Multiparton distributions from lattice QCD

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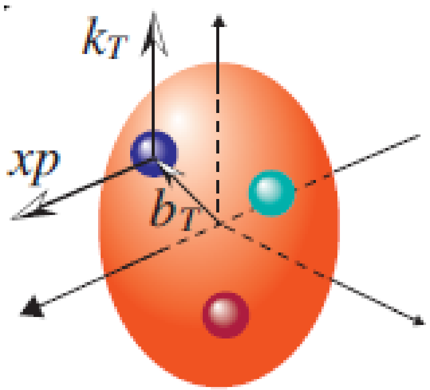
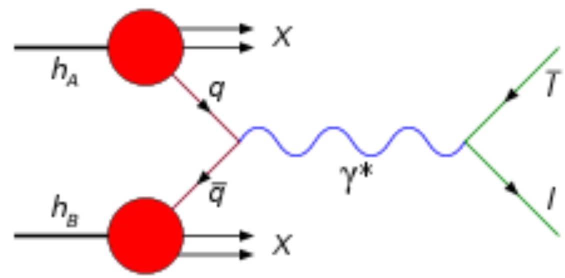
Introduction

- Parton physics plays an important role in mapping out the 3D structure of hadrons and interpreting the experimental data at hadron colliders





Example: Drell-Yan Process

- **Factorization**

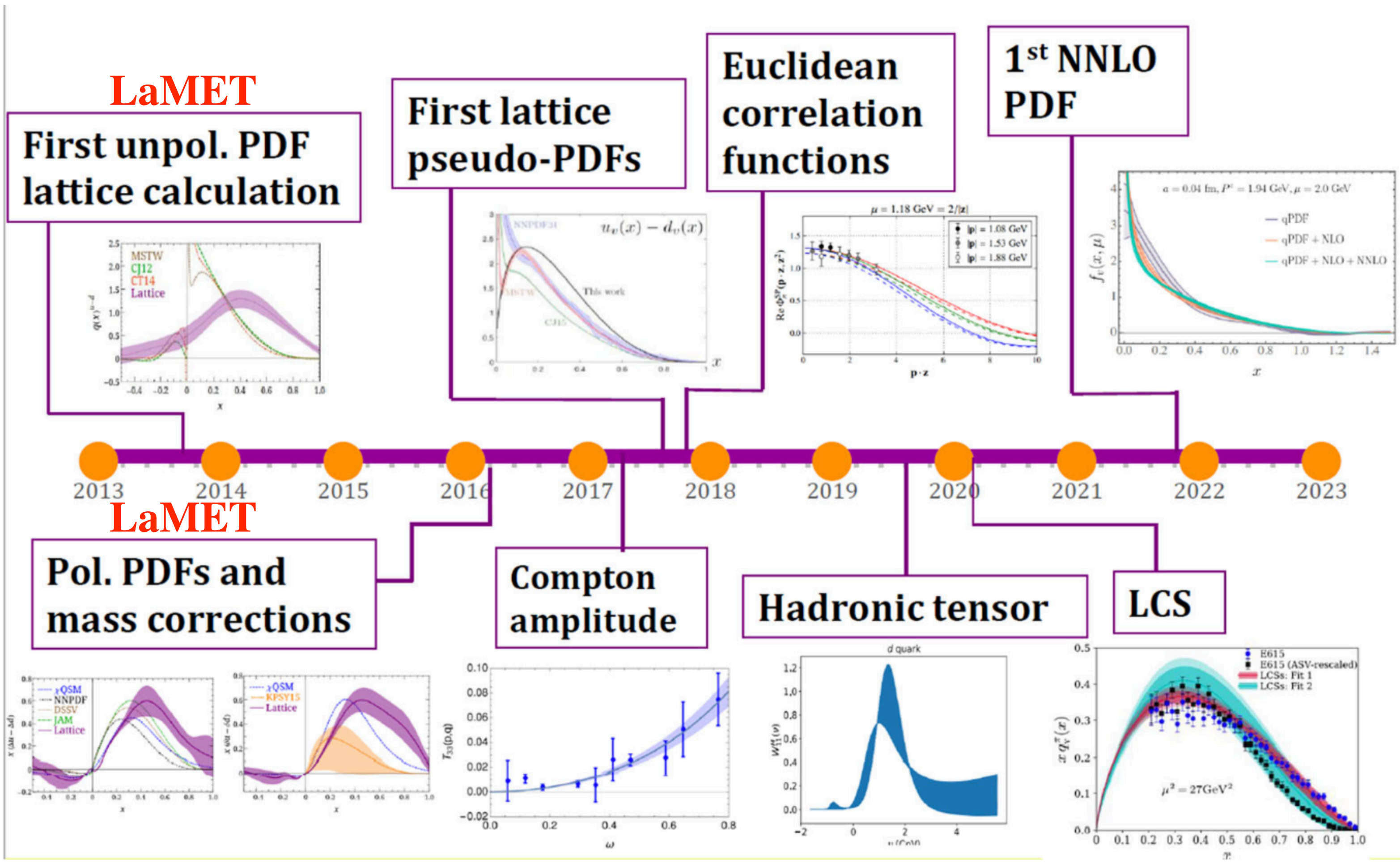
$$\frac{d\sigma}{dQ^2} = \sum_{i,j} \int_0^1 d\xi_a d\xi_b \, f_{i/P_a}(\xi_a) f_{j/P_b}(\xi_b) \frac{d\hat{\sigma}_{ij}(\xi_a, \xi_b)}{dQ^2} \times \left[1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)\right] \qquad Q = \sqrt{q^2}$$

$q_T \ll Q$:

$$\frac{d\sigma}{dQ^2 d^2\mathbf{q}_T} = \sum_{i,j} H_{ij}(Q) \int_0^1 d\xi_a d\xi_b \int d^2\mathbf{b}_T \, e^{i\mathbf{b}_T\cdot\mathbf{q}_T} \times f_{i/P}(\xi_a, \mathbf{b}_T) f_{j/P}(\xi_b, \mathbf{b}_T) \times \left[1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}, \frac{q_T}{Q}\right)\right]$$

Introduction

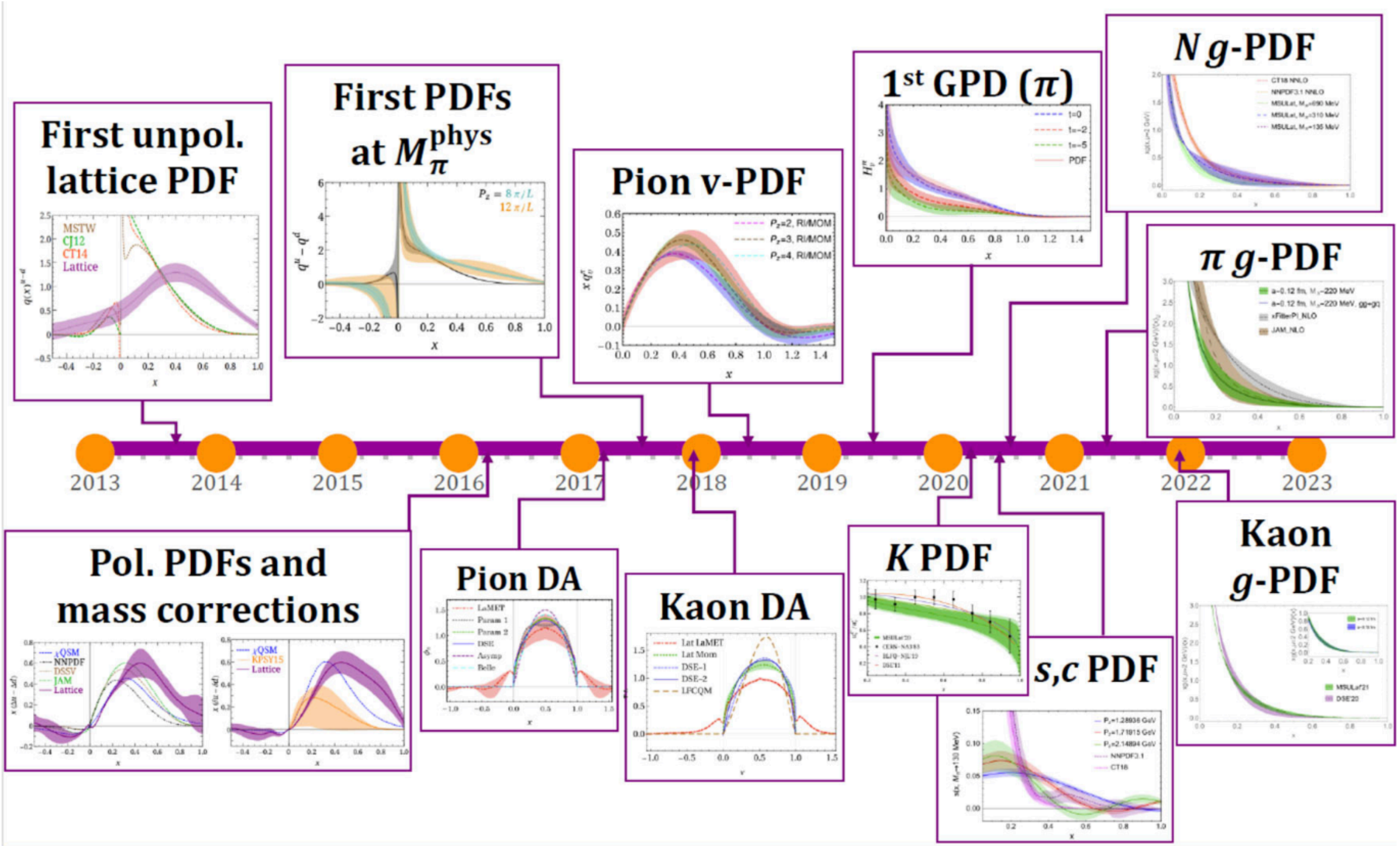
- Tremendous progress has been achieved on calculating the **x-dependent partonic structure** of hadrons from Euclidean lattice



H-W Lin, Few Body Syst. 23'

Introduction

- **A popular approach: Large-momentum effective theory (LaMET)**
  Ji, PRL 13' & SCPMA 14', Ji, Liu, Liu, JHZ, Zhao, Rev. Mod. Phys. 21'



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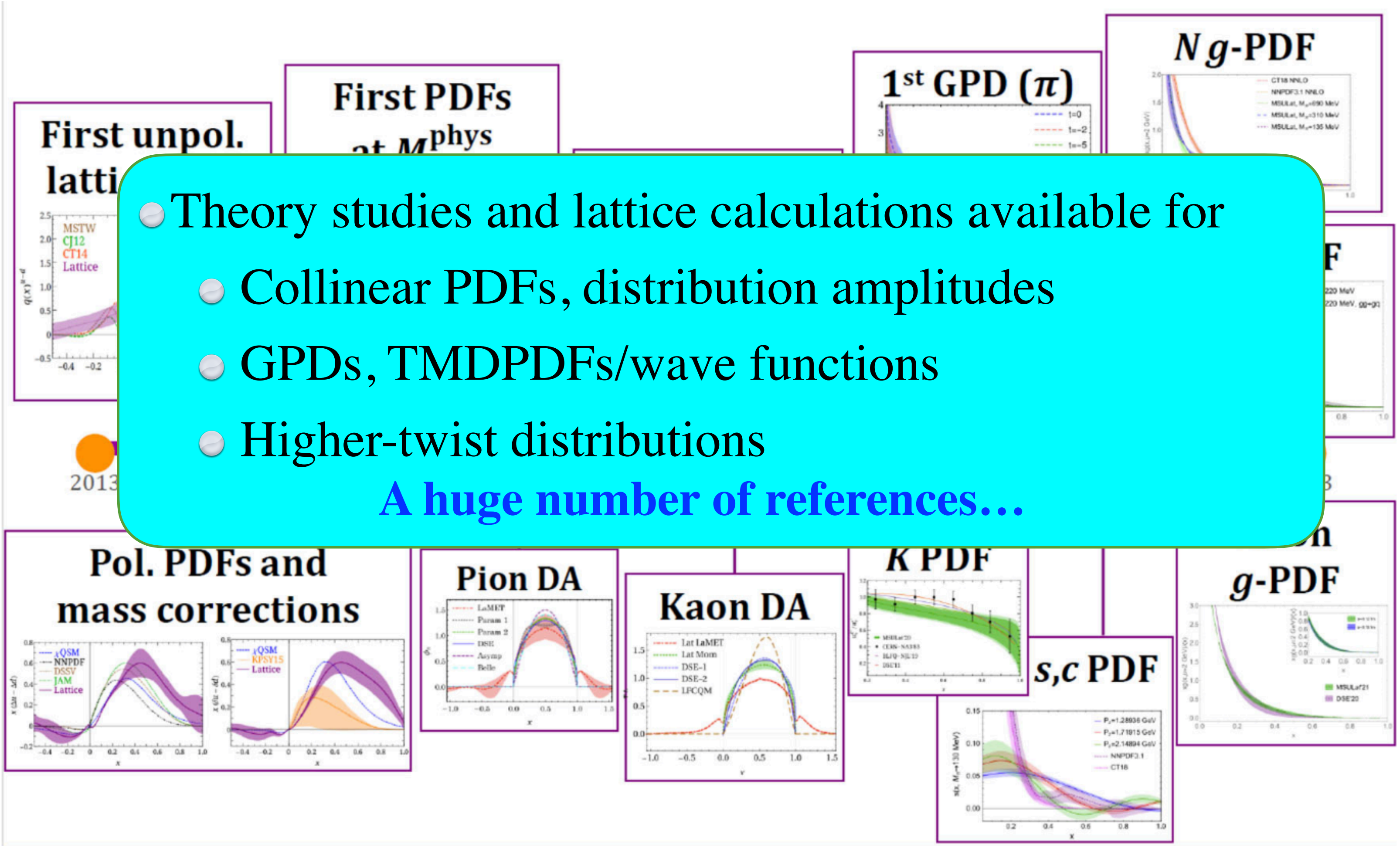


- Theory studies and lattice calculations available for
  - Collinear PDFs, distribution amplitudes
  - GPDs, TMDPDFs/wave functions
  - Higher-twist distributions

A huge number of references…

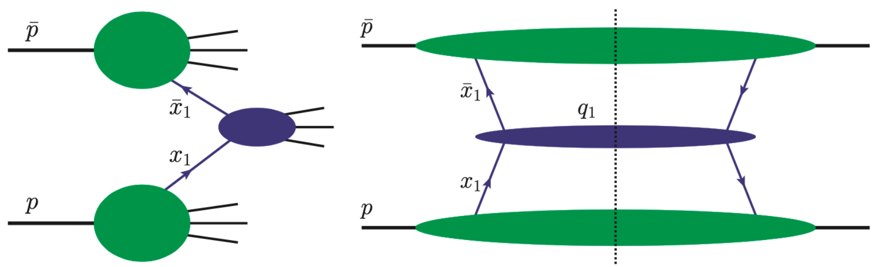
H-W Lin, Few Body Syst. 23'

Towards precision calculations

- Lattice calculations of partonic structure of hadrons (to the leading-power accuracy) now reach the stage of **precision control**
  - Higher-order perturbative correction
    - Unpol. quark PDF@NNLO **Li et al, PRL 21', Chen et al, PRL 21'**
    - Quark TMDPDF@NNLO **Del Rio et al, 2304.14440, Ji et al, JHEP 23'**
  - RG resummation **Su, JHZ et al, NPB 23'**
  - Threshold resummation **Gao et al, PRD 21', Ji et al, JHEP 23'**
  - Power correction, renormalon ambiguity
    Braun, JHZ et al, PRD 19', Liu et al, PRD 21', Zhang et al, PLB 23'
  - Control of lattice artifacts
    - ANL-BNL, ETMC, LPC, MSU…

From single- to multi-parton distributions

- The computational effort so far has been mainly focused on **single parton distributions** (relevant for single parton scattering)



$$\sigma = \sum_{ij} \int dx_1 \int d\bar{x}_1 f_i(x_1) f_j(\bar{x}_1) \hat{\sigma}_{ij}.$$

Single parton distributions

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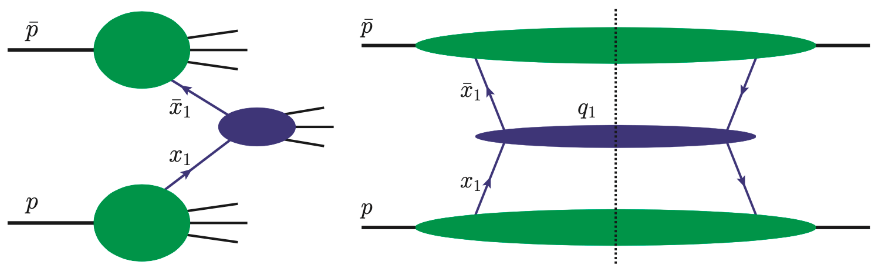
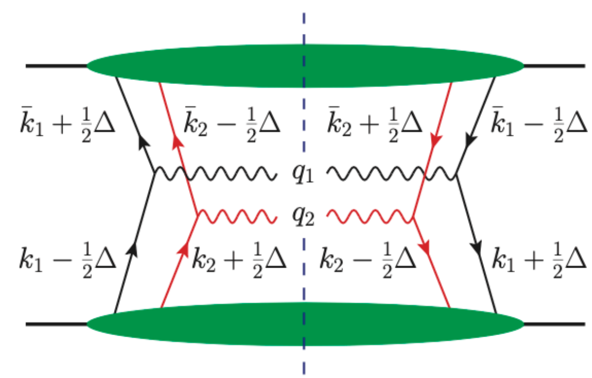


$$\sigma = \sum_{ij} \int dx_1 \int d\bar{x}_1 f_i(x_1) f_j(\bar{x}_1) \hat{\sigma}_{ij}.$$

Single parton distributions

- With the increasing energy of hadron colliders, **multiparton scattering** (e.g., double parton scattering) processes become increasingly important



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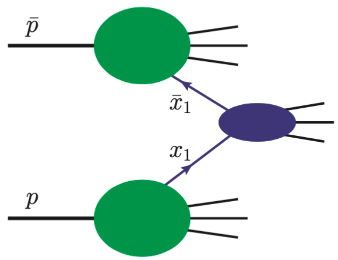

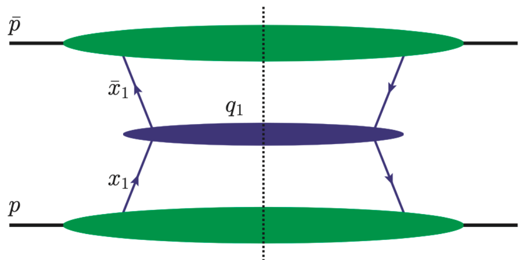

$$\sigma = \sum_{ij} \int dx_1 \int d\bar{x}_1 f_i(x_1) f_j(\bar{x}_1) \hat{\sigma}_{ij}.$$

Single parton distributions

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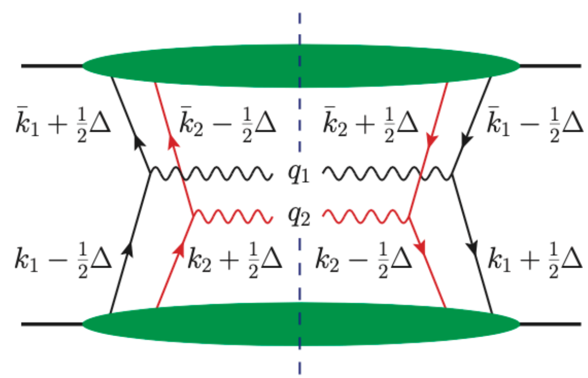
It can compete with single parton scattering in certain situations



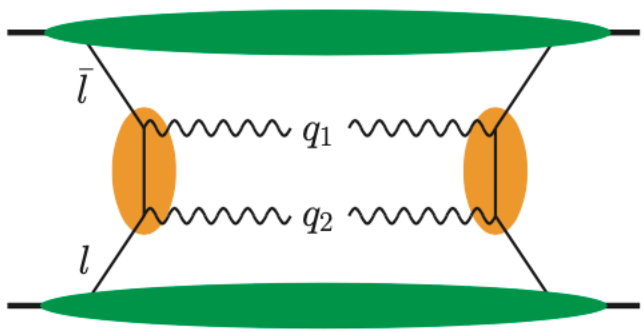
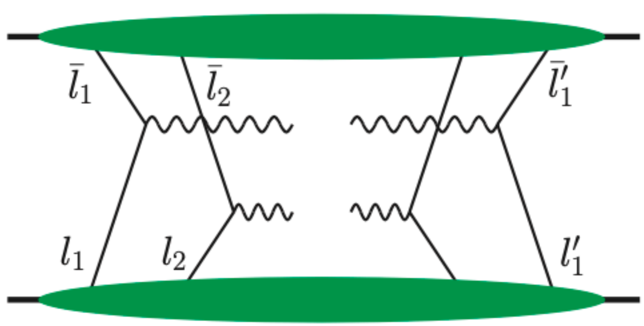
Longitudinal parton momenta fixed by final state kinematics

Transverse parton momenta can differ by Δ, conjugate to transverse separation of partons

From single- to multi-parton distributions

- The computational effort so far has been mainly focused on **single parton distributions** (relevant for single parton scattering)

Double parton scattering power counting

		$\frac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\frac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
a		$\frac{1}{\Lambda^2 Q^2}$	1
b		$\frac{1}{\Lambda^2 Q^2}$	$\frac{\Lambda^2}{Q^2}$

From single- to multi-parton distributions

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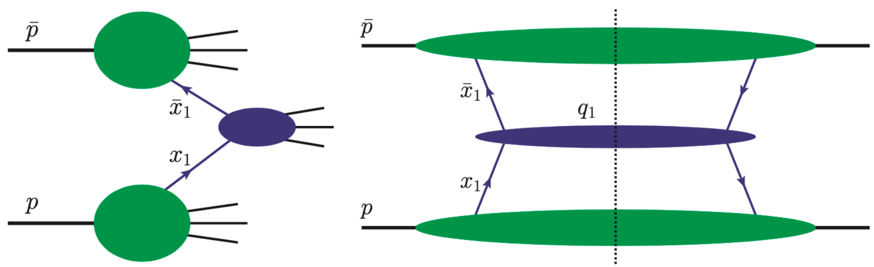

$$\sigma = \sum_{ij} \int dx_1 \int d\bar{x}_1 f_i(x_1) f_j(\bar{x}_1) \hat{\sigma}_{ij}.$$

Single parton distributions

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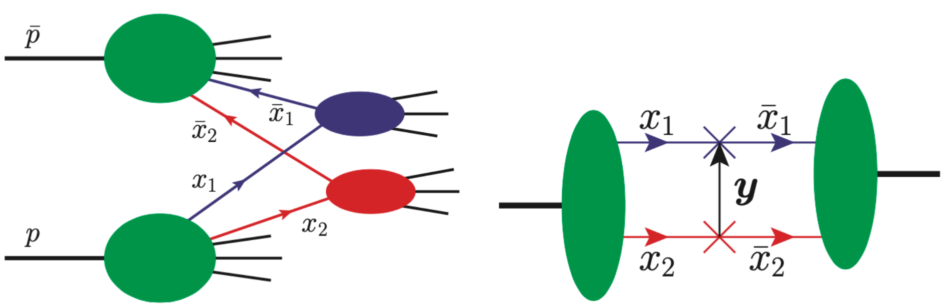


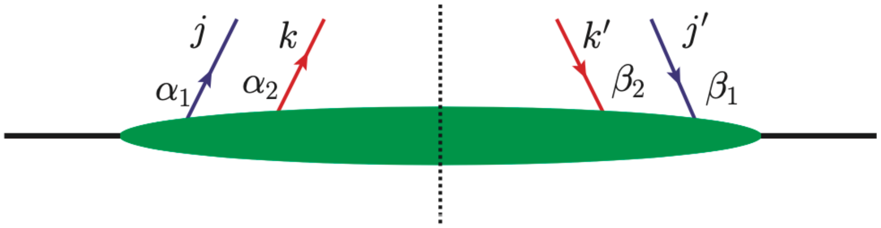
(a) (b)

Double parton distributions (the two partons can have transverse separations)

$$\sigma_{DPS} \sim \sum_{ijkl} \int dx_1 dx_2 \int d\bar{x}_1 d\bar{x}_2 \int d^2\boldsymbol{y} f_{ij}(x_1, x_2, \boldsymbol{y}) f_{kl}(\bar{x}_1, \bar{x}_2, \boldsymbol{y}) \hat{\sigma}_{ik} \hat{\sigma}_{jl}.$$

From single- to multi-parton distributions

- Double parton distributions (DPDs) **Diehl, Ostermeier, Schaefer, JHEP 12', Diehl, Gaunt, 17'**



- Two-quark correlation

$$\Phi_{\Sigma_1,\Sigma_1',\Sigma_2,\Sigma_2'}(k_1,k_2,r) = \int \frac{d^4z_1}{(2\pi)^4} \mathrm{e}^{iz_1k_1} \frac{d^4z_2}{(2\pi)^4} \mathrm{e}^{iz_2k_2} \frac{d^4y}{(2\pi)^4} \mathrm{e}^{-iyr}$$
$$\times \langle p| \bar{T} \left[\bar{\psi}_{\Sigma_1'}(y - \tfrac{1}{2}z_1)\bar{\psi}_{\Sigma_2'}(-\tfrac{1}{2}z_2)\right] T \left[\psi_{\Sigma_2}(\tfrac{1}{2}z_2)\psi_{\Sigma_1}(y + \tfrac{1}{2}z_1)\right] |p\rangle$$

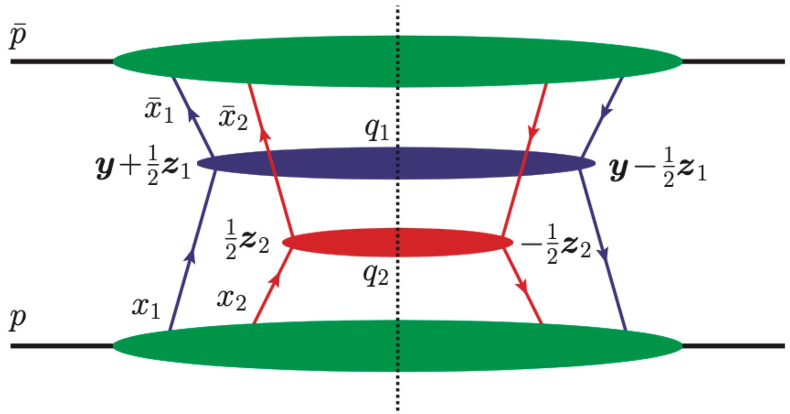
- Fourier transform to transverse position space

$$F_{\Sigma_1,\Sigma_1',\Sigma_2,\Sigma_2'}(x_1,x_2,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{y}) = 2p^+ \int \frac{dz_1^-}{2\pi}\frac{dz_2^-}{2\pi}\, dy^-\; e^{ix_1z_1^-p^+ + ix_2z_2^-p^+}$$
$$\times \langle p| \bar{\psi}_{\Sigma_2'}(-\tfrac{1}{2}z_2)\psi_{\Sigma_2}(\tfrac{1}{2}z_2)\bar{\psi}_{\Sigma_1'}(y - \tfrac{1}{2}z_1)\psi_{\Sigma_1}(y + \tfrac{1}{2}z_1)\,|p\rangle \Big|_{z_i^+ = y^+ = 0}.$$

- Σ_i denotes collectively the spin, color and flavor of the corresponding quark, non-trivial correlated structure

From single- to multi-parton distributions

- Double parton scattering in terms of DPDs



- Factorization

$$\frac{d\sigma}{\prod_{i=1}^{2} dx_i \, d\bar{x}_i \, d^2\boldsymbol{q}_i} = \frac{1}{C} \sum_{a_1 a_2 a_3 a_4} \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2} \frac{d^2\boldsymbol{z}_2}{(2\pi)^2} \, e^{-i\boldsymbol{z}_1\boldsymbol{q}_1 - i\boldsymbol{z}_2\boldsymbol{q}_2} \int d^2\boldsymbol{y}$$
$$\times \Big\{ d\hat{\sigma}_{a_1\bar{a}_3} \, d\hat{\sigma}_{a_2\bar{a}_4} \left[{}^{1}F_{a_1 a_2} \, {}^{1}\bar{F}_{\bar{a}_3\bar{a}_4} + c_8 \, {}^{8}F_{a_1 a_2} \, {}^{8}\bar{F}_{\bar{a}_3\bar{a}_4} \right]$$
$$+ d\hat{\sigma}^{I}_{a_1\bar{a}_3} \, d\hat{\sigma}^{I}_{a_2\bar{a}_4} \left[{}^{1}F^{I}_{a_1 a_2} \, {}^{1}\bar{F}^{I}_{\bar{a}_3\bar{a}_4} + c_8 \, {}^{8}F^{I}_{a_1 a_2} \, {}^{8}\bar{F}^{I}_{\bar{a}_3\bar{a}_4} \right] \Big\} \, .$$

DPDs in phenomenology

- Simplified modeling often ignores the correlation between partons in analyzing double parton scattering

- DPDs in terms of single parton impact parameter distributions

$$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) \stackrel{?}{=} \int d^2\boldsymbol{b}\, f_{a_1}(x_1, \boldsymbol{b}+\boldsymbol{y})\, f_{a_2}(x_2, \boldsymbol{b})$$

- Such a factorization has been investigated on the lattice for the pion, significant differences have been observed between l.h.s. and r.h.s.

- DPDs in a completely factorized form

$$F_{a_1 a_2}(x_1, x_2, \boldsymbol{y}) \stackrel{?}{=} f_{a_1}(x_1)\, f_{a_2}(x_2)\, G(\boldsymbol{y})$$

- Double parton scattering X-sec is given by

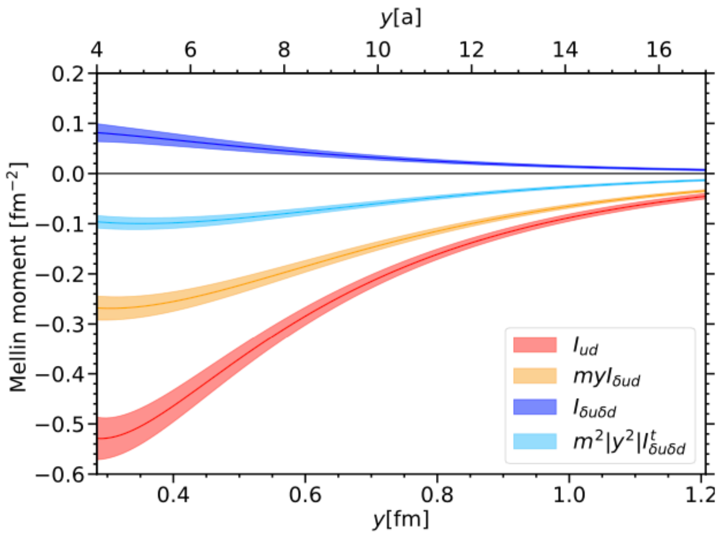
$$\sigma_{\mathrm{DPS},ij} = \frac{1}{C} \frac{\sigma_{\mathrm{SPS},i}\ \sigma_{\mathrm{SPS},j}}{\sigma_{\mathrm{eff}}}$$

DPDs from lattice

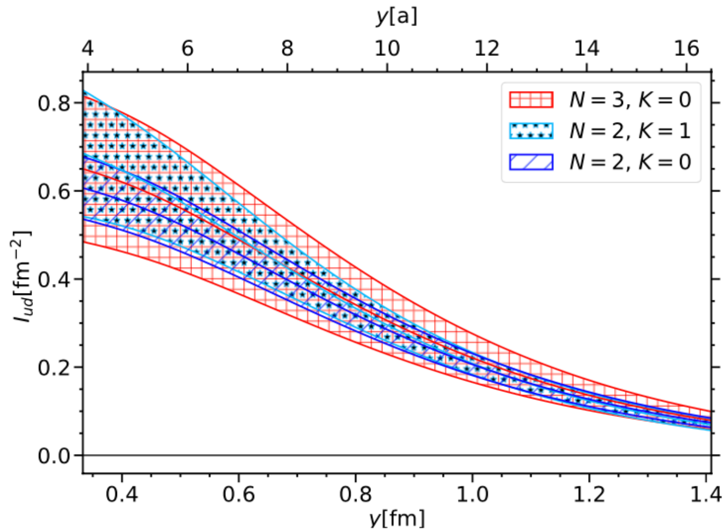
- DPDs involve fields at lightlike distances and thus are not well-suited for lattice calculations
- What can be studied on the lattice are two-current correlations at space like separations, e.g., **Bali et al, JHEP 21'**

$$M^{\mu_1\cdots\mu_2\cdots}_{q_1 q_2, i_1 i_2}(p,y) = \langle h(p)|\, J^{\mu_1\cdots}_{q_1,i_1}(y)\, J^{\mu_2\cdots}_{q_2,i_2}(0)\,|h(p)\rangle$$

- The lowest double Mellin moment



(a) Mellin moments $I(y^2, \zeta = 0)$



(a) fit comparison $I_{ud}(\zeta = 0, y^2)$

DPDs from lattice

- Accessing full DPD: simplest unpolarized color singlet case **JHZ, 23'**

$$f_{q_1 q_2}(x_1, x_2, y^2) = 2P^+ \int dy^- \int \frac{dz_1^-}{2\pi} \frac{dz_2^-}{2\pi} e^{i(x_1 z_1^- + x_2 z_2^-)P^+} h_0(y, z_1, z_2, P)$$

$$= 2 \int d\lambda \int \frac{d\lambda_1}{2\pi} \frac{d\lambda_2}{2\pi} e^{i(x_1 \lambda_1 + x_2 \lambda_2)} h(\lambda, \lambda_1, \lambda_2, y^2),$$

with

$$h_0(y, z_1, z_2, P) = \langle P | O_{q_1}(y, z_1) O_{q_2}(0, z_2) | P \rangle,$$

$$h(\lambda, \lambda_1, \lambda_2, y^2) = \frac{1}{(P^+)^2} h_0(y, z_1, z_2, P),$$

$$O_q(y, z) = \bar{\psi}_q\left(y - \frac{z}{2}\right) \frac{\gamma^+}{2} W\left(y - \frac{z}{2}; y + \frac{z}{2}\right) \psi_q\left(y + \frac{z}{2}\right),$$

$$\lambda = P \cdot y, \quad \lambda_1 = P \cdot z_1, \quad \lambda_2 = P \cdot z_2,$$

- Double Mellin moments are given by

$$M_{q_1 q_2}^{n_1 n_2}(y^2) = \int_{-1}^{1} dx_1 dx_2 \, x_1^{n_1 - 1} x_2^{n_2 - 1} f_{q_1 q_2}(x_1, x_2, y^2)$$

$$= \frac{(P^+)^{1 - n_1 - n_2}}{2} \int dy^- \langle P | \mathcal{O}_{q_1}^{+\cdots+}(y) \mathcal{O}_{q_2}^{+\cdots+}(0) | P \rangle$$

$$\mathcal{O}_q^{\mu_1 \cdots \mu_n}(y) = \bar{\psi}_q(y) \gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}(y) \cdots i\overleftrightarrow{D}^{\mu_n\}}(y) \psi_q(y),$$

DPDs from lattice

- Consider the correlation of equal-time nonlocal operators following the spirit of LaMET

$$\tilde{h}(z_1, z_2, y, P) = \frac{1}{N}\langle P|O_{q_1}(y, z_1)O_{q_2}(0, z_2)|P\rangle,$$

$$y^\mu = (0, \overrightarrow{y}_\perp, y^z), \quad z_i^\mu = (0, \overrightarrow{0}_\perp, z_i)$$

- From OPE **Izubuchi et al, PRD 18'**

$$\tilde{h}(z_i, \mu_i, y, P) = \frac{1}{4N}\sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\frac{(-iz_1)^{n_1-1}}{(n_1-1)!}\frac{(-iz_2)^{n_2-1}}{(n_2-1)!} \times C_{q_1}^{(n_1-1)}(\mu_1^2 z_1^2)C_{q_2}^{(n_2-1)}(\mu_2^2 z_2^2)\tilde{\mathcal{M}}_{q_1 q_2}^{n_1 n_2}(\mu_i, y, P) + \cdots,$$

$$\tilde{\mathcal{M}}_{q_1 q_2}^{n_1 n_2}(\mu_i, y, P) = n_{\mu_1}\cdots n_{\mu_{n_1}} n_{\nu_1}\cdots n_{\nu_{n_2}} \times \langle P|\mathcal{O}_{q_1}^{\mu_1\cdots\mu_{n_1}}(y, \mu_1)\mathcal{O}_{q_2}^{\nu_1\cdots\nu_{n_2}}(0, \mu_2)|P\rangle = 2(n\cdot P)^{n_1+n_2}\langle \mathcal{O}_{q_1}^{n_1}\mathcal{O}_{q_2}^{n_2}\rangle(\mu_i, \lambda, y^2) + \cdots,$$

- The same Lorentz invariant reduced matrix element appears both in correlations of lightcone and Euclidean nonlocal operators

DPDs from lattice

- Factorization

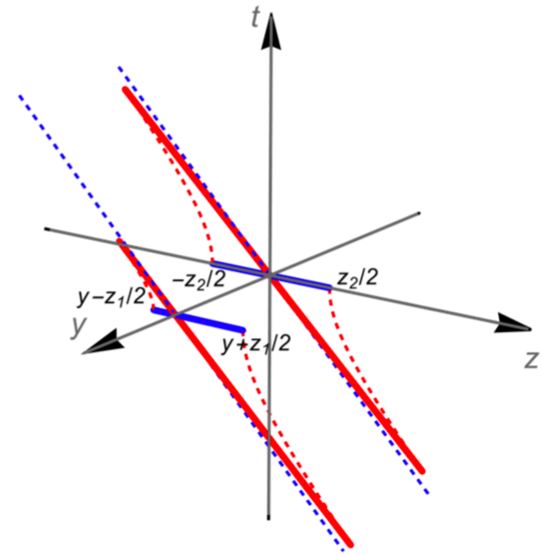
$$\tilde{H}(\lambda_i, \mu_i, z_i^2, y^2) = \int du_1 du_2\, \mathcal{C}_{q_1}(u_1, \mu_1^2 z_1^2)\mathcal{C}_{q_2}(u_2, \mu_2^2 z_2^2) H(u_i\lambda_i, \mu_i, y^2) + \cdots$$

$$\tilde{H}(\lambda_i, \mu_i, z_i^2, y^2) = \int d\lambda\, \tilde{h}(\lambda, \lambda_i, \mu_i, z_i^2, y^2),$$

$$H(\lambda_i, \mu_i, y^2) = \int d\lambda\, h(\lambda, \lambda_i, \mu_i, y^2).$$

- FT w.r.t. z_i with P fixed

$$\tilde{f}(x_1, x_2, \mu_i, y^2) = 2\int \frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi} e^{i(x_1\lambda_1 + x_2\lambda_2)} \times \tilde{H}\left(\lambda_i, \mu_i, -\frac{\lambda_i^2}{(P^z)^2}, y^2\right) = \int \frac{dx_1'}{|x_1'|}\frac{dx_2'}{|x_2'|} C_{q_1}\left(\frac{x_1}{x_1'}, \frac{\mu_1^2}{(x_1' P^z)^2}\right) C_{q_2}\left(\frac{x_2}{x_2'}, \frac{\mu_2^2}{(x_2' P^z)^2}\right) \times f(x_i', \mu_i^2, y^2) + \cdots,$$



JHZ, 23'

DPDs from lattice

- Factorization

$$\tilde{H}(\lambda_i, \mu_i, z_i^2, y^2) = \int du_1 du_2\, \mathcal{C}_{q_1}(u_1, \mu_1^2 z_1^2)\mathcal{C}_{q_2}(u_2, \mu_2^2 z_2^2) H(u_i\lambda_i, \mu_i, y^2) + \cdots$$

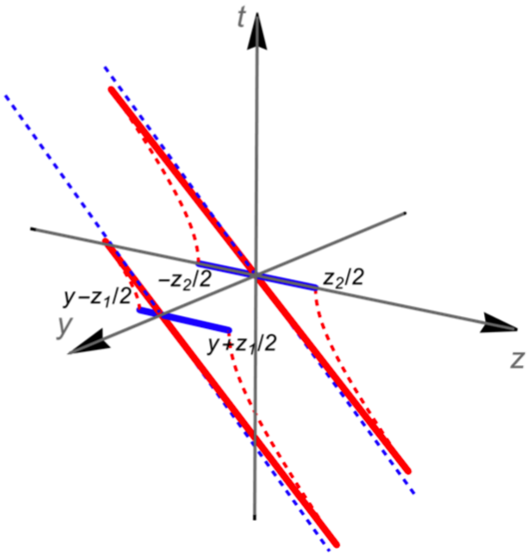
$$\tilde{H}(\lambda_i, \mu_i, z_i^2, y^2) = \int d\lambda\, \tilde{h}(\lambda, \lambda_i, \mu_i, z_i^2, y^2),$$

$$H(\lambda_i, \mu_i, y^2) = \int d\lambda\, h(\lambda, \lambda_i, \mu_i, y^2).$$

- FT w.r.t. λ_i with z_i^2 fixed

$$\mathcal{D}(x_i, \mu_i, z_i^2, y^2) = 2\int \frac{d\lambda_1}{2\pi}\frac{d\lambda_2}{2\pi} e^{i(x_1\lambda_1 + x_2\lambda_2)} \tilde{H}(\lambda_i, \mu_i, z_i^2, y^2)$$

$$= \int \frac{dx_1'}{|x_1'|}\frac{dx_2'}{|x_2'|} \mathcal{C}_{q_1}(\frac{x_1}{x_1'}, \mu_1^2 z_1^2)\mathcal{C}_{q_2}(\frac{x_2}{x_2'}, \mu_2^2 z_2^2) f(x_i', \mu_i, y^2) + \cdots,$$



JHZ, 23'

DPDs from lattice

- Color-correlated DPD **Jaarsma, Rahn, Waalewijn, 23', JHZ, in preparation**

$$
\begin{aligned}
&{}^{R_1R_2}\tilde{F}^{\mathrm{NS}}_{q_1q_2}(x_1,x_2,b_\perp,\mu,\tilde{\zeta}_p,\tilde{P}^z) \\
&= \sum_{R_1',R_2'}\sum_{q_1',q_2'}\int_0^1 \frac{\mathrm{d}x_1'}{x_1'}\frac{\mathrm{d}x_2'}{x_2'}\,{}^{R_1R_1'}C_{q_1q_1'}\left(\frac{x_1}{x_1'},x_1'\tilde{P}^z,\mu\right){}^{R_2R_2'}C_{q_2q_2'}\left(\frac{x_2}{x_2'},x_2'\tilde{P}^z,\mu\right) \\
&\qquad \times \exp\left[\frac{1}{2}{}^{R_{1/2}}J(b_\perp,\mu)\ln\left(\frac{\tilde{\zeta}_p}{\zeta_p}\right)\right]{}^{R_1'R_2'}F^{\mathrm{NS}}_{q_1'q_2'}(x_1',x_2',b_\perp,\mu,\zeta_p)\,.
\end{aligned}
$$

- Rapidity divergences show up in the collinear DPDs, and introduce rapidity scale dependence

- Checked by an explicit one-loop calculation, consistent with RG and rapidity evolution of DPDs

Summary and outlook

- Lattice calculations of single parton distributions have reached a stage of precision control

- Multiparton distributions are important both for new physics searches at hadron colliders and for understanding the correlated partonic structure of hadrons

- Very limited knowledge even on the simplest case (DPDs)

- The full DPD can now be accessed on lattice, providing important inputs for phenomenological analyses, very interesting to MPI@LHC

- The same development in single parton distributions (PDFs, TMDs, GPDs…) can be extended to DPDs, and generalized to multiparton distributions, a lot more to be explored…