

Generation of relative lattice energy spectrum using KK^* operators

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Introduction

LHCb collaboration recently report a doubly charmed tetraquark T_{cc}^+ in the $D^0 D^0 \pi^+$ mass spectrum, which is closed to threshold of $D^0 D^{*+}$ [1]. One explanation for T_{cc}^+ is a DD^* molecular state. The isospin breaking effect requires a resonance T_{cc}^+ between the $D^0 D^{*+}$ and $D^+ D^0$ mass thresholds [2]. The existence of T_{cc}^+ and T_{cc}^+ two states is directly related to the form of DD^* interaction. So we want to check the sign of the interaction of DD^* via the LQCD energy level. Here, due to limitations in data accuracy, we first construct the KK^* operator by dumbbell method [3], which is quite similar to the DD^* operator, and we obtain different energy levels of different irreducible representations. And this work will be helpful for our latter research of DD^* state.

FORMALISM

Base state for KK^*

$$|\Phi_{\mu R}(\mathbf{x})\rangle = |\Phi_{\mu}(R^{-1}\mathbf{x})\rangle$$

Representation matrix of the base state

$$\hat{P}_R |\Phi_{\mu R'}(\mathbf{x})\rangle = \sum_{\mu' R''} |\Phi_{\mu' R''}(\mathbf{x})\rangle \delta_{RR', R''} \Lambda_{\mu' \mu}(R)$$

where $\delta_{RR', R''}$ refers to a regular representation, $\Lambda_{\mu' \mu}$ refers to an irreducible representation T_1^+ of O_h group. So the total representation is the direct product of T_1^+ and the regular representation

Projection to the irreducible representation

$$|\Phi_{\mu R}(\mathbf{x})\rangle \Rightarrow |\Phi_{\mu i \Gamma_n}(\mathbf{x})\rangle \Rightarrow |\Psi_{i \Gamma_n}(\mathbf{x})\rangle$$

Here $\Phi_{\mu i \Gamma_n}$ refers to the direct product of $\Lambda_{\mu' \mu}$ and an irreducible representation of the regular representation, $\Psi_{i \Gamma_n}$ refers to an irreducible representation of the direct product of $\Lambda_{\mu' \mu}$ and the regular representation.

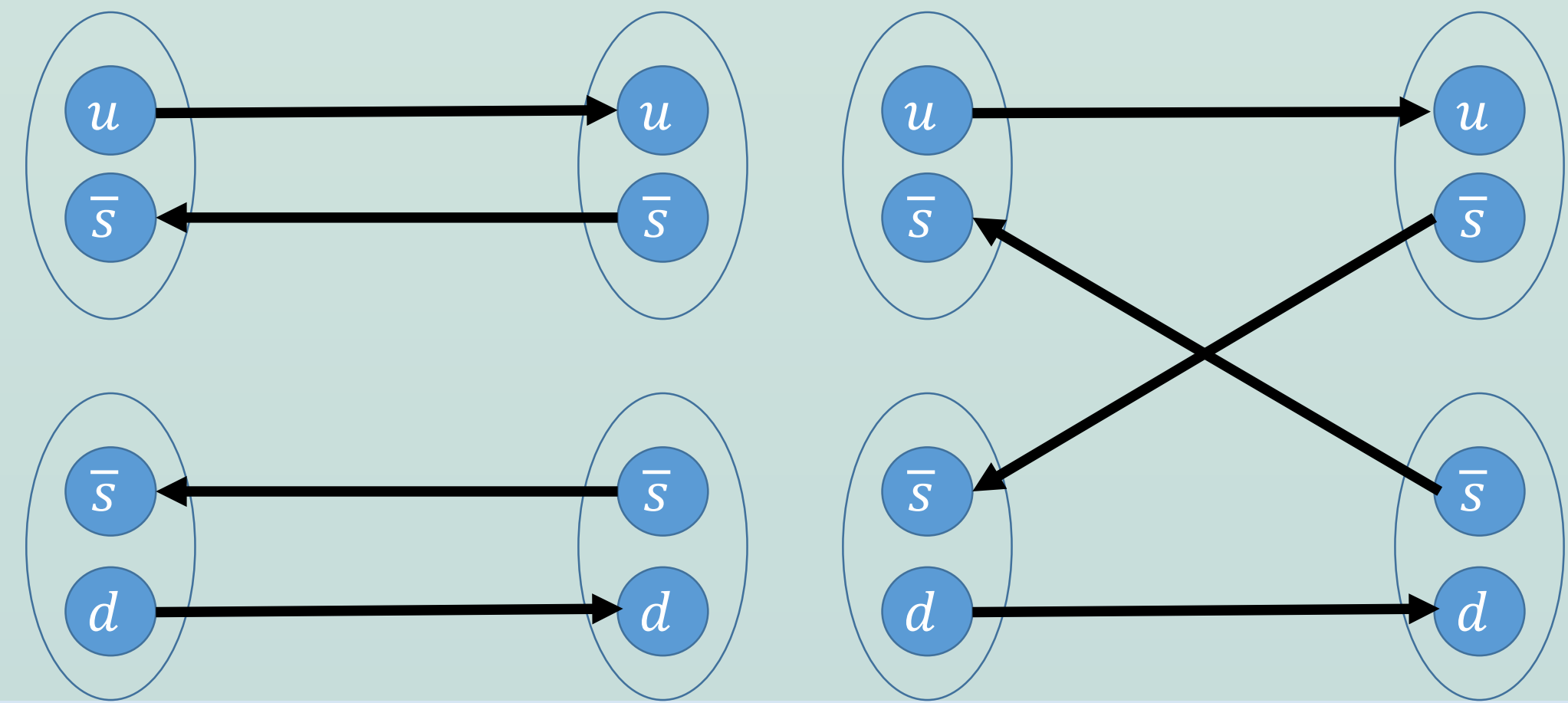
Correlation function of $\Phi_{\mu i \Gamma_n}$ and $\Psi_{i \Gamma_n}$

$$\sum_i \langle \Phi_{\mu i \Gamma_n}(\mathbf{y}) | \hat{O} | \Phi_{\mu' i \Gamma_n'}(\mathbf{x}) \rangle = \sum_{i R m l \tau \lambda} \bar{\Gamma}_{i, m}^*(\tilde{R}) \langle \Phi_{\tau \tilde{R}} | \hat{O} | \Phi_{\lambda} \rangle \sum_{R'} \Lambda_{\mu \tau}(R') \Lambda_{\mu' \lambda}(R') \bar{\Gamma}_{n, l}^*(R') \bar{\Gamma}_{n', m}(R')$$

Here, we project the $\Phi_{\mu R}$ to the $\Phi_{\mu i \Gamma_n}$, then by using the transition matrix we can obtain the correlation function of $\Psi_{i \Gamma_n}$.

$$\langle \Psi_{i' \Gamma_{n'}}(\mathbf{y}) | \hat{O} | \Psi_{i \Gamma_n}(\mathbf{x}) \rangle = \sum_{\mu' n'} \sum_{\mu i \Gamma_n} S_{\mu \Gamma_n', i' \Gamma_{n'}} \langle \Phi_{\mu' i' \Gamma_{n'}}(\mathbf{y}) | \hat{O} | \Phi_{\mu i \Gamma_n}(\mathbf{x}) \rangle S_{\mu \Gamma_n, i \Gamma_n}$$

Wick contraction

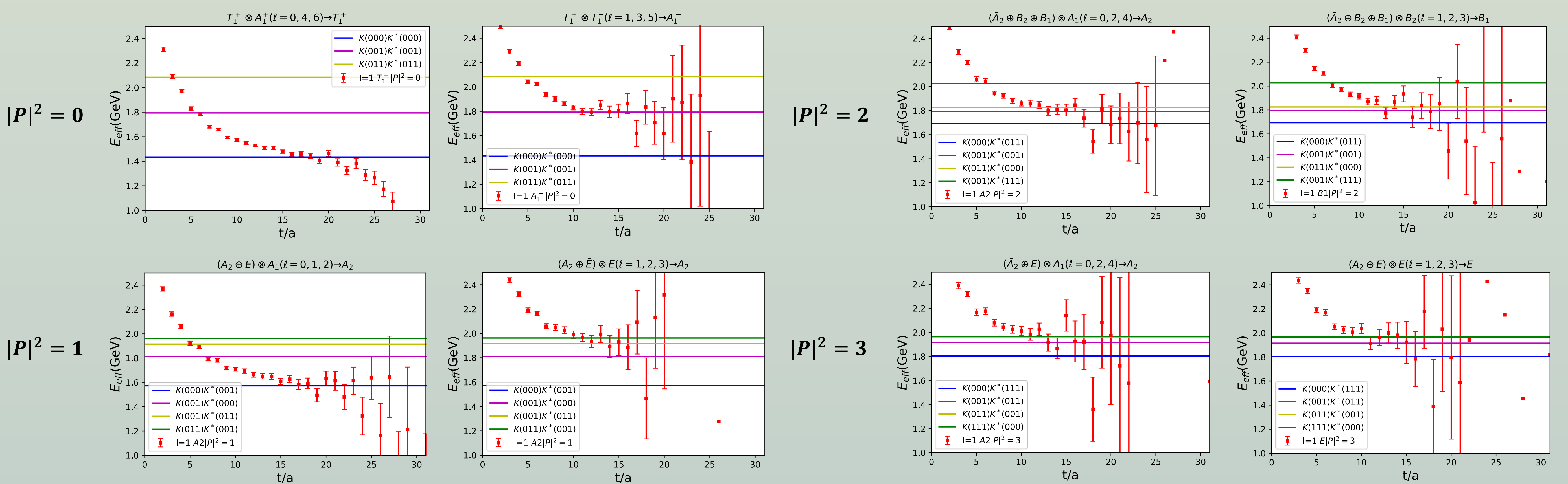


Main Result

Lattice setup

$L^3 \times T$	$a(\text{fm})$	$\kappa(u, d)$	$\kappa(s)$	$N_{c f g}$	$m_{\pi}(\text{MeV})$	$m_K(\text{MeV})$	$m_{K^*}(\text{MeV})$	$N_{t_{s r c}}$
$32^3 \times 64$	0.074	0.12104	0.12062	700	362.1(0.5)	506.6(1.6)	928(11)	16

Spectra



Summary

- The energy levels of distinct irreducible representation are separated clearly.
- As expected, the results belonging to the irreducible representation A_1^+ of the regular representation are generally cleaner statistically, nevertheless the signal quality degrades for the irreducible representation corresponding to the resolution of higher-spin partial waves.

Reference: [1] R. Aaij et al. (LHCb), Nature Phys. 18, 751 (2022), arXiv:2109.01038 [hep-ex].

[2] Chen, Rui and Huang, Qi and Liu, Xiang and Zhu, Shi-Lin, Phys. Rev. D 104, 114042 [arXiv:2108.01911 [hep-ph]]

[3] Wu, Jia-Jun and Kamleh, Waseem and Leinweber, Derek B. and Li, Yan and Schierholz, Gerrit and Young, Ross D. and Zanotti, James M, Phys. Rev. D 105.074506 [arXiv:2109.01557 [hep-lat]]