On the form factors of a spin-3/2 particle and its Generalized Parton Distributions (GPDs)

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Collaborative research center CRC 110

„Symmetries and the emergence of structure in QCD"

Status of the project B.1:

Partonic structure of nucleons and nuclei

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Outline

1, Introduction: Form factors (FFs) and GPDs

2, ★Spin-3/2 particle (selected) and the basic properties

3, ●Numerical calculation (Framework: Covariant quark-diquark model) and ●Results (EMFFs and GFFs, and some others)

4, Summary, discussions, and questions

1, Introduction: Form factors and GPDs

● Electromagnetic probes

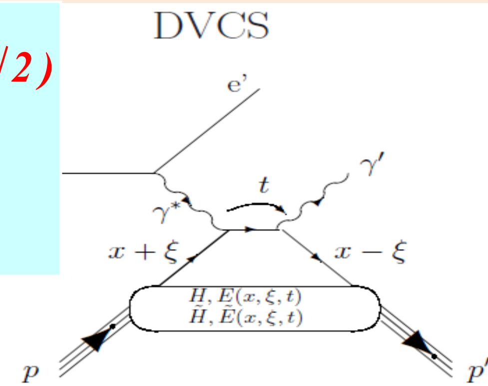
- Electric and magnetic proton form factors$_{2D}$
- Proton and Neutron charge distributions
- Nucleon spin structure$_{2D}$
- Nucleon-Delta transition (other resonances)
- Parton distributions$_{2D}$
- ★ Generalized parton distributions (GPDs$_{3D}$)

★ GPDs (generalized parton distributions)$_{nucleon}$

GPDs $H_{(q,g)}(x,\xi,Q^2)$ naturally embody the information of both PDFs and FFs, and, therefore, display the unique properties to present a "3-D" description for a system.

$$V^{S=1/2}_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^- e^{i\omega(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q\left(+\frac{z}{2}\right) | p',\lambda' \rangle \Big|_{z^+=0,\vec{z}=0} \quad \textit{for nucleon } (S=1/2)$$

$$= H^q(x,\xi,t)\bar{u}(p')\not{n}\, u(p) + \frac{i}{2M_N} E^q(x,\xi,t)\bar{u}(p')\sigma^{\alpha\beta} n_\alpha q_\beta u(p)$$



GPDs allow for a unified description of a number of hadronic properties; for example:

(1) In the forward limit they reduce to conventional PDFs

$$H_q(x,0,0) = q(x)\,,$$

$$\tilde{H}_q(x,0,0) = \Delta q(x)\,.$$

Parton distributions (PDFs)

★GPDs

Form Factors(FFs)

(2) When one integrates GPDs over x they reduce to the usual form factors, e.g. the Dirac form factors[a]

$$\sum_q e_q \int dx\, H_q(x,\xi,t) = F_1(t)\,,$$

$$\sum_q e_q \int dx\, E_q(x,\xi,t) = F_2(t)\,.$$

Other observables (Like Gravitational FFs) and a Global Description of nucleon:

last global unknown: How do we learn about hadrons?

$|N\rangle$ = strong interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\|J^\mu_{\text{em}}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\|J^\mu_{\text{weak}}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\|T^{\mu\nu}_{\text{grav}}\|N\rangle$	$\longrightarrow$	$M,\ J,\ D,$

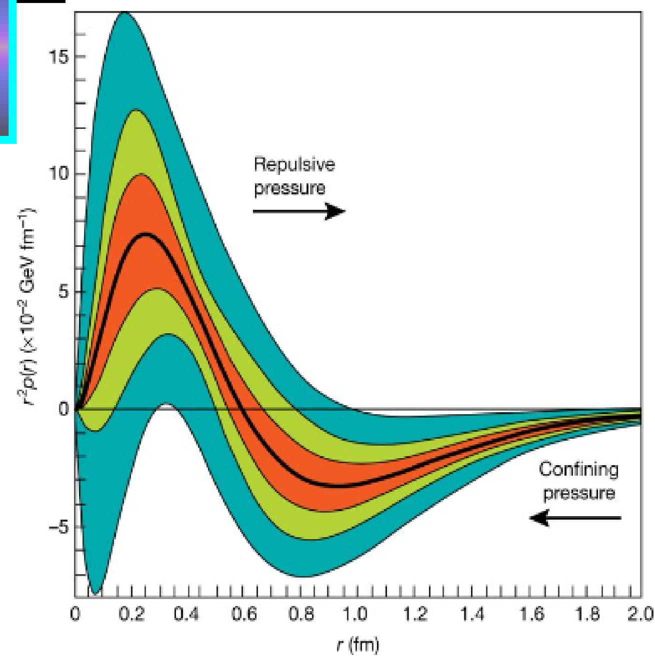
global properties:

Q_{prot}	=	$1.602176487(40) \times 10^{-19}$C
μ_{prot}	=	$2.792847356(23)\mu_N$
g_A	=	1.2694(28)
g_p	=	8.06(0.55)
M	=	938.272013(23) MeV
J	=	$\frac{1}{2}$
D	=	??

Gravitation form factors (GFFs) ★

[Polyakov Proposed, 1998]

1: Radial pressure distribution in the proton.



Burkert V D, Nature, 2018

GPDs → Gravitational FFs,

<|Energy-momentum tensors|>

Mechanics Observables:

Energy density, mass radius,

- ***"pressure",***
- ***"shear force",***

★***"D-term"***★

★ Spin-1 particle and basic properties *example*

- ***Form factor decomposition of Local current*** *EMFFS*

[Hoodbhoy '89, Frederico '97, Berger '01, Broniowski '08 Cosyn'17]

$$I^{\mu}_{\lambda'\lambda} = \langle p',\lambda' | \bar{q}(0)\gamma^{\mu} q(0) | p,\lambda\rangle = \varepsilon'^{\beta *}\left[-\left(G_1^q g_{\beta\alpha} + G_3^q \frac{P_\beta P_\alpha}{2M^2}\right)P^\mu + G_2^q\left(g^\mu_\alpha P_\beta + g^\mu_\beta P_\alpha\right)\right]\varepsilon^\alpha$$

$$\begin{cases} G_C(t) = G_1(t) + 2\eta/3\cdot G_Q(t) \\ G_M(t) = G_2(t) \\ G_Q(t) = G_1(t) - G_2(t) + (1+\eta)G_3(t) \end{cases}$$

Definitions of GPDs (spin -1)

- **Unpolarized** [PRL: Berger '01 , for the deuteron]

$$V^{S=1}_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)}\langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not{n} q\left(+\frac{z}{2}\right) | p,\lambda\rangle\Big|_{z^+=0,\vec{z}=0} \quad \text{for } S=1$$

$$= \sum_{i=1}^{5} (\varepsilon'^{\beta})^* V^i_{\beta\alpha}\varepsilon^\alpha H_i^q(x,\xi,t)$$

$$V_{\mu\nu} : \{g_{\mu\nu}, P_\mu n_\nu, P_\nu n_\mu, P_\mu P_\nu, n_\mu n_\nu\}$$

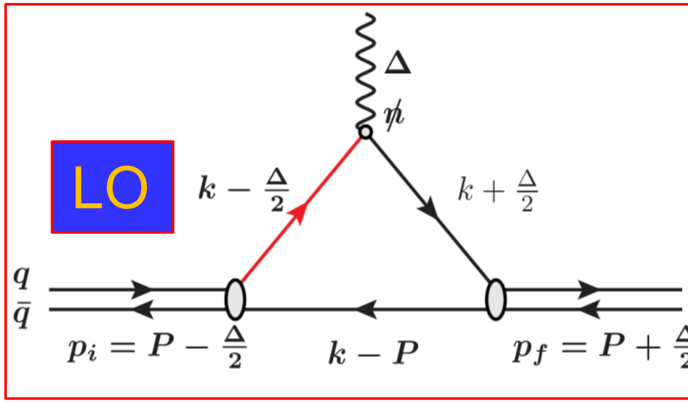
$$P = \frac{p'+p}{2}, \quad t = \Delta^2 = (p'-p)^2,$$

$$n^2 = 0, \text{ (lightlike four-vector)}$$

$$\xi = (n\cdot\Delta)/(n\cdot P), \quad \text{skewness parameter},$$

$$\epsilon = \epsilon(p,\lambda), \ \epsilon' = \epsilon'(p',\lambda'), \text{ polarizations},$$

A-Symmetry in Longitudinal direction



● ***EMT (Energy Momentum tensor), Gravitational Form Factors (GFFs)***

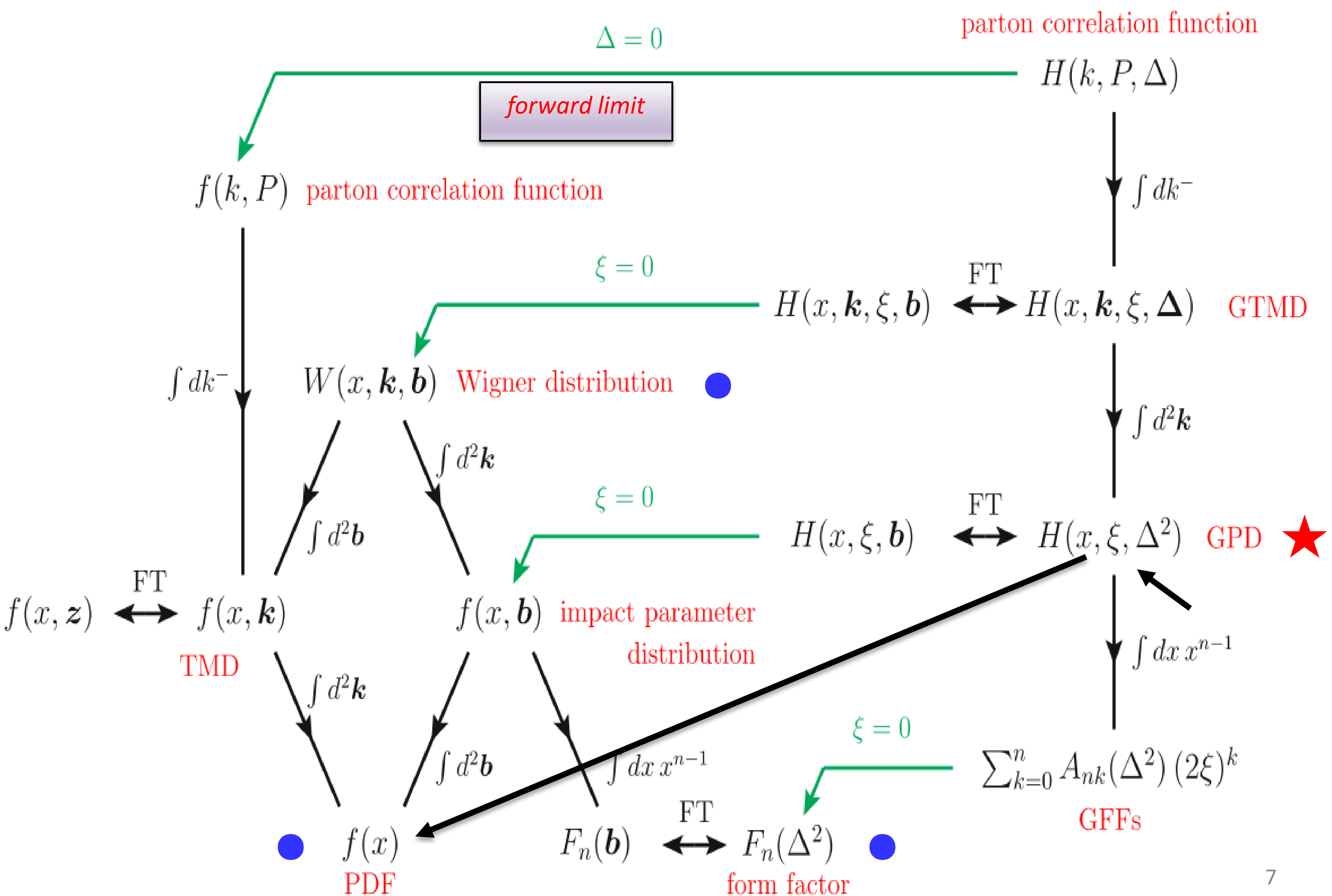
$$(P\cdot n)^{\alpha+1}\int dx\, x^\alpha \int \frac{dz^-}{2\pi} e^{ixP^+z^-}\left[\bar{\psi}\left(-\frac{z}{2}\right)\not{n}\psi\left(+\frac{z}{2}\right)\right]\Big|^{z^+=0}_{\vec{z}=0}$$

$$= \left(i\frac{d}{dz^-}\right)^\alpha \left[\bar{\psi}\left(-\frac{z}{2}\right)\not{n}\psi\left(+\frac{z}{2}\right)\right]\Big|_{z=0} = \bar{\psi}(0)\not{n}\left(i\overleftrightarrow{\partial}^+\right)^\alpha\psi(0)$$

Mellin Moment: $\begin{cases}\alpha = 0, & \text{EMFFs} \\ \alpha = 1, & \text{GFFs}\end{cases}$

3-D GPDs Schemes★: give rich information

[Diehl '16]



GPDs (*generalized parton distributions*)*Literature (S<3/2)*

GPDs ① *for pion* (*S=0*),

Broniowski, PLB 574, PRD78; Choi et al., PRD64; Fanelli, EPJC76; ……

② *for nucleon* (proton and neutron, *S=1/2*)

Diehl et al., EPJC 73; Kroll, EPJA53; Pire et al., PRD79; Selyugin, PRD91;……

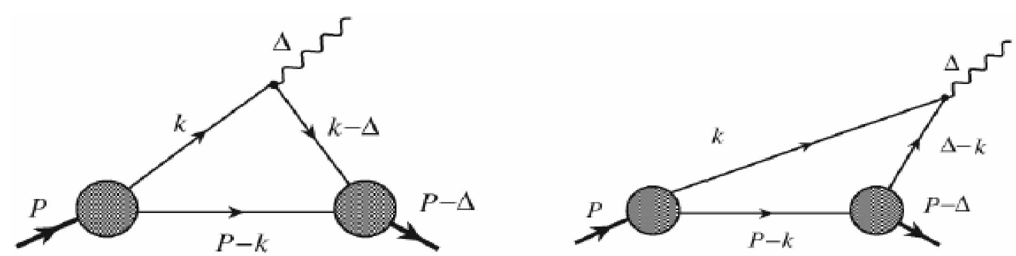
③ *Light Nuclei: He-3* (*S=1/2*)

Rinaldi et al., PRC87……

④ Deuteron *(S=1)*

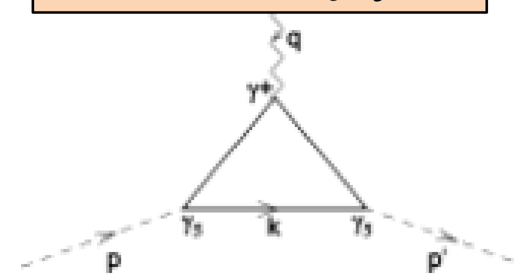
Cano et al., PRL87, YBD et al., JPG19,……

Generalized Parton distributions for pion, e. g.



Covariant amplitude with a reduced photon vertex for pion GPD (left diagram) and its nonvalence $x < \zeta$ part (right diagram).

Broniowski, PLB574,* *In the limit of ξ=0



. The diagram for the evaluation of the generalized parton distribution of the pion in chiral quark models.

●2, Spin (1->3/2)$_{\text{high-spin}}$ particles (selected) and the basic properties

★ Spin-1: deuteron target is accessible in some facilities

★ Spin-3/2: particles, theoretically necessary

★ Spin-3/2 beams may be accessible in future EIC (EicC) and some other Facilities

The comparison between the parameters of the electron-ion colliders proposed in China and in the US [redacted].

Facility	CoM energy	Lum./10^{33} ($\text{cm}^{-2}\cdot\text{s}^{-1}$)	Ions	Polarization
EicC	15–20	2–3	$p\to$U	e^-, p, and light nuclei
EIC-US	30–140	2–15	$p\to$U	e^-, p, ^{3}He

◆[Electron-ion collider in China Fronties of Physics, 16, 64701]

★ Spin-3⁻/2 (Ω hyperon) target might be possible

$$\begin{cases} e^+e^- \to (\Omega\ \bar{\Omega})\ \textit{pair} \\ p + A \\ \textit{Heavy ion collisions} \end{cases}$$

★ 2.1), Spin-3/2 particle and basic properties

Spin-3/2 -- Rarita-Schwinger spinor $u^\alpha(p,\lambda)$

DYF, BDS, YBD, PRD105, 096002, PRD106,116012, 2305.02680

$$u^\alpha(p,\lambda) = \sum_{\rho,\sigma} C^{\frac{3}{2}\lambda}_{1\rho,\frac{1}{2}\sigma} \epsilon^\alpha(p,\rho) u(p,\sigma) \qquad u(p,\sigma) = \frac{(\not p + M)}{\sqrt{2p\cdot n}} \not n \chi_\sigma ;$$

$$\epsilon^\alpha(p,0) = \frac{1}{M}\left(p^+, p^- - \frac{2M^2}{p^+}, \boldsymbol{\epsilon}_\perp(p,0)\right)^{\mathrm{T}}, \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,0) = (p_1, p_2),$$

$$\epsilon^\alpha(p,+1) = -\left(0, \frac{\sqrt{2}(p_1 + ip_2)}{p^+}, \boldsymbol{\epsilon}_\perp(p,+1)\right)^{\mathrm{T}}, \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,+1) = (\frac{1}{\sqrt{2}}, \frac{i}{\sqrt{2}}),$$

$$\epsilon^\alpha(p,-1) = \left(0, \frac{\sqrt{2}(p_1 - ip_2)}{p^+}, \boldsymbol{\epsilon}_\perp(p,-1)\right)^{\mathrm{T}}, \quad \text{with} \quad \boldsymbol{\epsilon}_\perp(p,-1) = (\frac{1}{\sqrt{2}}, \frac{-i}{\sqrt{2}}).$$

Light-Cone $v = (v^+, v^-, \mathbf{v}), \quad \text{with} \quad v^\pm = v^0 \pm v^3 \text{ and } \mathbf{v} = (v^1, v^2)$

$$(\not p - M)\, u^\alpha(p,\lambda) = 0, \quad \gamma_\alpha u^\alpha(p,\lambda) = 0, \quad \partial_\alpha u^\alpha(p,\lambda) = 0.$$

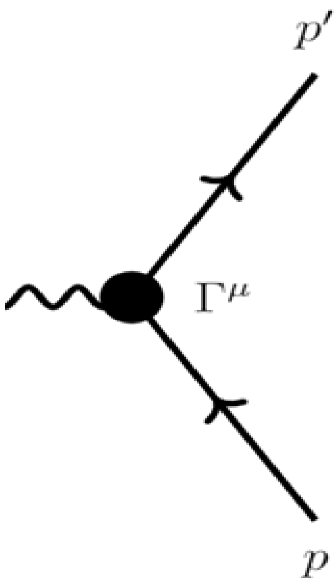
$$\bar{u}_\alpha(p,\lambda')u^\alpha(p,\lambda) = -2M\delta_{\lambda'\lambda}$$

$n^2 = 0$, *lightlike four vector*

EMFFs of a Spin-3/2 particle : (2S+1)=4

$$\langle p',\lambda'|\bar{\psi}(0)\gamma^{\mu}\psi(0)|p,\lambda\rangle = -\vec{u}_{\alpha'}(p',\lambda')\left[\frac{P^{\mu}}{M}\left(g^{\alpha'\alpha}F_{1,0}^{V,a}(t) - \frac{q^{\alpha'}q^{\alpha}}{2M^2}F_{1,1}^{V,a}(t)\right)\right.$$
$$\left. + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M}\left(g^{\alpha'\alpha}F_{2,0}^{V,a}(t) - \frac{q^{\alpha'}q^{\alpha}}{2M^2}F_{2,1}^{V,a}(t)\right)\right]u_{\alpha}(p,\lambda),$$

$$\langle p',\lambda'|\bar{\psi}(0)\gamma^{\mu}\gamma^{5}\psi(0)|p,\lambda\rangle = -\vec{u}_{\alpha'}(p',\lambda')\left[\gamma^{\mu}\left(g^{\alpha'\alpha}\tilde{F}_{1,0}^{V,a}(t) + \frac{P^{\alpha'}P^{\alpha}}{M^2}F_{1,0}^{V,a}(t)\right)\right.$$
$$\left. - \frac{q^{\mu}}{2M}\left(-g^{\alpha'\alpha}\tilde{F}_{2,0}^{V,a}(t) + \frac{P^{\alpha'}P^{\alpha}}{M^2}\tilde{F}_{2,1}^{V,a}(t)\right)\right]\gamma^{5}u_{\alpha}(p,\lambda)$$



$$\left(G_{E0}(t),G_{M1}(t),G_{E2}(t),G_{M3}(t)\right) \Leftarrow \left(F_{10}(t),F_{11}(t),F_{20}(t),F_{21}(t)\right)$$

$$G_{E0}(t) = \left(1+\frac{2}{3}\tau\right)[F_{2,0}^{V}(t) + (1+\tau)(F_{1,0}^{V}(t) - F_{2,0}^{V}(t))]$$
$$+ \frac{2}{3}\tau(1+\tau)[F_{2,1}^{V}(t) + (1+\tau)(F_{1,1}^{V}(t) - F_{2,1}^{V}(t))],$$
$$G_{E2}(t) = [F_{2,0}^{V}(t) + (1+\tau)(F_{1,0}^{V}(t) - F_{2,0}^{V}(t))] + (1+\tau)[F_{2,1}^{V}(t) + (1+\tau)(F_{1,1}^{V}(t) - F_{2,1}^{V}(t))],$$
$$G_{M1}(t) = \left(1+\frac{4}{5}\tau\right)F_{2,0}^{V}(t) + \frac{4}{5}\tau(\tau+1)F_{2,1}^{V}(t),$$
$$G_{M3}(t) = F_{2,0}^{V}(t) + (\tau+1)F_{2,1}^{V}(t),$$

One can select a reference frame to proceed a calculation (say Breit frame) for EM-multipole form factors

GPDs for spin-3/2 particles

$$\begin{cases}(1, \not n) & S=1/2 \\ (g^{\alpha\alpha'}, P^{\alpha}P^{\alpha'}, n^{[\alpha'}P^{\alpha]}, n^{[\alpha}P^{\alpha']}, n^{\alpha'}n^{\alpha}) & S=1\end{cases}$$

$$V^{S=3/2}_{\lambda'\lambda} = \frac{1}{2}\int \frac{dz^- e^{ix(P\cdot z)}}{(2\pi)} \langle p',\lambda' | \bar{q}\left(-\frac{z}{2}\right) \not n q\left(+\frac{z}{2}\right) | p,\lambda\rangle \Big|_{z^+=0,\vec{z}=0}$$

Direct product

$$(g^{\alpha\alpha'}, P^{\alpha}P^{\alpha'}, n^{[\alpha'}P^{\alpha]}, n^{[\alpha}P^{\alpha']}, n^{\alpha'}n^{\alpha}, g^{\alpha\alpha'}\not n, P^{\alpha}P^{\alpha'}\not n, n^{[\alpha'}P^{\alpha]}\not n, n^{[\alpha}P^{\alpha']}\not n, n^{\alpha'}n^{\alpha}\not n,)$$

$$a^{[\mu}b^{\nu]} = a^{\mu}b^{\nu} - a^{\nu}b^{\mu}$$

$$a^{\{\mu}b^{\nu\}} = a^{\mu}b^{\nu} - a^{\nu}b^{\mu}$$

$$\bar{u}_{\alpha'}(p',\lambda')\gamma^{\mu}\vec{u}_{\alpha'}(p,\lambda) = \bar{u}_{\alpha'}(p',\lambda')\left[\frac{P^{\mu}}{M} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M}\right]\gamma^{\mu}\vec{u}_{\alpha'}(p,\lambda) \Rightarrow \text{Gordon Identity}$$

$$i\varepsilon^{\mu\nu\rho\sigma}g^{\tau\lambda} + i\varepsilon^{\nu\rho\sigma\tau}g^{\mu\lambda} + i\varepsilon^{\rho\sigma\tau\mu}g^{\nu\lambda} + i\varepsilon^{\sigma\tau\mu\nu}g^{\rho\lambda} + i\varepsilon^{\tau\mu\nu\rho}g^{\sigma\lambda} = 0 \Rightarrow \text{Schouten Identity}$$

★ Some other on-shell relations +Conservations

$$\gamma^{\mu}\gamma_5 \doteq \frac{q^{\mu}\gamma_5}{2M} + \frac{i\sigma^{\mu P}}{M}, \qquad 0 \doteq P^{\mu}\gamma_5 + \frac{i\sigma^{\mu q}\gamma_5}{2},$$

$$1 \doteq \frac{\not P}{M}, \qquad 0 \doteq \not q,$$

$$i\sigma^{\mu\nu} \doteq -\frac{q^{[\mu}\gamma^{\nu]}}{2M} + \frac{i\epsilon^{\mu\nu P\lambda}\gamma_{\lambda}\gamma_5}{M}, \qquad 0 \doteq -P^{[\mu}\gamma^{\nu]} + \frac{i\epsilon^{\mu\nu q\lambda}\gamma_{\lambda}\gamma_5}{2},$$

$$\gamma_5 \doteq \frac{\not q\gamma_5}{2M}, \qquad 0 \doteq \not P\gamma_5,$$

$$\gamma^{\mu} \doteq \frac{P^{\mu}}{M} + \frac{i\sigma^{\mu q}}{2M}, \qquad 0 \doteq \frac{q^{\mu}}{2} + i\sigma^{\mu P}$$

$$i\sigma^{\mu\nu}\gamma_5 \doteq -\frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{M} + \frac{i\epsilon^{\mu\nu q\lambda}\gamma_{\lambda}}{2M}, \qquad 0 \doteq -\frac{q^{[\mu}\gamma^{\nu]}\gamma_5}{2} + i\epsilon^{\mu\nu P\lambda}\gamma_{\lambda},$$

● GPDs of a Spin-3/2 particle

Definitions of GPDs (spin -3/2)

★ **Unpolarized**

Conservations

$$V_{\lambda'\lambda}^{S=3/2} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \left\langle p',\lambda' \left| \bar{\psi}\left(-\frac{1}{2}z\right) \not{n}\, \psi\left(\frac{1}{2}z\right) \right| p,\lambda \right\rangle \Big|_{z^+=0,\vec{z}=0}$$

$$= -\bar{u}_{\alpha'}(p',\lambda') \mathrm{H}^{\alpha'\alpha}(x,\xi,t) u_\alpha(p,\lambda),$$

$$\mathrm{H}^{\alpha'\alpha} = H_1 g^{\alpha'\alpha} + H_2 \frac{P^{\alpha'}P^{\alpha}}{M^2} + H_3 \frac{n^{[\alpha'}P^{\alpha]}}{(P\cdot n)} + H_4 \frac{M^2 n^{\alpha'} n^{\alpha}}{(P\cdot n)^2} + H_5 \frac{M g^{\alpha'\alpha}\not{n}}{(P\cdot n)}$$

$$+ H_6 \frac{P^{\alpha'}P^{\alpha}\not{n}}{M(P\cdot n)} + H_7 \frac{M n^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2} + H_8 \frac{M^3 n^{\alpha'} n^{\alpha}\not{n}}{(P\cdot n)^3}$$

★ **Polarized**

$$\tilde{V}_{\lambda'\lambda}^{S=3/2} = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix(P\cdot z)} \left\langle p',\lambda' \left| \bar{\psi}\left(-\frac{1}{2}z\right) \not{n}\gamma_5 \psi\left(\frac{1}{2}z\right) \right| p,\lambda \right\rangle \Big|_{z^+=0,\vec{z}=0}$$

$$= -\bar{u}_{\alpha'}(p',\lambda') \tilde{\mathrm{H}}^{\alpha'\alpha}(x,\xi,t) u_\alpha(p,\lambda),$$

$$\tilde{\mathrm{H}}^{\alpha'\alpha} = \tilde{H}_1 g^{\alpha'\alpha}\gamma_5 + \tilde{H}_2 \frac{P^{\alpha'}P^{\alpha}}{M^2}\gamma_5 + \tilde{H}_3 \frac{n^{[\alpha'}P^{\alpha]}}{(P\cdot n)}\gamma_5 + \tilde{H}_4 \frac{M^2 n^{\alpha'} n^{\alpha}}{(P\cdot n)^2}\gamma_5 + \tilde{H}_5 \frac{M g^{\alpha'\alpha}\not{n}}{(P\cdot n)}\gamma_5$$

$$+ \tilde{H}_6 \frac{P^{\alpha'}P^{\alpha}\not{n}}{M(P\cdot n)}\gamma_5 + \tilde{H}_7 \frac{M n^{[\alpha'}P^{\alpha]}\not{n}}{(P\cdot n)^2}\gamma_5 + \tilde{H}_8 \frac{M^3 n^{\alpha'} n^{\alpha}\not{n}}{(P\cdot n)^3}\gamma_5$$

● GPDs, EMFFs, and EMT *Energy-Momentum Tensor*

Sum rules

$$M\int_{-1}^{1}\mathrm{d}x\,H_i(x,\xi,t)=G_i(t)\quad\text{with}\quad i=1,2,5,6,$$

$$M\int_{-1}^{1}\mathrm{d}x\,\tilde{H}_i(x,\xi,t)=\xi\tilde{G}_i(t)\quad\text{with}\quad i=1,2,$$

$$M\int_{-1}^{1}\mathrm{d}x\,\tilde{H}_i(x,\xi,t)=\tilde{G}_i(t)\quad\text{with}\quad i=5,6,$$

$$M\int_{-1}^{1}\mathrm{d}x\,H_j(x,\xi,t)=M\int_{-1}^{1}\mathrm{d}x\,\tilde{H}_j(x,\xi,t)=0\quad\text{with}\quad j=3,4,7,8,$$

Symmetry properties:

$$H_i(x,\xi,t)=H_i(x,-\xi,t)\quad\text{with}\quad i=1,2,4,5,6,8,$$
$$H_i(x,\xi,t)=-H_i(x,-\xi,t)\quad\text{with}\quad i=3,7,$$
$$\tilde{H}_i(x,\xi,t)=-\tilde{H}_i(x,-\xi,t)\quad\text{with}\quad i=1,2,3,4,$$
$$\tilde{H}_i(x,\xi,t)=\tilde{H}_i(x,-\xi,t)\quad\text{with}\quad i=5,6,7,8.$$

Decomposition of EMT

$$\left\langle p',\lambda'\left|\hat{T}^{\mu\nu}(0)\right|p,\lambda\right\rangle$$
$$=-\bar{u}_{\alpha'}(p',\lambda')\left[\frac{P^\mu P^\nu}{M}\left(g^{\alpha'\alpha}F^T_{1,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{1,1}(t)\right)\right.$$
$$+\frac{(q^\mu q^\nu-g^{\mu\nu}q^2)}{4M}\left(g^{\alpha'\alpha}F^T_{2,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{2,1}(t)\right)$$
$$+Mg^{\mu\nu}\left(g^{\alpha'\alpha}F^T_{3,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{3,1}(t)\right)+\frac{P^{\{\mu}i\sigma^{\nu\}q}}{2M}\left(g^{\alpha'\alpha}F^T_{4,0}(t)+\frac{2P^{\alpha'}P^\alpha}{M^2}F^T_{4,1}(t)\right)$$
$$\left.-\frac{1}{M}\left(2q^{\{\mu}g^{\nu\}[\alpha'}P^{\alpha]}+8g^{\mu\nu}P^{\alpha'}P^\alpha-g^{\alpha'\{\mu}g^{\nu\}\alpha}q^2\right)F^T_{5,0}(t)+Mg^{\alpha'\{\mu}g^{\nu\}\alpha}F^T_{6,0}(t)\right]u_\alpha(p,\lambda)$$

● GPDs and (EMT, GFFs, GMFFs)

Mellin Moment: $(P\cdot n)^{\alpha+1}\int dx x^{\alpha}\int \frac{dz^-}{2\pi}e^{ixP^+z^-}\left[\bar{\psi}\left(-\frac{z}{2}\right)\not{n}\psi\left(+\frac{z}{2}\right)\right]\Big|_{\vec{z}=0}^{z^+=0}$

$$=\left(i\frac{d}{dz^-}\right)^{\alpha}\left[\bar{\psi}\left(-\frac{z}{2}\right)\not{n}\psi\left(+\frac{z}{2}\right)\right]\Big|_{z=0}=\bar{\psi}(0)\not{n}\left(i\overleftrightarrow{\partial}^+\right)^{\alpha}\psi(0)$$

Mellin moment

$$\begin{cases}\alpha=0, & \text{EMFFs}\\ \alpha=1, & \text{GFFs}\end{cases}$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_1(x,\xi,t)=F^T_{1,0}(t)+\xi^2F^T_{2,0}(t)-2F^T_{4,0}(t),$$

$$M\int_{-1}^{1}\mathrm{d}x\,xH_2(x,\xi,t)=2F^T_{1,1}(t)+2\xi^2F^T_{2,1}(t)-4F^T_{4,1}(t),$$

One can select a reference frame ★ to proceed a calculation (say Breit frame) for gravitational multi-pole form factors

● GPDs and Structure Functions

Forward limit

$$F_1(x)=\frac{1}{2}\sum_q e_q^2\frac{q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x)+q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x)}{2}+\{q\to\bar{q}\},$$

$$b_1(x)=\frac{1}{2}\sum_q e_q^2\frac{(q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x))-(q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x))}{2}+\{q\to\bar{q}\},$$

$$2H_1(x,0,0)=\frac{q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x)+q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x)}{2},$$

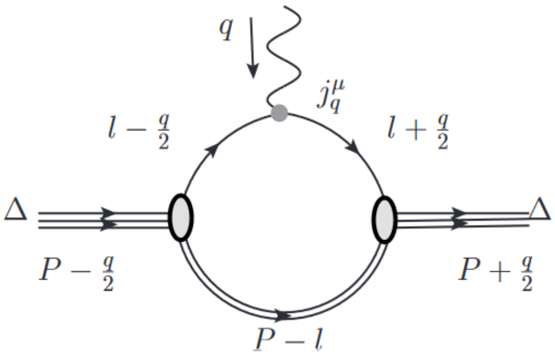
$$2H_4(x,0,0)=\frac{(q_\uparrow^{\frac{3}{2}}(x)+q_\uparrow^{-\frac{3}{2}}(x))-(q_\uparrow^{\frac{1}{2}}(x)+q_\uparrow^{-\frac{1}{2}}(x))}{2},$$

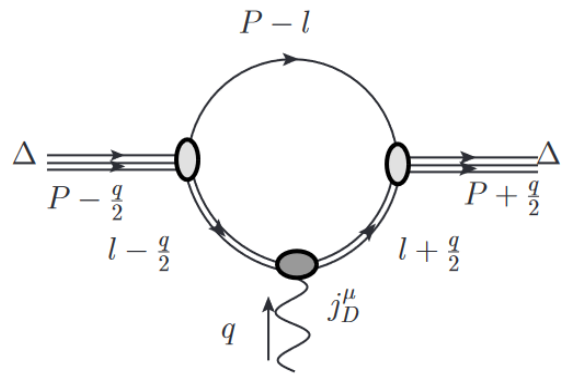
3. Numerical calculation and Results

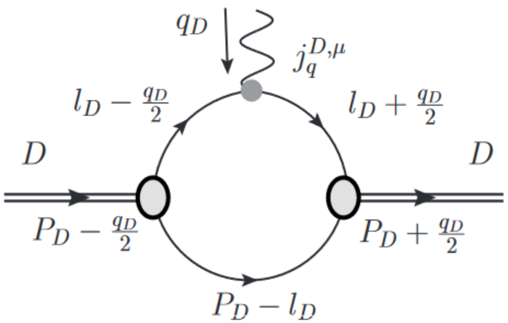
3.1), Framework: Covariant quark-diquark model (S=3/2)

$$\begin{cases} \Delta^+ \\ \Omega^- \end{cases} \Rightarrow \begin{pmatrix} q(1^+/2) \\ D_{qq}(1^+) \end{pmatrix}$$

$$\begin{cases} j_q^\mu = -iQe\gamma^\mu \\ T_q^{\mu\nu} = \dfrac{i}{4}\left(\bar{\psi}_q \gamma^\mu \overleftrightarrow{\partial}^\nu \psi_q + \bar{\psi}_q \gamma^\nu \overleftrightarrow{\partial}^\mu \psi_q\right) \end{cases}$$





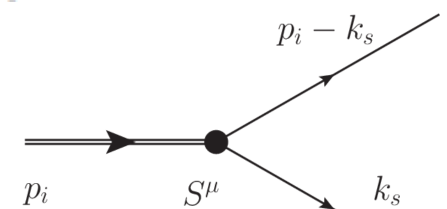


[Choi '04, Frederico '09]

Phenomenological vertex For ( ) :

$$\Gamma^{\alpha\beta} = c_1[g^{\alpha\beta} + g_2\gamma^\beta\Lambda^\alpha + g_3\Lambda^\beta\Lambda^\alpha],$$

[Scadon, PR165,1640]



Phenomenal vertex: $\tilde{\Gamma}^{\alpha\beta} = \Gamma^{\alpha\beta} \cdot \Xi(p_1, p_2; m_R)$

Bethe-Salpeter amplitude(BSA):

$$\Xi(p_1, p_2; m_R) = \frac{c}{\left[p_1^2 - m_R^2 + i\varepsilon\right]\left[p_2^2 - m_R^2 + i\varepsilon\right]}$$

3.2), Results: a), EMFFs of Δ

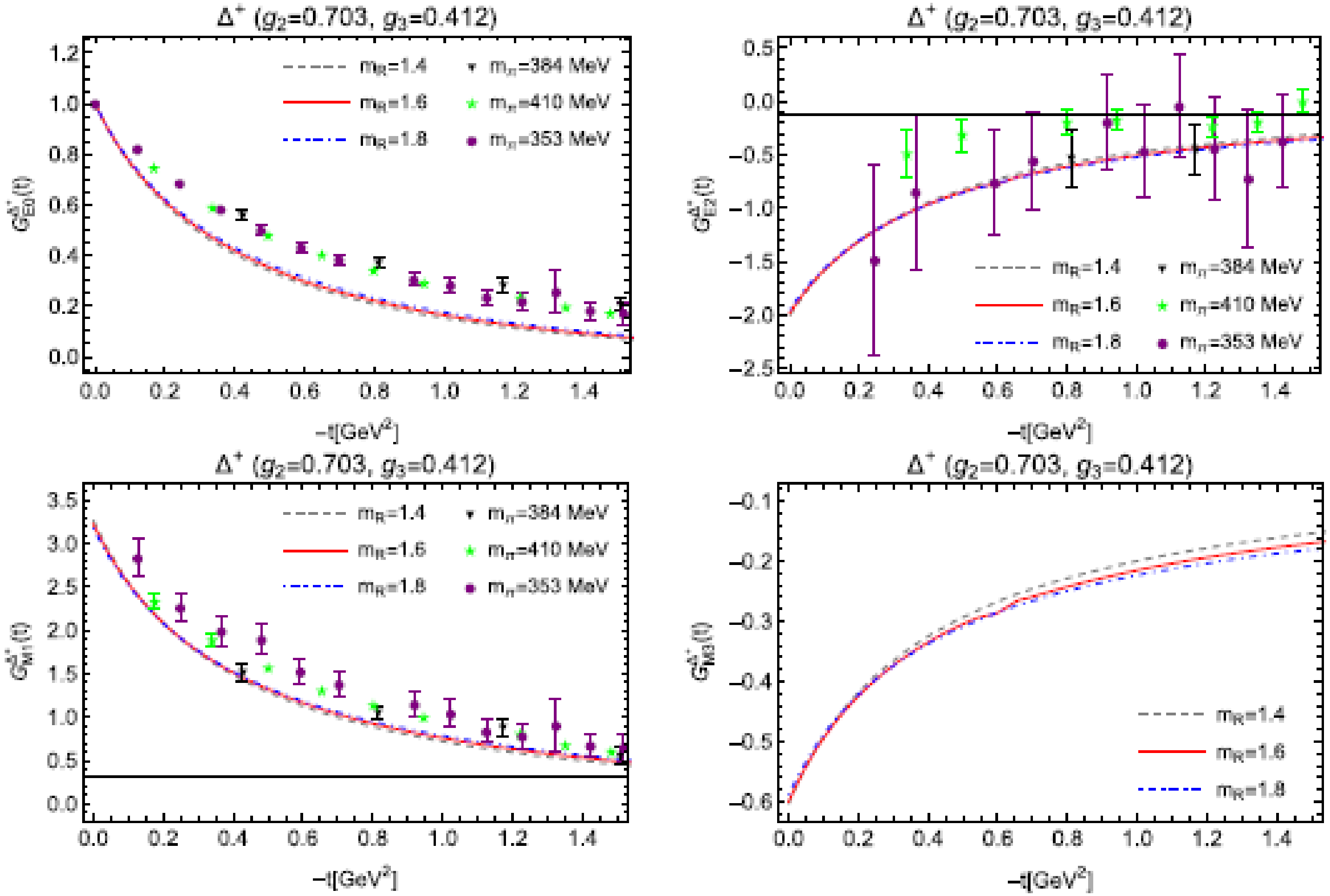
$$\begin{cases} \Delta \\ \Omega^- \end{cases} \Rightarrow \begin{pmatrix} q\left(\frac{1^+}{2}\right) \\ D_{qq}\left(1^+\right) \end{pmatrix}$$

No direct measurement (So far)

$\begin{cases} \text{*LQCDs} \\ \text{Models} \end{cases}$

TABLE I. The parameters used in our approach.

M/GeV	m_q/GeV	m_D/GeV	m_R/GeV	g_2/GeV^{-1}	g_3/GeV^{-2}
1.085	0.4	0.76	1.6	0.703	0.412



Our EMFFs v.s. LQCD Consistent

$G_{M1}(t)$

Δ

TABLE II. A comparison of our magnetic-dipole moment with other models.

$G_{M1}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
This work	6.04	3.02	0.00	−3.02
NQM [68]	5.56	2.73	−0.09	−2.92
RQM [71]	4.76	2.38	0.00	−2.38
QCDSR [72–74]	4.39 ± 1.00	2.19 ± 0.50	0.00	−2.19 ± 0.50
LCQSR [76]	4.4 ± 0.8	2.2 ± 0.4	0.0	−2.2 ± 0.4
Large N_c [77–79]	5.9(4)	2.9(2)	⋯	−2.9(2)
χQMEC[80,81]	6.93	3.47	0.00	−3.47
QCDQM [82,83]	5.689	2.778	−0.134	−3.045
CBM [84]	4.52	2.12	−0.29	−2.69
EMS [87,88]	4.56	2.28	0	−2.28
χPT [89,90]	5.390	2.383	−0.625	−3.632
LQCD [92–94]	4.91 ± 0.61	2.46 ± 0.31	0.00	−2.46 ± 0.3
χCQM[95]	5.82 ± 0.08	2.63 ± 0.06	−0.56 ± 0.09	−3.75 ± 0.0

Consistent

$r_E^2(\Delta^+) = 0.665\ fm^2$

$G_M^{\Delta^+} = -1.93(\mu_N)$

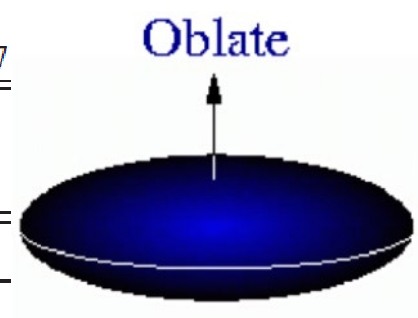
Oblate deformed

$G_A^{\Delta^+} = 0.727$

G_{E2}

TABLE III. A comparison of our electric-quadrupole moment with other models.

$G_{E2}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
This work	−3.86	−1.93	0.00	1.93
NQM [69]	−3.82	−1.91	0	1.91
NQM [70]	−3.63	−1.79	0	1.79
χPT [91]	−3.12 ± 1.95	−1.17 ± 0.78	0.47 ± 0.20	2.34 ± 1.17
χQSM [86]	⋯	−2.15	⋯	⋯
QCDSR [75]	−0.0452 ± 0.0113	−0.0226 ± 0.0057	0	0.0226 ± 0.0057



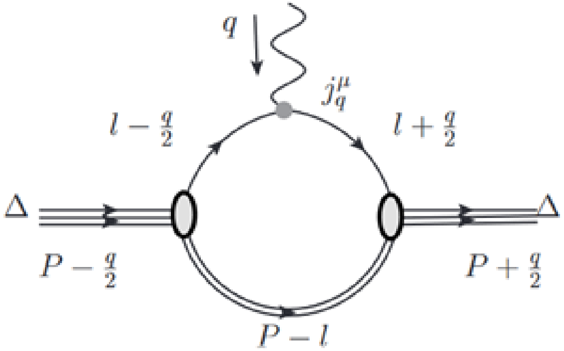
$G_{M3}(t)$

TABLE IV. A comparison of our magnetic-octupole moment with other model calculations.

$G_{M3}(0)$	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}
This work	−1.12	−0.56	0.00	0.56
GPQCD [85]	−11.68	−5.84	0	5.84
QCDSR [75]	−0.0925 ± 0.0234	−0.0462 ± 0.0117	0	0.0462 ± 0.0117

PRD105, 096002

3.2), Results: b), GPDs of Δ :



b.1), 3-D plots for d-quark unpolarized GPDs of Δ^+ (ξ=0, or -0.4)

$H_q(x,\xi,Q^2)$

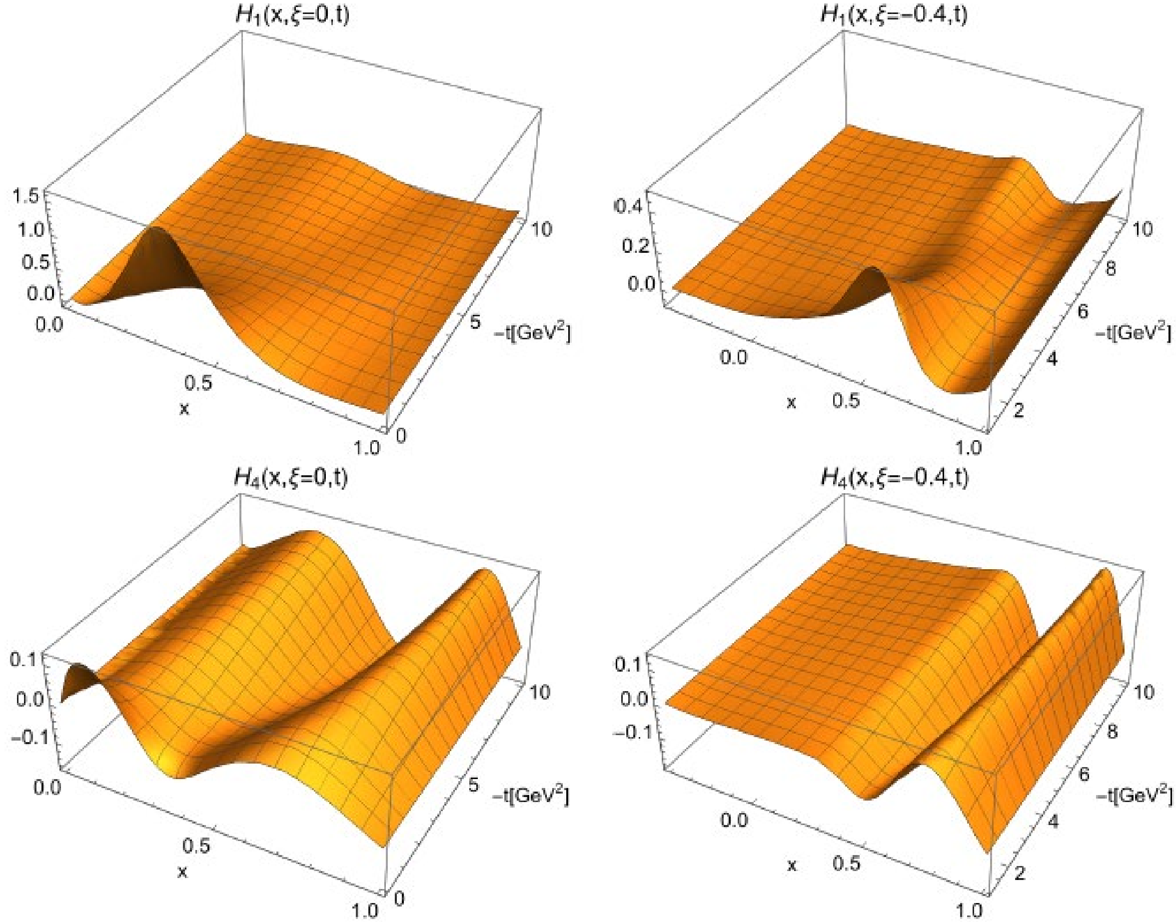


Figure 3: The 3D d quark unpolarized GPDs of Δ^+ H_1 and H_4 as functions of x and $-t$ at $\xi=0$ and $\xi=-0.4$.

c), GFFs of Δ

$r_M^2(\Delta) = 0.529\ fm^2$

$\varepsilon_0(t=0)$ □ 1

S □ $3/2$

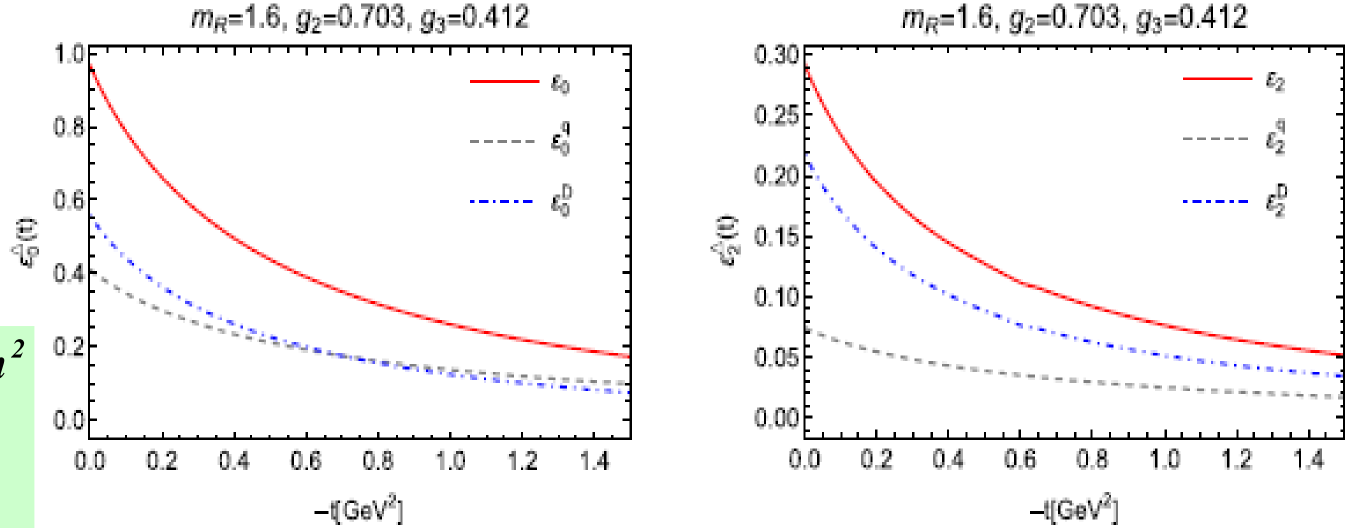


FIG. 7. The calculated energy-monopole form factor of the Δ as a function of $-t$ (left panel) and the energy quadrupole (right panel).

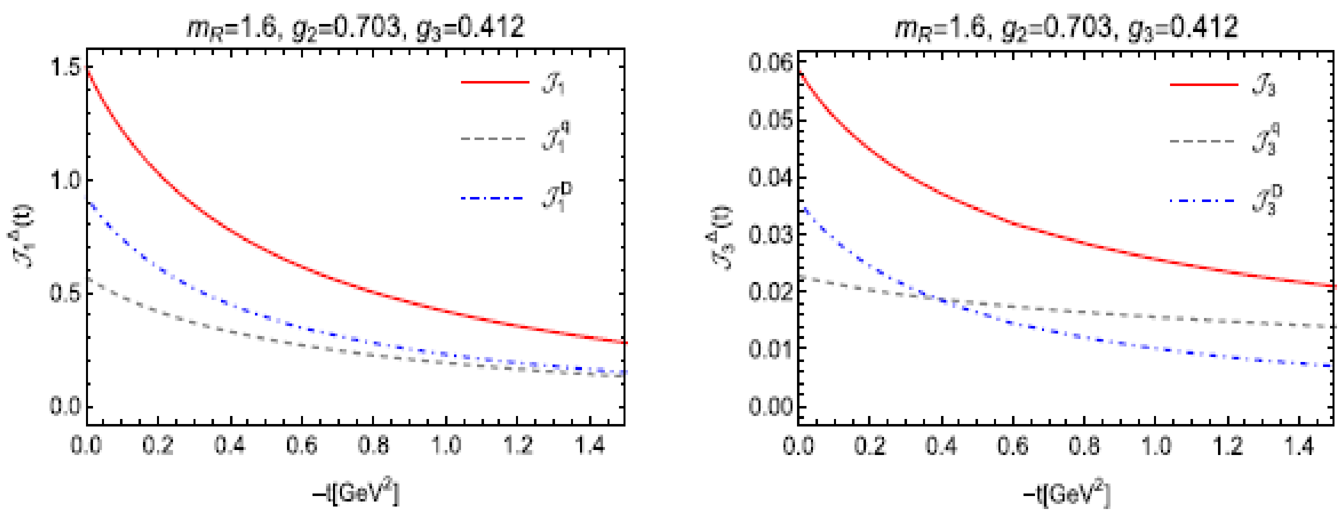


FIG. 8. The angular-momentum form factor of the Δ as a function of $-t$ (left panel), and the octupole-angular momentum form factor

4.1, Summary

① Focus on spin-3/2 particles (Δ &Ω) and their GPDs, EMFFs, GFFs, and some other properties;

★ GPDs of the systems with spin-3/2 are defined and given.

② Numerical calculation: (Quark-diquark approach or quark diquark spectator approach)

③ Results: electromagnetic form factors of the example look okay (at least qualitatively) √

④ Some properties (static) of the systems are obtained (√)

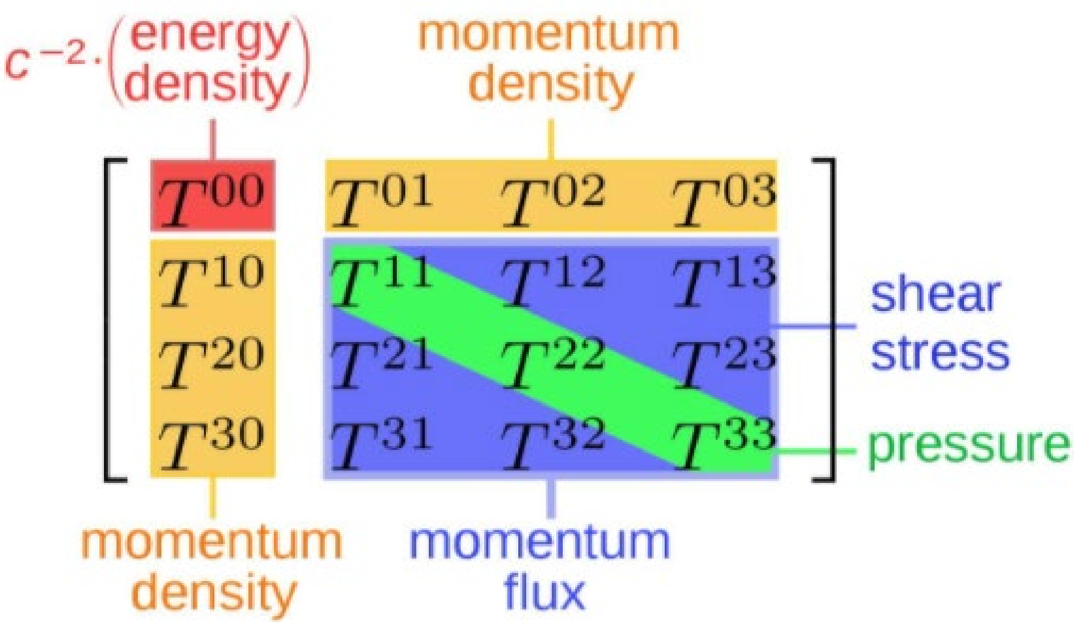
⑤ The calculations and analyses maybe useful for EicC (EIC)…

4.2, discussions and questions (GFFs)

I. Gravitational form factors of the systems (governed by the strong interaction) are also discussed through the GPDs and their moments.

II. Understanding the mechanical properties of the systems is necessary.

Energy-Momentum Tensor$_{EMT}$



Classical Systems

⇕

Quantum Systems

♣ **In continuum media theory**

4.2, discussions and questions (GFFs)

$D(t)\left(\tilde{D}_n(r)\right)-term$

$$\begin{cases} \text{"Shear Force"}: s_n(r) = -\dfrac{1}{4M} r \dfrac{d}{dr}\dfrac{1}{r}\dfrac{d}{dr}\tilde{D}_n(r) \\ \text{"Pressure"}: p_n(r) = \dfrac{1}{6M}\dfrac{1}{r^2}\dfrac{d}{dr} r^2 \dfrac{d}{dr}\tilde{D}_n(r) \end{cases}$$

Moreover, there is an equilibrium relation between the pressure and shear force densities

$$\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$$

von Laue condition is indeed satisfied $\int_0^\infty r^2 p_0(r)dr = 0$

■ × But not inequality $p_0(r) + \frac{2}{3}s_0(r) > 0$

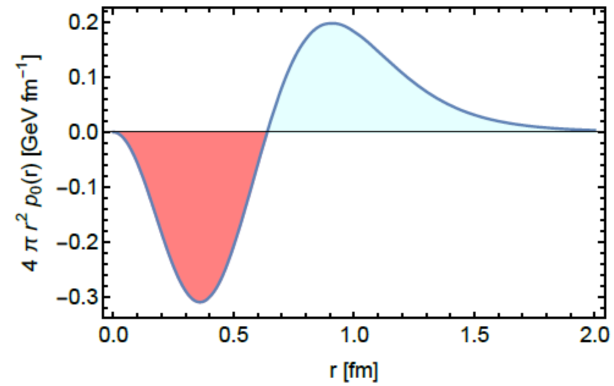


Figure 8: The physical quantity $4\pi r^2 p_0(r)$ as a function of r.

Questions:

0. Numerical results: model-dependent

1. The interpretation of "pressure" and "shear force" in this quantum few-body system?

T^{ij} momentum current

2. EMT and the $\vec{\nabla}_i \langle T^{ij} \rangle = 0$ is sufficient? momentum current density?

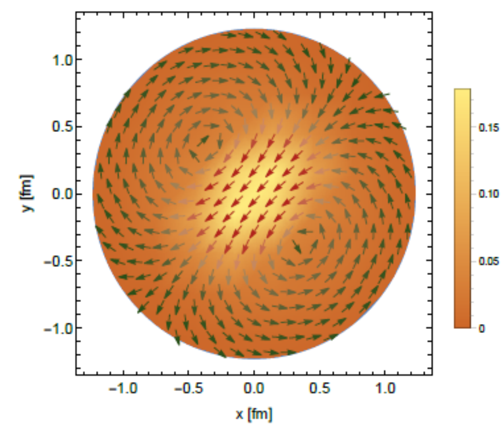


Figure 9: The momentum current with the unit

END – –Thanks!