Near-threshold Heavy-Strange mesons from Lattice QCD and Hamiltonian Effective Field Theory

王广娟
IPNS, KEK

Together with 杨智(UESTC), 吴佳俊(UCAS), Makoto Oka (JAEA&RIKEN) and 朱世琳(PKU)

Based on Phys. Rev. Lett. 128.112001, JHEP 01 058,1-20, (2023)

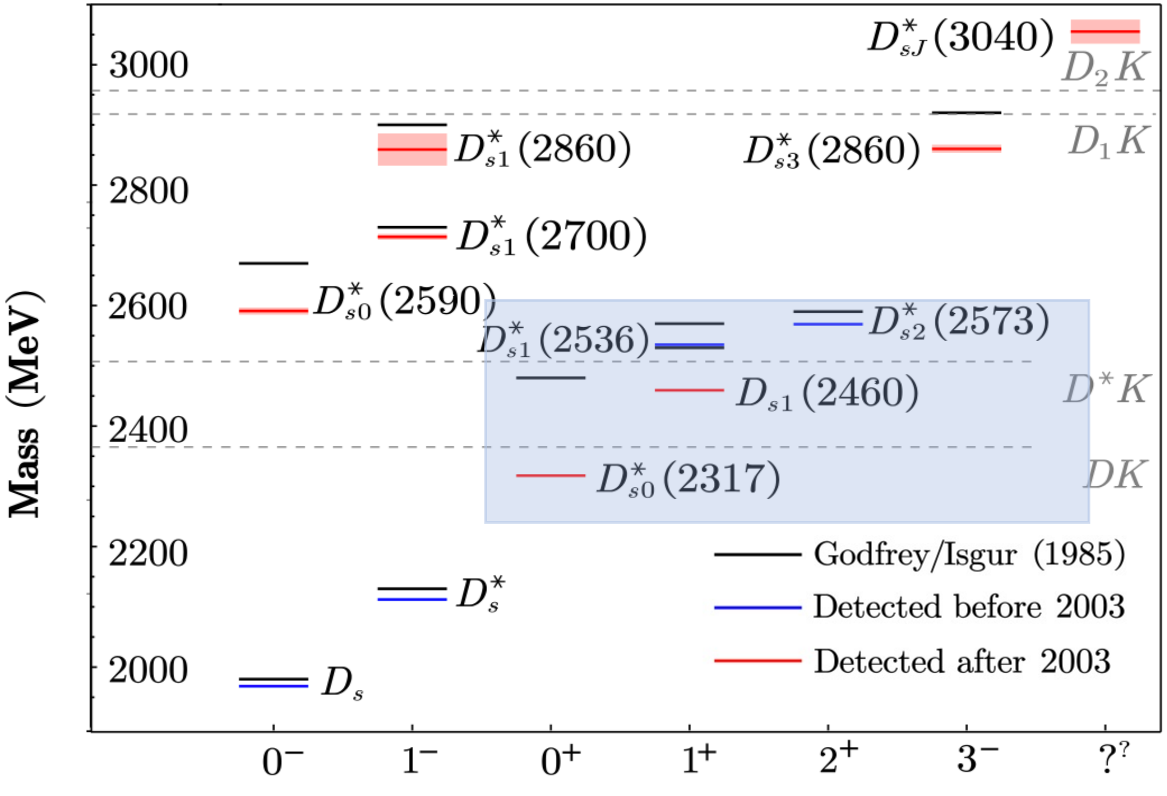
第三届中国格点量子色动力学研讨会

Puzzles of P-wave D_s mesons

- Four P-wave excited $c\bar{s}$ mesons in QM:

$$S_{\bar{c}s} = 0,\ J^P = 1^+$$
$$S_{\bar{c}s} = 1, J^P = 0^+, 1^+, 2^+$$

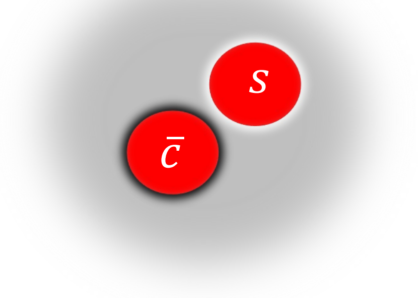
- $D_{s0}^*(2317)$ & $D_{s1}(2460)$: $m_{exp} < m_{the}$?
- $D_{s1}^*(2536)$ & $D_{s2}^*(2573)$: $m_{exp} \sim m_{the}$.
- Close to the $D^{(*)}K$ threshold.



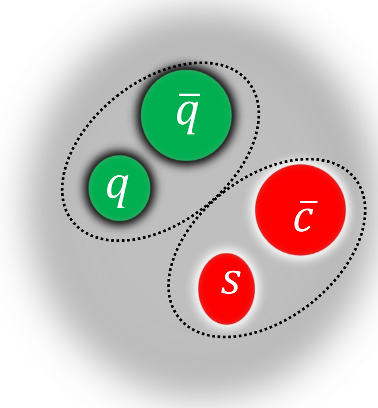
Lu *et al*,. Phys. Rept. 1019, 1-149, (2023)

$D_{s0}^*(2317)$& $D_{s1}(2460)$

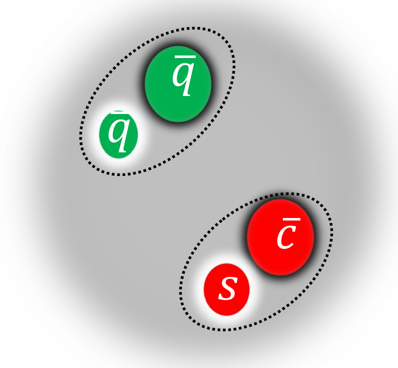
- Various theoretical models: molecule, tetraquark, quenched quark model …



$\bar{c}s$ meson



Compact $\bar{c}\bar{q}sq$



$D^{(*)}K$ molecule

S. Godfrey et al,. Phys. Rev. D 32, 189 (1985)
Y.-B. Dai et al,. Phys. Rev. D 68, 114011 (2003)
D. S. Hwang et al,. Phys. Lett. B 601, 137 (2004)
Y. A. Simonov et al,. Phys. Rev. D 70, 114013 (2004)
H.-Y. Cheng et al,. Phys. Rev. D 89, 114017 (2014)
Q.-T. Song et al,. Phys. Rev. D 91, 054031 (2015)
H.-Y. Cheng et al,. Eur. Phys. J. C 77, 668 (2017)
S.-Q. Luo, et al,. Phys. Rev. D 103, 074027 (2021)
Z.-Y. Zhou et al,. Eur. Phys. J. C 81, 551 (2021)
……

H.-Y. Cheng et al,., Phys. Lett. B 566, 193 (2003)
Y.-Q. Chen et al,. Phys. Rev. Lett. 93, 232001(2004)
V. Dmitrasinovic, Phys. Rev. Lett. 94, 162002 (2005)
H. Kim et al,. Phys. Rev. D 72, 074012 (2005)
J.-R. Zhang, Phys. Lett. B 789, 432 (2019)
……

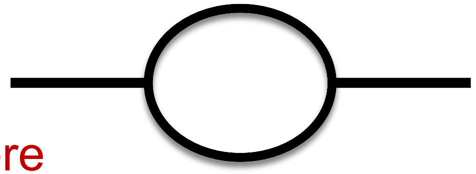
E. E. Kolomeitsev et al,. Phys. Lett. B 582, 39 (2004)
A. P. Szczepaniak, Phys. Lett. B 567, 23 (2003)
J. Hofmann et al,. Nucl. Phys. A 733, 142 (2004)
E. van Beveren et al,. Phys. Rev. Lett. 91, 012003 (2003)
T. Barnes et al,. Phys. Rev. D68, 054006 (2003)
D. Gamermann et al,. Phys. Rev. D 76,074016 (2007)
F.-K. Guo et al,. Phys. Lett. B 647, 133 (2007)
J. M. Flynn et al,. , Phys. Rev. D 75, 074024 (2007)
A. Faessler et al,. Phys. Rev. D 76, 014005 (2007)
F.-K. Guo et al,. Eur. Phys. J. A 40, 171 (2009)
Z.-X. Xie et al,. Phys. Rev. D 81, 036014 (2010)
……

Puzzles of P-wave D_s mesons

- Quark model successfully described other D_s mesons: $D_{s1}(2536)$ & $D_{s2}^*(2573)$.

$\bar{c}s$ core is there

- Excited hadrons: create $\bar{q}q$ pair from the vacuum - Hadronic loop



Hadron interaction is there

- Coupled-channel effect: extremely important for near-threshold states.

- $D_{s0}^*(2317)$ & $D_{s1}(2460)$ lie closely to the $D^{(*)}K$ channels.

Quark model + Coupled-channel effects to study the four P-wave D_s states.

Hamiltonian Framework

- The Hamiltonian reads

$$H = H_0 + H_I,$$

- Non-interacting Hamiltonian

$$H_0 = \sum_B \underline{|B\rangle} m_B \langle B| + \sum_\alpha \int d^3\vec{k} \underline{|\alpha(\vec{k})\rangle} E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

bare $\bar{c}s$ meson

two-meson state

Hamiltonian Framework

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bare $\bar{c}s$ meson

two-meson state

↓

Well established quark model

Hamiltonian Framework

- The Hamiltonian reads

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- Non-interacting Hamiltonian

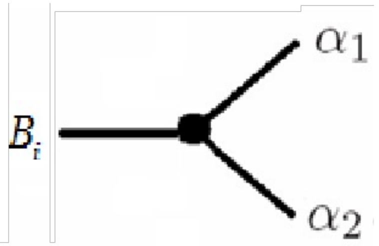
$$H_0 = \sum_B \underline{|B\rangle} m_B \langle B| + \sum_\alpha \int d^3\vec{k} \underline{|\alpha(\vec{k})\rangle} E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

bare $\bar{c}s$ meson — two-meson state

- Interacting Hamiltonian

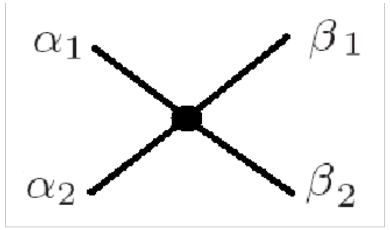
$$H_I = g + v$$

bare state core -> channel：



$$g = \sum_{\alpha,B} \int d^3\vec{k} \left\{ |\alpha(\vec{k})\rangle g_{\alpha B}(|\vec{k}|) \langle B| + h.c. \right\}$$

channel -> channel：



$$v = \sum_{\alpha,\beta} \int d^3\vec{k} d^3\vec{k}' |\alpha(\vec{k})\rangle V^L_{\alpha,\beta}(|\vec{k}|, |\vec{k}'|) \langle \beta(\vec{k}')|$$

Hamiltonian Framework

- The Hamiltonian reads

$$H = H_0 + H_I,$$

- Non-interacting Hamiltonian

$$H_0 = \sum_B \underline{|B\rangle} m_B \langle B| + \sum_\alpha \int d^3\vec{k} \underline{|\alpha(\vec{k})\rangle} E_\alpha(\vec{k}) \langle \alpha(\vec{k})|.$$

bare $\bar{c}s$ meson

two-meson state

- Interacting Hamiltonian

$$H_I = g + v$$

→ **Various theoretical models but lack of experimental data.**

↑ **Hamiltonian Effective field theory (HEFT)**

Lattice Spectrum

Hamiltonian effective field theory(HEFT)

1. **Finite-volume matrix Hamiltonian model for a Δ→Nπ system**
J.M.M. Hall, A.C.-P. Hsu, D.B. Leinweber, A.W.Thomas, R.D. Young [Phys.Rev. D87 (2013) no.9, 094510]
2. **Finite-volume Hamiltonian method for coupled-channels interactions in lattice QCD**
Jia-Jun Wu, T.-S.H.Lee, A.W.Thomas, R.D. Young [Phys.Rev. C90 (2014) no.5, 055206]
3. **Hamiltonian effective field theory study of the N∗(1535) resonance in lattice QCD**
Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
[Phys.Rev.Lett. 116 (2016) no.8, 082004]
4. **Lattice QCD Evidence that the Λ(1405) Resonance is an Antikaon-Nucleon Molecule**
J.M.M. Hall, Waseem Kamleh, Derek B. Leinweber, Benjamin J. Menadue, Benjamin J. Owen, A.W.Thomas, R.D. Young
[Phys.Rev.Lett. 114 (2015) no.13, 132002]
5. **Hamiltonian effective field theory study of the N∗(1440) resonance in lattice QCD**
Zhan-Wei Liu, Waseem Kamleh, Derek B. Leinweber, Finn M. Stokes, Anthony W. Thomas, Jia-Jun Wu
[Phys.Rev. D95 (2017) no.3, 034034]
6. **Structure of the Λ(1405) from Hamiltonian effective field theory**
Zhan-Wei Liu, Jonathan M.M. Hall, Derek B. Leinweber, Anthony W. Thomas, Jia-Jun Wu
[Phys.Rev. D95 (2017) no.1, 014506]
7. **Nucleon resonance structure in the finite volume of lattice QCD**
Jia-jun Wu, H. Kamano, T.-S.H.Lee , Derek B. Leinweber, Anthony W. Thomas [Phys.Rev. D95 (2017) no.11, 114507]
8. **Structure of the Roper Resonance from Lattice QCD Constraints**
Jia-jun Wu, Derek B. Leinweber, Zhan-wei Liu, Anthony W.Thomas [Phys.Rev. D97(2018) no.9, 094509]
9. **Kaonic Hydrogen and Deuterium in Hamiltonian Effective Field Theory**
Zhan-wei Liu, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas [Phys.Lett.B 808(2020),135652]
10. **Partial Wave Mixing in Hamiltonian Effective Field Theory**
Yan Li, Jia-jun Wu, Curtis D. Abell, Derek B. Leinweber, Anthony W. Thomas [Phys.Rev. D101(2020) no.11,114501]
11. **Hamiltonian effective field theory in elongated or moving finite volume**
Yan Li, Jia-jun Wu, Derek B. Leinweber, Anthony W. Thomas [Phys.Rev. D103(2021) no.9, 094518]
12. **Regularisation in Nonperturbative Extensions of Effective Field Theory**
Curtis D. Abell, Derek B. Leinweber, Anthony W. Thomas, Jia-jun Wu [arXiv: 2110.14113]
13. **Novel Coupled Channel Framework Connecting the Quark Model and Lattice QCD for the Near-threshold Ds States** Zhi Yang, Guang-Juan Wang, Jia-jun Wu, Shi-lin Zhu, Makoto Oka [Phys.Rev.Lett.128(2020),112001]

Hamiltonian Effective Field Theory (HEFT)

- In the finite volume, the momentum is discretized as

$$k_n = 2\pi\sqrt{n}/L, \;\; n = n_x^2 + n_y^2 + n_z^2, \;\; n = 0, 1, 2, \ldots$$

Continuous

↓

Discrete

$\int d\vec{k}$ and	$\lvert\alpha(\vec{k}_\alpha)\rangle$ and	$\langle\beta(\vec{k}_\beta)\lvert\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$
↓	↓	↓
$\sum_i \left(2\pi/L\right)^3$ and	$\left(2\pi/L\right)^{-3/2} \lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha$ and	${}_\beta\langle\vec{k}_j, -\vec{k}_j\lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha = \delta_{\alpha\beta}\delta_{ij}$

J.M.M. Hall, et al. Phys. Rev. D 87, 094510.
J.J. Wu, et al. Phys. Rev. C 90, 055206.
Z. W. Liu, et al. Phys. Rev. Lett.116,082004

$$H_0 = \sum_{i=1,n} \lvert B_i\rangle m_i \langle B_i\rvert + \sum_{\alpha,i} \lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha \left[\sqrt{m_{\alpha_B}^2 + k_\alpha^2} + \sqrt{m_{\alpha_M}^2 + k_\alpha^2}\right]_\alpha \langle\vec{k}_i, -\vec{k}_i\rvert$$

$$H_I = \sum_j \left(2\pi/L\right)^{3/2} \sum_\alpha \sum_{i=1,n} \left[\lvert\vec{k}_j, -\vec{k}_j\rangle_\alpha \, g_{i,\alpha}^+ \langle B_i\rvert + \lvert B_i\rangle g_{i,\alpha} \; {}_\alpha\langle\vec{k}_j, -\vec{k}_j\rvert\right]$$

$$+ \sum_{i,j} \left(2\pi/L\right)^3 \sum_{\alpha,\beta} \lvert\vec{k}_i, -\vec{k}_i\rangle_\alpha \, v_{\alpha,\beta} \; {}_\beta\langle\vec{k}_j, -\vec{k}_j\rvert$$

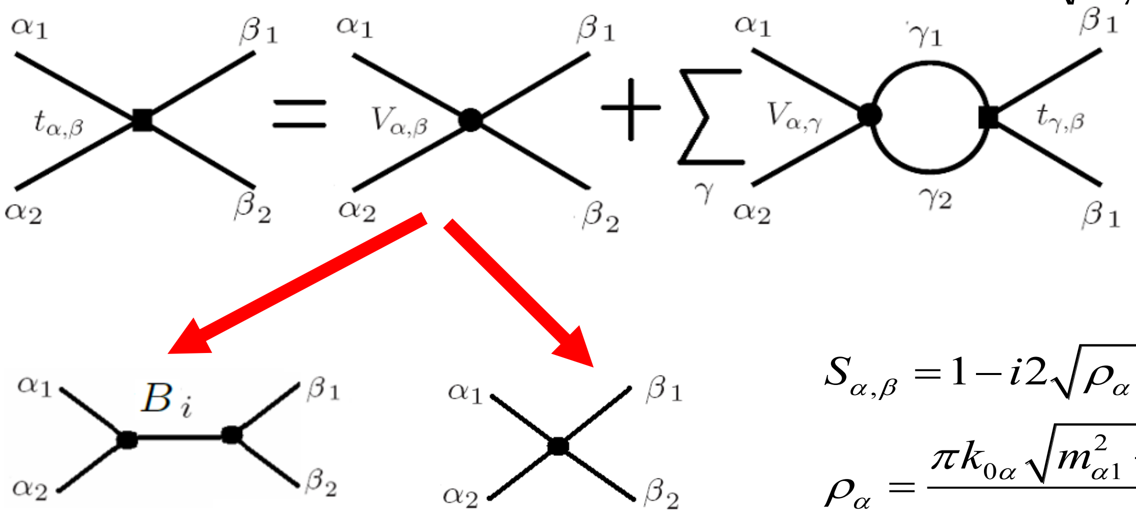
$$(H_0 + H_I)\lvert\Psi\rangle = \underline{E}\lvert\Psi\rangle$$

$\hat{g}$, $\hat{v}$ ⟵ Energy levels in lattice QCD

HEFT

- **T Matrix:**

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma{}^2dk_\gamma\,\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$



$$g_{i,\alpha}^*\frac{1}{E-m_i}g_{i,\beta}$$

$$v_{\alpha,\beta}$$

$$S_{\alpha,\beta}=1-i2\sqrt{\rho_\alpha}\,t_{\alpha,\beta}(k_{0\alpha},k_{0\beta},E)\sqrt{\rho_\beta}$$

$$\rho_\alpha=\frac{\pi k_{0\alpha}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha}=S_{\alpha,\alpha}$$

From Wu Jia-Jun's talk

Quark model: bare $\bar{c}s$ state

The relativized quark model:

$$H = H_0 + V$$

$$H_0 = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$

$$V = G_{\rm eff}(r) + S_{\rm eff}(r)$$

$$G_{\rm eff}(r) = \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2} \tilde{G}(r) \left[1 + \frac{p^2}{E_1 E_2}\right]^{1/2}$$
$$+ \left[\frac{\mathbf{S}_1 \cdot \mathbf{L}}{2{m_1}^2} \frac{1}{r} \frac{\partial \tilde{G}_{11}^{\rm so(v)}}{\partial r} + \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2{m_2}^2} \frac{1}{r} \frac{\partial \tilde{G}_{22}^{\rm so(v)}}{\partial r} + \frac{(\mathbf{S}_1 + \mathbf{S}_2) \cdot \mathbf{L}}{m_1 m_2} \frac{1}{r} \frac{\partial \tilde{G}_{12}^{\rm so(v)}}{\partial r}\right]$$
$$+ \frac{2\mathbf{S}_1 \cdot \mathbf{S}_2}{3 m_1 m_2} \nabla^2 \tilde{G}_{12}^{c} - \left[\frac{\mathbf{S}_1 \cdot \hat{r} \mathbf{S}_2 \cdot \hat{r} - \frac{1}{3} \mathbf{S}_1 \cdot \mathbf{S}_2}{m_1 m_2}\right] \left[\frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r}\right] \tilde{G}_{12}^{t}$$

$$S_{\rm eff}(r) = \tilde{S}(r) - \frac{\mathbf{S}_1 \cdot \mathbf{L}}{2{m_1}^2} \frac{1}{r} \frac{\partial \tilde{S}_{11}^{\rm so(s)}}{\partial r} - \frac{\mathbf{S}_2 \cdot \mathbf{L}}{2{m_2}^2} \frac{1}{r} \frac{\partial S_{22}^{\rm so(s)}}{\partial r}$$

Godfrey, Isgur, Phys. Rev. D 32,189 (1985)

- Relativized Modification:

- Relativistic kinematic energy

- Energy dependence

$$\omega_{ij} = 1 + \frac{p_i p_j}{E_i E_j} \qquad \rho_{ij} = \frac{m_i m_j}{E_i E_j}$$

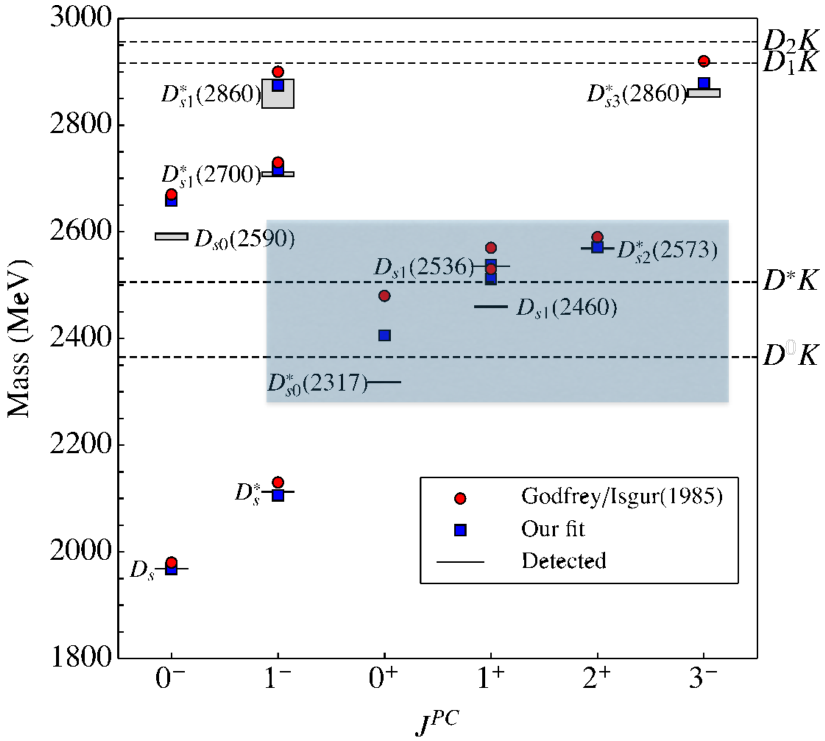
- Mass & wavefunction:

$$H|B\rangle = m_B\,|B\rangle$$

- Input of fit:

All the well-established mesons far away from two-meson thresholds as input (from π to Υ).

Quark model: bare $\bar{c}s$ state



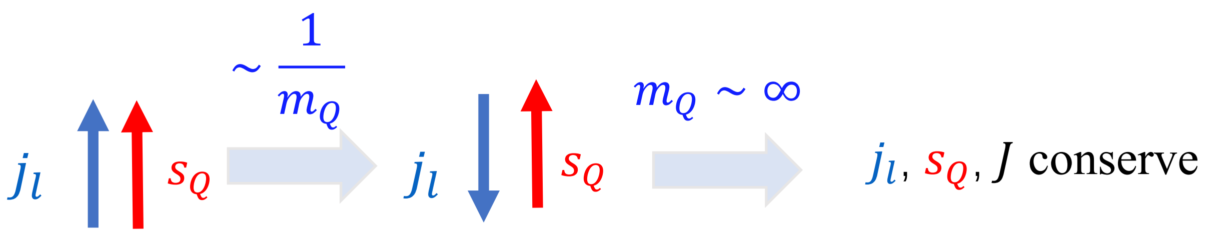
$\bar{c}s$ cores			channel	
	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\|^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\|^1P_1\rangle - 0.74\|^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\|^1P_1\rangle - 0.68\|^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

- The predicted lowest $0^+/1^+$ bare $\bar{c}s$ mesons
  -located above the $D_{s0}^*(2317)$&$D_{s1}(2460)$ states.

- Good heavy quark spin symmetry

 - Total Spin

$$J = s_Q \otimes s_q \otimes L = S(s_Q \otimes s_q) \otimes L = s_Q \otimes j_l(s_q \otimes L)$$



M. Neubert, Phys. Rept. 245 (1994) 259-396

0^+ $|\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_0$

1^+ $\phi_s = |\frac{1}{2}_l \otimes \frac{1}{2}_H\rangle_1$

→ S-wave $D^{(*)}K$

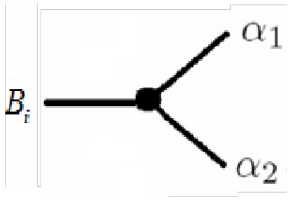
1^+ $\phi_d = |\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_1$

2^+ $|\frac{3}{2}_l \otimes \frac{1}{2}_H\rangle_2$

→ D-wave $D^{(*)}K$

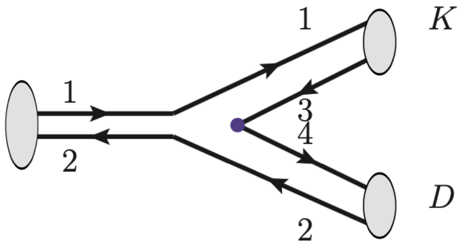
$H_I = g + v$

g: $\bar{c}s \to D^{(*)}K$



$^3P_0$ model

at quark level

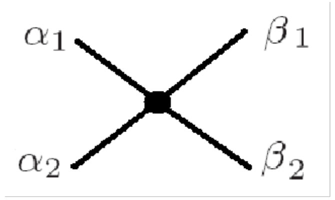


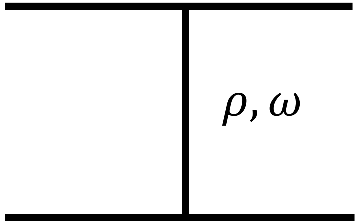
$$g_{\alpha B}(|\vec{k}|) = \gamma I_{\alpha B}(|\vec{k}|) e^{-\frac{\vec{k}^2}{2\Lambda'^2}}$$

P.G. Ortega, et al. Phys. Rev. D 94, 074037.

Undetermined γ & Λ'

v: $D^{(*)}K - D^{(*)}K$





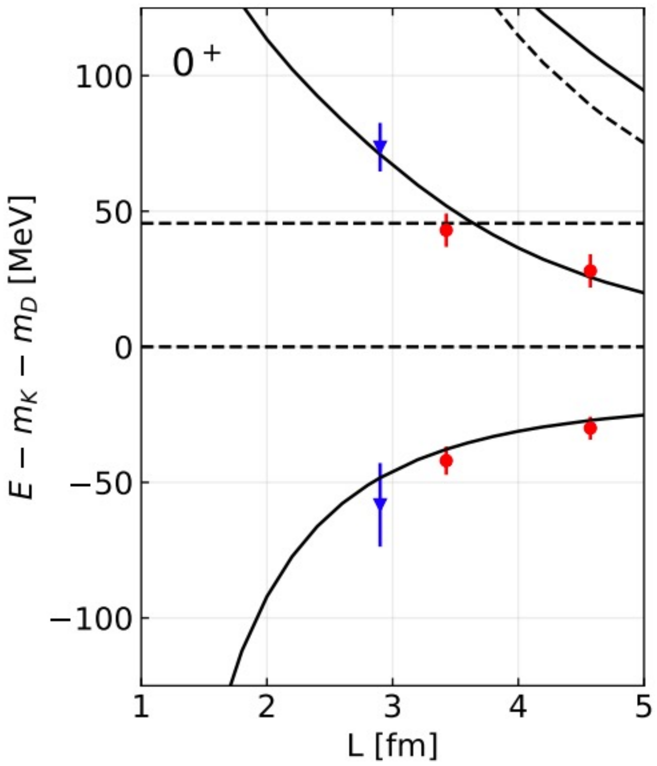
$$\mathcal{L} = \mathcal{L}_{PPV} + \mathcal{L}_{VVV} = ig_v \mathrm{Tr}(\partial^\mu P[P, V_\mu]) + ig_v \mathrm{Tr}(\partial^\mu V^\nu [V_\mu, V_\nu]),$$

$$\mathcal{V}(l, l'S, j) = \frac{1}{(2\pi)^3}\sqrt{\frac{1}{2E_D^i 2E_D^f 2E_K^i 2E_K^f}}\, 2\pi \int d\cos\theta V^v(\vec{p}_f, \vec{p}_i)\left(\frac{\Lambda^2}{\Lambda^2 + p_f^2}\right)^2\left(\frac{\Lambda^2}{\Lambda^2 + p_i^2}\right)^2$$

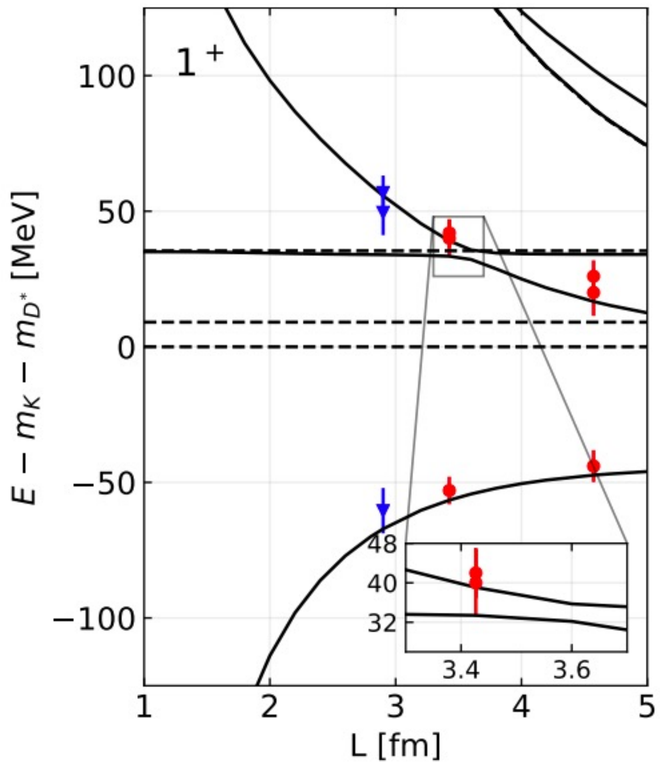
Form factor

Undetermined $g_c = g_{VDD}g_{VKK}(g_{VD^*D^*}g_{VKK})$

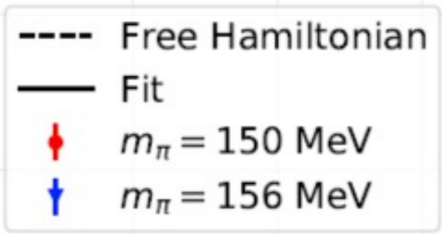
Fitting the Lattice data



$D_{s0}(2317)$



$D_{s1}(2460)$ & $D_{s1}(2536)$



- The data from Lattice QCD with $m_\pi = 150$ MeV [1] & $m_\pi = 156$ MeV [2].
- No chiral extrapolation.

2. We find a fit for the lattice result with χ^2/dof =0.95 with fixed $\Lambda = 1.0$ GeV.

[1] G. Bali, et al. (RQCD Collaboration) Phys. Rev. D 96, 074501 (2017).
[2] C. Lang, et al. Phys. Rev. D 90, 034510 (2014) .

Parameters

- Parameters

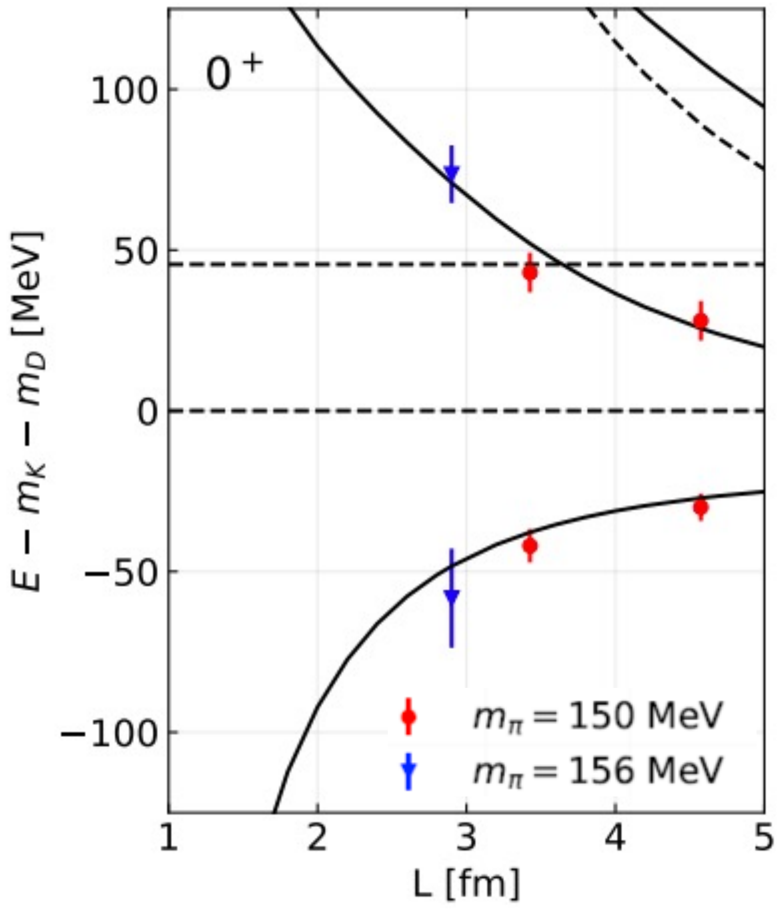
Parameters	g_c	Λ' [GeV]	γ
Best fit	$4.2^{+2.2}_{-3.1}$	$0.323^{+0.033}_{-0.031}$	$10.3^{+1.1}_{-1.0}$
Ref. [1]		0.84	6.5
Ref. [2]	-	-	6.9
Ref. [3]	18.2/9.8	-	-
Ref. [4]	8.4	-	-

[1] P. Ortega, et al. Phys. Rev. D 94 , 074037 (2016).
[2] S. Godfrey, et al. Phys .Rev. D 93 (2016) 3, 034035.
[3] C. W. Shen, et al. Phys. Rev. D 100, 056006 (2019).
[4] Z.W. Lin, et al. Phys. Rev. C 61, 024904 (2000).

- Pole mass: solving the scattering T-matrix in infinite limit,

$$T_{\alpha,\,\beta}(k,k';E) = \mathcal{V}_{\alpha,\,\beta}(k,k';E) + \sum_{\alpha'} \int q^2 dq \frac{\mathcal{V}_{\alpha,\,\alpha'}(k,q;E)T_{\alpha,\,\beta}(q,k';E)}{E-E_{\alpha'}(q)+i\epsilon}$$

Results: $D_{s0}^*(2317)$



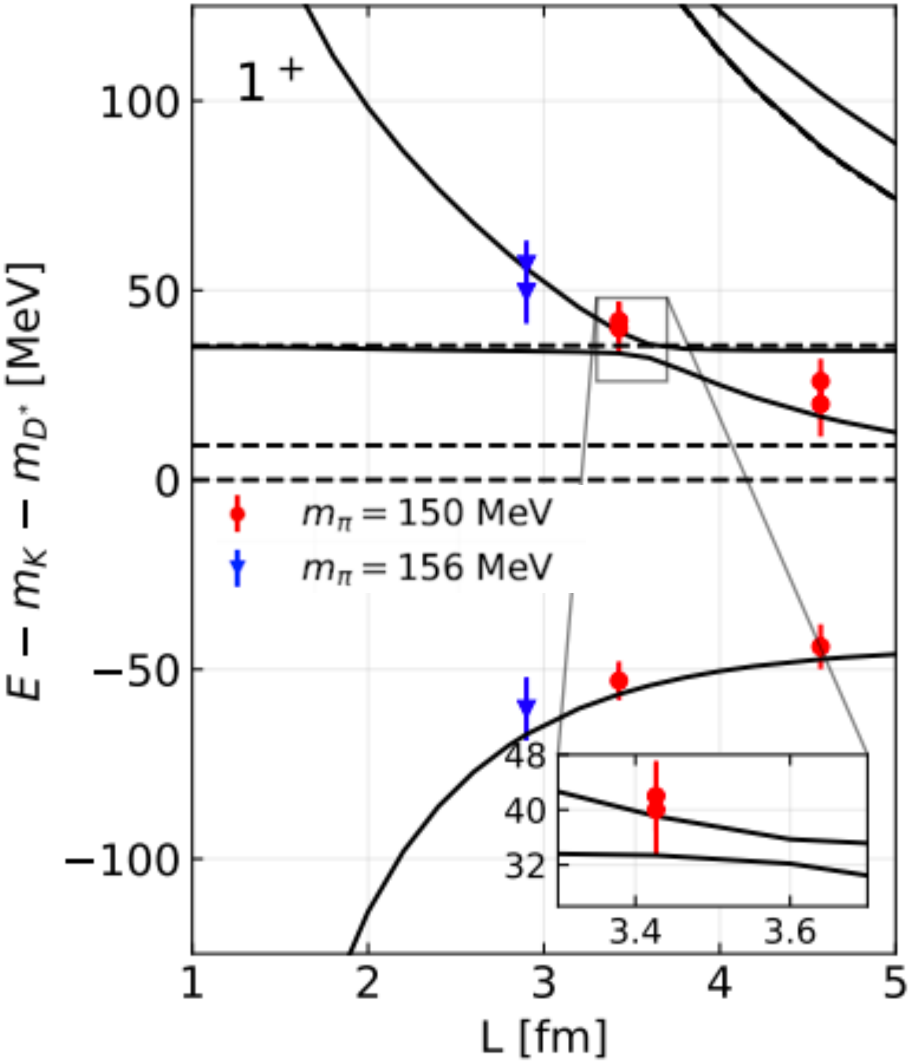
1. $J^P = 0^+$: $\bar{c}s(\,^3P_0)$ + S-wave DK.

2. $M^{th}_{D_{s0}(2317)}(2338.9^{+2.1}_{-2.7}) > M^{exp}(2317.8)$ due to larger Lattice data.

3. A mixture of the bare $\bar{c}s$ & DK component:

$$\mathrm{P}(DK) = 68.0\%, P(\bar{c}s) = 32.0\% \ (L = 4.57 \text{ fm}).$$

Results: $D_{s1}^*(2460)$ & $D_{s1}^*(2536)$



$J^P = 1^+$: D_{s1} (2460)&D_{s1}(2536)

1. $J^P = 1^+$: two $\bar{c}s$ cores + S-wave D*K + D-wave D*K.

2. Three energy levels.

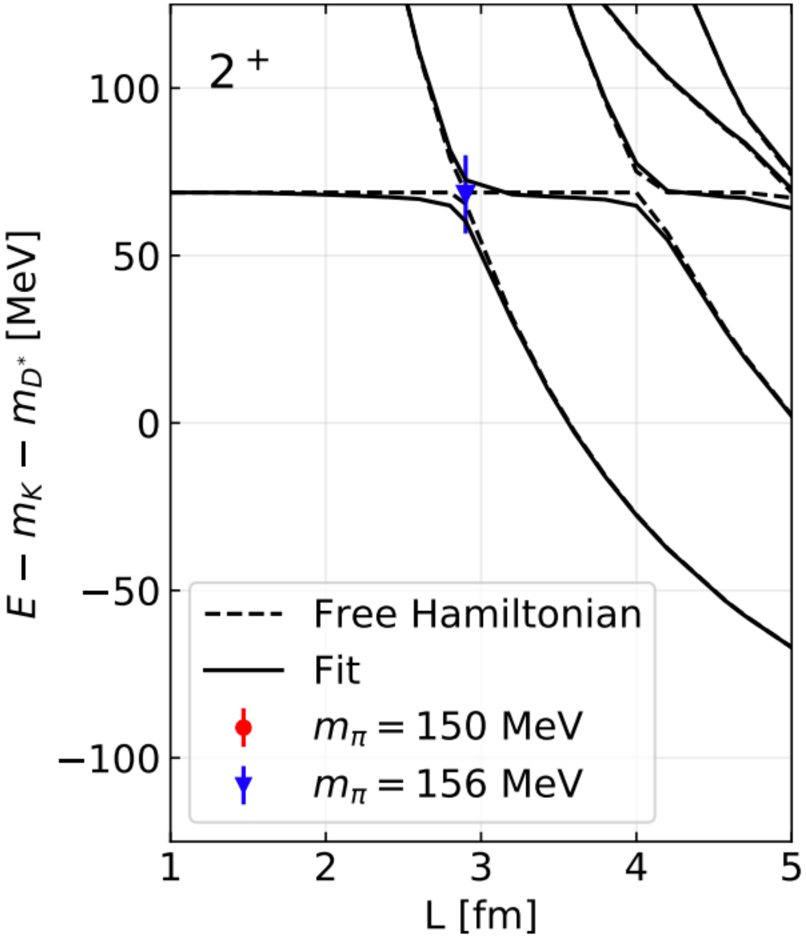
3. $D_{s1}^*(2460)$: the lowest state.

4. A special cross at L =3.5 fm: well approved by lattice QCD.

   - The dropping line: lowest excited D^*K channel, $E_{kin} \sim \left(\frac{2\pi}{L}\right)^2$.

   - The flat line represents the $D_{s1}^*(2536)$.

Prediction I: $D_{s2}(2573)$



1. $J^P = 2^+$: $\bar{c}s$ core + D-wave D^*K + D-wave DK.

2. The energy levels in the finite volume are predicted.

3. Mass and components: almost pure $\bar{c}s$ mesons.

	$P(c\bar{s})[\%]$	$P(D^{(*)}K)[\%]$	ours	exp
$D^*_{s2}(2573)$	$95.9^{+1.0}_{-1.5}$	4.1	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

Results &discussions

1. Different mass shifting pattern:

	$P(c\bar{s})[\%]$	$P(D^{(*)}K)[\%]$	ours	exp
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	68.0	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	47.6	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	1.8	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	4.1	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

	$B(\|^{2S+1}L_J\rangle)$	$B(\text{mass})$	α	L
$D_{s0}^*(2317)$	$\|^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\|^1P_1\rangle - 0.74\|^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\|^1P_1\rangle - 0.68\|^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

$D_{s0}^*(2317)$ & $D_{s1}(2460)$: S-wave $D^{(*)}K$

⟹ Sizable mass shift & mixing

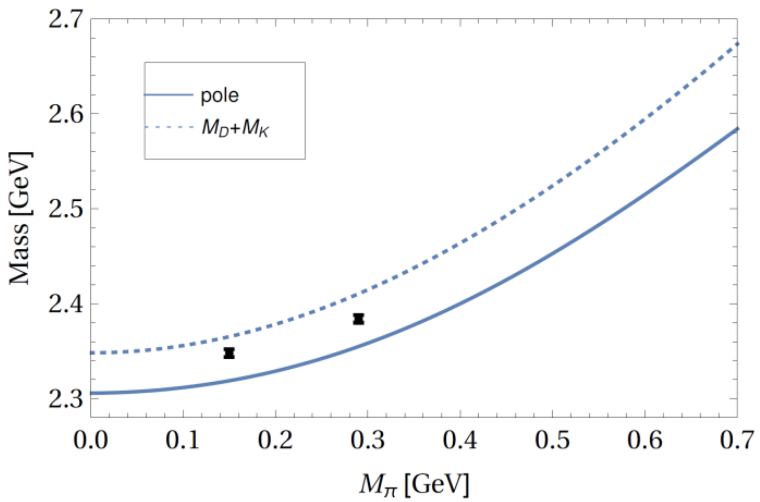
$D_{s1}(2536)$ & $D_{s2}(2573)$: D-wave $D^{(*)}K$

⟹ Small mass shift & tiny mixing

Prediction II: m_π- dependence

- DK molecule: Tends to become larger with larger m_π.

Latest lattice results in G. Bali et al., PRD96(2017)074501



curves: prediction in Du et al., EPJC77(2017)728

- Bare state ($c\bar{s}$): Tends to become stable with larger m_π.

"…for the lower lying pseudoscalar and vector D_s meson masses which decrease by 3 MeV (from 1980(1) MeV at m_π = 290 MeV to 1977(1) at m_π = 150 MeV) and 7 MeV (from 2101(1) MeV to 2094(1) MeV), respectively, hinting that the 0+ and 1+ states may have a more complicated internal structure."

G. Bali et al., PRD96(2017)074501

Prediction II: m_π- dependence

- Our prediction: the mass of $D_{s0}^*(2317)$ finally tends to become stable with increasing m_π.

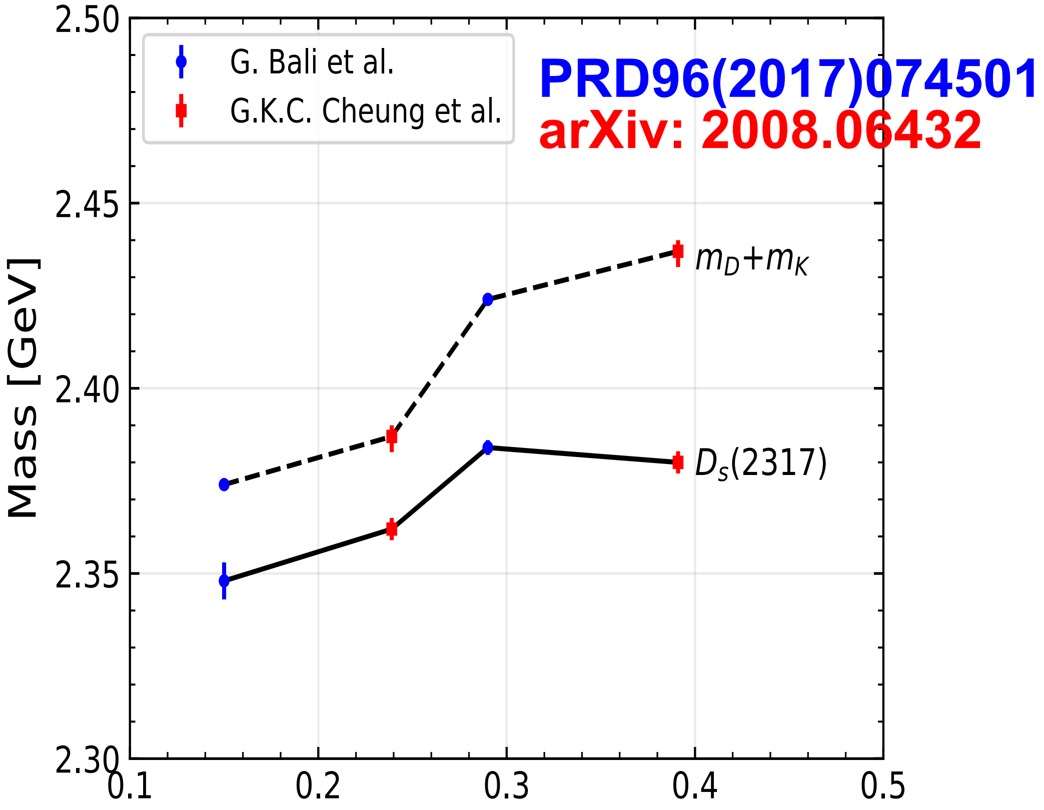
- m_π ↗ , m_{DK} ↗ , $m_{\bar{c}s}$ → stable

- $m_{DK} < m_{\bar{c}s}$

- $D_{s0}^*(2317)$: dominated by mainly $\bar{c}s$, increasing

- $m_{DK} >> m_{\bar{c}s}$:

- $D_{s0}^*(2317)$ is mainly $\bar{c}s$. $M_{D_{s0}^*(2317)}$ tends to be stable.



G. Bali et al., PRD96(2017)074501

Gavin K. C. Cheung et al., arXiv: 2008.06432

Extension to P-wave B_s mesons

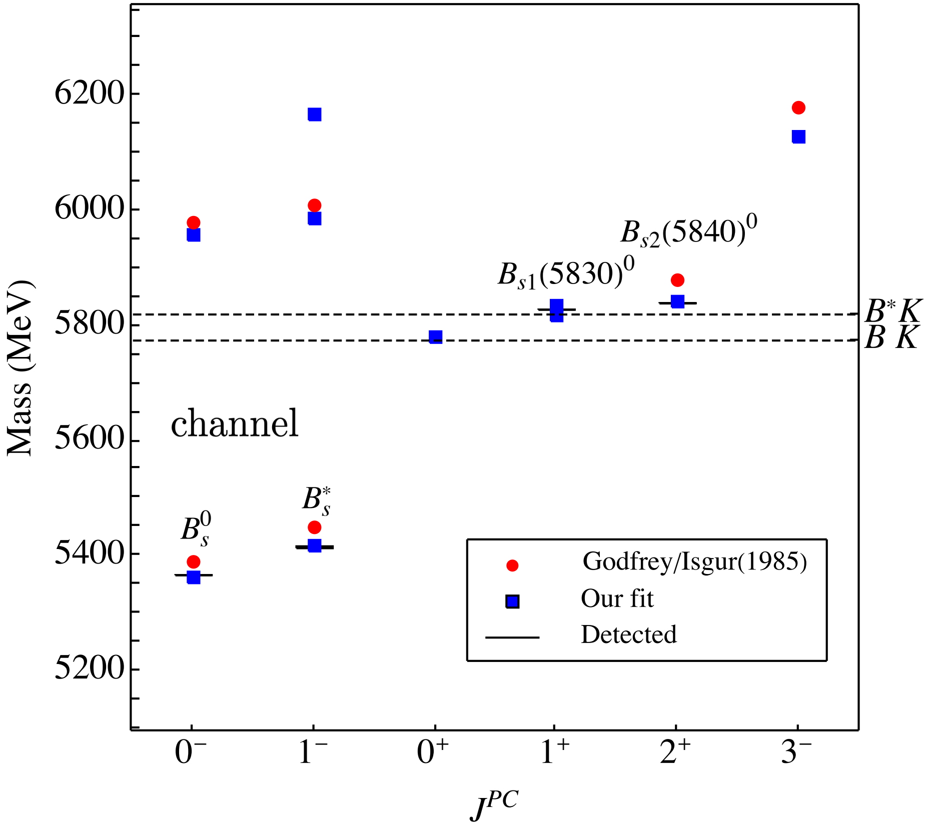
- Quark model

$$S_{\bar{b}s} = 0,\ J^P = 1^+$$
$$S_{\bar{b}s} = 1,\ J^P = 0^+, 1^+, 2^+$$

- Absence of 0^+ and lower 1^+ B_s states

$b\bar{s}$ cores channel

	$b\,(\vert^{2S+1}L_J\rangle)$	$b\,$(mass)	α	L
B_{s0}^*	$\vert^3P_0\rangle$	5780.9	$B\bar{K}$	S
B_{s1}^*	$-0.74\,\vert^1P_1\rangle + 0.67\,\vert^3P_1\rangle$	5818.5	$B^*\bar{K}$	S, D
	$= 0.98\,\phi_s - 0.22\,\phi_d$			
$B_{s1}^{*\prime}$	$0.67\,\vert^1P_1\rangle + 0.74\,\vert^3P_1\rangle$	5835.6	$B^*\bar{K}$	S, D
	$= 0.22\,\phi_s + 0.98\,\phi_d$			
$B_{s2}^{*\prime}$	$\vert^3P_2\rangle$	5842.7	$B\bar{K},\ B^*\bar{K}$	D



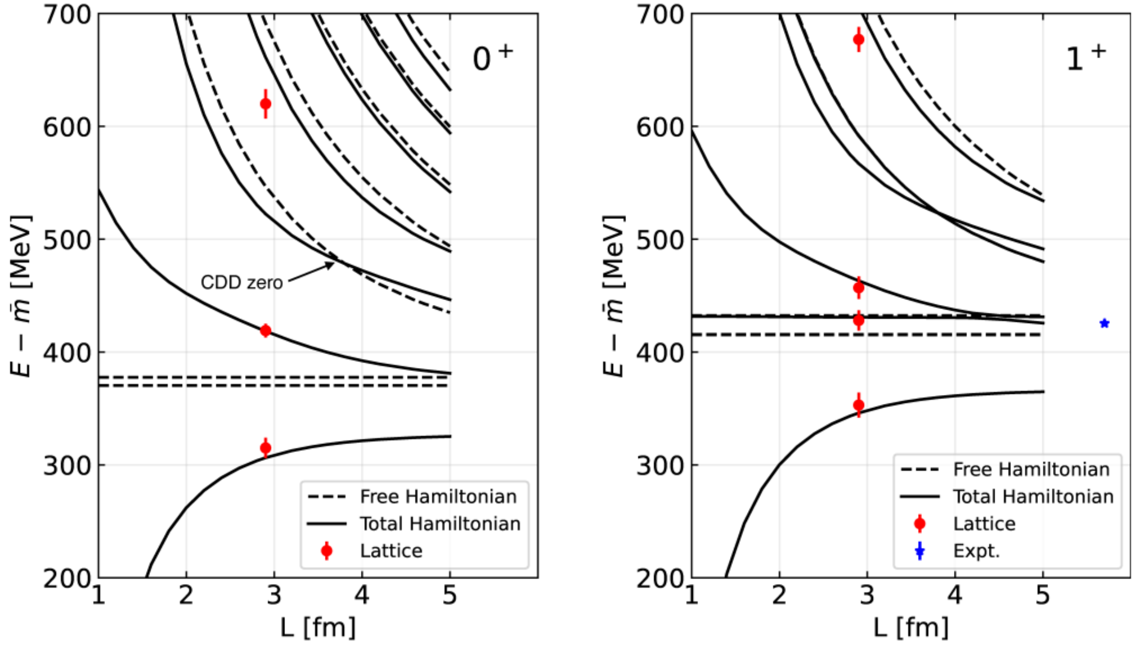
- Similar to D_s mesons: 0^+ and lower 1^+ B_s-S wave $B^{(*)}\bar{K}$ channels → Sizable mass shift & mixing

  higher 1^+ and 2^+ B_s-D wave $B^{(*)}\bar{K}$ channels → Small mass shift & tiny mixing

Extension to P-wave B_S mesons

- Heavy quark flavor symmetry: Using Previous Parameters

Postprediction, not a fit!



$B^*_{s0}(5730)$-$B\bar{K}$

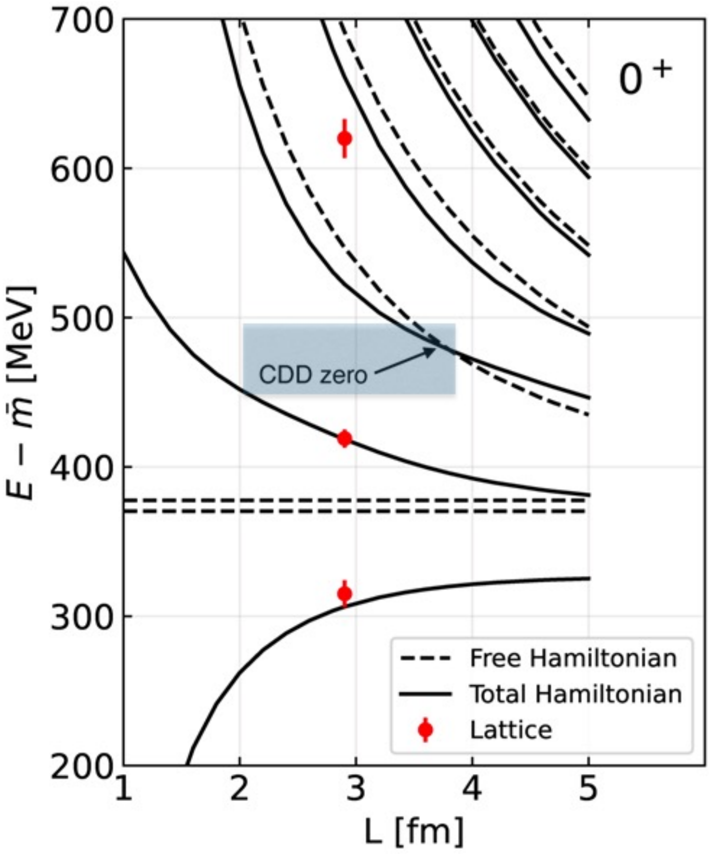
$B^*_{s1}(5770)$-$B^*\bar{K}$

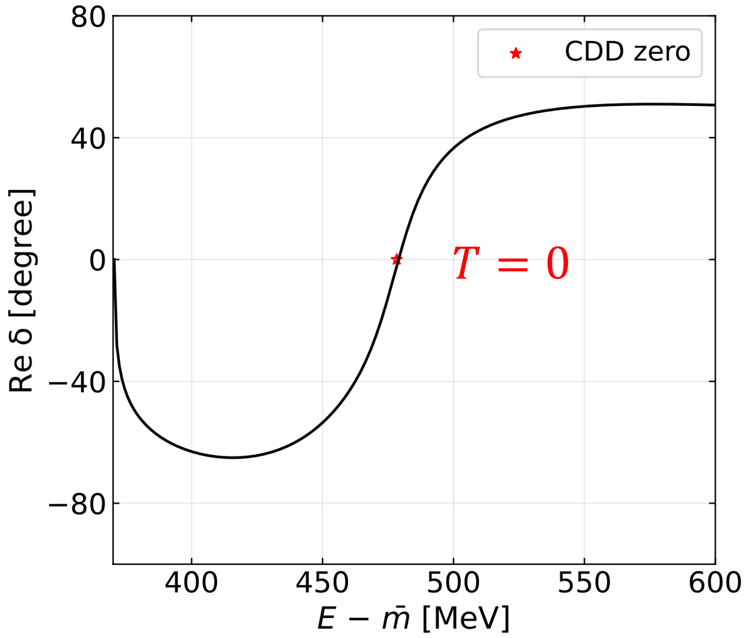
S-wave

Mass moving vs GI Model

J^P		0^+	1^+
mass [MeV]	rel. quark model [63]	5804	5842
	rel. quark model [64]	5833	5865
	rel. quark model [65]	5830	5858
	nonrel. quark model. [66]	5788	5810
	LO $\chi - SU(3)$ [18]	5643	5690
	Bardeen, Eichten, Hill [89]	5718 ± 35	5765 ± 35
	LO UChPT [24, 25]	5725 ± 39	5778 ± 7
	NLO UHMChPT [30]	$5696 \pm 20 \pm 30$	$5742 \pm 20 \pm 30$
	NLO UHMChPT [90]	5720^{+16}_{-23}	5772^{+15}_{-21}
	HQET + ChPT [67]	5706.6 ± 1.2	5765.6 ± 1.2
	Covariant ChPT [68]	5726 ± 28	5778 ± 26
	local hidden gauge [69]	$5475.4 \sim 5457.5$	$5671.2 \sim 5663.6$
	heavy meson chiral unitary [70]	5709 ± 8	5755 ± 8
	lattice QCD [91]	$5752 \pm 16 \pm 5 \pm 25$	$5806 \pm 15 \pm 5 \pm 25$
	lattice QCD [88]	$5713 \pm 11 \pm 19$	$5750 \pm 17 \pm 19$
	this work	$5730.2^{+2.4}_{-1.5}$	$5769.6^{+2.4}_{-1.6}$
$P(b\bar{s})[\%]$	heavy meson chiral unitary [70]	$48.2 \pm 1.5/54.2 \pm 1.1$	$50.3 \pm 1.4/51.7 \pm 1.3$
	this work	$54.7^{+5.2}_{-4.1}$	$56.7^{+4.6}_{-3.7}$

A CDD Zero

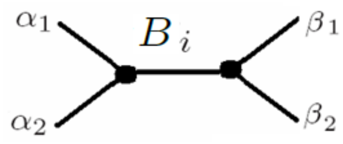




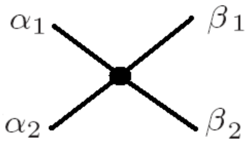
$\bar{m} = \frac{1}{4}(m_{B_s} + 3m_{B_s^*}) = 5403.3$ MeV

- The CDD zero indicates there are two mechanisms which will cancel at this energy.

g



v



- Give a new method to search CDD zero: LQCD.

Summary

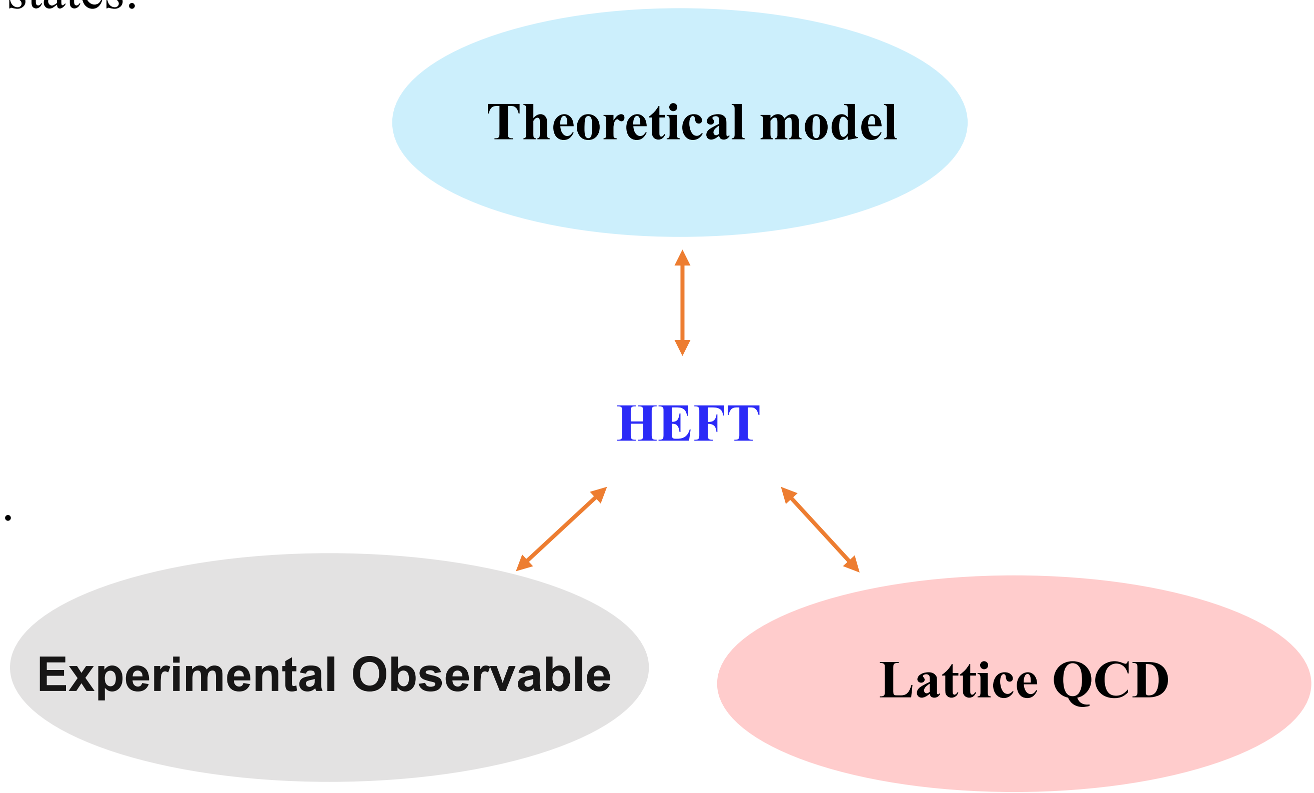
- Quark model + coupled channel effect +HEFT & Lattice QCD: two body problem.

- Investigation of inner structures of P-wave D_s states:
  $D_{s0}(2317)$ [$c\bar{s}$-DK(s-wave)],
  $D_{s1}(2460)$ [$c\bar{s}$-DK*(s-wave)],
  $D_{s1}(2536)$ [$c\bar{s}$](DK*(d-wave)),
  $D_{s2}(2573)$ [$c\bar{s}$] DK*(d-wave).

- Prediction of the $B_{s0}(5730)$ and $B_{s1}(5770)$.

- $\boldsymbol{q\bar{q}}$ and **hadron interactions are** always there.

- Extension to other near-threshold states.



Thank you for your attention!

Backup side

Outline

- Background
- Hamiltonian Effective Field Theory (HEFT)
- Study $D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, $D_{s2}(2573)$
- Predict $B_{s0}(5730)$, $B_{s1}(5770)$
- Summary

Our fit VS GI quark model

	$\bar{c}s$ cores		Coupled channels	
	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s0}^*(2317)$	$\|^3P_0\rangle$	2405.9	DK	S
$D_{s1}^*(2460)$	$0.68\|^1P_1\rangle - 0.74\|^3P_1\rangle$ $= -0.99\phi_s + 0.13\phi_d$	2511.5	D^*K	S, D
$D_{s1}^*(2536)$	$-0.74\|^1P_1\rangle - 0.68\|^3P_1\rangle$ $= -0.13\phi_s - 0.99\phi_d$	2537.8	D^*K	S, D
$D_{s2}^*(2573)$	$\|^3P_2\rangle$	2571.2	DK, D^*K	D

$J^P = 1^+$ ($D_{s1}^*(2460)$, $D_{s1}^*(2536)$)

	$\bar{c}s$ cores		Coupled channels	
$J^P = 1^+$	$B(\|^{2S+1}L_J\rangle)$	B(mass)	α	L
$D_{s1}^*(2460)$	$-0.97\|^1P_1\rangle + 0.24\|^3P_1\rangle$ $0.76\phi_s - 0.65\phi_d$	2549.7	D^*K	S, D
$D_{s1}^*(2536)$	$-0.24\|^1P_1\rangle - 0.97\|^3P_1\rangle$ $= -0.65\phi_s - 0.76\phi_d$	2559.46	D^*K	S, D

- GI model: Two 1^+ $\bar{c}s$ meson almost on the basis $^{2S+1}L_J$.
- Our fit: good HQS. Two 1^+ $\bar{c}s$ meson are almost on the heavy quark spin basis.

$$\phi_s = \left|\frac{1}{2}_l \otimes \frac{1}{2}_h\right\rangle \Rightarrow \text{S-wave } D^{(*)}K$$

$$\phi_d = \left|\frac{3}{2}_l \otimes \frac{1}{2}_h\right\rangle \Rightarrow \text{D-wave } D^{(*)}K$$

Quark model: bare $\bar{c}s$ meson

The relativized quark model:

$$V = G_{\text{eff}}(r) + S_{\text{eff}}(r)$$

1. $G(r) = -\frac{4\alpha_s(r)}{3r}$
$S(r) = br + c$

smearing →

$$\tilde{G}(r) = -\sum_k \frac{4\alpha_k}{3r}\left[\frac{2}{\sqrt{\pi}}\int_0^{\tau_{kij'}} e^{-x^2}dx\right]$$

$$\tilde{\mathbf{S}}(r) = br[\frac{e^{-\sigma_{ij}^2 r^2}}{\sqrt{\pi}\sigma_{ij} r} + \left[1 + \frac{1}{2\sigma_{ij}^2 r^2}\right]\frac{2}{\sqrt{\pi}}\int_0^{\sigma_{ij} r} e^{-x^2}dx + c$$

Smearing : $\tilde{f}_{ij}(r) \equiv \int d^3r' \rho_{ij}(\mathbf{r}-\mathbf{r}') f(r')$ with $\rho_{ij}(\mathbf{r}-\mathbf{r}') = \frac{\sigma_{ij}^3}{\pi^{3/2}} e^{-\sigma_{ij}^2(\mathbf{r}-\mathbf{r}')^2}$

$$\tilde{G}(r) \to \left(1 + \frac{p^2}{E\bar{E}}\right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E\bar{E}}\right)^{1/2}$$

$$\frac{\tilde{V}_i(r)}{m_1 m_2} \to \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon_i} \frac{\tilde{V}_i(r)}{m_1 m_2} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon_i}$$

Godfrey, Isgur, Phys. Rev. D 32,189 (1985)

Hamiltonian effective field theory (HEFT)

- Coupled channel effect: 2→2 scattering process,

$$D^{(*)}K \to D^{(*)}K$$

- The scattering amplitude cannot be extracted from experiments and need lattice QCD data.
- The result is helpful in the relevant analysis of experimental processes, e.g.,

$$B_s/B \to D^{(*)}D^{(*)}K \text{ or } D^{(*)}KK$$