Lattice QCD calculation of π^0-pole's contribution to HLbL

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Introduction

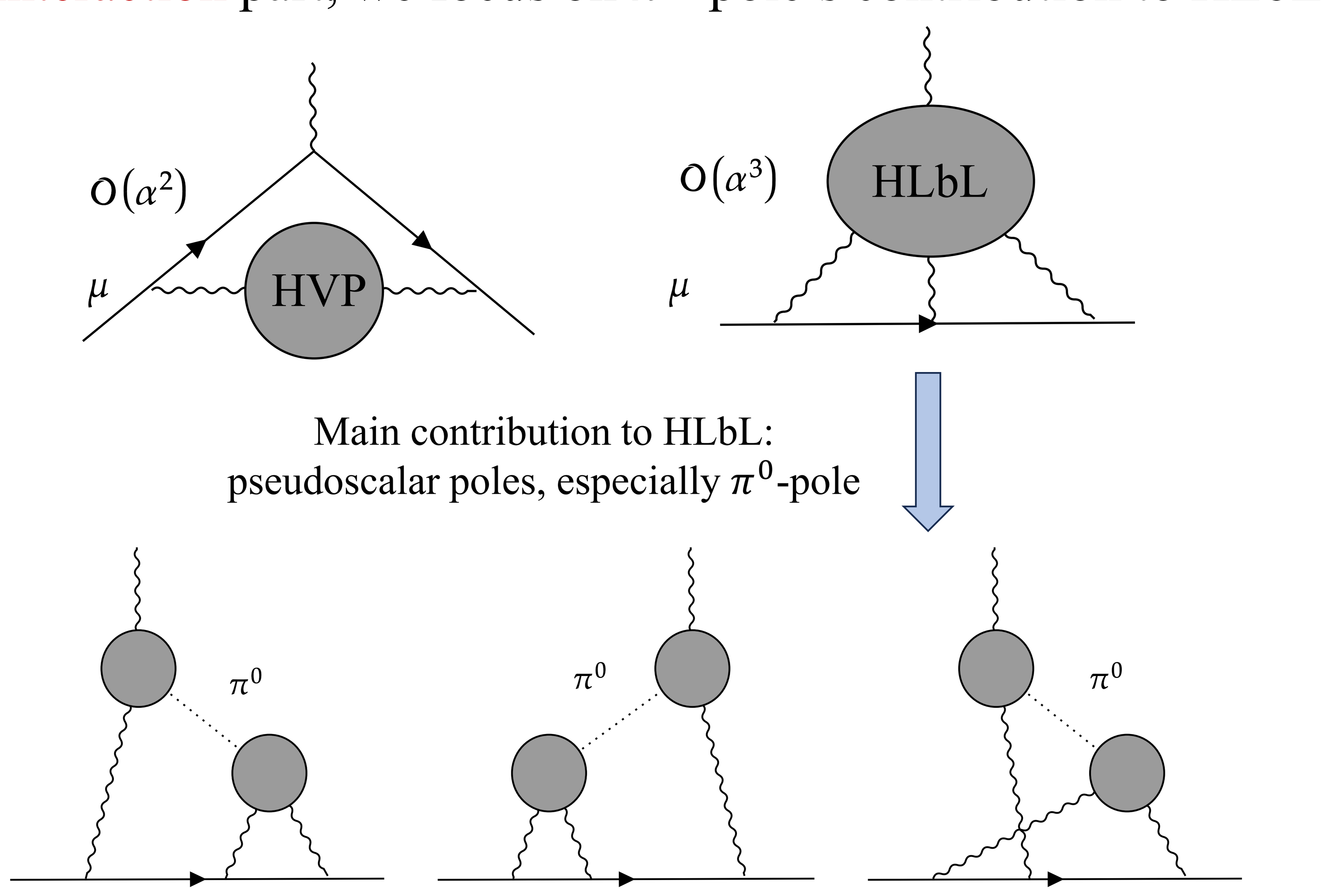
The study of muon g-2 has sparked strong interest within the academic community.

2023 Newest Experiment (Aguillard et al.) ⟷ **2020 White Paper (Aoyama et al.)**

A discrepancy of 5.0 σ

New Physics? More precise calculations are required!

The primary source of uncertainty stems from the strong interaction part, we focus on π^0-pole's contribution to HLbL.



We propose a new method to calculate π^0 transition form factor and π^0-pole's contribution to HLbL improved in the following aspects:

- Extract form factors of arbitrary spacelike momentum no need to fit form factor
- Long time contribution with lower noise no need for time truncation and tail correction.
- Well controlled model dependence
- Smaller lattice spacing error

Methodology

We begin from correlation function of $\pi \to \gamma^*\gamma^*$ in Euclidean space with infinity volume to extract π^0 form factor:

$$\varepsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta \mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,p'^2) = \int d^4x\, e^{\left(E-\frac{1}{2}m_\pi t\right)} e^{-i\vec{p}\cdot\vec{x}} \mathcal{H}_{\mu\nu}(x)$$

where $\mathcal{H}_{\mu\nu}(x) = \langle 0 \left| T\left\{ J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right)\right\} \right| \pi(q)\rangle$, q, p, p' are momentums of pion and two photons in rest frame. In a system with infinity volume, momentum of photon can be taken continuously so that the whole spacelike kinematic region can be reached.

By introducing pion distribution amplitude (PDA) $f(\chi, x^2)$, we can perform a SO(4) average of weight function to solve increasing noise catastrophe in long time region, our master formula can be written as:

Analytic known function — Lattice input

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(p^2,p'^2) = i\int d^4x \int_0^1 d\chi\, \omega(k,x,\chi) f(\chi,x^2) H(x)$$

Physical PDA

where $k = p - \chi q$ and $H(x)$ is a scalar hadronic function defined as: $H(x) = \varepsilon^{\mu\nu\alpha 0} x_\alpha q_0 \mathcal{H}_{\mu\nu}(x)$.

Finally, insert form factors to calculate pion's pole contribution to HLbL:

$$a_\mu^{\pi^0-pole} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau\, w_1(Q_1,Q_2,\tau) \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,0)$$
$$\times \mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_2^2,-Q_3^2) + w_2(Q_1,Q_2,\tau)\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2)\,\mathcal{F}_{\pi^0\gamma^*\gamma^*}(-Q_3^2,0)$$

w_1, w_2 are weight functions defined in (Guy, R. E. (1980)), $Q_3^2 = Q_1^2 + Q_2^2 + 2\tau Q_1 Q_2$.

Lattice Ensembles

Domain wall fermion + Iwasaki gauge action

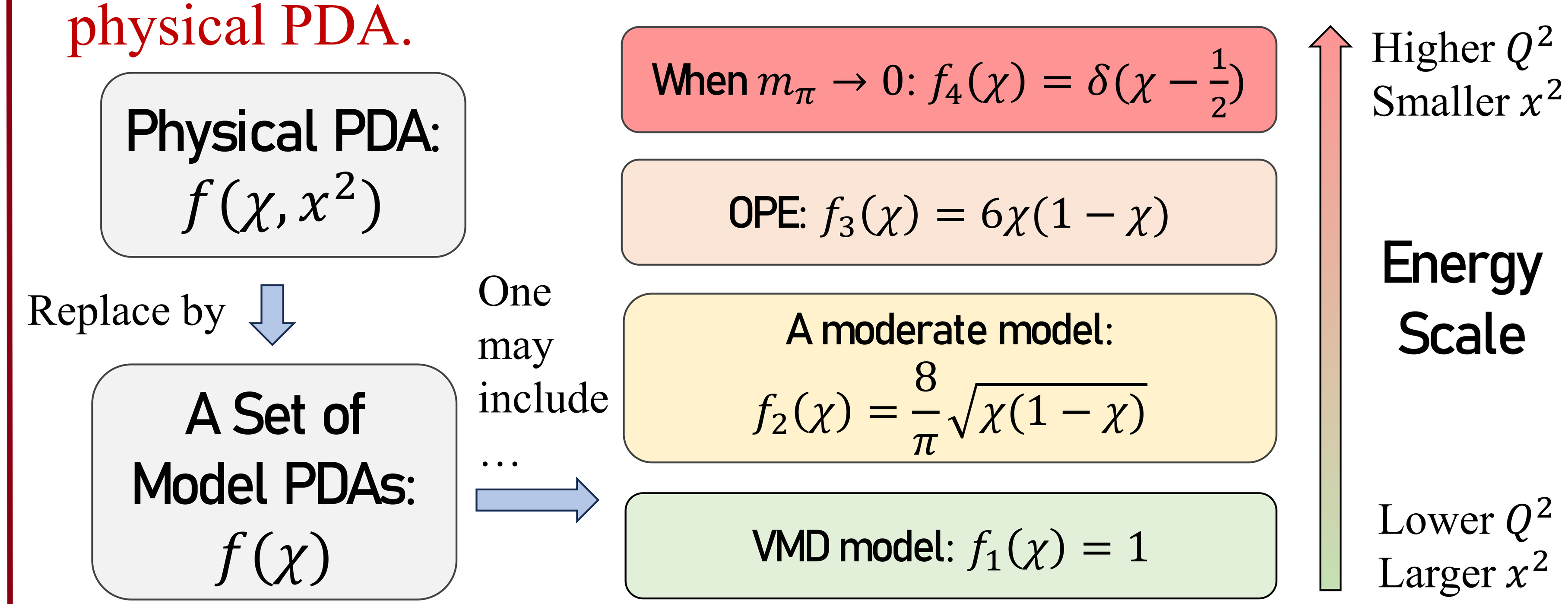
Physical pion mass with different lattice spacings.

Ensembles	$a[\mathrm{GeV}^{-1}]$	L/a	T/a	L/fm	$m_\pi[\mathrm{MeV}]$	# of confs
24D	1.015	24	64	4.7	141.56(22)	146
32D	1.015	32	64	6.2	141.38(20)	63
32Df	1.378	32	64	4.7	142.89(40)	69
48I	1.730	48	96	5.5	135.43(36)	33
64I	2.359	64	128	5.4	135.33(20)	65

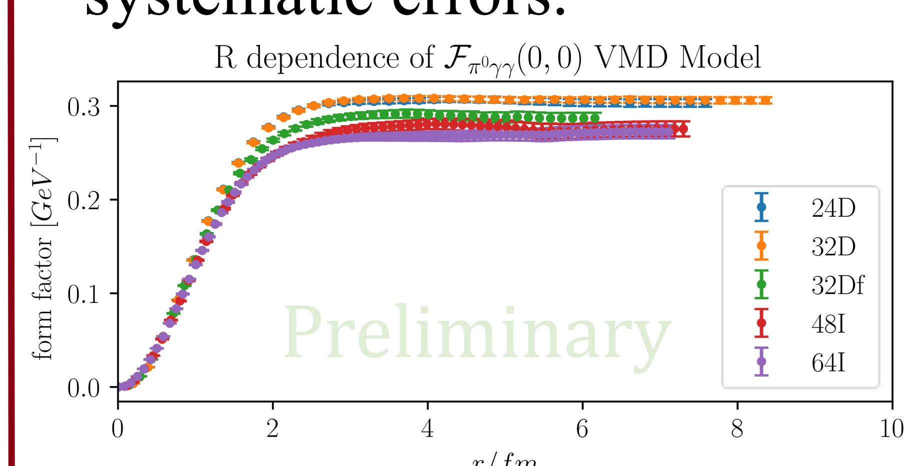
Generated by RBC/UKQCD (T. Blum et al. 2016)

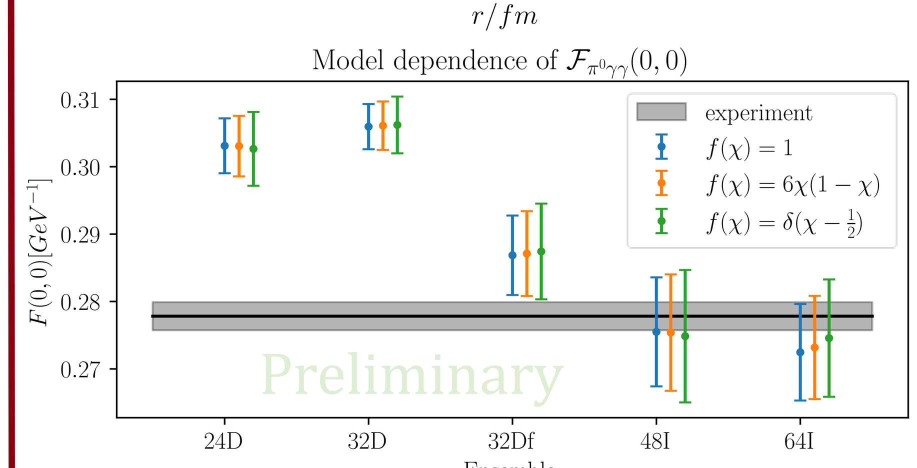
Lattice Calculation of π^0 form factor

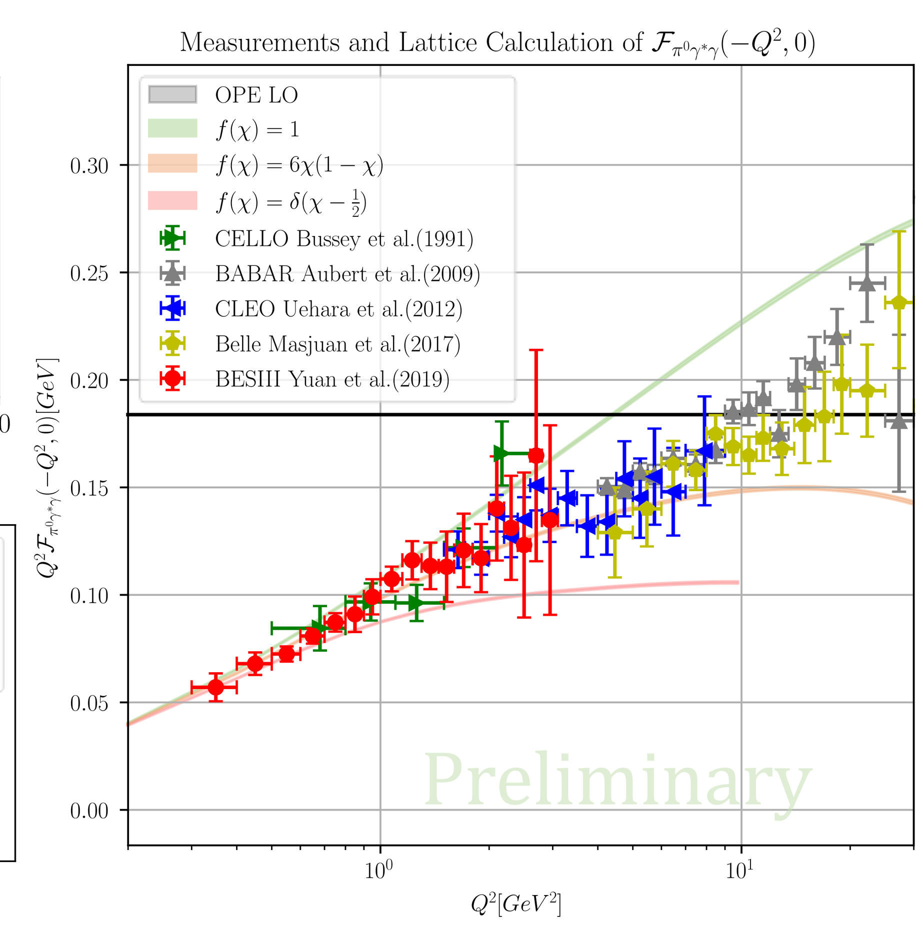
It's convenient to choose a set of model PDAs instead of physical PDA.



Different models will be applied to estimating model dependent systematic errors.







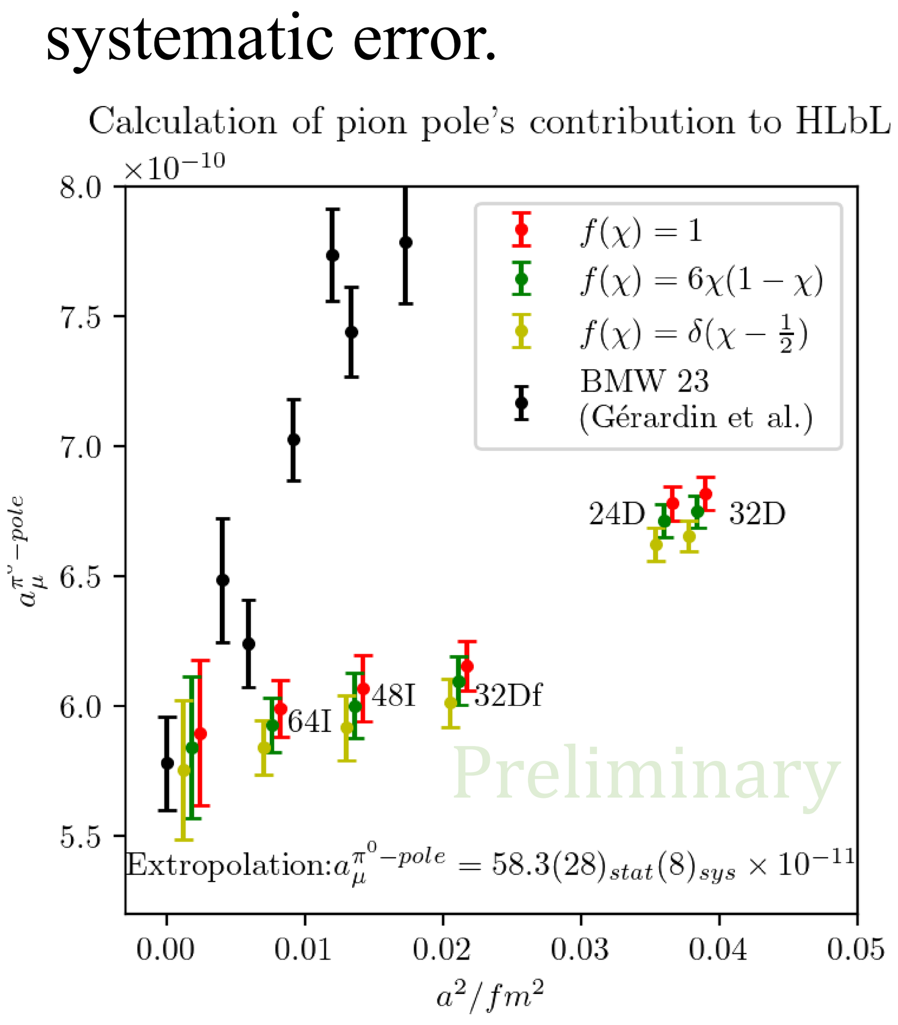
Form factor is almost model independent when p^2 is close to p'^2

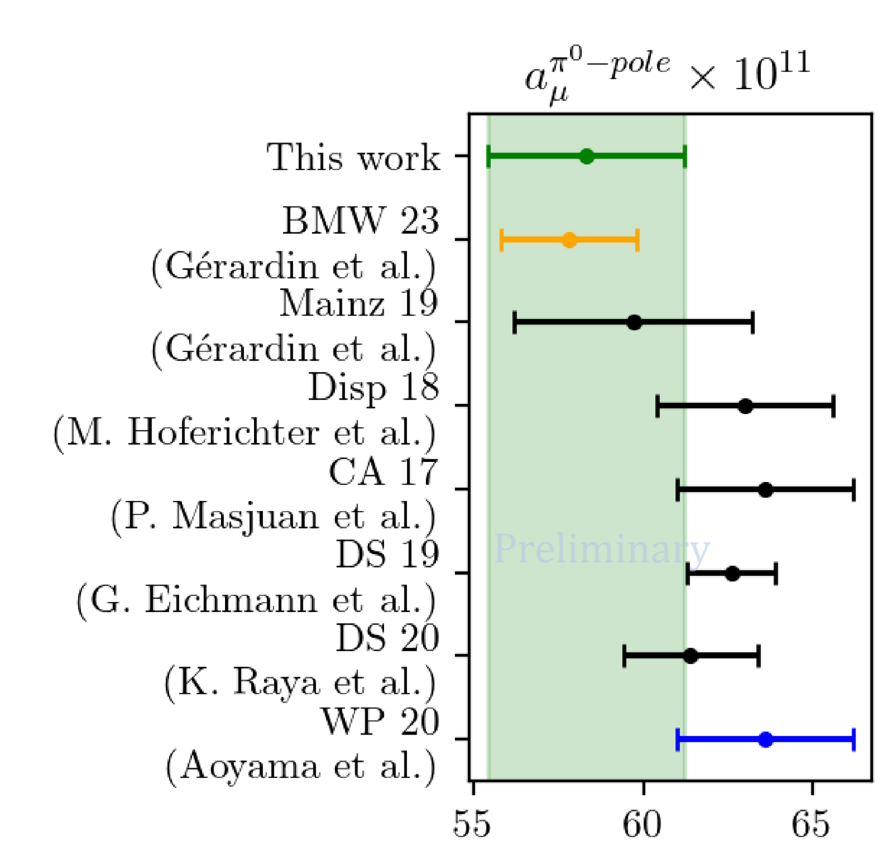
Experiment data helps to constrain choices of model PDAs.

Lattice Calculation of $a_\mu^{\pi^0-pole}$

$a_\mu^{\pi^0-pole}$ is not sensitive to pion's shape!

We choose two "bound" PDAs to estimate model dependent systematic error.





Lattice spacing error is improved a lot in our results.

We are going to calculate in more ensembles.

Contact

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