Lattice calculation of the intrinsic soft function and the Collins-Soper kernel

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Introduction

Transverse-momentum-dependent parton distribution functions (TMDPDFs), which give the probability density for 3D parton momenta in hadrons, have seen significant focus in recent hadron physics studies. These functions are universal, implying their consistency across Drell-Yan (DY) and semi-inclusive deep-inelastic scattering (SIDIS) processes. Past decades have seen numerous experiments in these areas, enriching our understanding of TMDPDFs. The study of TMDPDFs spans perturbative, phenomenological, and non-perturbative determinations. Phenomenological analyses often delve into the transverse momenta of the final state particles. This requires careful data selection. The intrinsic soft function, which helps remove regulator-scheme-dependence from the quasi TMDPDF/TMDWF, can be derived from either heavy quark effective theory or large-momentum-transfer form factors. Our work contrasts the intrinsic soft function derived from different lattice QCD ensembles.

LaMET (Large Momentum Effective Theory) offers insights into parton light cone correlations. It suggests that these correlations in the hadron's rest frame can be gleaned from spatial correlations in the infinite-momentum frame. LaMET paves a systematic path to discern TMDs using lattice simulations, emphasizing the role of the universal soft function.

The CS kernel, another crucial element, can be sourced from global fits of TMDPDFs data or lattice calculable ratios. Our work extracts the CS kernel for a specific ensemble within LaMET, aiming to incorporate one-loop contributions. We also explore the advantages of sourcing the CS kernel from different data types. Equipped with the CS kernel and the intrinsic soft function, lattice determinations of TMDWFs/TMDPDFs become achievable. We delve into their applications in TMD physics and evaluate potential uncertainties.

Theoretical framework

The LaMET factorization formula that relates the quasi TMDPDF $\tilde{f}$ to the light cone TMDPDF f reads

$$\tilde{f}\left(x, b_\perp, \zeta_z, \mu\right) \sqrt{S_I\left(b_\perp, \mu\right)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) \times e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)K(b_\perp,\mu)} f\left(x, b_\perp, \mu, \zeta\right) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta_z}\right), \quad (1)$$

where x denotes the longitudinal momentum fraction, $b_\perp$ is the transverse separation, M is the hadron mass and ζ is a reference rapidity scale. Therefore, by taking a ratio of $\tilde{\Psi}^\pm$ at different momentum the CS kernel $K\left(b_\perp, \mu\right)$ can be extracted through

$$K\left(b_\perp, \mu\right) = \frac{1}{\ln(P_1^z/P_2^z)} \times \ln\frac{H^\pm(xP_2^z,\mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_1^z)}{H^\pm(xP_1^z,\mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_2^z)}. \quad (2)$$

To determine the intrinsic soft function using the quasi TMDWFs $\tilde{\Psi}^\pm$, there needs a four quark form factor. Then the intrinsic soft function then reads

$$F\left(b_\perp, P_z, \Gamma, \mu\right) = \int dx_1 dx_2 H\left(x_1, x_2, \Gamma\right) S_I\left(b_\perp, \mu\right)\tilde{\Psi}^{\pm *}\left(x_2, b_\perp, P^z\right)\tilde{\Psi}^{\pm}\left(x_1, b_\perp, P^z\right). \quad (3)$$

This work is mainly based on the calculation of intrinsic soft function on MILC and CLS ensembles and reproduce CS kernel on CLS ensembles. Besides, with the intrinsic soft function CS kernel, quasi TMDWFs and quasi TMDPDFs at hand, we can capture the correct IR physics to all-orders and by a perturbative matching the physical TMDs can be obtained. We have illustrated this matching procedure, and obtain physical TMDWF and TMDPDF results.

Lattice results

1. Renormalization

Caused by the staple shaped Wilson loop $U_{c\pm}$, the bare matrix element in the numerator in the following equation contains both pinch pole singularity and linear divergence which can be removed by the Wilson loop $Z_E\left(2L + |z|, b_\perp, \mu\right)$.

$$\tilde{\Psi}^\pm\left(x, b_\perp, \mu, \zeta^z\right) = \lim_{L\to\infty}\frac{1}{-if_\pi P^z}\int\frac{dzP^z}{2\pi}e^{ixzP^z}\frac{\langle 0\left|\bar{q}\left(z\hat{n}_z + b_\perp\hat{n}_\perp\right)\gamma^t\gamma_5 U_{c\pm}q(0)\right|\pi\left(P^z\right)\rangle}{\sqrt{Z_E\left(2L+|z|, b_\perp, \mu\right)}Z_O(1/a,\mu)}. \quad (4)$$

The logarithmic divergences arising from the endpoints of the Wilson line need an additional quark Wilson line vertex renormalization factor $Z_O(1/a, \mu)$. A straightforward way to determine Z_O is to evaluate the quotient of the renormalized quasi-TMDWF calculated on the lattice in the small $b_\perp$ region and the quasi-TMDWF perturbatively calculated in $\overline{\mathrm{MS}}$ scheme.

2. Intrinsic soft function results

In the calculation of intrinsic soft function, we use proper normalization and investigate the Dirac structure in four quark form factor, and additionally we considered perturbative kernel at one-loop accuracy to obtain the results in MILC and CLS ensembles, and shown in Fig.1.

The intrinsic soft functions for CLS exhibit a more pronounced P^z-dependence compared to MILC. Transitioning from tree-level to 1-loop matching leads to a considerable increase in these functions for both sets, nearing the 1-loop perturbative figures, particularly at minor $b_\perp$. Our definitive intrinsic soft function estimates, derived from 1-loop matching with the $\gamma^\perp + \gamma^\perp\gamma_5$ combination, are presented in the above figure. Overall, the two ensemble results align well, with notable differences at smaller $b_\perp$ due to lattice discretization effects.

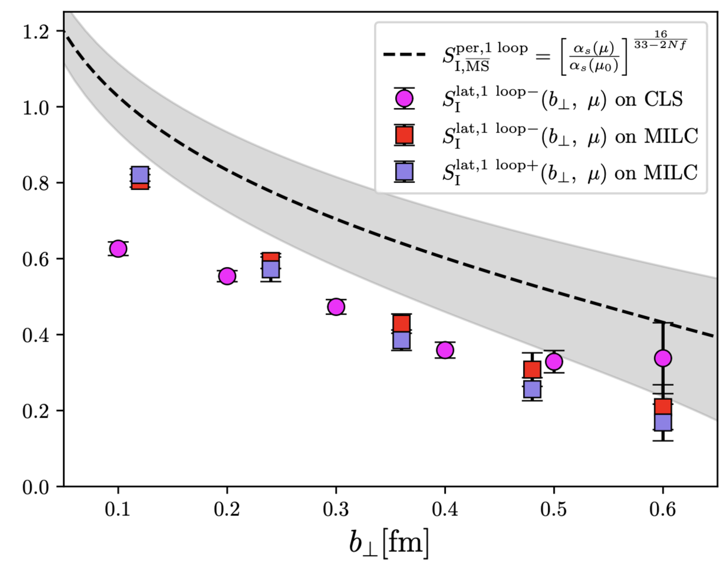

Figure 1: Results of intrinsic soft function.

3. CS kernel results:

In Fig.2, we juxtapose the CS kernel from this study with other analyses, including 3-loop perturbative outcomes, phenomenological extractions and various lattice computations.

In principle the CS kernel can also be obtained from the quasi TMDPDF via a similar equation compared with quasi TMDWF, by replacing the quasi TMDWF objects with the quasi TMDPDF objects, and replacing the hard kernel function for quasi TMDWF with that for quasi TMDPDF. Our tests show that using quasi TMDWF to extract the CS kernel will give better defined results, even though it suffers from systematic uncertainties induced by its imaginary part. To conclude, we will use the results obtained from quasi TMDWF as our final estimate for the CS kernel.

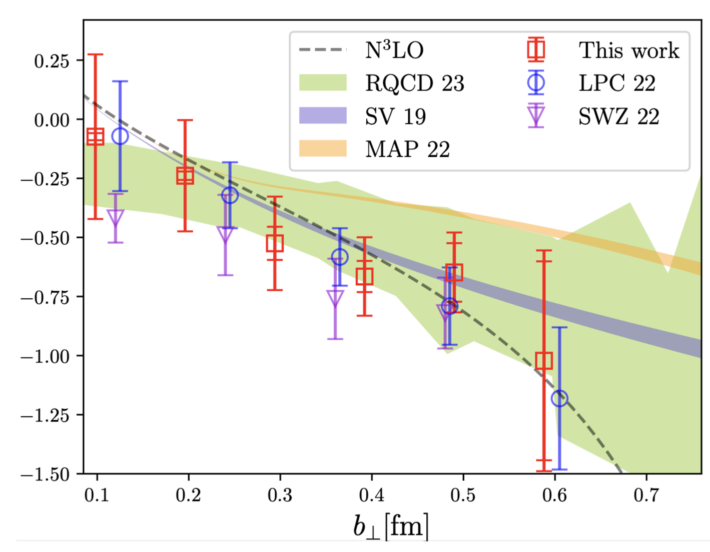

Figure 2: Results of CS kernel.

Conclusions and Outlook

We computed the intrinsic soft function and CS kernel on two lattice ensembles, incorporating one-loop contributions and normalizing light meson form factors. The intrinsic soft functions from the two ensembles are largely consistent, with discrepancies at low $b_\perp$ due to lattice effects. Our extraction of the CS kernel from TMDWFs, including the CLS ensemble, shows that this method is more favorable than using quasi TMDPDFs. Our results align well with prior studies, especially. Utilizing the soft function, we derived physical TMDWFs for pion and TMDPDFs for proton, confirming the feasibility of computing light cone values from lattice simulated quasi objects via TMD factorization. Our observations underscore the need for precise determinations of the soft function and thorough examination of discretization impacts in future research.