Nucleon Electric Polarizability and Nucleon-Pion Scattering at Physical Pion Mass

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Motivations

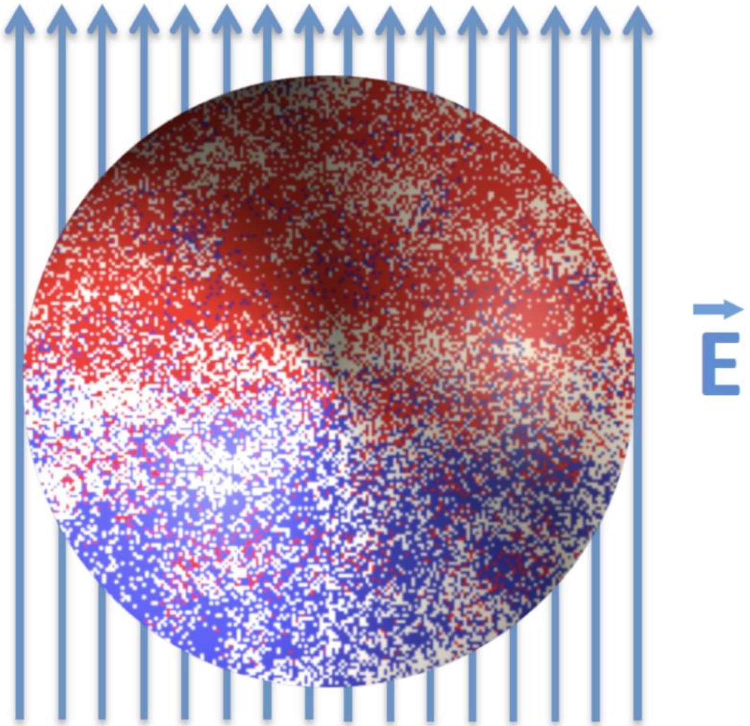
- Nucleon ➡ The fundamental blocks of visible matter in the universe.

- Polarizabilities ➡ Crucial properties of nucleon, akin to size and shape.

- The second-order response of a neutron to an EM field,

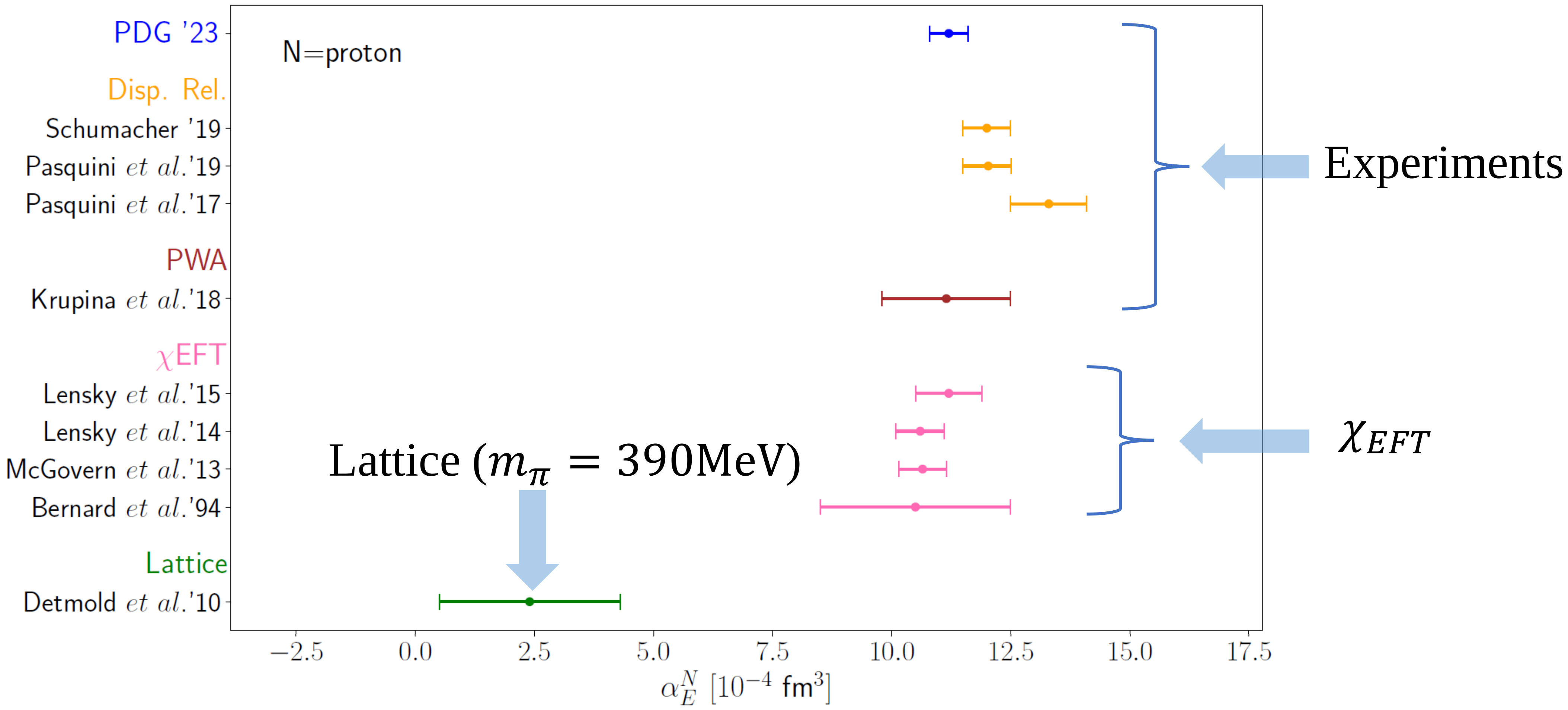
$$H_{eff}^{(2)} = -\frac{4\pi}{2}\alpha_E E^2 - \frac{4\pi}{2}\beta_M B^2$$

- Our Method: calculating a nucleon 4-point correlation function on lattice.



Recent Researches on Polarizabilities

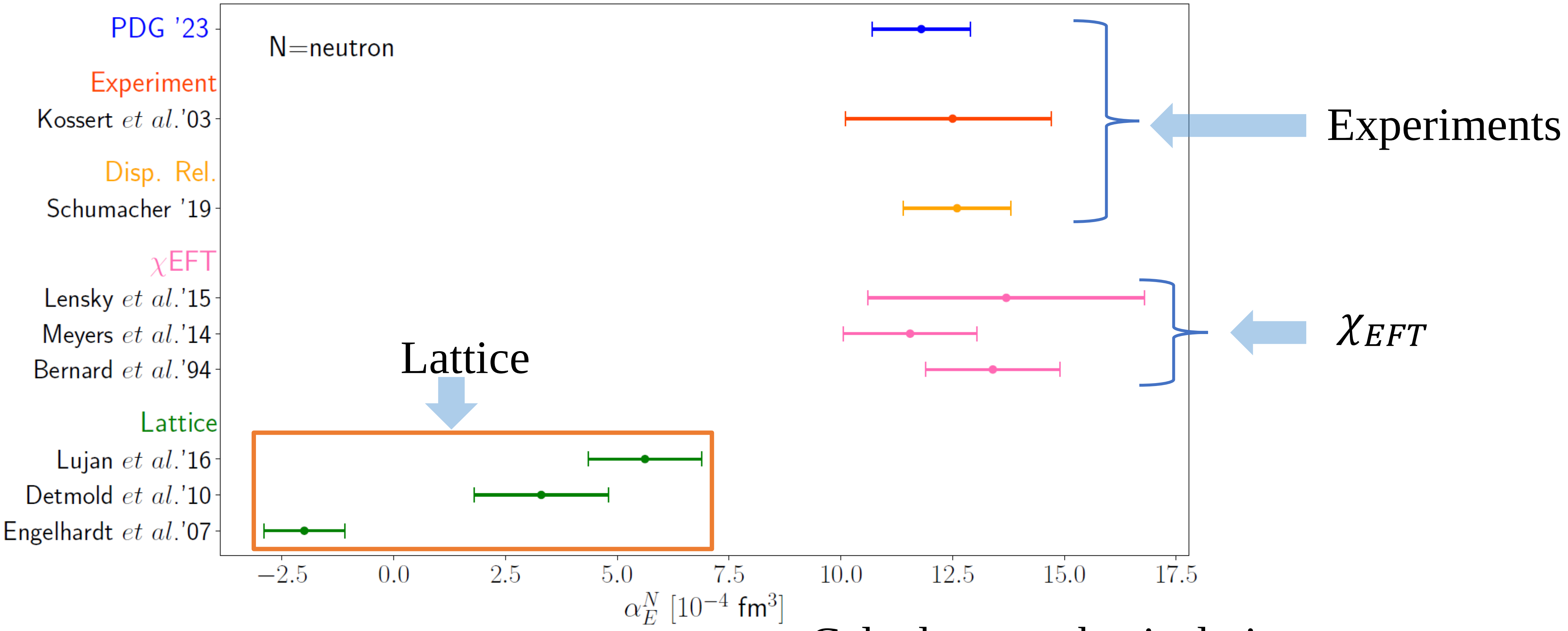
- Summary for electric polarizability α_E of proton



Only one former Lattice result for proton α_E^p

Recent Researches on Polarizabilities

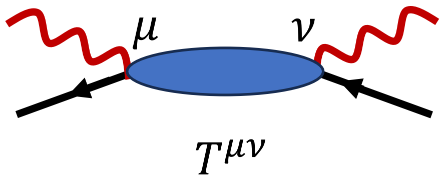
- Summary for electric polarizability α_E of neutron



Lattice results deviate from others?

- Calculate on physical pion mass
- Calculate with 4-point function
- Include nucleon-pion contribution

Doubly Virtual Compton Scattering



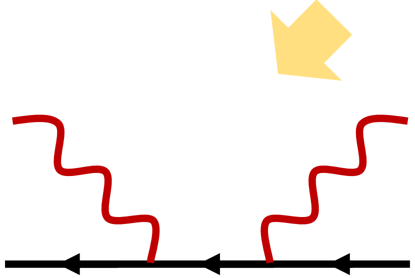
- Spin-averaged forward Compton scattering

$$T^{\mu\nu}(P,q) = \int d^4x\, e^{iqx} \langle N | J^\mu(x,t) J^\nu(0) | N \rangle = K_1^{\mu\nu} T_1(Q_0, Q^2) + K_2^{\mu\nu} T_2(Q_0, Q^2)$$

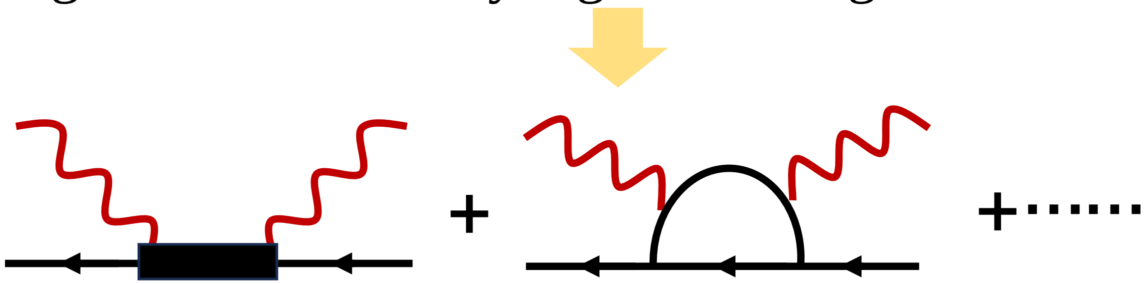
$$K_1^{\mu\nu} = q^\mu q^\nu - g^{\mu\nu} q^2,\ K_2^{\mu\nu} = \frac{1}{M^2}\left[(P^\mu q^\nu + P^\nu q^\mu) P \cdot q - g^{\mu\nu}(P \cdot q)^2 - P^\mu P^\nu q^2\right]$$

$$T_i = T_i^B + T_i^{NB}$$

- Born term:
  Thomson scattering
- Non-Born term:
  Rayleigh scattering



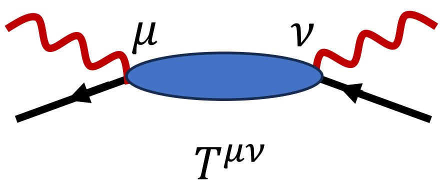
Intermeidate states: N



Intermeidate states: $N^*, N\pi, \Delta, \cdots$

- Polarizabilities: $T_1^{NB} = \frac{M}{\alpha_{em}}[-\beta_M + O(Q)],\ T_2^{NB} = \frac{M}{\alpha_{em}}[\alpha_E + \beta_M + O(Q)]$

Extraction from 4-point Function



- Compton tensor:

$H^{\mu\nu}(x)$

$$T^{\mu\nu}(P,q) = \int d^4x\, e^{iqx} \langle N|J^\mu(x)J^\nu(0)|N\rangle = K_1^{\mu\nu}(T_1^B + T_1^{NB}) + K_2^{\mu\nu}(T_2^B + T_2^{NB})$$

- 3 formulae available to calculate α_E

- $P = (M, 0), q = (0, \vec{\xi})$: $\quad \alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x \vec{x}^2 \left(H^{00}(x) - H^{00}_{GS}(x)\right) + \alpha_E^r$
- $P = (M, 0), q = (\xi, 0, 0, \xi)$: $\quad \alpha_E = \frac{\alpha_{em}}{4M}\int d^4x (t + x_i)^2 \left(H^{0i}(x) - H^{0i}_{GS}(x)\right) + \alpha_E^r$
- $P = (M, 0), q = (\xi, 0)$: $\quad \alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x t^2 H^{ii}(x) + \alpha_E^r$ → Our choice

$$H^{ii}(x,t) = \langle N|J^i(x)J^i(0)|N\rangle$$

magnetic moment charge radius

- Born term residue: $\alpha_E^r = \frac{\alpha_{em}}{M}\left(\frac{G_E^2(0)+\kappa^2}{4M^2} + \frac{G_E(0)\langle r_E^2\rangle}{3}\right)$, $\quad G_E(0) = 1$ (for p) / 0 (for n)

Ensemble for Calculation

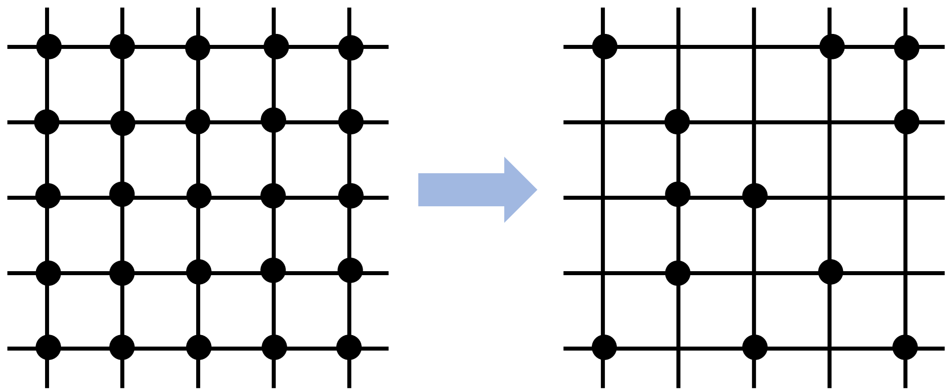
Ensembles	m_π[MeV]	L/a	T/a	a[fm]	N_{conf}
24D	142.6(3)	24	64	0.1929	207
32Dfine	143.6(9)	32	64	0.1432	69

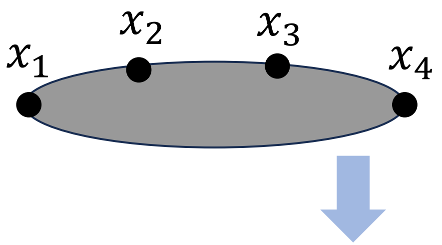
- Domain Wall Fermion ensemble generated by RBC/UKQCD[1].

[1] Blum T et al. PRD, 2016, 93(7):074505.

- Random field selection method[2,3] is used.

[2] Y Li et al. PRD,103 (2021) 1, 014514
[3] W Detmold et al. PRD, 104 (2021) 3, 034502





4-points: $\sum_{\{x_1,x_2,x_3\}} \sim L^9$

Lattice data: highly correlated

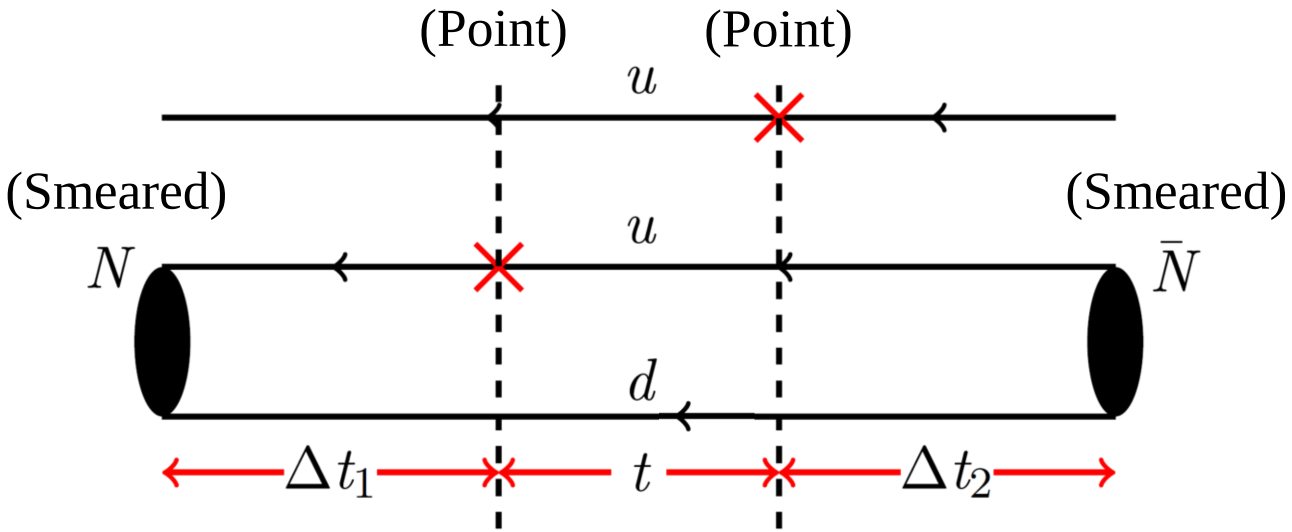
1000 times less points yields similar precision

4-point Function Calculated on Lattice

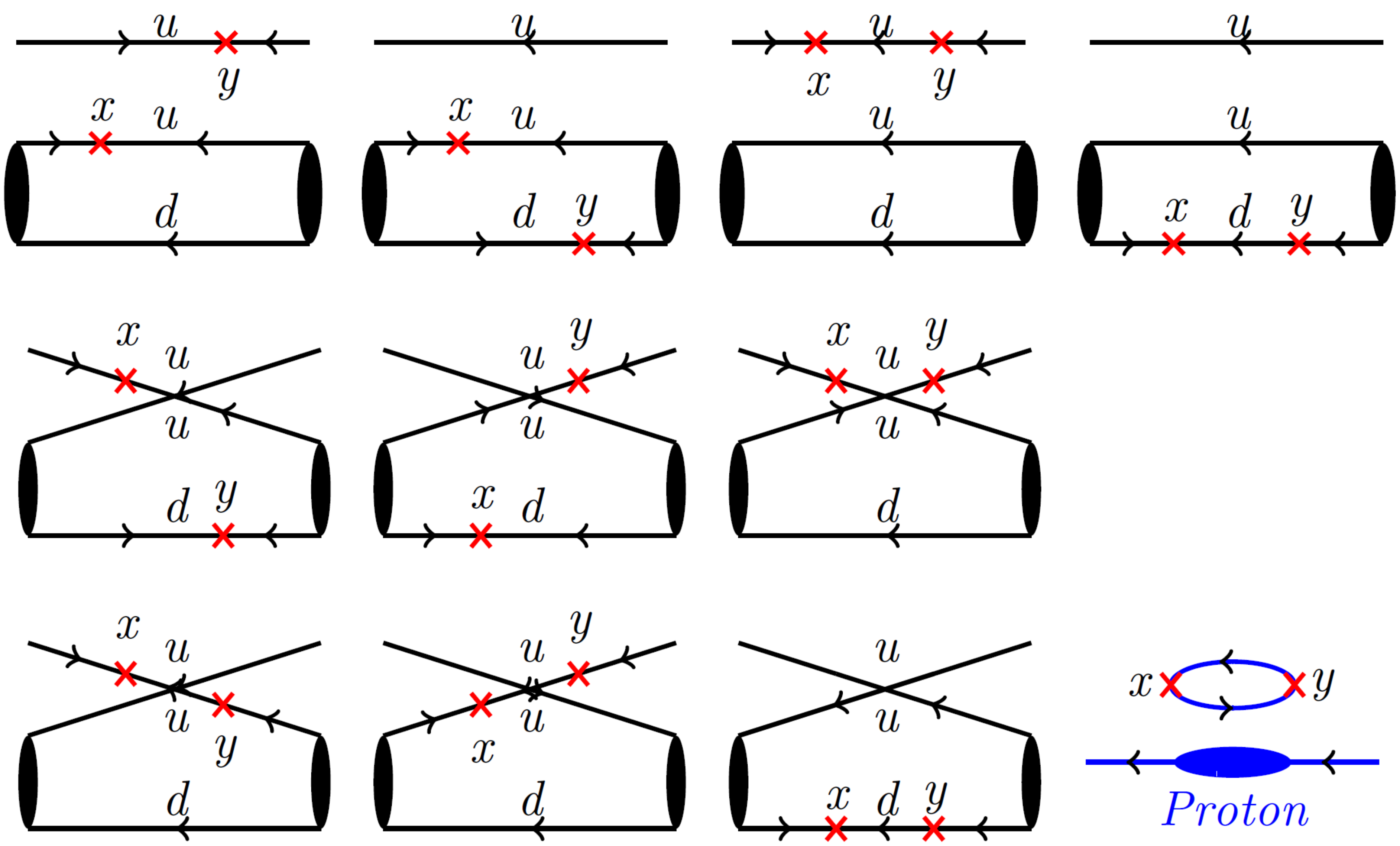
- Compton tensor extracted from nucleon 4-point function

$$\langle N|J^{\mu}(x)J^{\nu}(0)|N\rangle \Rightarrow \langle N(t+\Delta t_1)J^{\mu}(t,x)J^{\nu}(0)\bar{N}(-\Delta t_2)\rangle$$

- Two nucleon operators and two vector current operators placed on different time slices, separated by Δt and t.

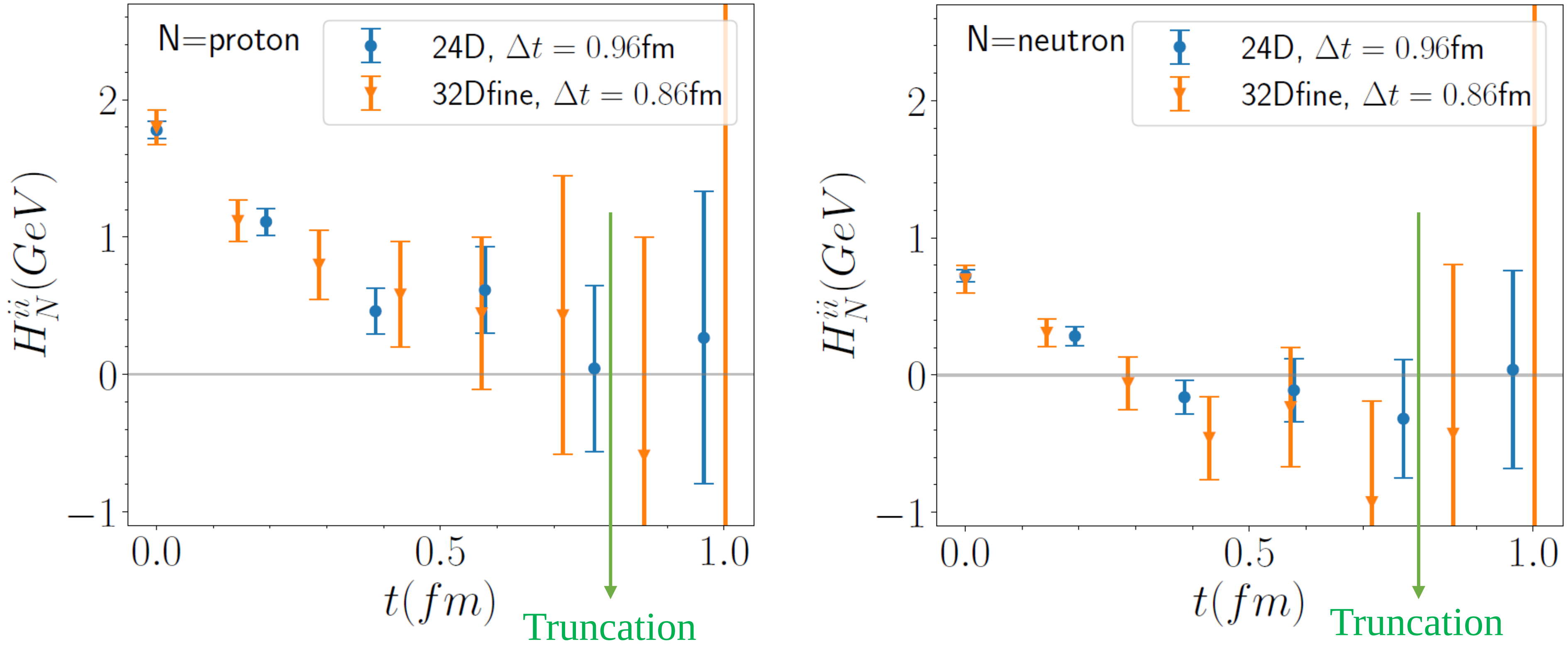


Feynman Diagrams in 4-point Function



Signal of $\sum_x H_{ii}(x,t)$

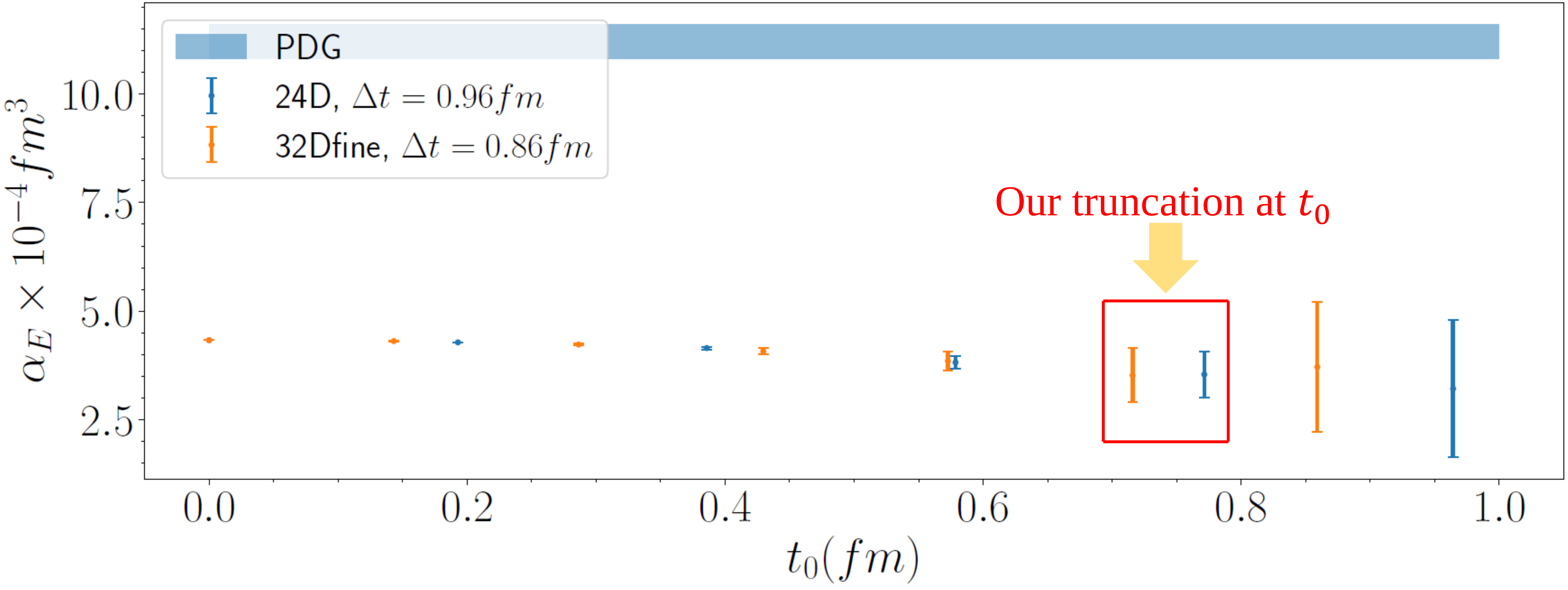
Structure of hadronic function $\sum_{\vec{x}} H^{ii}(t,\vec{x}) = \sum_k \langle p|J_i(0)|k\rangle e^{-(E_k-M)t}\langle k|J_i(0)|p\rangle$



We choose $\Delta t = \Delta t_1 + \Delta t_2 = \begin{cases} 0.96\ fm\ (24D) \\ 0.86\ fm\ (32Dfine) \end{cases}$, and truncate at $t_0 = \begin{cases} 0.77\ fm\ (24D) \\ 0.72\ fm\ (32Dfine) \end{cases}$

Polarizability α_E from $H_{ii}(x)$

Polarizability extraction, $\alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x t^2 H^{ii}(x) + \alpha_E^r$



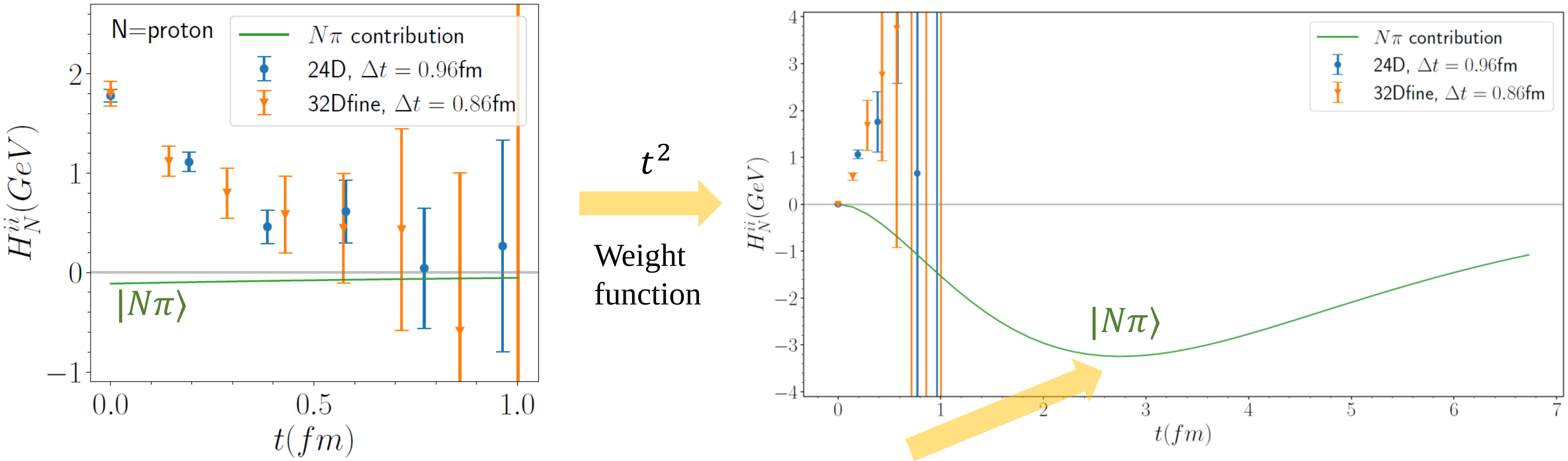
However, lattice predictions are significantly below the PDG value. Why?

Nucleon polarizabilities and $N\pi$ scattering

Structure of hadronic function $\int d^4x\, t^2 H_{ii}(x,t) = \int dt\, t^2 \sum_k \langle p|J_i(0)|k\rangle e^{-(E_k-M)t}\langle k|J_i(0)|p\rangle$

$$= 4\sum_k \frac{\langle p|J_i(0)|k\rangle\langle k|J_i(0)|p\rangle}{(E_k-M)^3}$$

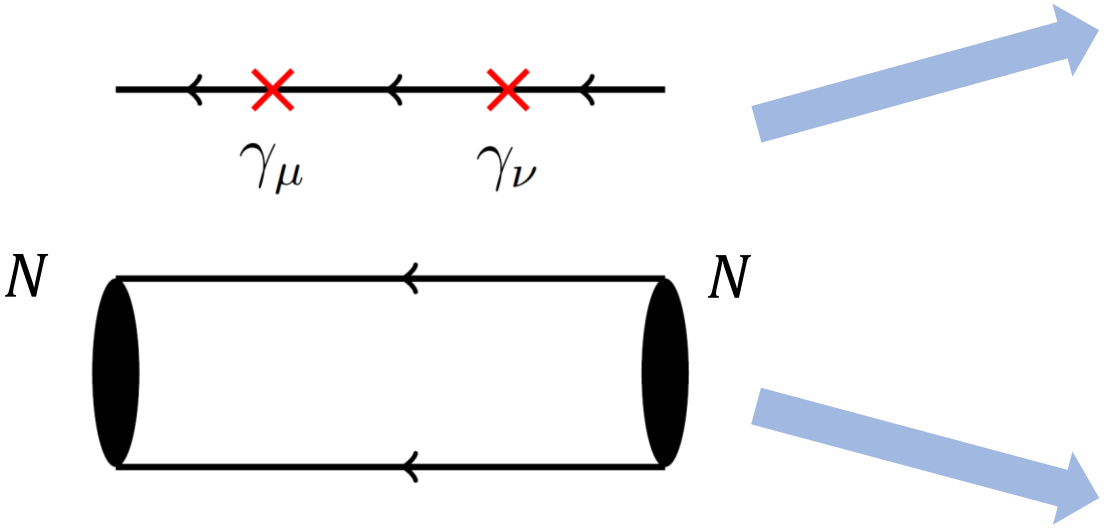
The dominant contribution is given by $|k\rangle = |N\pi\rangle$ states

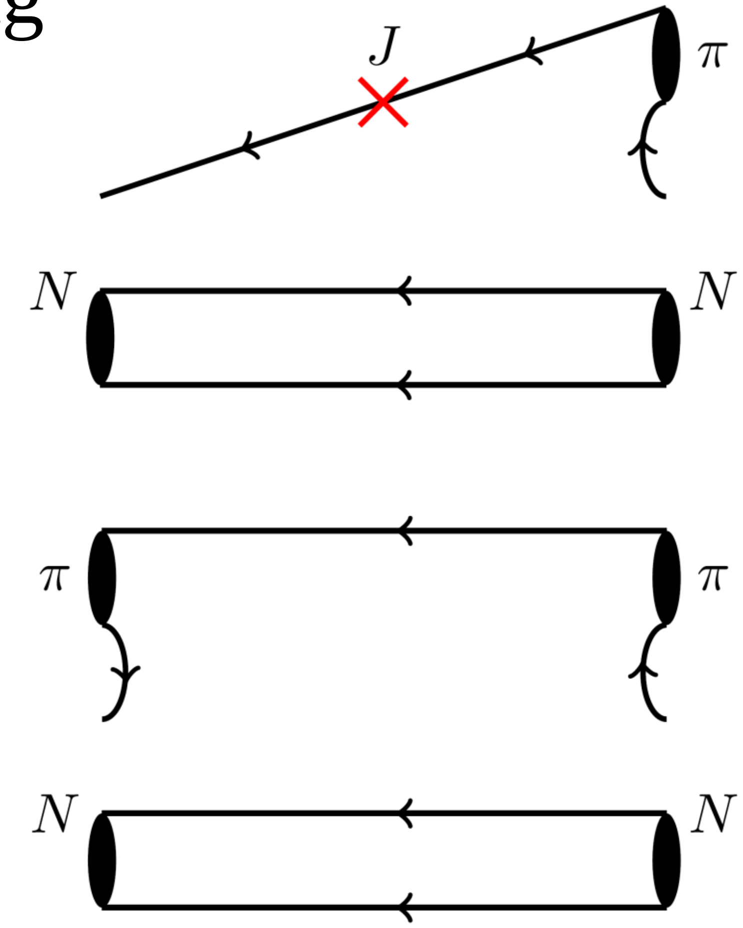


$|N\pi\rangle$ states contribution exhibits a peak at $t = 2.8\ fm$, far exceed our truncation at $t_0 \approx 0.8\ fm$

Must calculate $N\pi$ scattering directly!

Feynman Diagrams of $N\pi$ Rescattering





$$I = 1/2:\ O_{N\pi}^{I_3=+\frac{1}{2}} = O_p O_{\pi^0} - \sqrt{2} O_n O_{\pi^+},\ O_{N\pi}^{I_3=-\frac{1}{2}} = \sqrt{2} O_n O_{\pi^0} - O_p O_{\pi^-}$$

$$I = 3/2:\ O_{N\pi}^{I_3=+\frac{1}{2}} = \sqrt{2} O_p O_{\pi^0} + O_n O_{\pi^+},\ O_{N\pi}^{I_3=-\frac{1}{2}} = O_n O_{\pi^0} + \sqrt{2} O_p O_{\pi^-}$$

} Operators

Results of $N\pi$ Scattering

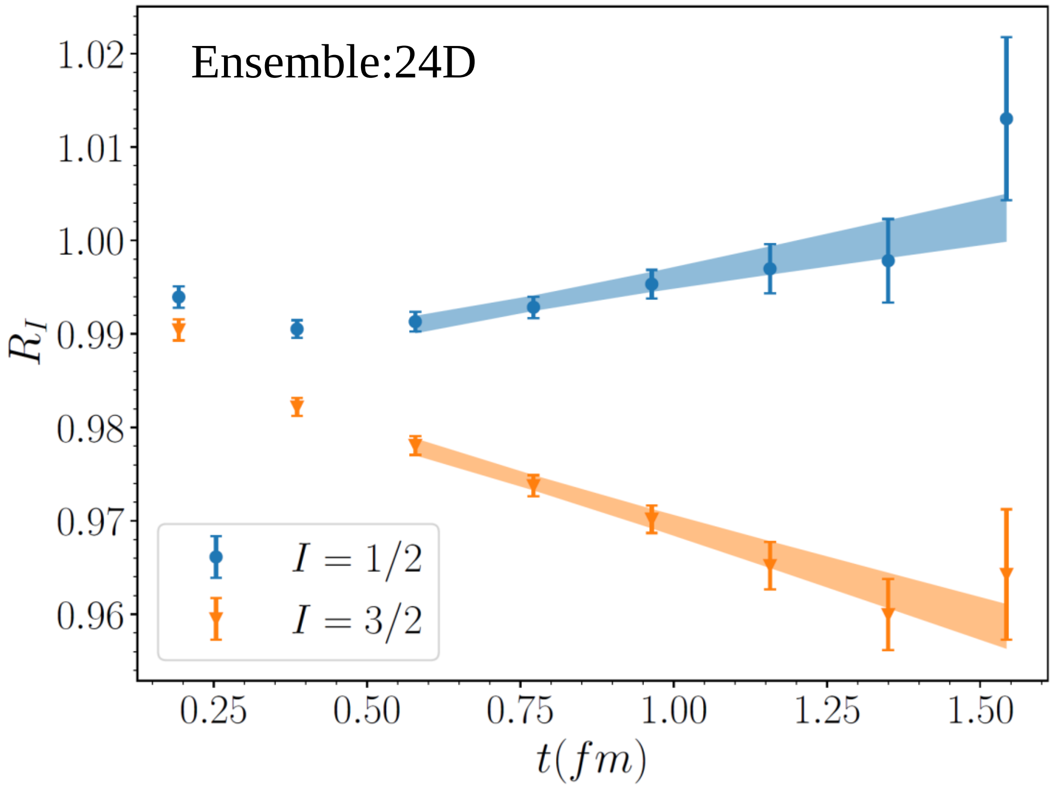
- $N\pi$ scattering for $I_3 = +1/2$

  (similar for $I_3 = -1/2$ case)

$$R = \frac{C_{N\pi}(t)}{C_N(t)C_\pi(t)}$$
$$= \frac{A_{N\pi}}{A_N A_\pi}\frac{e^{-E_{N\pi}t}}{e^{-(M_N+M_\pi)t}}$$
$$\approx R_0(1-\Delta E t)$$

with $\Delta E = E_{N\pi} - M_N - M_\pi$

- Scattering for different isospin channel
  - $I = 1/2$, $\Delta E < 0$, attractive interaction
  - $I = 3/2$, $\Delta E > 0$, repulsive interaction
- Combined fit for 24D and 32Dfine



[1] M. Hoferichter et al. PLB, 2023, 843:138001.

Phenomenology[1]:

$$a_0^{1/2}m_\pi = 0.170(2), a_0^{3/2}m_\pi = -0.087(2)$$

This work:

$$a_0^{1/2}m_\pi = 0.093(25)(2), a_0^{3/2}m_\pi = -0.128(15)(2)$$

Deviations might due to isospin breaking

Matrix Elements of $N\gamma \rightarrow N\pi$

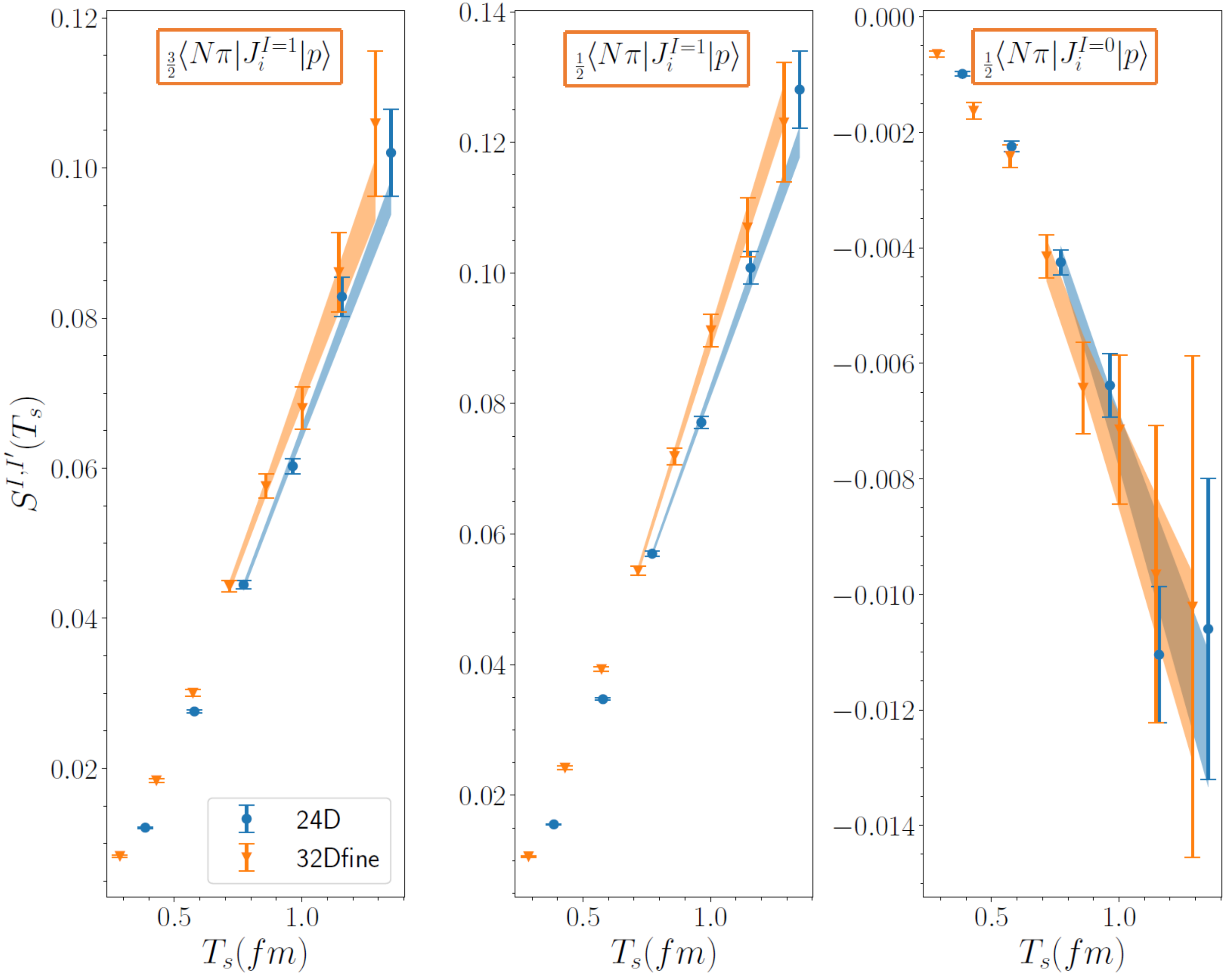
- Normalization for ${}_I\langle N\pi|J_i^{I'}|N\rangle$

$$R = \frac{C_{NJN\pi}(t_1,t_2)}{C_{N\pi}(t_1+t_2)} \times \sqrt{\frac{C_N(t_1)C_{N\pi}(t_2)C_{N\pi}(t_1+t_2)}{C_{N\pi}(t_1)C_N(t_2)C_N(t_1+t_2)}}$$

- Summed insertion

Maiani L. NPB, 293, 420(1987)

$$S(T_s) = \sum_{t_1+t_2=T_s} R(t_1,t_2) \xrightarrow{T_s\to\infty} c_0 + \frac{1}{\sqrt{2M}}\,{}_I\langle N\pi|J_i^{I'}|N\rangle \cdot T_s$$



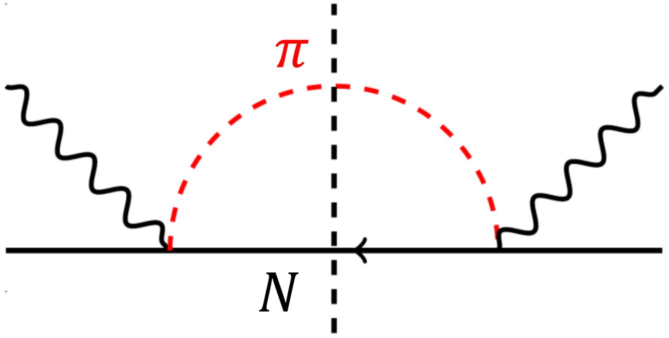
- Linear fit $S(T_s)$ with T_s to extract

Decomposition of α_E

Master formula: $\alpha_E = -\frac{\alpha_{em}}{12M}\int d^4x t^2 H^{ii}(x) + \alpha_E^r$

- The nucleon-pion contribution :

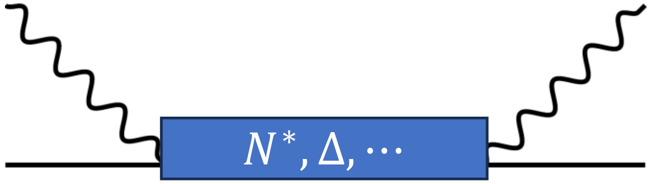
$$\alpha_E^{N\pi} = -\frac{\alpha_{em}}{3M}\frac{A_{N\pi}}{\Delta E_0^3}$$



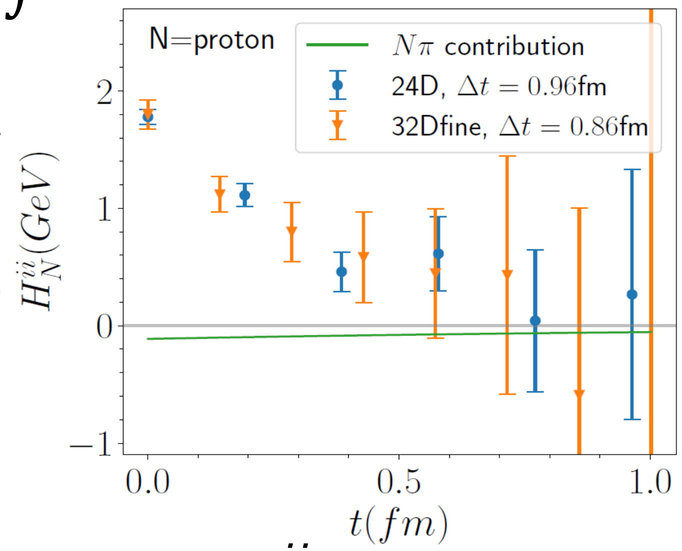
- Constructed from ${}_I\langle N\pi|J_i^{I'}|N\rangle$
- Long Distance

- The excited-states contribution:

$$\alpha_E^{es} = -\frac{\alpha_{em}}{12M}\int d^4x\, t^2 H_{es}^{ii}(x)$$



All excited-states



- Extracted from 4-pt function $H^{ii}(x)$
- Short Distance

- Final results: $\alpha_E = \alpha_E^{N\pi} + \alpha_E^{es} + \alpha_E^r$

Numerical Results

- Our results of α_E, in units of $10^{-4} fm^3$

		24D	**32Dfine**	**PDG**
Proton	$\alpha_E^{N\pi}$	6.51(45)	8.03(85)	
	α_E	11.3(1.4)(2.7)	11.8(2.3)(3.6)	11.2(4)
Neutron	$\alpha_E^{N\pi}$	8.93(57)	10.5(1.0)	
	α_E	11.1(1.4)(2.1)	12.0(2.3)(2.8)	11.8(1.1)

- Our results of α_E are consistent with PDG.
- Systematic error of truncation effects of $|N(\vec{p})\pi(-\vec{p})\rangle$ has been estimated.
- Contributions of $N\pi$ states: about 60% of α_E^p and 80% of α_E^n.

Outlooks

- Large uncertainties due to temporal truncation

  ⇨ Investigate additional low-lying $N\pi$ contribuitions

- Calculate magnetic polarizabilities with our method.

- Various other projects contains $N\pi$ scattering, like pion photoproduction……

Thanks!