Two bare states and multi-channel analysis in HEFT for Δ(1232) and N*(1535)

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University of Chinese Academy of Sciences

Outline

- Introduction of HEFT : 王广娟’s talk
- Analysis of N*(1535) and N*(1650)
- Summary

Motivation

Resonance
(Mass , Width, Pole position, Coupling)

Partial Wave Analysis
With various Model:
Such as Breit-Wigner Form, Flatte Form, Coupled Channel Form, and so on

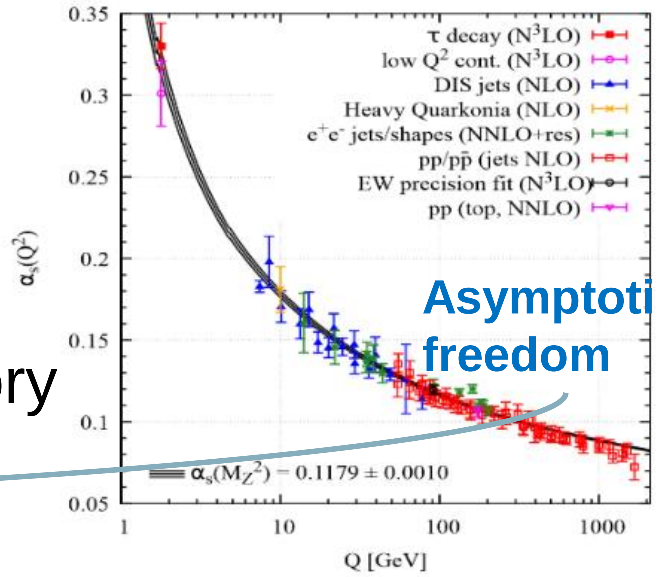
Confinement

?

Non-perturbative

Experimental Observable

QCD theory



Asymptotic freedom

Perturbative

Introduction of HEFT

J. M. M. Hall etc. PRD 87(2013), 094510
J.-j. Wu etc. PRC90 (2014), 055206
Y. Li etc. PRD 101(2020), 114501
PRD 103(2021), 094518

$$H = H_0 + H_I$$

$$H_0 = \sum_{i=1,n} |B_i\rangle m_i \langle B_i| + \sum_{\alpha} |\alpha(k_\alpha)\rangle \left[\sqrt{m_{\alpha 1}^2 + k_\alpha^2} + \sqrt{m_{\alpha 2}^2 + k_\alpha^2}\right] \langle \alpha(k_\alpha)|$$

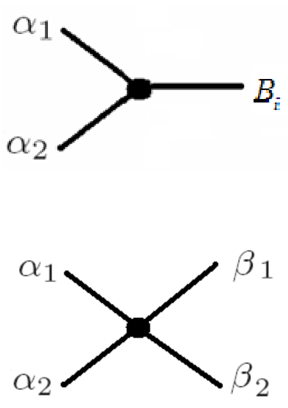
$|B_i>$ bare state， bare mass m_i

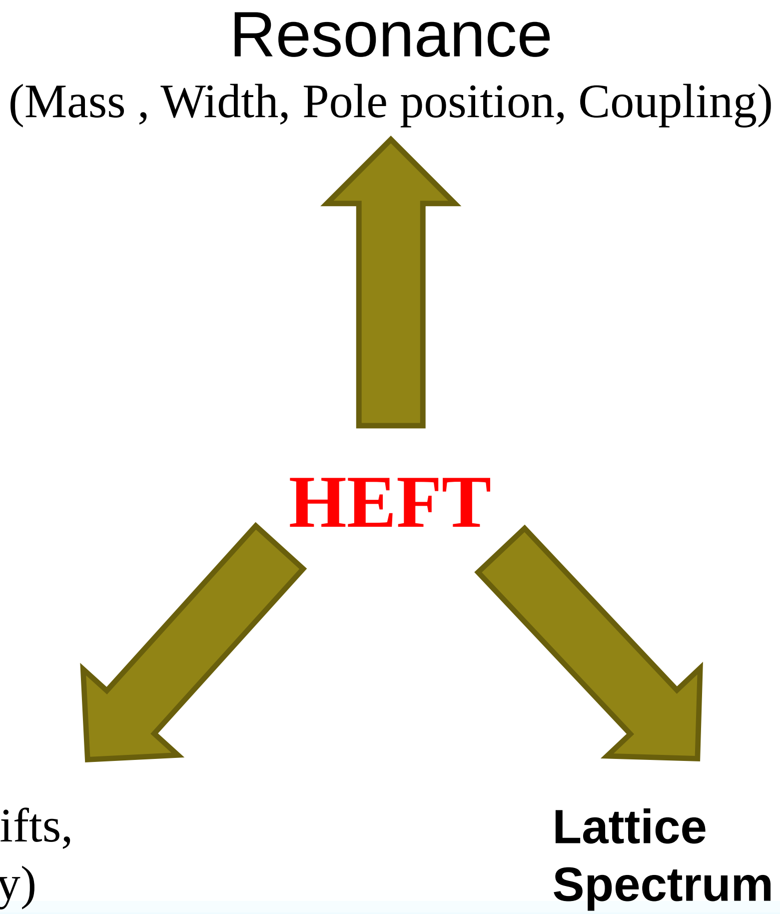
$|\alpha(k_\alpha)>$ non-interaction channels

$$H_I = \hat{g} + \hat{v}$$

$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[|\alpha(k_\alpha)\rangle g_{i,\alpha}^{+} \langle B_i| + |B_i\rangle g_{i,\alpha} \langle \alpha(k_\alpha)| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} |\alpha(k_\alpha)\rangle v_{\alpha,\beta} \langle \beta(k_\beta)|$$



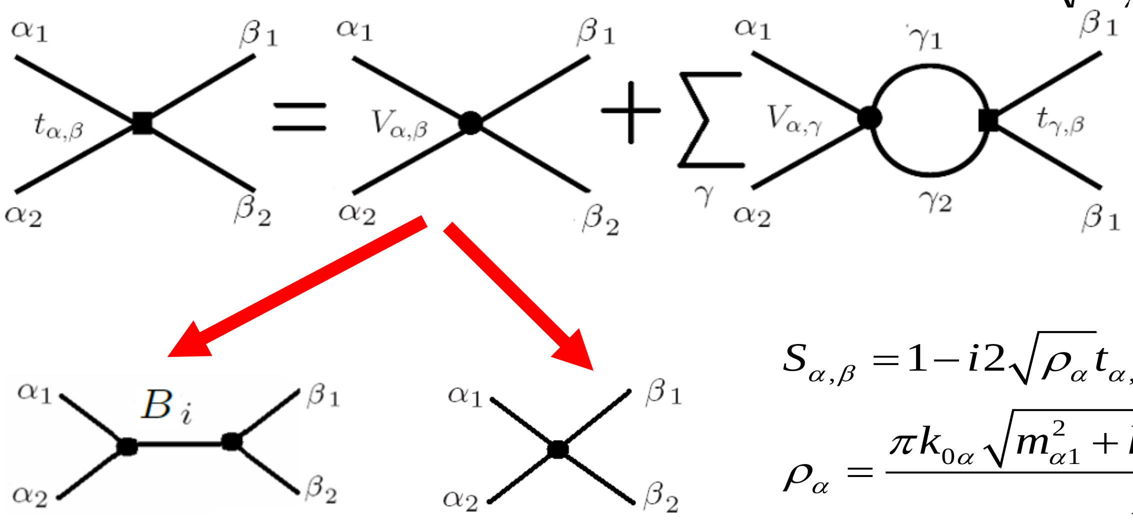


Introduction of HEFT

Argonne-Osaka Model

- **T Matrix:**

$$t_{\alpha,\beta}(k_\alpha,k_\beta,E)=V_{\alpha,\beta}(k_\alpha,k_\beta)+\sum_\gamma\int k_\gamma^{\ 2}dk_\gamma\frac{V_{\alpha,\gamma}(k_\alpha,k_\gamma)t_{\gamma,\beta}(k_\gamma,k_\beta,E)}{E-\sqrt{m_{\gamma1}^2+k_\gamma^2}-\sqrt{m_{\gamma2}^2+k_\gamma^2}+i\varepsilon}$$



$$g_{i,\alpha}^{*}\frac{1}{E-m_i}g_{i,\beta}$$

$$v_{\alpha,\beta}$$

$$S_{\alpha,\beta}=1-i2\sqrt{\rho_\alpha}t_{\alpha,\beta}(k_{0\alpha},k_{0\beta},E)\sqrt{\rho_\beta}$$

$$\rho_\alpha=\frac{\pi k_{0\alpha}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}\sqrt{m_{\alpha1}^2+k_{0\alpha}^2}}{E}$$

$$\eta e^{2i\delta_\alpha}=S_{\alpha,\alpha}$$

Resonance
(Mass , Width, Pole position, Coupling)

HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

Introduction of HEFT

J. M. M. Hall etc. PRD 87(2013), 094510
J.-j. Wu etc. PRC90 (2014), 055206

- **Hamiltonian with discrete momentum**

Continuous ↓ Discrete

$$\int d\vec{k} \quad \text{and} \quad |\alpha(\vec{k}_\alpha)\rangle \quad \text{and} \quad \langle\beta(\vec{k}_\beta)|\alpha(\vec{k}_\alpha)\rangle = \delta_{\alpha\beta}\delta(\vec{k}_\alpha - \vec{k}_\beta)$$

$$\downarrow$$

$$\sum_{\mathrm{i}}\left(\frac{2\pi}{\mathrm{L}}\right)^3 \quad \text{and} \quad \left(\frac{2\pi}{\mathrm{L}}\right)^{-3/2}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha \quad \text{and} \quad {}_\beta\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha = \delta_{\alpha\beta}\delta_{\mathrm{ij}}$$

$$H_0 = \sum_{i=1,n}|B_i\rangle m_i\langle B_i| + \sum_{\alpha,\mathrm{i}}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha\left[\sqrt{m_{\alpha_B}^2+k_\alpha^2}+\sqrt{m_{\alpha_M}^2+k_\alpha^2}\right]_\alpha\langle\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}|$$

$$H_I = \sum_{\mathrm{j}}\left(\frac{2\pi}{\mathrm{L}}\right)^{3/2}\sum_\alpha\sum_{i=1,n}\left[|\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}\rangle_\alpha g_{i,\alpha}^{+}\langle B_i| + |B_i\rangle g_{i,\alpha}\,{}_\alpha\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}|\right] + \sum_{\mathrm{i,j}}\left(\frac{2\pi}{\mathrm{L}}\right)^3\sum_{\alpha,\beta}|\vec{k}_{\mathrm{i}},-\vec{k}_{\mathrm{i}}\rangle_\alpha v_{\alpha,\beta}\,{}_\beta\langle\vec{k}_{\mathrm{j}},-\vec{k}_{\mathrm{j}}|$$

$$[H_0]_{N_c+1} = \begin{pmatrix} m_0 & 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & \epsilon_1(k_0) & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \epsilon_2(k_0) & \cdots & 0 & 0 & \cdots \\ \vdots & & & \ddots & & & \\ 0 & 0 & 0 & \ddots & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots & \epsilon_{n_c}(k_0) & 0 & \cdots \\ 0 & 0 & 0 & \cdots & 0 & \epsilon_1(k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$[H_I]_{N_c+1} = \begin{pmatrix} 0 & g_1^V(k_0) & g_2^V(k_0) & \cdots & g_{n_c}^V(k_0) & g_1^V(k_1) & \cdots \\ g_1^V(k_0) & v_{1,1}^V(k_0,k_0) & v_{1,2}^V(k_0,k_0) & \cdots & v_{1,n_c}^V(k_0,k_0) & v_{1,1}^V(k_0,k_1) & \cdots \\ g_2^V(k_0) & v_{2,1}^V(k_0,k_0) & v_{2,2}^V(k_0,k_0) & \cdots & v_{2,n_c}^V(k_0,k_0) & v_{2,1}^V(k_0,k_1) & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots \\ g_{n_c}^V(k_0) & v_{n_c,1}^V(k_0,k_0) & v_{n_c,2}^V(k_0,k_0) & \cdots & v_{n_c,n_c}^V(k_0,k_0) & v_{n_c,1}^V(k_0,k_1) & \cdots \\ g_1^V(k_1) & v_{1,1}^V(k_1,k_0) & v_{1,2}^V(k_1,k_0) & \cdots & v_{1,n_c}^V(k_1,k_0) & v_{1,1}^V(k_1,k_1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$(H_0+H_I)\,|\Psi\rangle = E\,|\Psi\rangle$$

Eigen-Value ↔ Lattice Spectrum

Eigen-Vector

Resonance
(Mass , Width, Pole position, Coupling)

Complex Continuum Momentum Space

HEFT

Real Continuum Momentum Space

T matrix
(Phase Shifts, inelasticity)

Real Discrete Momentum Space

Lattice Spectrum

Introduction of HEFT

T matrix
(Phase Shifts, inelasticity)

Lattice Spectrum

HEFT

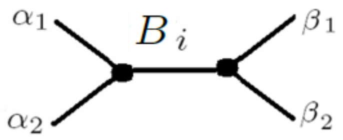
Resonance
(Mass , Width, Pole position, Coupling)

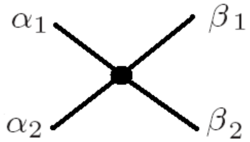
1. Bulid Model

2. Fix Para.

3. Extract Phys.

Unphysical π mass ??





HEFT:
1. Build a Hamiltonian model;

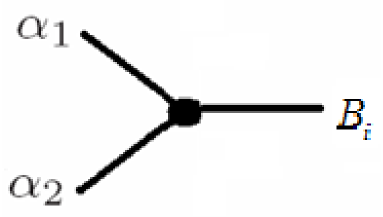
2. If Experimental data available, we fit Experimental data to fix the parameters in the model;
   If Lattice data available (close to physical pion mass), we fit these data;
   If both, we can use both of them constraint the model parameters.
   If we only have Lattice data with unphysical pion mass, we need another parameter for the mass dependence, such as mass slope.

3. From the fixed Hamiltonian, we can study the properties of Resonance. Especially, from the eigenvector in the finite volume, we can estimate the internal structure of the hadron.

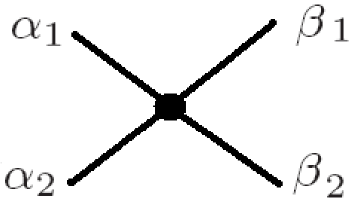
N*(1535)

Zhan-wei Liu etc. Phys.Rev.Lett. 116 (2016) no.8, 082004

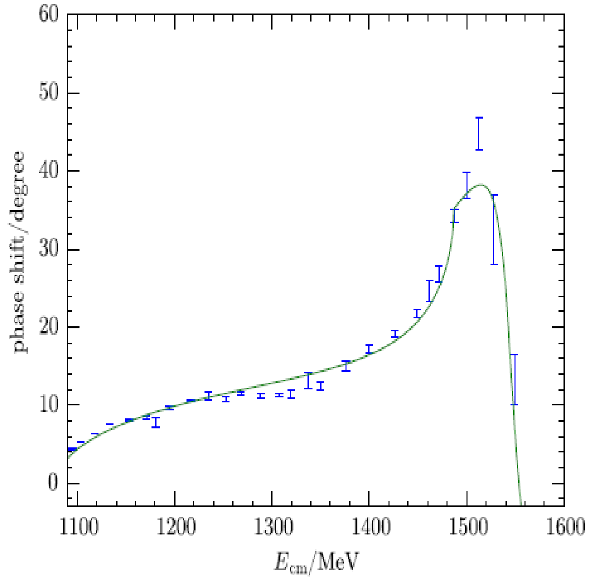
2 Channels: πN and ηN

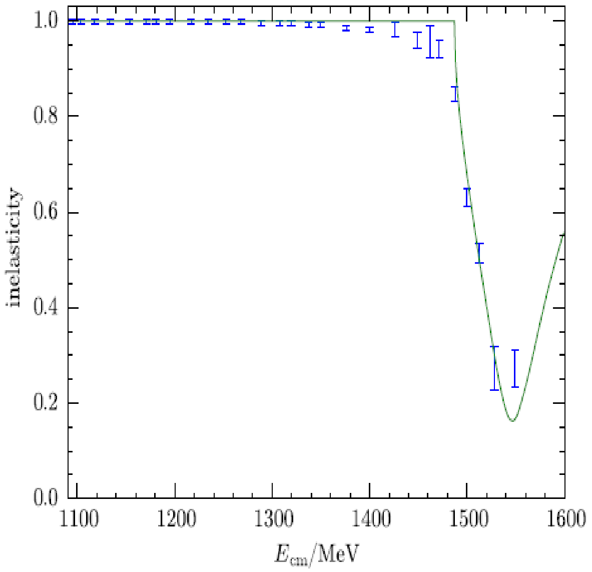


$$G_{iN}^2(k) = \left(3g_{N_0^* iN}^2/4\pi^2 f^2\right)\omega_i(k)u^2(k)$$



$$\frac{3g_{\pi N}^S \tilde{u}(k)\tilde{u}(k')}{4\pi^2 f^2}$$





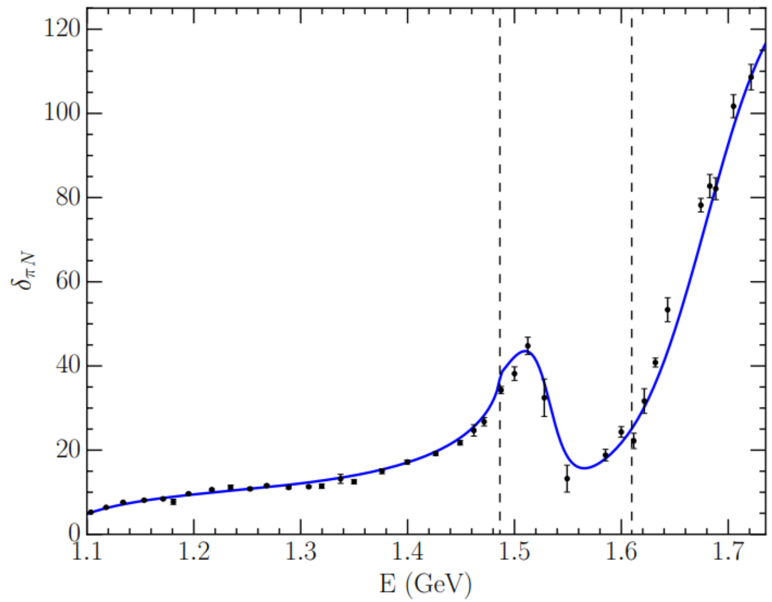
$$g_{\pi N}^S = -0.0608 \pm 0.0004$$
$$m_0 = 1601 \pm 14 \text{ MeV}$$
$$g_{N_0^* \pi N} = 0.186 \pm 0.006$$
$$g_{N_0^* \eta N} = 0.185 \pm 0.017,$$

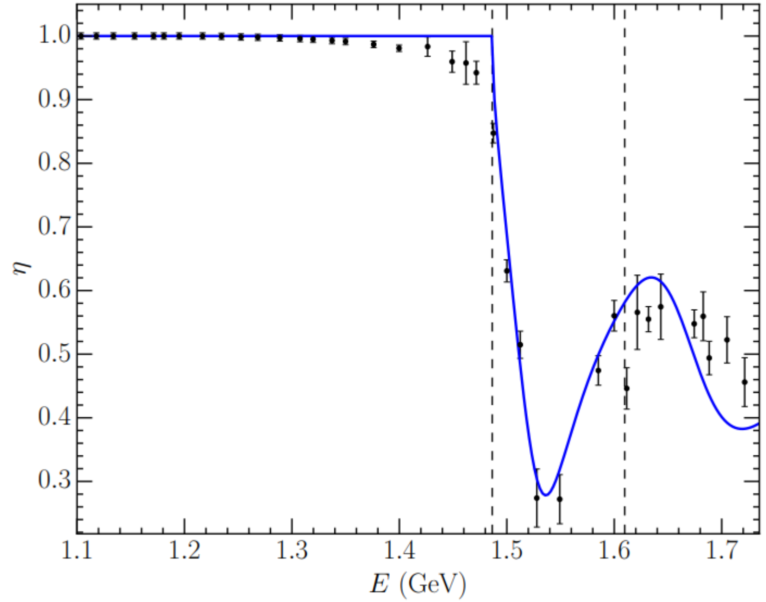
$$\chi^2_{\mathrm{DOF}} = 6.8$$

$$1531 \pm 29 - i88 \pm 2 \text{ MeV}$$

For N*(1535)

Parameter	Value	Parameter	Value
$m_{N_1}^{(0)}$ / GeV	1.6301	$m_{N_2}^{(0)}$ / GeV	1.8612
$g_{\pi N}^{N_1}$	0.0898	$g_{\pi N}^{N_2}$	0.2181
$g_{\eta N}^{N_1}$	0.1525	$g_{\eta N}^{N_2}$	0.0009
$g_{K\Lambda}^{N_1}$	0.0000	$g_{K\Lambda}^{N_2}$	-0.2367
$\Lambda_{\pi N}^{N_1}$ / GeV	1.2335	$\Lambda_{\pi N}^{N_2}$ / GeV	1.4000
$\Lambda_{\eta N}^{N_1}$ / GeV	1.2642	$\Lambda_{\eta N}^{N_2}$ / GeV	0.9521
$\Lambda_{K\Lambda}^{N_1}$ / GeV	...	$\Lambda_{K\Lambda}^{N_2}$ / GeV	0.7283
$v_{\pi N,\pi N}$	-0.0655	$v_{\eta N,\eta N}$	-0.0245
$v_{\pi N,\eta N}$	0.0388	$v_{\eta N,K\Lambda}$	0.0320
$v_{\pi N,K\Lambda}$	-0.0757	$v_{K\Lambda,K\Lambda}$	0.1371
$\Lambda_{v,\pi N}$ / GeV	0.6000	$\Lambda_{v,\eta N}$ / GeV	0.9036
$\Lambda_{v,K\Lambda}$ / GeV	0.6060		





$$G_\alpha^{N_i}(k) = \frac{\sqrt{3}\, g_\alpha^{N_i}}{2\pi f_\pi} \sqrt{\omega_{M_\alpha}(k)}\, u(k)\,, \quad V_{\alpha\beta}(k,k') = \frac{3\, v_{\alpha\beta}}{4\pi^2 f_\pi^2}\, \tilde{u}(k)\, \tilde{u}(k')\,, \quad \tilde{u}(k) = \frac{\omega_\pi(k) + m_\pi^{\text{phys}}}{\omega_\pi(k)}\, u(k)\,.$$

We consider three channels: $\pi N, \eta N, K\Lambda$

$$E_{N^*(1535)} = 1510 \pm 10 - (65 \pm 10)i \text{ MeV}\,,$$
$$E_{N^*(1650)} = 1655 \pm 15 - (67 \pm 18)i \text{ MeV}\,.$$

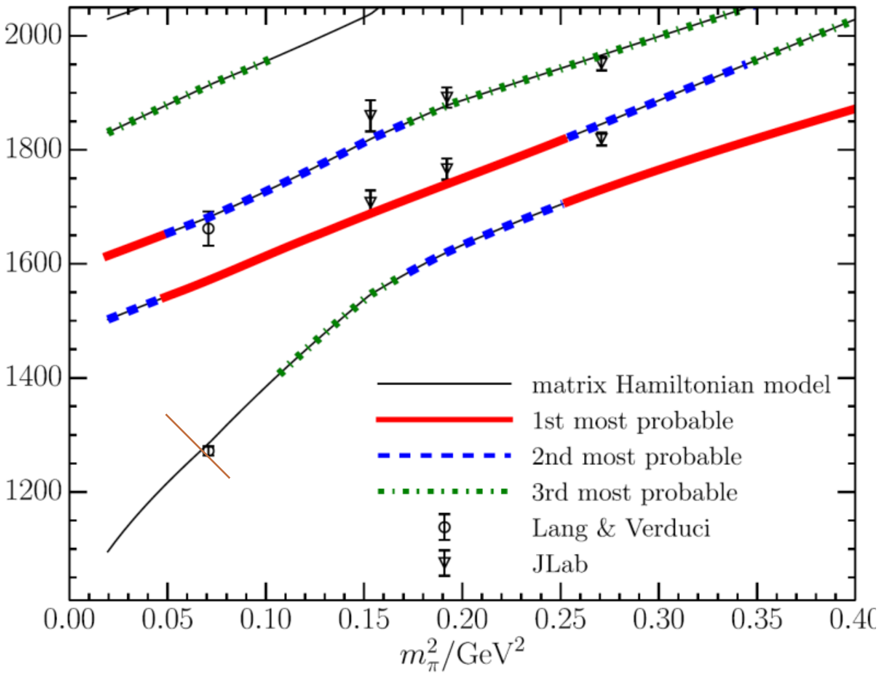
EXP

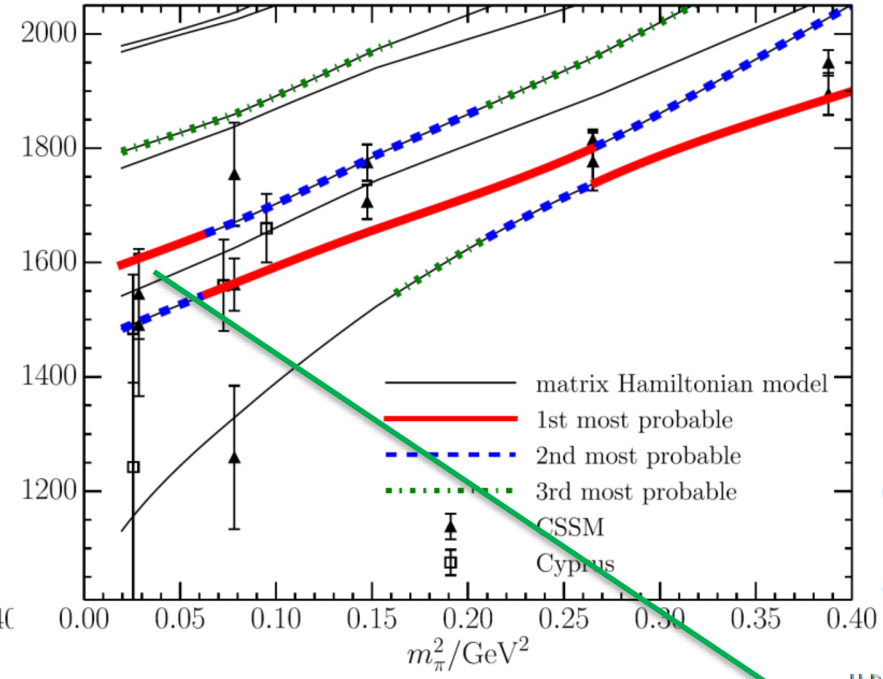
$$E_1 = 1500 - 50i \text{ MeV}\,,$$
$$E_2 = 1658 - 56i \text{ MeV}\,,$$

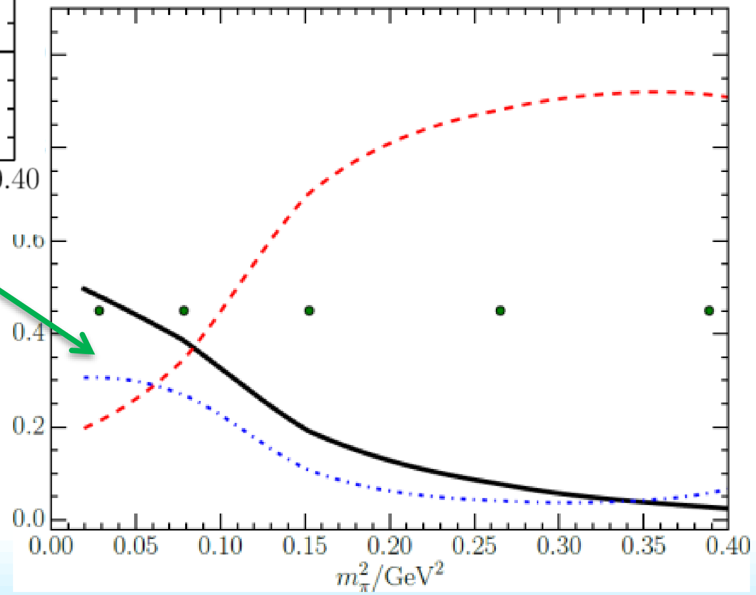
HEFT

N*(1535)

Zhan-wei Liu etc. Phys.Rev.Lett. 116 (2016) no.8, 082004







The main components (at least 50%) of N*(1535) is from the 3 quark core.

For N*(1535)

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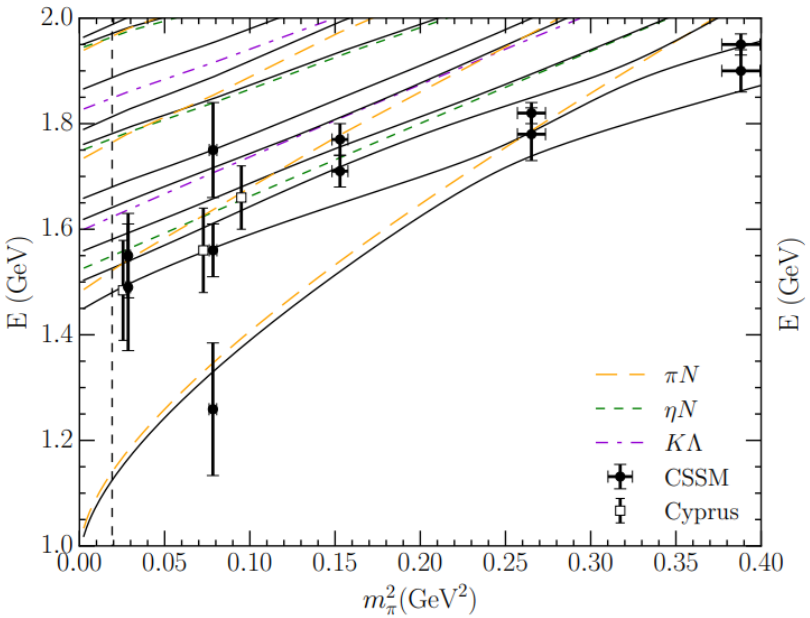
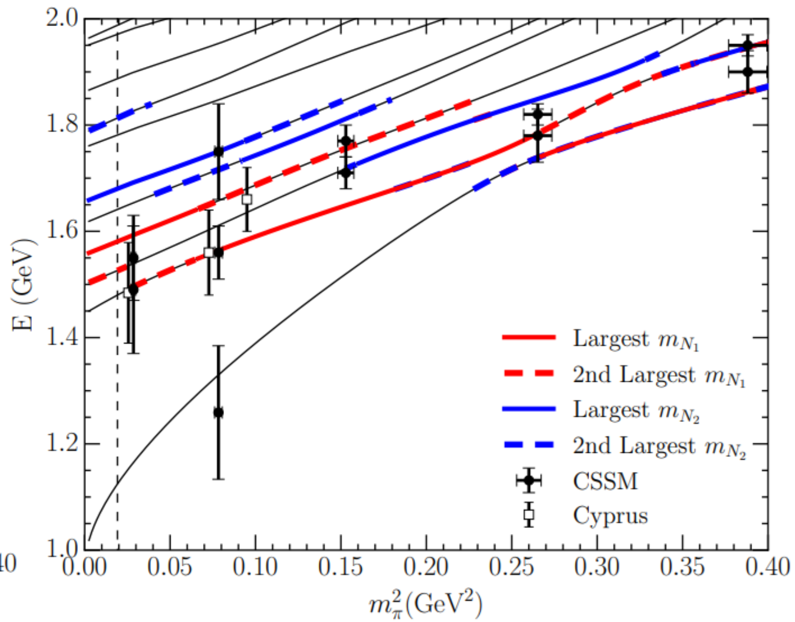
$$m_{N_i}(m_\pi^2) = m_{N_i}^{(0)} + \alpha_{N_i}\left(m_\pi^2 - m_\pi^2|_{\text{phys}}\right),$$

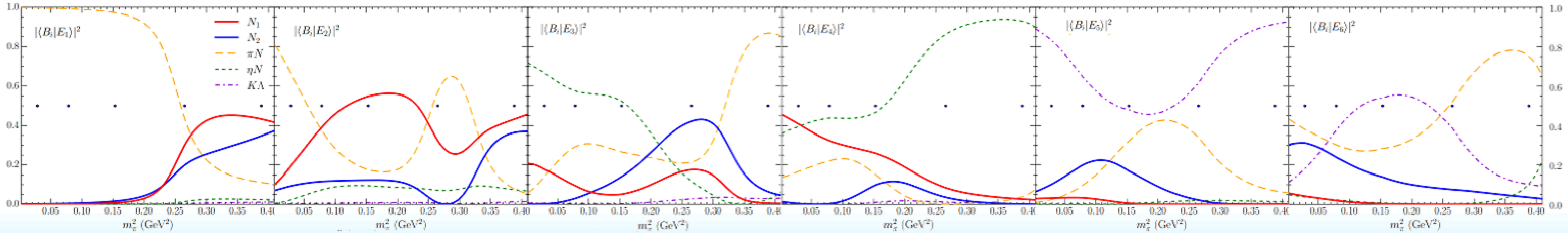
$$\alpha_{N_1} = 0.944\,\text{GeV}^{-1}, \quad \alpha_{N_2} = 0.611\,\text{GeV}^{-1}.$$

Fitting

L ~ 3 fm **Fitting**







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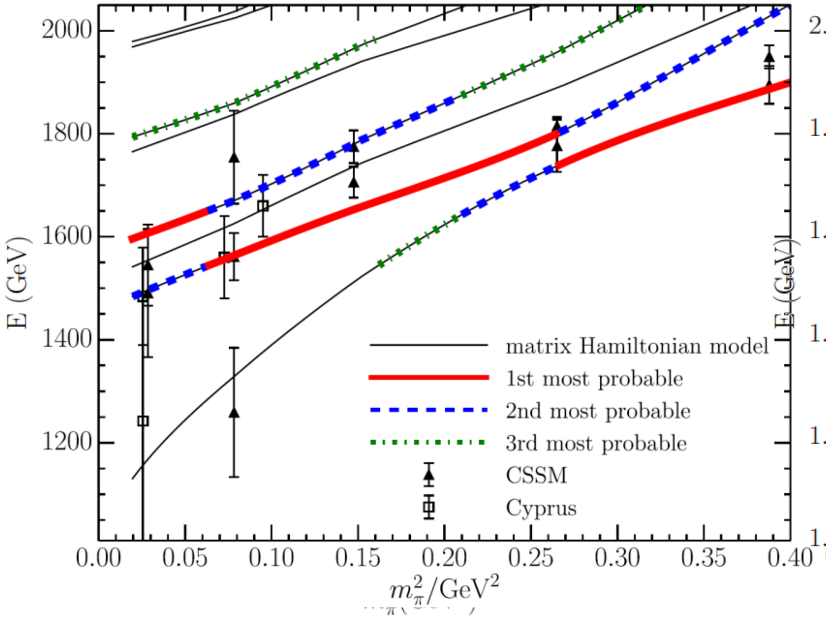
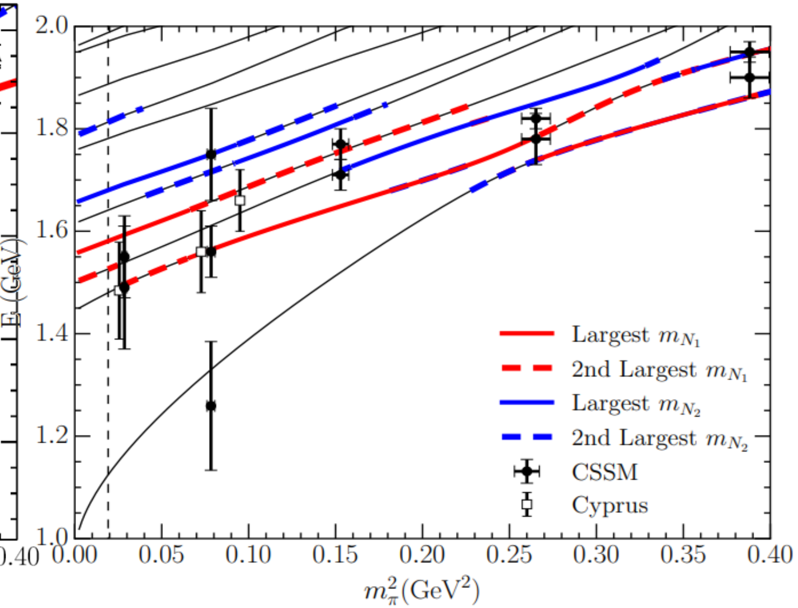
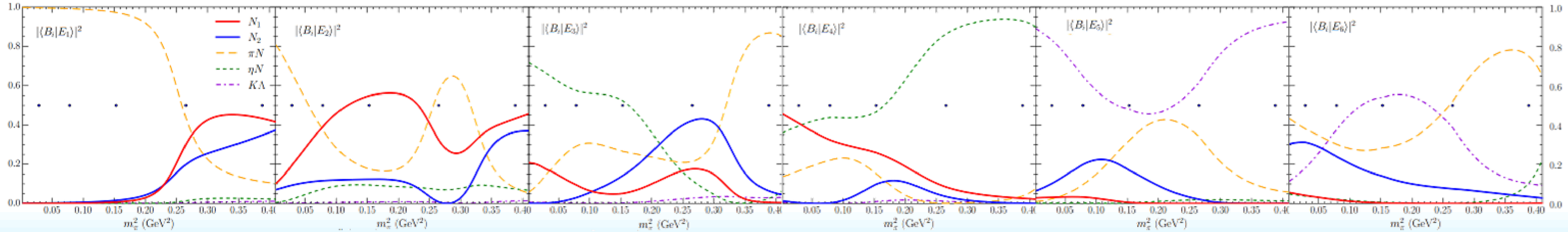


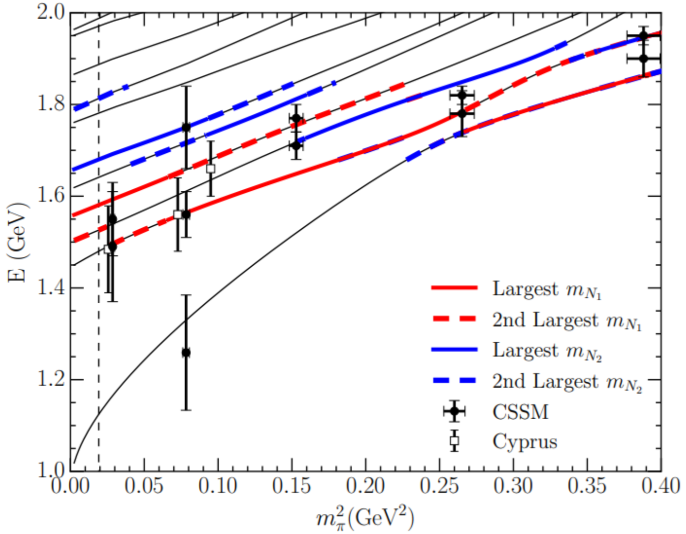




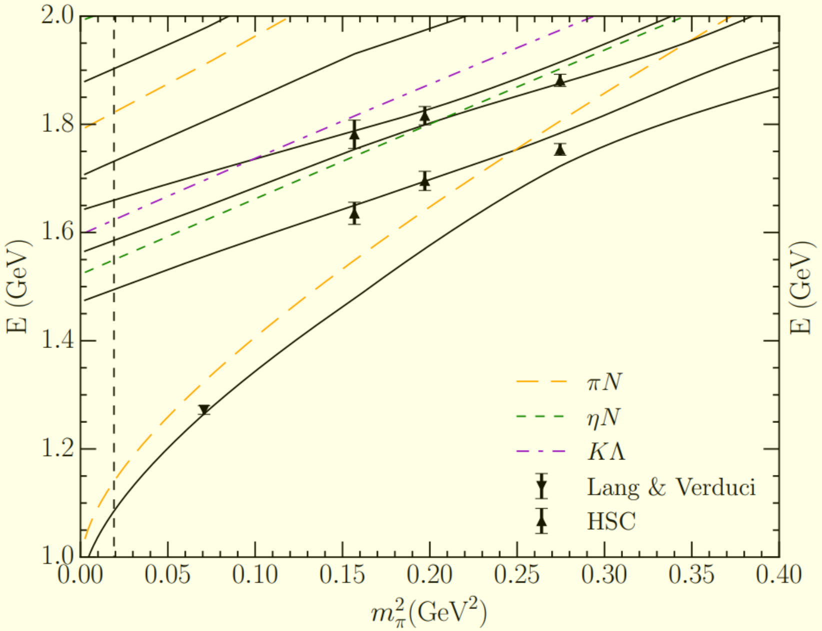
L ~ 3 fm **Fitting**

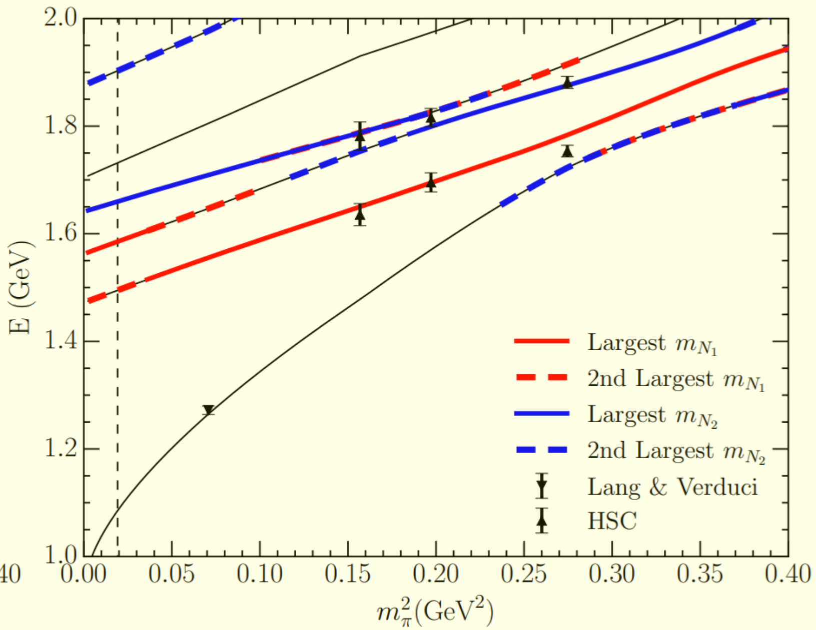
1. Scattering state & 3q state
2. Two clear states

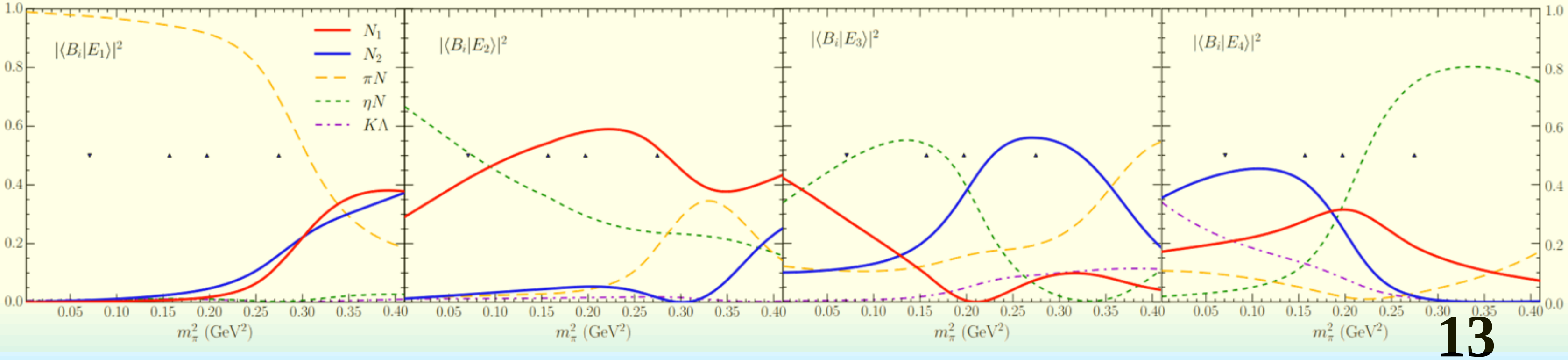
For N*(1535)



L ~ 2 fm **Not Fit**







L ~ 4.05 fm
Not Fit

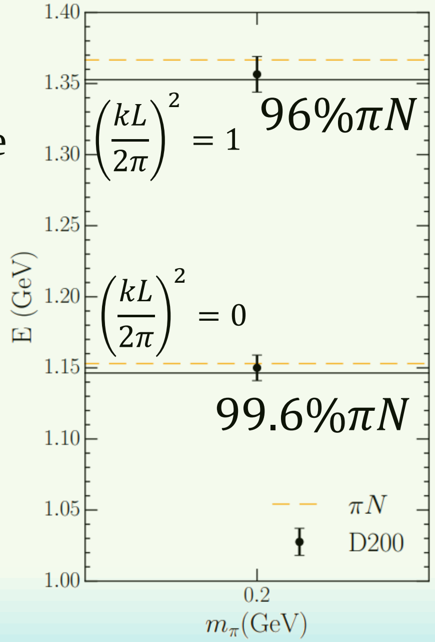
Coordinated Lattice Simulations (CLS) consortium

D200 ensemble

a[fm]	$(L/a)^3 \times T/a$
0.0633(4)(6)	$64^3 \times 128$

am_π	am_K	am_N
0.06617(33)	0.15644(16)	0.3148(23)

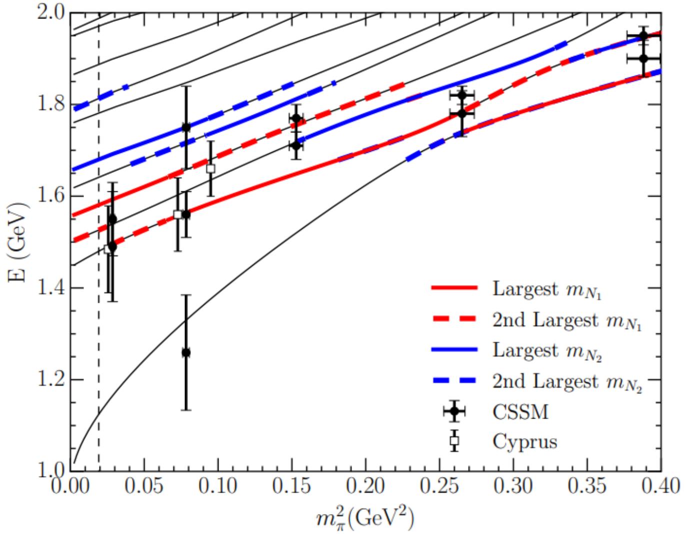
af_π	af_K
0.04233(16)	0.04928(21)



For N*(1535)

1. Scattering state & 3q state
2. Two clear states

L ~ 3 fm **Fitting**



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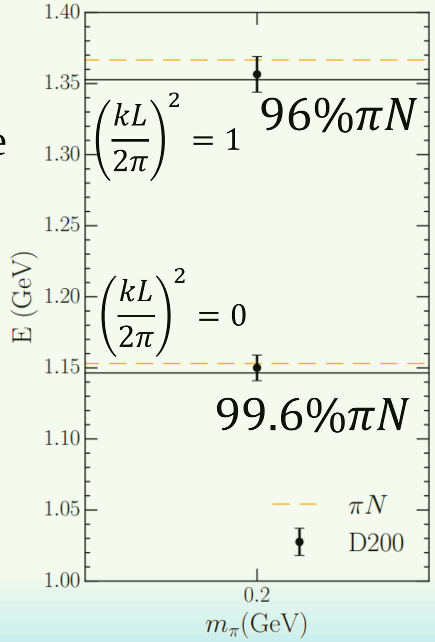
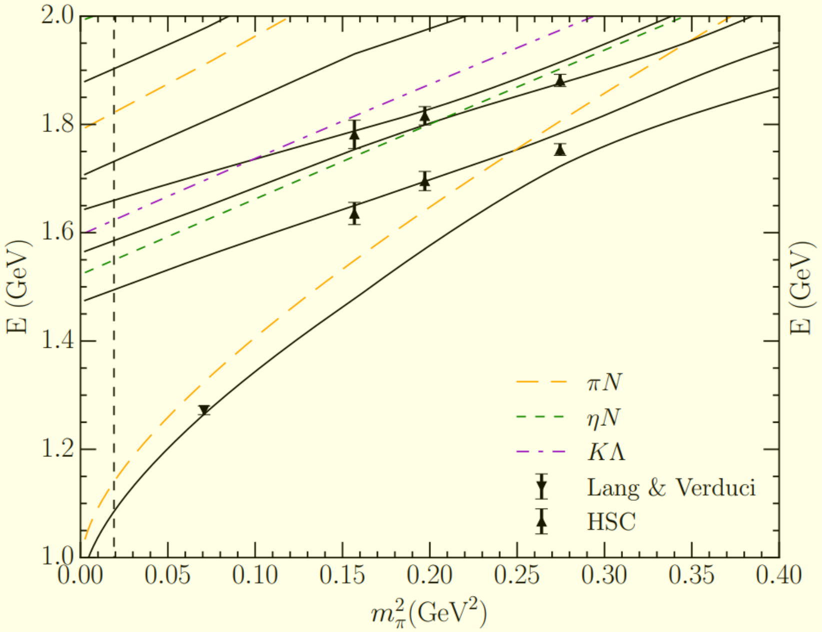
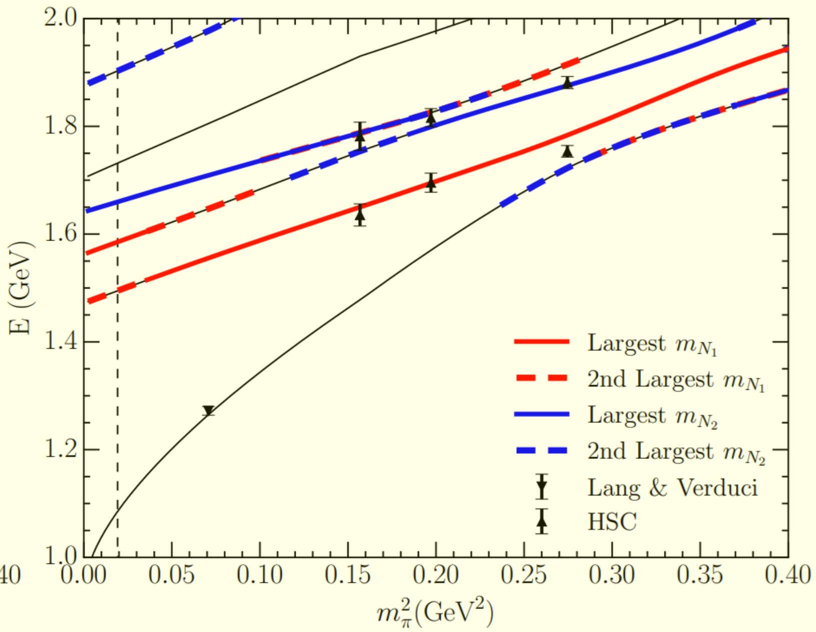
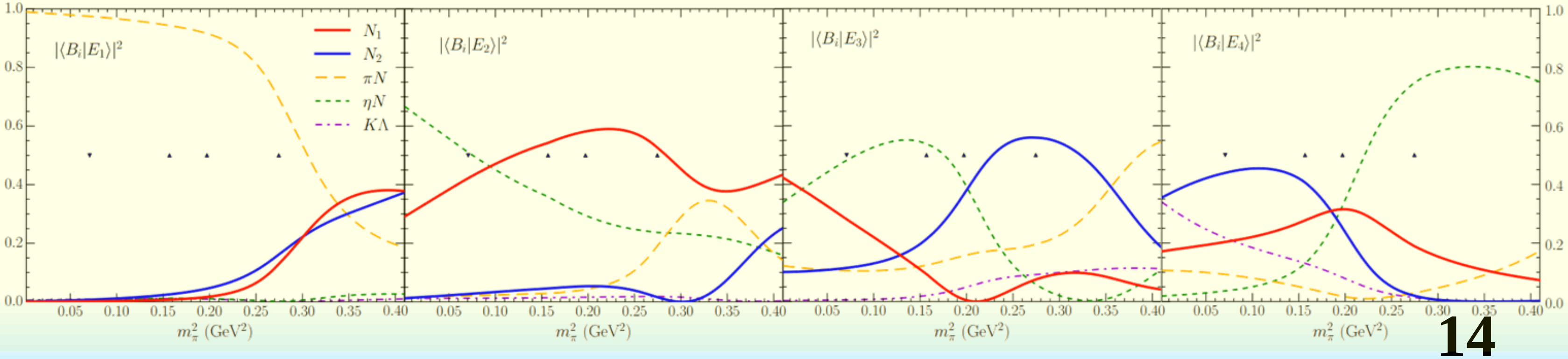


L ~ 2 fm **Not Fit**







For N*(1535)

Define "Contamination Function" to compare HEFT VS LQCD

For One bare states: $\bar{\chi}(0)\,|\Omega\rangle = |B_0\rangle$

Correlation Function: $G_\chi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \langle\bar{\Omega}|\chi(\boldsymbol{x},t)\,\bar{\chi}(0,0)|\Omega\rangle$,

$$G_\chi(t) = \sum_i |\langle\Omega|\chi|E_i\rangle|^2 e^{-E_i t}, \quad G_{B_0}(t) = \sum_i |\langle B_0|E_i\rangle|^2 e^{-E_i t}$$

Contamination function: $C_{B_0}(t) = \frac{1}{G_{B_0}(t)} \sum_{i\neq B_0} |\langle B_0|E_i\rangle|^2 e^{-E_i t}$

If $|E_{B0}\rangle$ is a ground state, $C_{B0} \sim 0$.

If $|E_{B0}\rangle$ is a excited state, $C_{B0}(t)$ will have a minimal value as function of t.

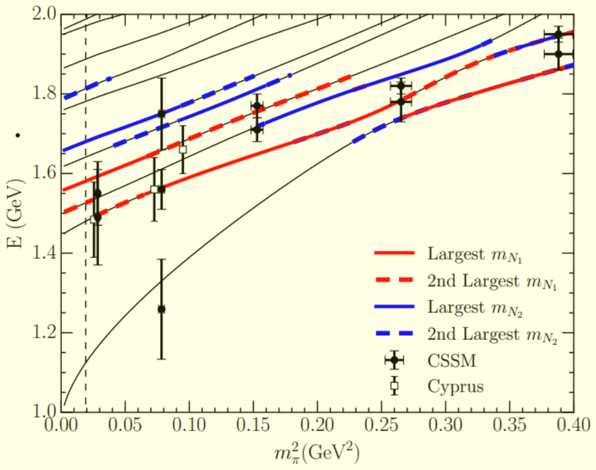
For Two bare states: $(\alpha^*\,\overline{\chi_1} + \beta^*\,\overline{\chi_2})\,|\Omega\rangle = \alpha^*\,|N_1\rangle + \beta^*\,|N_2\rangle$,

Correlation Function: $G_j(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \langle\bar{\Omega}|\,(\alpha_j\,\chi_1(\boldsymbol{x},t) + \beta_j\,\chi_2(\boldsymbol{x},t)\big(\alpha_j^*\,\bar{\chi}_1(0) + \beta_j^*\,\bar{\chi}_2(0)\big)\,|\Omega\rangle$.

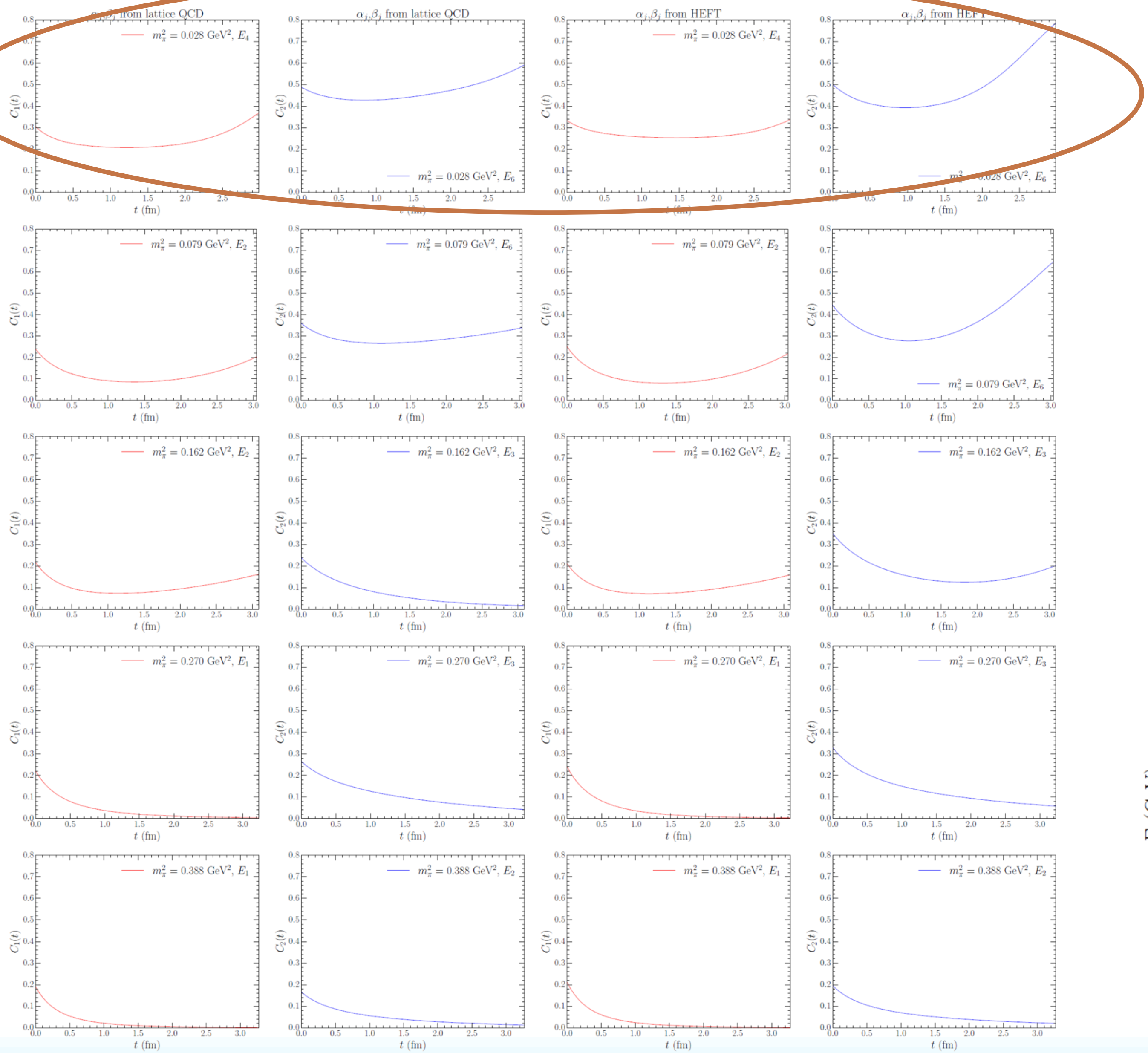
$$G_j(t) = \sum_i (\alpha_j\,\langle N_1| + \beta_j\,\langle N_2|)\,|E_i\rangle\,\langle E_i|\,\big(\alpha_j^*\,|N_1\rangle + \beta_j^*\,|N_2\rangle\big)\;e^{-E_i t},$$

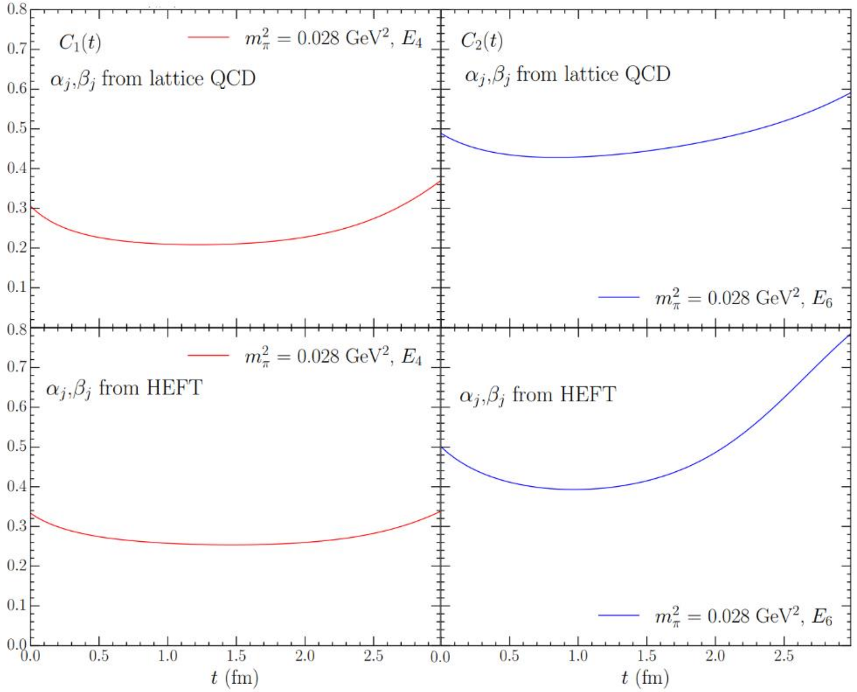
$$= \sum_i |\alpha_j\,\langle N_1|E_i\rangle + \beta_j\,\langle N_2|E_i\rangle|^2\;e^{-E_i t}.$$

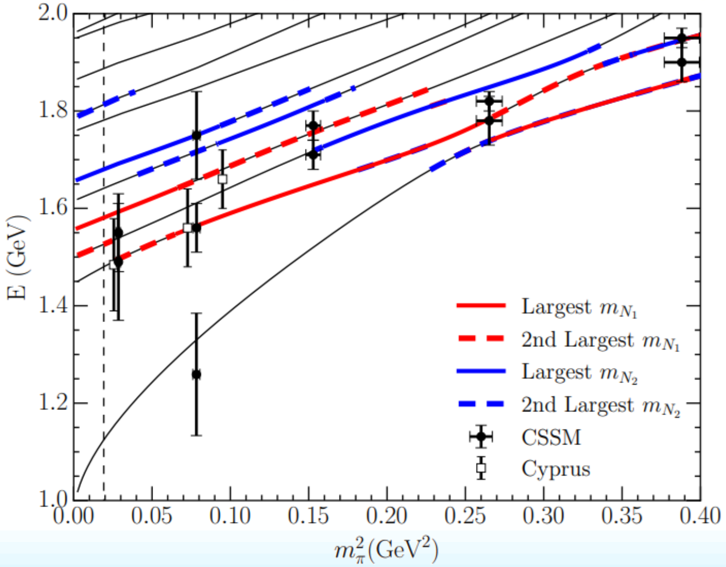
Contamination function: $C_j(t) = \frac{1}{G_j(t)} \sum_{i\neq N_1,N_2} (\alpha_j\,\langle N_1|E_i\rangle + \beta_j\,\langle N_2|E_i\rangle)^2\,e^{-E_i t}$



$$\alpha_1 = \langle N_1|E_{N_1}\rangle, \quad \beta_1 = \langle N_2|E_{N_1}\rangle,$$
$$\alpha_2 = \langle N_1|E_{N_2}\rangle, \quad \beta_2 = \langle N_2|E_{N_2}\rangle.$$







This result shows that HEFT is quiet consistent with Lattice data.

Summary

- **Here we find that the interpretation of the two resonances as three-quark cores dressed by scattering-state dynamics is consistent with the L ~ 3 fm lattice calculations.**

- **To extend to the L ~ 2 fm and 4 fm are both quiet good.**

- **We define a “contamination function” related to the overlap of bare states with eigenstates, then we compare this function by Lattice input and HEFT results.**

- **All of these consistent comparisons show that the results of HEFT correctly reflect the structure of hadron from experimental and lattice data.**

Thanks for attention!