

# $Z_c(3900)$ RELEVANT $J/\psi\pi - D\bar{D}^*$ COUPLED CHANNEL EFFECTS

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## Abstract

- Since its discovery in 2013, the nature of  $Z_c(3900)^\pm$  has been studied by phenomenological [6, 4, 2, 8] and lattice QCD studies [3, 7].
- We employ the finite volume scattering formalism to extract the scattering amplitudes from the energy levels of interacting two-meson systems from  $N_f = 2$  anisotropic lattice QCD at  $m_\pi \approx 350$  MeV.
- This study explores the **coupled-channel** effects of  $J/\psi\pi$  and  $D\bar{D}^*$ , with preliminary results presented by extracting phase-shifts and scattering amplitudes on two lattice volumes.

## Feature of this study

- Two Volume
- Multi irreps (moving frame)
- Energy level stability
- Realistic Coupled-Channel analysis
- Best Fitting  $\chi^2$
- Partial-wave mixing effects:  $L_{max} = 0$

## Lattice setup

distillation method + anisotropic lattice

ens.	size	$m_\pi$ (MeV)	LapH	$N_V$	$N_{cfg}$
L16	$16^3 \times 128$	348.5(1.0)	70	6991	
L24	$24^3 \times 192$	345.6(0.8)	160	489	

$m_D$ (GeV)	$m_{D^*}$ (GeV)	$m_{J/\psi}$ (GeV)	$m_\pi$ (GeV)
1.8819(5)	2.0216(9)	3.0991(1)	0.3471(4)

## Wave function

$$O_{DD^*}^{(l=1)}(\vec{p}) = \sum_{\vec{p}_1, \vec{p}_2} \frac{1}{\sqrt{2}} (O_{D^*} O_{D^0} - O_{D^0} O_{D^*})$$

$$O_{J/\psi\pi}^{(l=1)} = \sum_{\vec{p}_1, \vec{p}_2} O_{J/\psi} O_\pi$$

## Lattice irreps.

$$T_1^+(d = (0, 0, 0)) : O = P(\vec{p})V_z(-\vec{p}),$$

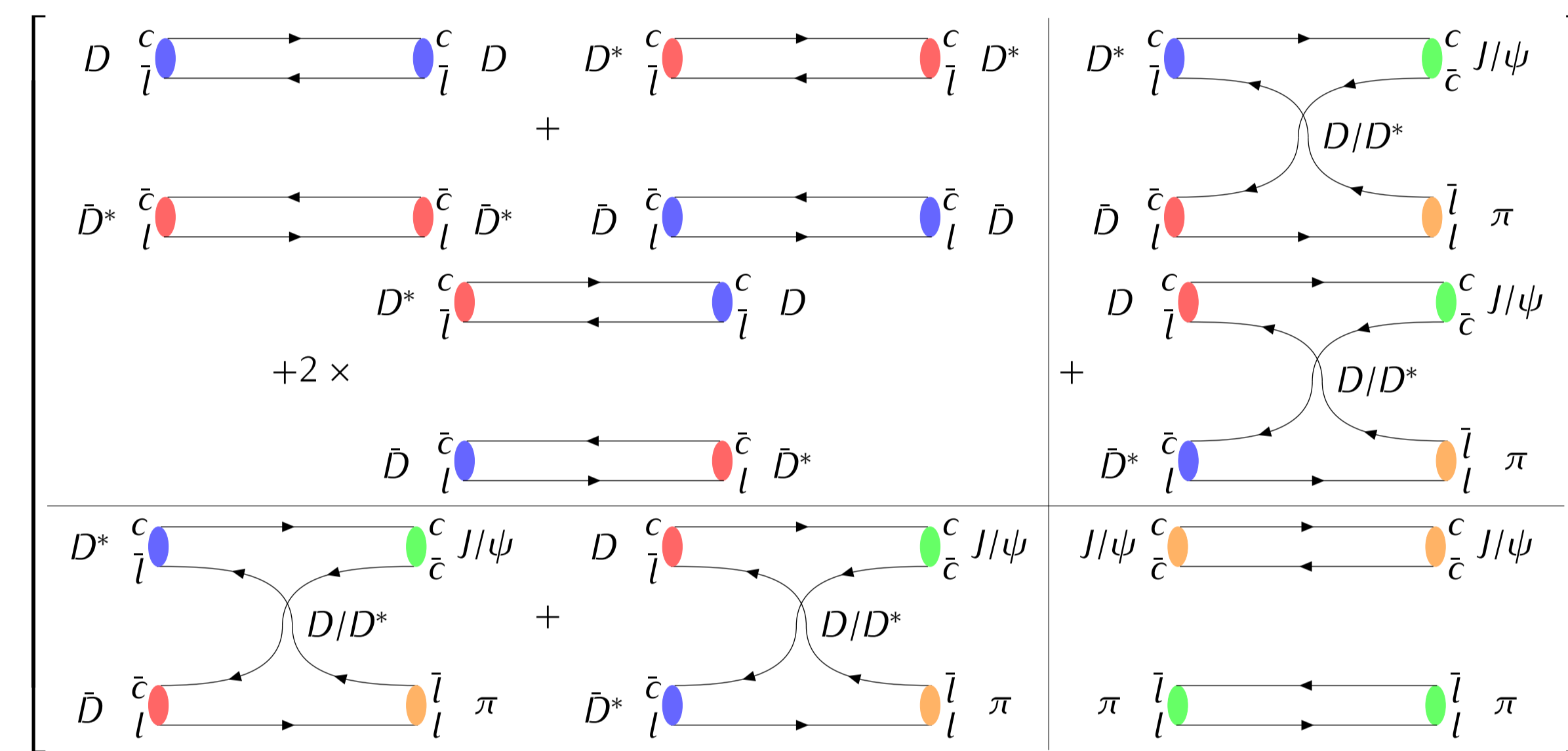
$$A_2^-(d = (0, 0, 1)) : O = P(\vec{0})V_z(\vec{p}), P(\vec{p})V_z(0),$$

$$A_2^-(d = (1, 1, 0)) : O = P(\vec{0})[V_x(\vec{p}) + V_y(\vec{p})].$$

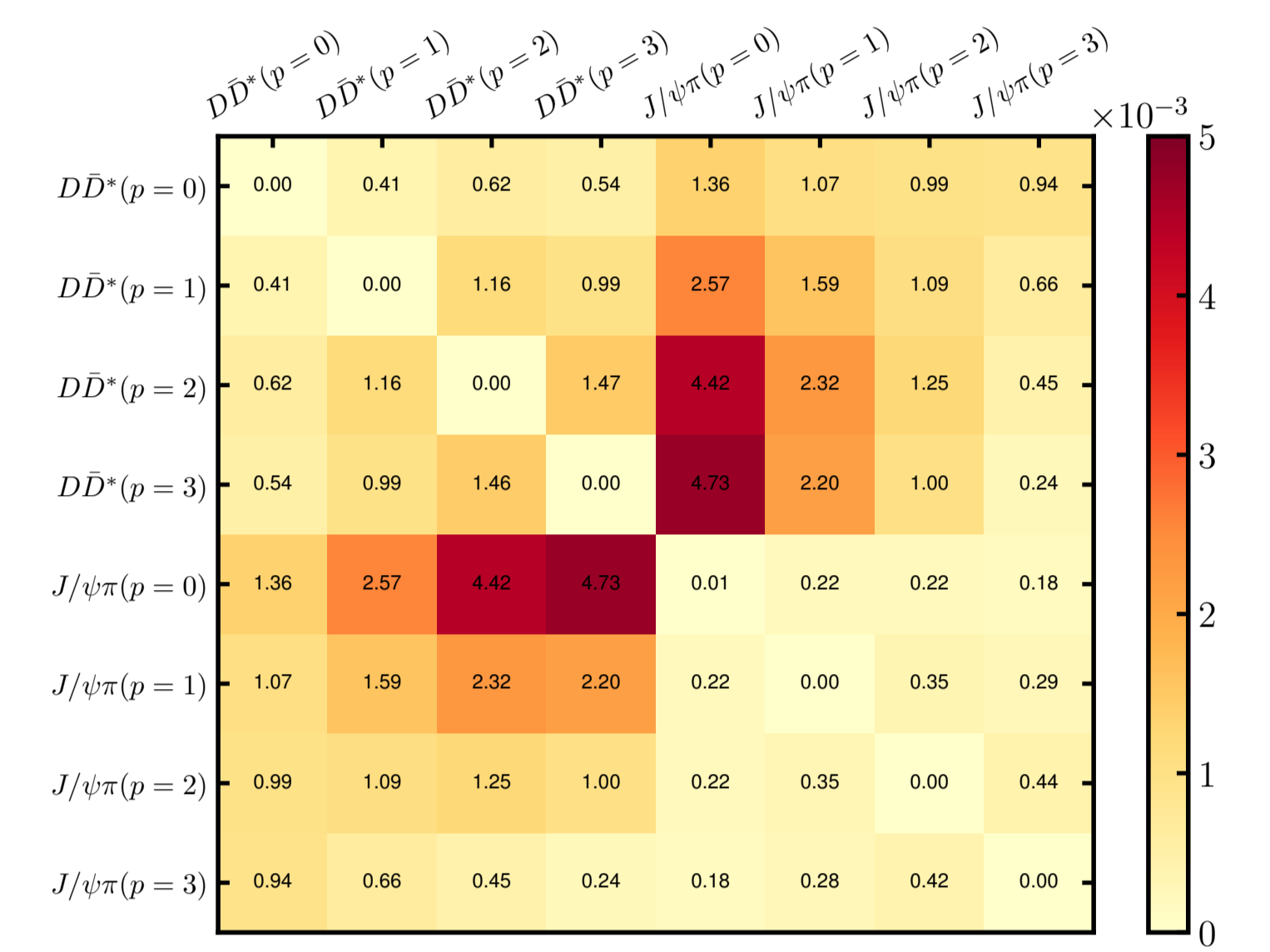
## Schematic diagrams and off-diagonal coupling

- We calculate the two-point correlation function matrix between  $S$ -wave  $J/\psi\pi$  and  $D\bar{D}^*$  operators, where the related momentum mode remains the specific lattice irrep. Then we solve the GEVP for the exact energy level. The off-diagonal correlation function also can be extracted in the following form.

$$C_{mn}(t) = \sum_{\tau} \langle 0 | O_m(t + \tau) O_n^\dagger(\tau) | 0 \rangle = \begin{bmatrix} C_{DD^* - DD^*} & C_{J/\psi\pi - DD^*} \\ C_{DD^* - J/\psi\pi} & C_{J/\psi\pi - J/\psi\pi} \end{bmatrix}$$



solving GEVP :  $C(t)v(t) = \lambda_n(t)C(t_0)v(t)$



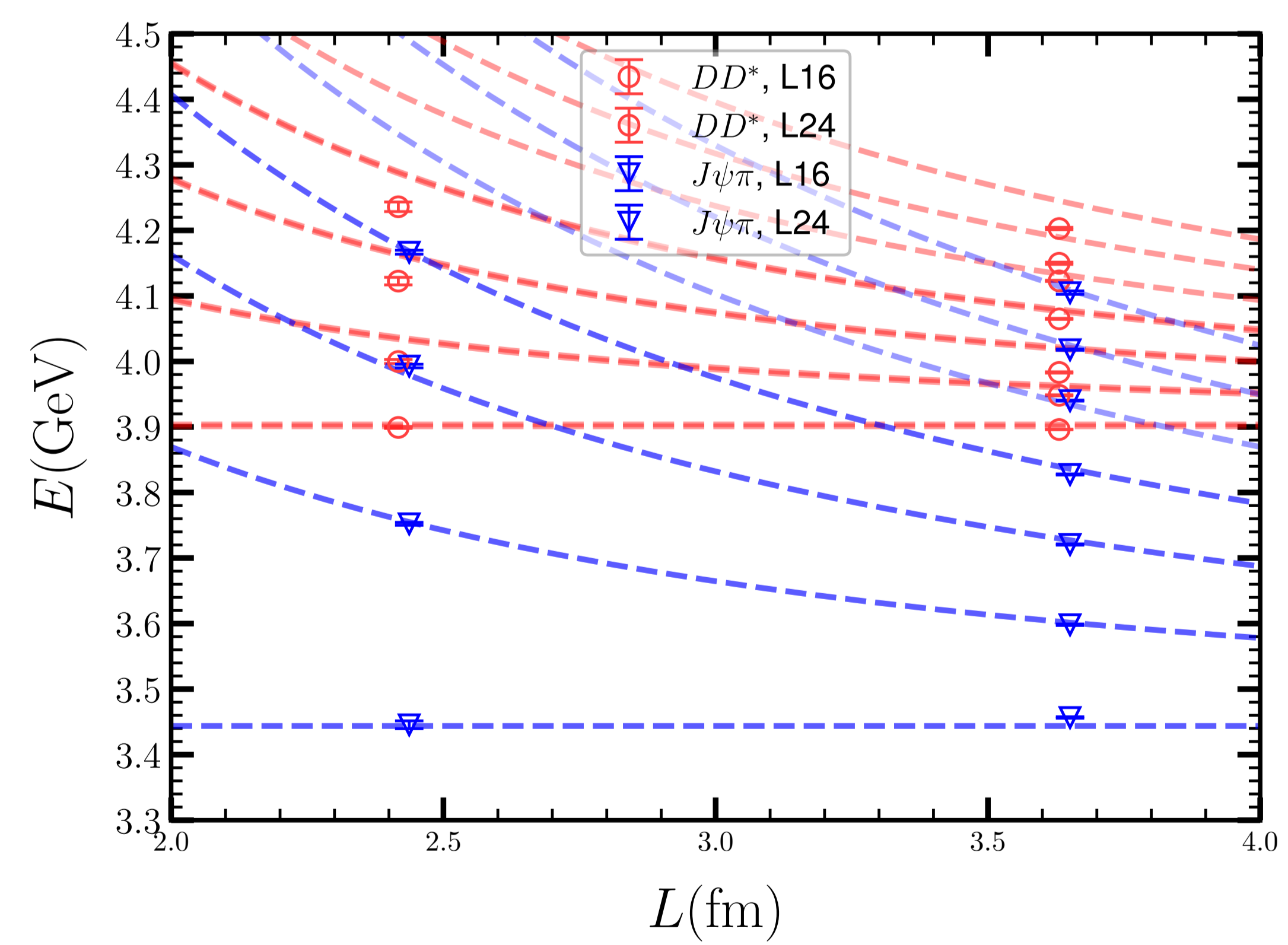
$$X_{mn} \equiv \langle m | \hat{H} | n \rangle \approx V_{mn}$$

$$\text{from } \frac{C_{DD^* \rightarrow J/\psi\pi}}{\sqrt{C_{DD^*} \times C_{J/\psi\pi}}} \sim xt(1 + \frac{1}{24}(\Delta Et)^2)$$

- The off-diagonal coupling  $J/\psi\pi - D\bar{D}^*$  is important and strongly **momentum dependent**.
- Therefore, a constant contact term interaction is not enough, which favors to the results of study [1?].

## Finite Volume Formalism

- After solving GEVP procedure, from two volumes, we use the precise energy level to implement the coupled-channel Lüscher formulae (or the Lüscher-Lellouch quantization condition), which involves the determinant of a  $2 \times 2$  matrix that includes the inverse of the transition matrix  $t(s)$  and other factors.



$$S = \begin{bmatrix} \eta e^{2i\delta_1} & i\sqrt{1 - \eta^2} e^{i(\delta_1 + \delta_2)} \\ i\sqrt{1 - \eta^2} e^{i(\delta_1 + \delta_2)} & \eta e^{2i\delta_2} \end{bmatrix}$$

$$S_{ij} = I + 2i\rho_{ij}^{\frac{1}{2}} \cdot t \cdot \rho_{ij}^{\frac{1}{2}}$$

## Lüscher-Lellouch formulism

$$\det [t_{ij}^{-1}(s) + i\rho_{ij} - \rho_{ij} M^{\Delta\Lambda}(k)] = 0.$$

or

$$\det [\mathcal{F}^{d,-1}(k) - t(s)] = 0.$$

- In this study, We try two methods to parameterize the transition matrix  $t(s)$ , which reminds the unitarity of the scattering matrix  $S$ . The first method involves the inverse of another matrix  $K(s)$ . The second method involves the inverse of the matrix  $K(s)$  with a different parameter.

## K-Matrix Parameterization

$$t^{-1}(s) = K^{-1}(s) + \begin{bmatrix} I_1 & 0 \\ 0 & I_2 \end{bmatrix} (s),$$

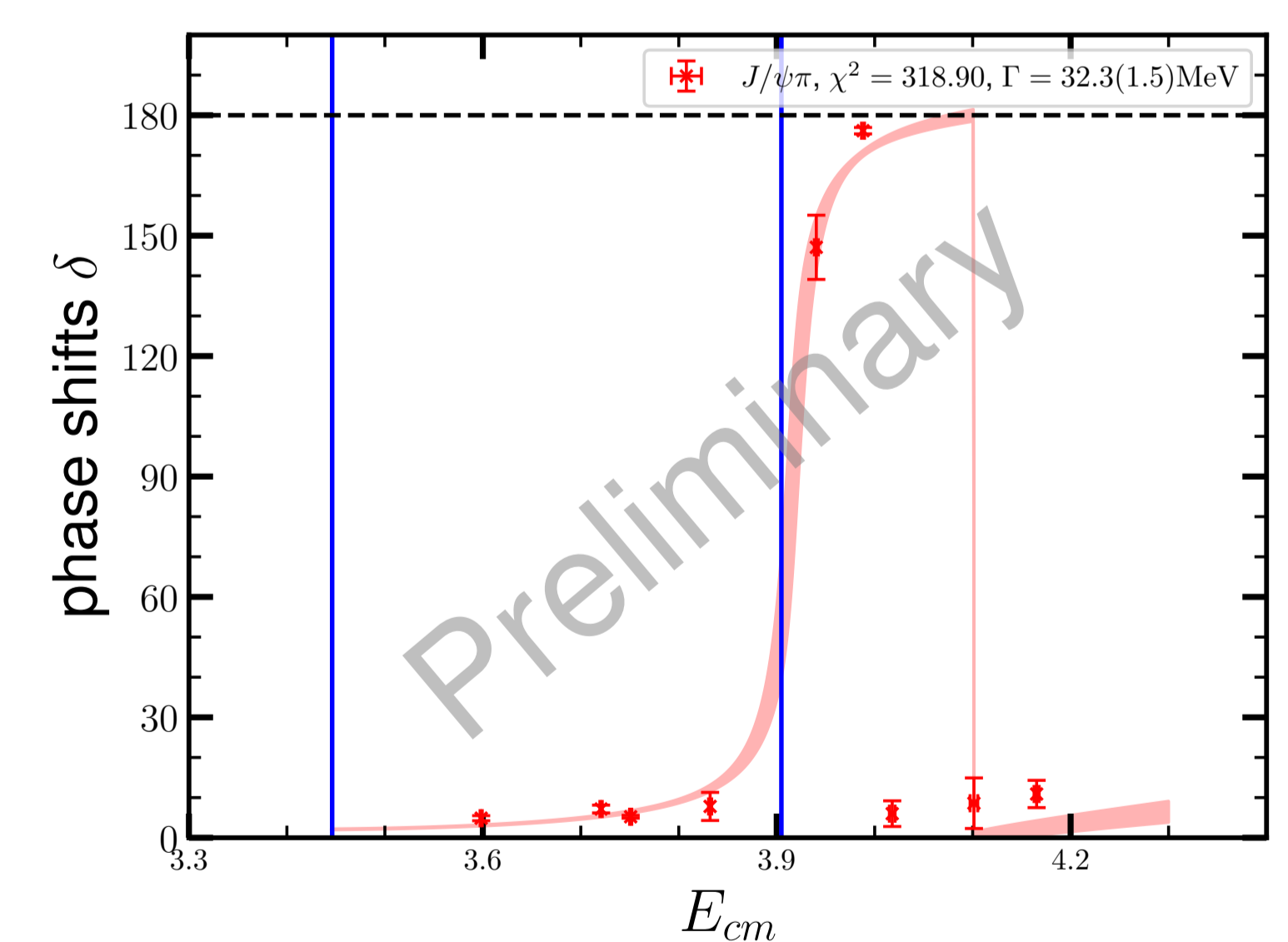
$$K(s) = \frac{1}{m^2 - s} \begin{bmatrix} g_1^2 & g_1 g_2 \\ g_1 g_2 & g_2^2 \end{bmatrix} + \begin{bmatrix} \gamma_{11}^2 & \gamma_{12} \\ \gamma_{12} & \gamma_{22}^2 \end{bmatrix}.$$

## Single-Channel analysis

- Within  $J/\psi\pi$  channel, though the single channel Lüscher formulae, the phase-shifts  $\delta_0$  are extracted from the scattering momentum  $q$ .

## Single channel Lüscher formulae

$$p \cot \delta(q) = \frac{2}{\gamma L \pi^2} Z_{00}^d(1, q^2).$$



- The possible phase-shift jump give a hint for a **resonance** near the  $D\bar{D}^*$  threshold.
- However, more reliable results should be give by **coupled-channel** analysis.

## Outlook

- We performed a preliminary single-channel analysis of the scattering amplitudes, yielding some insights into the resonance state.
- Currently, the fitting of the coupled-channel analysis is still **in progress**.

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