$\pi\pi$ scattering in P-wave and the ρ resonance from lattice QCD

Zheng-Li Wang [1]

[1]University of Chinese Academy of Sciences

Abstract

We present a lattice QCD investigation of the ρ-meson with $N_f = 2+1$ dynamical quark flavours. The calculation is performed based on Wilson-Clover configuration ensembles produced by the CLQCD Collaboration with two lattice spacing values and pion masses ranging from 220 to 300 MeV. Applying the Lüscher method phase-shift curves are determined for all ensembles separately. Assuming a Breit–Wigner form, the ρ-meson mass and width are determined by a fit to these phase-shift curves.

Introduction

The ρ-meson represents together with the famous σ- meson one of the most prominent meson resonances in the standard model. The ρ decays almost exclusively to two pions and the experimental phase-shift curve is a textbook example for a relativistic Breit–Wigner form. Moreover ρ plays a fundamental role in many processes within the context of vector meson dominance. Therefore, an investigation of the ρ-meson properties from first principles with lattice QCD is highly desirable.

However, unstable particles require special care in lattice QCD: interaction properties can only be computed using the by now famous Lüscher method. There are two major complications. The first one concerns the hadronic lattice interpolators used. The second challenge concerns the energy levels. One should extend the set of hadron interpolators to include both various versions of the quark- antiquark interpolator and meson-meson interpolators. The so-called distillation [or Laplacian-Heaviside (LapH) quark smearing] method helps us significantly to deal with that problems.

Configurations

The gauge ensembles used in this study:

configuration	volume	spacing	β	m_π/MeV	$m_\pi L$
C32P29	$32^3 \times 64$	$0.1053fm$	6.20	292	5.01
C32P23	$32^3 \times 64$	$0.1053fm$	6.20	228	3.91
F32P30	$32^3 \times 96$	$0.0775fm$	6.41	303	3.81
F48P30	$48^3 \times 96$	$0.0775fm$	6.41	303	5.72

Interpolators

The states with good helicity in the continuum scattering have been thoroughly discussed by Jacob and Wick. They have been considered for one-hadron states on the lattice by the Hadron-Spectrum collaboration. For ρ:

$$\mathbb{O}^{J,\lambda}(\vec{p}) = \sum_M \mathcal{D}^{(J)*}_{M\lambda}(R)\mathcal{O}^{J,M}(\vec{p})$$

$$\mathbb{O}^{[J,P,|\lambda|]}_{\Lambda,\mu}(\vec{p}) = \sum_{\hat{\lambda}=\pm|\lambda|} \mathcal{S}^{\tilde{\eta},\hat{\lambda}}_{\Lambda,\mu}\mathbb{O}^{J,P,\hat{\lambda}}(\vec{p})$$

For $\pi\pi$, we use the projection formula with the induced representation to construct the Clebsch-Gordan coefficients for $\Lambda_1 \otimes \Lambda_2 \to \Lambda$

$$(\pi\pi)^{\left[\vec{k}_1,\vec{k}_2\right]\dagger}_{\vec{P},\Lambda,\mu} = \sum_{\substack{\vec{k}_1,\vec{k}_2 \\ \vec{k}_1+\vec{k}_2=\vec{P}}} \mathcal{C}\left(\vec{P},\Lambda,\mu;\vec{k}_1;\vec{k}_2\right)\pi^\dagger\left(\vec{k}_1\right)\pi^\dagger\left(\vec{k}_2\right)$$

Distillation

Laplacian-Heaviside (LapH) quark smearing: The technique applies a low-rank operator to define smooth fields that are to be used in hadron creation operators. The resulting space of smooth fields is small enough that all elements of the reduced quark propagator can be computed exactly at reasonable computational cost.

$$-\nabla^2_{xy}(t) = 6\delta_{xy} - \sum_{j=1}^{3}\left(\tilde{U}_j(x,t)\delta_{x+\hat{\jmath},y} + \tilde{U}^\dagger_j(x-\hat{\jmath},t)\delta_{x-\hat{\jmath},y}\right)$$

$$\square(t) = V(t)V^\dagger(t) \longrightarrow \square_{xy}(t) = \sum_{k=1}^{N} v^{(k)}_x(t)v^{(k)\dagger}_y(t)$$

$$\chi^\dagger_M(\vec{p},t) = \bar{u}_x(t)\square_{xy}(t)\cdot e^{-ip\cdot y}\Gamma^A_{yz}(t)\cdot\square_{zw}(t)d_w(t)$$

GEVP

Our spectrum results follow from the application of a variational method to matrices of correlators.

$$C(t)v_\alpha = \lambda_\alpha(t)C(t_0)v_\alpha$$

Principal correlators, $\lambda(t)$, from the variational analysis:

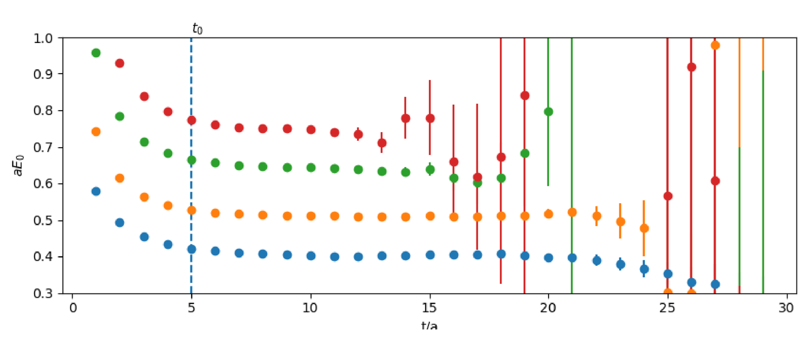


Figure 1. $P000, T_1$,C32P29

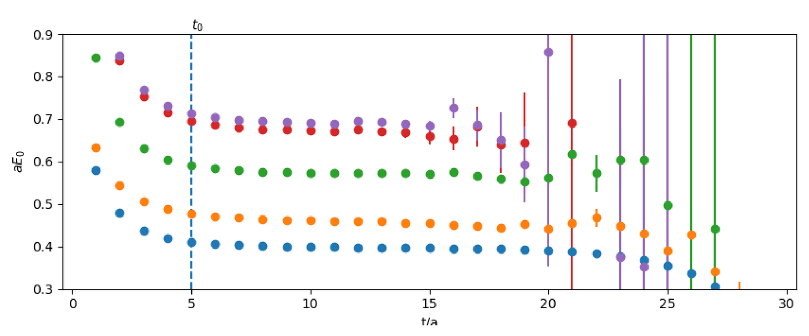


Figure 2. $P001, A_1$,C32P29

Finite volume spectra

Extracted center-of-momentum frame energy spectra for $P = [000]$, The curves represent noninteracting meson-meson energies

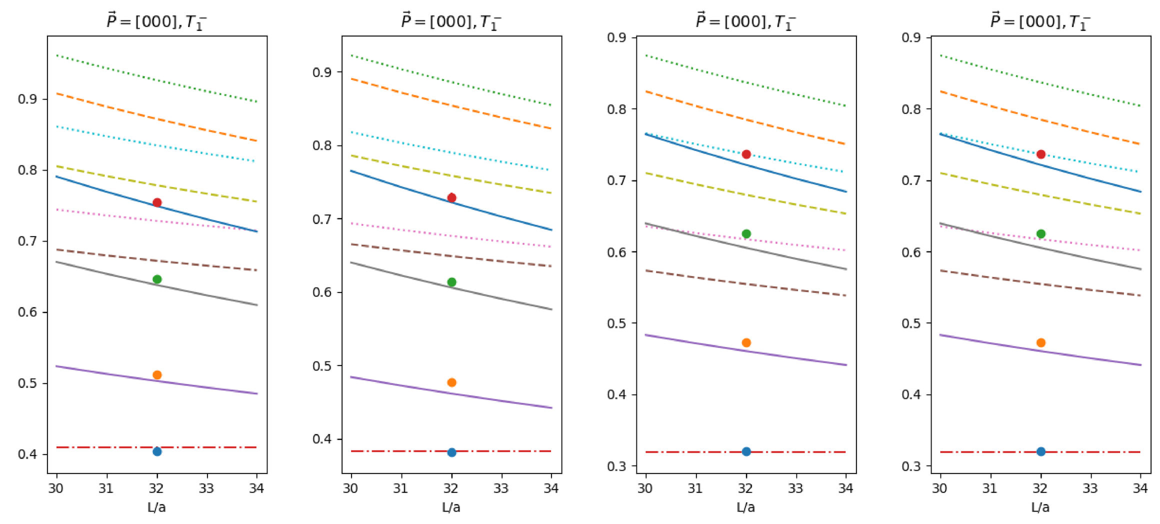


Figure 3. C32P29/C32P23/F32P30/F48P30

Scattering in finite volume

The extraction of scattering properties from lattice QCD in Euclidean space-time and a finite volume requires the application of the so-called Lüscher method.

$$\det\left[\mathbf{E}(p_{\rm cm}) - \mathbf{U}^{(\vec{P},\Lambda)}\left(\left(\frac{p_{\rm cm}L}{2\pi}\right)^2\right)\right] = 0$$

We presume we can truncate the matrices to a finite size since at low energy we expect $\delta_{\ell>3} \ll \delta_3 \ll \delta_1$ and we have

$$\delta_1 = \operatorname{arccot} M^\Gamma_{11,11}$$

with

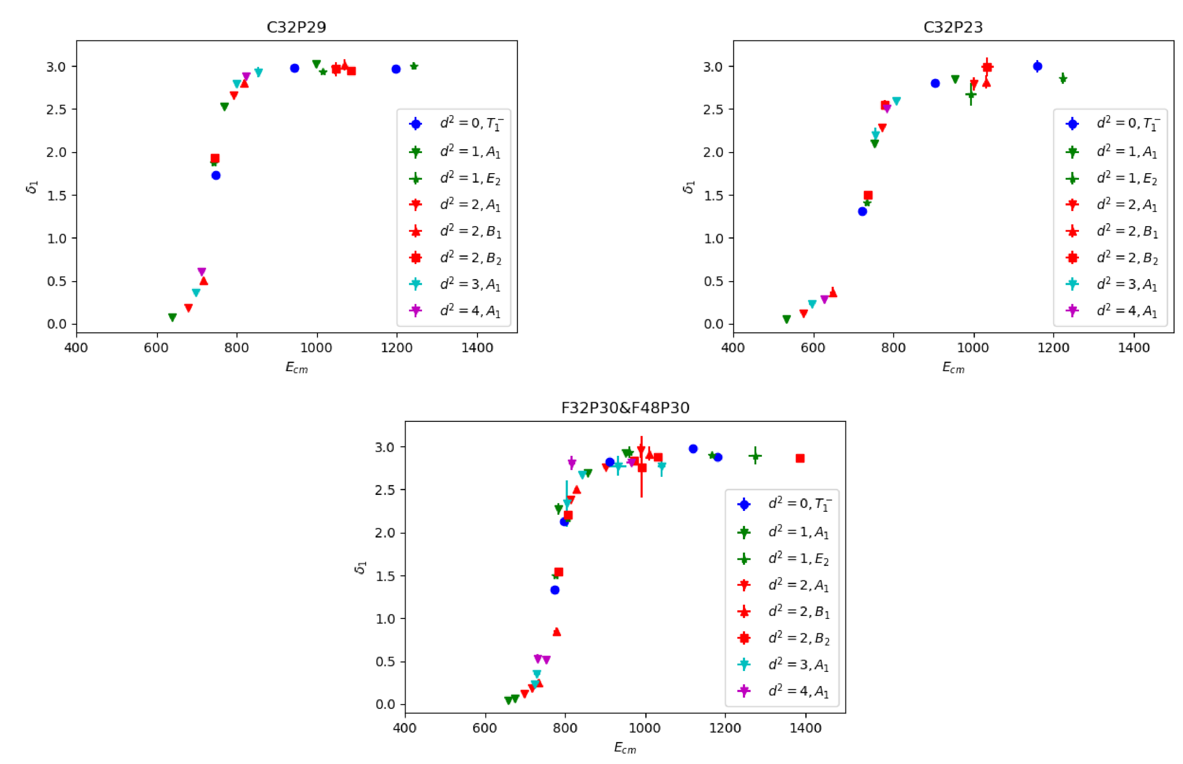
$\boldsymbol{d}^2$	Γ,μ	$M^\Gamma_{11,11}$
0	$T_1^-,2$	$w_{0,0}$
1	$A_1,1$	$w_{0,0}+2w_{2,0}$
1	$E_2,1$	$w_{0,0}-w_{2,0}$
1	$E_2,2$	$w_{0,0}-w_{2,0}$
2	$A_1,1$	$w_{0,0}+\frac{1}{2}w_{2,0}+i\sqrt{6}w_{2,1}-\frac{\sqrt{6}}{2}w_{2,2}$
2	$B_1,1$	$w_{0,0}+\frac{1}{2}w_{2,0}-i\sqrt{6}w_{2,1}-\frac{\sqrt{6}}{2}w_{2,2}$
2	$B_2,1$	$w_{0,0}-w_{2,0}+\sqrt{6}w_{2,2}$
3	$A_1,1$	$w_{0,0}-i2\sqrt{6}w_{2,2}$
3	$E_2,1$	$w_{0,0}+i\sqrt{6}w_{2,2}$
3	$E_2,2$	$w_{0,0}+i\sqrt{6}w_{2,2}$

and

$$w_{js} = \frac{\mathcal{Z}_{js}\left(1,q^2\right)}{\pi^{3/2}\sqrt{2j+1}\gamma q^{j+1}}, \quad q = \frac{kL}{2\pi}$$

Phase shifts and resonance parameters

Here we give the P-wave $\pi\pi$ elastic scattering phase shift, $\delta_1(E_{cm})$ under the assumption that $\delta_{\ell>1} = 0$



The resulting phase shift is related to the relativistic Breit-Wigner form for the elastic P-wave amplitude in the resonance region

$$a_1 = \frac{-\sqrt{s}\Gamma(s)}{s - m_\rho^2 + \mathrm{i}\sqrt{s}\Gamma(s)} = \mathrm{e}^{\mathrm{i}\delta(s)}\sin\delta(s)$$

Conclusions

Extracting scattering phase shifts and resonance properties is one of the most challenging problems in hadron spectroscopy based on lattice QCD. We combine several sophisticated tools to approach this problem: Lüscher's phase-shift relations for finite-volume lattices, moving frames, and variational analysis of correlation matrices, where a number of quark-antiquark and $\pi\pi$ interpolators with quantum numbers $I(J^{PC}) = 1(1^{--})$ are used. All needed contractions are evaluated using the distillation method with the Laplacian-Heaviside smearing of quarks. We find that these tools lead to precise values of the P-wave phase shift for $\pi\pi$ scattering at three values of pion relative momenta in the vicinity of the resonance. This allows a determination of the ρ resonance parameters m_ρ and Γ_ρ at our value of m_π.

Future work to explore the hadron resonance spectrum will need to consider inelastic scattering—we can reliably extract the finite-volume spectrum above inelastic thresh- olds by including the appropriate meson-meson-like op- erators (e.g., $K\bar{K}$ . . .), following the general methodology.