THE IMAGINARY-PART DISTRIBUTION OF LATTICE QCD DATA AND SIGNAL IMPROVEMENT

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Introduction

Lattice QCD is the most important theoretical method to solve the strong interaction non-perturbatively from first principles. In recent, some studies began to focus on the imaginary part distribution and signal noise ratio (SNR) problems of lattice data [1–3]. This work gives a mathematical description that can reasonably explain the lattice data, and then gives a self-consistent SNR improvement scheme.

Real and imaginary parts distributions

In lattice studies, the meson two-point correlation function is one of the most basic computations, which can be formally expressed as

$$C_2(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \langle O(\boldsymbol{x},t)O^\dagger(\boldsymbol{0},0)\rangle = \sum_n |A_n|^2 e^{-E_n t}. \quad (1)$$

To extract the distribution of real and imaginary parts, we use an ensemble composed of 4000 pure gauge configurations with lattice size $16^3 \times 192$. By observing the real and imaginary parts distributions of the boosted pseudoscalar two-point function (Fig. 1), we can learn

- The width of the real part distribution is much larger than that of the imaginary part distribution when t is small. The real part distribution is close to the imaginary part distribution when t is large.
- The imaginary part of the distribution is always close to a symmetric, zero-centered normal distribution.
- When t is small, the correlation function is always positive and the distribution has a lower bound of zero. The lower bound significantly affects the shape of the distribution function, which is similar to log-normal.

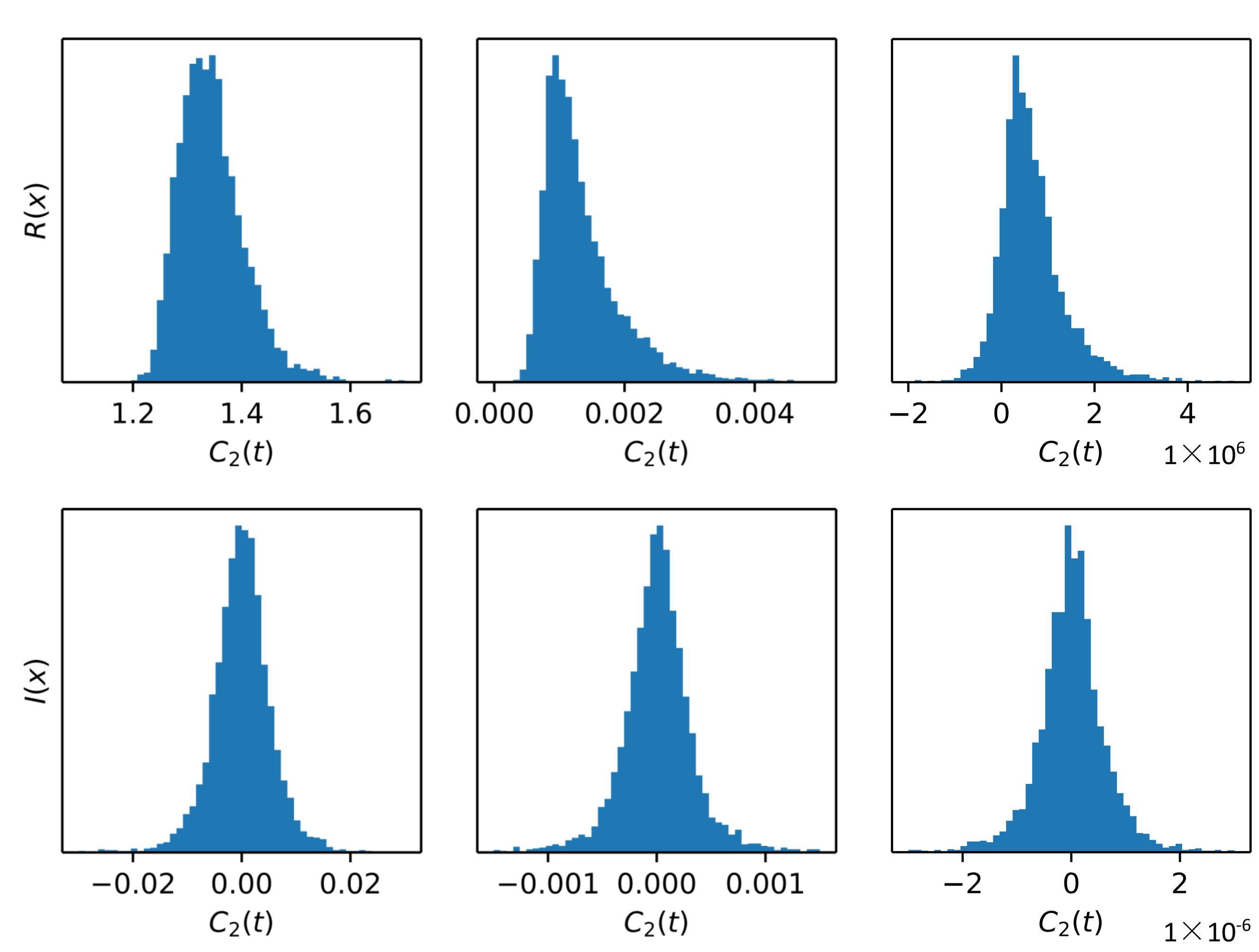

Figure 1: The real-part (upper panel) and imaginary-part (lower panel) distributions of the pseudoscalar two-point functions with $p^2 = 1$ over gauge configurations. From left to right, the figures are for $t/a_t = 1$, 30 and 90, respectively.

Statistical correlation

In order to explain the above distribution behavior of real and imaginary parts of correlation function, we propose

$$R(x) = \int dy S(y-x)\left[I(y)K(U_y)\right], \quad (2)$$

where R, S and I are real part distribution, signal and imaginary part distribution, respectively. $K(U_x)$ is a kernel function associated with the gauge field. And first, we assume $K(U_x)$ is trivial, such that

$$R(x) = S(x) \otimes I(x). \quad (3)$$

One self-consistent example of this assumption is the zero-momentum pseudoscalar case, where the imaginary part is strictly zero and its SNR does not decay with time [4].

From Eq. (3), one can prove that there should be a non-zero statistical correlation between the real and imaginary parts

$$R(R,I) = C(R,I)/\sqrt{V(R)V(I)}, \quad (4)$$

where C denotes the covariance and V is the variance. However, as shown by the blue dashed line in the left panel of Fig. 2, no significant statistical correlation is found. So we abandon the assumption of $K(U_x)$ being trivial and propose instead $K(U_x) = \mathrm{sgn}(U_x)$, where function sgn denotes a sign correction for different gauge samples. Specifically, this sign function changes the sign of the imaginary parts to be the same as the sign of the mean of the real parts in each jackknife ensemble. After this correction, as shown by the orange line in the left panel of Fig. 2, significant statistical correlation is observed. The non-triviality of such a sign correction can be investigated by adding random perturbations on the magnitude of the imaginary parts. As shown in the right panel of Fig. 2, a small disturbance makes the correlation vanish, which indicates that the sign function is for the moment a good approximation of the kernel $K(U_x)$.

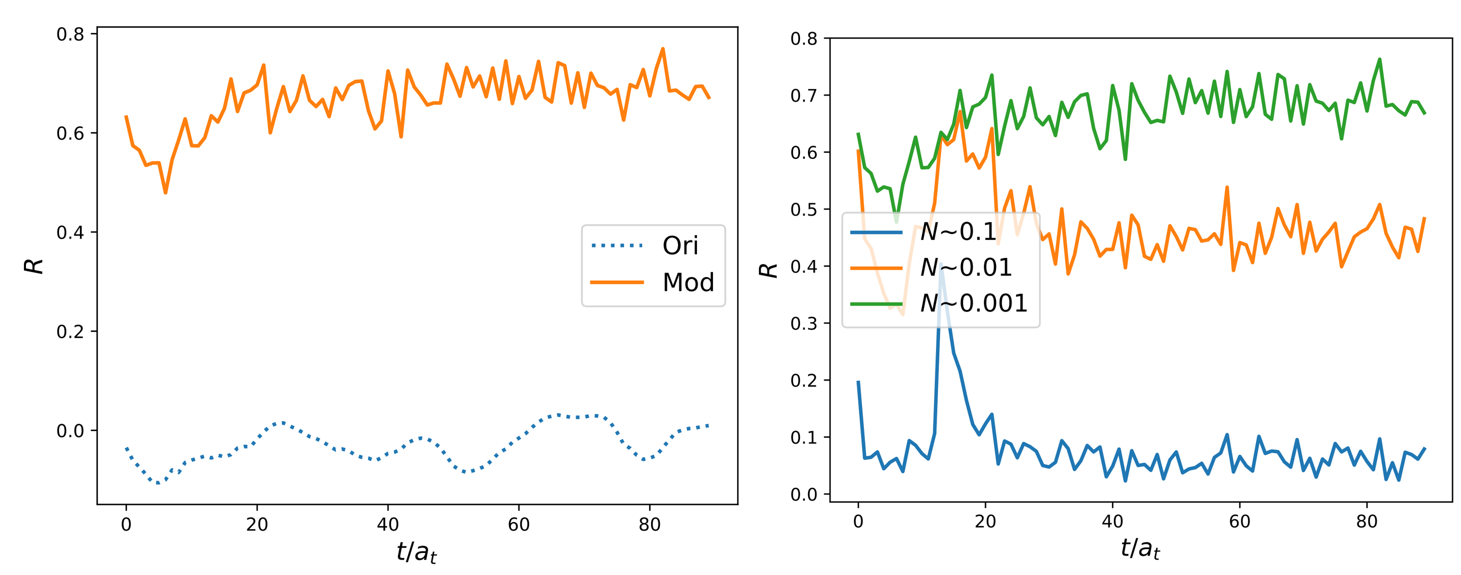

Figure 2: The statistical correlation between the real and imaginary part for different cases.

Utilizing Eq. (2) and the correlation between the real and (corrected) imaginary parts, the signal of the correlation function can be improved. The variance improvement for different quantum numbers and different momenta is about 60% as show in Fig. 3, indicating that the correlation between the real and imaginary part distributions is roughly independent of the specific form of the physical observables.

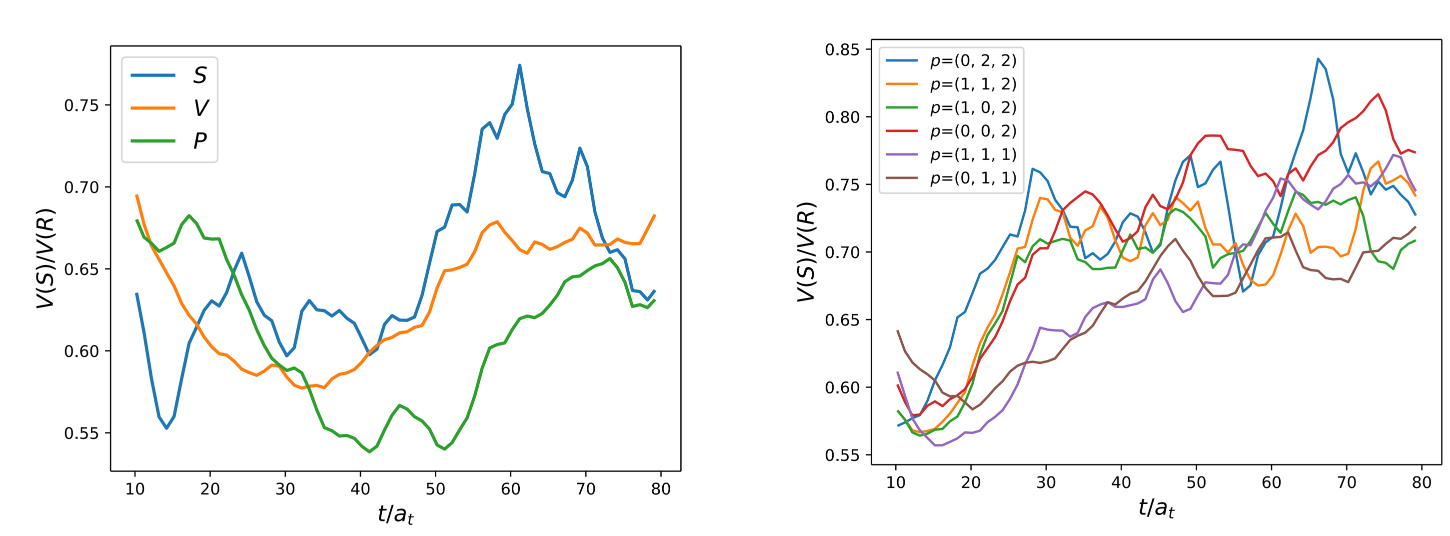

Figure 3: The variance improvement of two-point correlation functions with different quantum numbers and momenta.

Conclusions and outlook

In this work, we propose a conjecture about the distribution of real and imaginary parts as $R(x) = S(x) \otimes \left[I(y)K(U_y)\right]$. We find that the kernel function $K(U_y)$ can be chosen approximately as a sign function, and thus the statistical variance of the two-point functions is reduced to 60%. If we have more rigorous constraints on the kernel function, we can further get stronger statistical correlation and more effective error improvement. It is our ongoing work to further explore the relationship between the real and imaginary parts of lattice data by using machine learning, and to better understand the error of lattice calculation.

References

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