Collaborative research center CRC 110

"Symmetries and the emergence of structure in QCD"

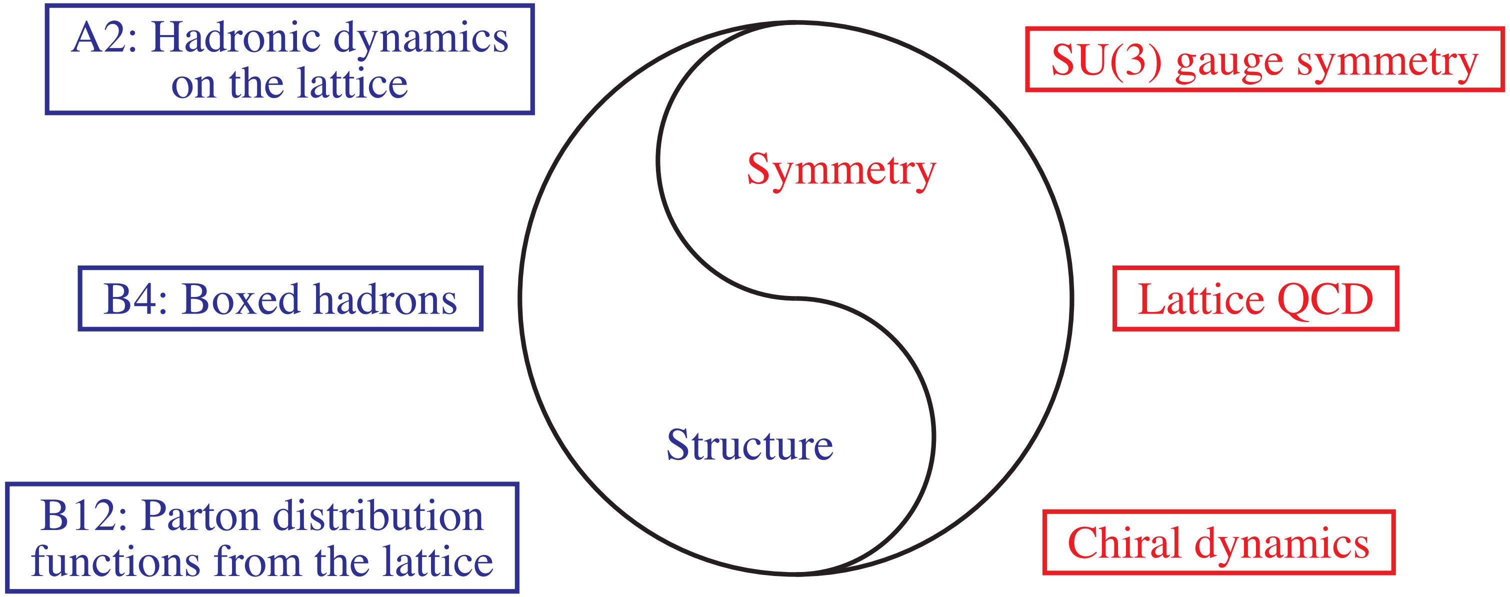
Hadron Dynamics and Lattice QCD

Projects A.2, B.4 and B.12

Xu Feng & Chuan Liu

2023.07.20

Summary of projects



- A.2 Hadronic Dynamics ➡ Multi-particle and multi-channel scattering

  PLs: C. Liu (PKU), T. Luu (FZJ), C. Urbach (UBO)

- B.4 Boxed hadrons ➡ Finite-volume effects for multi-particle system

  PLs: C. Liu (PKU), A. Rusetsky (UBO)

- B.12 PDFs ➡ Partonic structure of hadrons

  PLs: X. Feng (PKU), F. Steffens (UBO)

Work supported by CRC

- Paper published since CRC 3rd term funded on Dec. 17, 2020

  4 PRL, 8 PRD, 1 EPJC, 1 CPC

- Approval of Major Program of NSFC last year

  (Led by C. Liu, including H. Ding, Y. Chen, L. Liu, Y. Yang)

  Key issues in lattice studies based on domestic supercomputers

- Plan for annual domestic lattice conference (2023.10.06 – 2023.10.09)

Lattice team @ PKU

P. I.

X. Feng

Professor

C. Liu

Professor

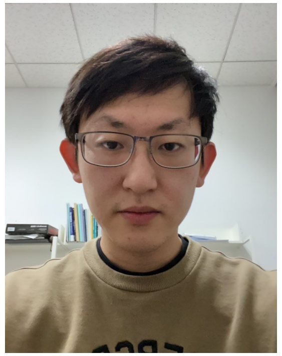
PhD students

Y. Fu (2018)

X.Y. Tuo (2018)

Z.Y. Wang (2019)

X.Y. Wang (2019)



X.H. Wang (2020)

P. Ma (2020)

Y.S. Gao (2021)

T. Wang (2021)

H.B. Yan (2021)

T. Lin (2022)

C. Lu (2022)

J.L. Dang (2022)

Project (I) – multi-hadron scattering

- Coupled-channel study on $Z_c(3900)$ C. Liu, L. Liu, K. Zhang, PRD 101 (2020) 054502
  - Can $Z_c(3900)$ be obtained via multi-channel scattering on a FV lattice
  - Relevant channels: $J/\psi\pi,\ D\bar{D}^*,\ \eta_c\rho,\ D^*\bar{D}^*,\ \ldots$
  - Single-channel Lüscher approach does not find a $D\bar{D}^*$ bound state

    S. Prelovsek et. al. 2015
  - HALQCD claims to reproduce $Z_c(3900)$ on lattice @m_π=410-700 MeV
    - Strong coupling between three lightest channels reported

      Y. Ikeda et. al. 2016

Important to verify using multi-channel Lüscher formula

  - In this work, most strongly coupled channels $J/\psi\pi,\ D\bar{D}^*$ are singled out
    - @m_π=320 MeV, results do not support a narrow resonance close to threshold

Project (I) – multi-hadron scattering

- $D\pi$ scattering and D_0^*

H.-B. Yan with C. Liu, L. Liu, P. Sun, J.-J. Wu et. al.

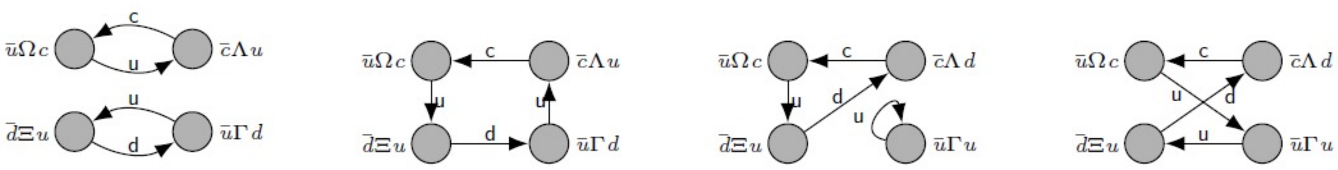
Correlation functions

- Create $D\pi$ from a spacetime point, and annihilate them later

$$\langle \mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]}_{D^{(*)}\pi,\Gamma,p}(t')\mathcal{O}^{[I=\frac{1}{2},I_z=\frac{1}{2}]\dagger}_{D^{(*)}\pi,\Gamma,p}(t)\rangle = \sum_{\beta\alpha ji}(6\mathbb{E}+9\mathbb{F}-3\mathbb{G})^{[\beta,P-\beta;-\alpha,-(P-\alpha)]}_{[\gamma_j,\gamma_5;\gamma_i,\gamma_5]}$$

$$\mathbb{F} = \langle \bar{u}\Box e^{-ip_\delta\cdot x}\Omega\Box c(t')\cdot \bar{d}\Box e^{-ip_\gamma\cdot x}\Xi\Box u(t')\cdot \bar{c}\Box e^{-ip_\beta\cdot x}\Lambda\Box u(t)\cdot \bar{u}\Box e^{-ip_\alpha\cdot x}\Gamma\Box d(t)\rangle$$

- The Wick contractions contain the following diagrams



(a) $\mathbb{E}$ (b) $\mathbb{F}$ (c) $\mathbb{J}$ (d) $\mathbb{G}$

We apply the distillation method[12] to make the calculation possible

$$\Box(t) = V(t)\,V^\dagger(t) \longrightarrow \Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t)v_y^{(k)\dagger}(t)$$

[12] Peardon *et al.*, PRD 80 (2009) 054506.

H.-B. Yan (2nd year PhD)

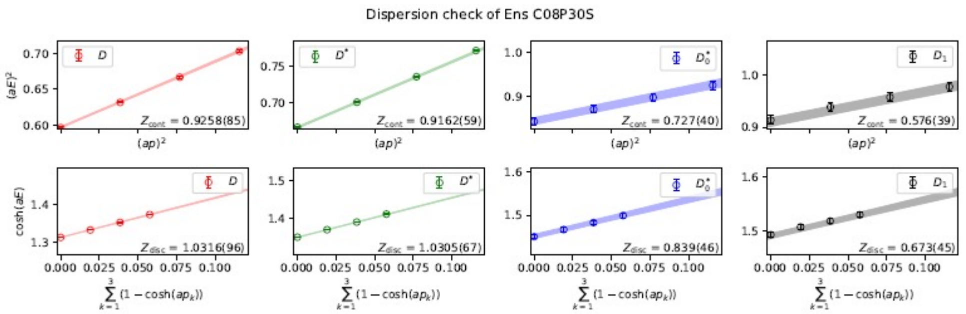
Project (I) – multi-hadron scattering

➢ Dπ scattering and D_0^*

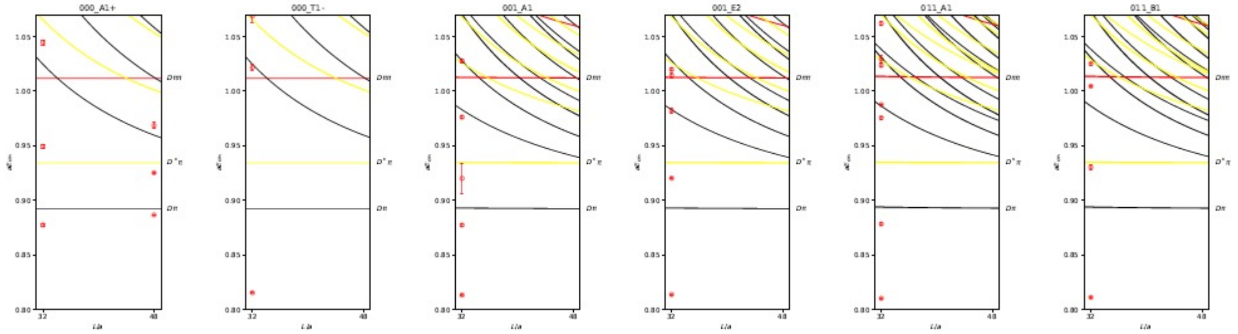
H.-B. Yan with C. Liu, L. Liu, P. Sun, J.-J. Wu et. al.

The spectra

- The dispersion relation



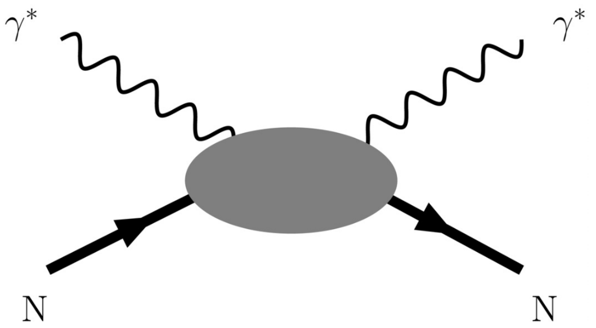
- The extracted finite volume scattering spectra



Operator built, contraction code tested ➡ Continue to push forward

Project (I) – multi-hadron scattering

- Nucleon E&M polarizability and Nπ scattering
  - Polarizability describes the size of dipole moments induced by external E&M fields
  - It is most central quantity relevant for Compton scattering



☐ Unpolarized doubly virtual Compton scattering

$$T^{\mu\nu} = \int d^4x e^{i\mathbf{q}\cdot\mathbf{x}} \langle p|\mathcal{T}[J^\mu(t,\mathbf{x})J^\nu(0)]|p\rangle = T^{\mu\nu}_{Born} + \frac{2M}{\alpha_{em}}[-\beta_M \mathcal{K}_1^{\mu\nu} + (\alpha_E + \beta_M)\mathcal{K}_2^{\mu\nu}]$$

☐ Set up momentum for proton $P = (M, \mathbf{0})$ and photon $q = (0, \boldsymbol{\xi})$

$$\frac{2M}{\alpha_{em}}\alpha_E^N = \frac{1}{3}\left(\frac{\partial T^{ii}}{\partial \xi^2} - \frac{\partial T^{ii}_{Born}}{\partial \xi^2}\right)\Bigg|_{\xi\to 0} \;\Rightarrow\; \frac{2M}{\alpha_{em}}\alpha_E = \frac{1}{2M^2} + \frac{2}{3}\langle r_E^2\rangle + \frac{\kappa^2}{2M^2} + \int_{|t|<t_s} d^4x \left(-\frac{t^2}{6}\right) H(x,t)$$

magnetic moment

charge radius

lattice QCD input

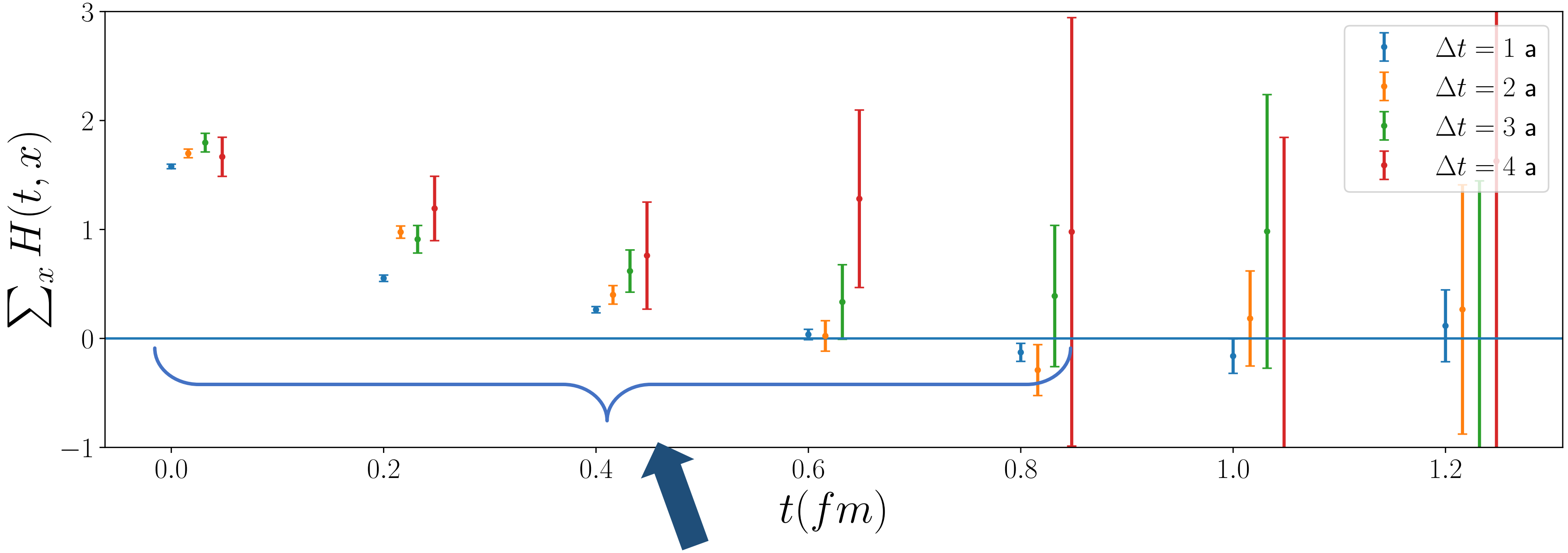
Project (I) – multi-hadron scattering

➢ Nucleon E&M polarizability and Nπ scattering

X.-H. Wang with C. Fan, XF, L. Jin

$$\frac{2M}{\alpha_{em}}\alpha_E = \frac{1}{2M^2} + \frac{2}{3}\langle r_E^2\rangle + \frac{\kappa^2}{2M^2} + \int_{|t|<t_s} d^4x \left(-\frac{t^2}{6}\right) H(x,t)$$

□ Hadronic function $\sum_x H(t,x)$ as a function of time separation t



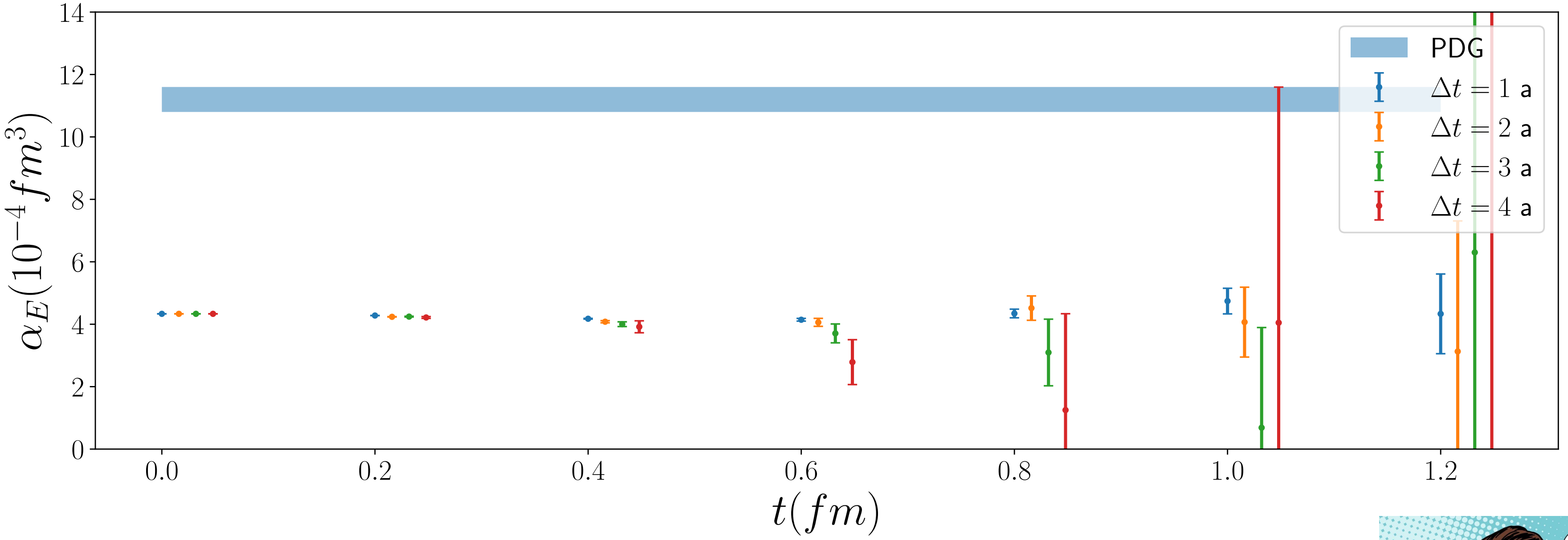
Hadronic function mainly contribute in the region of t<0.8 fm

X.-H. Wang (PhD, 3rd year)

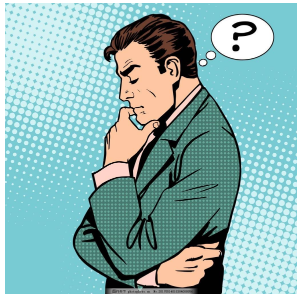
Project (I) – multi-hadron scattering

➢ Nucleon E&M polarizability and Nπ scattering

$$\frac{2M}{\alpha_{em}}\alpha_E = \frac{1}{2M^2} + \frac{2}{3}\langle r_E^2\rangle + \frac{\kappa^2}{2M^2} + \int_{|t|<t_s} d^4x \left(-\frac{t^2}{6}\right) H(x,t)$$



However, lattice predictions are significantly below the PDG value. Why?



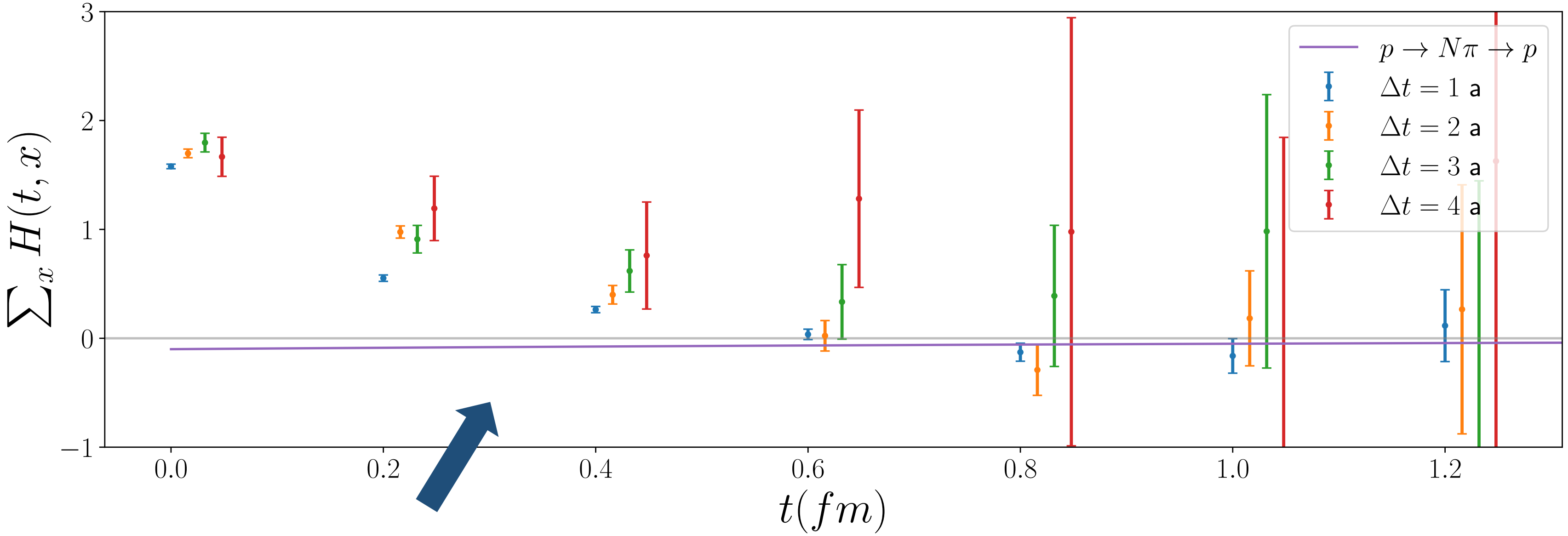
Need new insight to turn the decent to the magic!

Project (I) – multi-hadron scattering

- Nucleon E&M polarizability and Nπ scattering

Structure of hadronic function $$\int d^4x \left(-\frac{t^2}{6}\right) H(x,t) = \int dt \left(-\frac{t^2}{6}\right) \sum_k \langle p|J(0)|k\rangle e^{-(E_k-M)t} \langle k|J(0)|p\rangle$$
$$= -\frac{2}{3} \sum_k \frac{\langle p|J(0)|k\rangle\langle k|J(0)|p\rangle}{(E_k-M)^3}$$

The dominant contribution is given by $|k\rangle = |N\pi\rangle$ ground intermediate states



Nπ contribution seems negligible compared to the total contribution, and is complete hidden by noise at large time separtion

Project (I) – multi-hadron scattering

- Nucleon E&M polarizability and Nπ scattering

Structure of hadronic function $\int d^4x \left(-\frac{t^2}{6}\right) H(x,t) = \int dt \left(-\frac{t^2}{6}\right) \sum_k \langle p|J(0)|k\rangle e^{-(E_k-M)t} \langle k|J(0)|p\rangle$

$$= -\frac{2}{3} \sum_k \frac{\langle p|J(0)|k\rangle \langle k|J(0)|p\rangle}{(E_k - M)^3}$$

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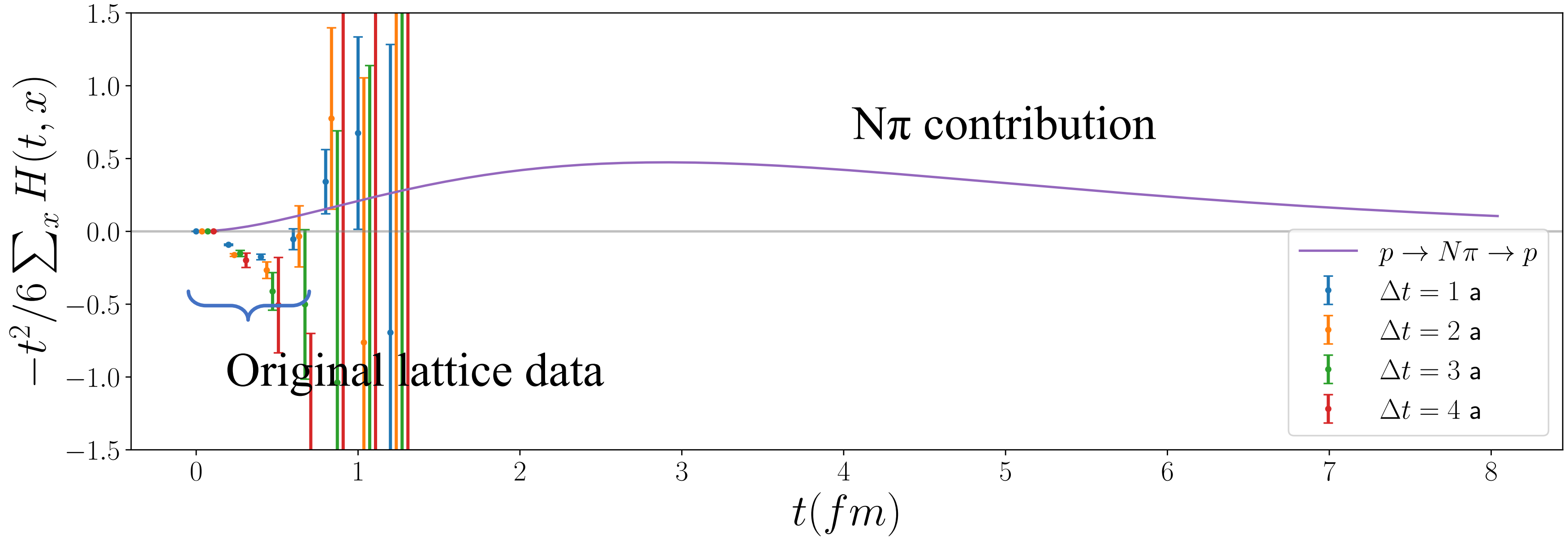


Nπ contribution significantly enhanced by factor of t^2 ➡ Need to calculate Nπ rescattering on lattice

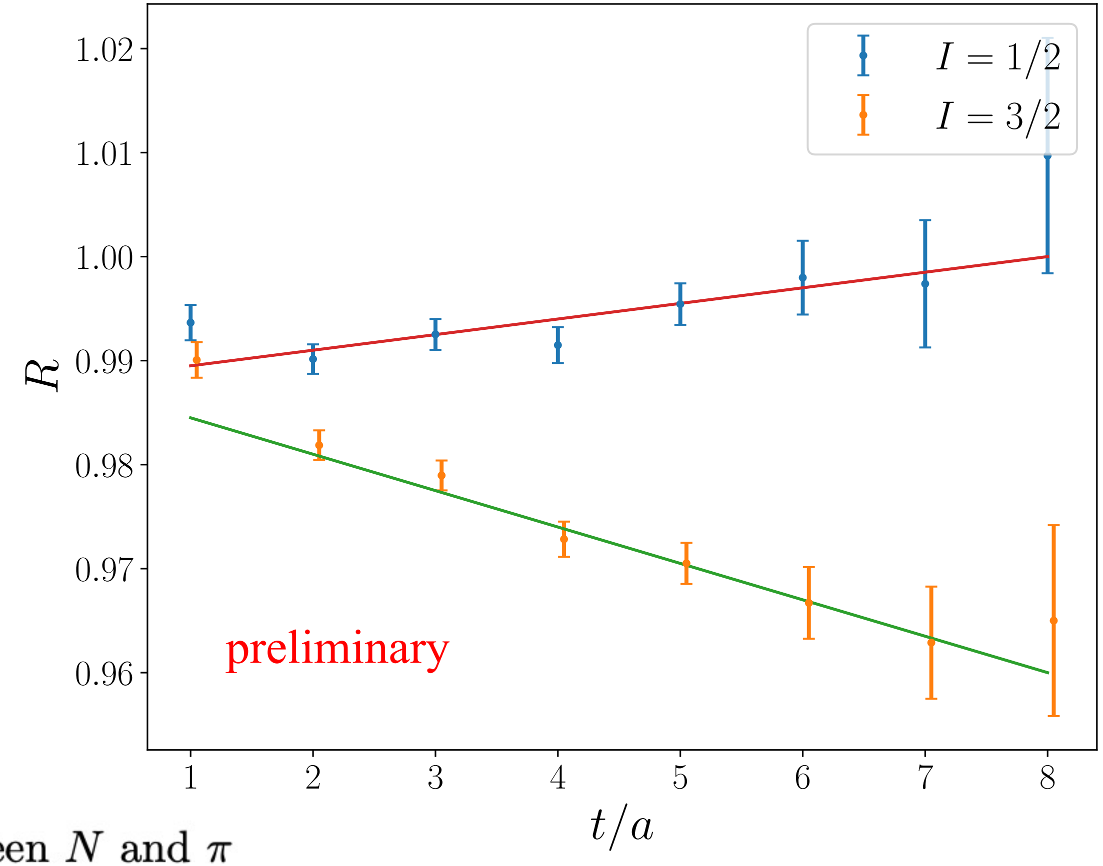
Project (I) – multi-hadron scattering

➢ Nucleon E&M polarizability and Nπ scattering

- Nπ scattering at m_π=140 MeV

$$
\begin{aligned}
R &= \frac{C_2^{N\pi}(t)}{C_2^N(t) C_2^\pi(t)} \\
&= \frac{A_{N\pi}}{A_N A_\pi} \frac{e^{-E_{N\pi}t}}{e^{-(M_N+M_\pi)t}} \\
&\approx R_0(1-\Delta E t)
\end{aligned}
$$

with $\Delta E = E_{N\pi} - M_N - M_\pi$



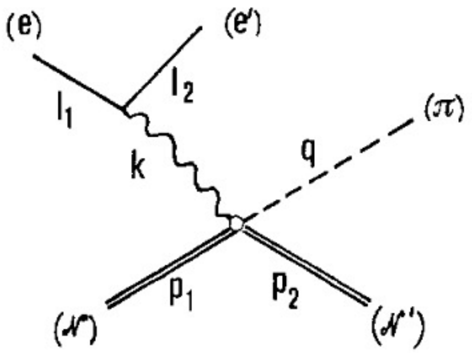
- Scattering for different isospin channel

  - $I = 1/2$, $\Delta E < 0$, attractive interaction between N and π
  - $I = 3/2$, $\Delta E > 0$, repulsive interaction between N and π

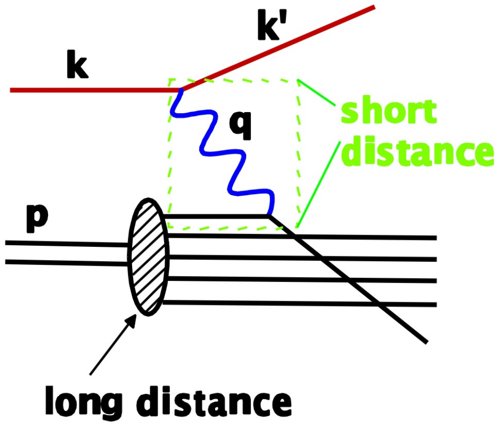
Project (I) – multi-hadron scattering

- Nucleon E&M polarizability and Nπ scattering

  - Extended projects



Pion electroproduction



Neutrino induced shallow inelastic scattering

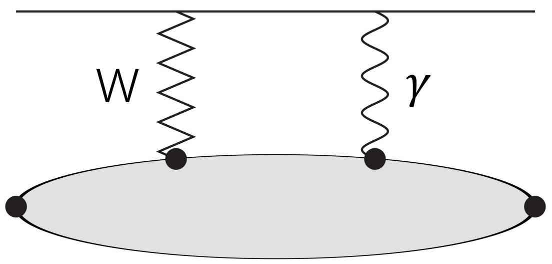
Y.-S. Gao (PhD, 2nd year)

Project (II) – Photon-W box diagram

➢ Pion & Kaon semileptonic decays

German collaborators: M. Gorchtein (U. Mainz), U.-G. Meißner (UBO/FZJ), C.-Y. Seng (UBO)

- Precise determination of CKM matrix elements is the central themes in modern high energy physics
- V_{ud} is the most accurately-determined element from the study of superallowed nuclear β decay

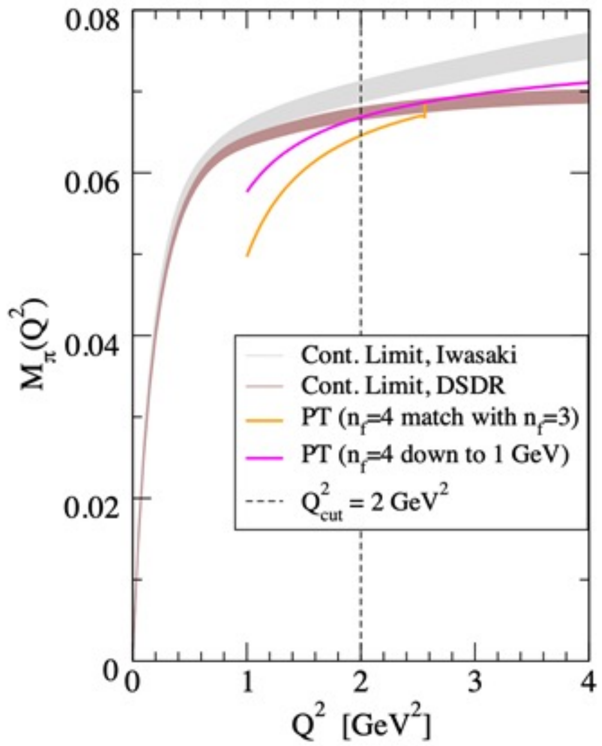


- Photon-W box diagram contributes the largest theoretical uncertainties

A. Sirlin, Rev.Mod.Phys. 50 (1978) 573

- In pion decays, lattice QCD can reduce theoretical uncertainty for the non-perturbative QCD part by 10 times

XF, M. Gorchtein, L. Jin, P. Ma, C. Seng
PRL 124 (2020) 19, 192002



Project (II) – Photon-W box diagram

➢ Pion & Kaon semileptonic decays

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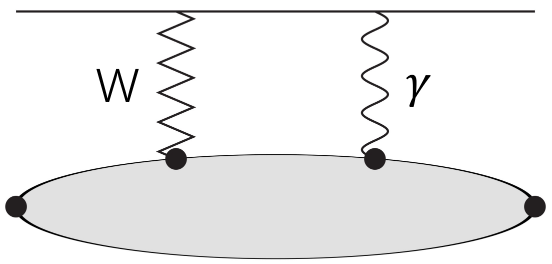


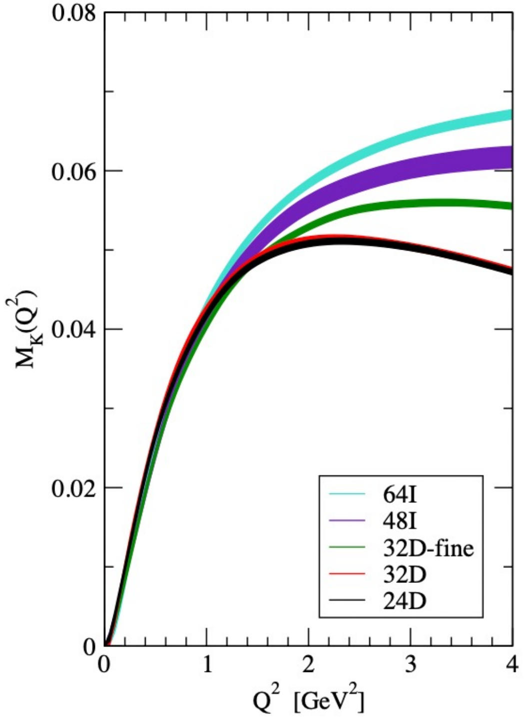
- Photon-W box diagram contributes the largest theoretical uncertainties

A. Sirlin, Rev.Mod.Phys. 50 (1978) 573

- In kaon decays, we use lattice QCD to determine the LECs for ChPT, and then give the prediction for γ-W box diagram

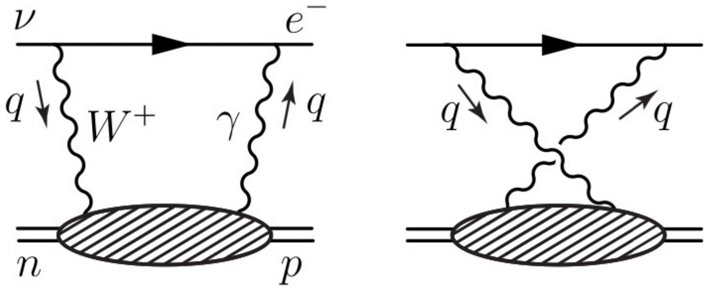
C. Seng, XF, M. Gorchtein, L. Jin, U.-G. Meißner, JHEP 10 (2020) 179

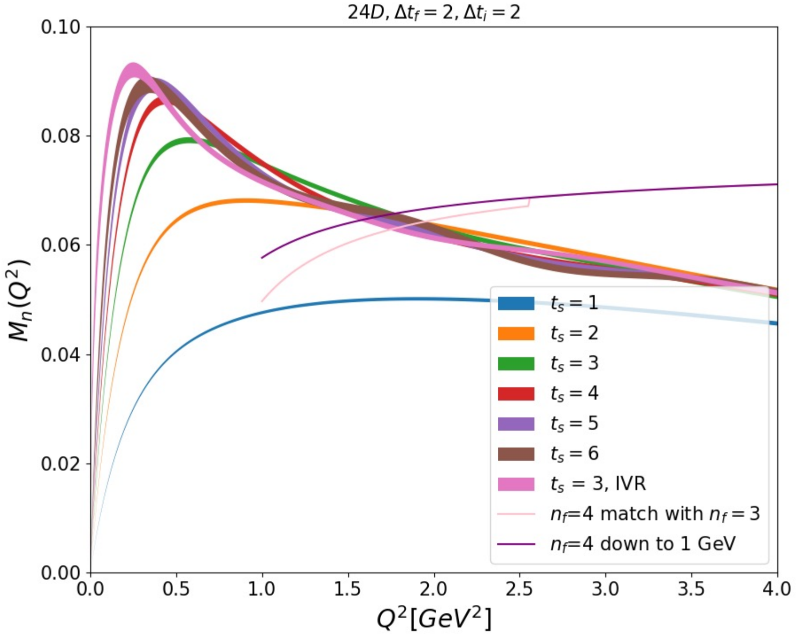
P. Ma, XF, M. Gorchtein, L. Jin, PRD 103 (2021) 114503



Project (II) – Photon-W box diagram

- Neutron beta decays
  - Project has now moved to the neutron system





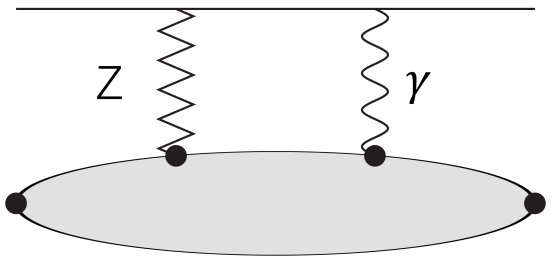
P. Ma (PhD, 3rd year)

- Ultra Cold Neutron Experiment, UCNτ+@LANL, targets on precise measurement of neutron lifetime

- Lattice QCD results of γW box diagram

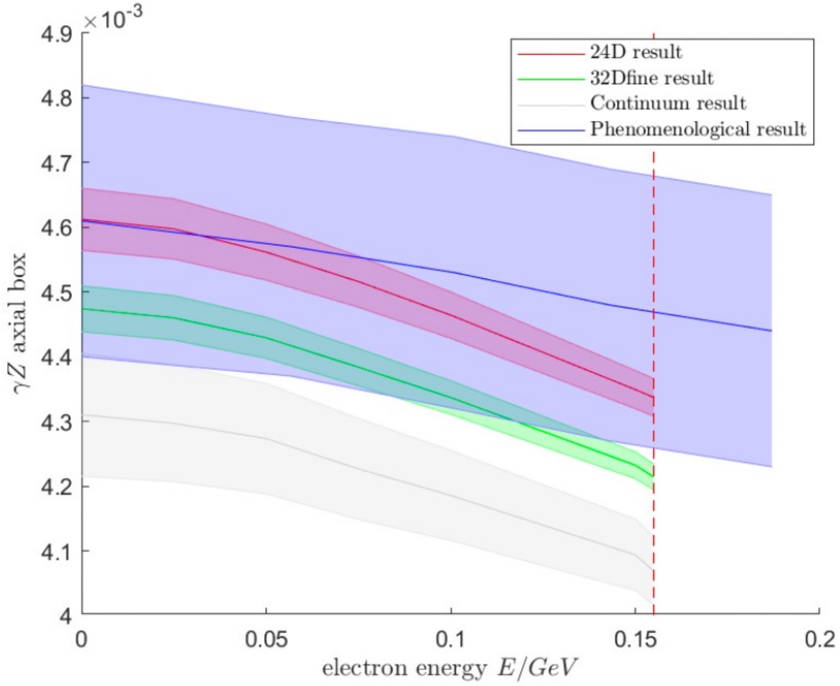
Project (II) – Photon-W box diagram

- Parity-violating e-p scattering
  - ☐ Q_{weak} experiment @ JLab: polarized electron and unpolarized proton scattering
  - ☐ Extract the interference between E&M interaction and Z-boson exchange
    → used for accurate determination of Weinberg angle θ_W



☐ γ-Z box diagram contributes the largest theoretical uncertainties

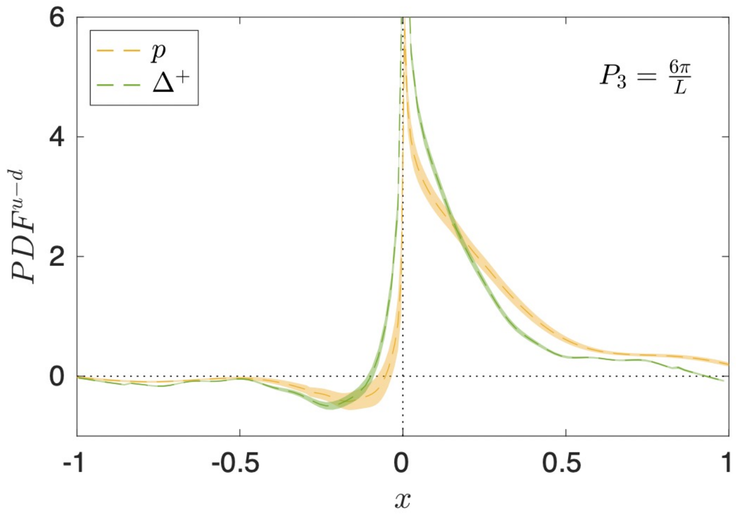
☐ Electron energy dependence of γ-Z contribution @ m_π=140 MeV



Z.-L. Zhang (new PhD)

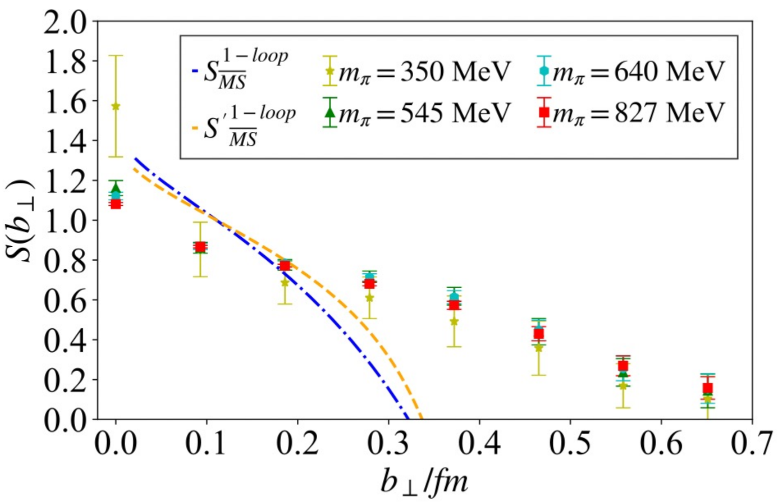
Project (III) – Parton distribution function

- Parton structure from lattice QCD German side: K. Jansen (DESY), F. Steffens (UBO), A. Sen (UBO)
  - Lattice methodology: LaMET proposed by X. Ji in 2013
  - Current project involves calculation of Δ baryon PDFs, soft function and renormalization of TMDPDF operators



Δ baryon PDFs

Y. Chai, Y. Li, S. Xia, et. al.
PRD 102 (2020) 014508

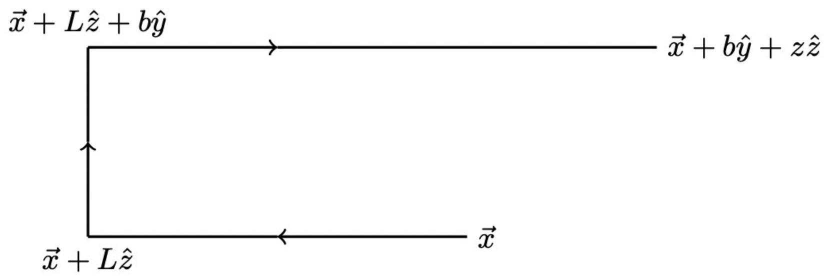


Intrinsic soft function

Y. Li, S. Xia, et. al.
PRL 128 (2022) 062002

$$\mathcal{W}(b+z;L) = W_z(\vec{x};L)W_\perp(\vec{x}+L\hat{z};b)W_z(\vec{x}+L\hat{z}+b\hat{y};-L+z)\,,$$

$$W_z(\vec{x};L) = \mathcal{P}\exp\left[-ig\int_0^L d\lambda\hat{z}\cdot A(\vec{x}+\hat{z}\lambda)\right].$$

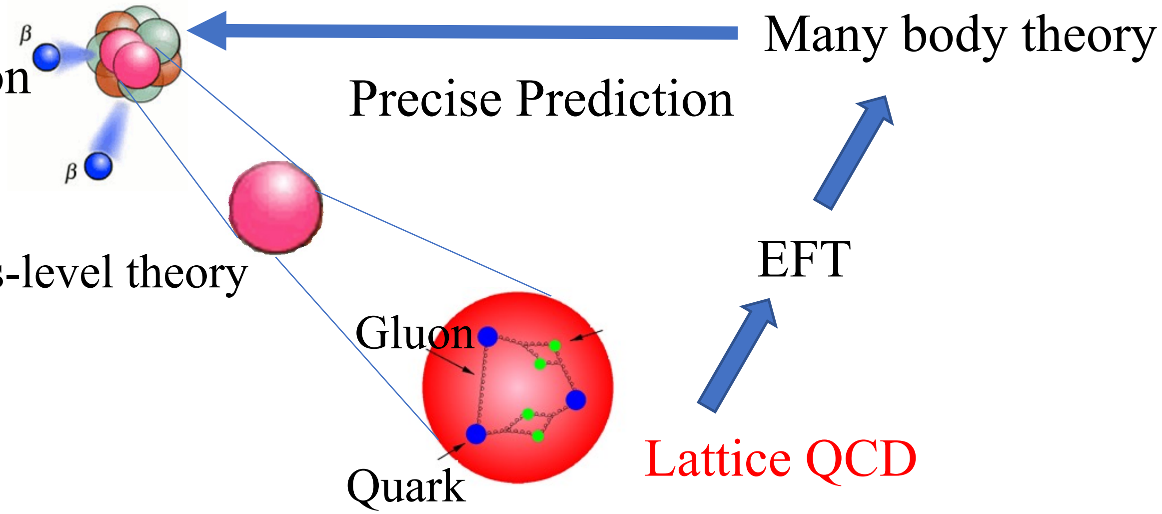


Renormalization of staple shaped operator

C. Alexandrou, et. al.
arXiv: 2305.11824

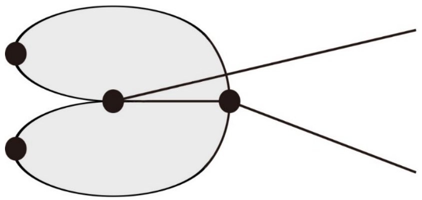
Other projects – 0ν2β decays

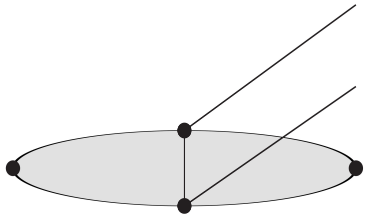
0ν2β once detected – confirm existence of Majorana fermion



- Requires the interplay among various theoretical fields
- Target: Seamless connection between quark-level and nucleus-level theory
- Lattice QCD is the starting point

0ν2β decays in pion system





$\pi^-\pi^- \to ee$ $g_\nu^{\pi\pi} = -12.0(3)$

$\pi^- \to \pi^+ ee$ $g_\nu^{\pi\pi} = -10.9(3)(7)$

XF, L. Jin, X. Tuo, et.al PRL 122 (2019) 022001
X. Tuo, XF, L. Jin, PRD 100 (2019) 094511

X.-Y. Tuo (PhD, 5th year)
→ BNL postdoctor

Later, MIT group confirms our results with $g_\nu^{\pi\pi} = -10.8(1)(5)$

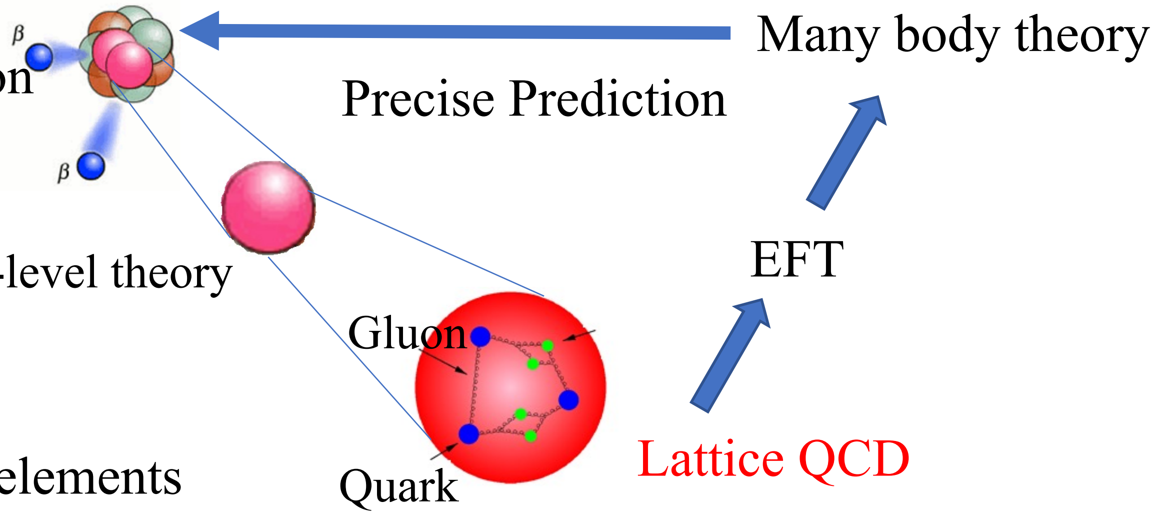
W.Detmold, D. Murphy, et. al. arXiv:2004.07404

See Xin-Yu's talk on Saturday morning

Other projects – 0ν2β decays

If 0ν2β decay detected – confirm existence of Majorana fermion

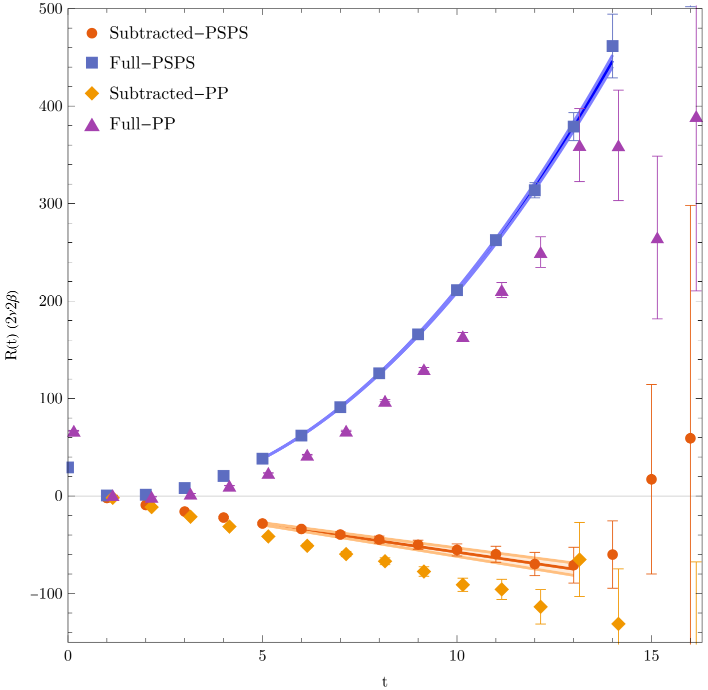
- Interplay between different fields
- Target: seamless connection between quark-level and nucleus-level theory
- Lattice QCD is the starting point

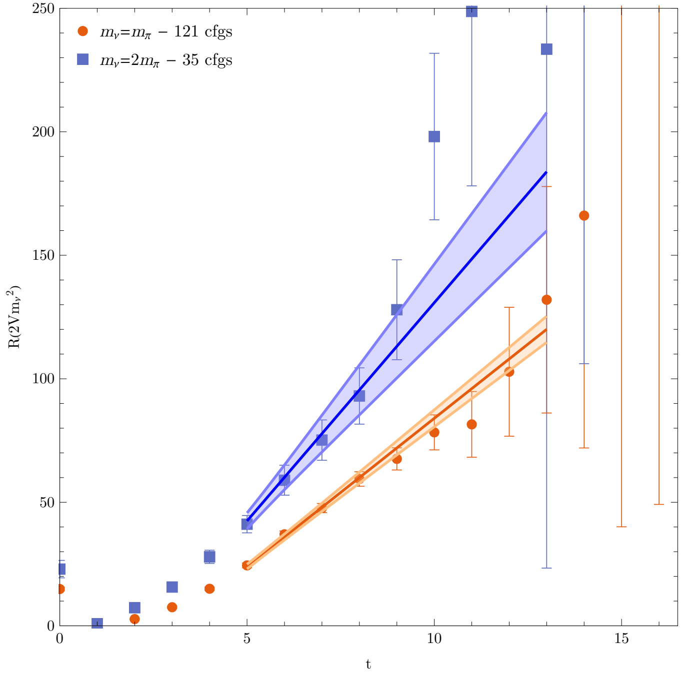


Dibaryon nn→pp: 2ν2β (left) and 0ν2β (right) nucleon matrix elements

Z.-Y. Wang (PhD, 4th year)

T. Wang (PhD, 2nd year)

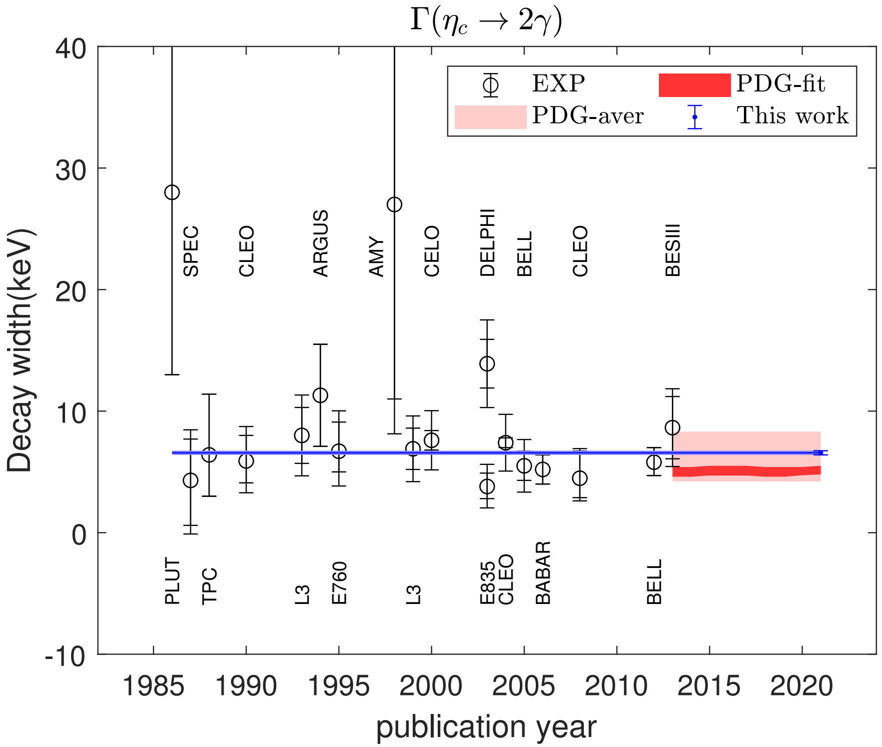




Other projects – Meson radiative decays

- Develop a new method to calculate meson radiative decays

  - $\eta_c \to \gamma\gamma$ decays   Y. Meng, XF, C. Liu, T. Wang, Z. Zuo, accepted by Science Bulletin



Lattice results exhibit a tension with PDG fit

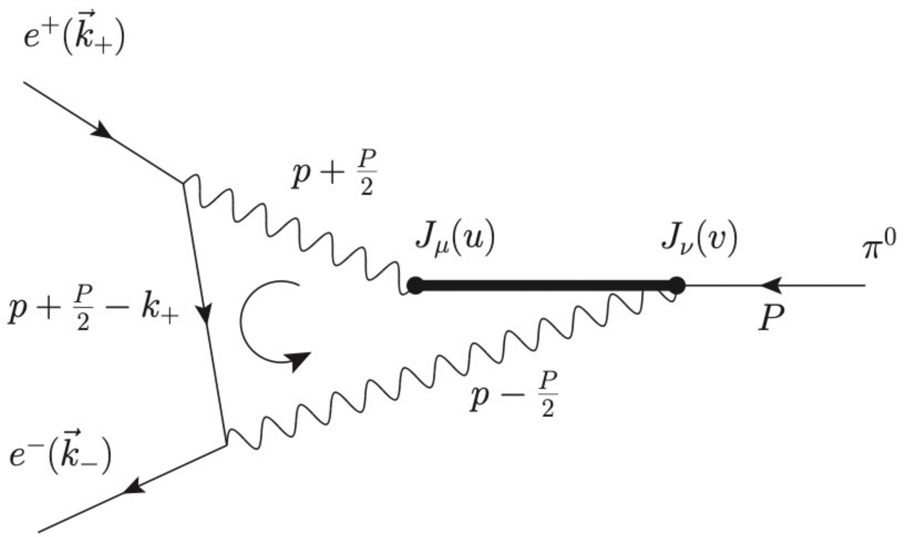
Rumor from BESIII ➡ Larger decay width favored!

Y. Meng (Postdoctor 2021-2022)
→ faculty @ Zhengzhou U.

See Yu's talk on Saturday morning

Other projects – Meson radiative decays

- Develop a new method to calculate meson radiative decays
  - $\eta_c \to \gamma\gamma$ decays — Y. Meng, XF, C. Liu, T. Wang, Z. Zuo, accepted by Science Bulletin
  - $\chi_{c0} \to \gamma\gamma$ decays — Z. Zuo, Y. Meng, C. Liu, CPC 46 (2022) 053102
  - $\pi^0 \to e^+e^-$ decays — N. Christ, XF, L. Jin, C. Tu, PRL 130 (2023) 191901

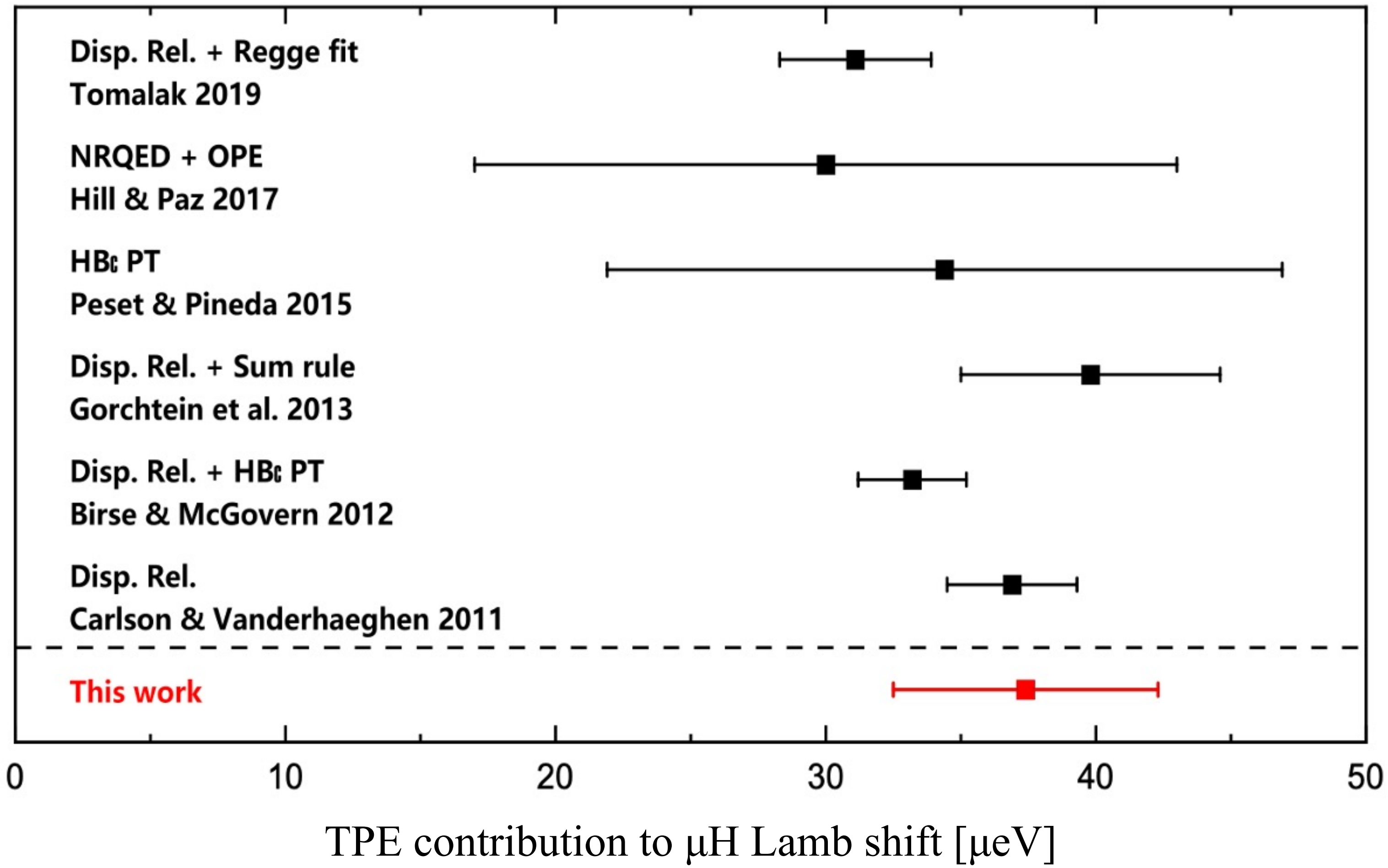


Other projects – μH Lamb shift & TPE

- 2010: μH experiment@PSI yields 4% smaller proton charge radius

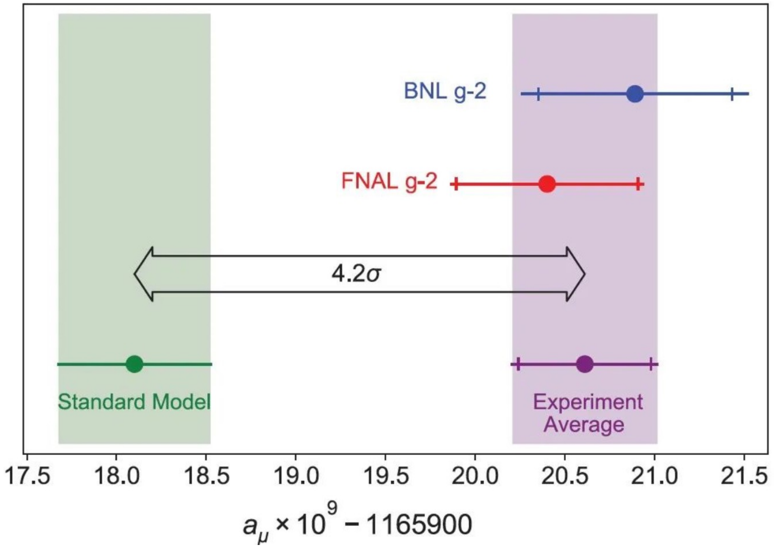
Y. Fu (PhD, 5th year)
→ MIT postdoctor

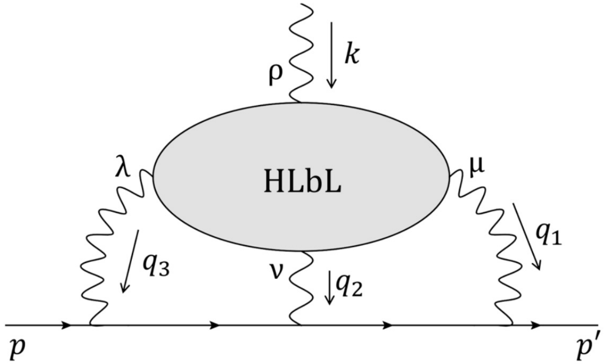
C. Lu (PhD, 1st year)

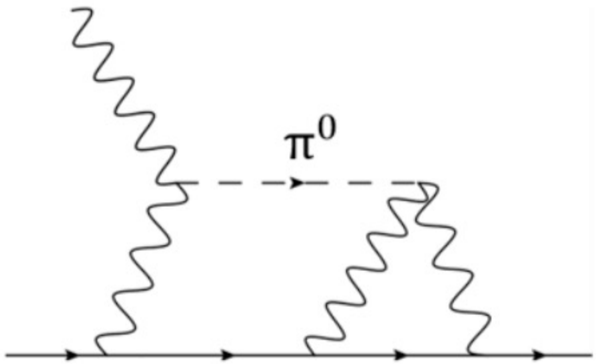


Y. Fu, XF, L. Jin, C. Lu, PRL 128 (2022) 172002

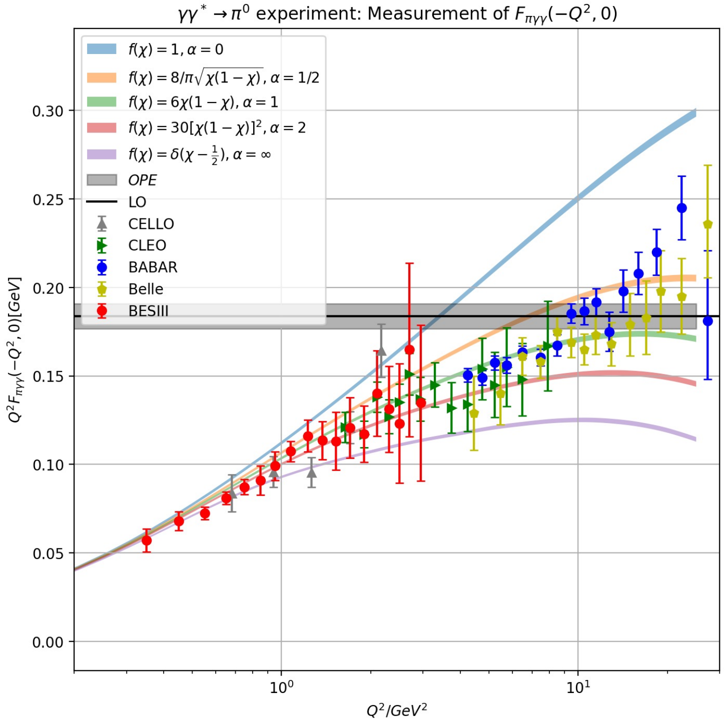
Muon g-2 and hadronic light-by-light

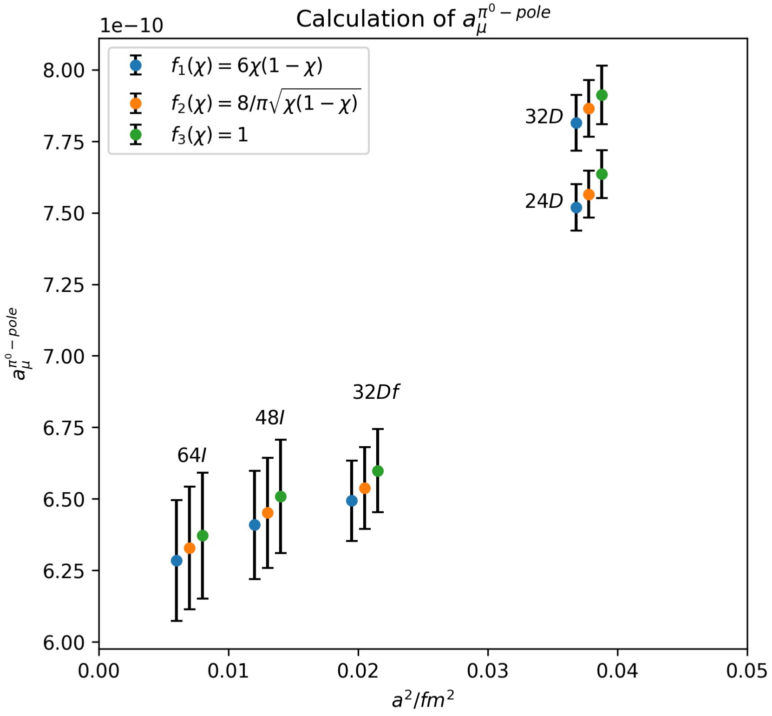






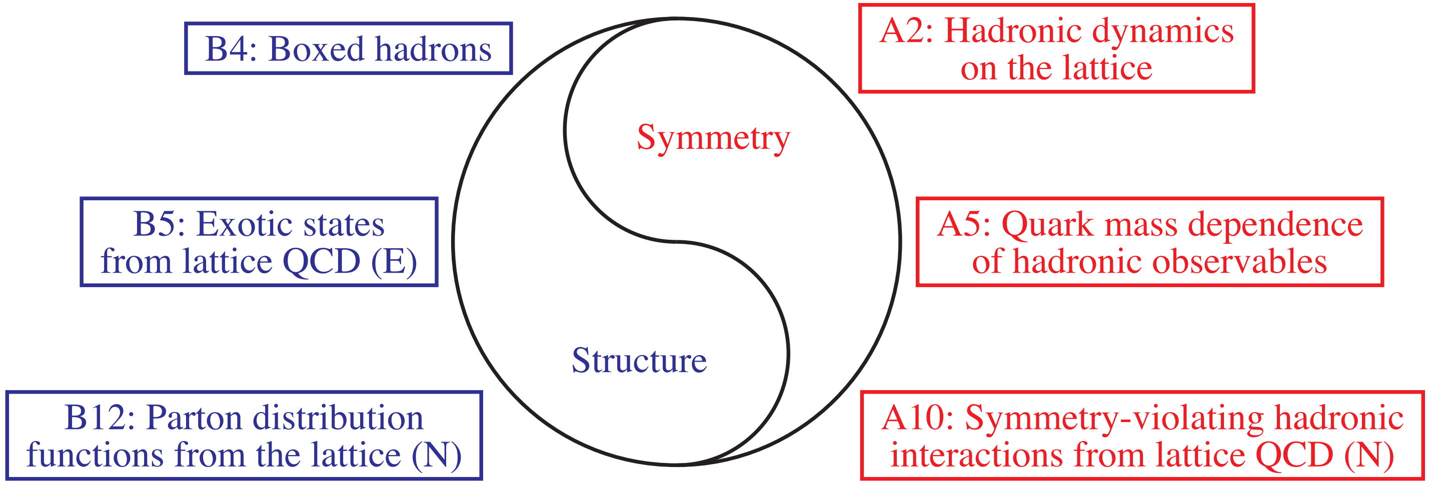
T. Lin (PhD, 1st year)





Conclusion and outlook

- Lattice QCD is an exciting field
- We got a lot of support from CRC program (in total 6 projects relevant for LQCD)



- For 3rd funding period, unfortunately, project significantly overlaps with Covid19

  e.g. Lattice 2022 at Bonn (Thanks for the effort from C. Urbach! But difficult for us to get visa)
- Now, situation changes! ➡ Look forward to more exchange and communication!

Welcome to Chinese Lattice 2023

- October 6 – 9, 2023
- Chunhui Yuan Hot Spring Resort Hotel, Beijing, Shunyi