Lattice QCD study of the Exotic states

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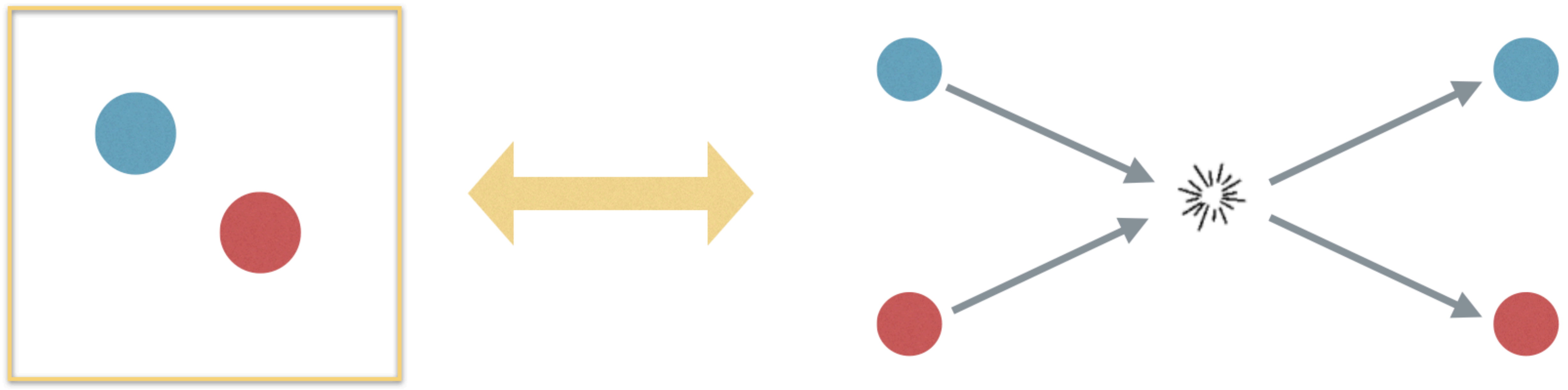
Introduction

- ✦ Since 2003, large amount of new hadronic states beyond the conventional quark model are discovered in experiments.
- ✦ Nearly all of the exotic candidates are close to the thresholds of two hadrons.
- ✦ Study of the hadron scattering is a main approach to probe the structure of the exotic hadrons.
  - Scattering on lattice.
  - Preliminary results on the hidden-charm pentaquarks.

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)



Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

Scattering on lattice

CLQCD

Finite volume spectrum:

- build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- Solve the generalized eigenvalue problem(GEVP): $C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$
- Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$
- Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

- Computational technique: distillation quark smearing.
  - Improve precision
  - Disconneted diagrams
  - Efficient for large numbers of ops

Lattice QCD configurations

- 2+1 flavor Wilson-clover configurations generated by CLQCD.

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of confs
~0.108fm	$24^3 \times 72$	290	1000
	$32^3 \times 64$	290	1000
	$32^3 \times 64$	220	450
	$48^3 \times 96$	220	200
	$48^3 \times 96$	140	200
~0.080fm	$32^3 \times 96$	300	480
	$48^3 \times 96$	300	200
	$32^3 \times 64$	220	460
	$48^3 \times 96$	220	200
~0.055fm	$48^3 \times 144$	300	200

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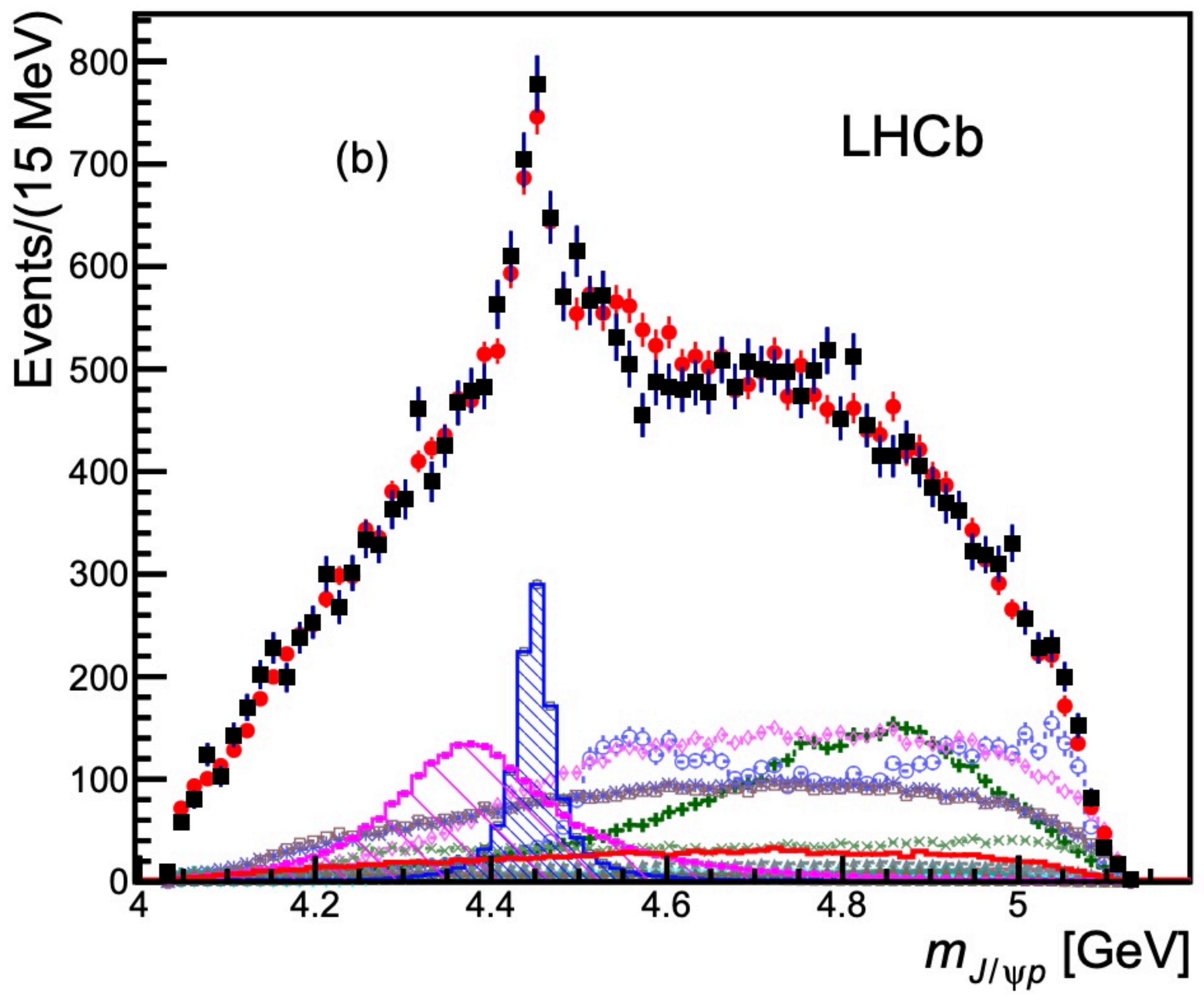
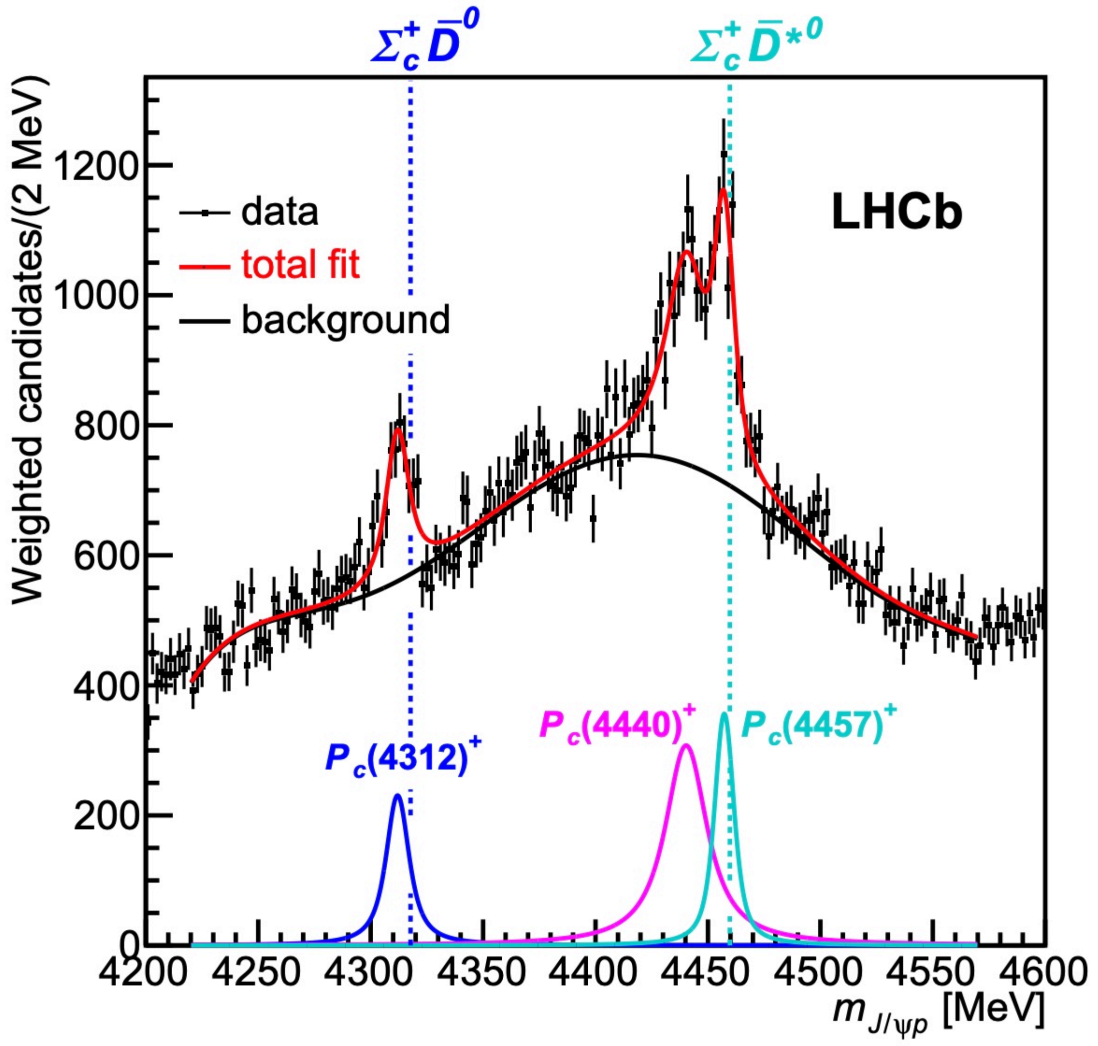
P_c Pentaquarks





$P_c(4380)$

$P_c(4450)$ → $P_c(4440)$, $P_c(4457)$

$P_c(4312)$

R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 072001 (2015)

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Theory interpretations:

- Molecule bound states
- Compact pentaquark states
- ……

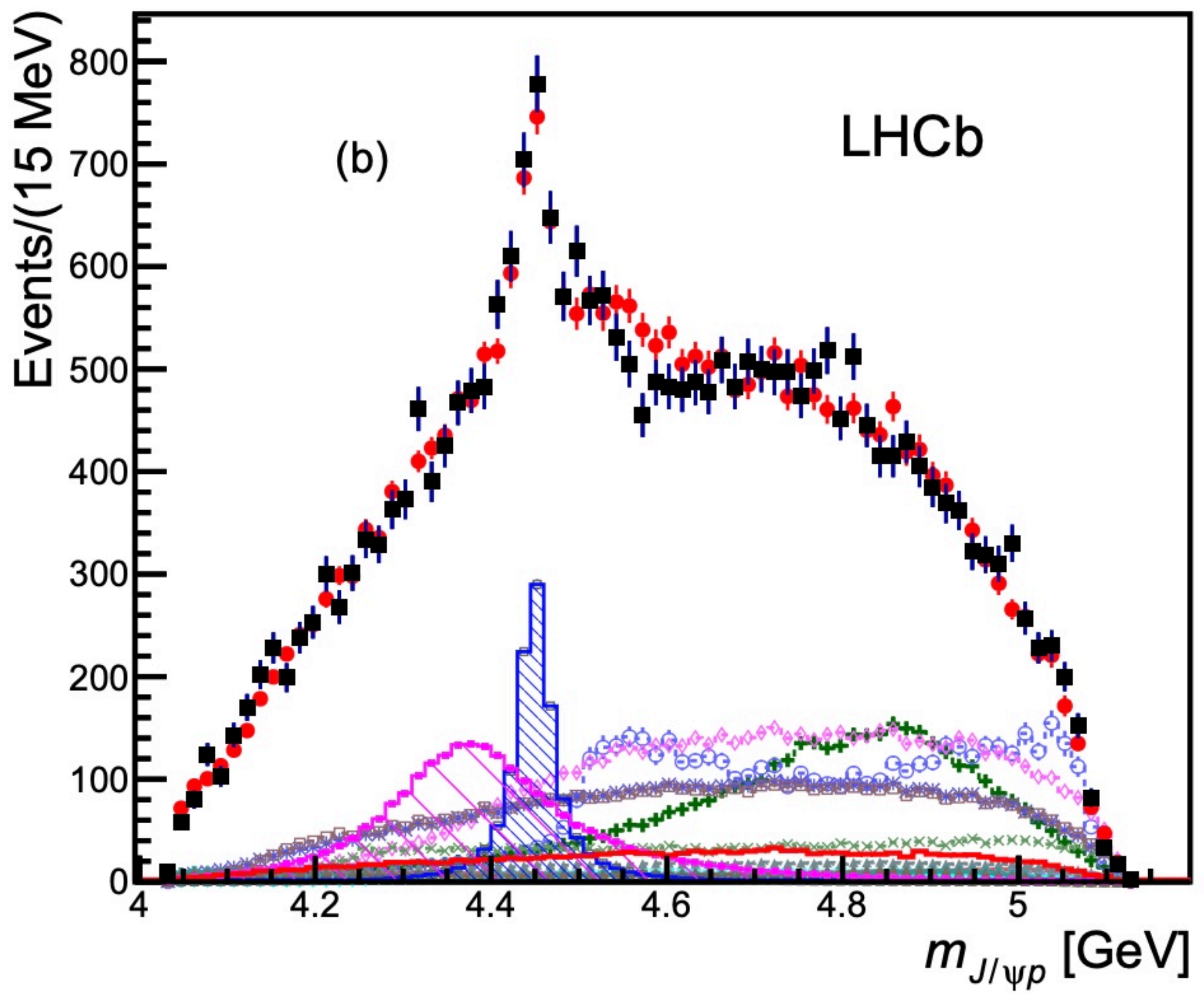
$\Sigma_c^{(*)}\bar{D}^{(*)}$ molecules:

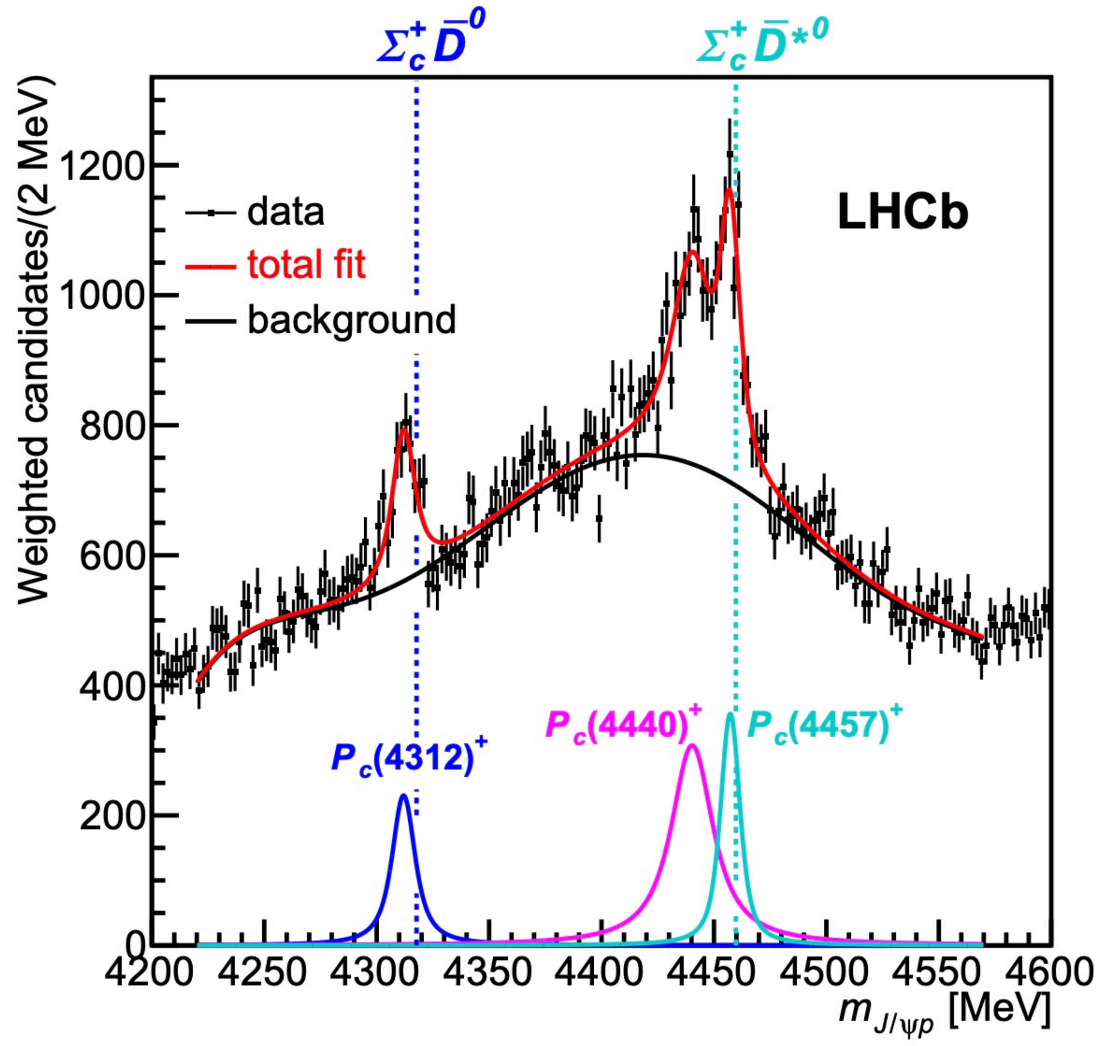
$$\Sigma_c\bar{D}, J^P = \frac{1}{2}^-, P_c(4312)$$

$$\Sigma_c\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-), P_c(4440)/P_c(4457)$$

$$\Sigma_c^*\bar{D}, J^P = \frac{3}{2}^-, \quad \Sigma_c^*\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-),$$

P_c Pentaquarks





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Results

CLQCD

$\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ scattering$(J^P = \frac{1}{2}^-)$:

✦ Five operators:

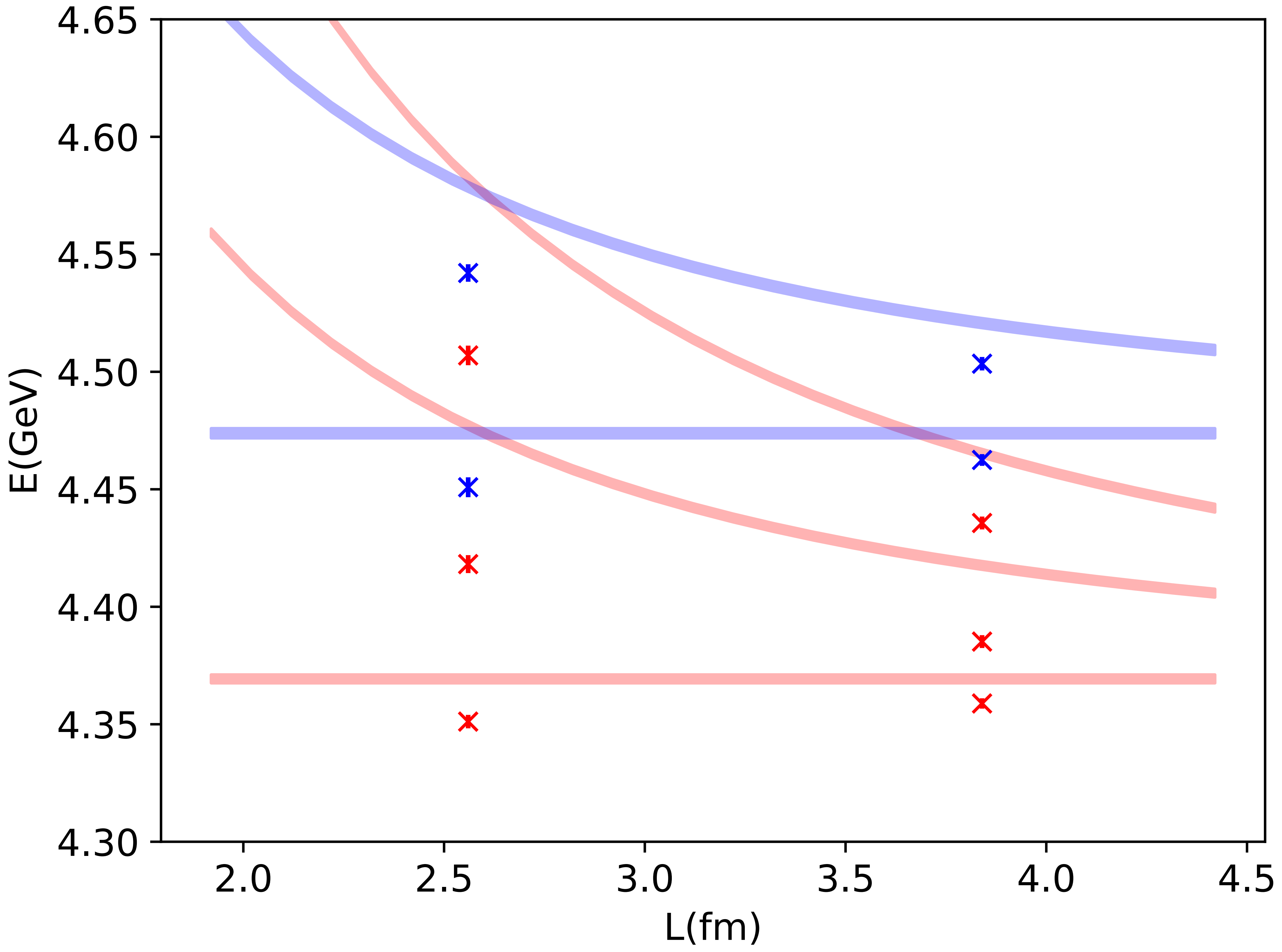
$\mathcal{O}_1 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = 0)$

$\mathcal{O}_2 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = 1)$

$\mathcal{O}_3 = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (|\mathbf{p}| = \sqrt{2})$

$\mathcal{O}_4 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (|\mathbf{p}| = 0)$

$\mathcal{O}_5 = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (|\mathbf{p}| = 1)$



✦ The finite-volume energies lie below the free energies, indicating rather strong attractive interactions.

Results

Scattering amplitude:

$$T \sim \frac{1}{p\cot\delta - ip}$$

Bound state pole:

$$p = i|p_B|$$

Effective range expansion:

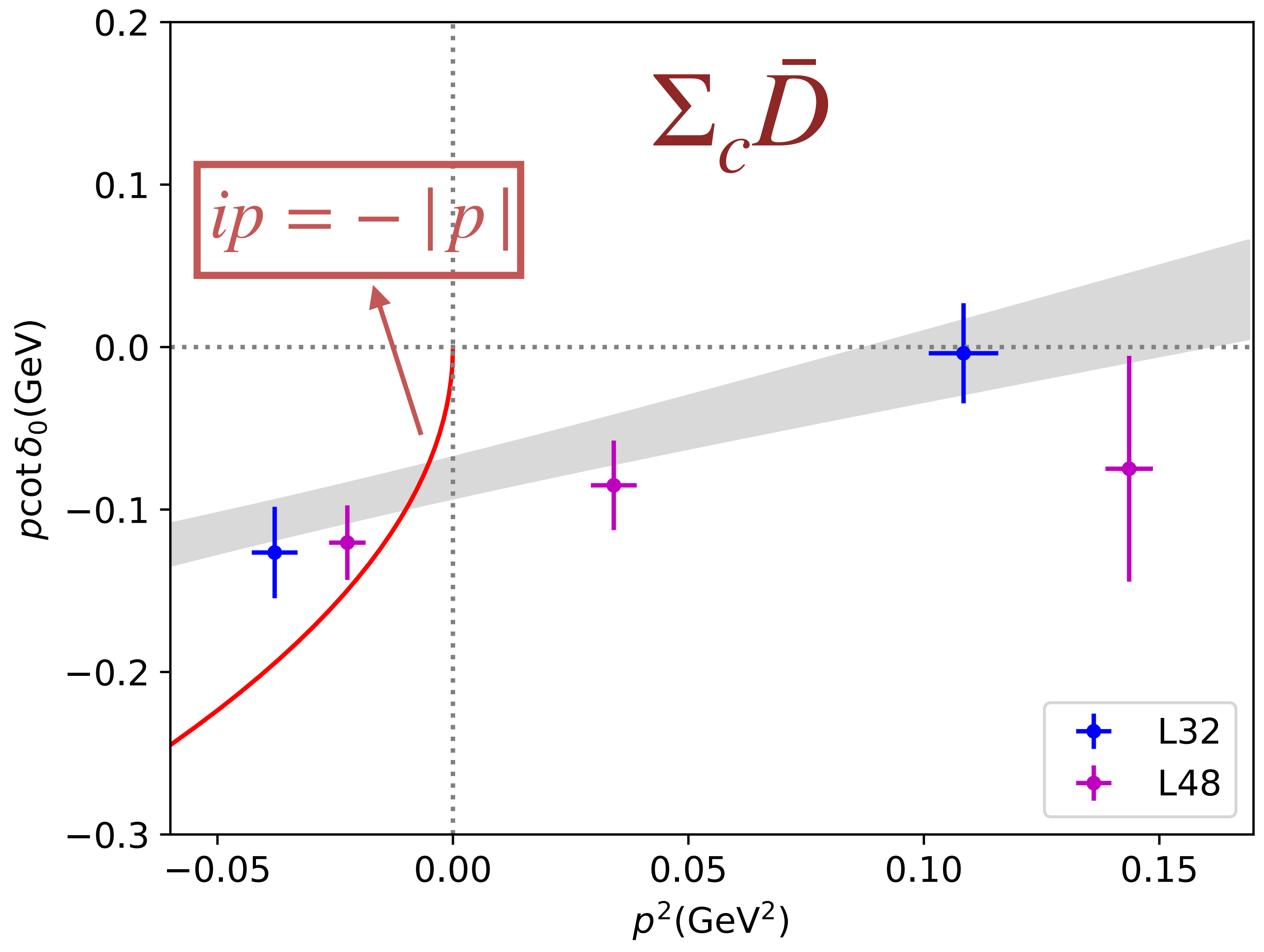
$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \cdots$$

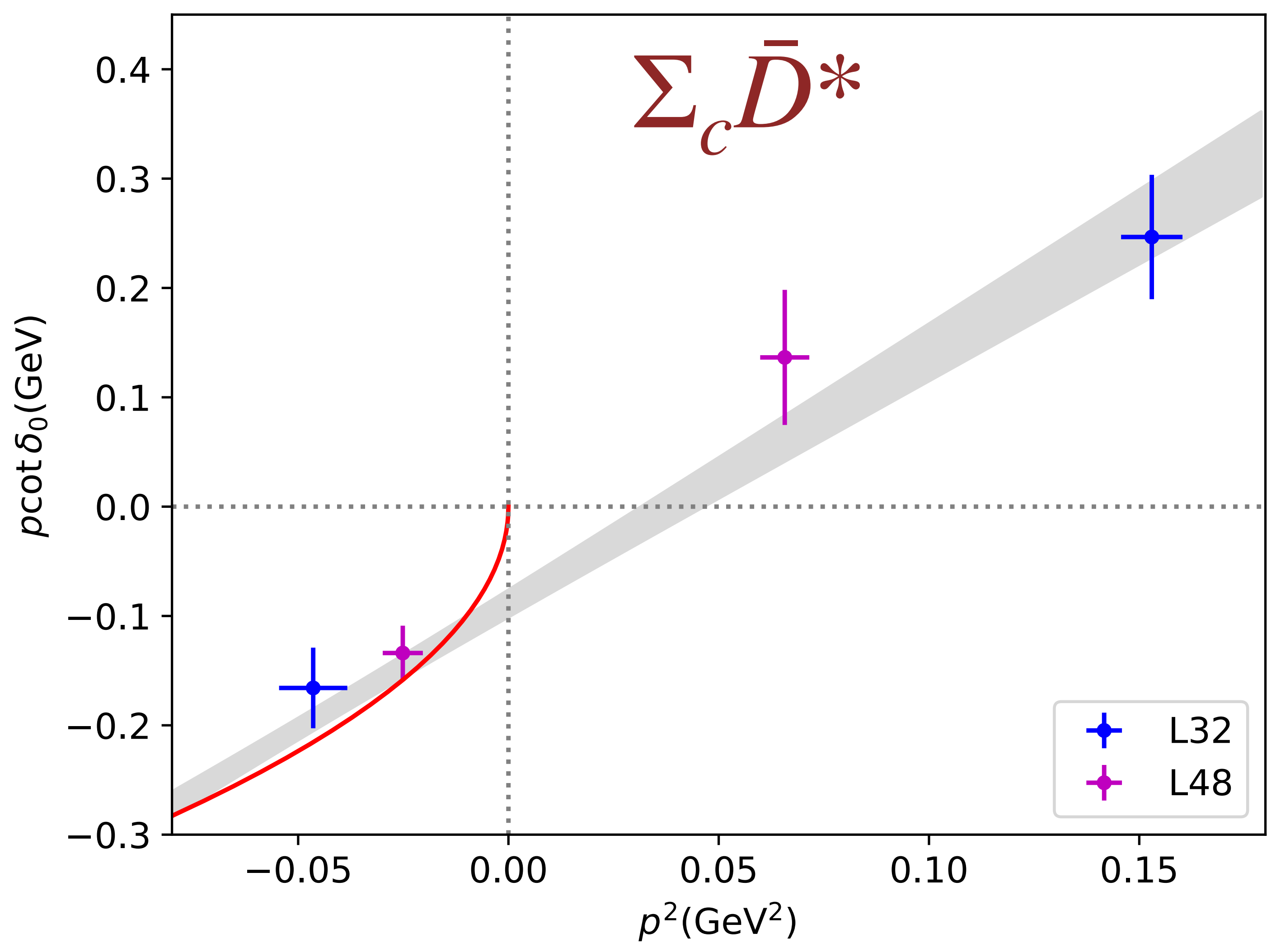
$\Sigma_c\bar{D}$: $P_c(4312)$?
$a_0 = -2.0(3)(5)\text{fm}$
$E_B = 6(2)(2)\text{MeV}$

Luscher's formula:

$$p\cot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$\Sigma_c\bar{D}^*$: $P_c(4440)/P_c(4457)$?
$a_0 = -2.3(5)(1)\text{fm}$
$E_B = 7(3)(1)\text{MeV}$



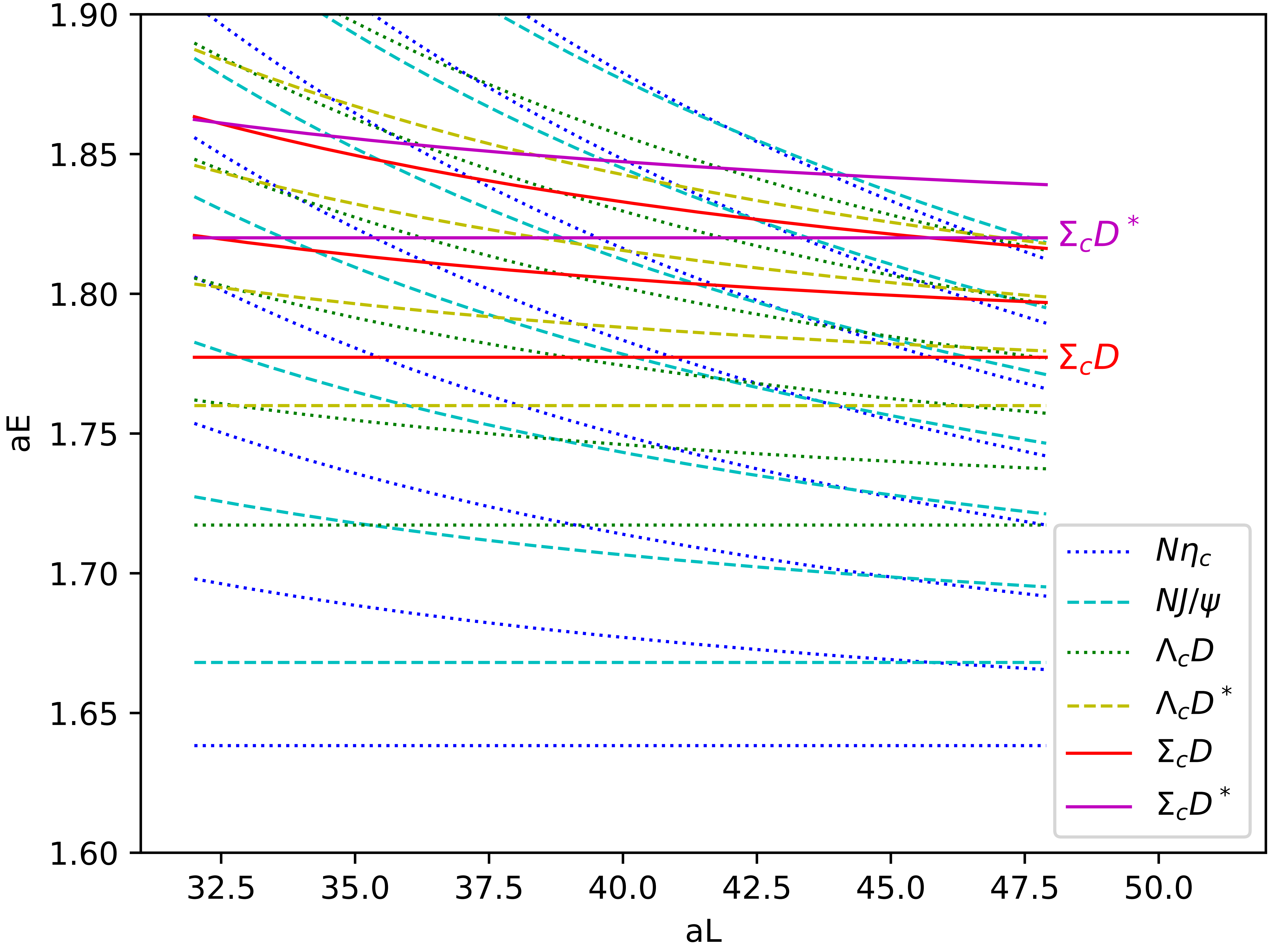


Coupled channels

Coupled channels: $\eta_c N, J/\psi N, \Lambda_c \bar{D}, \Lambda_c \bar{D}^*, \Sigma_c \bar{D}, \Sigma_c \bar{D}^*$

Non-interacting energy levels:

$$E_{free} = \sqrt{m_1^2 + p_1^2} + \sqrt{m_1^2 + p_2^2}$$



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✦ 15 operators for the $L = 32$ ensemble:

$\mathcal{O}_{1,2,3} = N(\mathbf{p})\eta_c(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2)$

$\mathcal{O}_{4,5} = N(\mathbf{p})J/\psi(-\mathbf{p})\ (\mathbf{p}^2 = 0,1)$

$\mathcal{O}_{6,7,8} = \Lambda_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2)$

$\mathcal{O}_{9,10} = \Lambda_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (\mathbf{p}^2 = 0,1)$

$\mathcal{O}_{11,12,13} = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2)$

$\mathcal{O}_{14,15} = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (\mathbf{p}^2 = 0,1)$

✦ 23 operators for the $L = 48$ ensemble:

$\mathcal{O}_{1,2,3,4,5} = N(\mathbf{p})\eta_c(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2,3,4)$

$\mathcal{O}_{7,8,9,10} = N(\mathbf{p})J/\psi(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2,3)$

$\mathcal{O}_{10,11,12,13,14} = \Lambda_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2,3,4)$

$\mathcal{O}_{15,16,17,18} = \Lambda_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2,3)$

$\mathcal{O}_{19,20,21} = \Sigma_c(\mathbf{p})\bar{D}(-\mathbf{p})\ (\mathbf{p}^2 = 0,1,2)$

$\mathcal{O}_{22,23} = \Sigma_c(\mathbf{p})\bar{D}^*(-\mathbf{p})\ (\mathbf{p}^2 = 0,1)$

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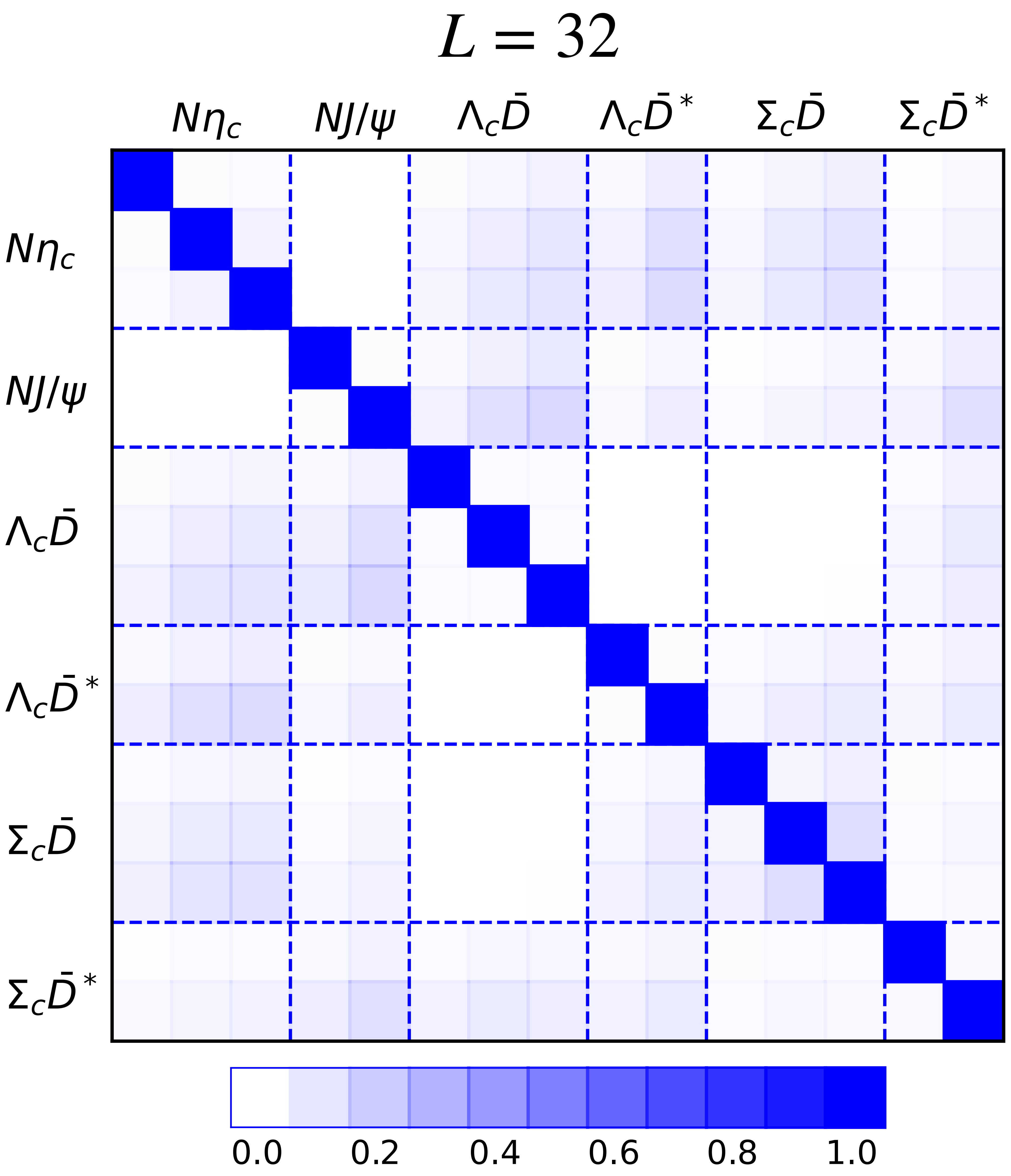

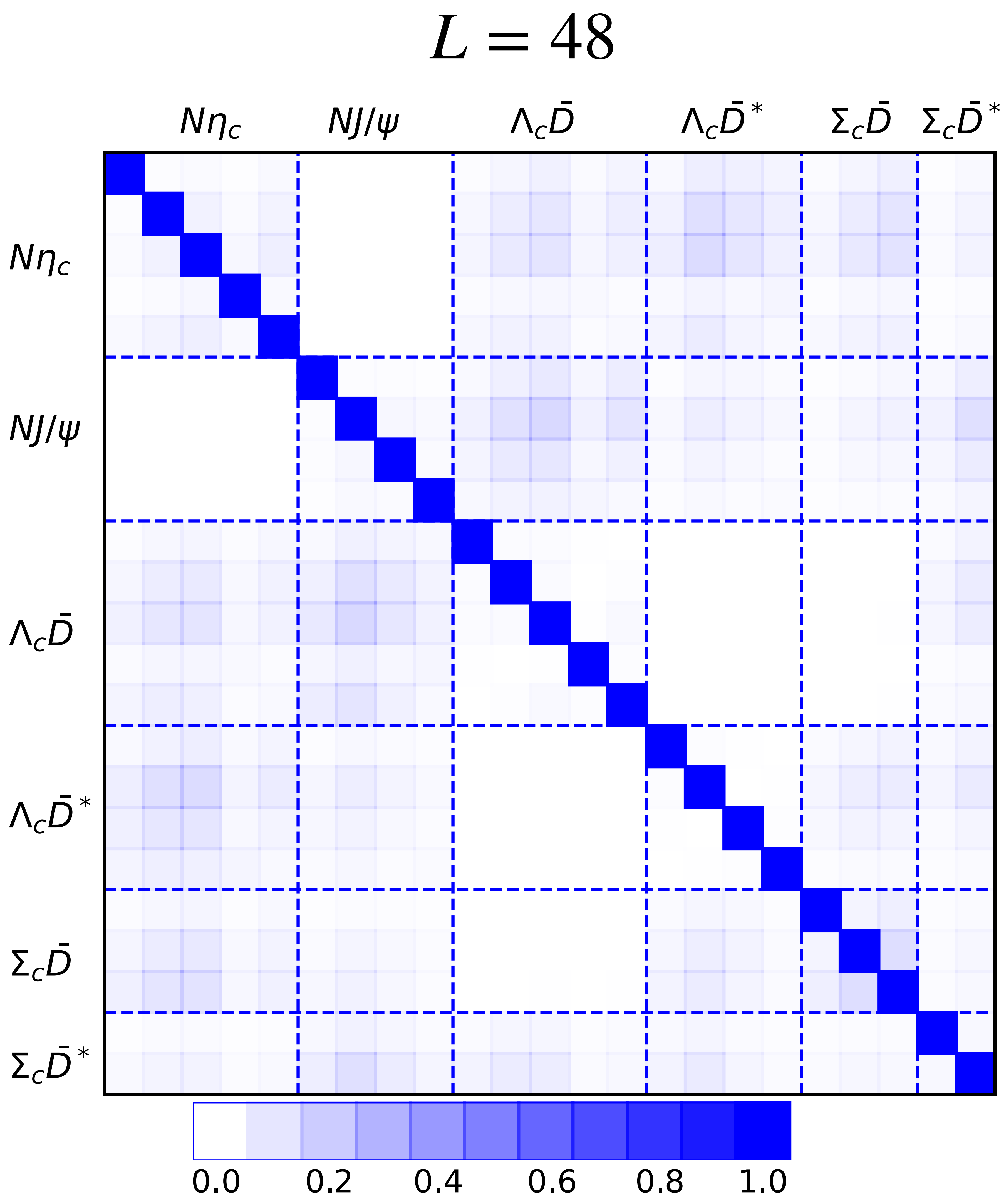

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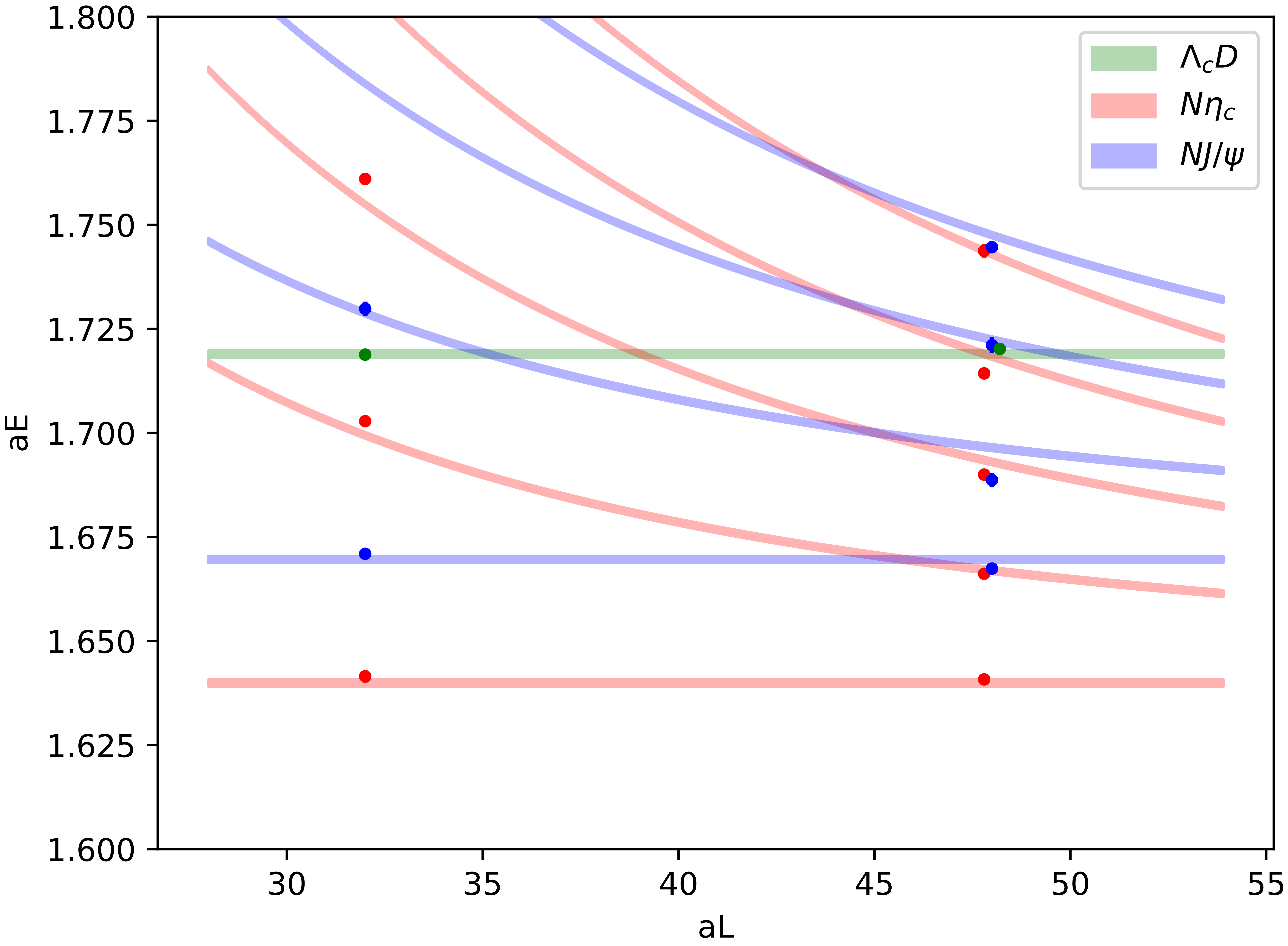

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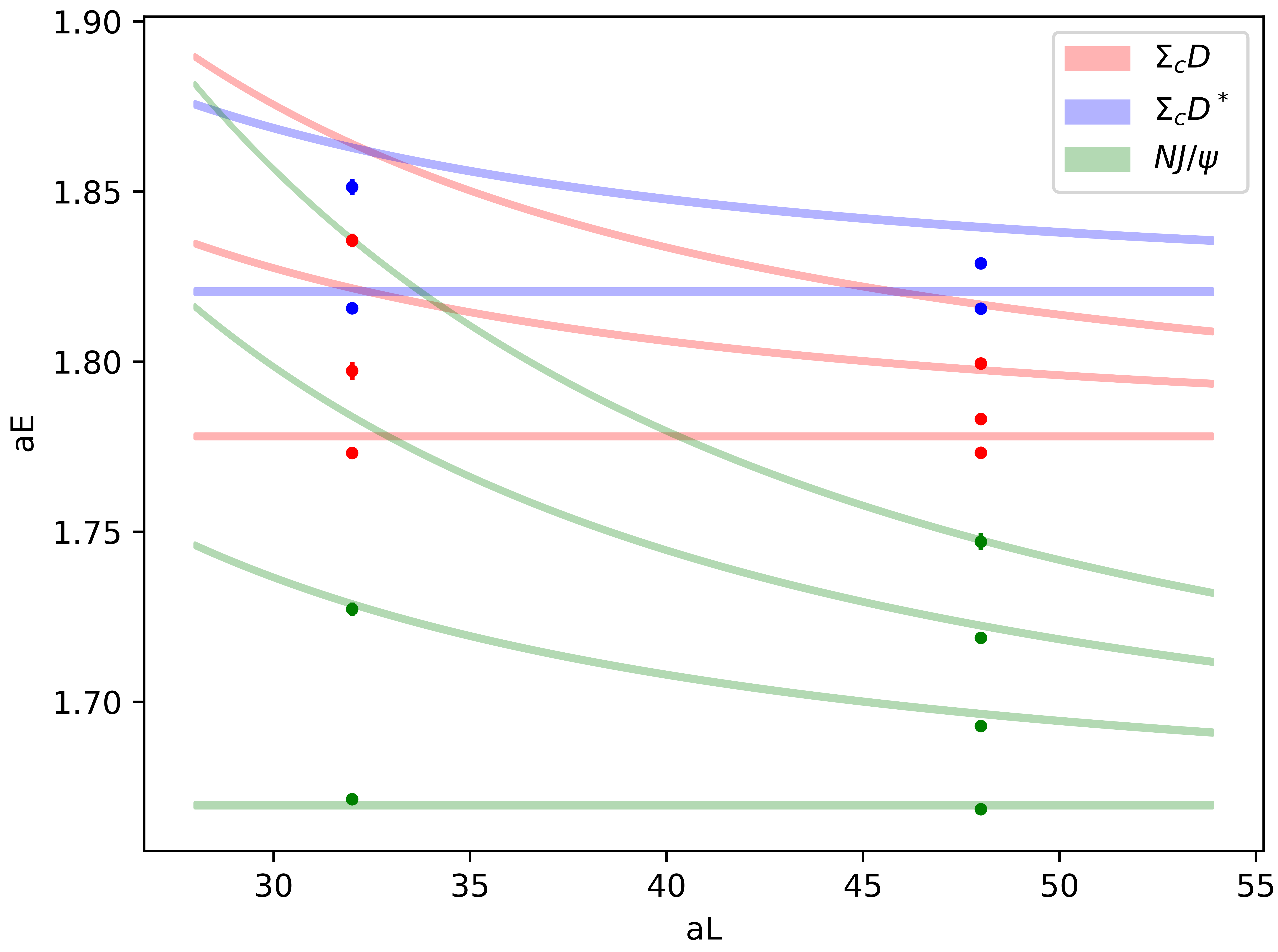

Summary

- Single channel analysis indicates bound states in $\Sigma_c\bar{D}$ and $\Sigma_c\bar{D}^*$ channel at $m_\pi \sim 300\text{MeV}$.
- Coupled channels:
  - Need robust determination of the spectrum with a complete set of interpolating operators.
  - Coupled channel scattering analysis.