

# Recent progress on the molecular tetraquarks, pentaquarks and di-baryons

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第三届LHCb前沿物理研讨会

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# Outline

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## ✓ Background : experimental observations

- $Z_c(3900), Z_c(4020), Z_b(10610), Z_b(10650)$
- $Z_{cs}(3985), Z_{cs}(4000)$
- $P_c(4312), P_c(4440), P_c(4457)$
- $P_{cs}(4459), P_{cs}(4338)$
- $X_{0,1}(2900), T_{cc}(3875)$  and  $T_{cs}(2900)$

## ✓ Theoretical aspects:

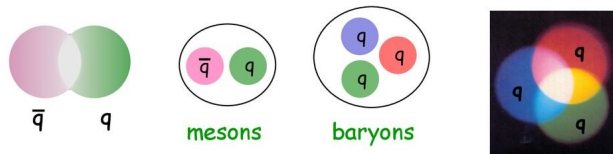
- Molecular tetraquarks
- Molecular pentaquarks
- Molecular hexaquarks (dibaryons)

## ✓ Summary and outlook

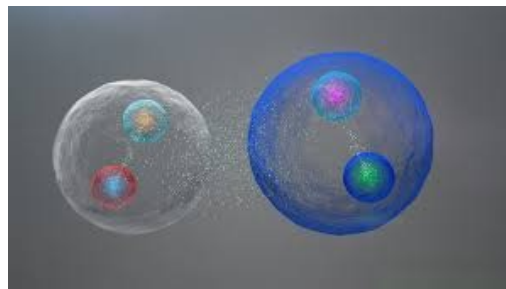
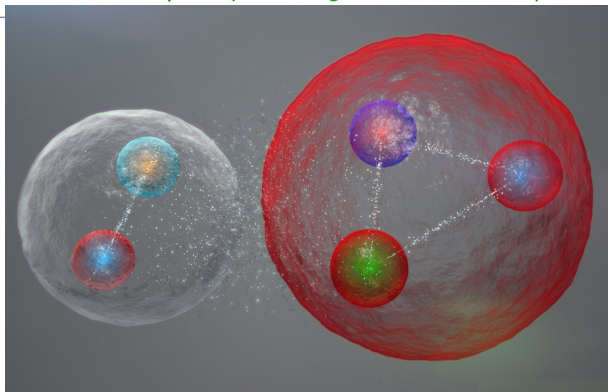
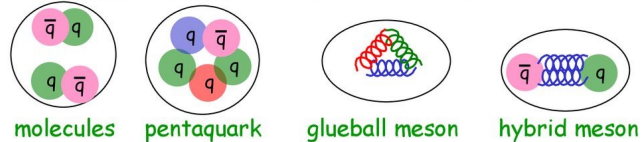
# Conventional and exotic hadrons

## Quarks are confined inside colorless hadrons

Quarks combine to “neutralize” color force



Configurations outside the standard quark model



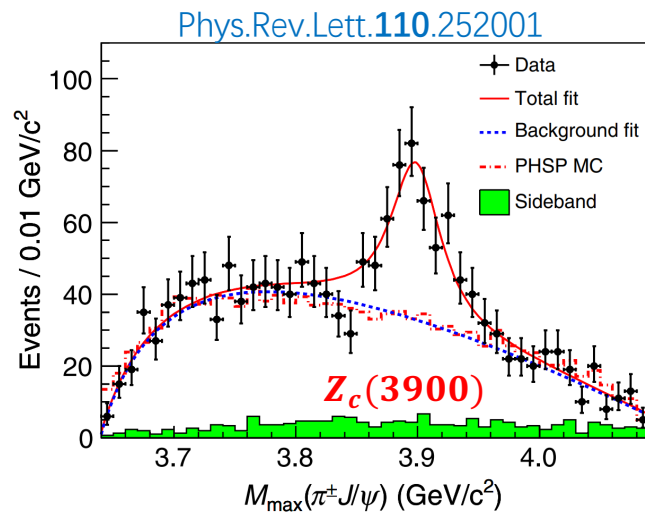
- $X(3872)$
- $Z_c(3900)$
- $Z_c(4020)$
- $Z_b(10610)$
- $Z_b(10650)$
- $P_c(4312)$
- $P_c(4440)$
- $P_c(4457)$
- .....

## Recent reviews:

- ✓ **H.-X. Chen** et al, *Phys. Rept.* 639, 1 (2016)
- ✓ **R. Lebed** et al, *Prog. Part. Nucl. Phys.* 93, 143 (2017)
- ✓ **A. Esposito** et al, *Phys. Rept.* 668, 1(2017)
- ✓ **F.-K. Guo** et al, *Rev. Mod. Phys.* 90, 015004 (2018)
- ✓ **Y.-R. Liu** et al, *Prog. Part. Nucl. Phys.* 107, 237 (2019)
- ✓ **N. Brambilla** et al, *Phys. Rept.* 873, 1 (2020)
- ✓ **S. Chen** et al, *Front. Phys.* 18, 44601 (2023)
- ✓ **H.-X. Chen** et al, *Rept. Prog. Phys.* 86, 026201 (2023)
- ✓ **L. Meng** et al, *arXiv:2204.08716*

# Experimental observations

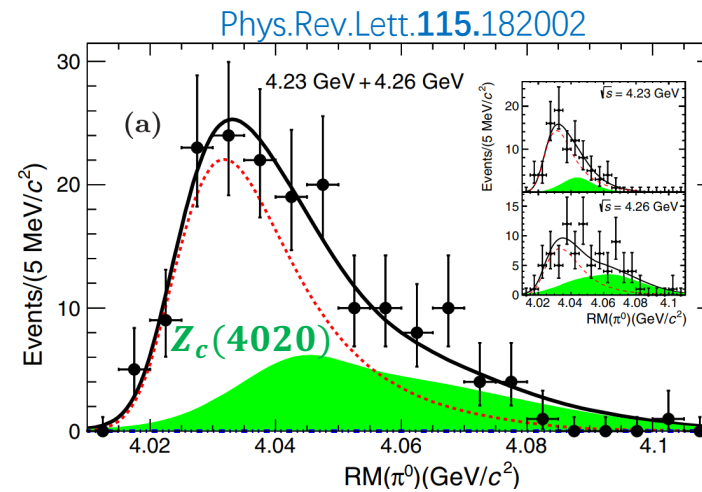
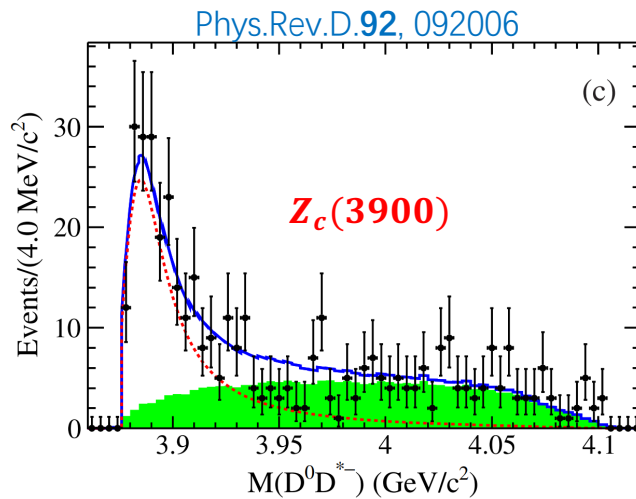
- Charmonium energy region:  $Z_c(3900)$  ( $Z_c$ ) and  $Z_c(4020)$  ( $Z'_c$ )



- BESIII:**  $e^+e^- \rightarrow J/\psi\pi^+\pi^-$  and  $e^+e^- \rightarrow h_c\pi^+\pi^-$ , respectively.
- $Z_c(3900)$ : subsequently confirmed by the Belle [Phys. Rev. Lett. **110**, 252002] and Xiao *et al* [Phys. Lett. B **727**, 366].

# Experimental observations

- Charmonium energy region:  $Z_c(3900)$  and  $Z_c(4020)$

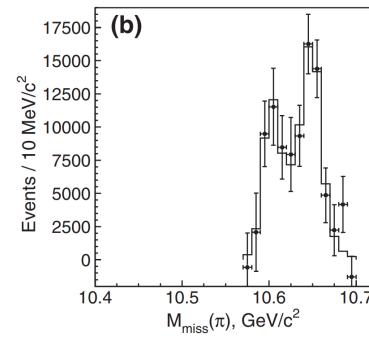
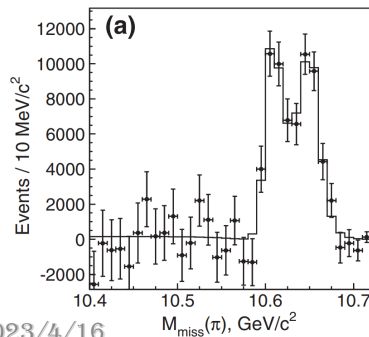
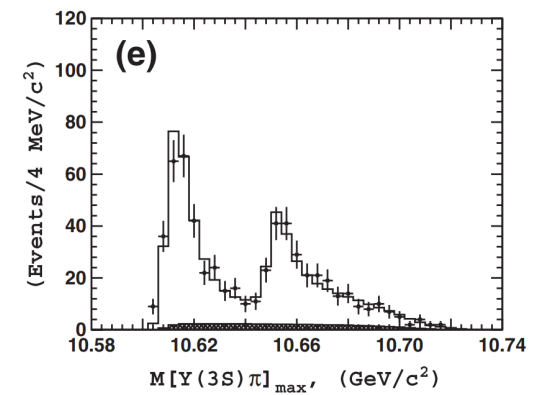
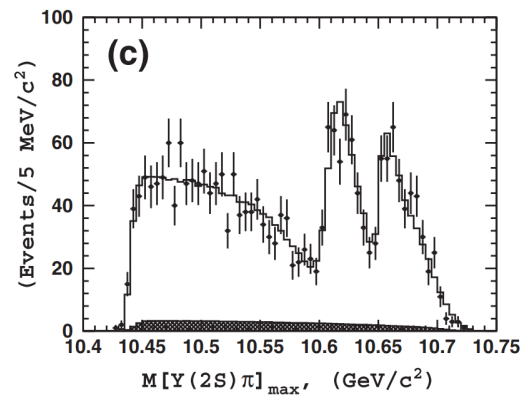
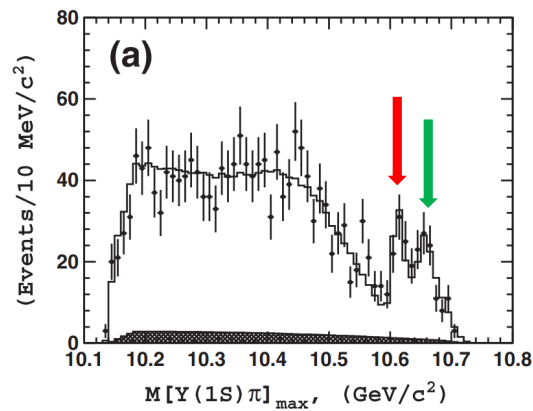


- BESIII:  $e^+e^- \rightarrow \bar{D}D^*\pi$  and  $e^+e^- \rightarrow \bar{D}^*D^*\pi$ , respectively.

$Z_c(3900): I^G(J^{PC}) = 1^+(1^{+-})$  is measured  
 $Z_c(4020): I^G(J^{PC}) = 1^+(1^{+-})$  is favored

# Experimental observations

- Bottomonium energy region:  $Z_b(10610)$  ( $Z_b$ ) and  $Z_b(10650)$  ( $Z'_b$ )

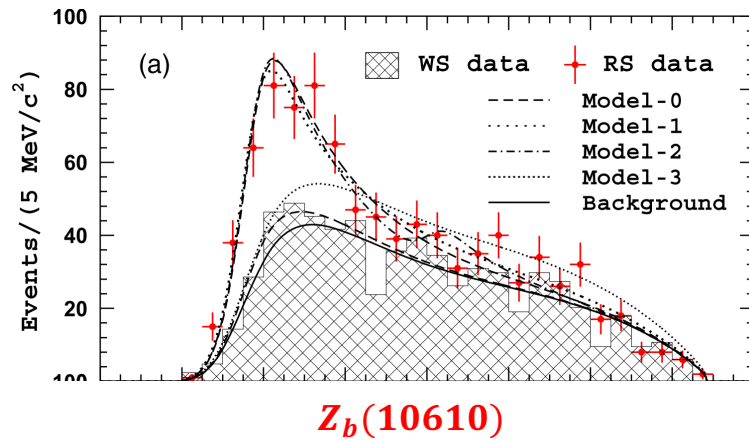


Phys.Rev.Lett. **108**.122001

Belle:  $e^+e^- \rightarrow \Upsilon(nS)\pi^+\pi^-$  ( $n = 1,2,3$ ) and  $e^+e^- \rightarrow h_b(mP)\pi^+\pi^-$  ( $m = 1,2$ ), respectively.

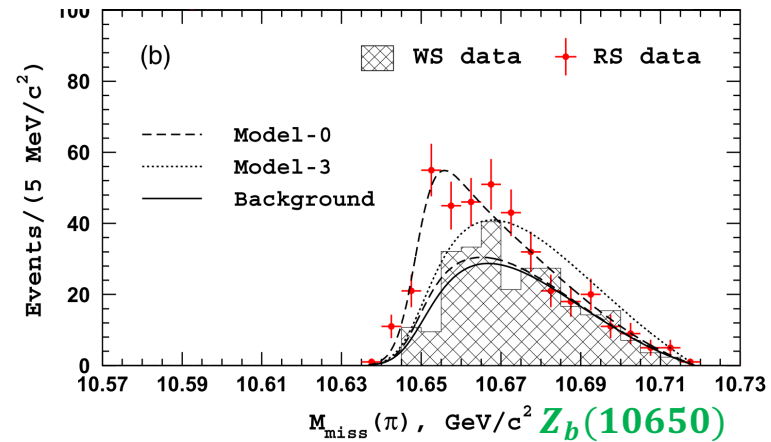
# Experimental observations

- Bottomonium energy region:  $Z_b(10610)$  and  $Z_b(10650)$



Phys.Rev.Lett. **116**.212001

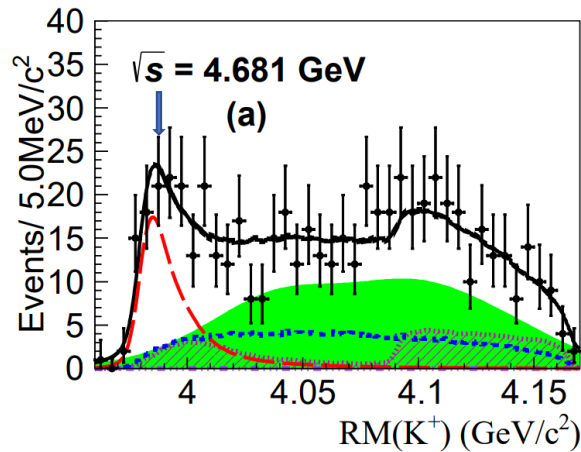
Belle:  $e^+e^- \rightarrow \bar{B}B^*\pi$  and  $e^+e^- \rightarrow \bar{B}^*B^*\pi$ , respectively.



$Z_b(10610): I^G(J^{PC}) = 1^+(1^{+-})$  is measured  
 $Z_b(10650): I^G(J^{PC}) = 1^+(1^{+-})$  is measured

# Experimental observations

- Charmonium energy region:  $Z_{cs}(3985)$  and  $Z_{cs}(4000)$

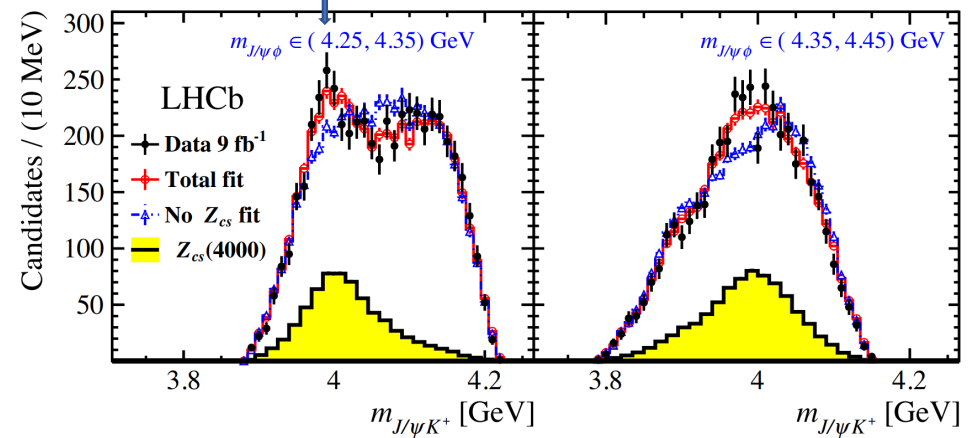


[PhysRevLett.126.102001](https://arxiv.org/abs/1206.1020)

BESIII:  $e^+e^- \rightarrow K^+(D_s^-D^{*0} + D_s^{*-}D^0)$

$$m_{Z_{cs}} = 3982.5^{+1.8}_{-2.6} \pm 2.1 \text{ MeV}$$

$$\Gamma_{Z_{cs}} = 12.8^{+5.3}_{-4.4} \pm 3.0 \text{ MeV}$$



[PhysRevLett.127.082001](https://arxiv.org/abs/1207.0820)

LHCb:  $B^+ \rightarrow J/\psi\phi K^+$

$$m_{Z_{cs}} = 4003 \pm 6^{+4}_{-14} \text{ MeV}$$

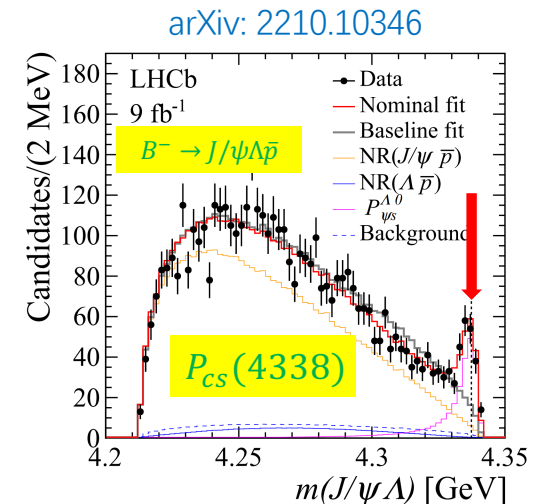
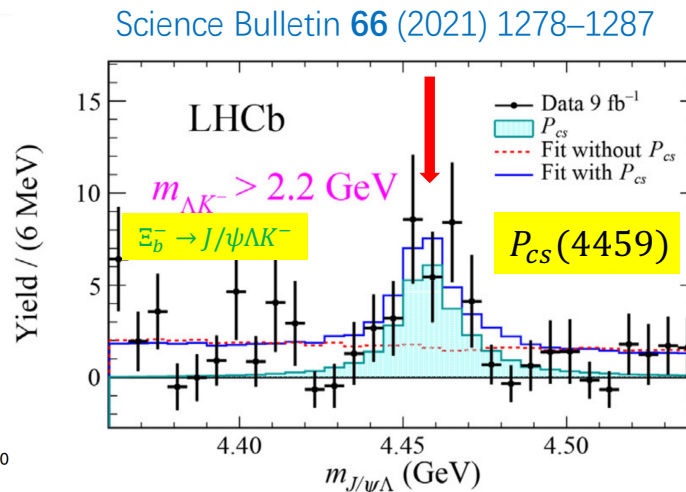
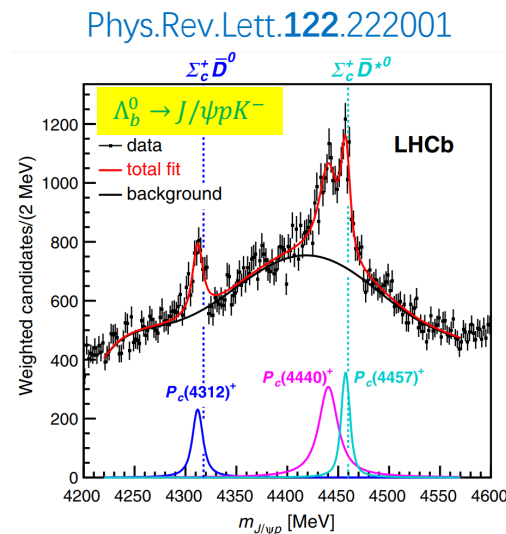
$$\Gamma_{Z_{cs}} = 131 \pm 15 \pm 26 \text{ MeV}$$

A broader  $Z_{cs}(4220)$  was also reported



# Experimental observations

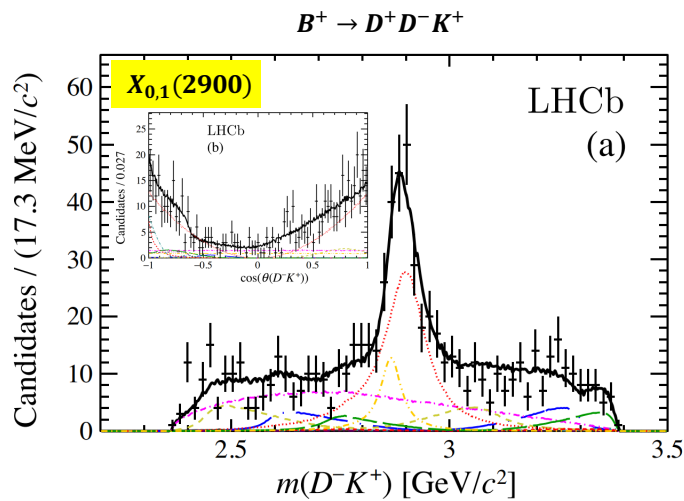
- Charmonium energy region:  $P_c(4312)$ ,  $P_c(4440)$ ,  $P_c(4457)$  and  $P_{cs}$



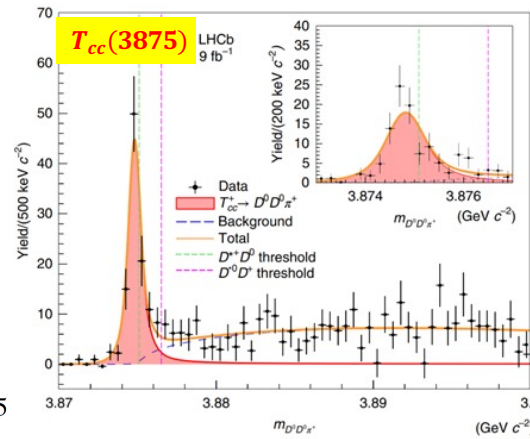
- A  $P_c(4337)$  was also reported by the LHCb in  $B_s^0 \rightarrow J/\psi p \bar{p}$  decay [Phys.Rev.Lett. 128 (2022) 062001].
- LHCb: the  $J^P$  quantum numbers of  $P_c$ s and  $P_{cs}(4459)$  are undetermined yet, while  $\frac{1}{2}^-$  is preferred for  $P_{cs}(4338)$  with 90% CL.

# Experimental observations

$X_{0,1}(2900)$ ,  $T_{cc}(3875)$  and  $T_{c\bar{s}}(2900)$



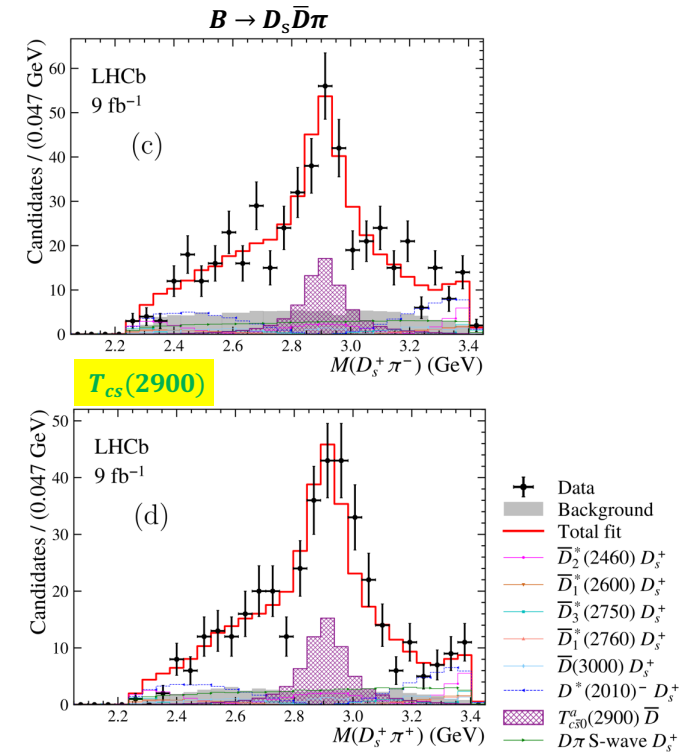
PhysRevD.102.112003



Nature Phys. 18 (2022) 7, 751-754

The fits of the lineshapes of  $X_{0,1}(2900)$  is given in **B. Wang et al**, Eur.Phys.J.C 82 (2022) 419. For  $T_{c\bar{s}}(2900)$ , see **W. Chen's** talk.

2023/4/16



arXiv: 2212.02716

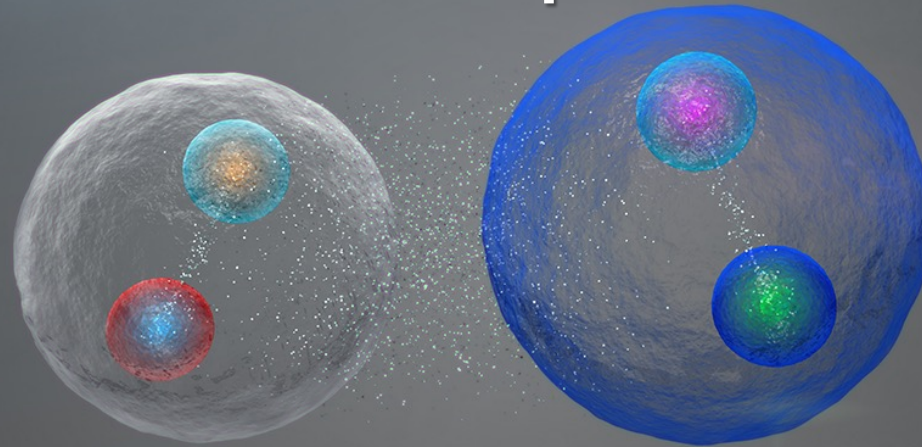
# Possible combinations

$$\begin{aligned}
 & \begin{bmatrix} N \\ M_Q \\ B_Q \\ B_{QQ} \end{bmatrix} \hat{\otimes} \begin{bmatrix} N \\ M_Q \\ B_Q \\ B_{QQ} \\ \bar{N} \\ \bar{M}_Q \\ \bar{B}_Q \\ \bar{B}_{QQ} \end{bmatrix}^T \Rightarrow \begin{bmatrix} NN & NM_Q & NB_Q & NB_{QQ} & N\bar{N} & & & & \\ & M_Q M_Q & M_Q B_Q & M_Q B_{QQ} & M_Q \bar{N} & M_Q \bar{M}_Q & & & \\ & & B_Q B_Q & B_Q B_{QQ} & B_Q \bar{N} & B_Q \bar{M}_Q & B_Q \bar{B}_Q & & \\ & & & B_{QQ} B_{QQ} & B_{QQ} \bar{N} & B_{QQ} \bar{M}_Q & B_{QQ} \bar{B}_Q & B_{QQ} \bar{B}_{QQ} & \end{bmatrix} \\
 & \Rightarrow \begin{bmatrix} \text{deuteron} & \Lambda(2940)?, \dots & \square & \square & X(1835)? & & & & \\ & T_{cc}, \dots & \square & \square & \square & X(3872), Z_{c(s)}^{(\prime)}, Z_b^{(\prime)}, \dots & & & \\ & & \square & \square & \square & P_{c(s)}, \dots & \square & & \\ & & & \square & \square & \square & \square & \square & \square \end{bmatrix},
 \end{aligned}$$

- Sometimes, the  $K^*$  meson may be regarded as the heavy matter field to some extent ( $m_{K^*} \sim m_N$ ). The  $X_{0,1}(2900)$  [1, 2] and  $T_{cs}(2900)$  [3] observed by the LHCb are very close to the  $\bar{D}^* K^*$  and  $D^* K^*$  thresholds, respectively.

[1] Phys. Rev. D 102 (2020) 112003 [2] Phys. Rev. Lett. 125 (2020) 242001 [3] arXiv: 2212.02716 [hep-ex] [4] arXiv: 2212.02717 [hep-ex]

# Molecular tetraquarks



# X(3872) and its possible partners

In Refs. [J. Nieves *et al*, Phys.Rev.D 86 (2012) 056004; F.-K. Guo *et al*, Phys.Rev.D 88 (2013) 054007; V. Baru *et al*, Phys.Lett.B 763 (2016) 20-28], the heavy quark spin symmetry (HQSS) partners of X(3872) with the  $J^{PC}$  quantum numbers  $2^{++}$  was proposed.

$$\begin{aligned} \left| \frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D}) \right\rangle_{1^{++}} &= -|1_{\bar{h}}^- \otimes 1_{\bar{\ell}}^- \rangle \\ |D^*\bar{D}^*\rangle_{2^{++}} &= -|1_{\bar{h}}^- \otimes 1_{\bar{\ell}}^- \rangle \end{aligned} \quad \xrightarrow{\hat{H}_{\text{eff}} \equiv \tilde{c}_1 + \tilde{c}_2 \boldsymbol{\ell}_1 \cdot \boldsymbol{\ell}_2} \quad \begin{aligned} V_{1^{++}}^\alpha &= C_1^\alpha = \tilde{c}_1 + \frac{1}{4}\tilde{c}_2, \\ V_{2^{++}}^\alpha &= C_1^\alpha = \tilde{c}_1 + \frac{1}{4}\tilde{c}_2. \end{aligned}$$

$SU(3)_F$  symmetry and HQSS for di-meson systems L. Meng, B. Wang, S.-L. Zhu, Sci.Bull. 66 (2021) 1288-1295

$$\begin{aligned} D_s^{(*)} - D^{(*)} &\simeq 100 \text{ MeV}, \\ D_{(s)}^* - D_{(s)} &\simeq 140 \text{ MeV}, \end{aligned} \quad V_{q\bar{q}} = C_1 + C_2 \mathbf{S}_1 \cdot \mathbf{S}_2 + C_3 \mathbb{C}_2 + C_4 (\mathbf{S}_1 \cdot \mathbf{S}_2) \mathbb{C}_2$$

$m_{D^0} + m_{D^{*0}} - m_{X(3872)} = (0.00 \pm 0.18) \text{ MeV}$ . It can be approximately regarded as a pure  $D^0\bar{D}^{*0}/\bar{D}^0D^{*0}$  dimeson, then its flavor wave function in the light part will be  $|\bar{u}u\rangle$ .

$$\begin{aligned} \langle \mathbb{C}_2 \rangle_{\bar{u}u} &= \langle \mathbb{C}_2 \rangle_{\bar{s}s} \\ \langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle_{\{\text{PP}, \text{VV}\}}^{0^{++}} &= \begin{bmatrix} 0 & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{1}{2} \end{bmatrix}, \\ \langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle_{\{\text{PV}, \text{VV}\}}^{1^{+-}} &= \begin{bmatrix} -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}, \\ \langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle_{\{\text{PV}\}}^{1^{++}} &= \frac{1}{4}, \quad \langle \mathbf{S}_1 \cdot \mathbf{S}_2 \rangle_{\{\text{VV}\}}^{2^{++}} = \frac{1}{4}, \end{aligned} \quad \xrightarrow{\quad} \quad \begin{aligned} V_{q\bar{q}} &= \tilde{c}_1 + \tilde{c}_2 \mathbf{S}_1 \cdot \mathbf{S}_2, \\ (V_{\text{PV}}^{1^{++}} - V_{\text{PP}}^{0^{++}}) : (V_{\text{VV}}^{0^{++}} - V_{\text{PP}}^{0^{++}}) : (V_{\text{PV/VV}}^{1^{+-}} - V_{\text{PP}}^{0^{++}}) &= 1 : -2 : -1. \end{aligned}$$

X(3872)

Bound state

$[\bar{D}_s^* D_s^*]^{2^{++}}$   
 $[\bar{D}_s D_s]^{0^{++}}$

$[\bar{D}_s^* D_s^*]^{1^{+-}}$   
 $[\bar{D}_s D_s]^{0^{++}}$

$[\bar{D}_s^* D_s / \bar{D}_s D_s^*]^{1^{++}}$

$[\bar{D}_s^* D_s / \bar{D}_s D_s^*]^{1^{+-}}$

More attractive

# X(3872) and its possible partners

## Two prerequisites:

- The X(3872) is the molecular state with its mass coinciding exactly with the  $\bar{D}_0^* D_0$  threshold;
- There exist the  $\bar{D}_s D_s$  bound states with  $J^{PC} = 0^{++}$ .

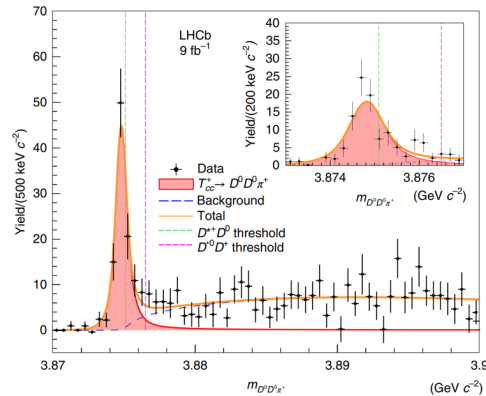
The recent lattice QCD calculation yielded a shallow  $[D_s \bar{D}_s]^{0^{++}}$  bound state with  $\Delta E = -6.2^{+3.8}_{-2.0}$  MeV [JHEP 06 (2021) 035]

$$T(p', p; E) = V(p', p) + \int \frac{d^3 p''}{(2\pi)^3} \frac{V(p', p'') T(p'', p; E)}{E - p^2/(2\mu) + i\epsilon},$$

$X(3872)_{\text{input}}$	$[\bar{D}_s D_s]_{\text{input}}^{0^{++}}$	$[\bar{D}_s^* D_s^*]^{0^{++}}$	$[\bar{D}_s^* D_s / \bar{D}_s D_s^*]^{1^{+-}}$	$[\bar{D}_s^* D_s^*]^{1^{+-}}$				
$\Delta E$ (MeV)	$\Delta E$ (MeV)	$M$ (MeV)	$\Delta E$ (MeV)	$M$ (MeV)	$\Delta E$ (MeV)	$M$ (MeV)	$\Delta E$ (MeV)	$M$ (MeV)
0.0	-2.4	3934.3	-20.3	4204.1	-9.5	4071.0	-11.4	4213.0
0.0	-6.2	3930.5	-45.5	4178.9	-22.5	4058.0	-25.2	4199.2
0.0	-8.2	3928.5	-57.6	4166.8	-29.0	4051.5	-32.0	4192.4
0.0	-12.9	3923.8	-84.3	4140.1	-43.7	4036.8	-47.2	4177.2
-1.0	-2.4	3934.3	-8.3	4216.1	-4.9	4075.6	-6.3	4218.1
-1.0	-6.2	3930.5	-28.9	4195.5	-15.9	4064.6	-18.2	4206.2
-1.0	-8.2	3928.5	-39.6	4184.8	-21.7	4058.8	-24.4	4200.0
-1.0	-12.9	3923.8	-64.1	4160.3	-35.2	4045.3	-38.5	4185.9
Cutoff-I [49]	-13	3924	-84	4140	-46	4035	-47	4177
Cutoff-II [49]	-9	3928	-84	4140	-41	4040	-44	4180

[49] C. Hidalgo-Duque *et al*, Phys.Rev.D 87 (2013) 7, 076006.

# $T_{cc}$ and its decays



**Table 1 |** Parameters obtained from the fit to the  $D^0 D^0 \pi^+$  mass spectrum: signal yield,  $N$ , BW mass relative to the  $D^{*+} D^0$  mass threshold,  $\delta m_{BW}$ , and width,  $\Gamma_{BW}$ . The uncertainties are statistical only

Parameter	Value
$N$	$117 \pm 16$
$\delta m_{BW}$	$-273 \pm 61 \text{ keV } c^{-2}$
$\Gamma_{BW}$	$410 \pm 165 \text{ keV}$

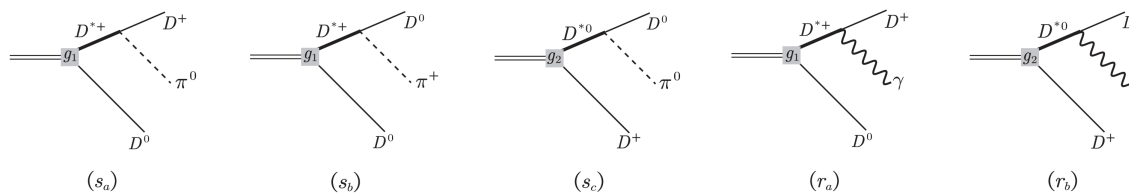
Nature Phys. 18 (2022) 7, 751-754

✓ Within the contact EFT [L. Meng *et al*, Phys.Rev.D 104 (2021) , L051502]  $|T_{cc}^+\rangle = \cos \theta |D^{*+} D^0, \phi_1\rangle + \sin \theta |D^{*0} D^+, \phi_2\rangle$ ,

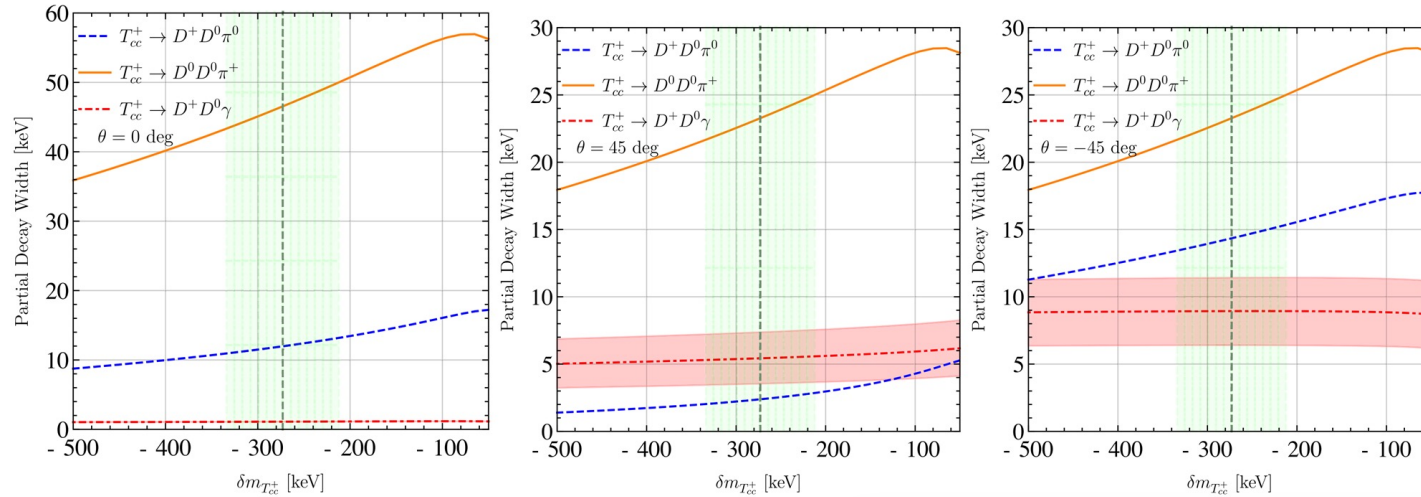
$$\lim_{E \rightarrow E_0} (E - E_0) t_{ij} = \lim_{E \rightarrow E_0} \left[ \frac{d(t_{ij})^{-1}}{dE} \right]^{-1} = \frac{1}{8M_T^2 \mu} g_i g_j$$

$$g_1 = \frac{4M_T \sqrt{\pi \kappa_1}}{\sqrt{\mu}} \cos \theta, \quad g_2 = \frac{4M_T \sqrt{\pi \kappa_2}}{\sqrt{\mu}} \sin \theta.$$

$$\theta = \begin{cases} 0 & \text{pure } D^{*+} D^0 \\ \frac{\pi}{4} & I = 1, I_3 = 0. \\ -\frac{\pi}{4} & I = 0, I_3 = 0 \end{cases}$$



# $T_{cc}$ and its decays



Single-channel limit :  $\Gamma_{\text{str}} + \Gamma_{\text{EM}} = 59.7^{+4.6}_{-4.4}$  keV.

Isospin singlet :  $\Gamma_{\text{str}} + \Gamma_{\text{EM}} = 46.7^{+2.7}_{-2.9}$  keV,

Isospin triplet :  $\Gamma_{\text{str}} + \Gamma_{\text{EM}} = 31.2^{+2.2}_{-2.4}$  keV.

The pole parameters are found to be

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}/c^2,$$

Nature Commun. 13 (2022) 1, 3351

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV},$$

**isoscalar assignment for  $T_{cc}$  is supported!**

The improved calculations for the  $DD^*$  interactions that based on the  $\chi\text{EFT}$  up to  $N^2\text{LO}$  was given in **B. Wang et al**, [arXiv:2212.08447](https://arxiv.org/abs/2212.08447) [accepted by PRD].

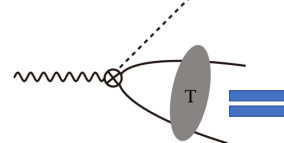
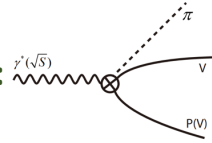
Isospin violating decays of  $X(3872)$  was revisited in **L. Meng et al**, [PhysRevD.104.094003](https://arxiv.org/abs/1004.09400)



# $Z_{c,b}$ and their strange partners

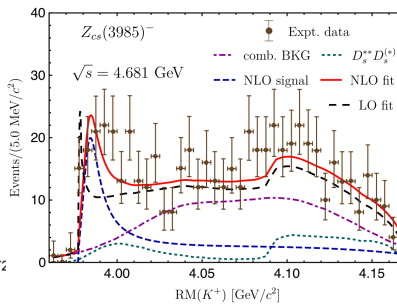
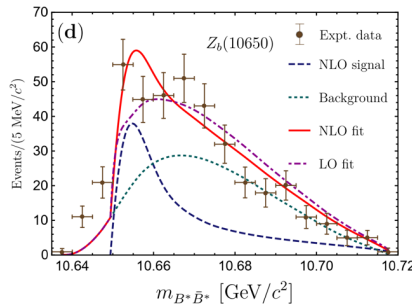
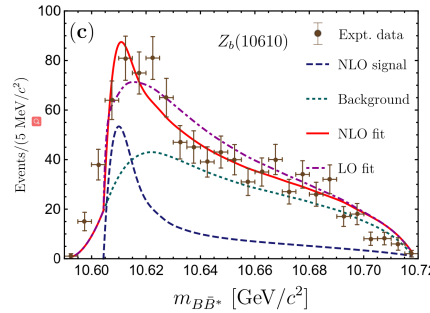
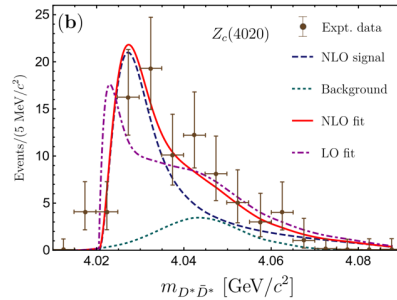
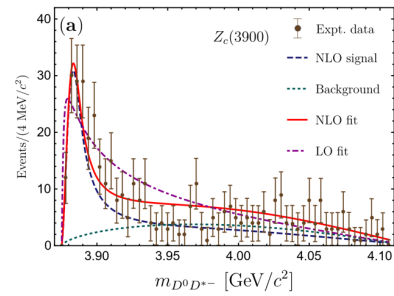
Comprehensive review on  $\chi$ EFTs in heavy sectors: [arXiv:2204.08716](https://arxiv.org/abs/2204.08716)

1. Simulate the production process:

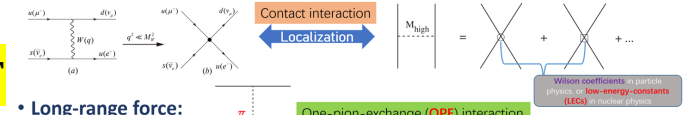


$\chi$ EFT

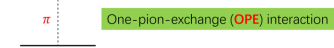
2. Fit the lineshapes in experiments:



• Short-range force:



• Long-range force:

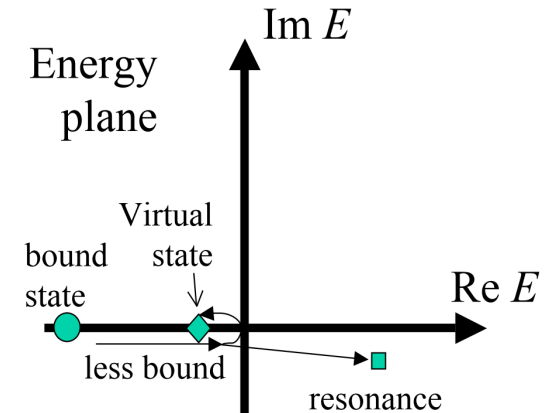


• Mid-range force:



$$\mathcal{U}(E, \mathbf{p}) = \mathcal{M}(E, \mathbf{p}) + \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \mathcal{V}(E, \mathbf{p}, \mathbf{q}) \mathcal{G}(E, \mathbf{q}) \mathcal{U}(E, \mathbf{q}),$$

Search for the poles in the Riemann sheets:



# $Z_{c,b}$ and their strange partners

B. Wang *et al*, PhysRevD.102.114019  
B. Wang *et al*, PhysRevD.103.L021501

## • Fitted parameters and predicted states

Poles in the second Riemann sheet

Strangness  $S = 0$

States	Thresholds	$\tilde{C}_s$ [GeV <sup>-2</sup> ]	$C_s$ [GeV <sup>-4</sup> ]	$C_{sd}$ [GeV <sup>-4</sup> ]	$\Lambda$ [GeV]	$[m, \Gamma]_{\text{pole}}$	$[m, \Gamma]_{\text{expt}}$
$\frac{1}{\sqrt{2}} [D\bar{D}^* + D^*\bar{D}]$	3875.8	$3.6^{+1.2}_{-1.2}$	$-76.9^{+6.2}_{-6.2}$	$1.1^{+5.8}_{-5.8}$	$0.33^{+0.024}_{-0.024}$	$[3881.3^{+3.0}_{-3.0}, 12.4^{+5.0}_{-5.0}]$	$[3881.7^{+2.3}_{-2.3}, 26.6^{+3.0}_{-3.0}]$
$D^*\bar{D}^*$	4017.1	$4.0^{+1.6}_{-1.6}$	$-78.1^{+8.7}_{-8.7}$	$1.7^{+6.3}_{-6.3}$	$0.34^{+0.031}_{-0.031}$	$[4026.5^{+4.5}_{-4.5}, 10.1^{+7.2}_{-7.2}]$	$[4025.5^{+3.7}_{-5.6}, 26.0^{+6.0}_{-6.0}]$
$\frac{1}{\sqrt{2}} [B\bar{B}^* + B^*\bar{B}]$	10604.4	$2.2^{+0.2}_{-0.2}$	$-9.9^{+1.0}_{-1.0}$	$3.6^{+4.7}_{-4.7}$	$0.51^{+0.014}_{-0.014}$	$[10607.9^{+2.2}_{-2.2}, 10.9^{+3.0}_{-3.0}]$	$[10607.2^{+2.0}_{-2.0}, 18.4^{+2.4}_{-2.4}]$
$B^*\bar{B}^*$	10649.4	$2.2^{+0.3}_{-0.3}$	$-9.9^{+1.2}_{-1.2}$	$3.3^{+6.6}_{-6.6}$	$0.51^{+0.015}_{-0.015}$	$[10652.8^{+2.7}_{-2.7}, 10.9^{+3.4}_{-3.4}]$	$[10652.2^{+1.5}_{-1.5}, 11.5^{+2.2}_{-2.2}]$

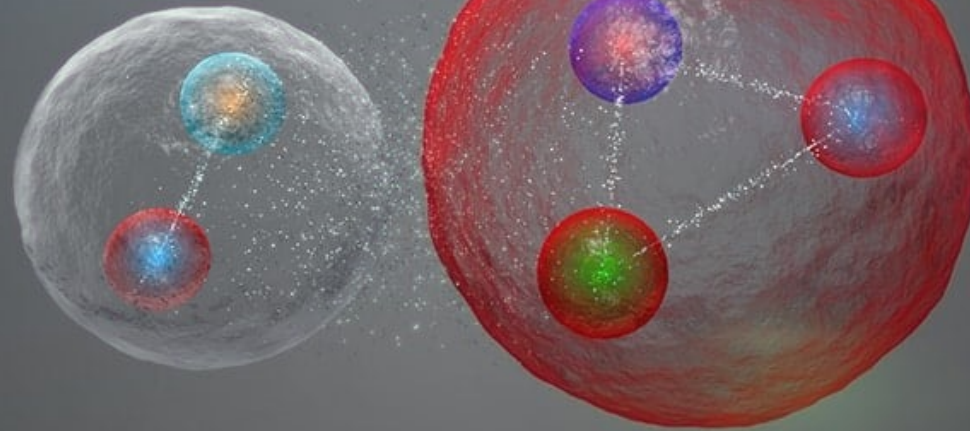
Strangness  $S = -1$

$$(m, \Gamma) = (3982.4^{+4.8}_{-3.4}, 11.8^{+5.5}_{-5.2}) \text{ MeV,}$$

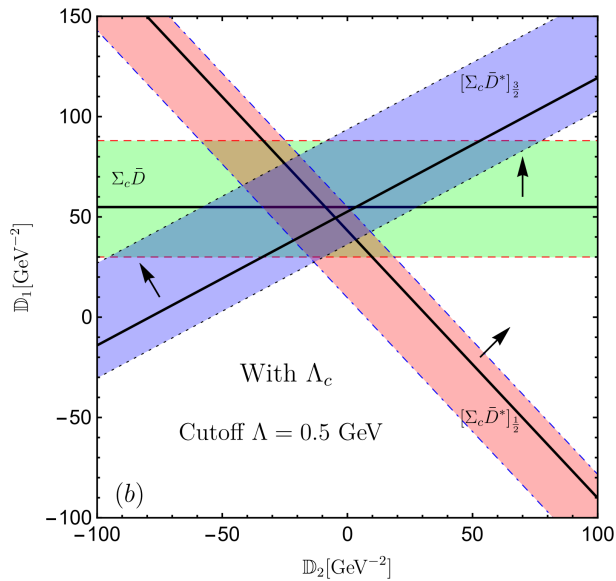
Systems	$I(J^P)$	Thresholds (MeV)	Masses (MeV)	Widths (MeV)	$\Delta m$ (MeV)	States
$\frac{1}{\sqrt{2}} [\bar{D}_s^* D + \bar{D}_s D^*]$	$\frac{1}{2} (1^+)$	3977.0	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$	$5.5^{+1.8}_{-2.6} \pm 2.1$	$Z_{cs}(3985)^\dagger$
$\bar{D}_s^* D^*$	$\frac{1}{2} (1^+)$	4119.1	$4124.2^{+5.6}_{-3.7}$	$9.8^{+5.2}_{-4.8}$	$5.1^{+5.6}_{-3.7}$	$Z_{cs}(4125)$
$\frac{1}{\sqrt{2}} [B_s^* \bar{B} + B_s \bar{B}^*]$	$\frac{1}{2} (1^+)$	10694.7	$10701.9^{+3.9}_{-2.7}$	$7.4^{+3.6}_{-4.4}$	$7.2^{+3.9}_{-2.7}$	$Z_{bs}(10700)$
$B_s^* \bar{B}^*$	$\frac{1}{2} (1^+)$	10740.1	$10747.0^{+4.3}_{-3.1}$	$7.3^{+3.7}_{-4.6}$	$6.9^{+4.3}_{-3.1}$	$Z_{bs}(10745)$

- ✓ New measurement from BESIII ( $e^+e^- \rightarrow K^+ D_s^{*-} D^{*0} + c.c.$ ):  $Z'_{cs}$ ,  $m \sim 4123.5$  MeV, with a significance of  $2.1 \sigma$ . Chin.Phys.C 47,033001 (2023).
- ✓ Implications of  $Z_{cs}(4000)$  and  $Z_{cs}(3985)$  as two different states are given in Ref. [L. Meng *et al*, Sci.Bull. 66 (2021) 2065-2071].

# Molecular pentaquarks



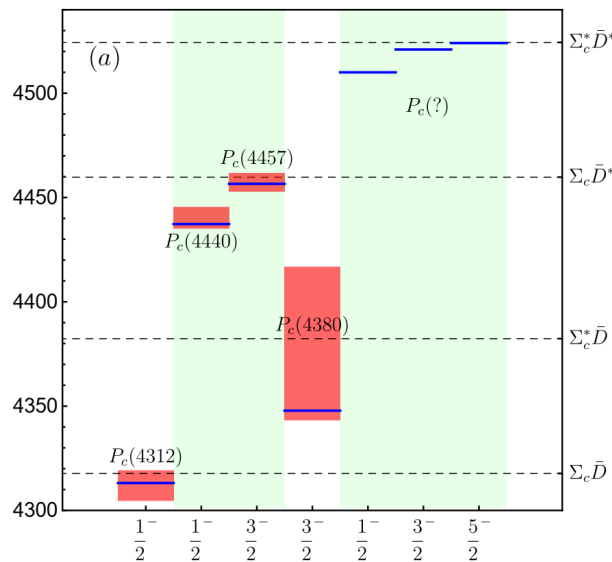
# $P_c$ s and their strange partners



B. Wang *et al*, JHEP 1911 (2019) 108

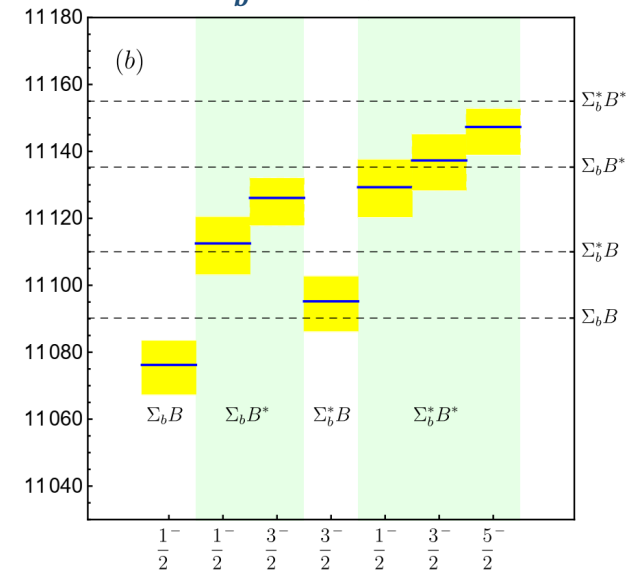
2023/4/16

## $\Sigma_c^{(*)} \bar{D}^{(*)}$ systems



Hidden-charm spectra

## $\Sigma_b^{(*)} B^{(*)}$ systems



Hidden-bottom spectra

### How to observe?

- Production mechanism is unknown.
  - Reconstruction channels:  $Y(nS)N$  ( $n = 1, 2$ ),  $\Lambda_b B^{(*)}$
- "There is plenty of room at the bottom" - R. P.

Feynman

20

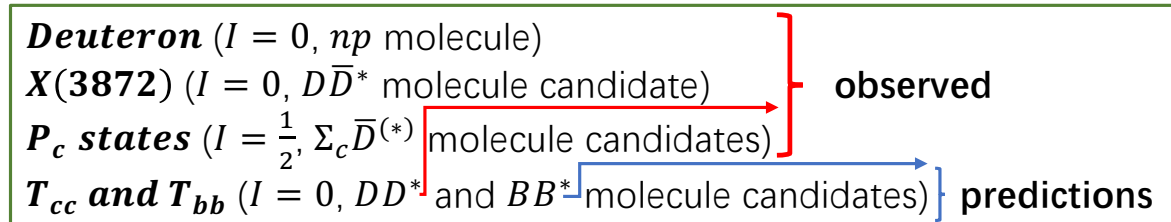
# $P_c$ s and their strange partners

- $\Xi_c^{(\prime,*)} \bar{D}^{(*)}$  systems

$XYZ: Q\bar{Q}q\bar{q}$ ;

$P_c: Q\bar{Q}qqq$ .

1. The **heavy quark core** plays an important role in stabilizing the exotic clusters [Phys. Rev. D **84**, 014031, Phys. Rev. D **86**, 014020, Eur. Phys. J. C **74**, 3198].
2. Hydrogen molecule: two protons plus two electrons, stably exists in the nature.
3. Existence of  $P_c \rightarrow$  more hadronic molecules in **SU(3) symmetry**?
4. Two heavy matter fields tend to form the bound states in the **lowest isospin channels**?



5. Whether the  $\Xi_c^{(\prime,*)} \bar{D}^{(*)}$  systems can form bound states in the  $I = 0$  channels?
6. May be observed in  $J/\psi\Lambda$  final states of the decays  $\Lambda_b(\Xi_b) \rightarrow J/\psi\Lambda K(\eta)$ ?

# $P_c$ s and their strange partners

## • $P_{cs}$ spectra

B. Wang *et al*, Phys. RevD. **101**.034018



System	$[\Xi'_c \bar{D}]_{\frac{1}{2}}$	$[\Xi'_c \bar{D}^*]_{\frac{1}{2}}$	$[\Xi'_c \bar{D}^*]_{\frac{3}{2}}$	$[\Xi_c^* \bar{D}]_{\frac{3}{2}}$	$[\Xi_c^* \bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c^* \bar{D}^*]_{\frac{3}{2}}$	$[\Xi_c^* \bar{D}^*]_{\frac{5}{2}}^{\#}$	$[\Xi_c \bar{D}]_{\frac{1}{2}}$	$[\Xi_c \bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c \bar{D}^*]_{\frac{3}{2}}$
$\Delta E$	$-18.5^{+6.4}_{-6.8}$	$-15.6^{+6.4}_{-7.2}$	$-2.0^{+1.8}_{-3.3}$	$-7.5^{+4.2}_{-5.3}$	$-17.0^{+6.7}_{-7.5}$	$-8.0^{+4.5}_{-5.6}$	$-0.7^{+0.7}_{-2.2}$	$-13.3^{+2.8}_{-3.0}$	$-17.8^{+3.2}_{-3.3}$	$-11.8^{+2.8}_{-3.0}$
$M$	$4423.7^{+6.4}_{-6.8}$	$4568.7^{+6.4}_{-7.2}$	$4582.3^{+1.8}_{-3.3}$	$4502.9^{+4.2}_{-5.3}$	$4635.4^{+6.7}_{-7.5}$	$4644.4^{+4.5}_{-5.6}$	$4651.7^{+0.7}_{-2.2}$	$4319.4^{+2.8}_{-3.0}$	$4456.9^{+3.2}_{-3.3}$	$4463.0^{+2.8}_{-3.0}$

1. Predicted ten  $P_{cs}$  states in the **isoscalar** channels.

Taken from Science Bulletin **66** (2021) 1278–1287

2. Three new ones in  $\Xi_c \bar{D}^{(*)}$  systems.

State	$M_0$ (MeV)	$\Gamma_0$ (MeV)
$P_{cs}(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$

3. The new  $[\Xi_c \bar{D}^*]_{1/2}$  state is **VERY consistent** with the newly LHCb result.

## • What about the $\Lambda_c \bar{D}^{(*)}$ and other systems?

1. The  $\Lambda_c \bar{D}^{(*)}$  systems: No isospin-isospin interaction, contact (repulsive)+TPE (couple-channel, attractive) $\simeq 0 \rightarrow$  **no bound states (estimation)**.

2. Other systems:  $\Lambda_c \bar{D}_s^{(*)}$ ,  $\Sigma_c \bar{D}_s^{(*)}$ ,  $\Sigma_c^* \bar{D}_s^{(*)}$  ( $s = -1$ ): **attractive**, but **too weak** to form bound states.  $\Omega_c^{(*)} \bar{D}_s^{(*)}$  ( $s = -3$ ): **repulsive**. *It is hard to form bound states in these systems!*

# Doubly charmed $P_{cc}$ states

From  $\Sigma_c^{(*)}\bar{D}^{(*)}$  to  $\Sigma_c^{(*)}D^*$  systems

The low energy constants of the  $\Sigma_c^{(*)}D^*$  systems are estimated from the  $N\bar{N}$  scattering data by introducing a quark level Lagrangian:

$$\mathcal{L} = g_s \bar{q} \mathcal{S} q + g_a \bar{q} \gamma_\mu \gamma^5 \mathcal{A}^\mu q,$$

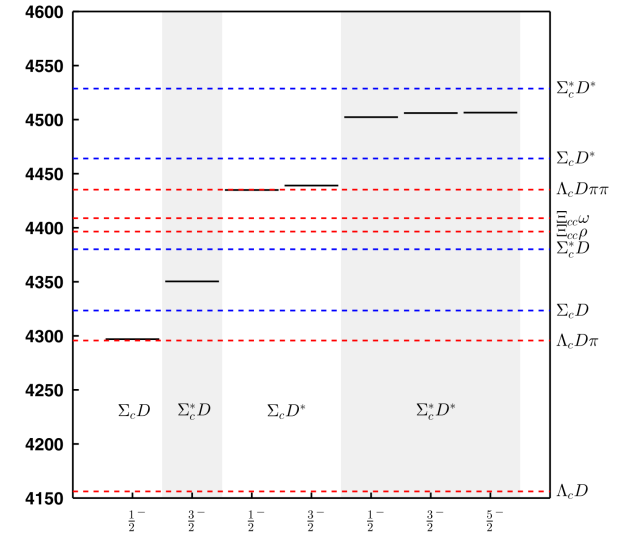
$$V_{q\bar{q}} = c_s(1 - 3\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + c_t(1 - 3\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2.$$

$$V_{\Sigma_c D^*} = 2c_s - 12c_s \mathbf{I}_1 \cdot \mathbf{I}_2 - \frac{4}{3}c_t \boldsymbol{\sigma} \cdot \mathbf{T} + 8c_t(\mathbf{I}_1 \cdot \mathbf{I}_2)(\boldsymbol{\sigma} \cdot \mathbf{T}).$$

		$[\Sigma_c D]_{\frac{1}{2}^-}$	$[\Sigma_c^* D]_{\frac{3}{2}^-}$	$[\Sigma_c D^*]_{\frac{1}{2}^-}$	$[\Sigma_c D^*]_{\frac{3}{2}^-}$	$[\Sigma_c^* D^*]_{\frac{1}{2}^-}$	$[\Sigma_c^* D^*]_{\frac{3}{2}^-}$	$[\Sigma_c^* D^*]_{\frac{5}{2}^-}$
Case 1	BE (MeV)	-15.4	-25.0	-31.8	-8.0	-32.8	-18.2	-3.5
	$R_{rms}$ (fm)	1.45	1.25	1.20	1.65	1.20	1.38	1.91
Case 2	BE (MeV)	-31.3	-42.9	-30.3	-31.7	-26.6	-25.4	-29.7
	$R_{rms}$ (fm)	1.23	1.11	1.22	1.20	1.26	1.27	1.22
Case 3	BE (MeV)	-26.5	-37.7	-29.1	-25.0	-26.4	-22.6	-22.2
	$R_{rms}$ (fm)	1.27	1.14	1.23	1.27	1.26	1.31	1.30

All the  $\Sigma_c^{(*)}D^*$  systems with isospin  $I = 1/2$  can form bound states. In addition, we also investigate the interactions of the charmed-bottom  $\Sigma_c^{(*)}\bar{B}^{(*)}$ ,  $\Sigma_b^{(*)}D^{(*)}$  and  $\Sigma_b^{(*)}\bar{B}^{(*)}$  systems. Among the obtained bound states, the bindings become deeper when the reduced masses of the corresponding systems are heavier.

K. Chen *et al*, PhysRevD.103.116017



# More systems

Within the same framework, we also covered more systems

K. Chen *et al*, [Eur.Phys.J.C 82 \(2022\) 7, 581](#)

Meson-meson	$[\bar{D}\bar{D}]_0^1$ $[\bar{D}^*\bar{D}^*]_1^0$	$[\bar{D}\bar{D}^*]_1^{0,1}$	$[\bar{D}^*\bar{D}^*]_{0,2}^1$
Baryon-meson	$[\Lambda_c\bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}$ $[\Sigma_c\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{\frac{1}{2},\frac{3}{2}}$ $[\Xi_c\bar{D}]_{\frac{1}{2}}^{0,1}$ $[\Xi_c'\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{0,1}$	$[\Lambda_c\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{\frac{1}{2},\frac{3}{2}}$ $[\Sigma_c^*\bar{D}]_{\frac{3}{2}}^{\frac{1}{2},\frac{3}{2}}$ $[\Xi_c\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{0,1}$ $[\Xi_c^*\bar{D}]_{\frac{3}{2}}^{0,1}$	$[\Sigma_c\bar{D}]_{\frac{1}{2}}^{\frac{1}{2},\frac{3}{2}}$ $[\Sigma_c^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{\frac{1}{2},\frac{3}{2}}$ $[\Xi_c'\bar{D}]_{\frac{1}{2}}^{0,1}$ $[\Xi_c^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{0,1}$
Baryon-baryon	$[\Lambda_c\Lambda_c]_0^0$ $[\Sigma_c\Sigma_c]_1^1$ $[\Sigma_c^*\Sigma_c^*]_{1,3}^1$ $[\Xi_c\Xi_c]_1^0$ $[\Xi_c'\Xi_c']_0^0$ $[\Xi_c^*\Xi_c^*]_{1,3}^0$	$[\Lambda_c\Sigma_c]_{0,1}^1$ $[\Lambda_c\Sigma_c^*]_{1,2}^1$ $[\Sigma_c^*\Sigma_c^*]_{0,2}^{0,2}$ $[\Xi_c\Xi_c']_{0,1}^{0,1}$ $[\Xi_c'\Xi_c']_1^0$ $[\Xi_c^*\Xi_c^*]_{0,2}^1$	$[\Sigma_c\Sigma_c]_0^{0,2}$ $[\Sigma_c\Sigma_c^*]_{1,2}^{0,1,2}$ $[\Xi_c\Xi_c]_0^1$ $[\Xi_c\Xi_c^*]_{1,2}^{0,1}$ $[\Xi_c'\Xi_c^*]_{1,2}^{0,1}$

	Mass (Expt.)	BE (Expt.)	Mass (Our)	BE (Our)
$T_{cc}(3875)^+$	3874.8	-1.0	$3874.5_{-1.1}^{+1.7}$	$-1.8_{-1.1}^{+1.7}$
$P_c(4312)^+$	$4311.9 \pm 0.7_{-0.6}^{+6.8}$	$-8.9_{-0.9}^{+6.8}$ (input)	$4311.9_{-2.8}^{+6.8}$	$-8.9_{-2.8}^{+6.8}$
$P_c(4380)^+$	$4380 \pm 8 \pm 29$	$-6.2 \pm 30.1$	$4376.2_{-2.8}^{+6.9}$	$-9.1_{-2.8}^{+6.9}$
$P_c(4440)^+$	$4440.3 \pm 1.3_{-4.7}^{+4.1}$	$-21.8_{-4.9}^{+4.3}$ (input)	$4440.2_{-5.3}^{+13.8}$	$-21.8_{-5.3}^{+13.8}$
$P_c(4457)^+$	$4457.3 \pm 0.6_{-1.7}^{+4.1}$	$-4.8_{-1.8}^{+4.1}$ (input)	$4457.3_{-1.9}^{+4.1}$	$-4.8_{-1.9}^{+4.1}$
$P_{cs}(4459)^0$	$4458.8 \pm 2.9_{-1.1}^{+4.7}$	$-19.7_{-3.1}^{+5.5}$	$4468.1_{-3.0}^{+7.3}$	$-10.0_{-3.0}^{+7.3}$

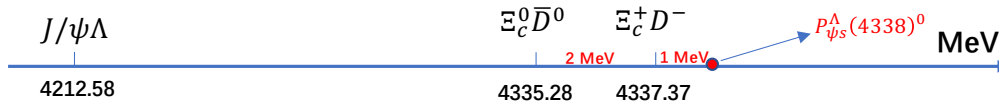
K. Chen *et al*, [Phys.Rev.D 105 \(2022\) 9, 096004](#)

$M-M$	$DD$ $D_s D_s$	$DD^*$ $D_s D_s^*$	$D^*D^*$ $D_s^* D_s^*$	$DD_s$	$DD_s^*$ ( $D_s D^*$ )	$D^*D_s^*$
$B-M$	$\Lambda_c\bar{D}$ $\Lambda_c\bar{D}_s$ $\Xi_c\bar{D}$ $\Xi_c\bar{D}_s$ $\Omega_c\bar{D}$	$\Lambda_c\bar{D}^*$ $\Lambda_c\bar{D}_s^*$ $\Xi_c\bar{D}^*$ $\Xi_c\bar{D}_s^*$ $\Omega_c\bar{D}^*$	$\Sigma_c\bar{D}$ $\Sigma_c\bar{D}_s$ $\Xi_c'\bar{D}$ $\Xi_c'\bar{D}_s$ $\Omega_c\bar{D}$	$\Sigma_c\bar{D}^*$ $\Sigma_c\bar{D}_s^*$ $\Xi_c'\bar{D}^*$ $\Xi_c'\bar{D}_s^*$ $\Omega_c\bar{D}^*$	$\Sigma_c^*\bar{D}$ $\Sigma_c^*\bar{D}_s$ $\Xi_c^*\bar{D}$ $\Xi_c^*\bar{D}_s$	$\Sigma_c^*\bar{D}^*$ $\Sigma_c^*\bar{D}_s^*$ $\Xi_c^*\bar{D}^*$ $\Xi_c^*\bar{D}_s^*$
$B-B$	$\Lambda_c\Lambda_c$ $\Xi_c\Xi_c$ $\Lambda_c\Xi_c$ $\Sigma_c\Xi_c^*$ $\Xi_c\Omega_c$	$\Lambda_c\Sigma_c$ $\Xi_c\Xi_c'$ $\Lambda_c\Xi_c^*$ $\Sigma_c\Omega_c$ $\Xi_c'\Omega_c$	$\Lambda_c\Sigma_c^*$ $\Xi_c\Xi_c^*$ $\Lambda_c\Xi_c^*$ $\Sigma_c\Xi_c$ $\Xi_c^*\Omega_c$	$\Sigma_c\Sigma_c$ $\Xi_c\Xi_c'$ $\Lambda_c\Omega_c$ $\Sigma_c\Xi_c'$ $\Xi_c\Omega_c$	$\Sigma_c\Sigma_c^*$ $\Xi_c\Xi_c^*$ $\Sigma_c\Xi_c$ $\Sigma_c\Xi_c^*$ $\Omega_c\Omega_c$	$\Sigma_c^*\Sigma_c^*$ $\Xi_c^*\Xi_c^*$ $\Lambda_c\Omega_c$ $\Sigma_c^*\Omega_c$ $\Omega_c\Omega_c$



# Lineshapes of the $P_{\psi_s}^{\Lambda}(4338)^0$

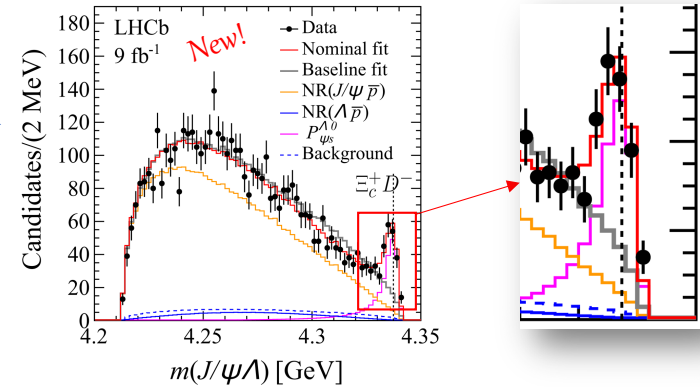
$m = 4338.3 \pm 0.7 \pm 0.4 \text{ MeV}, \Gamma = 7.0 \pm 1.2 \pm 1.3 \text{ MeV}.$



If a pole is close to the physical sheet and far away from the threshold,

Breit-Weigner (BW):  $T \propto 1/(E - M + i\Gamma/2)$

The lineshape of the resonance would be **distorted from the conventional BW distribution** if it appears near the threshold and strongly couples to the threshold at the same time [Phys. Lett. B 63,224(1976), Phys. Rev. D 76, 034007(2007), Phys. Rev. Lett. 115, 202001(2015)]. See also J.-J. Wu's talk.



therapies → K-matrix parameterization or LS equation

Two models for the  $J/\psi\Lambda-\Xi_c^0\bar{D}^0-\Xi_c^+D^-$  interactions: L. Meng *et al*, PhysRevD.107.014005

Model-I  $V_I = \frac{1}{2} \begin{bmatrix} 0 & -\tilde{c} & \tilde{c} \\ -\tilde{c} & c_1 + c_0 & c_1 - c_0 \\ \tilde{c} & c_1 - c_0 & c_1 + c_0 \end{bmatrix},$

Model-II  $V_{II} = \frac{1}{2} \begin{bmatrix} 0 & -\tilde{c} & \tilde{c} \\ -\tilde{c} & \frac{g^2}{E^2 - m_0^2} & -\frac{g^2}{E^2 - m_0^2} \\ \tilde{c} & -\frac{g^2}{E^2 - m_0^2} & \frac{g^2}{E^2 - m_0^2} \end{bmatrix},$

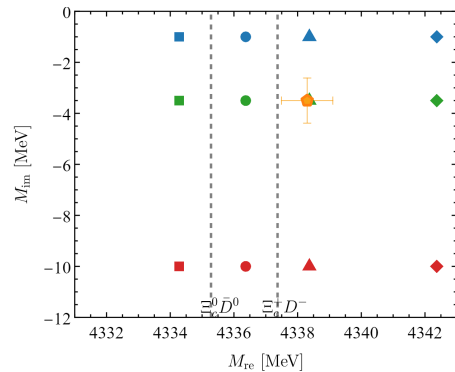
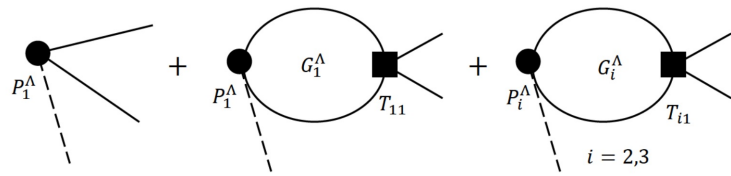
# Lineshapes of the $P_{\psi_s}^\Lambda(4338)^0$

$$T = V + VGT, G = \text{diag}\{G_1, G_2, G_3\}$$

$$G_i(E) = \int_0^\Lambda \frac{l^2 d^2 l}{(2\pi)^2} \frac{\omega_{i1} + \omega_{i2}}{\omega_{i1} \omega_{i2} [E^2 - (\omega_{i1} + \omega_{i2})^2 + i\epsilon]}, \quad \omega_{ia} = (\mathbf{l}^2 + m_{ia}^2)^{1/2}.$$

$$\text{Analytical continuation: } G_i \rightarrow G_i + i \frac{k_i}{4\pi E}$$

Production diagrams: L. Meng *et al*, PhysRevD.107.014005



“Synthetic” poles:

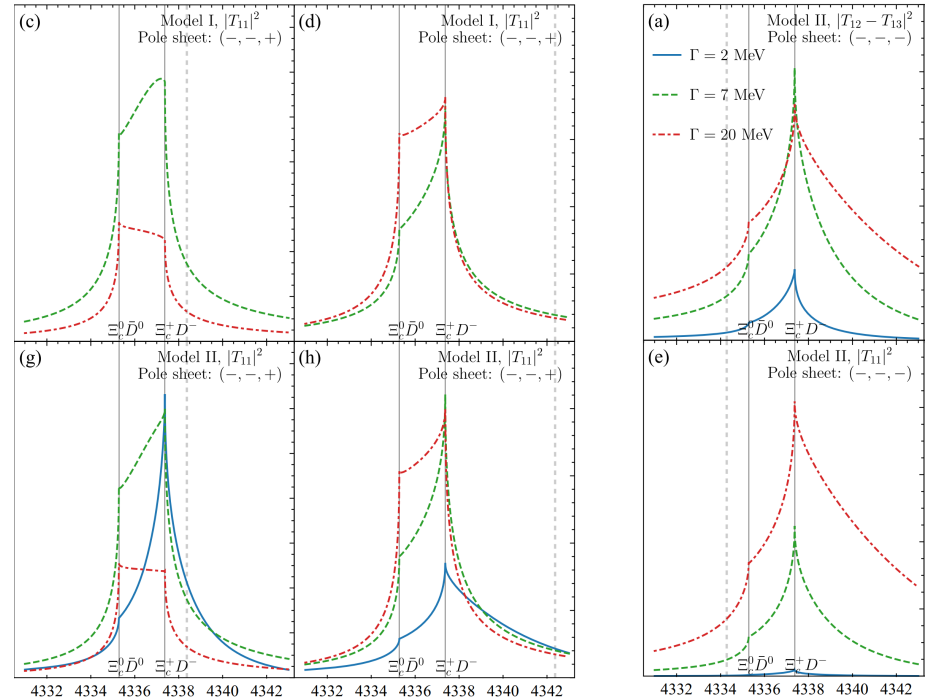
$$M_{ij} = M_i - i\Gamma_j/2$$

- $M_i = \{m_{\Xi_c^0 D^0} - 1, m_{\Xi_c^+ D^-} - 1, m_{\Xi_c^+ D^-} + 1, m_{\Xi_c^+ D^-} + 5\}$  MeV
- $\Gamma_i = \{2, 7, 20\}$  MeV

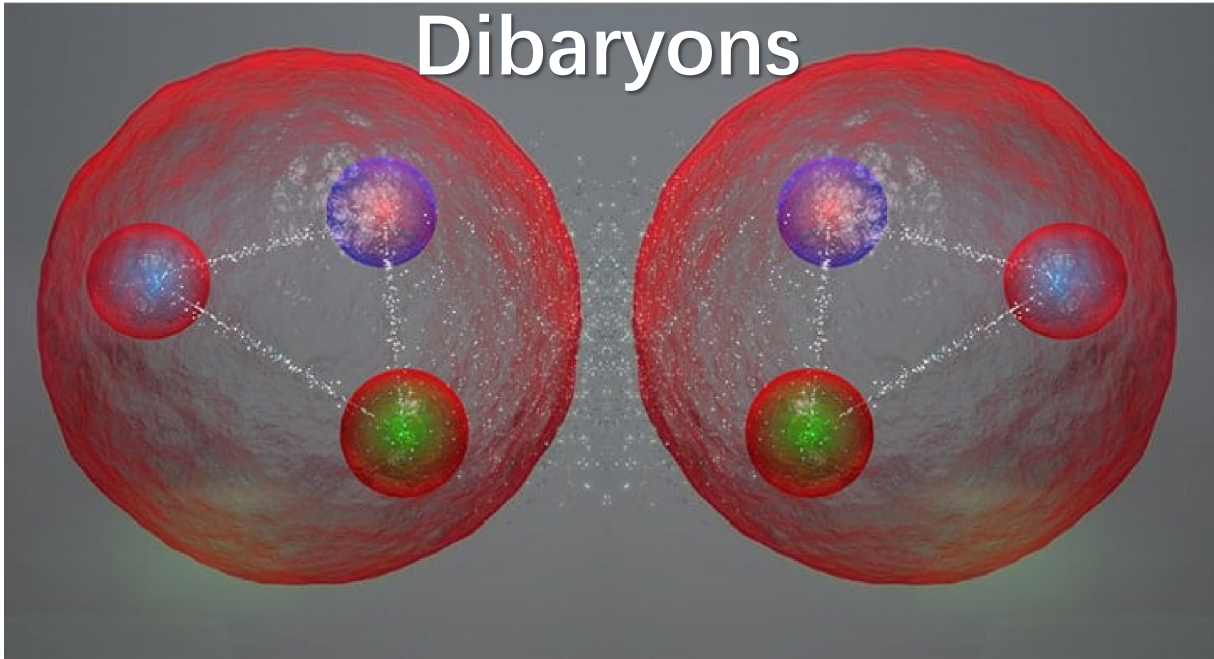
2023/4/16

Poles well above the  $\Xi_c^+ D^-$  threshold

Poles well below the  $\Xi_c^+ D^-$  threshold



# Dibaryons

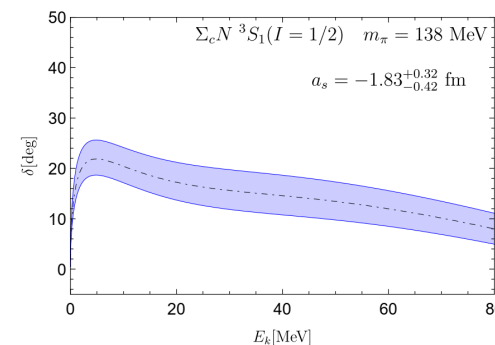
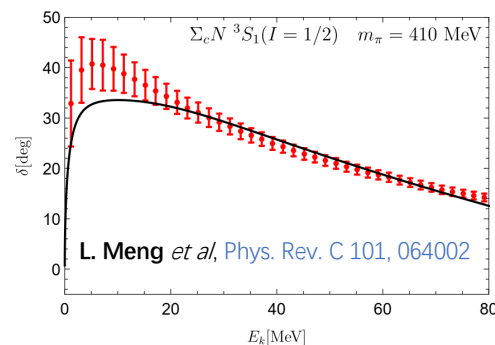
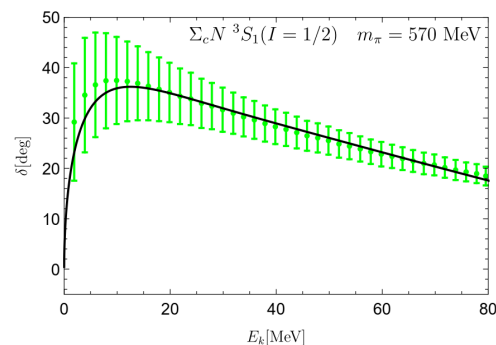


# Dibaryons (molecular hexaquark)

A dibaryon is essentially a system with two baryons. There is one known dibaryon in nature- deuteron, another possible one is the  $\Delta\Delta$  dibaryon- $d^*(2380)$  (disputed).

## NB<sub>Q</sub> and NB<sub>QQ</sub> systems

- ✓ The  $NY_c$  ( $Y_c = \Sigma_c, \Lambda_c$ ) interactions are essential for understanding the in-medium properties of the charmed baryons. The experimental proposals at the J-PARC [arXiv:1706.07916] and GIS-FAIR [Prog. Part. Nucl. Phys. 66 (2011) 477–518] have stimulated many investigations on the  $NY_c$  interactions.
- ✓ In Refs. [Nucl. Phys. A 971 (2018) 113–129, PoS Hadron2017 (2018) 146], the HAL QCD Collaboration calculated the phase shifts of the  $N\Lambda_c$  and  $N\Sigma_c$  scatterings from lattice QCD at the unphysical pion mass  $m_\pi = 410 - 570$  MeV.



- ✓ In Ref. [L. Meng *et al*, Eur. Phys. J. A 54 (9) (2018) 143], the authors predicted the bound states in the  $N\Xi_{cc}$  and  $\bar{N}\Xi_{cc}$  systems from the OBE model.

# Dibaryons (molecular hexaquark)

## $B_Q B_Q$ and $B_Q \bar{B}_Q$ systems

- ✓ In Ref. [N. Lee *et al*, [PhysRevD.84.014031](#)], the authors calculated the  $\Lambda_c \Lambda_c (\bar{\Lambda}_c)$ ,  $\Xi_c \Xi_c (\bar{\Xi}_c)$ ,  $\Sigma_c \Sigma_c (\bar{\Sigma}_c)$ ,  $\Xi'_c \Xi'_c (\bar{\Xi}'_c)$ ,  $\Omega_c \Omega_c (\bar{\Omega}_c)$  systems within the OBE model, they obtained: the H-dibaryonlike state  $\Lambda_c \Lambda_c$  does not exist; there may exist loosely bound deuteronlike states for the other systems
- ✓ In Ref. [J.-B. Cheng *et al*, [PhysRevD.107.054018](#)], the authors investigated the double-charm and hidden-charm hexaquarks as molecules in complex scaling method with explicit three-body effect.
- ✓ In Ref. [J.-X. Lu *et al*, [PhysRevD.99.074026](#)], the authors found that the isoscalar  $\Lambda_c \bar{\Lambda}_c$ ,  $\Sigma_c^{(*)} \bar{\Sigma}_c^{(*)}$  and isovector  $\Lambda_c \bar{\Sigma}_c^{(*)}$  as well as their doubly charmed and doubly bottom counterparts are good candidates of the molecular hexaquarks.
- ✓ In Ref. [X. Z. Ling *et al*, [Eur. Phys. J. C \(2021\) 81:1090](#)], the masses and strong decays of the  $\Sigma_c^{(*)} \Sigma_c^{(*)}$  dibaryons were calculated.
- ✓ Calculations from other approaches, see [H. Huang *et al*, [PhysRevC.89.035201](#); T. F. Carames *et al*, [PhysRevD.92.034015](#); H. Garcilazo *et al*, [Eur. Phys. J. C 80 \(8\) \(2020\) 720](#); Z. Liu *et al*, [Phys.Rev.D 105 \(2022\) 3, 034006](#); X.-K. Dong *et al*, [Commun. Theor. Phys. 73 \(12\) \(2021\) 125201](#); X.-K. Dong *et al*, [Progr. Phys. 41 \(2021\) 65–93](#)].

## $B_{QQ} B_Q$ and $B_{QQ} \bar{B}_{QQ}$ systems

The  $\Xi_{cc}^{(*)}$  [ $\bar{\Xi}_{cc}^{(*)}$ ] can be related to the  $\bar{D}^{(*)}$  [ $D^{(*)}$ ] with the heavy diquark-antiquark symmetry (HDAS),

$$\Xi_{cc}^{(*)} \xrightarrow{\text{HDAS}} \bar{D}^{(*)} \quad \bar{\Xi}_{cc}^{(*)} \xrightarrow{\text{HDAS}} D^{(*)}$$

# Dibaryons (molecular hexaquark)

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## $B_{QQ}B_Q$ and $B_{QQ}\bar{B}_{QQ}$ systems

As a consequence of the HDAS, the  $\Xi_{cc}^{(*)}D^{(*)}$ ,  $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$  and  $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^{(*)}$  systems can be related to the  $\bar{D}^{(*)}D^{(*)}$ ,  $\bar{D}^{(*)}\Sigma_c^{(*)}$  and  $\bar{D}^{(*)}D^{(*)}$  systems, respectively.

Thus, the existence of the molecular states in the  $\bar{D}^{(*)}D^{(*)}$  and  $\bar{D}^{(*)}\Sigma_c^{(*)}$  systems should also imply the existence of the molecular states in the  $\Xi_{cc}^{(*)}D^{(*)}$ ,  $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$  and  $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^{(*)}$  systems.

- ✓ In Ref. [B. Yang *et al*, *Eur. Phys. J. A56* (2) (2020) 67], Yang et al investigated the possible bound states in the  $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^{(*)}$  ( $\bar{\Xi}_{cc}^{(*)}$ ) systems, and predicted the molecular candidates in the isoscalar and isovector channels.
- ✓ In Ref. [F.-K. Guo *et al*, *PhysRevD.88.054014*], the authors predicted the triply heavy pentaquarks with  $I(J^P) = 0(3/2^-)$ ,  $0(5/2^-)$  with the  $X(3872)$  as input, as well as the  $1(1/2^-)$  and  $1(3/2^-)$  ones with the  $Z_b(10650)$  as input. see also R. Chen *et al*, *PhysRevD.96.114030*.
- ✓ In Ref. [Y.-W. Pan *et al*, *PhysRevD.102.011504*], the authors proposed an alternative way to determine the spins of the  $P_c(4440)$  and  $P_c(4457)$  from the spectrum of the  $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$  systems with the help of lattice QCD.

## $B_{QQQ}B_{QQQ}$ systems

- ✓ **Lattice:** *Phys. Rev. Lett.*127.072003; *Phys. Rev. Lett.*130.111901
- ✓ **Models:** *Chin. Phys. Lett.* 38, 101201; *Eur. Phys. J. C* 82, 805; *Int. J. Mod. Phys. A* 37, 2250166; arXiv: 2207.05505; arXiv: 2208.03041

# Summary and outlook

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1. Many near-threshold states have been observed in experiments.
2. Their spectra and decays were intensively studied within various models.
3. **Most of the nowadays observed exotic states have the same origin?** —The dynamically generated resonances (bound states) from the analogue of nuclear forces in different sectors.
4. What forces govern the formations of these states—the “**general nuclear forces**”?
5. Weak(er) model-dependent approaches need to be developed.

Thank you!