SOUTH CHINA NORMAL UNIVERSITY

Lattice Parton Collaboration

TMD Wave Functions of Pion from Lattice QCD

arXiv:2302.09961; JHEP.08.172(2023)

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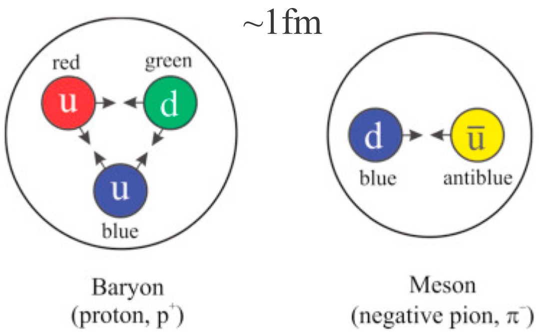
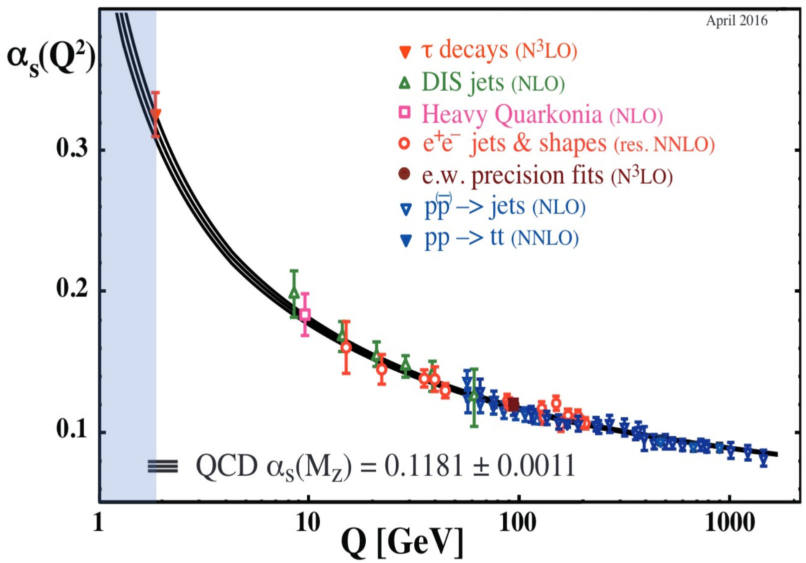
2023.10.08

第三届中国格点量子色动力学研讨会

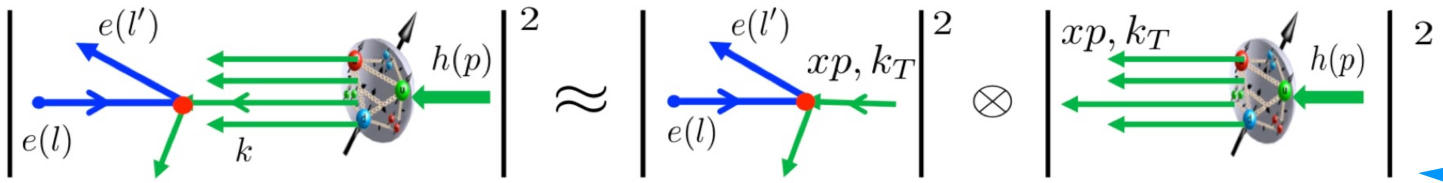
Out Line

Limited by QCD confinement





- **QCD factorization (1982)**



parton density amplitude (PDF, LCDA)

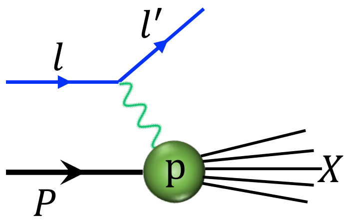
Measured cross section

Asymptotic freedom (calculated hard kernel)

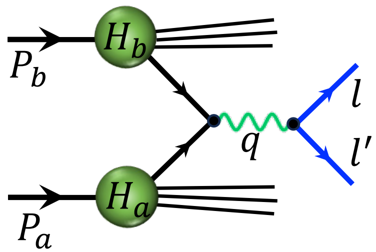
Color confinement (Parton in hadron)

- **PDFs: the probability distribution of partons (quarks and gluons) within a hadron ——— Inclusive process**

DIS

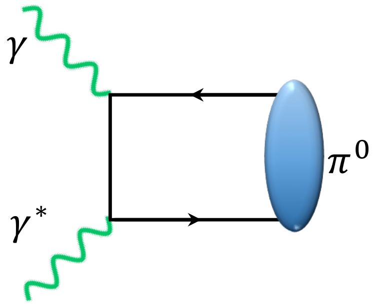


Drell-Yan

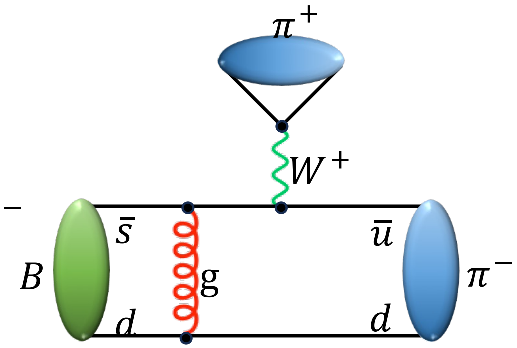


- **LCDAs: the probability amplitude for partons within a hadron ———Exclusive process**

$\gamma\gamma^* \to \pi$

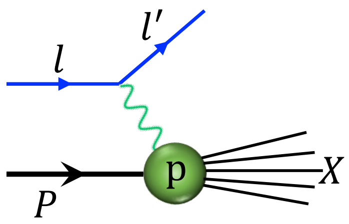


$B \to \pi^+\pi^-$

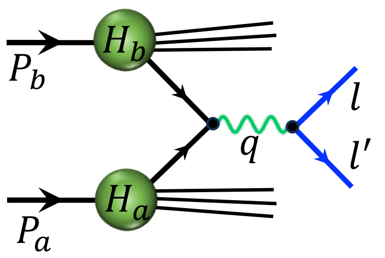


➢ PDF

DIS

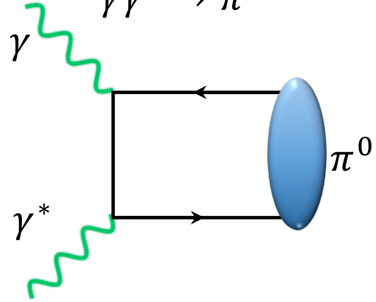


Drell-Yan

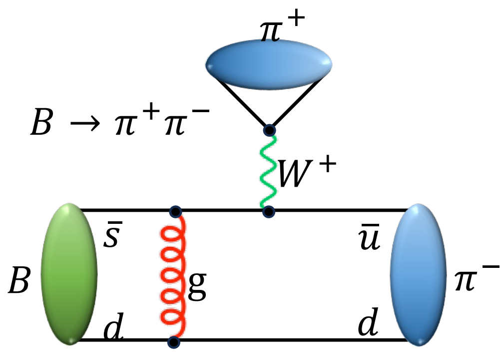


➢ LCDA

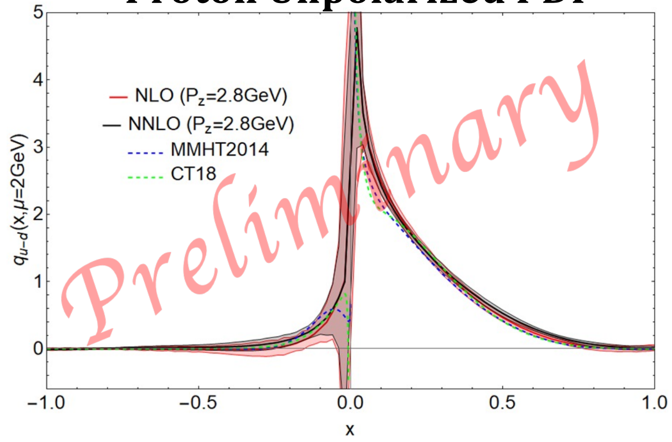
$\gamma\gamma^* \to \pi$



$B \to \pi^+\pi^-$

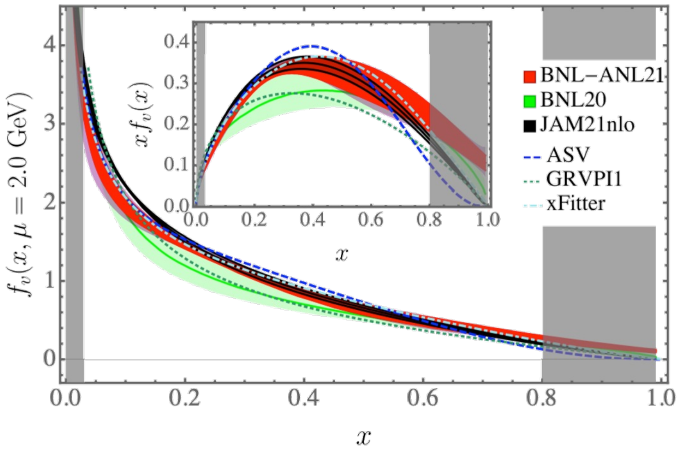


Proton Unpolarized PDF



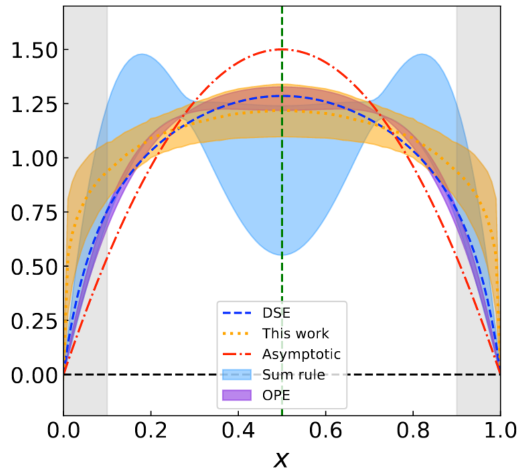
Preliminary result from LPC

Pion valancePDF



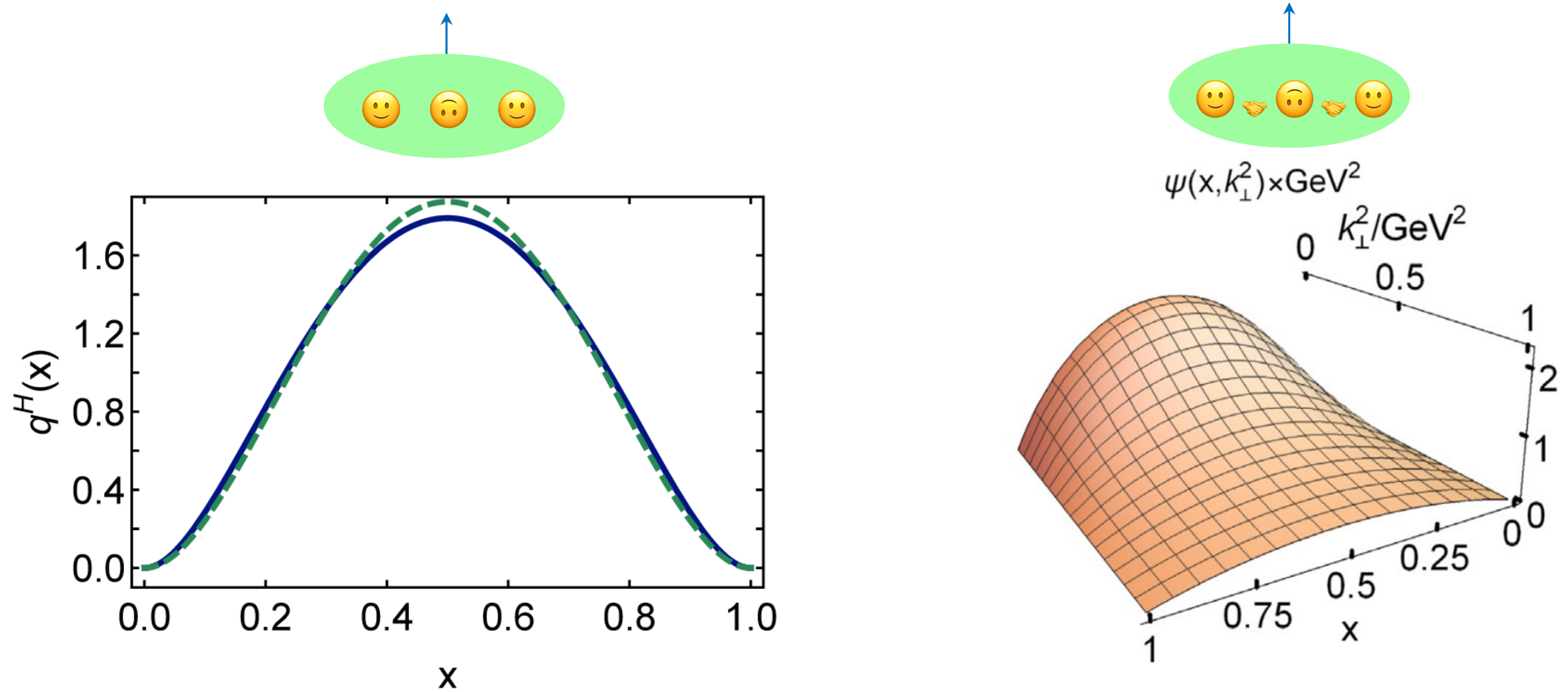
X.Gao, et.al. PRL.128.142003 (2022)

Pion LCDA

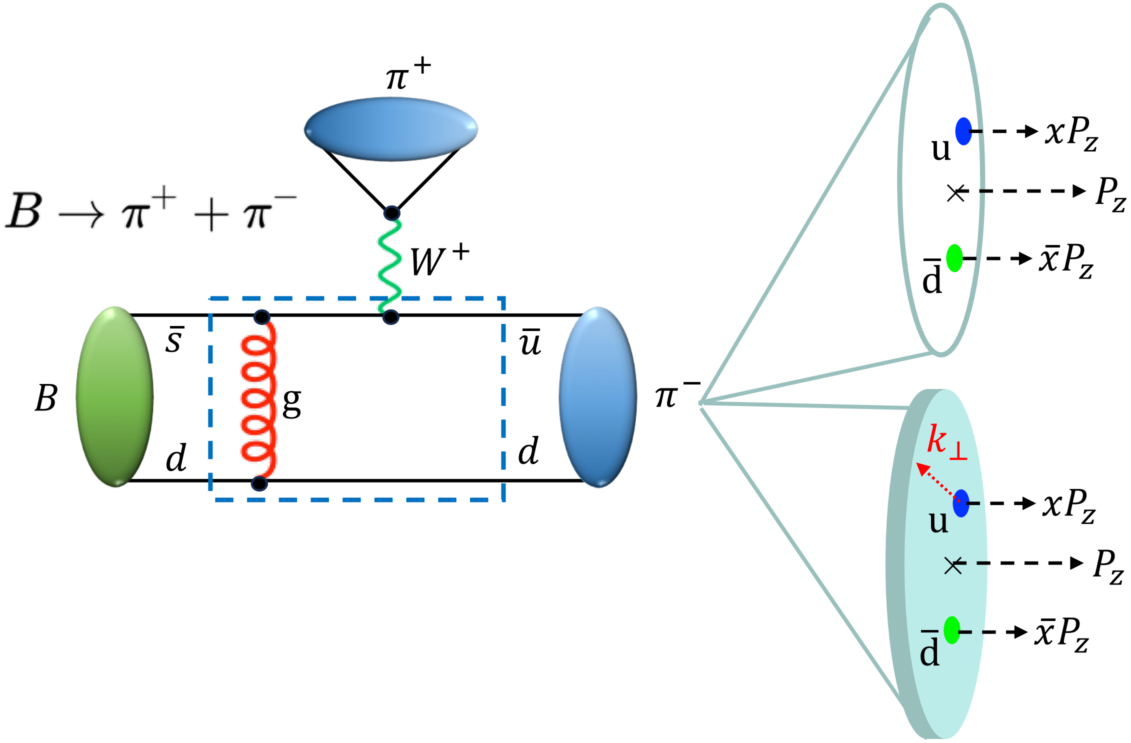


(LPC) J.Hua et.al. PRL.129,132001 (2022)

- **One dimensional LCDA**
- **Three dimensional TMDWF**

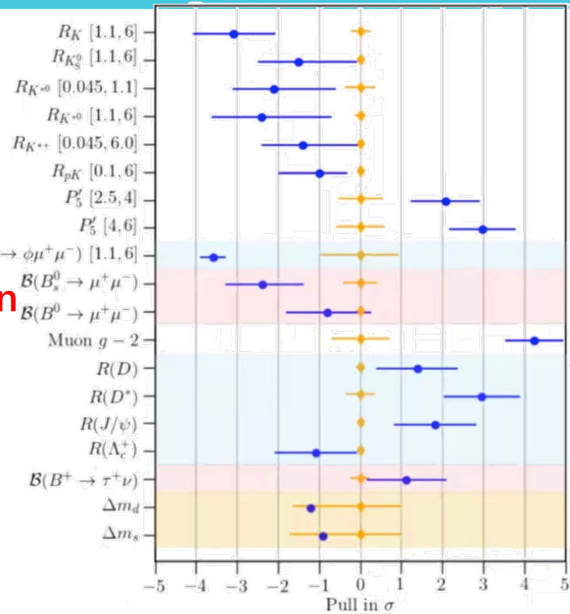


C.D.Roberts et.al. PPNP.120, 138883 (2021)



Collinear Factorization

TMD Factorization



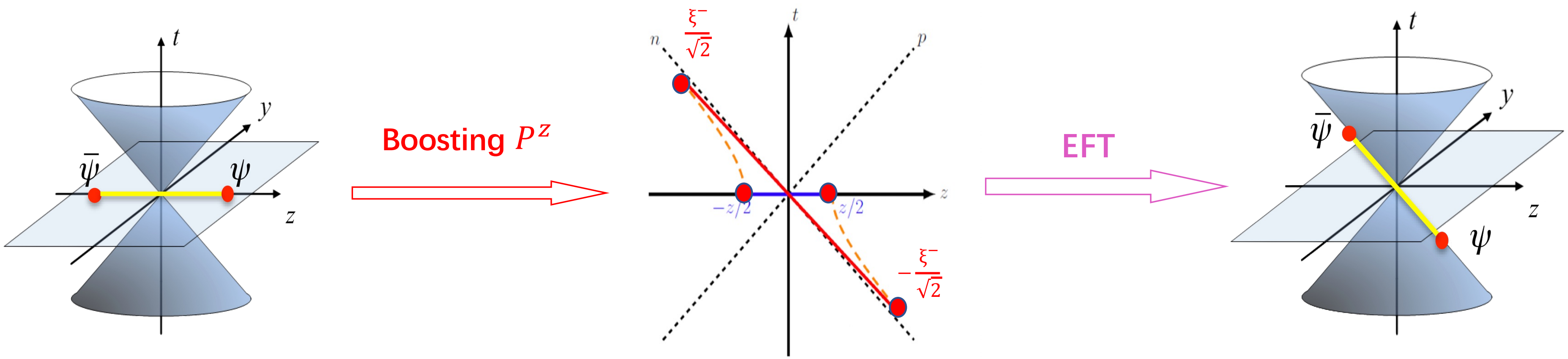
Y.Y.Keum et.al. PLB. 504, 6(2001)

C.D.Lu et.al. PRD.63,074009(2001)

A.Ali et.al. PRL.76, 074018(2007)

$$\mathscr{A} = \langle \pi^+\pi^- | H | B \rangle \sim \int d^4k_1 d^4k_2 d^4k_3 \, \mathrm{Tr}[C(t)\psi_B(p_1)\psi_{\pi^+}(p_2)\psi_{\pi^-}(p_3)H(k_1,k_2,k_3,t)]$$

➢ Large Momentum Effective Theory:



LaMET is capable for Entire x dependence distributions

Recent Progress on LCDA

R.Zhang et.al. PRD. 125, 094519(2020)
Pion LCDA with 3 lattice spacings

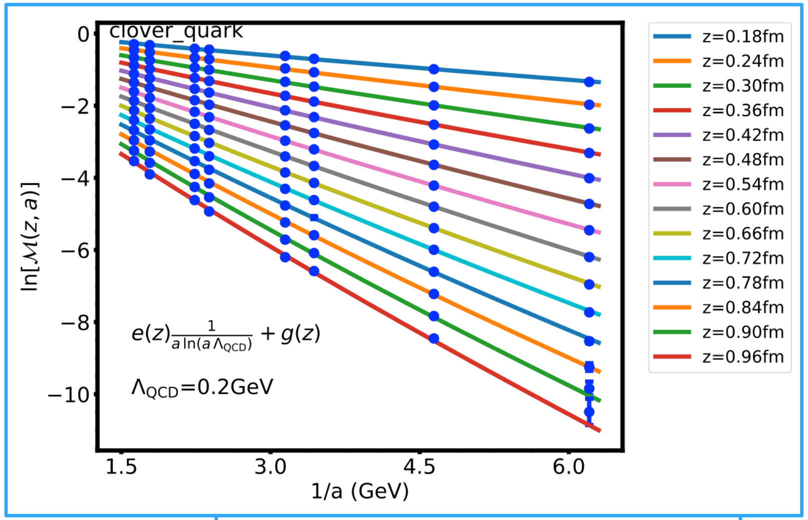
X.Ji et.al. NPB.964,115311(2021)
A hybrid renormalization scheme

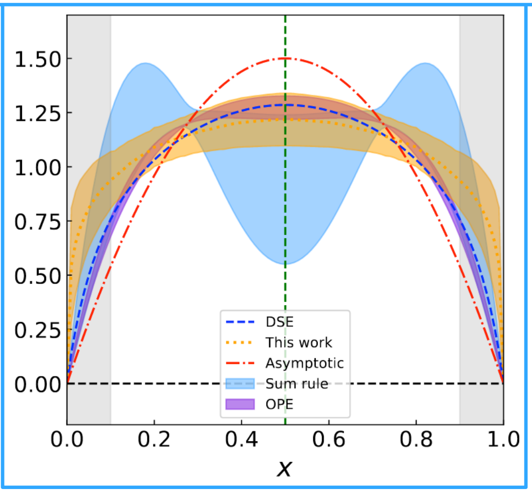
(LPC) J.Hua et.al. PRL.127, 062002(2021)
K^*, ϕ LCDA at physical with hybrid

(LPC) Y.K.Huo et.al. NPB. 969, 115443 (2021)
Solve linear divergence by self renormalization

(LPC) J.Hua et.al. PRL.129,132001 (2022)
π, K LCDA with self renormalization

R.Zhang et.al. RPB.844,138081(2023)
Resummation to improve endpoint region





Recent Progress on TMDWF

(LPC) Q.A. Zhang et.al. PRL. 125, 192001 (2020)
Soft function and CS kernel (First)

M. Schlemmer et.al. JHEP.08,004(2021)
CS kernel by different TMDs

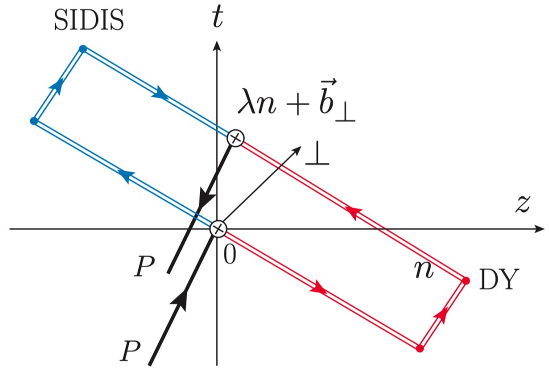
P. Shanahan et.al. PRD.104, 114502(2021)
CS kernel from quasi-TMDPDFs (1-loop)

L.Yuan, X.Feng et.al. PRL. 128, 062002 (2022)
Twists' effects on soft function

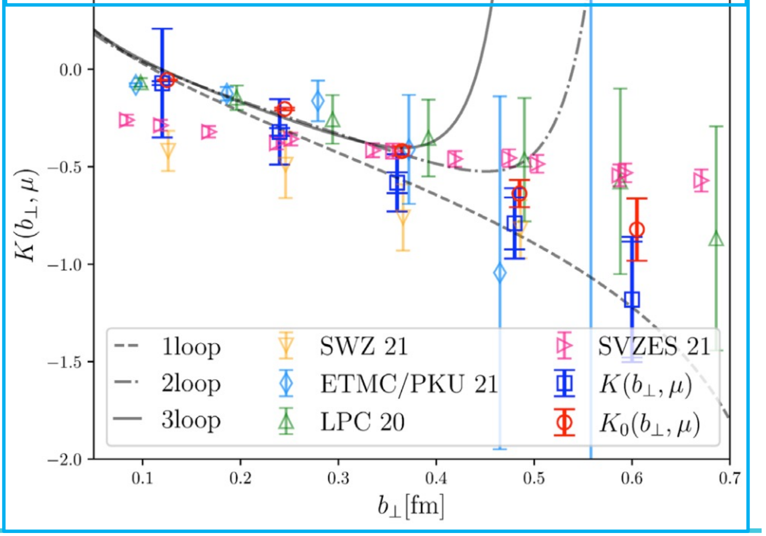
(LPC) K.Zhang PRL.129,082002 (2022)
Renormalization of TMDs on lattice

(LPC) M.H.Chu et.al. PRD.106, 034509 (2022)
CS kernel from quasi-TMDWFs (1-loop)

A. Avkhadiev et.al. arXiv2307.12359 (2023)
CS kernel at physical pion mass



(LPC) M.H.Chu et.al. PRD.106, 034509 (2022)



TMD Factorization in LaMET

➢ **Multiplicative factorization of quasi-TMDWF in LaMET**

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z)\, S_I^{\frac{1}{2}}(b_\perp, \mu)$$

$$= H^{\pm}(x, \zeta^z, \mu) \exp\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\pm\zeta^z + i\epsilon}{\zeta}\right] \Psi^{\pm}(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

X.D.Ji et.al. Rev.Mod.Phys. 93, 035005 (2021)

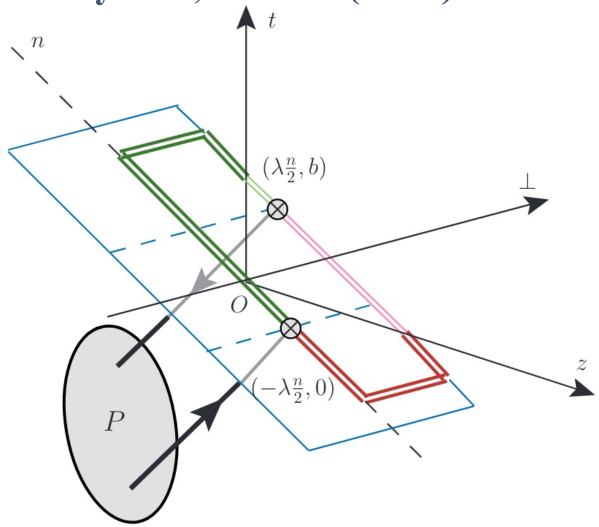
$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta_z)$: **Quasi-TMDWF,**

$S_r(b_\perp, \mu)$: **Intrinsic soft function,**

$H^{\pm}(\zeta_z, \bar{\zeta}_z, \mu^2)$: **Matching coefficient,**

$K(b_\perp, \mu)$: **Collins-Soper kernel,**

$\Psi^{\pm}(x, b_\perp, \mu, \zeta)$: **TMDWF.**



Lattice ensembles

$L^3 \times T$	a (fm)	m_π^{sea} (MeV)	m_π^v (MeV)
$24^3 \times 64$	0.12	310	670
			measurement
			1053×4

- 2+1+1 flavors of HISQ action (MILC)
- Momenta: 1.72, 2.15, 2.58, 3.01GeV
- Coulomb gauge fixed wall source

$L^3 \times T$	a (fm)	m_π^{sea} (MeV)	m_π^v (MeV)
$48^3 \times 48$	0.098	333	662
			measurement
			952×4

- 2+1 flavors of Symanzik gauge action (CLS)
- Momenta: 1.58, 2.11, 2.64, 3.16GeV
- Coulomb gauge fixed wall source

Soft Function by LaMET

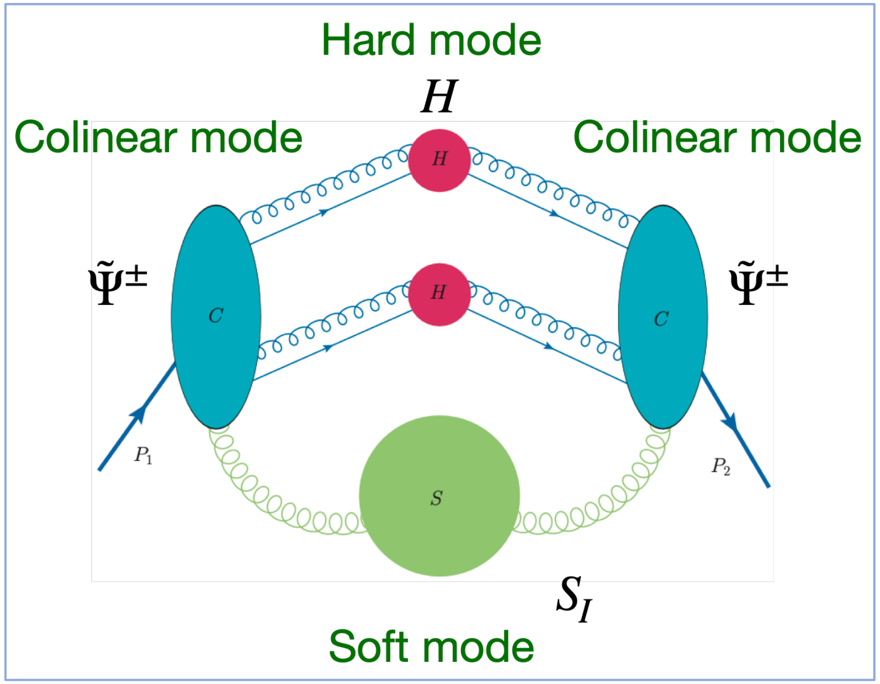
➢ Four quark form factor:

$$F(b_\perp, P_1, P_2, \Gamma, \mu) = \frac{\langle P_2|\bar{q}(b_\perp)\Gamma q(b_\perp)\bar{q}(0)\Gamma' q(0)|P_1\rangle}{\langle 0|\bar{q}(0)\gamma^\mu\gamma^5 q(0)|P_1\rangle\langle P_2|\bar{q}(0)\gamma_\mu\gamma^5 q(0)|0\rangle}$$

Normalization factor: $f_\pi^2 P_1 P_2$

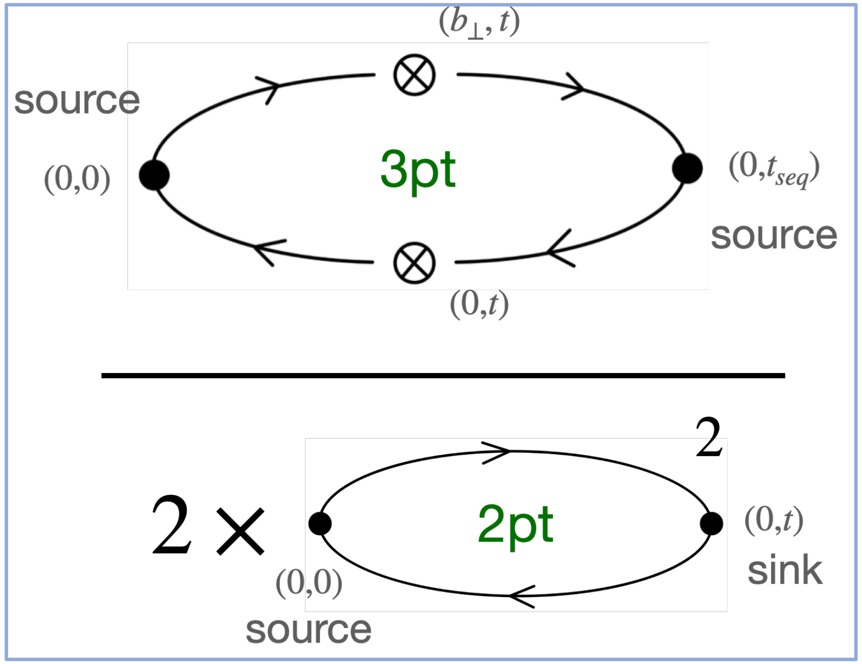
➢ Factorization of form factor:

$$S_I(b_\perp, \mu) = \frac{F(b_\perp, P_1, P_2, \Gamma, \mu)}{\int dx_1 dx_2 H(x_1, x_2, \Gamma)\tilde{\Psi}^{\pm *}(x_2, b_\perp, \zeta^z)\tilde{\Psi}^{\pm}(x_1, b_\perp, \zeta^z)}$$

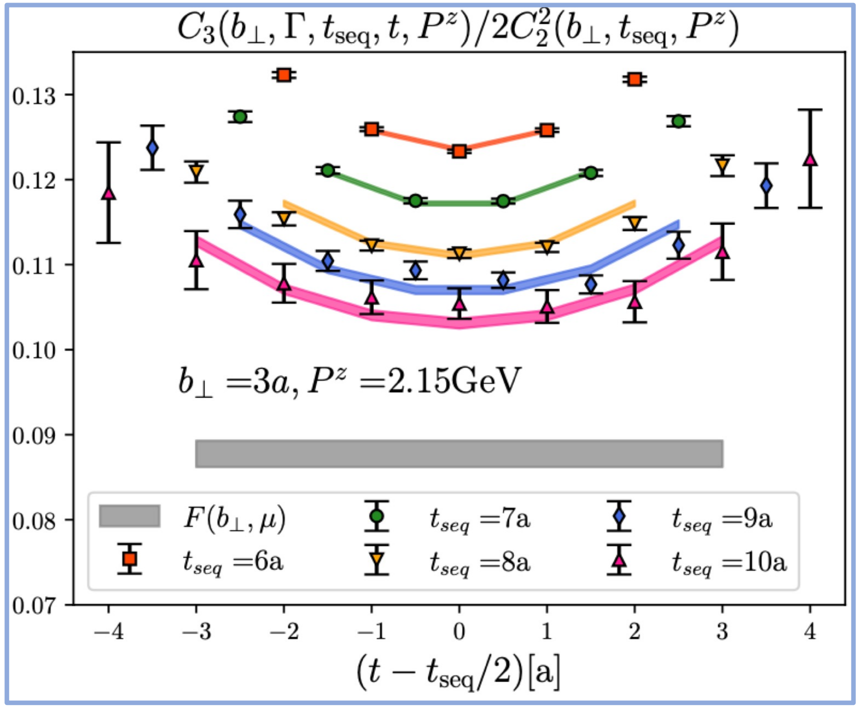


Soft Function by LaMET

$$\frac{C_3(b_\perp,\Gamma,t_{\rm sep},t,P^z)}{2C_2^2(t_{\rm sep}/2,P^z)} = F(b_\perp,\Gamma,P^z)\frac{1+c_1(e^{-\Delta E t}+e^{-\Delta E(t_{\rm sep}-t)})}{1+c_2 e^{-\Delta E t_{\rm sep}/2}}$$

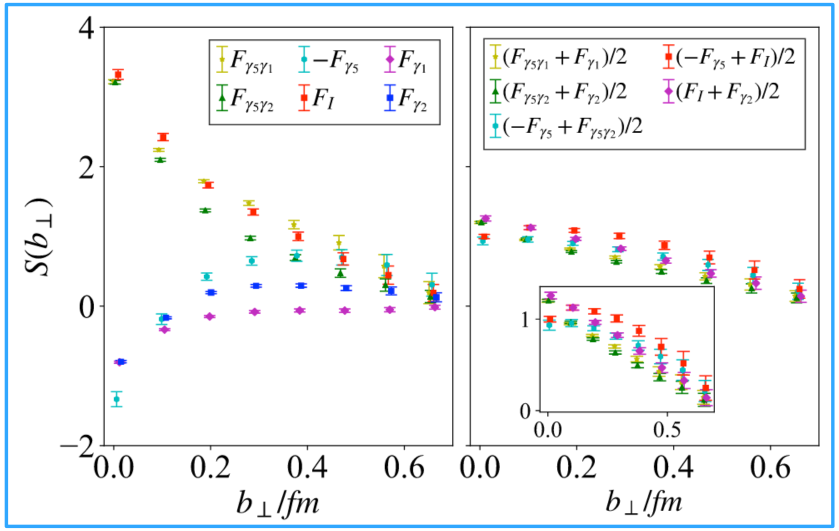


Joint fit for t_seq = (7,8,9,10)



Soft Function by LaMET

Operator mixing in Soft function:



Y.Li et.al. PRL.128, 062002 (2022)

By Fierz rearrangement analysis, these combination can suppress high twists contribution:

$$\sum F(\Gamma = \gamma^\mu) + F(\Gamma = \gamma^\mu\gamma_5)$$
$$= (\bar{\psi}_a\gamma^{x,y}\psi_b)(\bar{\psi}_c\gamma_{x,y}\psi_d) + (\bar{\psi}_a\gamma^{x,y}\gamma_5\psi_b)(\bar{\psi}_c\gamma_{x,y}\gamma_5\psi_d)$$
$$= \bar{\psi}_c\gamma^\mu\gamma_5\psi_b\bar{\psi}_a\gamma_\mu\gamma_5\psi_d + \bar{\psi}_c\gamma^\mu\psi_b\bar{\psi}_a\gamma_\mu\psi_d$$

$$F(\Gamma = I) - F(\Gamma = \gamma_5)$$
$$= (\bar{\psi}_a\psi_b)(\bar{\psi}_c\psi_d) - (\bar{\psi}_a\gamma_5\psi_b)(\bar{\psi}_c\gamma_5\psi_d)$$
$$= \frac{1}{2}\bar{\psi}_c\gamma^\mu\gamma_5\psi_b\bar{\psi}_a\gamma_\mu\gamma_5\psi_d - \frac{1}{2}\bar{\psi}_c\gamma^\mu\psi_b\bar{\psi}_a\gamma_\mu\psi_d$$

- The UV divergence in the I and γ_5 form factor. can be removed by the renormalization constant of scalar density operator

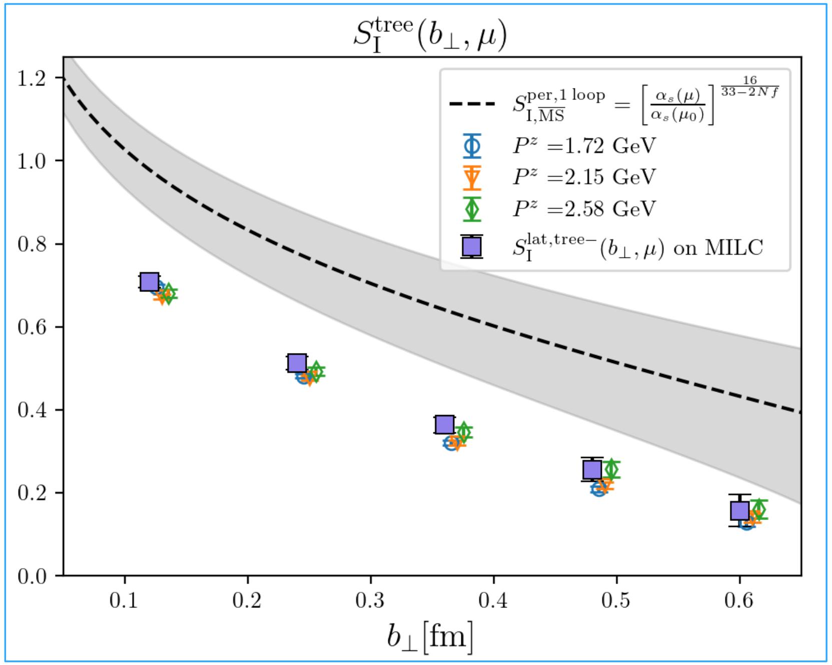
Z.F.Deng et.al. JHEP.09, 046 (2022)

$$Z_S = 1 + \frac{\alpha_s C_F}{4\pi}\frac{3}{\epsilon_{\rm UV}}. \qquad (59)$$

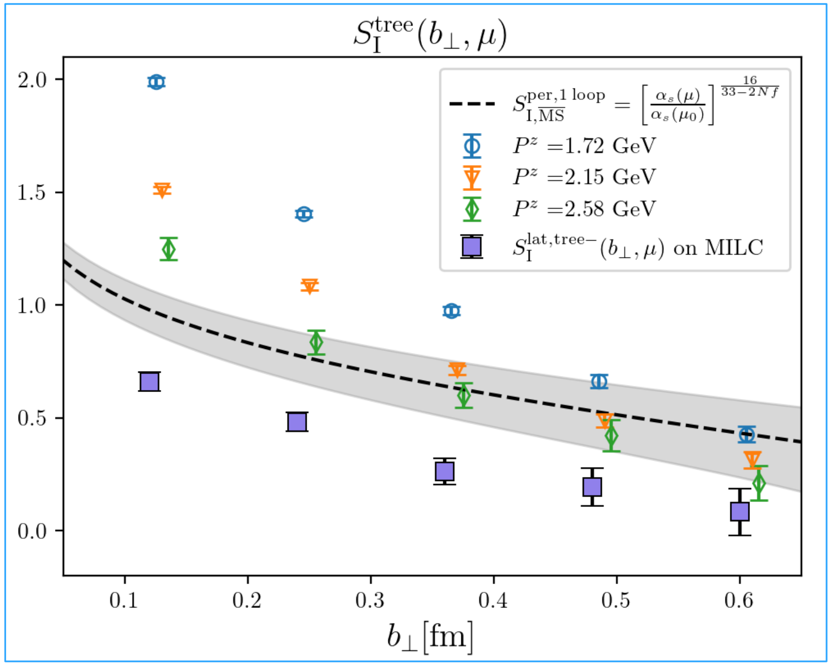
Soft Function by LaMET

➢ **Pz dependence of soft function for 2 combination:**

$\sum F(\Gamma = \gamma^\mu) + F(\Gamma = \gamma^\mu \gamma_5)$

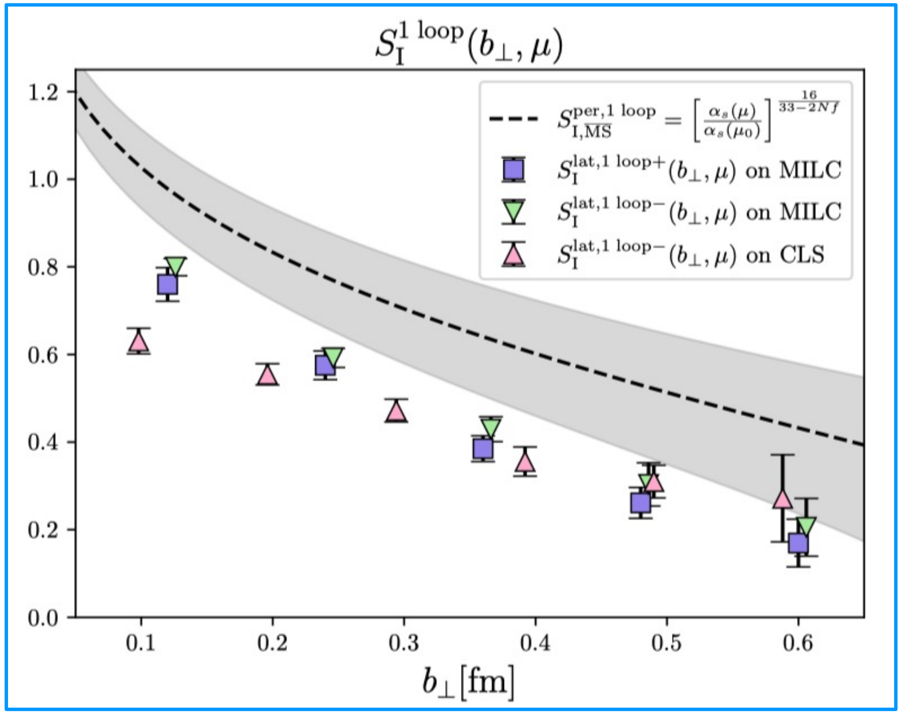


$F(\Gamma = I) - F(\Gamma = \gamma_5)$



Soft Function by LaMET

- **1-loop matching soft function extracted by MILC and CLS ensemble**
- **Consistent for '+/-' cases: soft function is universal**
- **Discrete effects are significant**



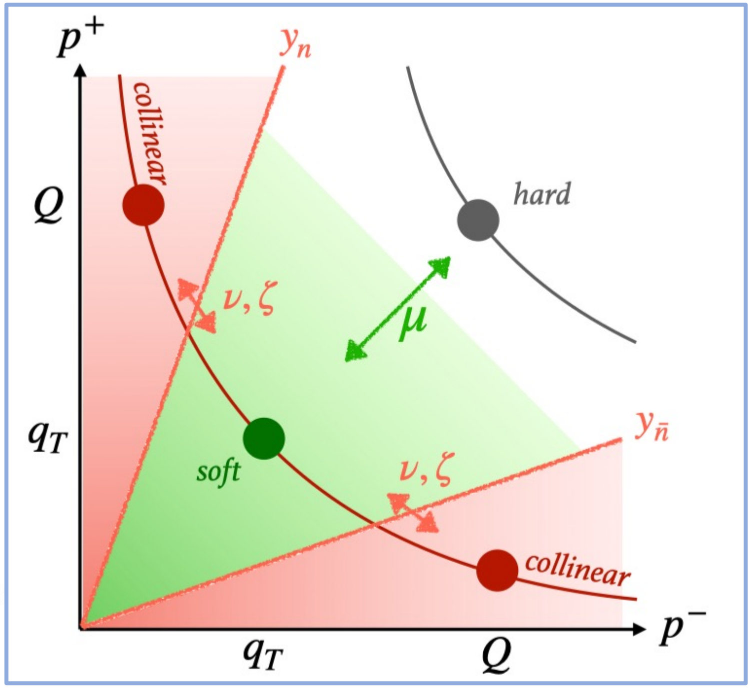
CS-Kernel by LaMET

- **Collins-Soper kernel: describe the evolution for rapidity scale:**

$$2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \ln \Psi(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu),$$

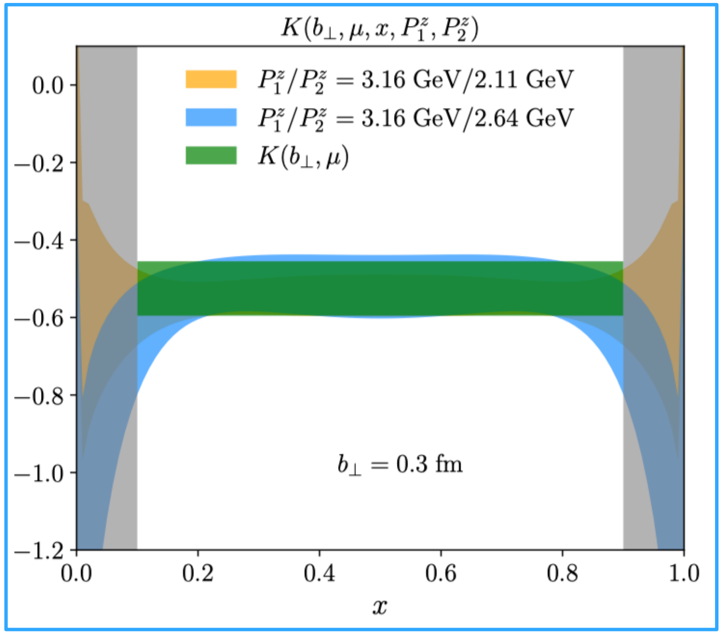
- **In LaMET factorization, CS-kernel can be extracted by ratio:**

$$K(b_\perp, \mu, x, P_1^z, P_2^z) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{H^\pm(xP_2^z, \mu)\tilde{\Psi} \pm (x, b_\perp, \mu, P_1^z)}{H^\pm(xP_1^z, \mu)\tilde{\Psi}^\pm(x, b_\perp, \mu, P_2^z)},$$

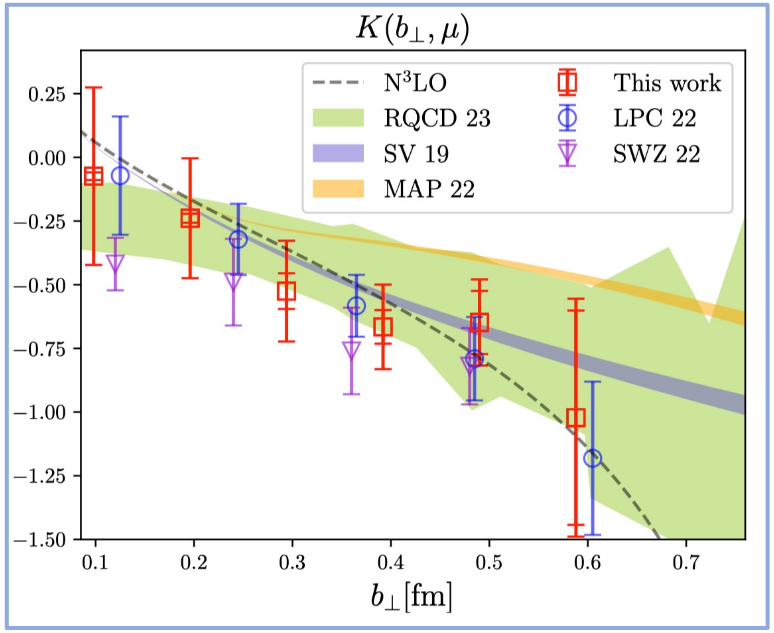


CS-Kernel by LaMET

$$K(b_\perp,\mu,x,P_1^z,P_2^z)=\frac{1}{\ln(P_1^z/P_2^z)}\ln\frac{H^\pm(xP_2^z,\mu)\tilde{\Psi}\pm(x,b_\perp,\mu,P_1^z)}{H^\pm(xP_1^z,\mu)\tilde{\Psi}^\pm(x,b_\perp,\mu,P_2^z)},$$



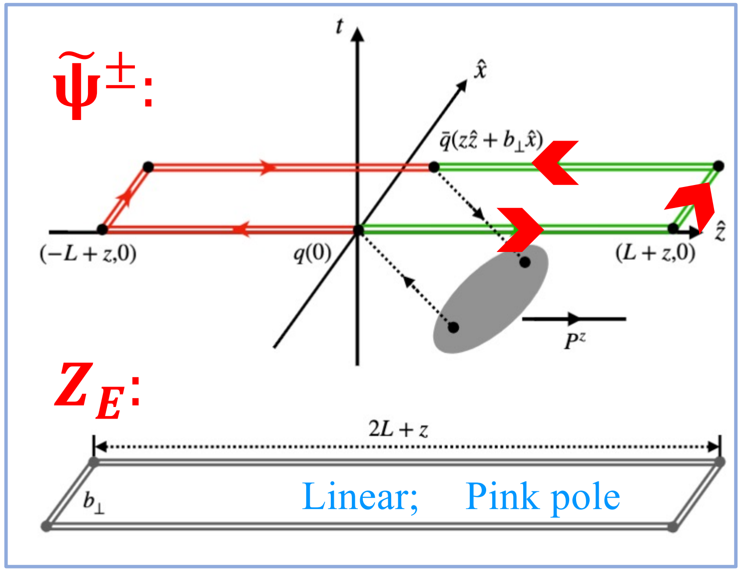
1-loop CS-kernel on CLS ensemble



Quasi TMDWFs

➢ **Quasi TMDWF in Euclidean lattice:**

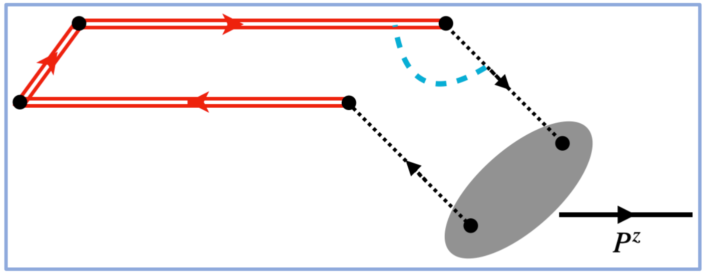
$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right)=\lim_{L \to \infty} \int \frac{P^{z} dz}{2\pi} e^{ixzP^{z}} \times \frac{\left\langle 0\left|\bar{q}\left(z\hat{n}_{z}+b_{\perp}\hat{n}_{\perp}\right) \gamma^{t}\gamma_{5} U_{c\pm} q(0)\right| \pi\left(P^{z}\right)\right\rangle}{\sqrt{Z_{E}\left(2L \pm z, b_{\perp}, \mu\right)} Z_{O}(1/a, \mu, \Gamma)}.$$



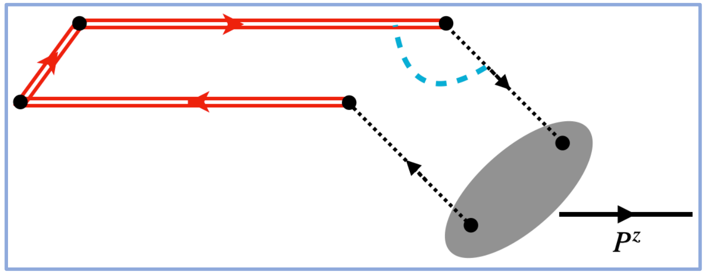
➢ **Staple-shaped gauge-link:**

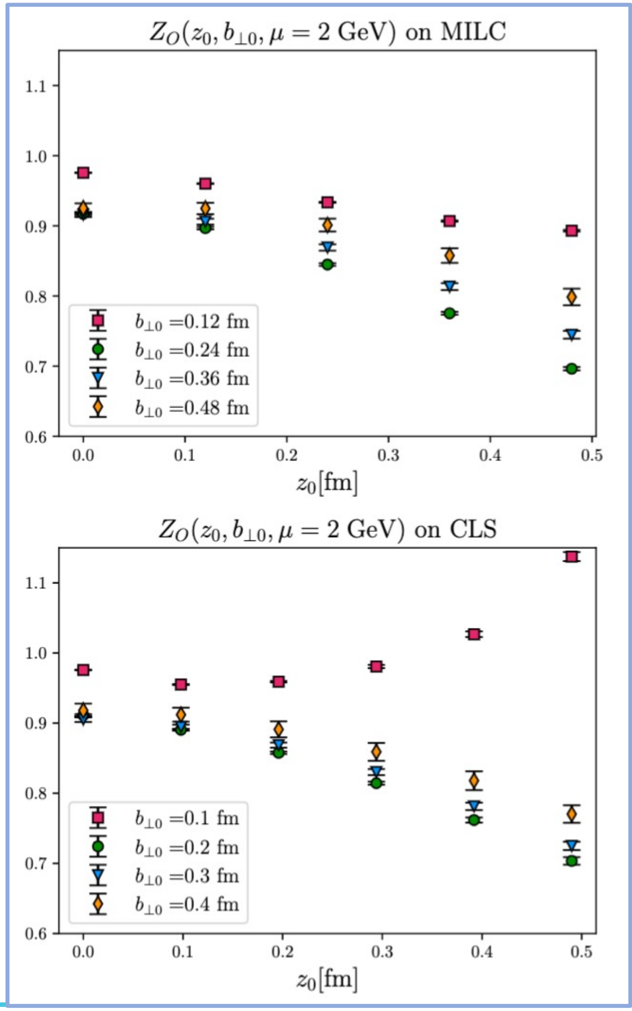
$$U_{c\pm}=U_{z}^{\dagger}(z\hat{n}_{z}+b_{\perp}\hat{n}_{\perp};L)U_{\perp}(\pm L\hat{n}_{z}+z\hat{n}_{z};b_{\perp}) \times U_{z}(0\hat{n}_{z};\pm L+z).$$

Logarithm divergence



Quasi TMDWFs





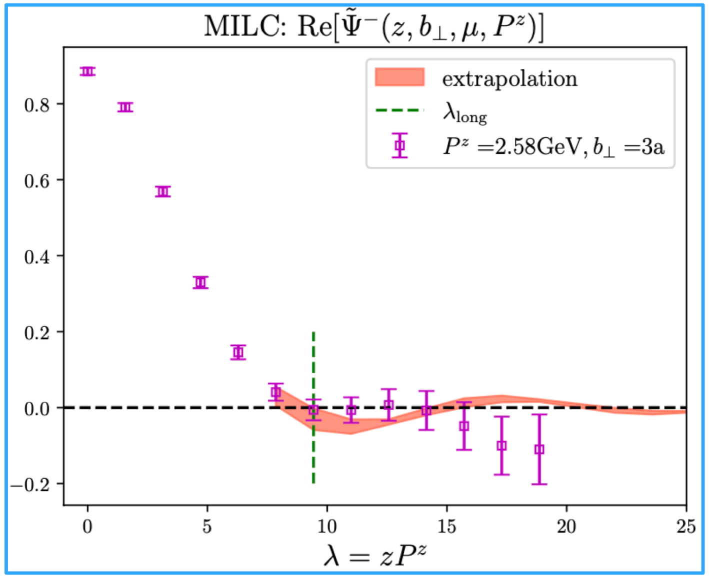
- Quark Wilson line vertex renormalization:

$$Z_O(1/a, \mu) = \frac{\tilde{\Psi}^{\pm,0}(z_0, b_{\perp 0}, \zeta^z = 0, L)}{\sqrt{Z_E(2L + |z_0|, b_{\perp 0}, \mu)}\tilde{\psi}^{\overline{\rm MS}}(z_0, b_{\perp 0}, \mu)}.$$

(LPC) K.Zhang PRL.129,082002 (2022)

Quasi TMDWFs

$$\tilde{\Psi}^{\pm}(x,b_\perp,\mu,\zeta^z)=\lim_{L\to\infty}\int\frac{P^z dz}{2\pi}e^{ixzP^z}\frac{\langle 0|\bar{q}(z\hat{n}_z+b_\perp\hat{n}_\perp)\gamma^t\gamma_5 U_{c\pm}q(0)|\pi(P^z)\rangle}{\sqrt{Z_E(2L\pm z,b_\perp,\mu)}Z_O(1/a,\mu,\Gamma)}$$

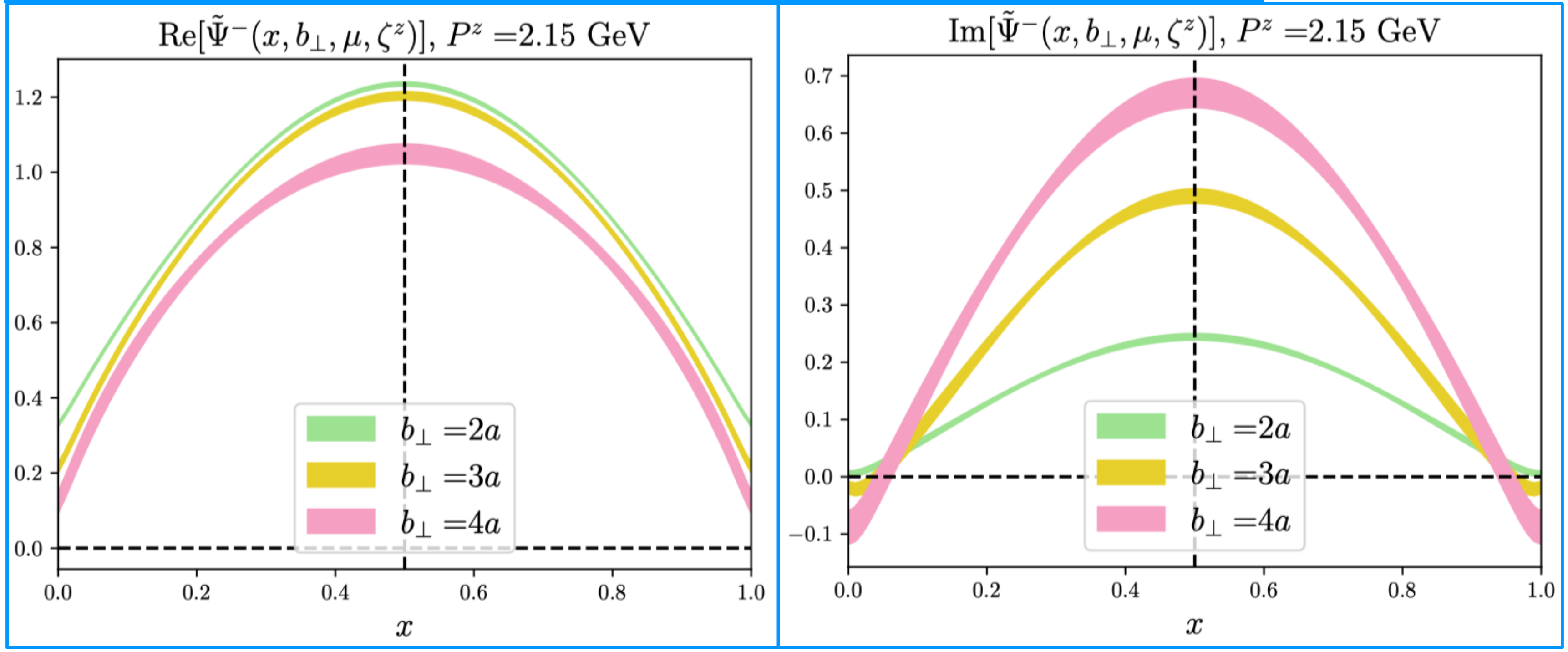


- **Quasi TMDWF in coordinate space and extrapolation in large λ**
- **Physical based parameterization：**

$$\tilde{\Psi}(z,b_\perp,\mu,P^z)=f(b_\perp)\left[\frac{c_1}{(-i\lambda)^d}+e^{i\lambda}\frac{c_2}{(i\lambda)^d}\right]e^{-\frac{\lambda}{\lambda_0}}$$

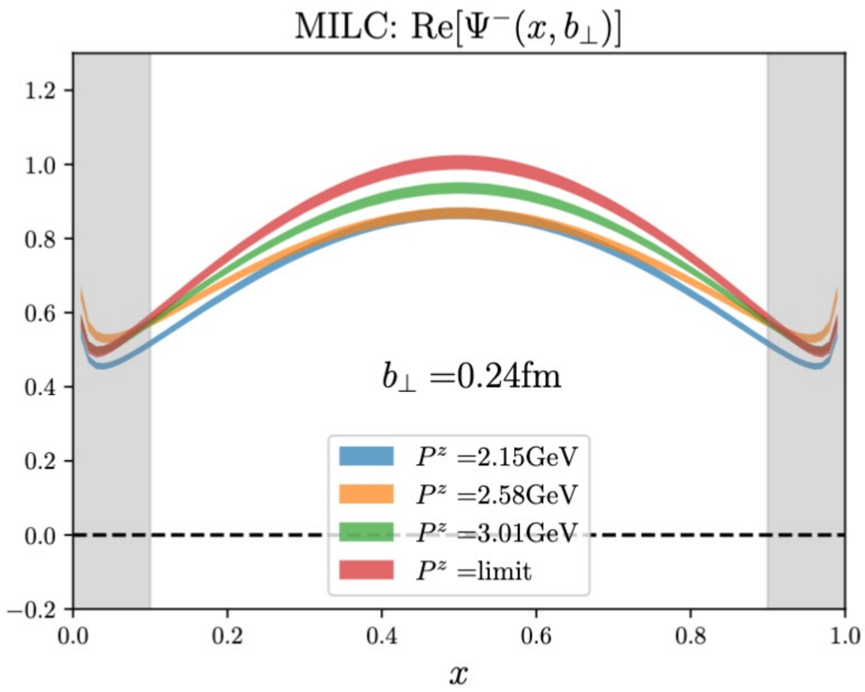
Quasi TMDWFs

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\langle 0|\bar{q}(z\hat{n}_z + b_\perp \hat{n}_\perp)\gamma^t\gamma_5 U_{c\pm} q(0)|\pi(P^z)\rangle}{\sqrt{Z_E(2L \pm z, b_\perp, \mu)} Z_O(1/a, \mu, \Gamma)}$$



TMDWFs by LaMET

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z)\, S_I^{\frac{1}{2}}(b_\perp, \mu) = H^{\pm}(x, \zeta^z, \mu)\, \mathbf{exp}\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\pm\zeta^z + i\epsilon}{\zeta}\right] \Psi^{\pm}(x, b_\perp, \mu, \zeta)$$



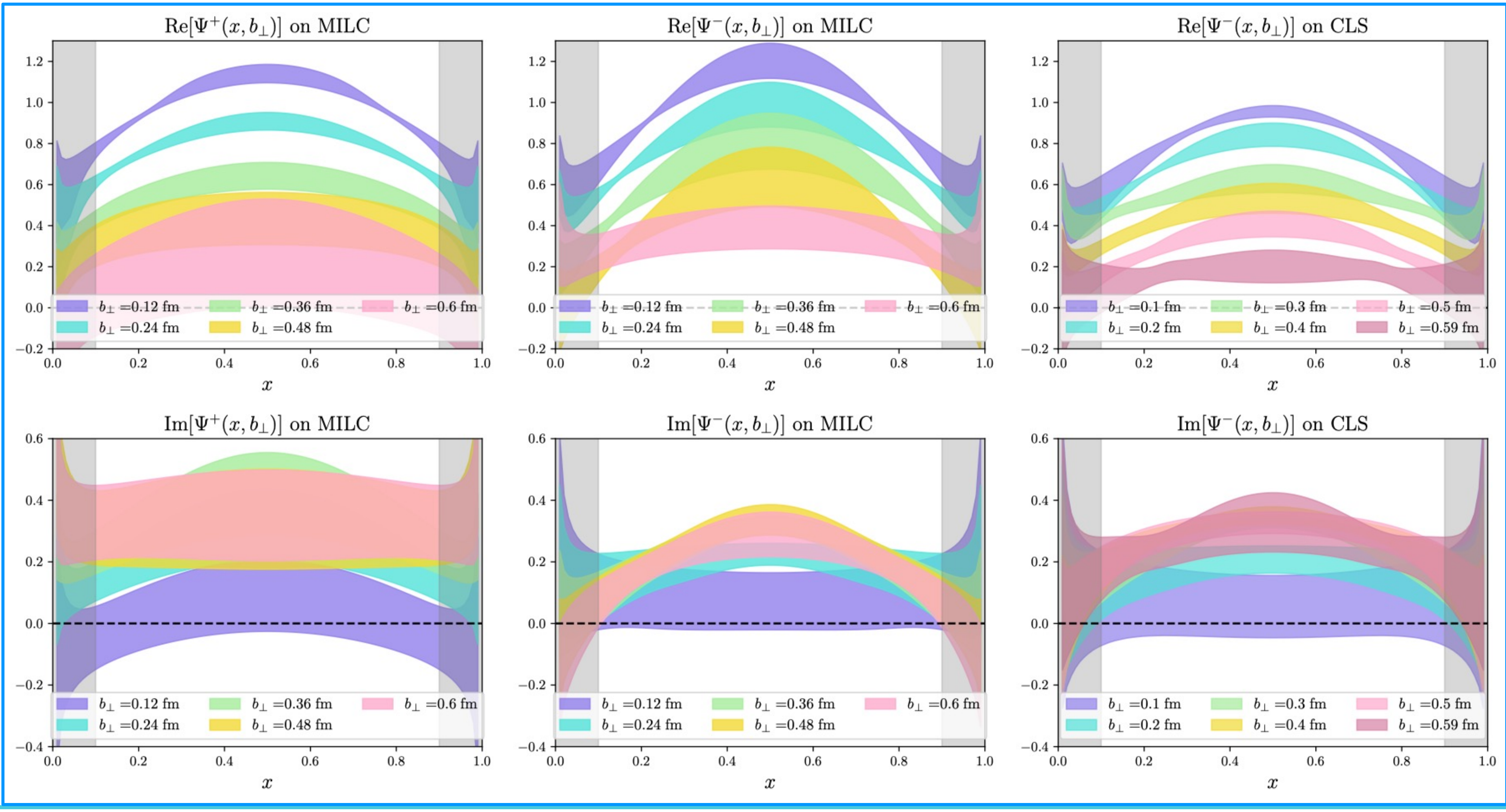
$$H^{\pm}(x, \zeta^z, \mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{5\pi^2}{6} - 4 + l_{\pm} + \bar{l}_{\pm} - \frac{1}{2}(l_{\pm}^2 + \bar{l}_{\pm}^2)\right)$$

$$l_{\pm} = \ln[(-x\zeta^z \pm i\epsilon)/\mu^2]$$

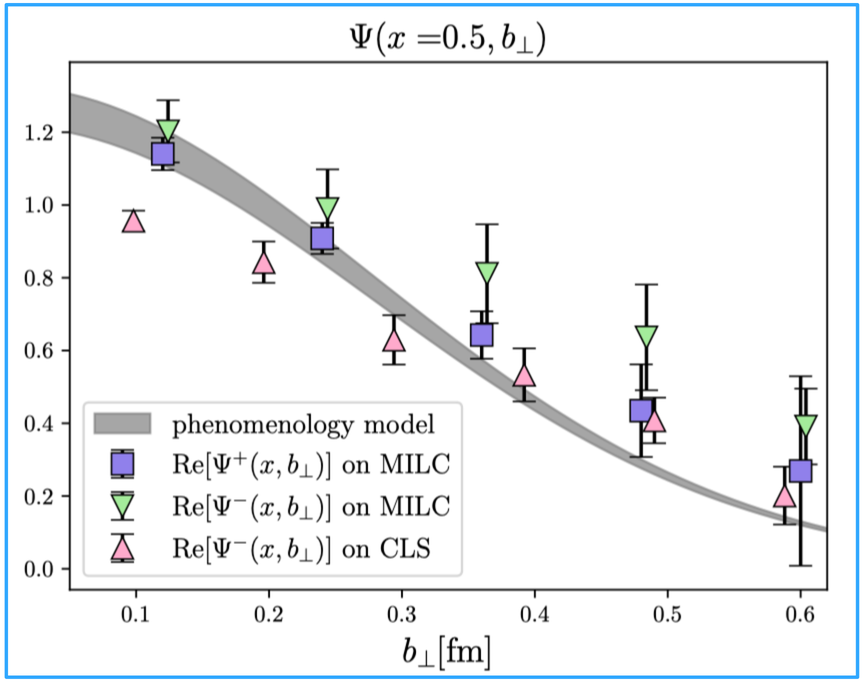
- **Pz dependence of TMDWF after mathing**
  - **Pz extrapolation:**

$$\Psi^{\pm}(x, P_z) = \Psi^{\pm}(x, P_z \to \infty) + \frac{c_2(x)}{P_z^2} + \mathcal{O}\left(\frac{1}{P_z^4}\right)$$

TMDWFs by LaMET



TMDWFs by LaMET



- **Decay behavior of $b_\perp$ at $x = 0.5$**
- **A comparison with a phenomenological model at $x = 0.5$**

$$\Psi(x, b_\perp) = 6x(1-x)\left[1 + \frac{3}{2} a_2^\pi \left(5(2x-1)^2 - 1\right)\right] \exp\left[-\frac{x(1-x)b_\perp^2}{4a^2}\right]$$

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Summary

∗. We calculate the one-loop intrinsic soft function and TMDWF with LaMET on MILC and CLS ensembles.

∗. Fierz rearrangement analysis can be adopted to suppress high twist's effect in soft function.

∗. The MILC and CLS results show good agreement, but discrete errors are still relatively significant in current results.

∗. Future calculations with more $b_\perp$ on smaller lattice spacings are necessary to get more complete TMDWF results.

Thanks for your attentions!