega}{\omega_0}}.$$

- Local duality model:

$$f(\omega) = \frac{5}{8\omega_0^5}(2\omega_0 - \omega)^3 \theta(2\omega_0 - \omega).$$

- Three-parameter model of $\phi_B^+(\omega, \mu_0)$ [Beneke,Braun,Ji,Wei,2018]:

$$\phi_B^+(\omega, \mu_0) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} \mathrm{U}\left(\beta - \alpha, 3 - \alpha, \omega/\omega_0\right).$$

where the three parameters are related to the shape parameters of $\phi_B^+(\omega, \mu_0)$

$$\lambda_B = \frac{\alpha - 1}{\beta - 1}\omega_0, \qquad \hat{\sigma}_1 = \psi(\beta - 1) - \psi(\alpha - 1) + \ln\left(\frac{\alpha - 1}{\beta - 1}\right),$$

$$\hat{\sigma}_2 = \frac{\pi^2}{6} + \left[\psi(\beta - 1) - \psi(\alpha - 1) + \ln\left(\frac{\alpha - 1}{\beta - 1}\right)\right]^2 - \left[\psi'(\beta - 1) - \psi'(\alpha - 1)\right].$$

- The representation of $\phi_B^+(\omega, \mu)$ and $\phi_B^-(\omega, \mu)$ in dual space [Bell,Feldmann,Wang,Yip,2013]:

$$\phi_B^+(\omega, \mu) = \int_0^\infty ds \sqrt{\omega s} J_1(2\sqrt{\omega s}) \eta_+(s, \mu)$$

$$\phi_B^-(\omega, \mu) = \int_\omega^\infty \frac{d\omega'}{\omega'} \int_0^\infty ds \sqrt{\omega' s} J_1(2\sqrt{\omega' s}) \left(\eta_+(s, \mu) + \eta_3^{(0)}(s, \mu)\right).$$

B-meson LCDAs and RGE

- RG equation at one-loop in the dual space [Bell,Feldmann,Wang,Yip,2013;Beneke,Braun,Ji,Wei,2018]:

$$\eta_+(s,\mu) = U_+(s;\mu,\mu_0)\eta_+(s,\mu_0);$$
$$\eta_3^{(0)}(s,\mu) = r^{N_c/\beta_0} U_+(s;\mu,\mu_0)\eta_3^{(0)}(s,\mu_0).$$

- Scale-dependence LCDAs:

$$\phi_B^+(\omega,\mu) = \frac{\omega}{\omega_0^2}\hat{U}_\phi^{\rm tw2}(\omega,\mu,\mu_0)\frac{\Gamma(\beta)}{\Gamma(\alpha)}\mathrm{G}_{23}^{21}\left(\frac{\omega}{\omega_0}\Big|_{\kappa,\alpha-2,\kappa-1}^{-1,\beta-2}\right),$$

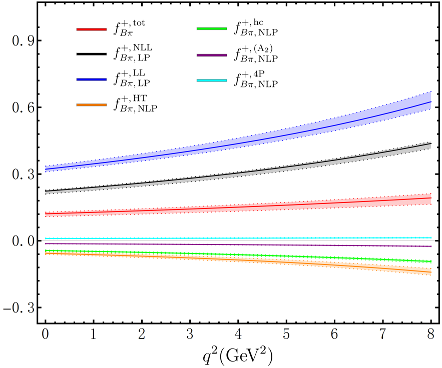
$$\phi_B^-(\omega,\mu) = \frac{1}{\omega_0}\hat{U}_\phi^{\rm tw2}(\omega,\mu,\mu_0)\frac{\Gamma(\beta)}{\Gamma(\alpha)}\mathrm{G}_{23}^{21}\left(\frac{\omega}{\omega_0}\Big|_{\kappa,\alpha-1,\kappa}^{0,\beta-1}\right) - \hat{U}_\phi^{\rm tw3}(\omega,\mu,\mu_0)\frac{\lambda_E^2-\lambda_H^2}{18\omega_0^3}\frac{\Gamma(\beta+2)}{\Gamma(\alpha+2)}$$
$$\left[(\beta-1)\beta\mathrm{G}_{23}^{21}\left(\frac{\omega}{\omega_0}\Big|_{\kappa,\alpha-1,\kappa}^{0,\beta-1}\right) - 2(\beta-1)\mathrm{G}_{23}^{21}\left(\frac{\omega}{\omega_0}\Big|_{\kappa,\alpha-1,\kappa}^{0,\beta-2}\right) + \mathrm{G}_{23}^{21}\left(\frac{\omega}{\omega_0}\Big|_{\kappa,\alpha-1,\kappa}^{0,\beta-3}\right)\right].$$

Numerical results

The LCSR for $B \to P$ form factors:

$$f^{i}_{B\,P}(q^2) = f^{i,\mathrm{NLL}}_{B\,P,\,\mathrm{LP}}(q^2) + f^{i,\mathrm{HT}}_{B\,P,\,\mathrm{NLP}}(q^2) + f^{i,(\mathrm{A}_2)}_{B\,P,\,\mathrm{NLP}}(q^2) + f^{i,\mathrm{hc}}_{B\,P,\,\mathrm{NLP}}(q^2) + f^{i,4\mathrm{p}}_{B\,P\,\mathrm{NLP}}(q^2).$$

△ Input $\lambda_B = (350 \pm 150)\mathrm{MeV}\,, \qquad \lambda_{B_s} = (400 \pm 150)\mathrm{MeV}\,;$
$\{\hat\sigma^{(1)}_{B_{d,s}}, \hat\sigma^{(1)}_{B_{d,s}}\} = \{0.7, 6\}, \quad \{0, \pi^2/6\}, \quad \{-0.7, -6\}\,.$

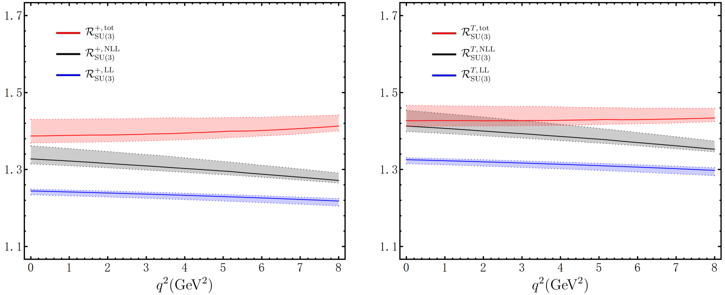


▽ Subleading power corrections from

- higher-twist LCDAs - 25%
- b-quark field in HQET - 6%
- hard-collinear propagator - 19%
- four-particle LCDA 4%
- total -46%

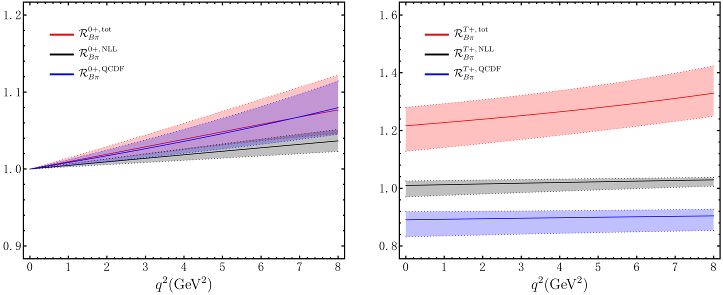
- SU(3)-flavour symmetry breaking

$$R^i_{\mathrm{SU}(3)}(q^2) = \frac{f^i_{B\,K}(q^2)}{f^i_{B\,\pi}(q^2)}, \qquad (i = +, 0, T).$$



- Large recoil symmetry breaking for $B \to \pi$ form factors

$$R^{0,+}_{B\,\pi}(q^2) = \frac{m_B}{n \cdot p} \frac{f^0_{B\,\pi}(q^2)}{f^+_{B\,\pi}(q^2)}, \qquad R^{T,+}_{B\,\pi}(q^2) = \frac{m_B}{m_B + m_\pi} \frac{f^T_{B\,\pi}(q^2)}{f^+_{B\,\pi}(q^2)}.$$



- Dominant breaking effect for $R^{T,+}_{B\,\pi}$ due to the subleading term of heavy-to-light current.

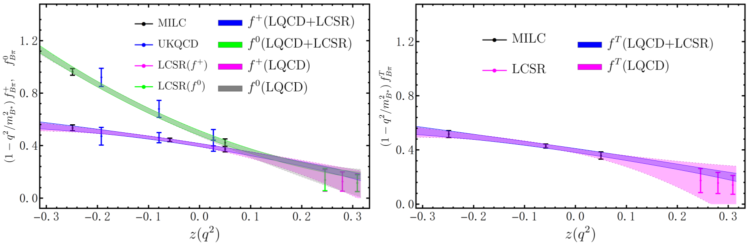
Numerical results

- Performing the combined fit from the LCSR and Lattice QCD [MILC,UKQCD,HPQCD] with the BCL-parameterize($N = 3$) [Bourrely,Caprini and Lellouch,2010]

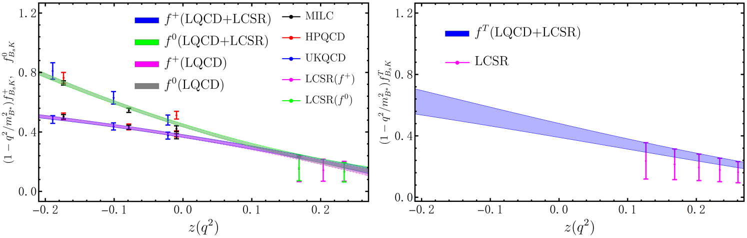
$$f^{+,T}_{BM}(q^2) = \frac{1}{1 - q^2/M^2} \sum_{n=0}^{N-1} a^{+,T}_{n,BM} \left[z(q^2, t_0)^n - (-1)^{n-N} \frac{n}{N} z(q^2, t_0)^N \right],$$

$$f^0_{BM}(q^2) = \frac{1}{1 - q^2/M_0^2} \sum_{n=0}^{N-1} a^0_{n,BM}\, z(q^2, t_0)^n \,.$$

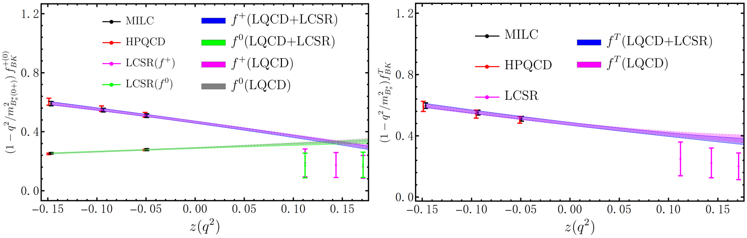
- The predicted $B \to \pi$ form factors from the combined fit



- The predicted $B_s \to K$ form factors from the combined fit

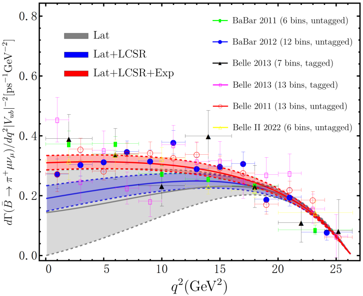


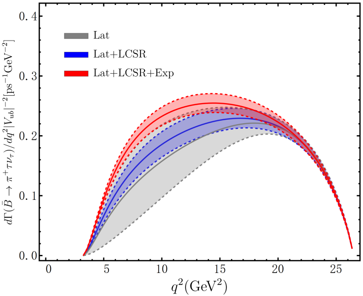
- The predicted $B \to K$ form factors from the combined fit



Numerical results

- Differential decay rate of the $B \to \pi \ell \nu_\ell$ decay



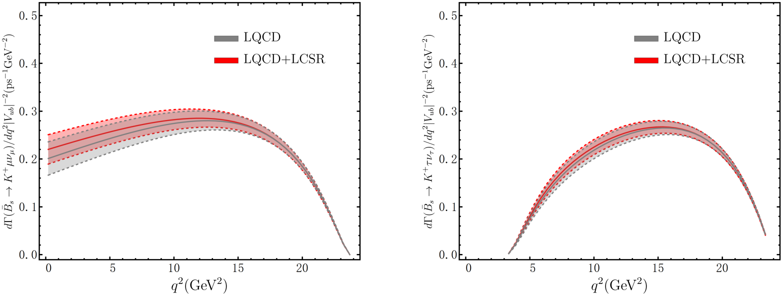


- Determined of $|V_{ub}|$ by fitting with experimental data from BaBar and Belle Collaboration:

$$|V_{ub}| = (3.76 \pm 0.13) \times 10^{-3}.$$

Confronted with $|V_{ub}|_{\text{Lat}} = (3.74 \pm 0.17) \times 10^{-3}$ from $B \to \pi \ell \nu_\ell$[FLAG, 2021].

- Differential decay rate of the $B_s \to K\ell\nu_\ell$ decay



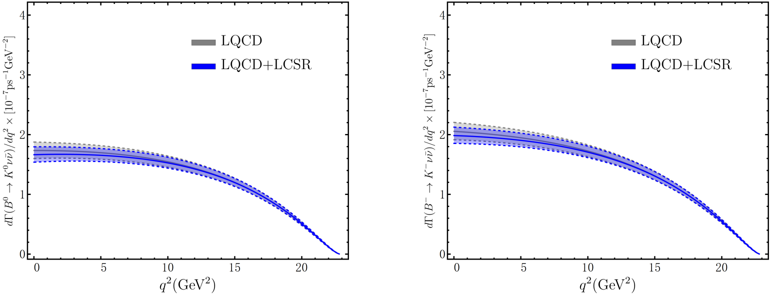
- The predicted branching radio for $B_s \to K\ell\nu_\ell$ decay

$$\mathcal{B}r(B_s \to K\ell\nu_\ell) = (1.20 \pm 0.13) \times 10^{-4} \qquad \text{(Our work)};$$

$$\mathcal{B}r(B_s \to K\ell\nu_\ell) = (1.06 \pm 0.05_{\rm stat} \pm 0.08_{\rm syst}) \times 10^{-4} \qquad \text{[LHCb, 2021]}.$$

Numerical results

- Differential decay rate of the $B^0 \to K^0 \nu\bar{\nu}$ and $B^- \to K^- \nu\bar{\nu}$

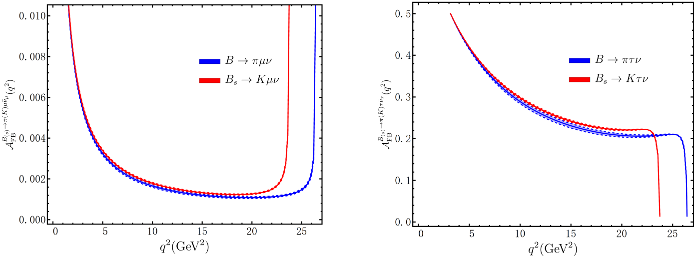


- The predicted branching radio for $B^- \to K^- \nu\bar{\nu}$ decay

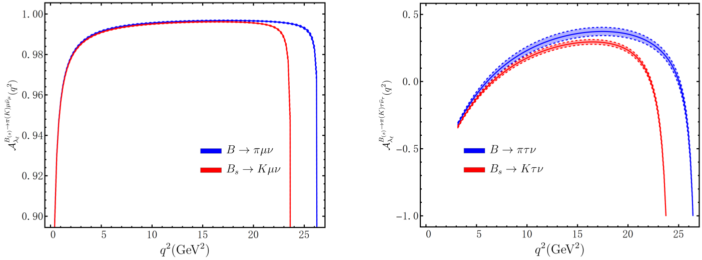
$$\mathcal{B}r(B^- \to K^- \nu\bar{\nu}) = 4.315^{+0.271}_{-0.248} \times 10^{-6} \qquad \text{(Our work)}$$

$$\mathcal{B}r(B^- \to K^- \nu\bar{\nu}) < 2.6 \times 10^{-5} (90\%\mathrm{CL}) \qquad \text{[PDG, 2022]}$$

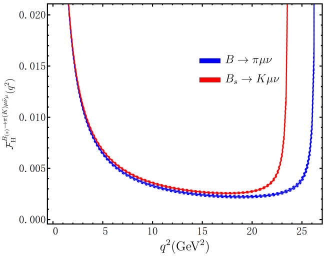
- Angular distributions
    - The forward-backward asymmetry

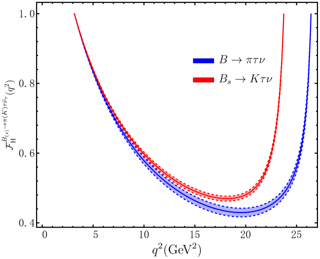


- The lepton polarization asymmetry



- The flat term





	$B \to \pi$		$B_s \to K$	
	μ	τ	μ	τ
A_{FB}^{P}	$3.99(35) \times 10^{-3}$	$0.248(5)$	$4.49(21) \times 10^{-3}$	$0.267(2)$
F_{H}^{ℓ}	$8.04(72) \times 10^{-3}$	0.514(12)	$9.10(43) \times 10^{-3}$	0.555(6)
$A_{\lambda_\ell}^{P}$	0.988(1)	0.266(29)	0.987(1)	0.191(14)

Conclusions and Outlook

- Subleading power contributions to the form factors reduce the leading power contribution 33%-50% numerically.
- One-loop level resummation for subleading power contributions is required to improve the precision and reduce the scale dependence.
- Best understanding the RG evolution of the higher-twist LCDAs is necessary.
- A global analysis on the determination of $|V_{ub}|$ is required.

Thank you for your attention!