Determining quark masses and low-energy constants using Overlap fermions

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(χQCD callabration)

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Introduction

The overlap fermion [1, 2, 3], which satisfies the Ginsburg-Wilson relation, would be the optimal choice for the discretized Dirac operator, but requires a factor of $\mathcal{O}(100)$ cost of computational resources comparing to the widely used Wilson-like fermions. The results presented in this paper, based on overlap fermions lattice QCD simulations, calculate the low-energy constants (LECs) and light quark masses, serve as important references for future theoretical and experimental investigations. The precise knowledge of these quantities contributes to our understanding of the strong nuclear force and provides valuable guidance in the quest for new physics beyond the Standard Model.

Theoretical Framework

We fit the meson mass M_m and decay constant F_m at different valence quark masses for each ensemble with a two-state fitting form, which is:

$$\begin{aligned} C(t) =& A_0 \exp(-M_m t)(1 + A_1 \exp(-\delta M_m t)), \\ F_m =& ((m^v_{q1} + m^v_{q2})\sqrt{M_m A_0})/(Z_S(M_m a)^2), \end{aligned} \tag{1}$$

where $C(t)$ is the two-point function of meson, while δM_m term represents the contribution of excited states.

After obtaining the pion masses under different ensembles and different valence quark masses, we consider fitting the light quark mass and other LECs under the continuum limit. Under different ensembles, the global fitting form is

$$\begin{aligned} M^2_\pi =& \Lambda^2_\chi 2y_v\{1 + \frac{2}{N_f}[(2y_v - y_s)\ln(2y_v) + (y_v - y_s)] \\ &+ 2y_v(2\alpha_8 - \alpha_5) + 2y_s N_f(2\alpha_6 - \alpha_4)\}(1 + c_m a^2), \\ F_\pi =& F(1 - \frac{N_f}{2}(y_v + y_s)\ln(y_v + y_s) + \alpha_5 y_v + \alpha_4 N_f y_s)(1 + c_f a^2), \end{aligned} \tag{2}$$

the square of the pion masses and decay constants are considered as functions of the valence and sea quark masses. Here, chiral scale Λ_χ is defined as $4\pi F$. The variables $y_{v,s}$ represents the ratios $\frac{\Sigma m_l^{v,s}}{(F\Lambda_\chi)^2}$ and $m_l^{v,s}$ is the valence/sea quark mass, respectively. We impose the condition that the physical pion mass $M_{\pi,phys}$ is equal to $134.9768(5)$MeV. Consequently, we obtain the averaged light quark mass, $m_l^{v,s} = m_{l,phys}$, by enforcing $M_\pi = M_{\pi,phys}$.

After obtaining simulated kaon masses, we consider fitting up (u), down (d) and strange (s) quark masses under the continuum limit. Under different ensembles, the fitting form is

$$M^2_K = (c^v_{1,l} m^v_l + c^v_{1,s} m^v_s + c^s_{1,l} m^s_l + c^s_{1,s} m^s_s)(1 + c_{2,l} m^v_l + c_{2,a} a^2). \tag{3}$$

By employing the previously obtained value of $m_{l,phys}$, we impose the condition $M_K = M_K^{iso,QCD} = 494.53(7)$ to determine the physical strange quark mass $m_{s,phys}$. Moreover, we employ the values of $M_{K^0_{QCD}} = 497.44(2)$MeV and $M_{K^+QCD} = 491.61(15)$MeV to determine the physical masses of $m_{u,phys}$, $m_{d,phys}$. Finally, we use the equation $m_{u,phys} + m_{d,phys} = 2m_{l,phys}$ to check the obtained results.

Simulation Setup

We use valance overlap fermion on $(2+1)$ flavor RBC/UKQCD DWF gauge configurations on four ensembles [4]. The parameters of the ensembles used are:

Action	Symbol	$L^3 \times T$	a (fm)	m_π (MeV)	N_{conf}	N_s
DWF+I	48I	$48^3 \times 96$	0.1141(2)	139	43	8
DWF+I	64I	$64^3 \times 128$	0.0837(2)	139	39	6
DWF+I	48If	$48^3 \times 96$	0.0711(3)	280	178	1
DWF+I	24I	$24^3 \times 64$	0.1105(2)	340	47	6
DWF+I	32I	$32^3 \times 64$	0.0837(2)	302	41	6
DWF+I	32If	$32^3 \times 64$	0.0626(4)	371	241	1

(4)

N_{conf} is the number of configurations and N_s represents the number of sources selected for each configuration. Also, we use valence overlap fermion actions with 1-step HYP smearing and the grid source for the meson 2pt functions to improve the signal.

We conducted a comprehensive global fitting analysis for the aforementioned scenarios. The fitting procedure was performed considering four distinct cases, accounting for errors in the scalar currents renormalization constants(RCs) Z_S [5]. These cases include: (1) fitting without any errors present (pf1), (2) fitting with solely matching errors (pf2), (3) fitting with solely statistical errors (pf3), and (4) fitting with both matching and statistical errors coexisting (pf4). Then the RCs of the scalar operators in the $\bar{M}S$ scheme at $\mu = 2$GeV obtained through the RI/MOM schemes are summarized:

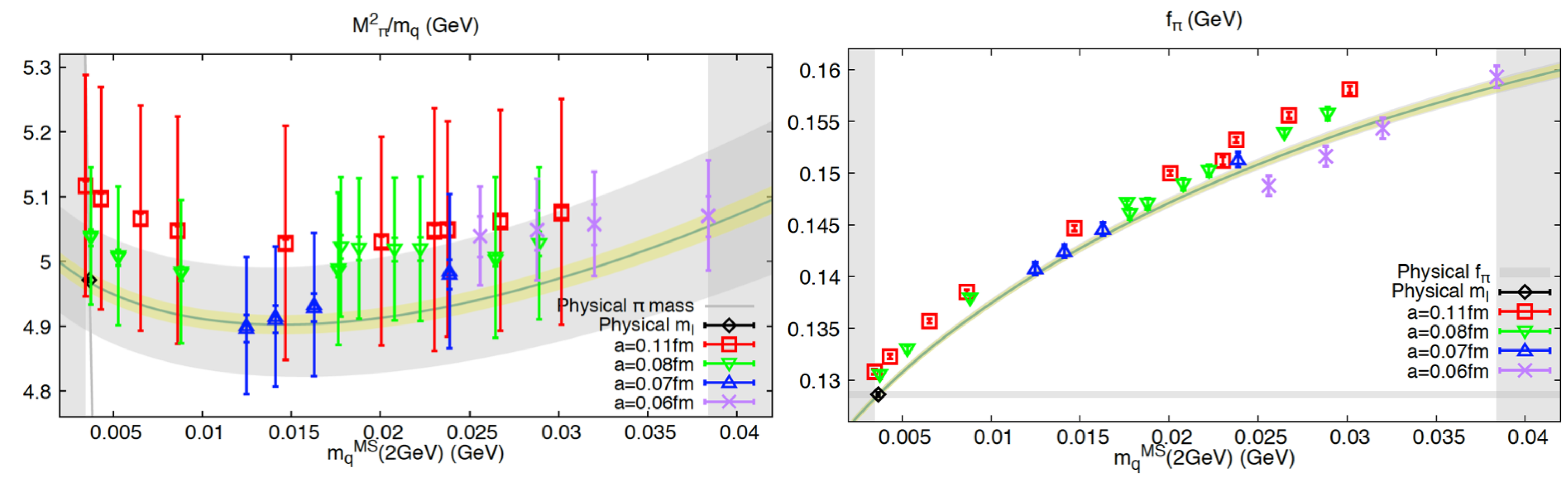
Ensemble	$Z_S^{\bar{M}S}$(2GeV)
48I	1.202(02)(40)(09)
64I	1.069(01)(22)(06)
48If	1.023(01)(22)(05)
32If	0.985(01)(13)(05)

(5)

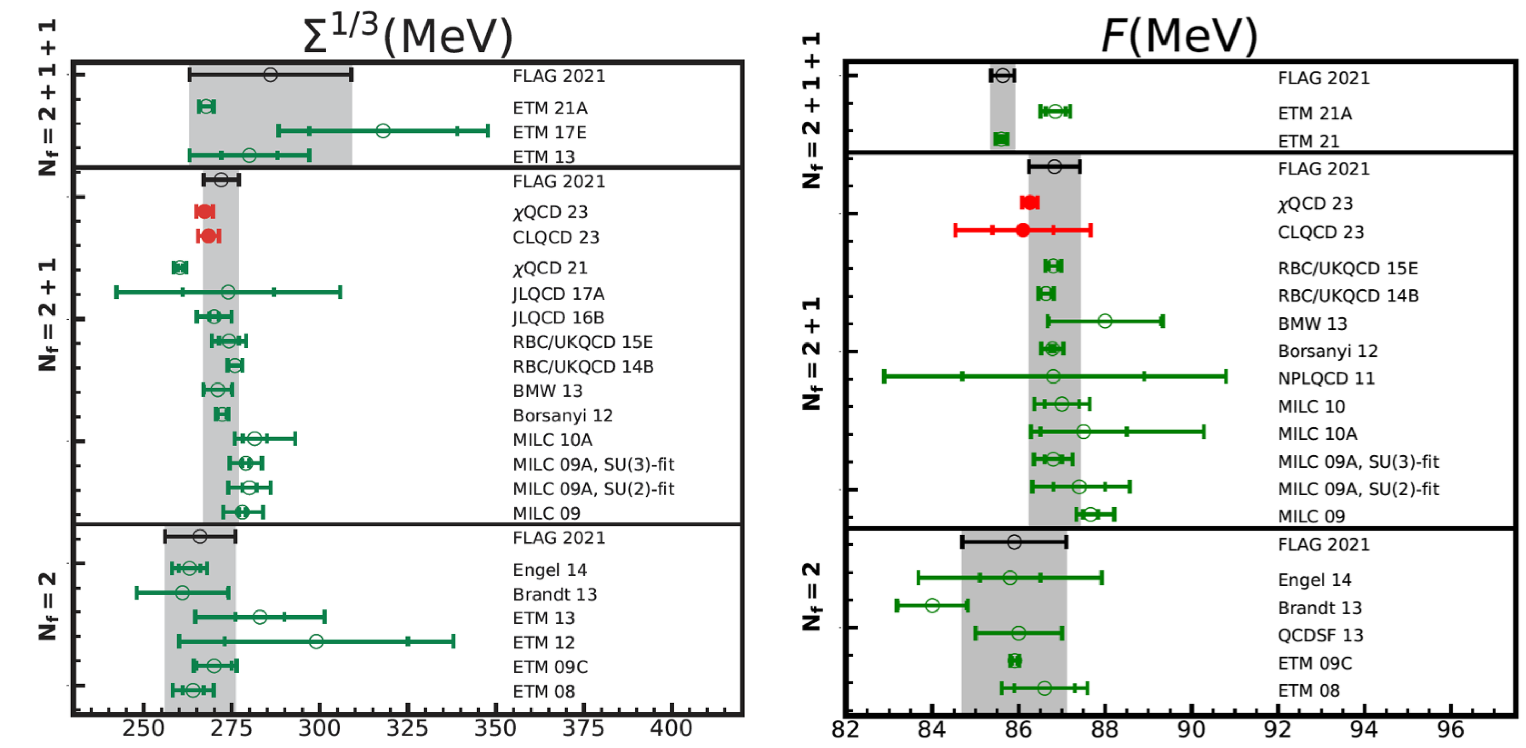
Note that since ensembles 24I and 48I have little difference in lattice spacing within the error, we use the same scalar current RCs for both ensembles. Similar treatment is applied to the two bunches of ensembles which are 32I and 64I.

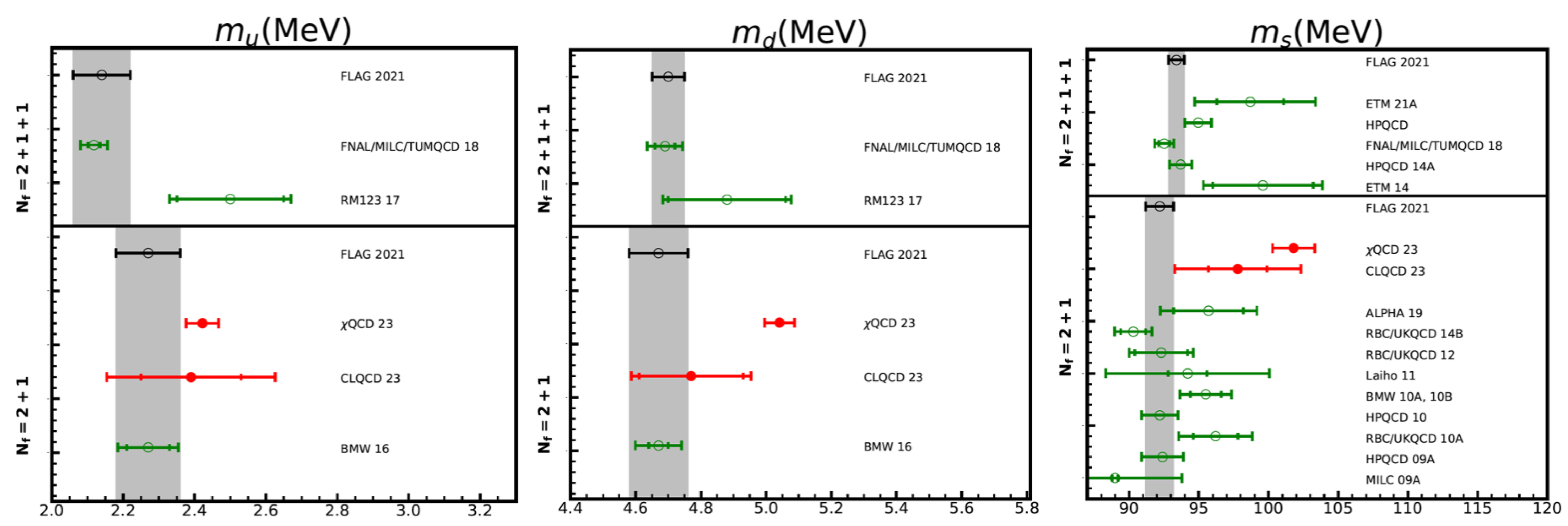
Result and Summary

We overlay the unitary measurements of M_π^2/m_l and $f_\pi = \sqrt{2}F_\pi$ on each lattice spacing with the ChPT prediction obtained using the LECs from each fit. The fit results have been used to correct each lattice measurement from the simulated point to the continuum and unitary light quark mass limit. Our global fit range is depicted between the two gray areas, while the error bands depict the results given by chiral limit extrapolation.



We also give the comparison results of LECs ($\Sigma^{1/3}$, F) and quark mass of three light flavors (u, d, s) with FLAG2021 [6].





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