Fast Fermion Smearing Scheme with Gaussian-like Profile

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Abstract

We propose a novel smearing scheme which gives a Gaussian-like profile and is more efficient than the traditional Gaussian smearing [1] in terms of computer time consumption. We also carry out a detailed analysis of the profiles, smearing sizes, and the behaviors of hadron effective masses of different smearing schemes, and point out that having a sufficient number of gauge paths in a smearing scheme is essential to produce strong smearing effects. For a moderate smearing size $\bar{r} \sim 10a$, the time cost for the novel smearing is less than $1/8$ of that for the traditional Gaussian smearing. In practical lattice calculations with larger smearing sizes or finer lattice spacings the improvement will be more substantial.

Novel smearing: definition and profile

We define the smearing operator as the product of shift operations in different orders of (i, j, k),

$$\hat{N} \sim \prod_{(i,j,k)\in P(i,j,k)}^{p} \left[1+\sum_{m=1}^{n}\left((\mathcal{S}_i^+)^m+(\mathcal{S}_i^-)^m\right)\right] \left[1+\sum_{m=1}^{n}\left((\mathcal{S}_j^+)^m+(\mathcal{S}_j^-)^m\right)\right]\left[1+\sum_{m=1}^{n}\left((\mathcal{S}_k^+)^m+(\mathcal{S}_k^-)^m\right)\right], \quad (1)$$

where $\prod_{(i,j,k)\in P(i,j,k)}^{p}$ means we randomly take p (i, j, k) orders from the 6 permutations $P(i, j, k)$, and this guarantees that rotation symmetry is restored after configuration averaging. We can tune the parameter n and p to control the effects and computer time consumption.

The shape of profile looks like Gaussian. This can be understood intuitively by the illustration of the smearing procedure on a one-dimensional unit-gauge lattice in the right two panels in Fig. 1. The two panels represent the novel smearing with $n = 1$ and 2, respectively.

The evolution of the profiles with increasing p is very similar to the Yang Hui's triangle (Pascal's triangle). The k_{th} number of the p_{th} line of the Yang Hui's triangle equals to the binomial coefficient C_p^k and the corresponding distribution is a binomial distribution with probability $P = 1/2$. The binomial distribution approximates Gaussian distribution very well with large p.

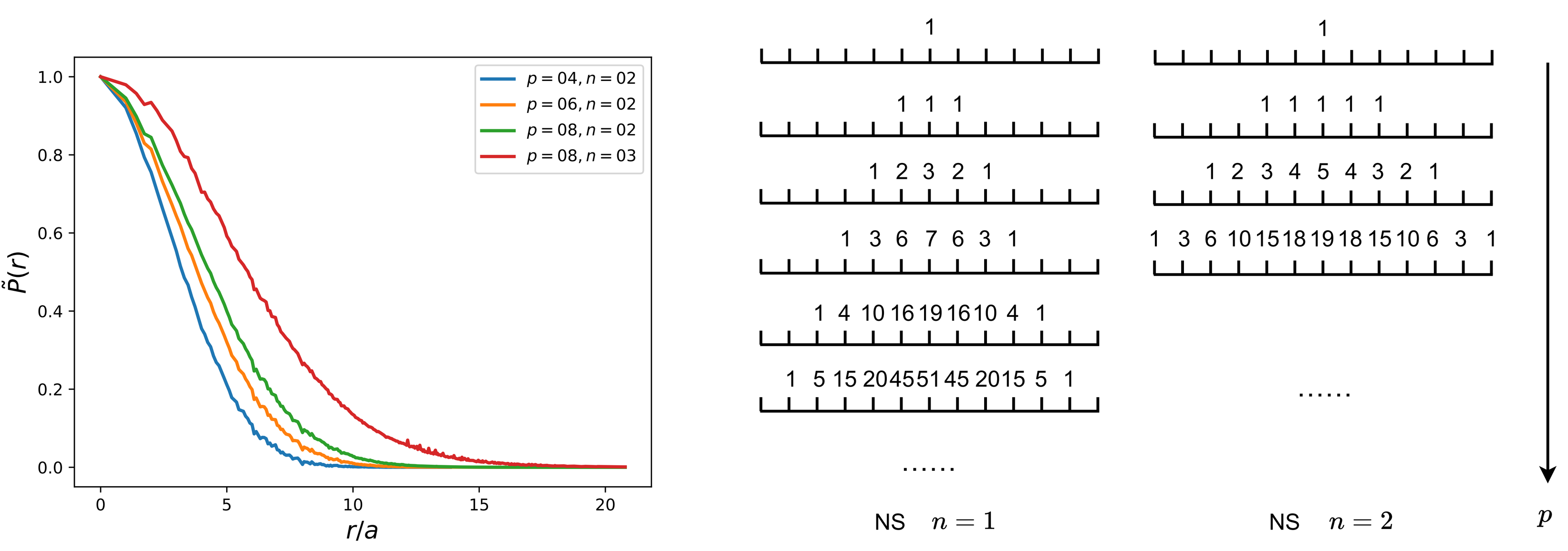

Figure 1: Left: profile examples of the novel smearing scheme with different p and n. Right: the profile evolution with increasing p for the novel smearing (NS) with $n = 1$ and 2.

A plot of the number of shift operations ($N_s = 6n \times p$) as a function of smearing size $\bar{r}$ and its comparison with different smearings is shown in Fig.2. Besides, we define $\bar{r}$ the mean-squared radius of profiles as

$$\bar{r} = \sqrt{\frac{\int \tilde{P}(r) r^2 dr}{\int \tilde{P}(r) dr}}. \quad (2)$$

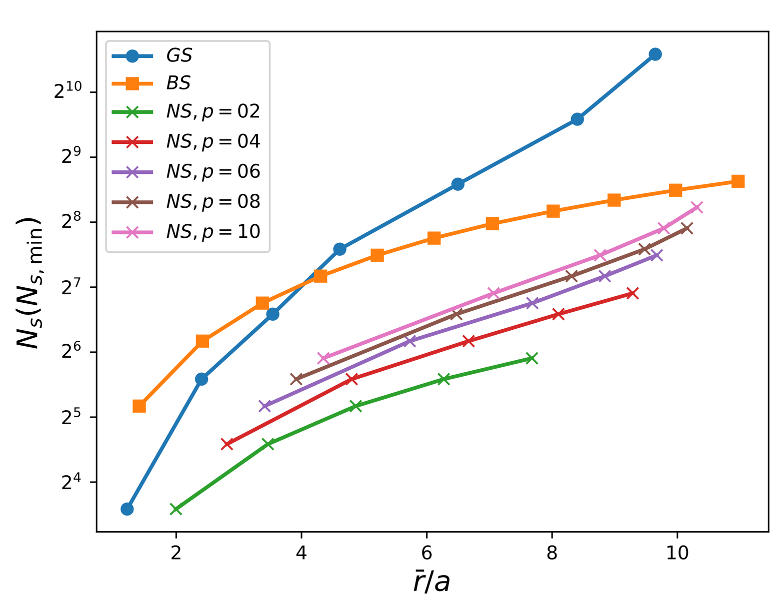

Figure 2: (Minimum required) number of shift operations as a function of smearing size $\bar{r}$.

By comparing all smearing schemes' numbers of shifts, we can conclude that the cost of the novel smearing is significantly less than that of the Gaussian smearing for the same $\bar{r}$, and for the novel smearing smaller p seems more efficient.

Advantage compared to Gaussian

To further analyze the smearing effects of novel smearing scheme, we compare the effective mass results of Gaussian and novel smearings.

The top left panel of Fig. 3 shows the representative pion effective masses with selected n and p, from which we do observe small and even negative excited-state contributions at large n or p, indicating that this novel smearing scheme can produce strong smearing effects with limited number of gauge links. The top right panel collects some p and n combinations that lead to nearly the same proton effective masses. According to Fig. 2, the combinations with smaller p and bigger n are more effective.

From the last two panels of Fig. 3 and Fig. 2, we find that when producing the same smearing effects, the cost of the novel smearing is 4 to 8 times improved than the Gaussian smearing, and the larger the smearing size is the greater the improvement will be.

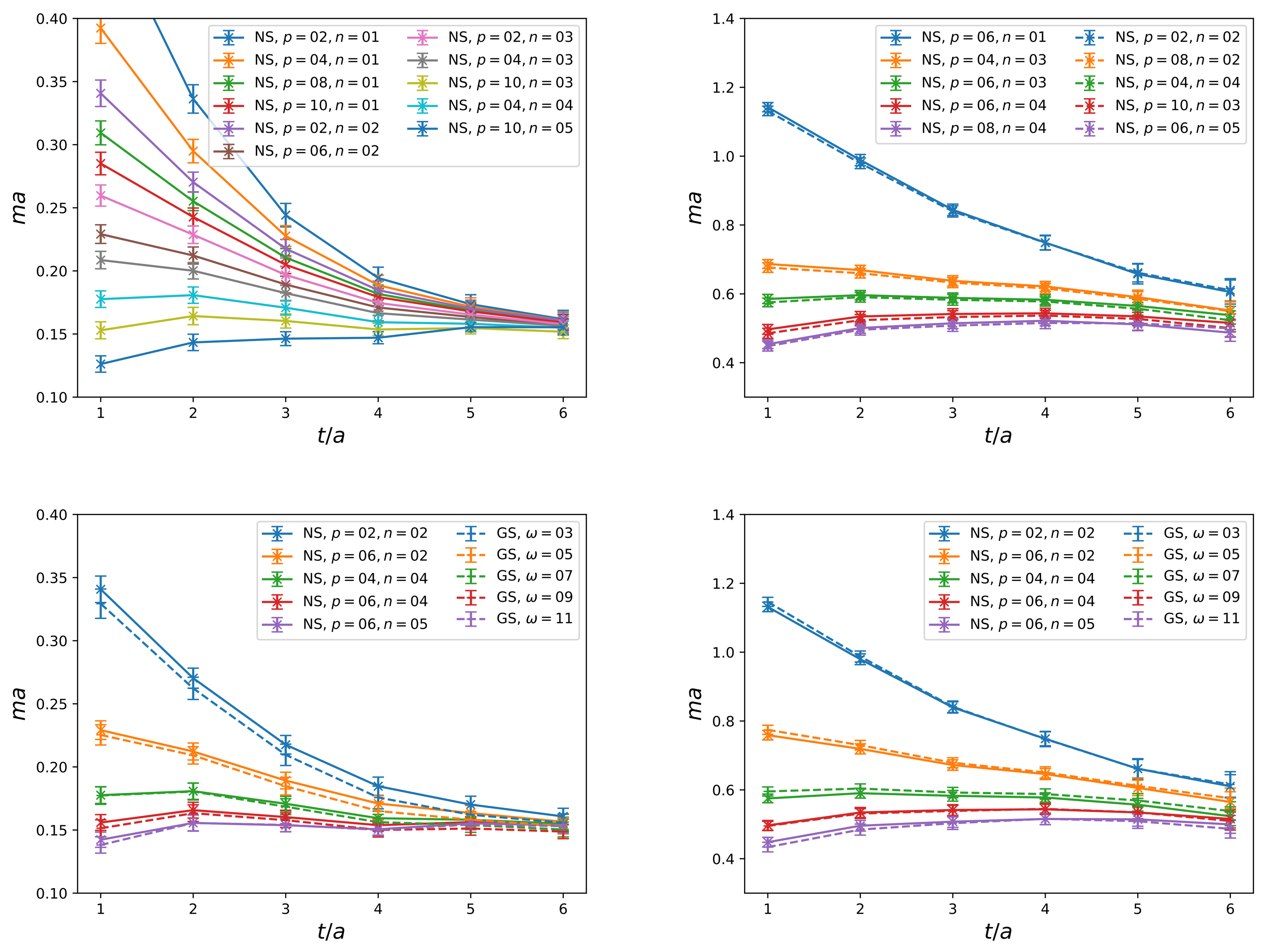

Figure 3: The top left panel shows the representative pion effective masses with selected n and p and the top right panel collects some p and n combinations that lead to nearly the same proton effective masses. Several n, p combinations that give very similar pion and proton effective masses to the Gaussian smearing with typical ω's are plotted in the lower two panels.

Discussion and Outlook

We propose a novel smearing scheme which gives a Gaussian-like profile and is more efficient than the traditional Gaussian smearing in terms of time cost. With close smearing effects, the improvement of the novel smearing is 4 to 8 times compared to the Gaussian smearing for smearing size $\bar{r} \leq 10a$. For practical lattice studies with larger smearing sizes or finer lattice spacings the improvement will be more substantial. Besides, by a detailed analysis of the profiles, smearing sizes and hadron effective masses, we point out that having a sufficient number of gauge paths is essential for a smearing scheme to produce strong smearing effects. An important point is that for not too large smearing sizes, the number of gauge paths in the novel smearing scheme is well enough for $p \sim 6$ as indicated by the effective mass behaviors. Therefore a qualitative conclusion is that the larger the smearing size is, the more gauge paths we need. And this scheme can easily accommodate any large smearing sizes by using a larger parameter p.

It is also worth mentioning that our novel smearing is tested to be compatible with the momentum smearing [2]. We believe that this smearing scheme or its variants should be inspiring to the community and beneficial for lattice studies of hadron spectra and structures.

References

[1] S. Gusken. A Study of smearing techniques for hadron correlation functions. *Nucl. Phys. B Proc. Suppl.*, 17:361–364, 1990.

[2] Gunnar S. Bali, Bernhard Lang, Bernhard U. Musch, and Andreas Schäfer. Novel quark smearing for hadrons with high momenta in lattice QCD. *Phys. Rev. D*, 93(9):094515, 2016.