



Amplitude analysis and its application in the hadrons observed by LHCb

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湖南大学
HUNAN UNIVERSITY

Outlines

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4

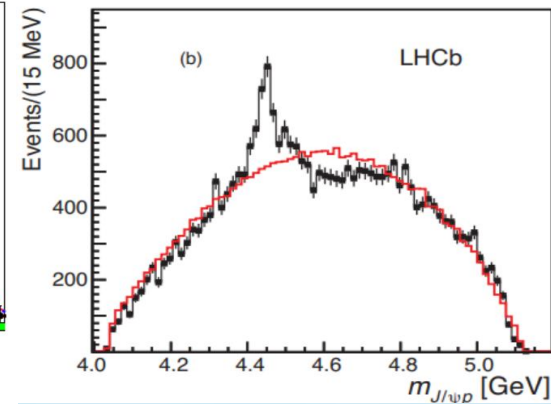
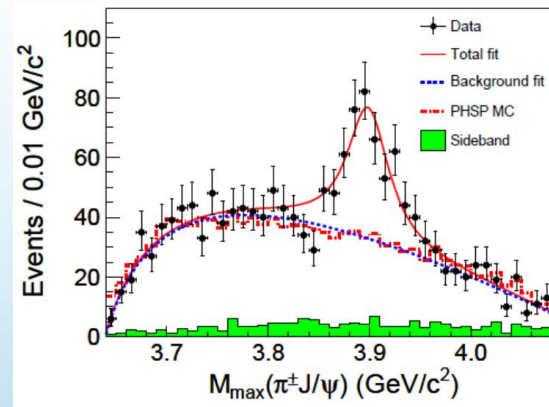
Some other resonances

5

Summary

Introduction

- $Z_c(3900)$ by BESIII and Belle, in 2013
- P_c states by LHCb, in 2015
-
- Their nature?
 - Quantum number?
 - The inner structure?



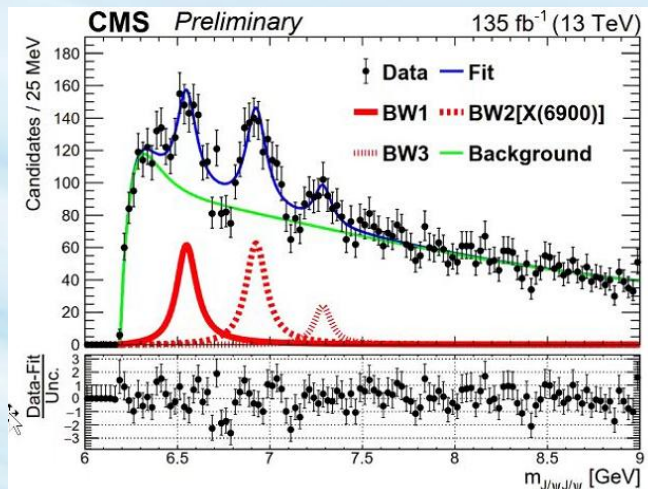
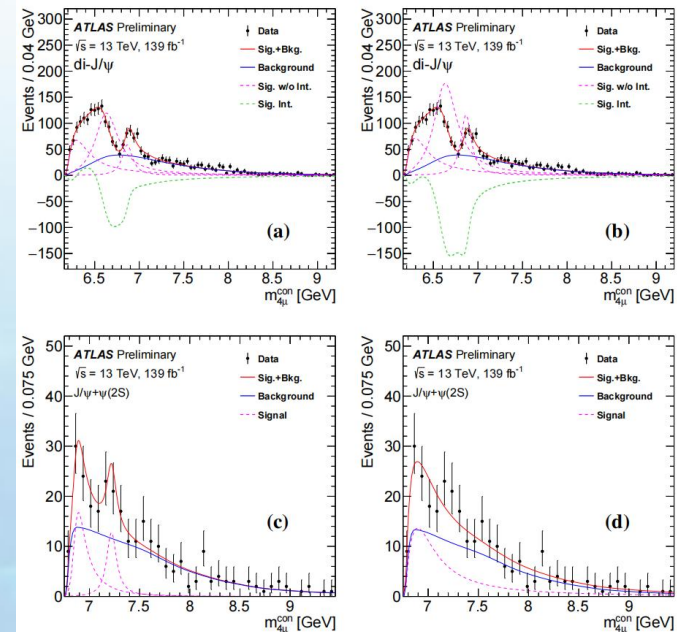
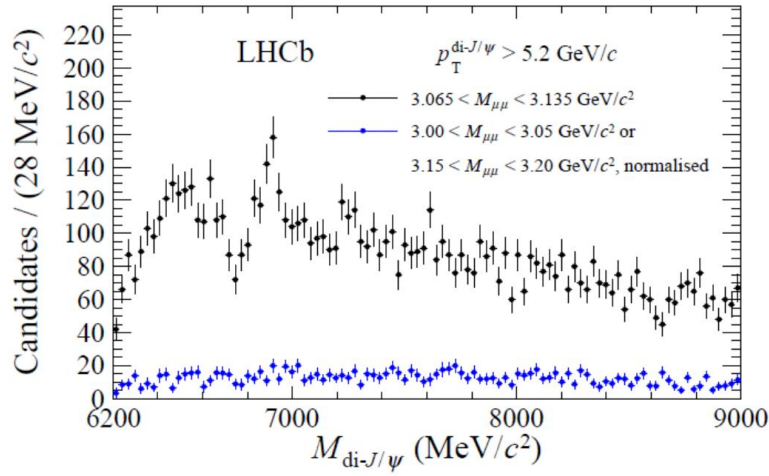
Amplitude analysis

- Only one physical amplitude!
- It should satisfy the fundamental QFT principles
- It should be compatible with the data

X(6900)

- Fully heavy tetra-quark state(s)?

LHCb, Sci.Bull. 65 (2020) 23, 1983

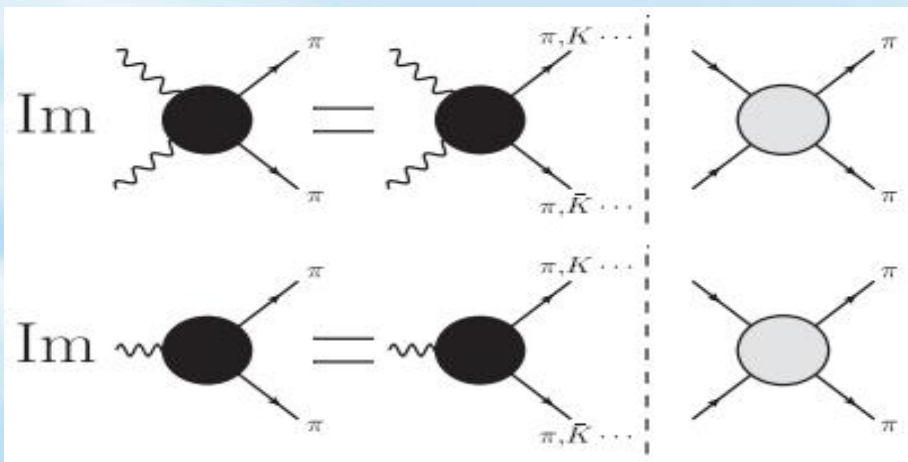


ATLAS, ATLAS-CONF-2022-040

CMS, Kai Yi's talk at ICHEP 2022.

why FSI ?

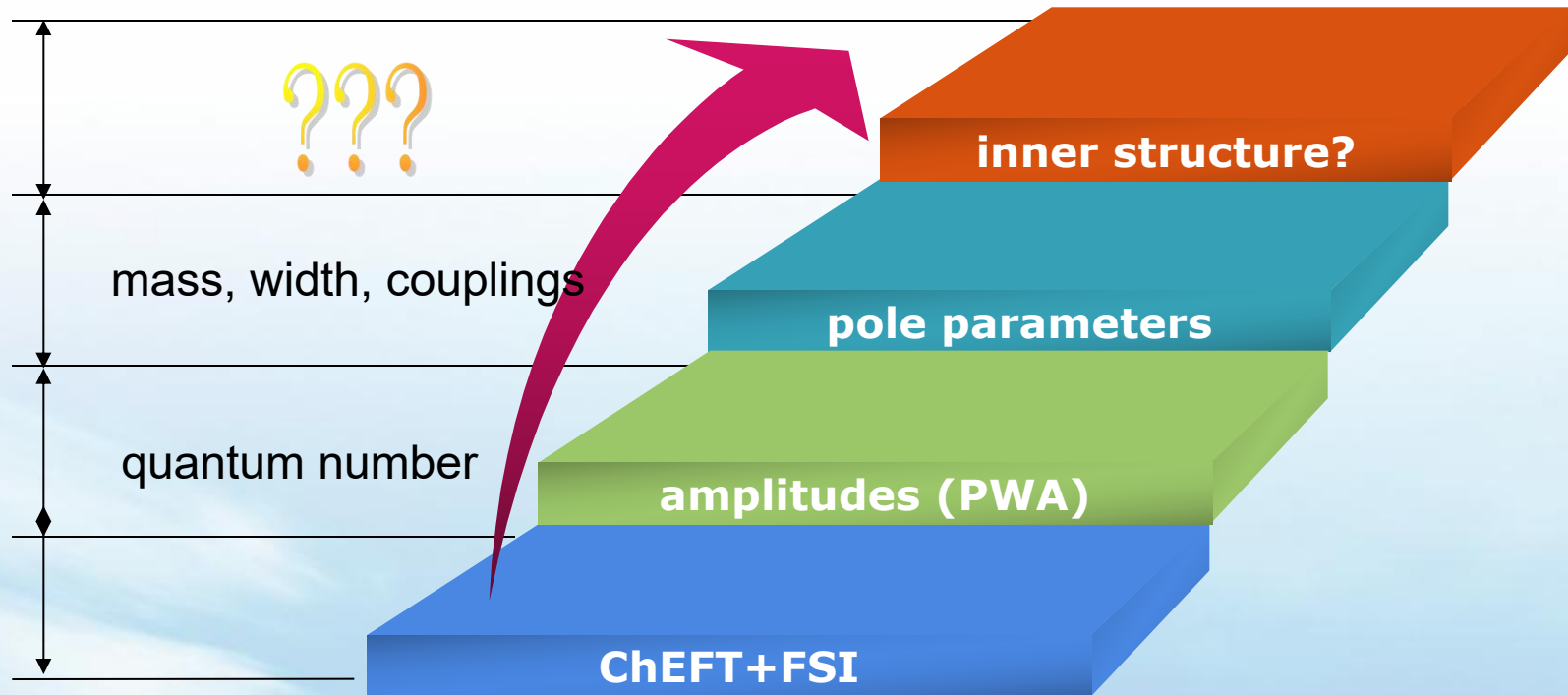
- FSI needs to be taken into account to perform an amplitude analysis
- Methods: KM, N/D, AMP, Roy equation, PKU, Pade, LSE, BSE, ChEFT, *et.al.*
- Fixed scattering, decaying amplitudes: extracting resonance information



Yao, Dai#, Zheng, Zhou,
RPP84(2021)076201

Inner structure?

- Interpretation these amplitudes, poles?



Pole counting rule

- Pole counting rule: distinguish molecule and BW resonance.
- At the very beginning, applied to light mesons

Morgan NPA543 (1992) 632.

Dai, Wang and Zheng

CTP57 (2012) 841, CTP58 (2012) 410

- Applied to heavier mesons

Dai, Sun, Kang, Szczepaniak, Yu,
PRD 105 (2022) 5, L051507

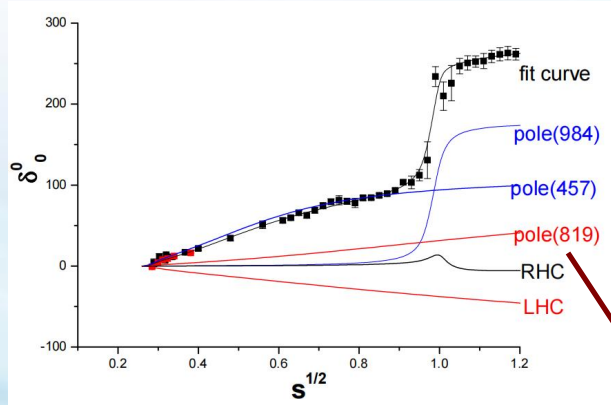
Kuang, **Dai**, Kang, Yao,
EPJC 80 (2020) 5, 433

- Any tools else?

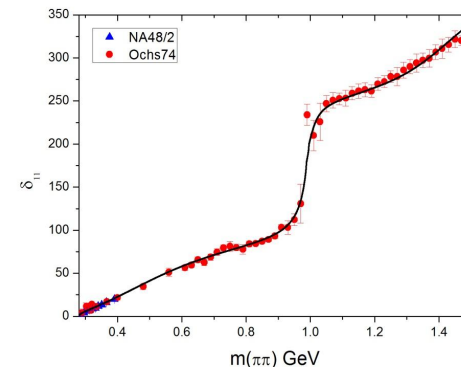
Dai, Shi, Tang, Zheng,
PRD 92 (2015) 1, 014020

phase shifts?

- Phase shifts help to study hadronic scatterings and resonances therein
- Successful in study of light scalars: PKU, Roy equation, Omnes representations, etc.



Zhou, Qin, Zhang, Xiao,
Zheng, JHEP 02 (2005)



Dai, Pennington, PLB736(2014)11;
PRD90 (2014) 036004;

- The phase shifts of a narrow BW resonance rise from 0 to 180 degrees, crossing 90 degrees at $E_{cm}=M$

2、Formalism

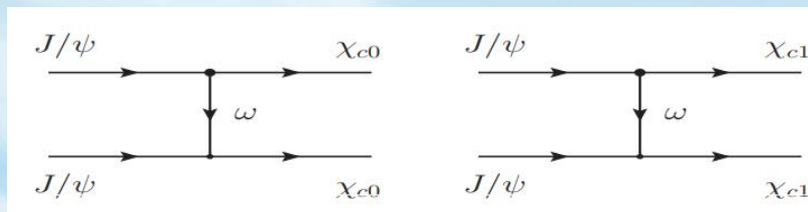
- coupled channels: $J/\psi J/\psi$ 、 $\psi(2s)J/\psi$ 、 $\psi(3770)J/\psi$

Dong, Baru, Guo, Hanhart, Nefediev,
PRL126, 132001(2021)

Gong, Du, Zhao, PRD, 106(5):054011, 2022

	J/ψ	$\psi(2s)$	$\psi(3770)$	η_c	h_c	χ_{c0}	χ_{c1}	χ_{c2}
J^{PC}	1^{--}	1^{--}	1^{--}	0^{-+}	1^{+-}	0^{++}	1^{++}	2^{++}
质量 (MeV)	3096.9	3686.1	3773.7	2983.9	3525.4	3414.7	3510.7	3556.2

- $\eta_c\eta_c$ 、 h_ch_c channels are suppressed by HQSS
- $\chi_{c0}\chi_{c0}$, $\chi_{c1}\chi_{c1}$, l.h.c. are faraway



$$\begin{aligned} \mathcal{L}_{\text{HQSS}}^{\text{LO}} &= g_1 \langle J\bar{J}J\bar{J} \rangle \\ &= \left\langle \frac{1+\not{\epsilon}}{2} \not{H} \frac{1-\not{\epsilon}}{2} \not{H}^\dagger \frac{1+\not{\epsilon}}{2} \not{H} \frac{1-\not{\epsilon}}{2} \not{H}^\dagger \frac{1+\not{\epsilon}}{2} \right\rangle \\ &= 2N_C g_1 V_\mu V_\alpha V^\mu V^\alpha, \end{aligned}$$

- [6.2,7.2] GeV

Kuang,Zhou,Guo,Yang,Dai#, arxiv:
2302.03968 [hep-ph], accepted by EPJC
Cao, Yang, EPJC 82 (2022) 2, 161

Effective Lagrangians

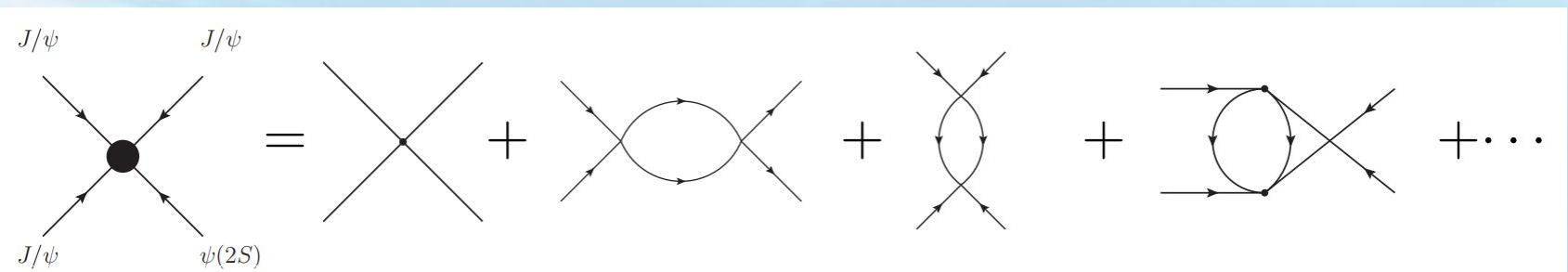
- Four vector interactions, similar to HGLS formalism

Geng, Molina, Oset.
CPC41(12):124101, 2017.

$$\begin{aligned} \mathcal{L} = & c_1 V_\mu V_\alpha V^\mu V^\alpha + c_2 V_\mu V_\alpha V^\mu V'^\alpha + c_3 V_\mu V'_\alpha V^\mu V'^\alpha \\ & + c_4 V_\mu V'^\mu V_\alpha V'^\alpha + c_5 V_\mu V_\alpha V^\mu V''^\alpha + c_6 V_\mu V''_\alpha V^\mu V''^\alpha \\ & + c_7 V_\mu V''^\mu V_\alpha V''^\alpha + c_8 V_\mu V'_\alpha V^\mu V''^\alpha + c_9 V_\mu V'^\mu V_\alpha V''^\alpha \end{aligned}$$

Zhou, Guo, Kuang, Yang, Dai,
PRD106 (2022) L111502.

- Feynman diagrams up to NLO



Building amplitudes

- PWA

$$T_{\mu_1\mu_2;\mu_3\mu_4}^{ij}(s, z_s) = 16\pi N_{ij} \sum_J (2J + 1) T_{\mu_1\mu_2;\mu_3\mu_4}^{J,ij}(s) d_{\mu\mu'}^J(z_s).$$

- Time reversal and parity invariance reduce the amplitudes
- Transfer amplitudes from helicity to the JMLS representation

$$T_{\mu_1\mu_2;\mu_3\mu_4}^{ij}(s, z_s) = 16\pi N_{ij} \sum_{J=M}^{\infty} (2J + 1) d_{\mu\mu'}^J(z_s) \sum_{LS, L'S'} \frac{\sqrt{(2L+1)(2L'+1)}}{2J+1} \langle LS0\mu | J\mu \rangle \langle J\mu' | L'S'0\mu' \rangle \\ \langle s_1 s_2 \mu_1, -\mu_2 | S\mu \rangle \langle S'\mu' | s_3 s_4 \mu_3, -\mu_4 \rangle T_{LS, L'S'}^{J,ij}.$$

$$T_{3P_2}^{ij}(s) = \frac{2}{5}T_{++++}^2(s) + \frac{\sqrt{3}}{5}T_{+++0}^2(s) + \frac{\sqrt{3}}{5}T_{+0++}^2(s) + \frac{\sqrt{3}}{5}T_{++0+}^2(s) + \frac{\sqrt{3}}{5}T_{0+++}^2(s) - \frac{2}{5}T_{++--}^2(s) \\ - \frac{\sqrt{3}}{5}T_{+++0}^2(s) - \frac{\sqrt{3}}{5}T_{-0++}^2(s) - \frac{\sqrt{3}}{5}T_{++0-}^2(s) - \frac{\sqrt{3}}{5}T_{0-++}^2(s) + \frac{3}{10}T_{+0+0}^2(s) + \frac{3}{10}T_{+00+}^2(s) \\ + \frac{3}{10}T_{0++0}^2(s) + \frac{3}{10}T_{0+0+}^2(s) - \frac{3}{10}T_{0+0-}^2(s) - \frac{3}{10}T_{+00-}^2(s) - \frac{3}{10}T_{0-+0}^2(s) - \frac{3}{10}T_{-0+0}^2(s).$$

perturbation amplitudes

- LO partial wave amplitudes
- containing threshold factors and momentum dependences

$$T_{1S_0}^{11,LO}(s) = \frac{c_1(68m_1^4 + 8sm_1^2 + 5s^2)}{144\pi m_1^4},$$

$$T_{5S_2}^{11,LO}(s) = \frac{c_1(s + 6m_1\sqrt{s} + 14m_1^2)^2}{1800\pi m_1^4},$$

$$T_{3P_0}^{11,LO}(s) = 0,$$

$$T_{3P_1}^{11,LO}(s) = -\frac{c_1(s - 4m_1^2)}{12\pi m_1^2},$$

$$T_{3P_2}^{11,LO}(s) = 0,$$

$$T_{1S_0}^{12,LO}(s) = \frac{-10c_2(m_1^3 - m_2^2m_1)^2 - 2s^2(3m_2^2 - 8m_2m_1 + m_1^2) + s(m_2^4 + 4m_2^2m_1^2 + 56m_2m_1^3 + 7m_1^4) + 5s^3}{288\sqrt{2}\pi m_2m_1^3s},$$

$$T_{5S_2}^{12,LO}(s) = \frac{-c_2(14m_1^2 + 6m_1\sqrt{s} + s)((m_1^2 - m_2^2)^2 - 3s^{3/2}(m_1 + m_2) - 14m_1m_2s + 3\sqrt{s}(m_1 - m_2)^2(m_1 + m_2) - s^2)}{3600\sqrt{2}\pi m_1^3m_2s}$$

PW amplitudes

$$T_{3P_2}^{11} = \frac{1}{64\pi} \int_{-1}^1 \frac{1}{20m_1^2} \{ F_{(b)}^{11} [(-3(E_1 E_3 - 2E_1 E_4 + E_2 E_4) + 12(E_1 + E_2)m_1 - 4m_1^2)z_s$$

$$+ 3(3E_1 E_3 - 2E_1 E_4 + 3E_2 E_4 - 4(E_1 + E_2)m_1 + 4m_1^2)z_s^3 + 6p_{cm} p'_{cm} (1 - 3z_s^2)]$$

$$+ F_{(c)}^{11} [6p_{cm} p'_{cm} (1 - 3z_s^2) + z_s (-3E_1 E_3 + 6E_1 E_4 - 3E_2 E_4 + 4m_1 (-3(E_1 + E_2) + m_1)$$

$$+ 3(E_1 E_3 - 6E_1 E_4 + E_2 E_4 + 4(E_1 + E_2 - m_1)m_1)z_s^2)] \} dz_s,$$

- Five PWs at last


L	$S = 0$	$S = 1$			$S = 2$					
0	$0^{++} ({}^1S_0)$	1^{+-}			$2^{++} ({}^5S_2)$					
1	1^{--}	$0^{-+} ({}^3P_0)$	$1^{-+} ({}^3P_1)$	$2^{-+} ({}^3P_2)$	1^{--}	2^{--}	3^{--}			
2	2^{++}		1^{+-}	2^{+-}	3^{+-}	0^{++}	1^{++}	2^{++}	3^{++}	4^{++}
\vdots	\vdots		\vdots				\vdots			

- Ignoring PWs with $L \geq 2$
- No S-D coupled channel scatterings

Unitarization

- unitarity: ignore the S-D coupled waves

$$\langle L'S'|T^J - T^{J\dagger}|LS\rangle = i \frac{4|\vec{p}''|}{E''_{cm}} \sum_{L''S''} \langle L'S'|T^{J\dagger}|L''S''\rangle \langle L''S''|T^J|LS\rangle$$


$$\text{Im}T_{JLS}^{ij} = \sum_{k=1}^a T_{JLS}^{ik} \rho_k T_{JLS}^{kj*}$$

- Pade approximation: successful in confirming existence of light scalars, restore unitarity and perturbative amplitudes up to NLO

$$T = T^{LO} \cdot [T^{LO} - T^{NLO}]^{-1} \cdot T^{LO}$$

Dai, Wang and Zheng

CTP57 (2012) 841, CTP58 (2012) 410

Invariant mass spectra

- Each channel contributes a ratio α_i

$$\frac{d \text{ Events}^{1,2}}{d\sqrt{s}} = \tilde{N}_{1,2} p_{cm}(s) \sum_{\mu_1\mu_2\mu_3\mu_4} \int_{-1}^1 dz_s \left| \sum_{i=1}^a \alpha_i T_{\mu_1\mu_2\mu_3\mu_4}^{i1,i2}(s, z_s) \right|^2$$

- Into the forms of PWs

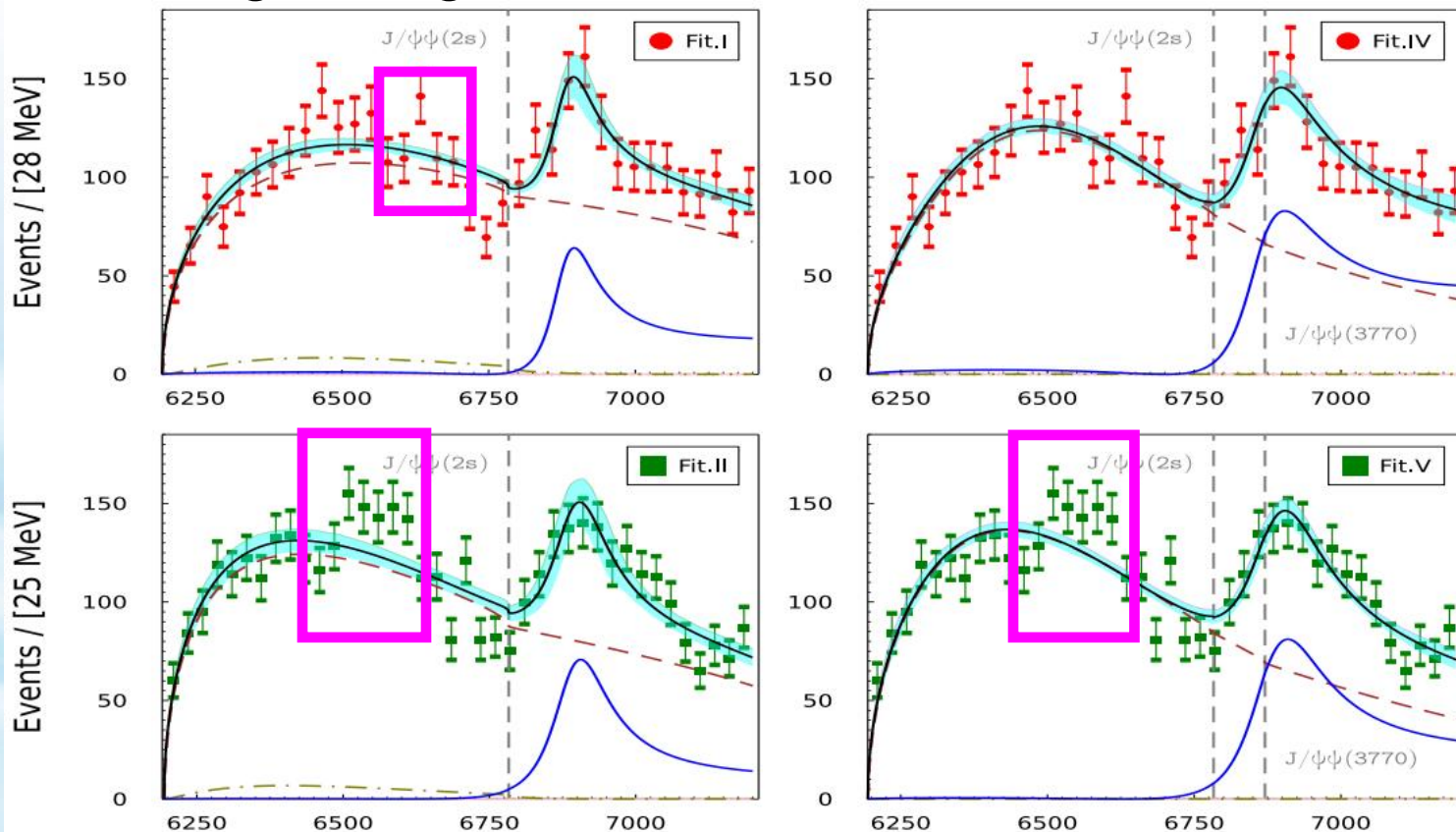
$$\sum_{\mu_1\mu_2\mu_3\mu_4} \int_{-1}^1 \left| \sum_{i=1}^a \alpha_i T_{\mu_1\mu_2\mu_3\mu_4}^{i1,i2}(s, z_s) \right|^2 dz_s = 512\pi^2 \left[|F_{1S_0}^{1,2}(s)|^2 + 5|F_{5S_2}^{1,2}(s)|^2 + |F_{3P_0}^{1,2}(s)|^2 + 3|F_{3P_1}^{1,2}(s)|^2 + 5|F_{3P_2}^{1,2}(s)|^2 \right]$$

- Compatible with AMP method
 - absorbing l.h.c. and distant r.h.c.
 - Fulfill FSI theorem

$$F_{JLS}^{1,2}(s) = \sum_{i=1}^a \alpha_i N_i T_{JLS}^{i1,i2}(s)$$

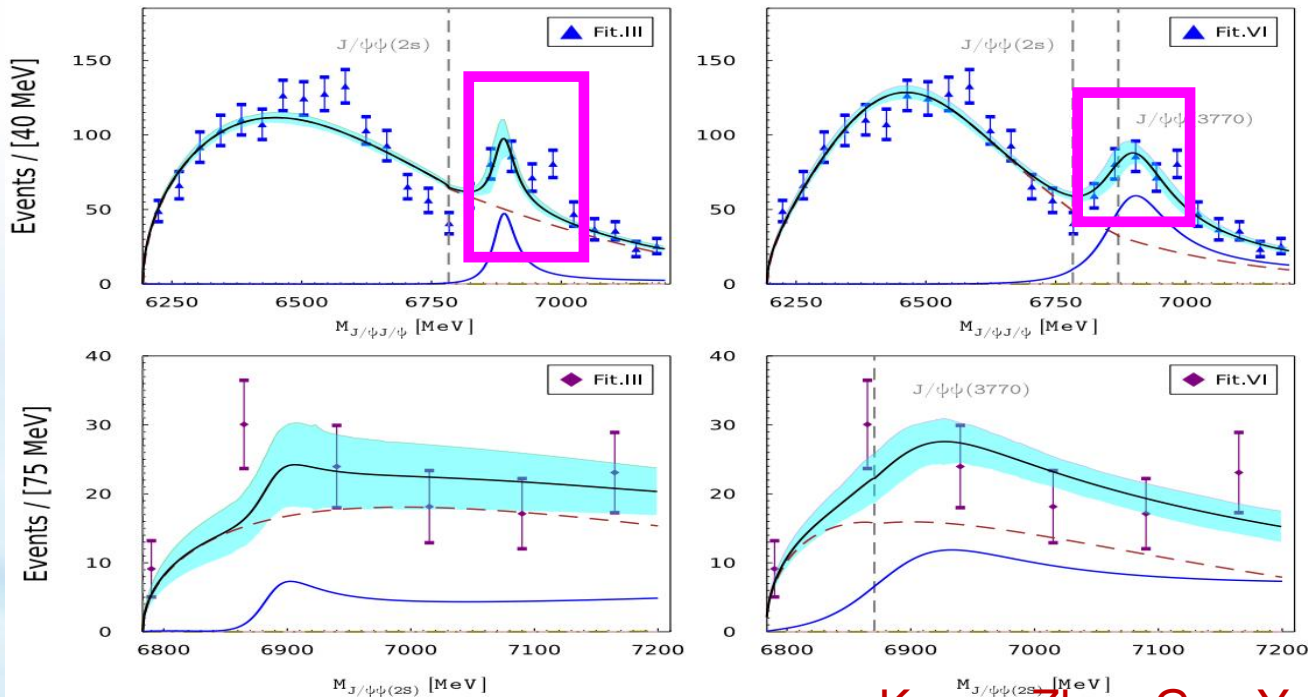
3. Nature of the X(6900)

- High quality Fits for all the data-sets
- Individual contributions of each PW
- 1S_0 dominates around 6900 MeV
- 5S_2 Contributes a large background



Fits

- Coupled channel fits on the data-sets of ATLAS, better constrains on coupled channel scattering



Kuang,Zhou,Guo,Yang,Dai#, arxiv:
2302.03968 [hep-ph], accepted by EPJC

- 1S_0 dominates around 6900 MeV for both $J/\psi J/\psi$ 、
 $\psi(2S)J/\psi$ channels

Couplings

- Couplings are reasonable
- So far, so good....

parameter	Fit.I(LHCb)	Fit.II(CMS)	Fit.III(ATLAS)
c_1	$-0.1236^{+0.0001}_{-0.0001}$	$-0.1504^{+0.0001}_{-0.0002}$	$-0.0618^{+0.0001}_{-0.0001}$
c_2	$-0.5336^{+0.0021}_{-0.0001}$	$-0.6203^{+0.0001}_{-0.0001}$	$-0.3369^{+0.0004}_{-0.0001}$
c_3	$-0.3180^{+0.0171}_{-0.0001}$	$-0.3492^{+0.0004}_{-0.0001}$	$-0.3171^{+0.0091}_{-0.0003}$
c_4	$-0.6178^{+0.0234}_{-0.0002}$	$-0.6835^{+0.0004}_{-0.0001}$	$-0.5386^{+0.0078}_{-0.0001}$
\tilde{N}_1	$1.5600^{+0.6284}_{-0.0850}$	$0.5336^{+0.1404}_{-0.0193}$	$0.1888^{+0.1109}_{-0.1934}$
\tilde{N}_2	$0.2200^{+0.0721}_{-0.0198}$
α_1	$0.3831^{+0.0104}_{-0.0052}$	$0.3510^{+0.0012}_{-0.0001}$	$0.1812^{+0.0473}_{-0.0032}$
α_2	$-0.9237^{+0.0089}_{-0.0022}$	$-0.9364^{+0.0001}_{-0.0009}$	$0.9834^{+0.1861}_{-0.0088}$
$\chi^2_{d.o.f.}$	1.31	1.77	2.53

- Lack of angular distributions!

Pole analysis

- One resonance found in 1S_0 wave: $X(6900)----0^{++}$
- Couple channels case: a pair of accompanying poles
- Triple channels case: Four poles in unphysical sheets, implying the BW origin.

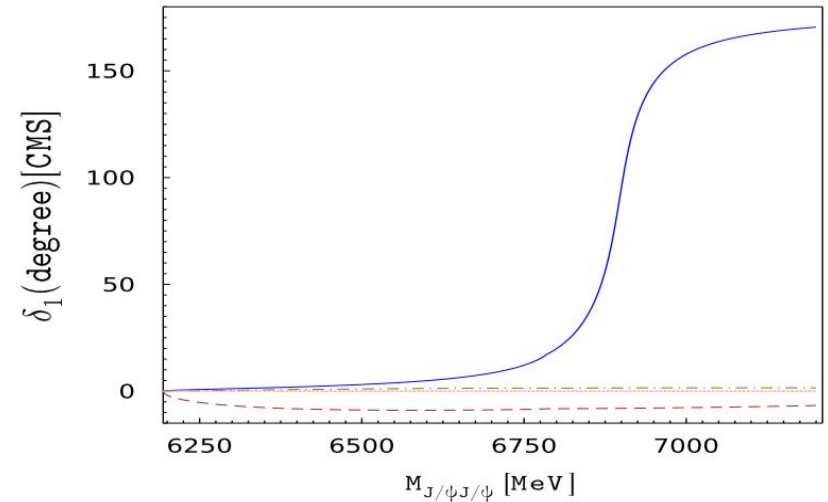
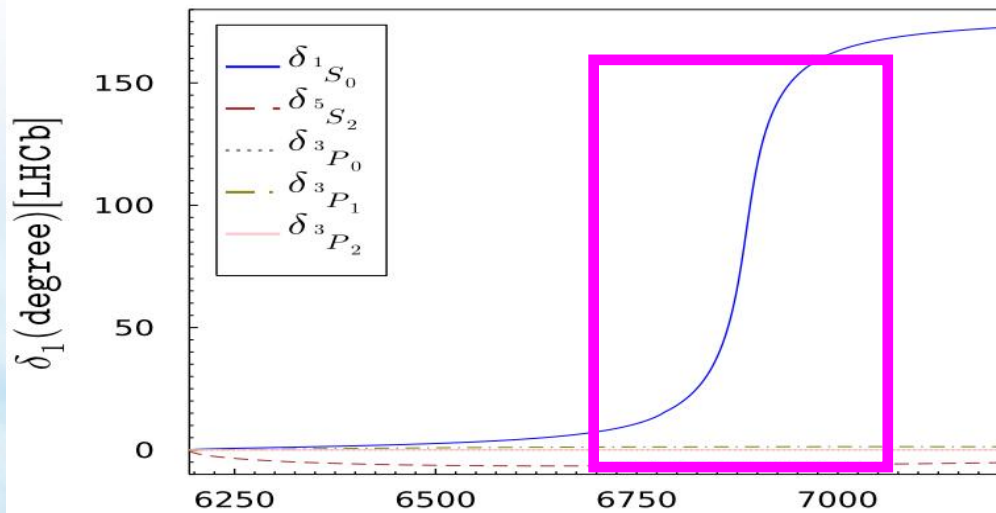


pole counting rule:
Morgan,
NPA543 (1992) 632.
Dai, Wang and Zheng
CTP57 (2012) 841

Data	RS	pole location (MeV)	$g_{J/\psi J/\psi} = g e^{i\varphi}$		$g_{J/\psi\psi(2S)} = g e^{i\varphi}$		$g_{J/\psi\psi(3770)} = g e^{i\varphi}$	
			$ g_1 (\text{MeV})$	$\varphi_1(^{\circ})$	$ g_2 (\text{MeV})$	$\varphi_2(^{\circ})$	$ g_3 (\text{MeV})$	$\varphi_3(^{\circ})$
LHCb(Fit.IV)	II(- + +)	$6874.8^{+5.0}_{-5.8} - i50.4^{+1.7}_{-1.1}$	$1398.5^{+21.6}_{-15.8}$	$85.9^{+0.3}_{-0.1}$	$962.1^{+14.9}_{-10.9}$	$84.6^{+0.1}_{-0.1}$	$18.2^{+0.7}_{-0.4}$	$-79.9^{+1.2}_{-0.2}$
	III(- - +)	$6862.0^{+4.3}_{-6.2} - i68.9^{+1.9}_{-2.0}$	$1364.7^{+20.1}_{-12.2}$	$80.6^{+0.3}_{-0.1}$	$927.4^{+13.4}_{-8.3}$	$77.5^{+0.5}_{-0.1}$	$19.3^{+0.7}_{-0.4}$	$-79.0^{+0.6}_{-0.3}$
	IV(- - -)	$6862.0^{+4.3}_{-6.2} - i68.9^{+1.9}_{-2.0}$	$1361.6^{+19.0}_{-12.4}$	$80.7^{+0.2}_{-0.1}$	$925.3^{+12.5}_{-8.7}$	$77.5^{+0.4}_{-0.1}$	$19.4^{+0.7}_{-0.4}$	$-78.6^{+0.5}_{-0.2}$
	VII(- + -)	$6874.8^{+5.0}_{-5.8} - i50.4^{+1.7}_{-1.1}$	$1394.3^{+17.7}_{-17.5}$	$85.9^{+0.2}_{-0.1}$	$959.2^{+11.7}_{-12.1}$	$84.5^{+0.1}_{-0.1}$	$18.4^{+0.7}_{-0.4}$	$-79.2^{+1.0}_{-0.3}$
CMS(Fit.V)	II(- + +)	$6888.4^{+11.3}_{-7.2} - i59.4^{+1.7}_{-0.5}$	$1452.8^{+23.1}_{-6.8}$	$85.6^{+0.1}_{-0.1}$	$795.8^{+12.2}_{-4.3}$	$83.3^{+0.1}_{-0.1}$	$38.8^{+2.1}_{-0.1}$	$82.2^{+0.3}_{-0.1}$
	III(- - +)	$6878.9^{+11.3}_{-7.4} - i73.1^{+2.6}_{-1.1}$	$1430.3^{+29.4}_{-5.7}$	$82.0^{+0.1}_{-0.1}$	$773.9^{+15.5}_{-4.2}$	$77.8^{+0.2}_{-0.1}$	$36.4^{+2.2}_{-0.1}$	$65.0^{+1.6}_{-0.4}$
	IV(- - -)	$6878.9^{+11.3}_{-7.4} - i73.1^{+2.6}_{-1.1}$	$1430.5^{+18.8}_{-5.0}$	$82.0^{+0.1}_{-0.1}$	$773.8^{+8.7}_{-3.1}$	$77.8^{+0.2}_{-0.1}$	$36.7^{+2.1}_{-0.1}$	$65.6^{+1.6}_{-0.4}$
	VII(- + -)	$6888.4^{+11.5}_{-7.2} - i59.4^{+1.7}_{-0.5}$	$1452.3^{+24.4}_{-5.6}$	$85.6^{+0.1}_{-0.1}$	$795.4^{+13.6}_{-3.4}$	$83.3^{+0.1}_{-0.1}$	$39.4^{+2.2}_{-0.1}$	$83.4^{+0.4}_{-0.2}$
ATLAS(Fit.VI)	II(- + +)	$6897.7^{+19.1}_{-4.3} - i50.9^{+0.9}_{-0.2}$	$1409.8^{+12.0}_{-1.9}$	$86.2^{+0.1}_{-0.1}$	$997.0^{+8.8}_{-1.8}$	$85.0^{+0.1}_{-0.1}$	$5.7^{+0.1}_{-0.1}$	$56.7^{+0.8}_{-0.3}$
	III(- - +)	$6883.8^{+18.3}_{-4.0} - i73.4^{+2.8}_{-0.7}$	$1373.6^{+7.3}_{-2.7}$	$80.8^{+0.1}_{-0.1}$	$960.0^{+5.6}_{-1.3}$	$77.5^{+0.1}_{-0.2}$	$7.2^{+0.1}_{-0.1}$	$21.6^{+1.1}_{-1.0}$
	IV(- - -)	$6883.8^{+18.3}_{-4.0} - i73.4^{+2.8}_{-0.7}$	$1379.0^{+10.0}_{-2.0}$	$80.8^{+0.1}_{-0.1}$	$963.8^{+7.1}_{-1.2}$	$77.5^{+0.1}_{-0.1}$	$7.3^{+0.1}_{-0.1}$	$22.1^{+1.1}_{-1.0}$
	VII(- + -)	$6897.7^{+19.1}_{-4.3} - i50.9^{+0.9}_{-0.2}$	$1406.7^{+10.4}_{-2.0}$	$86.2^{+0.1}_{-0.1}$	$994.9^{+7.4}_{-2.3}$	$85.0^{+0.1}_{-0.1}$	$5.8^{+0.2}_{-0.1}$	$57.6^{+0.9}_{-0.2}$

Phase shifts

- Phase shifts of 1S_0 is very likely to be the one generated by a narrow BW resonance: tetra-quark
- Other waves should contribute backgrounds



- other points of view.
 - excited tetraquark?

4、 Some other resonances: Tcc

- Obviously contain four quarks
- Chew-Mandelstam K-matrix

$$T^{-1}(s) = K^{-1}(s) - C(s) \qquad C_i(s) = \frac{s}{\pi} \int_{s_{th_i}}^{\infty} ds' \frac{\rho_i(s')}{s'(s' - s)}$$

- D^{*+} , unstable particles

$$C_2(s) \rightarrow \frac{1}{\pi} \int_{s_{tr, D\pi}}^{\infty} ds' C(s; \sqrt{s'}, m_2) \text{Im} f_{D\pi}(s')$$

- FSI: AMP method

$$F_i(s) = \sum_{k=1}^2 \alpha_k(s) T_{ki}(s),$$

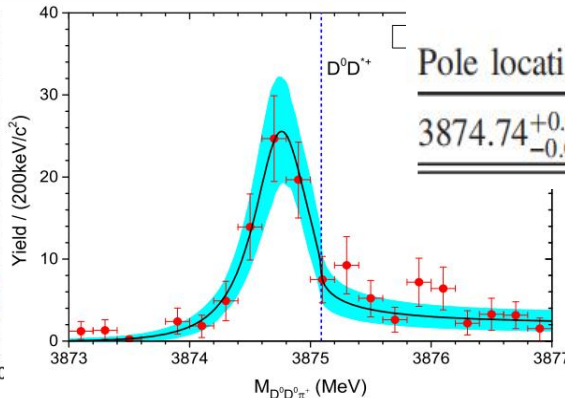
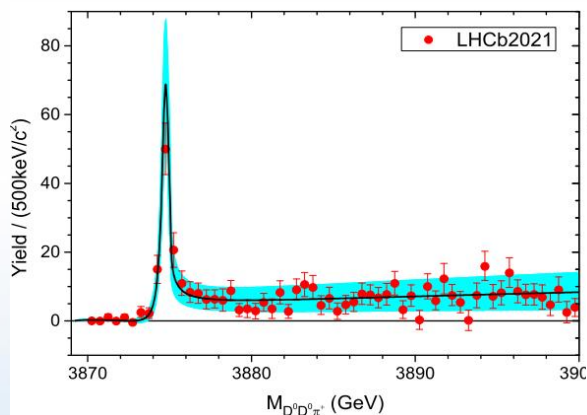
$$\begin{aligned} K_{11} &= -0.01204 \pm 0.00691^{+0.03280}_{-0.07039} \\ K_{12} = K_{21} &= 0.5080 \pm 0.0025^{+0.0348}_{-0.0700} \\ K_{22} &= 1.4447 \pm 0.0015^{+0.0235}_{-0.0477} \\ \alpha_2 &= -0.3024 \pm 0.0016^{+0.0261}_{-0.0583} \\ N_a &= 1434.0 \pm 129.8^{+662.0}_{-964.8} \text{ GeV}^{-2} \\ N_b &= 516.0 \pm 49.3^{+225.6}_{-363.4} \text{ GeV}^{-2} \\ \chi^2_{\text{d.o.f.}} &= 0.92 \end{aligned}$$

- one term in K-matrix: similar to effective range expansion

Dai, Sun, Kang, Szczepaniak, Yu,
PRD105 (2022) L051507

Tcc: virtual state origin?

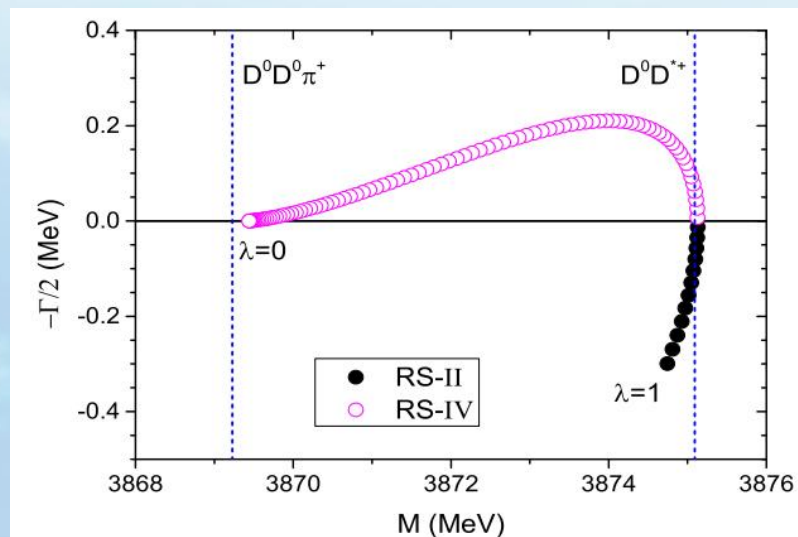
■ Pole parameters extracted



Pole location (MeV)	$g_{D^0 D^0 \pi^+}^{\text{II}} = g e^{i\varphi}$		$g_{D^0 D^{*+}}^{\text{II}} = g e^{i\varphi}$	
	$ g_1 $ (GeV)	φ_1 ($^\circ$)	$ g_2 $ (GeV)	φ_2 ($^\circ$)
$3874.74^{+0.11}_{-0.04} - i0.30^{+0.05}_{-0.09}$	$0.22^{+0.03}_{-0.04}$	9_{-5}^{+11}	$0.69^{+0.04}_{-0.02}$	10_{-5}^{+11}

- Pole analysis by switching the inelastic channel
- pole-counting, one pole, molecule?
- $\lambda=0$ RS-IV corresponds to RS-II in $D^0 D^{*+}$ single channel, virtual state origin!

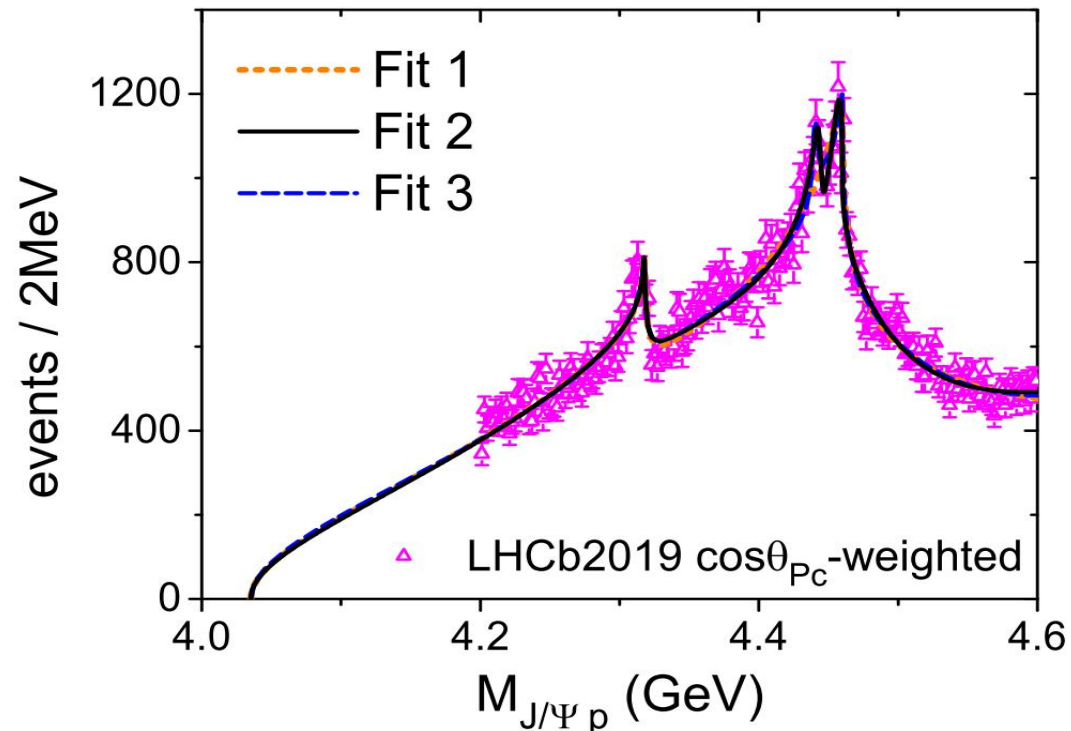
Ling, Geng, Xie *et.al.*,
PLB 826 (2022) 136897



Pc states

- Candidates for pentaquarks
- CM K-matrix+AMP
- $J/\psi p - \bar{D}^0 \Sigma_c^+ - \bar{D}^{*0} \Sigma_c^+$ triple channels
- Fit 1: no pole in KM
- Fit 2: a pole in KM around Pc(4440)
- Fit 3: P-wave instead of KM pole around Pc(4440)

Kuang, Dai#, Kang, Yao,
EPJC 80 (2020) 433



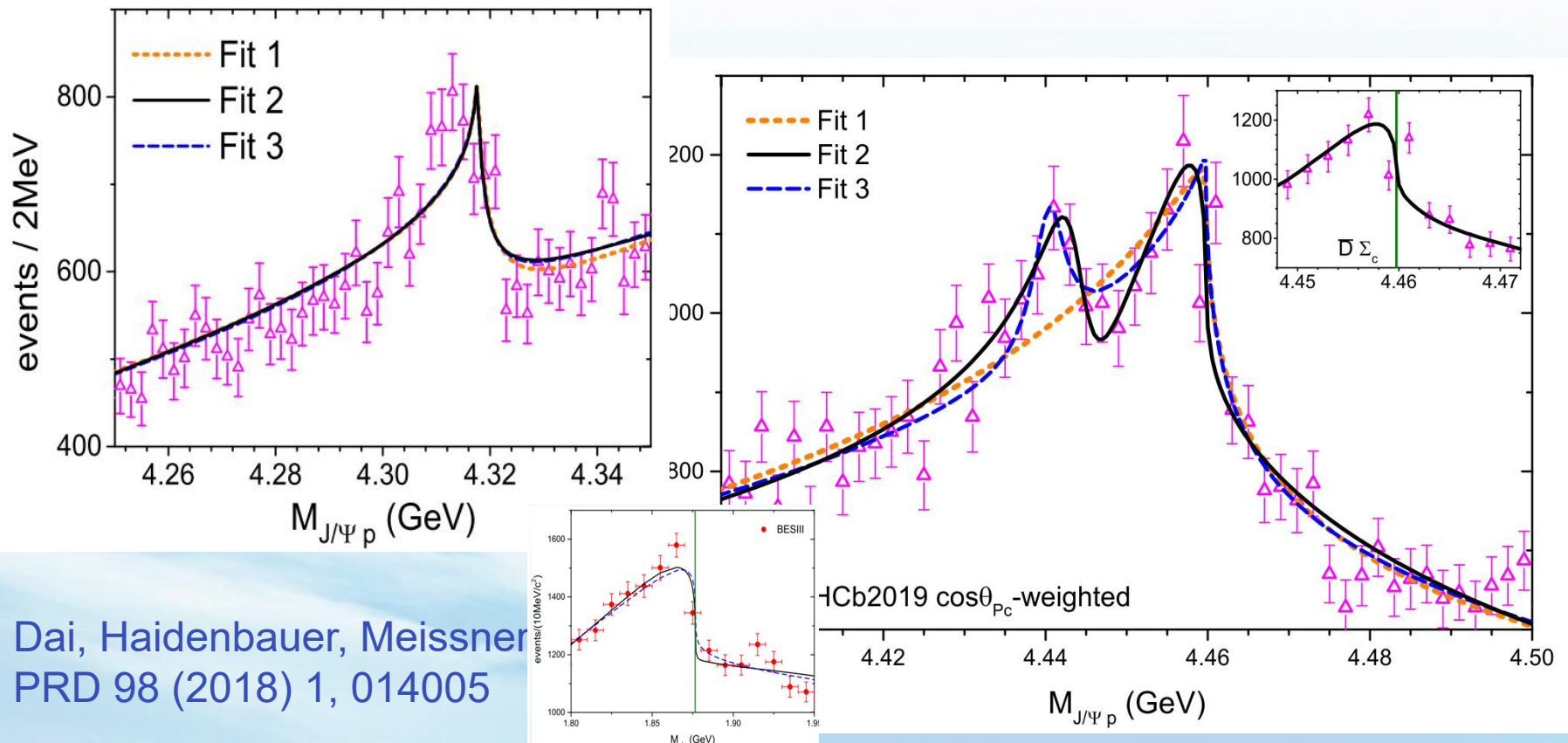
Pc states

- pole parameters
- pole counting rule
- Pc(4312): molecule?
- Pc(4440): compact tetraquark
- Pc(4457): threshold effect?

State	Pole locations (MeV)					
	RS	Fit.1	RS	Fit.2	RS	Fit.3
$P_c(4312)$	III	$4296.93^{+2.48}_{-3.00}$ $-i5.12^{+2.44}_{-1.06}$	III	...	V*	$4313.38^{+2.52}_{-5.73}$ $-i2.05^{+1.65}_{-0.75}$
	V*	$4312.74^{+1.69}_{-0.67}$ $-i3.33^{+2.91}_{-1.25}$	V*	$4314.31^{+2.06}_{-1.10}$ $-i1.43^{+1.50}_{-0.57}$	VIII	$4313.11^{+3.86}_{-4.76}$ $-i3.11^{+1.63}_{-2.02}$
$P_c(4440)$	III*	$4444.09^{+2.53}_{-1.48}$ $-i3.10^{+0.53}_{-1.33}$	III*	$4440.53^{+0.47}_{-0.31}$ $-i2.42^{+0.22}_{-0.22}$
	IV	$4443.69^{+2.89}_{-1.34}$ $-i0.32^{+1.23}_{-0.04}$	IV	$4440.38^{+0.41}_{-0.19}$ $-i1.40^{+0.59}_{-0.50}$
	V	$4444.22^{+2.72}_{-1.41}$ $-i2.48^{+0.57}_{-0.67}$	V	$4440.53^{+0.37}_{-0.30}$ $-i2.32^{+0.27}_{-0.61}$
	VII	$4443.84^{+1.93}_{-1.91}$ $-i1.02^{+1.05}_{-0.92}$	VIII	$4440.38^{+3.31}_{-0.52}$ $-i1.30^{+4.45}_{-0.50}$
	III	$4466.53^{+2.13}_{-4.75}$ $-i3.88^{+6.95}_{-0.93}$
$P_c(4457)$	VII	$4456.77^{+3.10}_{-8.89}$ $-i7.77^{+11.07}_{-4.41}$	VIII	$4453.44^{+7.11}_{-3.34}$ $-i21.58^{+8.01}_{-6.36}$

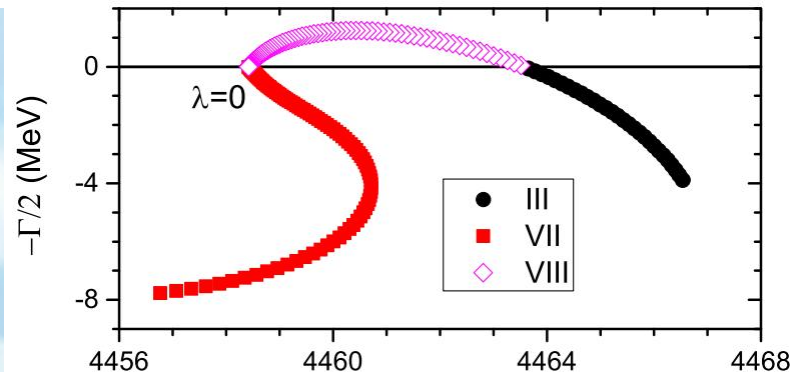
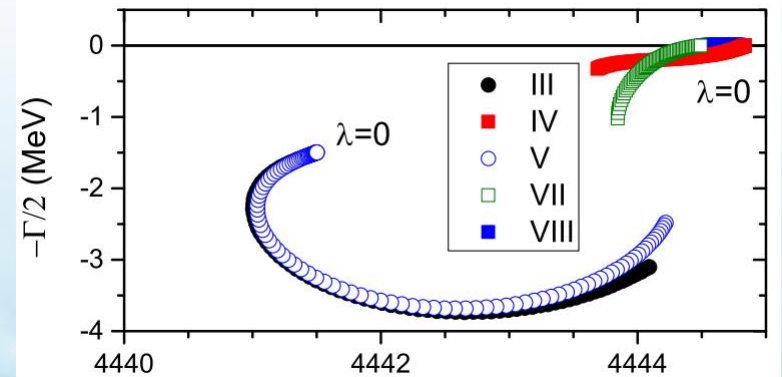
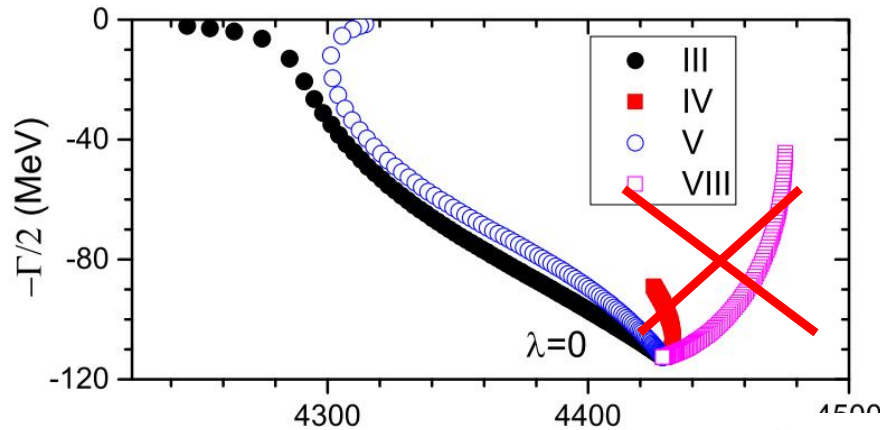
Pc states: inner structure?

- pole-counting, one pole is molecule type
- Fit 2 better than Fit 3, Pc(4440) prefers to be S-wave
- Pc(4457): quite similar to $p\bar{p}$ threshold behaviour



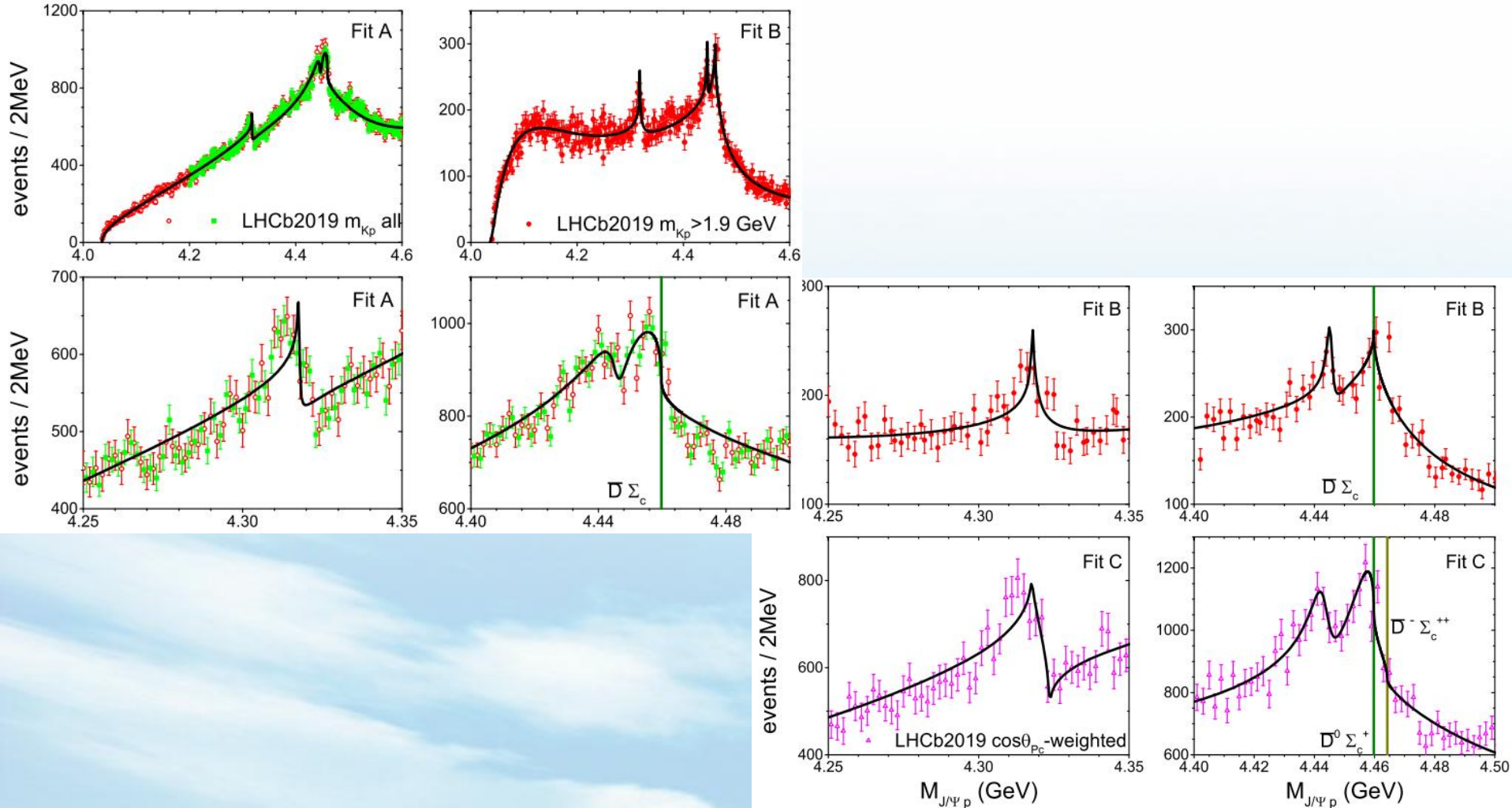
Pole trajectories?

- Pc(4312): support molecule.
- Pc(4440): compact tetraquark
- Pc(4457): threshold behaviour



other datasets

- The same conclusion as the previous analysis



4、 Summary

PWA

One needs PWA to extract pole parameters for the resonance: quantum number, mass, width, residues...

FSI

Amplitude analysis connects QFT principles and Exp. FSI needs to be considered when performing amplitude analysis.

structure

X6900: 0^{++} , likely to be a compact tetraquark;
Tcc: virtual state origin; Pc(4312): molecule, Pc(4440):
pentaquark, Pc(4457): threshold behaviour.

Next?

Angular distribution? Being essential to separate the partial waves. Other scatterings? Better method to classify nature of hadrons?



Thank You For your patience !

