The impact of imprecise gauge fixing

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Introduction

Non-perturbative lattice gauge-fixing becomes unavoidable to extract information from gauge dependent correlators [1]. It is also necessary in some non-perturbative renormalization schemes [2] which use gauge dependent matrix elements to renormalize composite operators. In the standard formulation of lattice gauge theories proposed by Wilson [3] the link $U_\mu(x)$ are the fundamental gauge fields of the theory, they are group elements of $SU(N)$ in the fundamental (N-dimensional) representation and they transform under a gauge transformation $g(x)$ as

$$U^g_\mu(x) = g(x)U_\mu(x)g^\dagger(x+\hat{\mu}a). \tag{1}$$

For Landau gauge on lattice, the discretized gauge condition is,

$$\Delta^g(x) \equiv \sum_\mu (A^g(x) - A^g(x-\hat{\mu}a)) = 0,\ A_\mu(x) \equiv \left[\frac{U_\mu(x) - U^\dagger_\mu(x)}{2ig_0}\right]_{\text{Traceless}}, \tag{2}$$

g_0 is the bare coupling constant. Non-perturbative gauge fixing is impossible to be perfect and imprecise gauge fixing is unavoidable, so the effect of imprecise gauge fixing is necessary to be investigated. Recently, we find that gauge dependent non-local measurements are much more sensitive to the precision of gauge fixing than we expect, especially for long distance and small lattice spacing.

Simulation Setup

We use valence clover fermion on $(2+1+1)$ flavor MILC gauge configurations on five ensembles [4]. The parameters of the ensembles used are:

Tag	$6/g^2$	L	T	a(fm)	$m^{\rm w}_q a$	$c_{\rm SW}$
a12m310	3.60	24	64	0.1213(9)	-0.0695	1.0509
a09m310	3.78	32	96	0.0882(7)	-0.0514	1.0424
a06m310	4.03	48	144	0.0574(5)	-0.0398	1.0349
a045m310	4.20	64	192	0.0425(5)	-0.0365	1.0314
a03m310	4.37	96	288	0.0318(5)	-0.0333	1.0287

We use the Coulomb wall source and pion external state in rest frame for 2pt and 3pt functions to improve the signal. One step of HYP smearing is applied on the Wilson link to improve the signal. The off-shell momentum for quark matrix element is set to be $(p_x,p_y,p_z,p_t) = \frac{2\pi}{La}(5,5,0,0)$. And for the gluon propagator, the momentum is selected by the condition $\sum_\mu p^4_\mu / \left(\sum_\mu p^2_\mu\right)^2 < 0.260$ for a09m310 and 0.251 for a045m310 to suppress the discretization error.

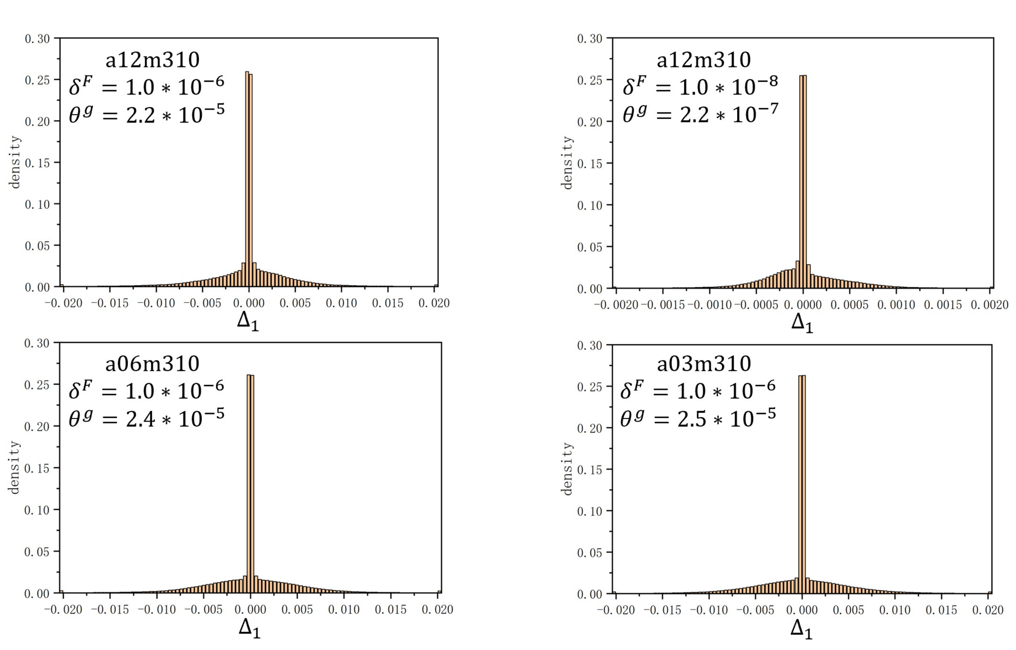
Result and Summary

The standard way of fixing the Landau gauges on the lattice [5, 6] is based on the numerical minimization of a functional, its extrema g^* are the gauge fixing transformation corresponding to the discretized gauge condition

$$F_U[g] = -\frac{1}{12V}\text{Re Tr}\sum_x\sum_{\mu=1}^{4} U^g_\mu(x),\ \left.\frac{\delta F}{\delta g}\right|_{g*} = 0. \tag{3}$$

The precision of gauge fixing can be defined as the change of the functional per renewal step, and we call it δ^F. Another one to estimate the precision, denoted by θ, is defined as folows,

$$\theta^g \equiv \frac{1}{V}\sum_x \theta^g(x) \equiv \frac{1}{V}\sum_x \text{Tr}[(\Delta^g)^\dagger(x)(\Delta^g)(x)]. \tag{4}$$

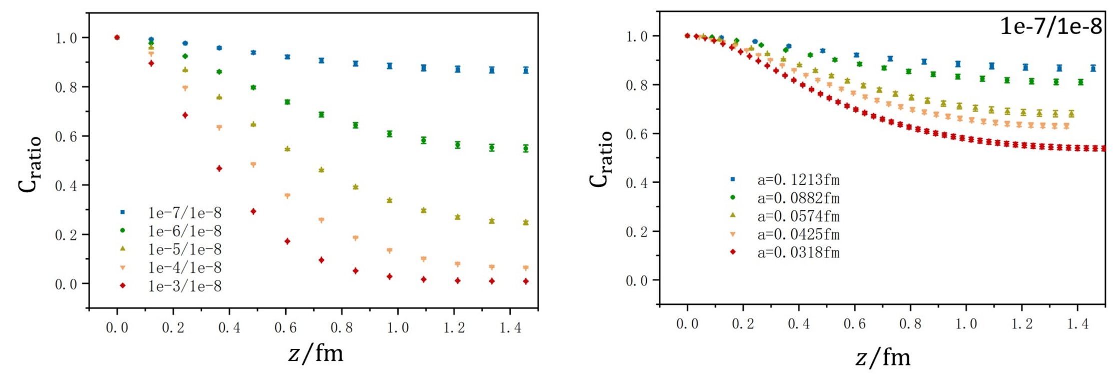


The distribution of $\Delta(x)$ in Eq. 2 is symmetric for different precision and different ensembles. The tail of the distribution will be longer as the precision becomes lower (note that the x axis is different for different precision), but it does not show a strong dependence of lattice spacing. And it is hard for us to learn something non-local by just look at $\Delta(x)$.

To extract some non-local information, we define a correlation function of gauge transformation as,

$$C_{1,0}(z) = \frac{1}{3V}\sum_x \text{Tr}\left[\left[g_1(x+z)g^\dagger_0(x+z)\right]^\dagger\left[g_1(x)g^\dagger_0(x)\right]\right]. \tag{5}$$

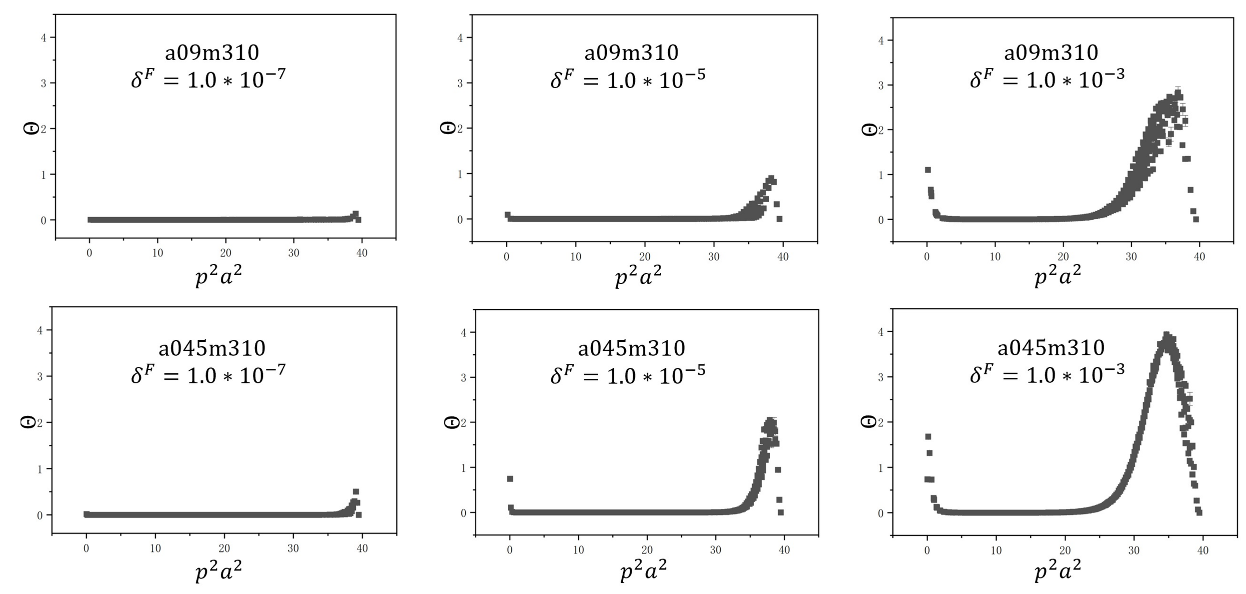
Typically, δ^F_0 of gauge fixing 0 is small enough and gauge fixing 1 is not so precise. Then the ratio of the gauge transformation will reveal the unphysical contribution from imprecise gauge fixing. And the correlation of the ratio will help us to get more knowledge of the unphysical contribution in long distance.



We choose gauge fixing with $\delta^F = 10^{-8}$ as the denominator. The correlation $C_{1,0}$ defined in Eq. 5 will decay faster as the gauge fixing becomes worse, the distance becomes longer and the lattice spacing becomes smaller. The effect of imprecise gauge fixing is more obvious in long distance and small lattice spacing.

To have a wider understanding of imprecise gauge fixing, we consider a longitudinal component related to gluon propagator in momentum space:

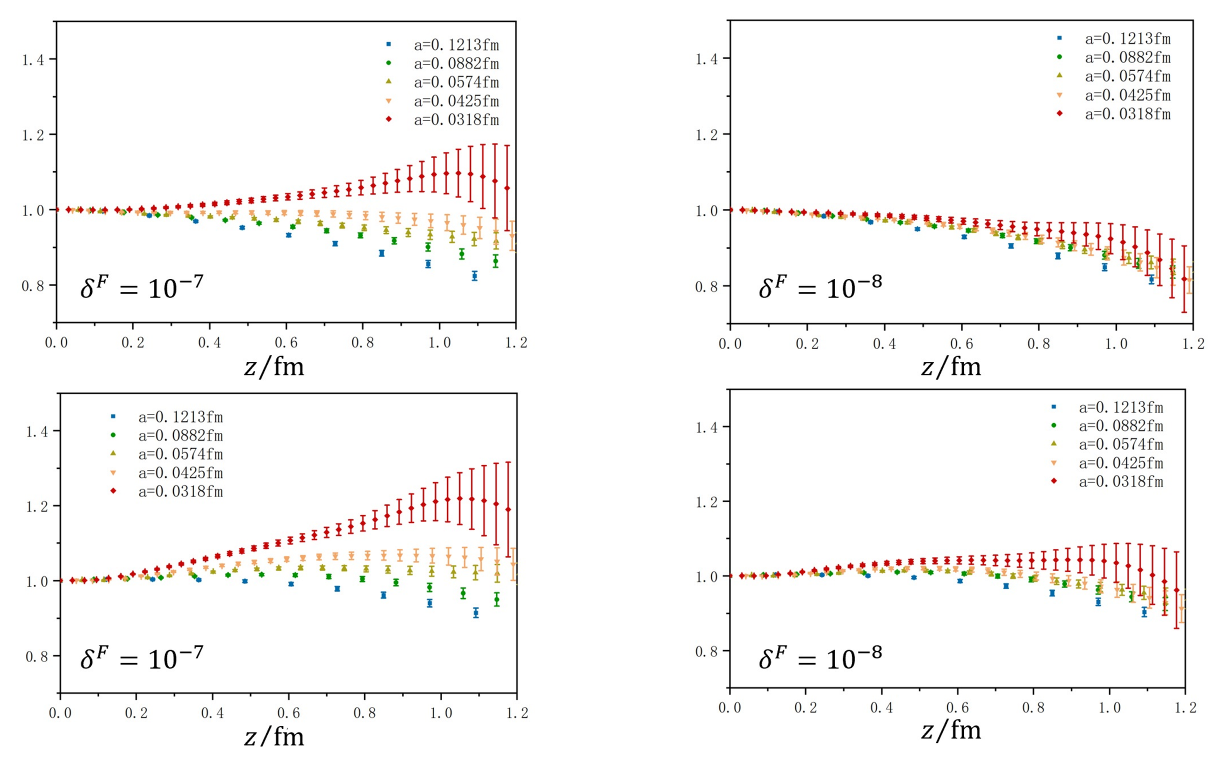
$$\Theta(p) \equiv \frac{1}{8V}\text{Tr}\left[(p\cdot A(p))^\dagger(p\cdot A(p))\right]. \tag{6}$$



Our result shows that, in the region of large momentum and small momentum, Θ will deviate from zero if the precision of gauge fixing is not so high, and the deviation is larger for finer lattice.

Now we focus on some more popular measurements. In RI/MOM scheme, we need to calculate quark matrix element, which is gauge dependent. If we just concentrate on local operator, the difference between $\delta^F = 10^{-7}$ and $\delta^F = 10^{-8}$ is negligible within uncertainty. But quasi-PDF operator is a non-local operator. Wwith small lattice spacing in long distance, it should have a much stronger dependence of the precision of gauge fixing. Wilson link is another non-local gauge dependent measurement. We try to use them to cancel the linear divergence in hadron matrix element of quasi-PDF operator. Note that hadron matrix element is gauge independent. Wilson link and quasi PDF operator are defined as follow,

$$U_\mu(x,x+na) = \prod_{k=0}^{n-1} U_\mu(x+k\hat{\mu}a),\ W_\mu(z) = \frac{1}{3V}\text{Tr}\Big[\sum_x U_\mu(x,x+z)\Big],\ O_\Gamma(z) = \bar{\psi}(0)\Gamma U(0,z)\psi(z). \tag{7}$$



For quasi PDF, no matter it is renormalized by Wilson link (lower panel) or in RI/MOM scheme (upper panel), the results with $\delta^F = 10^{-7}$ are not consistent between different lattice spacings. The discrepancy is larger for finer lattice. And the discrepancy grows larger as the link becomes longer. But as the precision becomes higher, such as $\delta^F = 10^{-8}$, the results show a convergent behavior which means linear divergence has been eliminated. Although in short distance, the results between $\delta^F = 10^{-7}$ and $\delta^F = 10^{-8}$ are consistent within error, the effect of imprecise gauge fixing will be huge in non-local case, especially for small lattice spacing in long distance.

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