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Axion-meson mixing in light of recent lattice light-flavor pseudoscalar meson simulations

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Background

Strong CP problem

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \to e^{i\gamma_5\alpha}q \quad \longrightarrow \quad \theta_q \to \theta_q + 2\alpha\,, \qquad \theta \to \theta - 2\alpha$$

implying the invariant quantity: $\bar{\theta} = \theta + \theta_q$

$$\Rightarrow \quad \mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Constraints from neutron electric dipole moment (nEDM)

$$|\bar{\theta}| \lesssim 10^{-10}$$

- **Strong CP problem: why is $\bar{\theta}$ so unnaturally tiny ?**

Axion: an elegant solution to strong CP

[Peccei,Quinn,PRL'77]

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

minium: $\langle a(x) + \bar{\theta} f_a \rangle = 0$ $\quad a(x) \to a(x) - \langle a(x) \rangle$

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- **f_a: the axion decay constant. Invisible axion: $f_a >> v_{\rm EW}$. Axion interactions with SM particles are accompanied with the factors of $(1/f_a)^n$.**

- **It is possible to introduce other model-dependent axion interactions, such as axion-quark and axion-photon. We will focus on the MODEL INDEPENDENT QCD axion interactions.**

- **Axion-hadron and axion-photon interactions are relevant at low energies.**

Axion chiral perturbation theory (AχPT)

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the *a*GG term via the quark axial transformation

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$${\rm Tr}(Q_a) = 1$$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q \qquad M_q \to M_q(a) = e^{i\frac{a}{2f_a}Q_a} M_q e^{i\frac{a}{2f_a}Q_a}$$

Mapping to χPT

$$\mathcal{L}_2 = \frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a}J_A^\mu\big|_{\rm LO}$$

$$\chi_a = 2B_0 e^{i\frac{a}{2f_a}Q_a} M_q e^{i\frac{a}{2f_a}Q_a} \qquad J_A^\mu\big|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a\left(D^\mu U U^\dagger + U^\dagger D^\mu U\right)\rangle$$

- $Q_a = \frac{M_q^{-1}}{\langle M_q^{-1}\rangle}$ **is usually taken to eliminate the LO mass mixing between axion and pion** [Georgi,Kaplan,Randall,'86], **though any other hermitian Q_a should lead to the same physical quantities.**

(2) Explicitly keep the aGG term and match it to χPT

Reminiscent:

QCD U(1)$_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 via the U(3) χPT:

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u\chi^\dagger u\,,$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log\left(\det U\right) + i\frac{a}{f_a} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- **Q_a is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$.**
- **Relevant NLO operators to the axion-meson mixing:**

$$\mathcal{L}^{\rm NLO} = L_5\langle u^\mu u_\mu \chi_+\rangle + \frac{L_8}{2}\langle \chi_+\chi_+ + \chi_-\chi_-\rangle - \frac{F^2\,\Lambda_1}{12}D^\mu X D_\mu X - \frac{F^2\,\Lambda_2}{12}X\langle\chi_-\rangle$$

Lagrangians including two-photon couplings of π, η, η' and axion

$$\mathcal{L}_{\mathrm{WZW}}^{\mathrm{LO}} = -\frac{3\sqrt{2}e^2}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\Phi\rangle$$

$$\mathcal{L}_{\mathrm{WZW}}^{\mathrm{NLO}} = it_1\varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\chi_-\rangle + k_3\varepsilon_{\mu\nu\rho\sigma}\langle f_+^{\mu\nu} f_+^{\rho\sigma}\rangle X$$

- **Our goal: to use the experimental inputs from $g_{\pi\gamma\gamma}$, $g_{\eta\gamma\gamma}$, $g_{\eta'\gamma\gamma}$ to predict $g_{a\gamma\gamma}$.**
- **Before doing this, one needs to solve the π-η-η'-*a* mixing problem first.**

π-η-η'-a mixing in U(3) AχPT

Expansion schemes:

Strong isospin breaking (IB) effects: leading corrections will be kept.

Effects of F/f_a : safe to keep the leading order.

δ expansion: the same as the standard U(3) χPT

Leading order results (mass mixing only)

$$\begin{pmatrix} \overline{\pi}^0 \\ \overline{\eta} \\ \overline{\eta}' \\ \overline{a} \end{pmatrix} = \begin{pmatrix} 1+O(v^2) & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1+O(v^2) & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1+O(v^2) & -v_{34} \\ v_{14} & v_{24} & v_{34} & 1+O(v^2) \end{pmatrix} \begin{pmatrix} \pi^0 \\ \mathring{\eta} \\ \mathring{\eta}' \\ a \end{pmatrix}$$

$$\begin{pmatrix} \mathring{\eta} \\ \mathring{\eta}' \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

For the reason to choose $\mathring{\eta}$ $\mathring{\eta}'$, see [ZHG,Oller,PRD'11].

$$m_{\overline{\eta}}^2 = \frac{M_0^2}{2} + m_{\overline{K}}^2 - \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2}\,, \qquad m_{\overline{\eta}'}^2 = \frac{M_0^2}{2} + m_{\overline{K}}^2 + \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2}\,,$$

$$\sin\theta = -\left(\sqrt{1 + \frac{\left(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4}\right)^2}{32\Delta^4}}\right)^{-1} \qquad \Delta^2 = m_{\overline{K}}^2 - m_{\overline{\pi}}^2$$

$$\begin{pmatrix} \overline{\pi}^0 \\ \overline{\eta} \\ \overline{\eta}' \\ \overline{a} \end{pmatrix} = \begin{pmatrix} 1+O(v^2) & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1+O(v^2) & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1+O(v^2) & -v_{34} \\ v_{14} & v_{24} & v_{34} & 1+O(v^2) \end{pmatrix} \begin{pmatrix} \pi^0 \\ \mathring{\eta} \\ \mathring{\eta}' \\ a \end{pmatrix}$$

$$\epsilon \equiv B(m_u - m_d) = m_{K^+}^2 - m_{K^0}^2 - (m_{\pi^+}^2 - m_{\pi^0}^2)$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2} s_\theta}{m_{\overline{\pi}}^2 - m_{\overline{\eta}}^2}, \qquad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2} c_\theta + s_\theta}{m_{\overline{\pi}}^2 - m_{\overline{\eta}'}^2},$$

$$v_{14} = \frac{M_0^2 \epsilon}{3\sqrt{2}(m_{a,0}^2 - m_{\overline{\pi}}^2)} \frac{F}{f_a} \left[\frac{s_\theta (c_\theta - \sqrt{2} s_\theta)(m_{a,0}^2 + m_{\overline{\pi}}^2 - m_{\overline{\eta}}^2)}{(m_{a,0}^2 - m_{\overline{\eta}}^2)(m_{\overline{\eta}}^2 - m_{\overline{\pi}}^2)} - \frac{c_\theta (\sqrt{2} c_\theta + s_\theta)(m_{a,0}^2 + m_{\overline{\pi}}^2 - m_{\overline{\eta}'}^2)}{(m_{a,0}^2 - m_{\overline{\eta}'}^2)(m_{\overline{\eta}'}^2 - m_{\overline{\pi}}^2)} \right],$$

$$v_{24} = -\frac{M_0^2 s_\theta}{\sqrt{6}(m_{a,0}^2 - m_{\overline{\eta}}^2)} \frac{F}{f_a}, \qquad v_{34} = \frac{M_0^2 c_\theta}{\sqrt{6}(m_{a,0}^2 - m_{\overline{\eta}'}^2)} \frac{F}{f_a}$$

$$m_{\overline{a}}^2 = m_{a,0}^2 + \frac{M_0^2 F^2}{6 f_a^2} \left[1 + \frac{c_\theta^2 M_0^2 (2 m_{a,0}^2 - m_{\overline{\eta}'}^2)}{(m_{a,0}^2 - m_{\overline{\eta}'}^2)^2} + \frac{s_\theta^2 M_0^2 (2 m_{a,0}^2 - m_{\overline{\eta}}^2)}{(m_{a,0}^2 - m_{\overline{\eta}}^2)^2} \right] + O(\epsilon),$$

For the QCD axion with $m_{a,0}=0$:

$$m_{\overline{a}}^2 = \frac{M_0^2 F^2}{6 f_a^2} \left[1 - \frac{c_\theta^2 M_0^2}{m_{\overline{\eta}'}^2} - \frac{s_\theta^2 M_0^2}{m_{\overline{\eta}}^2} \right] + O(\epsilon) \quad \Longrightarrow \quad m_{\overline{a}}^2 = \frac{F^2 m_{\overline{\pi}}^2}{f_a^2} + O\left(\frac{m_{\overline{\pi}}^2}{m_{\overline{K}}^2}\right) + O\left(\frac{m_{\overline{\pi}}^2}{M_0^2}\right) + O(\epsilon)$$

[Weinberg, PRL'78]

Discussions with the LO expressions

$$m_{\bar{a}}^2 = \frac{M_0^2 F^2}{6 f_a^2}\left[1 - \frac{c_\theta^2 M_0^2}{m_{\bar{\eta}'}^2} - \frac{s_\theta^2 M_0^2}{m_{\bar{\eta}}^2}\right]$$

(a) $M_0 = 820.0\,\mathrm{MeV}, \quad m_{\bar{\eta}} = 493.6\,\mathrm{MeV}, \quad m_{\bar{\eta}'} = 957.7\,\mathrm{MeV}, \quad \theta = -19.6^\circ$ **[Gu,Duan,ZHG,PRD'18]**

$$m_{\bar{a}} = 6.1\,\mu\mathrm{eV}\frac{10^{12}\,\mathrm{GeV}}{f_a}, \quad v_{14} = \frac{-0.012}{f_a}, \quad v_{24} = \frac{-0.035}{f_a}, \quad v_{34} = \frac{-0.026}{f_a}$$

(b) Physical masses for η and η' and others are the same as (a):

$$m_{\bar{a}} = 9.7\,\mu\mathrm{eV}\frac{10^{12}\,\mathrm{GeV}}{f_a}, \quad v_{14} = \frac{-0.011}{f_a}, \quad v_{24} = \frac{-0.028}{f_a}, \quad v_{34} = \frac{-0.026}{f_a}$$

(c) θ=-10^{o} and others are the same as (b)

$$m_{\bar{a}} = 14.5\,\mu\mathrm{eV}\frac{10^{12}\,\mathrm{GeV}}{f_a}, \quad v_{14} = \frac{-0.008}{f_a}, \quad v_{24} = \frac{-0.015}{f_a}, \quad v_{34} = \frac{-0.027}{f_a}$$

- **It seems necessary to dig into the higher-order calculations.**

NLO case: kinetic+mass mixing terms

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = \begin{pmatrix} 1 & -y_{12} & -y_{13} & -y_{14} \\ y_{12} & 1 & -y_{23} & -y_{24} \\ y_{13} & y_{23} & 1 & -y_{34} \\ y_{14} & y_{24} & y_{34} & 1 \end{pmatrix} \times \begin{pmatrix} 1-x_{11} & -x_{12} & -x_{13} & -x_{14} \\ -x_{12} & 1-x_{22} & -x_{23} & -x_{24} \\ -x_{13} & -x_{23} & 1-x_{33} & -x_{34} \\ -x_{14} & -x_{24} & -x_{34} & 1-x_{44} \end{pmatrix} \begin{pmatrix} \overline{\pi}^0 \\ \overline{\eta} \\ \overline{\eta}' \\ \overline{a} \end{pmatrix}$$

take care of mass mixing

take care of kinetic mixing and canonicalization

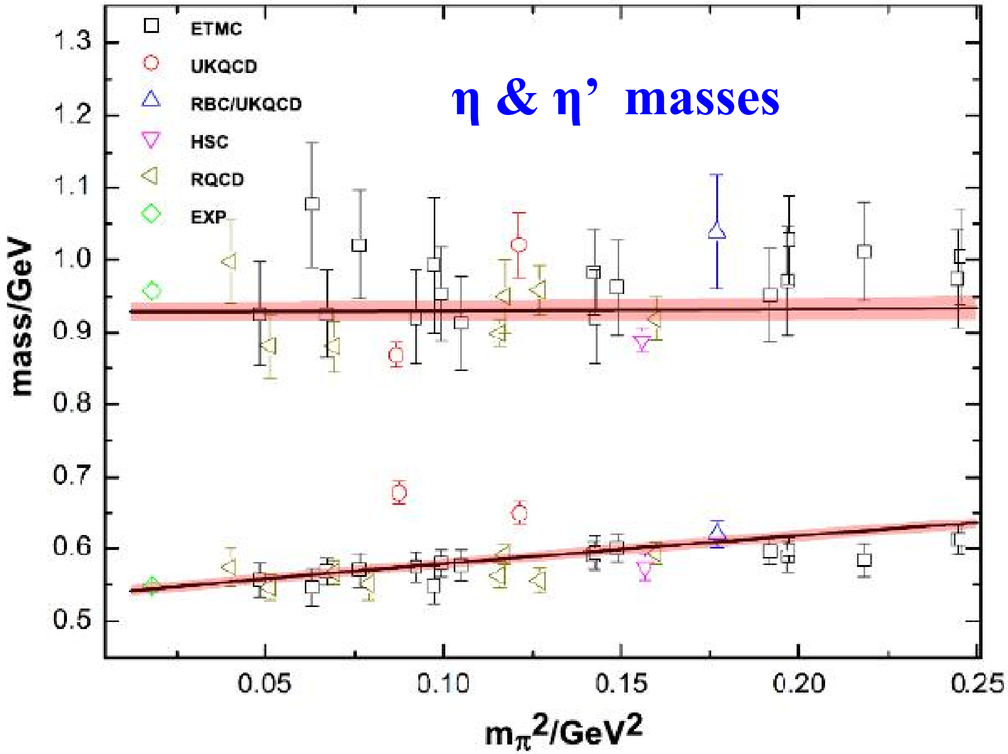
For details, see [Gao,ZHG,Oller,Zhou,JHEP'23]

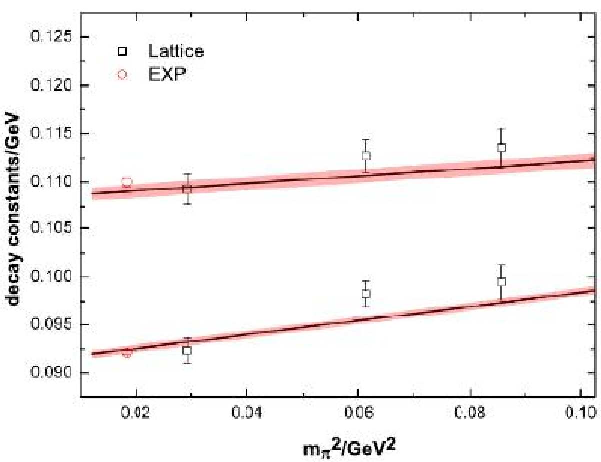
Free parameters to fix : F, L_5, L_8, Λ_1, Λ_2 , which also appear in

- **the masses of π, K, η , η'**
- **decay constants of π, K**
- **η - η' mixing related quantities**

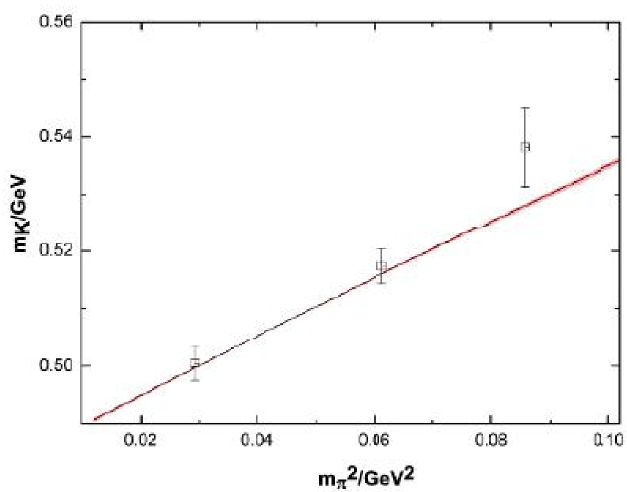
➢ **Lattice simulations provide valuable data, especially the m_π dependence quantities.**

Parameters	NLO Fit
F(MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
$\chi^2/(\text{d.o.f.})$	219.9/(111–5)

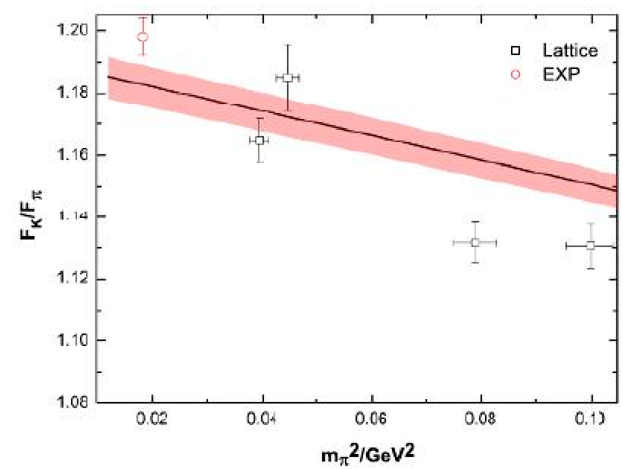




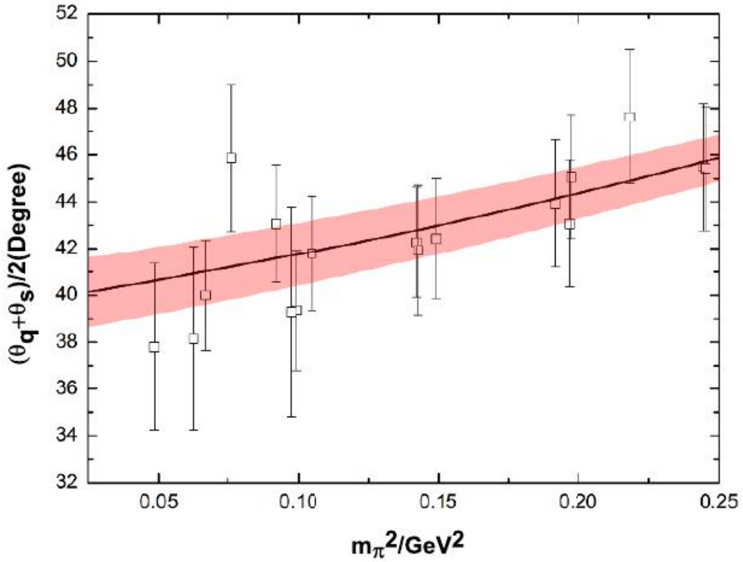
[RBC/UKQCD,PRD'11]



[RBC/UKQCD,PRD'13]



[Durr et al., PRD'10]

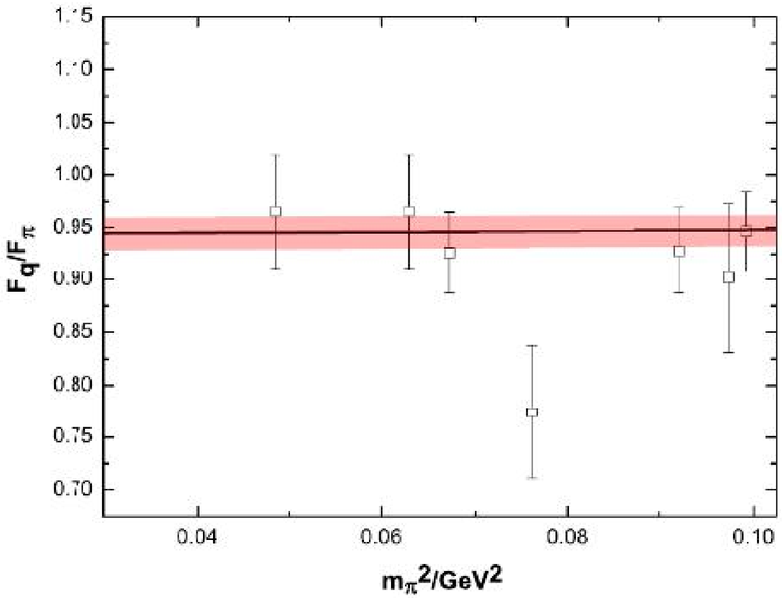


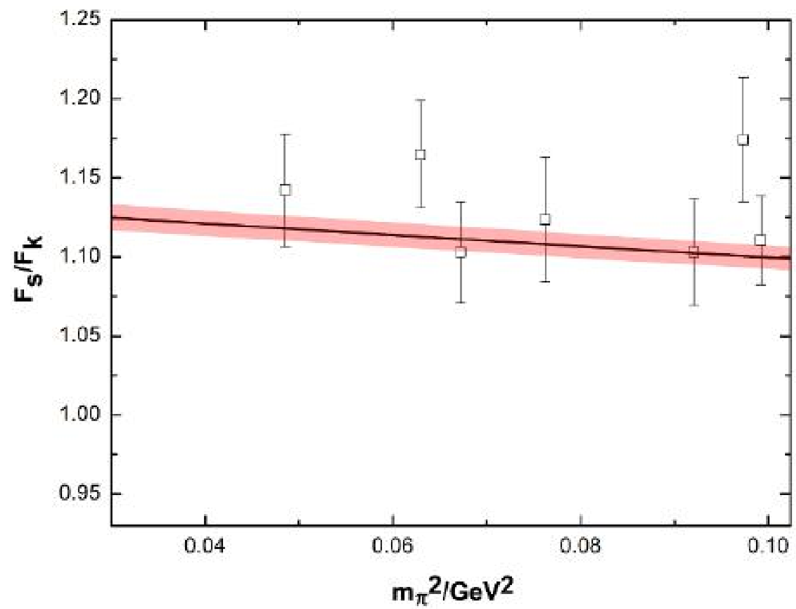
$$\begin{pmatrix} \hat{\eta} \\ \hat{\eta}' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

$$\begin{pmatrix} \hat{\eta} \\ \hat{\eta}' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_q \cos\theta_q & -F_s \sin\theta_s \\ F_q \sin\theta_q & F_s \cos\theta_s \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \\ \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

[ETMC,PRD'18]





Predictions to the mixing pattern and axion masses

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\rm LO+NLO} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

[Gao,ZHG,Oller,Zhou,JHEP'23]

$$M^{\rm LO+NLO} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.010\pm0.001) & 0.009+(-0.007\pm0.001) & \frac{12.1+(0.48\pm0.08)}{f_a} \\ -0.019+(0.007\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.22\pm0.03) & \frac{34.3+(0.9\pm0.2)}{f_a} \\ -0.003+(-0.003\pm0.000) & -0.33+(-0.18\pm0.03) & 0.94+(0.13\pm0.02) & \frac{25.9+(-0.5\pm0.1)}{f_a} \\ \frac{-12.1+(-0.20\pm0.03)}{f_a} & \frac{-23.8+(1.6^{+0.8}_{-0.8})}{f_a} & \frac{-35.7+(-5.7^{+1.6}_{-1.7})}{f_a} & 1+\frac{27.6\pm1.0}{f_a^2} \end{pmatrix}$$

$$\begin{aligned} m_{\hat{\pi}} &= [134.90+(0.10\pm0.07)]\ \text{MeV}\,, \\ m_{\hat{K}} &= [489.2+(5.0^{+3.4}_{-3.5})]\ \text{MeV}\,, \\ m_{\hat{\eta}} &= [490.2+(60.9^{+10.2}_{-10.0})]\ \text{MeV}\,, \\ m_{\hat{\eta}'} &= [954.3+(-28.4^{+11.9}_{-12.6})]\ \text{MeV}\,, \\ m_{\hat{a}} &= [5.96+(0.12\pm0.02)]\ \mu\text{eV}\,\frac{10^{12}\ \text{GeV}}{f_a}\,, \end{aligned}$$

Two-photon couplings

$$\mathcal{L}_{\rm WZW}^{\rm LO} = -\frac{3\sqrt{2}e^2}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\Phi\rangle$$

- **LO $a\gamma\gamma$ coupling is purely caused by the mixing.**

$$\begin{aligned} F_{a\gamma\gamma} &= \frac{(c_\theta - 2\sqrt{2}s_\theta)v_{24} + (s_\theta + 2\sqrt{2}c_\theta)v_{34}}{4\sqrt{3}\pi^2 F} \\ &+ \frac{(c_\theta - 2\sqrt{2}s_\theta)(x_{24}+y_{24}) + (s_\theta + 2\sqrt{2}c_\theta)(x_{34}+y_{34})}{4\sqrt{3}\pi^2 F} - \frac{64\sqrt{6}}{3F}k_3\left(\frac{F}{f_a} - \sqrt{6}s_\theta v_{24} + \sqrt{6}c_\theta v_{34}\right) \\ &- \frac{64}{27F}t_1\Big[-\sqrt{3}s_\theta(4\sqrt{2}v_{24}m_\pi^2 - 7v_{34}m_\pi^2 + 2\sqrt{2}v_{24}m_K^2 + 4v_{34}m_K^2) \\ &\qquad + \sqrt{3}c_\theta(4\sqrt{2}v_{34}m_\pi^2 + 7v_{24}m_\pi^2 + 2\sqrt{2}v_{34}m_K^2 - 4v_{24}m_K^2)\Big]. \end{aligned}$$

$$F_{\pi^0\gamma\gamma} = \frac{1}{4\pi^2 F} + \frac{1}{4\pi^2 F}x_{11} - \frac{64}{3F}t_1 m_\pi^2,$$

$$\begin{aligned} F_{\eta\gamma\gamma} &= \frac{c_\theta - 2\sqrt{2}s_\theta}{4\sqrt{3}\pi^2 F}(1+x_{22}) + \frac{s_\theta + 2\sqrt{2}c_\theta}{4\sqrt{3}\pi^2 F}(x_{23}-y_{23}) + \frac{64\sqrt{6}}{3F}s_\theta k_3 \\ &- \frac{64\sqrt{3}}{27F}t_1\left[c_\theta(7m_\pi^2 - 4m_K^2) - 2\sqrt{2}s_\theta(m_K^2 + 2m_\pi^2)\right], \end{aligned}$$

$$\begin{aligned} F_{\eta'\gamma\gamma} = &\frac{s_\theta + 2\sqrt{2}c_\theta}{4\sqrt{3}\pi^2 F}(1+x_{33}) + \frac{c_\theta - 2\sqrt{2}s_\theta}{4\sqrt{3}\pi^2 F}(x_{23}+y_{23}) - \frac{64\sqrt{6}}{3F}c_\theta k_3 \\ &- \frac{64\sqrt{3}}{27F}t_1\left[s_\theta(7m_\pi^2 - 4m_K^2) + 2\sqrt{2}c_\theta(m_K^2 + 2m_\pi^2)\right], \end{aligned}$$

$$F^{\mathrm{Exp}}_{\pi^0\gamma\gamma} = 0.274 \pm 0.002\,\mathrm{GeV}^{-1}\,,$$
$$F^{\mathrm{Exp}}_{\eta\gamma\gamma} = 0.274 \pm 0.006\,\mathrm{GeV}^{-1}\,,$$
$$F^{\mathrm{Exp}}_{\eta'\gamma\gamma} = 0.344 \pm 0.008\,\mathrm{GeV}^{-1}\,.$$

$$t_1 = -(4.4 \pm 2.3) \times 10^{-4}\,\mathrm{GeV}^{-2}\,,$$
$$k_3 = (1.25 \pm 0.23) \times 10^{-4}$$

This allows us to predict

$$F_{a\gamma\gamma} = -\frac{[20.1 + (0.5 \pm 0.1)] \times 10^{-3}}{f_a}$$

$$g_{a\gamma\gamma} = 4\pi\alpha_{em} F_{a\gamma\gamma} = -\frac{\alpha_{em}}{2\pi f_a}(1.63 \pm 0.01)$$

which can be compared to 1.92 ± 0.04 **and** 2.05 ± 0.03

[Grilli de Cortona, et al., JHEP'16]

[Lu, et al., JHEP'20]

- **IB corrections could cause visible effects (working in progress).**

Summary

- **U(3) chiral perturbation theory provides a systematical framework to include the axion together with the QCD light-flavor pseudoscalars π, K, η, η' .**

- **π-η-η'-*a* mixing is worked out at NLO and lattice data are found to be very useful to constrain the low energy constants.**

- **$g_{\pi\gamma\gamma}$, $g_{\eta\gamma\gamma}$, $g_{\eta'\gamma\gamma}$ and the π-η-η'-*a* mixing are used to predict $g_{a\gamma\gamma}$ within U(3) AχPT up to NLO.**

- **NNLO and IB corrections are being worked out.**

Thanks you very much!