Hyperon electromagnetic form factors in VMD model

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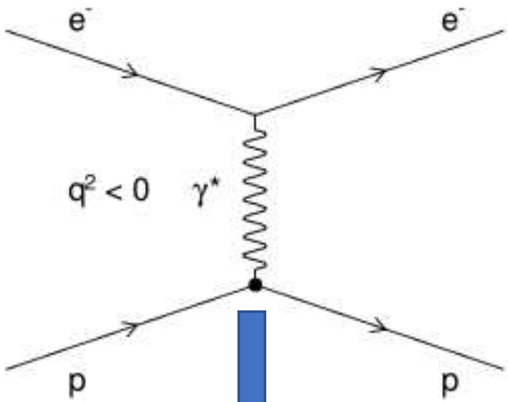
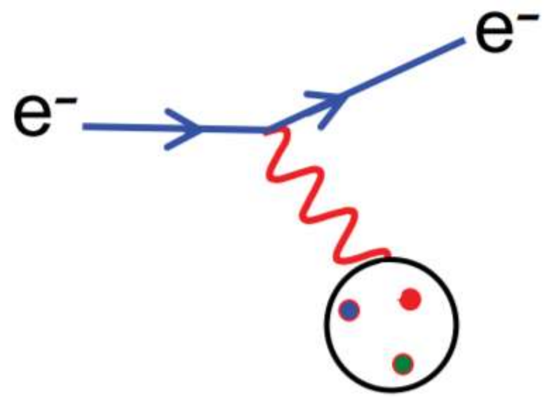
第七届强相互作用量子色动力学对称性及其物质结构学术研讨会
The 7th Symposium on “Symmetries and the emergence of Structure in QCD”
山东 日照

Outline

- Introduction： electromagnetic form factors
- The model: Vector Meson Dominance
- Hyperon electromagnetic form factors
- Summary

Electromagnetic form factors (space-like)





$$\langle p_f | \hat{J}^\mu(0) | p_i \rangle = \overline{u}(p_f)\left[F_1(q^2)\gamma^\mu - F_2(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2M}\right]u(p_i)$$

$$\Gamma^\mu(q^2) = \gamma^\mu F_1^p(q^2) + i\frac{F_2^p(q^2)}{2M_p}\sigma^{\mu\nu}q_\nu$$

F_1^N : Dirac form factor

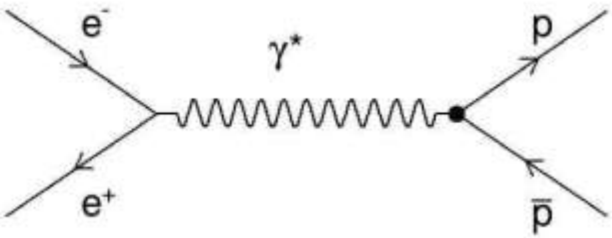
F_2^N: Pauli form factor

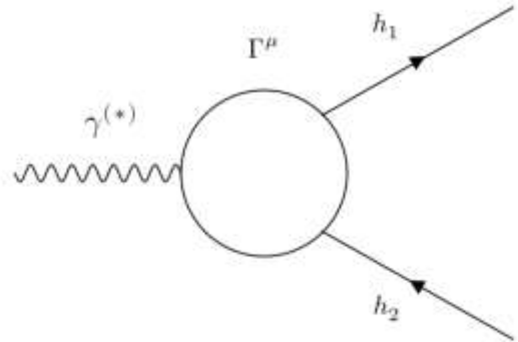
$$G_E^N(Q^2) = F_1^N(Q^2) - \tau F_2^N(Q^2)\ ,\ \ G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2)\ ,\ \ \tau = \frac{Q^2}{4M_N^2}$$

$$F_1^p(0) = 1\ , F_1^n(0) = 0\ ,\ \ F_2^p(0) = \kappa_p\ , F_2^n(0) = \kappa_n$$

S. Pacetti, R. Baldini Ferroli and E. Tomasi-Gustafsson, ``Proton electromagnetic form factors: Basic notions, present achievements and future perspectives," **Phys. Rept. 550-551, 1-103 (2015).**

Electromagnetic form factors (time-like)

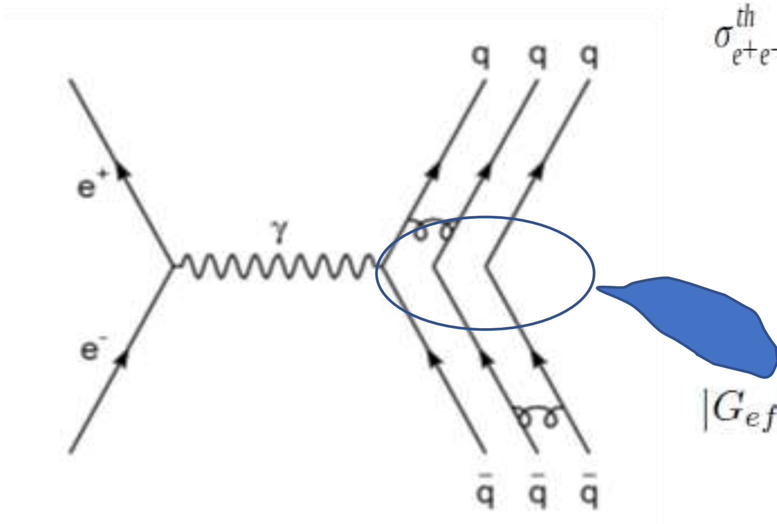




$$\left(\frac{d\sigma}{d\Omega}\right)^{th}_{e^+e^-\to N\bar{N}} = \frac{\alpha^2\beta}{4q^2} C_N(q^2)\left\{|G_M^N(q^2)|^2(1+\cos^2\theta)+|G_E^N(q^2)|^2\frac{1}{\tau}\sin^2\theta\right\}$$

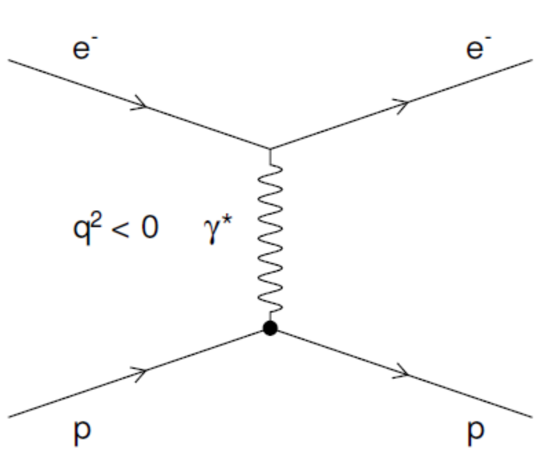
$$\sigma^{th}_{e^+e^-\to N\bar{N}} = \frac{\alpha^2\beta}{4q^2} C_N(q^2)\int d\Omega\left[|G_M^N(q^2)|^2(1+\cos^2\theta)+|G_E^N(q^2)|^2\frac{\sin^2\theta}{\tau}\right]$$

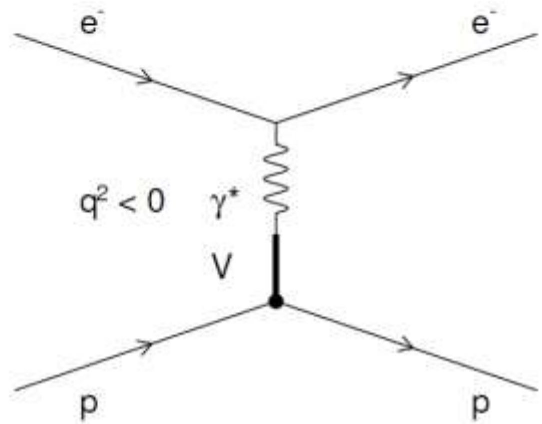
$$= \frac{4\pi\alpha^2\beta}{3q^2} C_N(q^2)\left[|G_M^N(q^2)|^2+\frac{|G_E^N(q^2)|^2}{2\tau}\right].$$



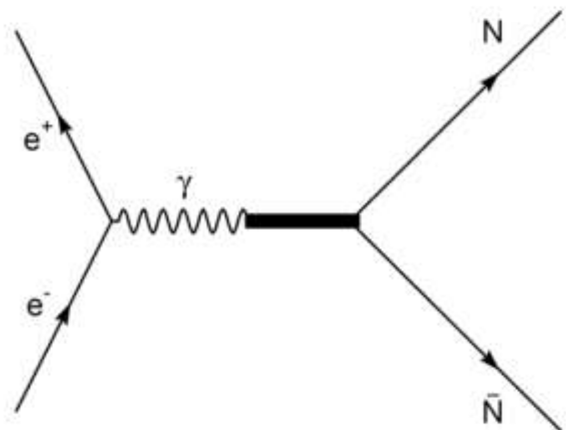
$$|G_{eff}(q^2)| = \sqrt{\frac{\sigma(q^2)}{\sigma_{point}(q^2)}} = \sqrt{\frac{|G_M(s)|^2+\frac{2M^2}{s}|G_E(s)|^2}{1+\frac{2M^2}{s}}}$$

VMD: vector meson dominance model



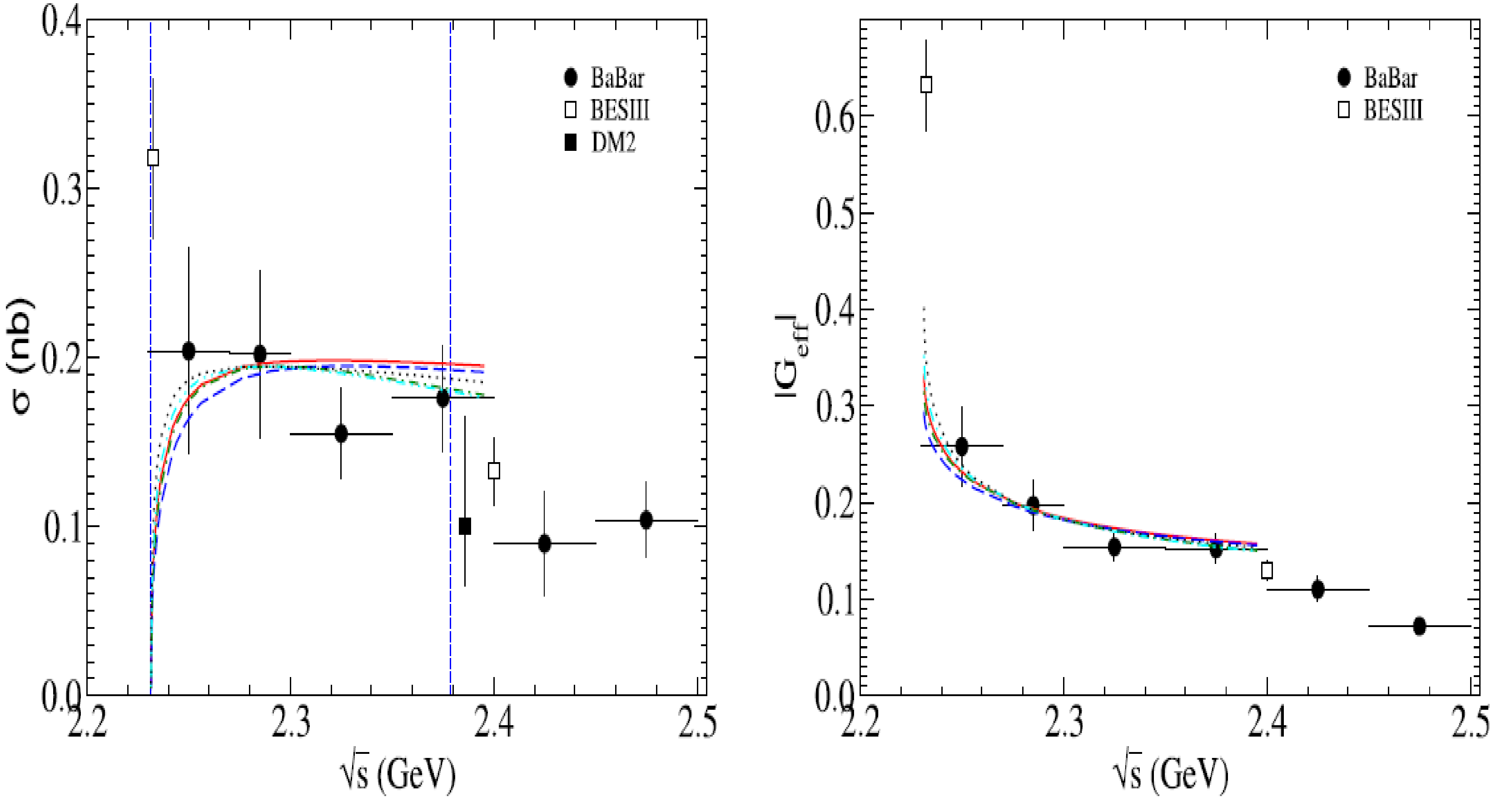


$$\mathcal{L}_{V\gamma} = \sum_V \frac{eM_V^2}{f_V} V_\mu A^\mu$$



$$L_{\mathrm{NNV}} = g\bar{\psi}\gamma_\mu\psi\varphi_V^\mu + \frac{\kappa}{4m}\bar{\psi}\sigma_{\mu\nu}\psi\left(\partial^\mu\varphi_V^\nu - \partial^\nu\varphi_V^\mu\right)$$

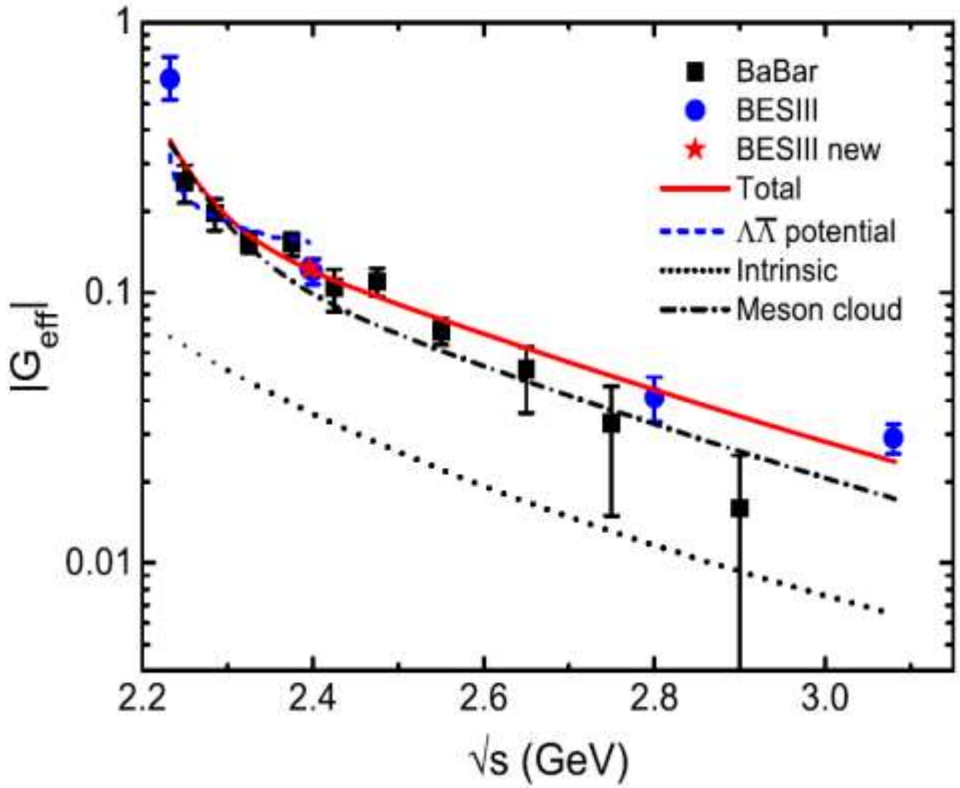
Λ EMFFs in final state interactions



J. Haidenbauer and U. G. Meißner, Phys. Lett. B 761, 456-461(2016).

Λ EMFFs in VMD

Y. Yang, D. Y. Chen and Z. Lu,
Phys. Rev. D 100, 073007 (2019).

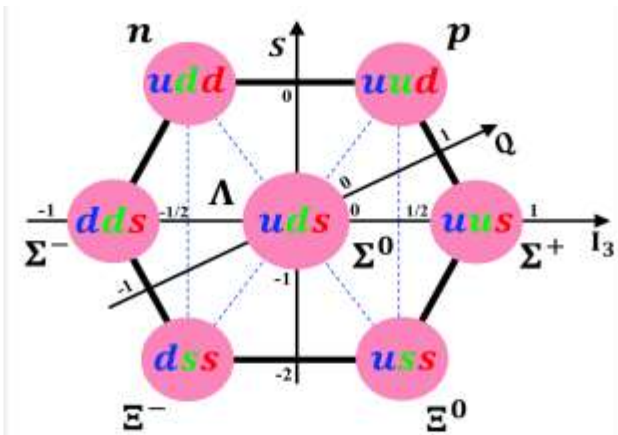


fit. In the present scenario, there are 16 experimental data and 10 free parameters. The value of intrinsic parameter γ is fitted to be 0.336 GeV^{-2} and the other parameters are summarized in Table II. It should be noticed that $g(q^2)$

$$g(q^2) = \frac{1}{(1-\gamma q^2)^2}$$

$$\gamma_N = \frac{1}{0.71\ \mathrm{GeV}^2} = 1.408\ \mathrm{GeV}^{-2}$$

State	Mass	Width	State	Mass	Width
$\omega(782)$ [55]	782	8.1	$\phi(1020)$ [56]	1019	4.2
$\omega(1420)$ [57]	1418	104	$\phi(1680)$ [57]	1674	165
$\omega(1650)$ [57]	1679	121	$\phi(2170)$ [58]	2171	128



Λ EMFFs in VMD (New proposal)

$$F_1(Q^2) = g(Q^2)[-\beta_\omega - \beta_\phi + \beta_\omega \frac{m_\omega^2}{m_\omega^2+Q^2} + \beta_\phi \frac{m_\phi^2}{m_\phi^2+Q^2} + \beta_x \frac{m_x^2}{m_x^2+Q^2}]$$

$$F_2(Q^2) = g(Q^2)[(\mu_\Lambda - \alpha_\phi)\frac{m_\omega^2}{m_\omega^2+Q^2} + \alpha_\phi \frac{m_\phi^2}{m_\phi^2+Q^2} + \alpha_x \frac{m_x^2}{m_x^2+Q^2}]$$

$$g(Q^2) = 1/(1+\gamma Q^2)^2$$

$$Q^2 \to -q^2$$

$$\frac{m_\omega^2}{m_\omega^2+Q^2} \to \frac{m_\omega^2}{m_\omega^2 - q^2 - i m_\omega \Gamma_\omega}$$

$$\frac{m_\phi^2}{m_\phi^2+Q^2} \to \frac{m_\phi^2}{m_\phi^2 - q^2 - i m_\phi \Gamma_\phi}$$

$$\frac{m_x^2}{m_x^2+Q^2} \to \frac{m_x^2}{m_x^2 - q^2 - i m_x \Gamma_x},$$

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

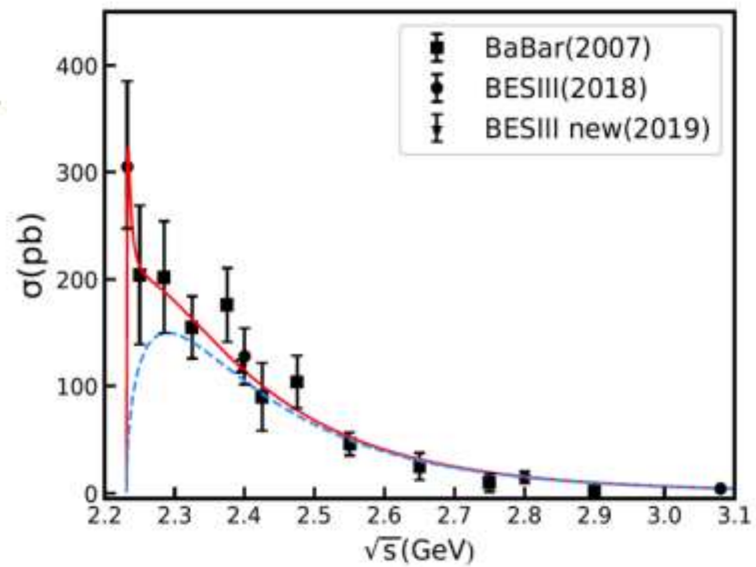
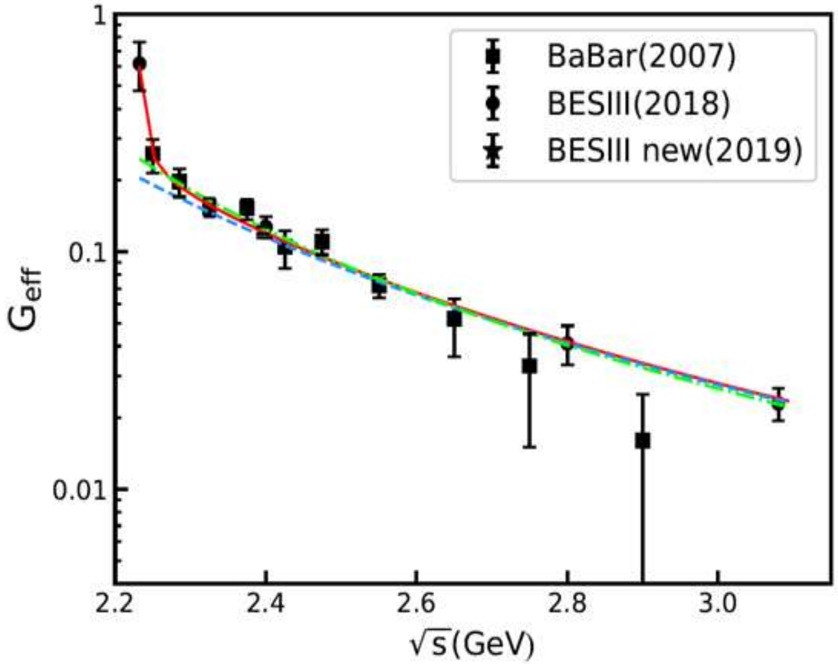


Figure: Cross section of the reaction $e^+e^- \to \bar{\Lambda}\Lambda$.

Z. Y. Li, A. X. Dai and J. J. Xie,
Chin. Phys. Lett. 39, 011201 (2022).



The red solid curve represents the total contributions from ω, ϕ and $X(2231)$, while the blue dashed curve stands for the results without the contribution from the new $X(2231)$ state. The green-dash-dotted curve stands for the fitted results with the effective form factor as in .

$$G_{\rm eff} = C_0 g(q^2) = \frac{C_0}{(1-\gamma q^2)^2}.$$

Table: Values of model parameters determined in this work.

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.43	β_ω	-1.13
β_ϕ	1.35	α_ϕ	-0.40
β_x	0.0015	m_x (MeV)	2230.9
Γ_x(MeV)	4.7		

New state X(2231) ?

Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

Flatte function

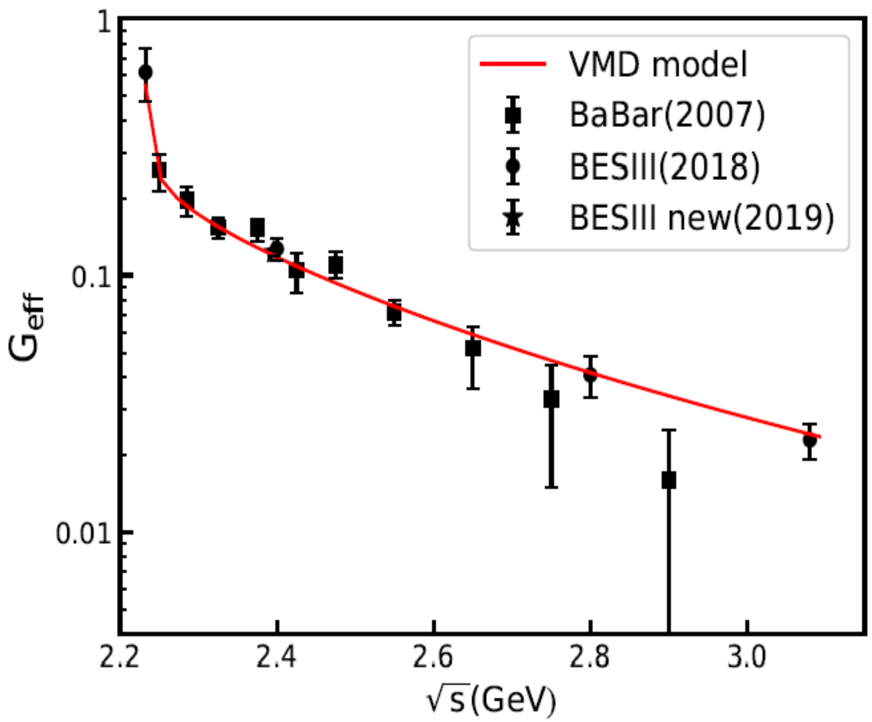


Figure: Fitting result of $|G_{eff}|$ with Flatte.

S.M. Flatte, Phys. Lett. B 63, 224-227 (1976).

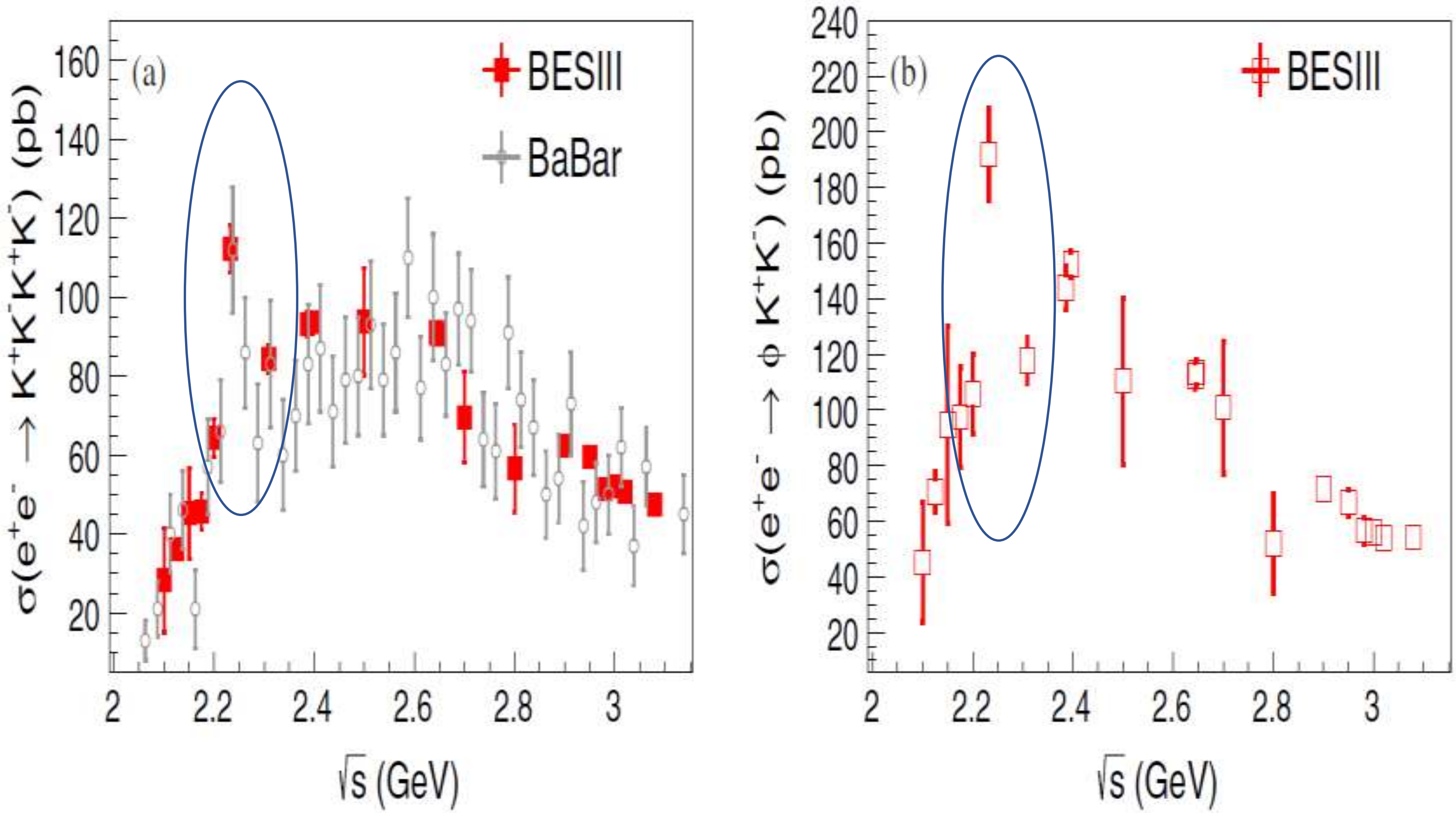
On the other hand, if one takes a Flatté form for the total decay width of $\omega(1420)$, $\omega(1650)$, $\phi(1680)$, and $\phi(2170)$, the experimental data can also be well reproduced with a strong coupling of these resonances to the $\Lambda\bar{\Lambda}$ channel.

$$\Gamma_x = \Gamma_0 + \Gamma_{\Lambda\bar{\Lambda}}(s) \qquad \Gamma_{\Lambda\bar{\Lambda}} = \frac{g^2}{4\pi}\sqrt{\frac{s}{4} - M_\Lambda^2}$$

Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.57 ± 0.21	$\beta_{\omega\phi}$	-0.3 ± 0.31
β_x	-0.03 ± 0.09	m_x(MeV)	2237.7 ± 50.2
Γ_0 (MeV)	$8.8^{+75.9}_{-8.8}$	$g_{\Lambda\bar{\Lambda}}$	3.0 ± 1.9

Z. Y. Li, A. X. Dai and J. J. Xie, Chin. Phys. Lett. 39, 011201 (2022).

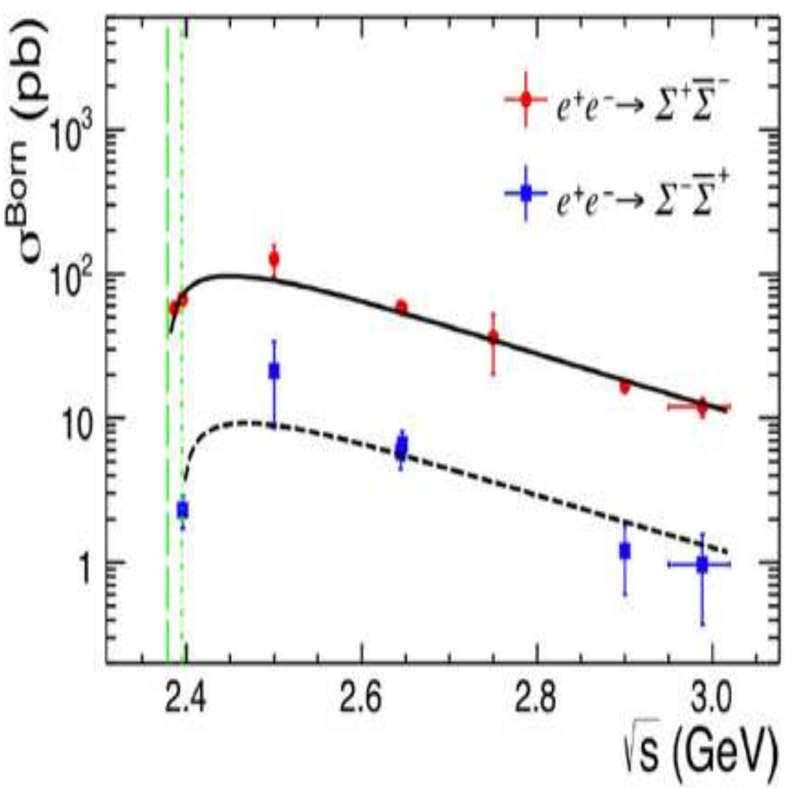
Where is X(2231)?

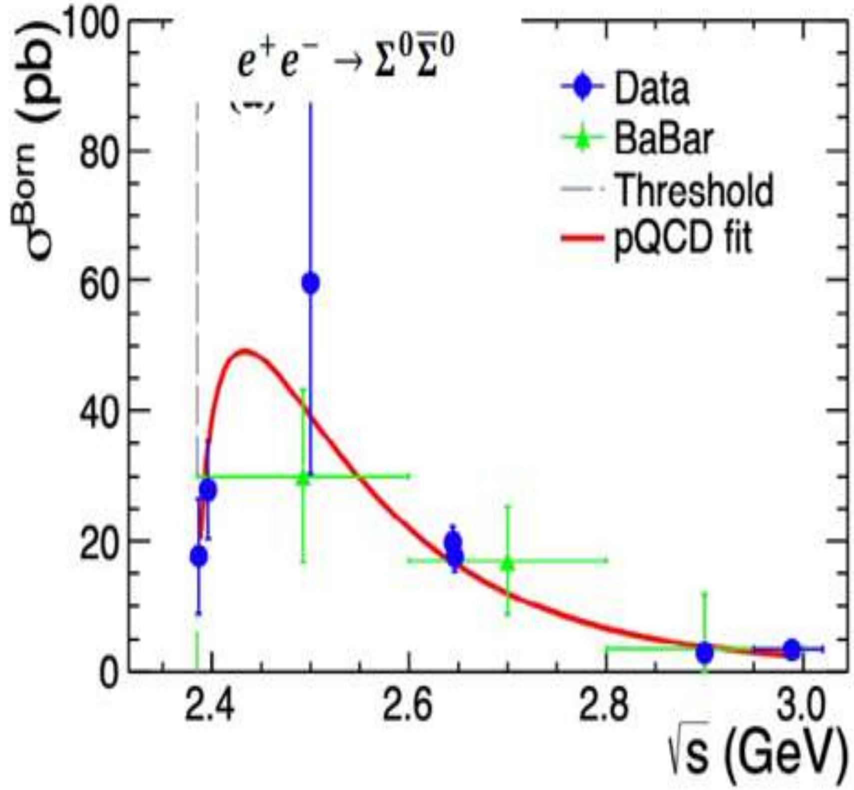


M. Ablikim, et al., Phys. Rev. D 100, 032009(2019).

Σ EMFFs

$$\sigma(s) = \frac{4\pi\alpha^2\beta}{3s}C(s)\left[1+\frac{2M^2}{s}\right]|G_{eff}(s)|^2 = \sigma_{point}(s)|G_{eff}(s)|^2.$$





BESIII, **Phys. Lett. B 814, 136110 (2021)；Phys. Lett. B 831, 137187 (2022).**

The ratio $\Sigma^+\bar{\Sigma}^-$: $\Sigma^0\bar{\Sigma}^0$: $\Sigma^-\bar{\Sigma}^+$ is about 9.7 ± 1.3 : 3.3 ± 0.7 : 1.

Σ^+, Σ^-, and Σ^0 EMFFs （VMD）

$$|\Sigma^+\bar{\Sigma}^-\rangle = \frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle + \frac{1}{\sqrt{6}}|2,0\rangle$$

$$|\Sigma^-\bar{\Sigma}^+\rangle = -\frac{1}{\sqrt{2}}|1,0\rangle + \frac{1}{\sqrt{3}}|0,0\rangle + \frac{1}{\sqrt{6}}|2,0\rangle$$

$$|\Sigma^0\bar{\Sigma}^0\rangle = -\frac{1}{\sqrt{3}}|0,0\rangle + \sqrt{\frac{2}{3}}|2,0\rangle$$

Isospin decomposition

$$F_1^{\Sigma^+} = g(q^2)(f_1^{\Sigma^+} + \frac{\beta_\rho}{\sqrt{2}}B_\rho - \frac{\beta_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_2^{\Sigma^+} = g(q^2)(f_2^{\Sigma^+}B_\rho - \frac{\alpha_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_1^{\Sigma^-} = g(q^2)(f_1^{\Sigma^-} - \frac{\beta_\rho}{\sqrt{2}}B_\rho - \frac{\beta_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_2^{\Sigma^-} = g(q^2)(f_2^{\Sigma^-}B_\rho - \frac{\alpha_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_1^{\Sigma^0} = g(q^2)(\frac{\beta_{\omega\phi}}{\sqrt{3}} - \frac{\beta_{\omega\phi}}{\sqrt{3}}B_{\omega\phi}),$$

$$F_2^{\Sigma^0} = g(q^2)\mu_{\Sigma^0}B_{\omega\phi},$$

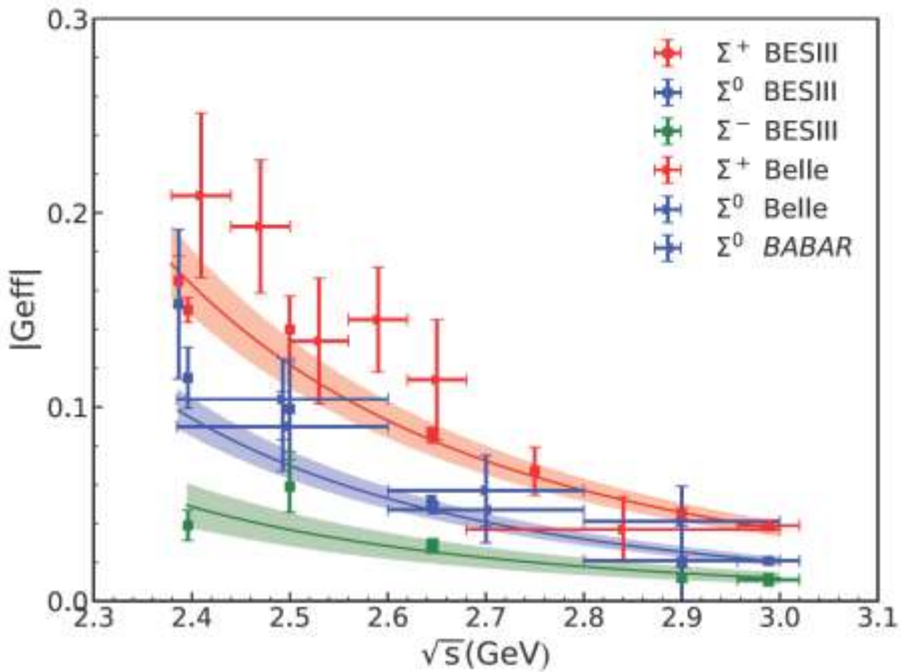
$$B_\rho = \frac{m_\rho^2}{m_\rho^2 - q^2 - im_\rho\Gamma_\rho},$$

$$B_{\omega\phi} = \frac{m_{\omega\phi}^2}{m_{\omega\phi}^2 - q^2 - im_{\omega\phi}\Gamma_{\omega\phi}},$$

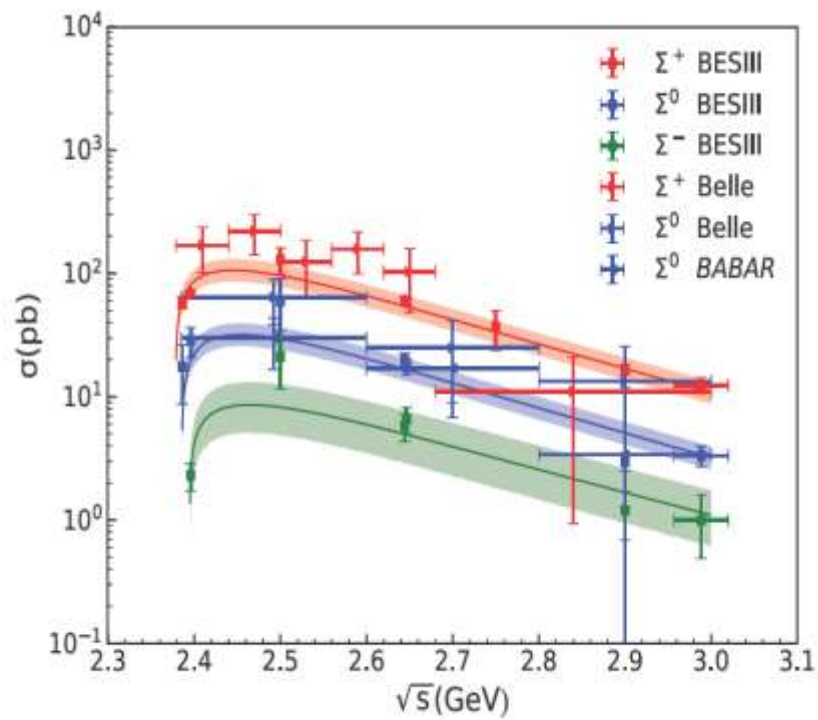
$$f_1^{\Sigma^+} = 1 - \frac{\beta_\rho}{\sqrt{2}} + \frac{\beta_{\omega\phi}}{\sqrt{3}}, \quad f_2^{\Sigma^+} = 2.112 + \frac{\alpha_{\omega\phi}}{\sqrt{3}},$$

$$f_1^{\Sigma^-} = -1 + \frac{\beta_\rho}{\sqrt{2}} + \frac{\beta_{\omega\phi}}{\sqrt{3}}, \quad f_2^{\Sigma^-} = -0.479 + \frac{\alpha_{\omega\phi}}{\sqrt{3}}$$

Σ^{+}, Σ^{-}, and Σ^{0} EMFFs （VMD）



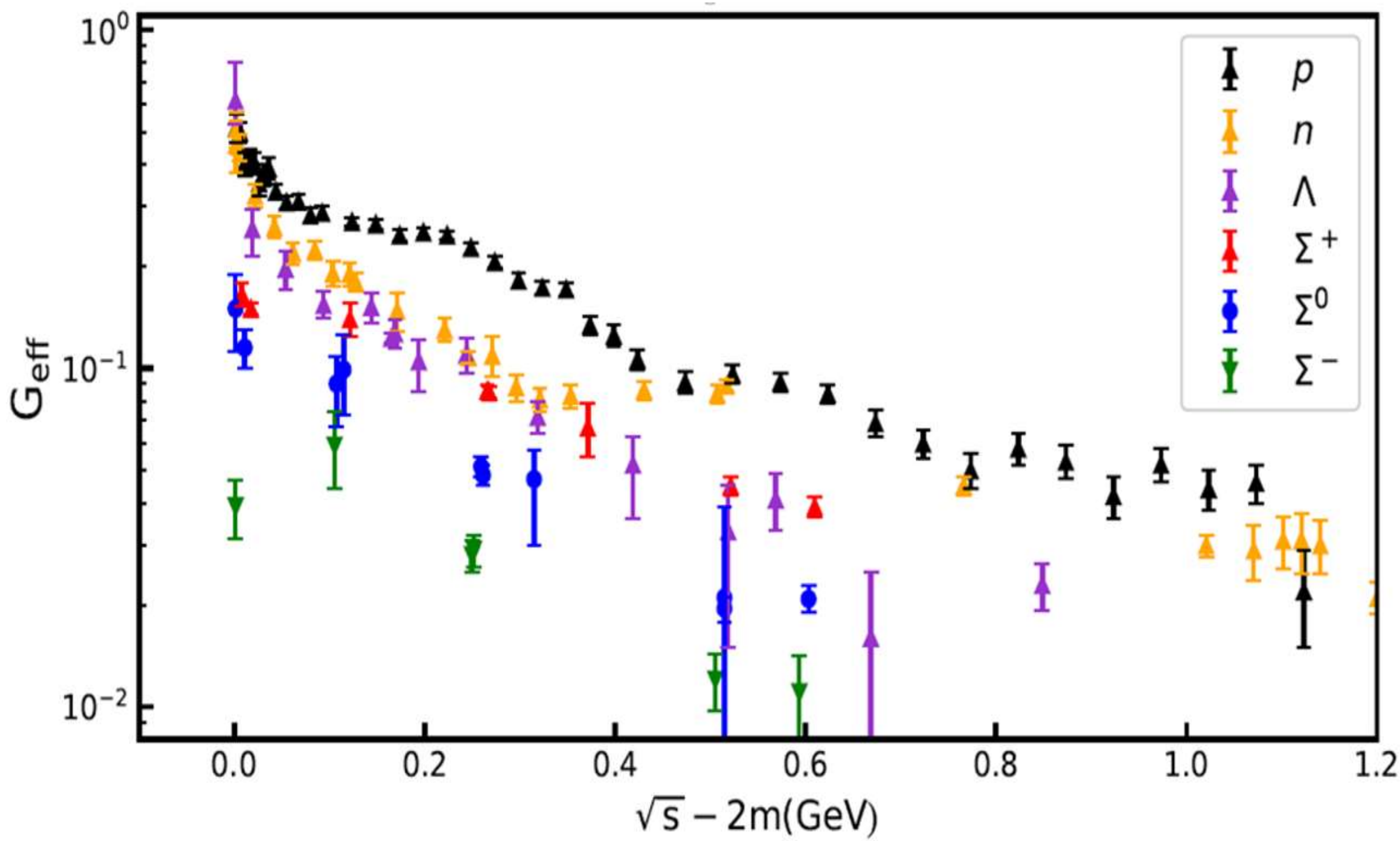
Parameter	Value	Parameter	Value
γ (GeV^{-2})	0.527 ± 0.024	$\alpha_{\omega\phi}$	-3.18 ± 0.77
$\beta_{\omega\phi}$	-0.08 ± 0.06	β_{ρ}	1.63 ± 0.07



With one γ, we can describe all the current experimental data on Σ^{+}, Σ^{-}, and Σ^{0} EMFFs .

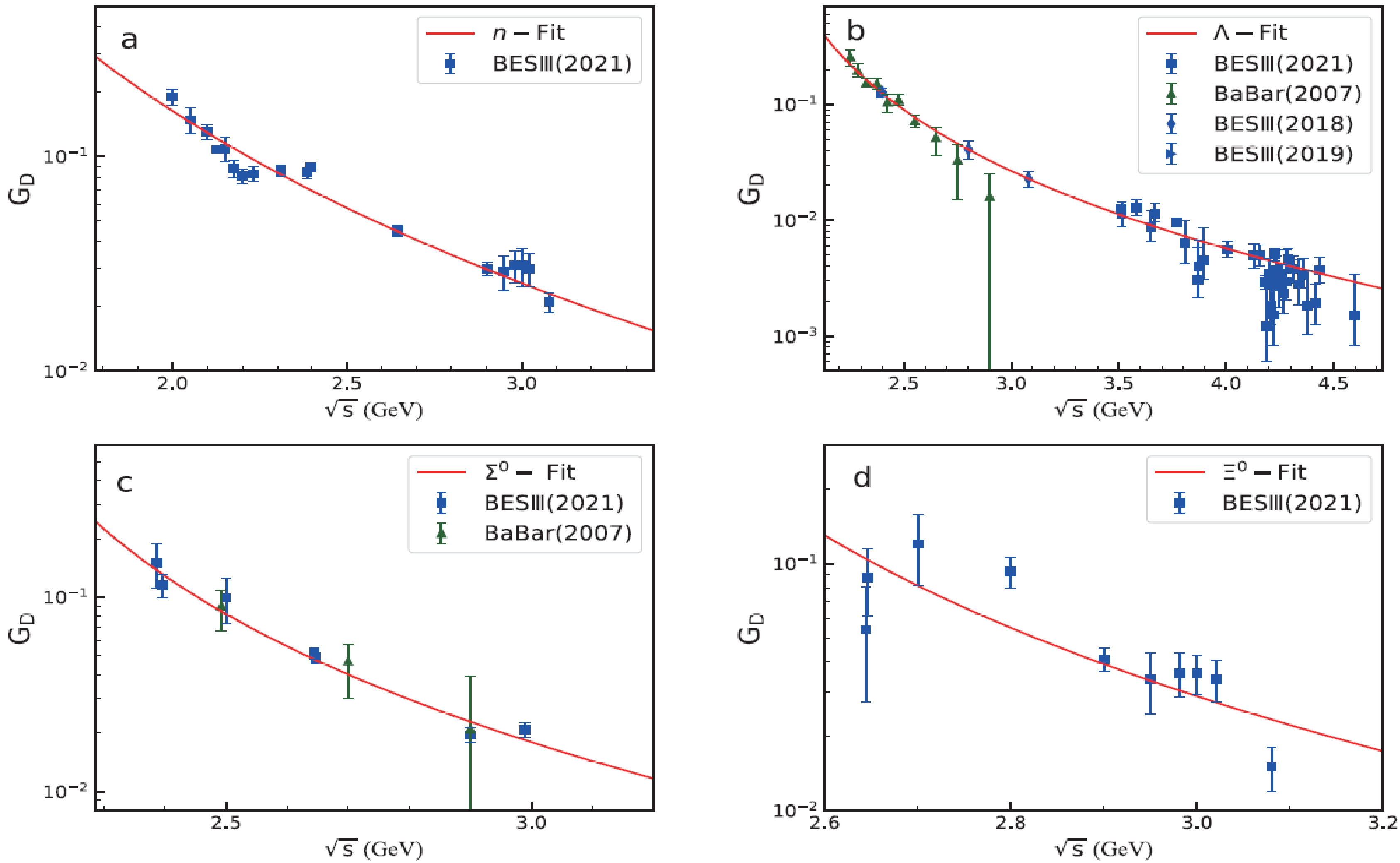
Bing Yan, Cheng Chen, and J. J. Xie, **Phys. Rev. D107, 076008 (2023).**

Dipole behavior of baryon effective form factors



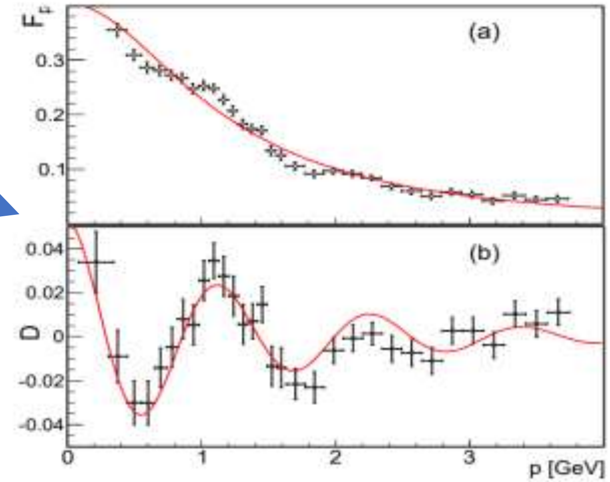
$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

Parameter	n	Λ	Σ^0	Ξ^0
γ	1.41 (fixed)	0.34 ± 0.08	0.26 ± 0.01	0.21 ± 0.02
c_0	3.48 ± 0.06	0.11 ± 0.01	0.033 ± 0.007	0.023 ± 0.008
χ^2/dof	4.3	2.4	1.1	3.0



Oscillation of baryon effective form factors

2015, Andrea Bianconi et al., Phys. Rev. Lett., 2015, 114(23): 232301.

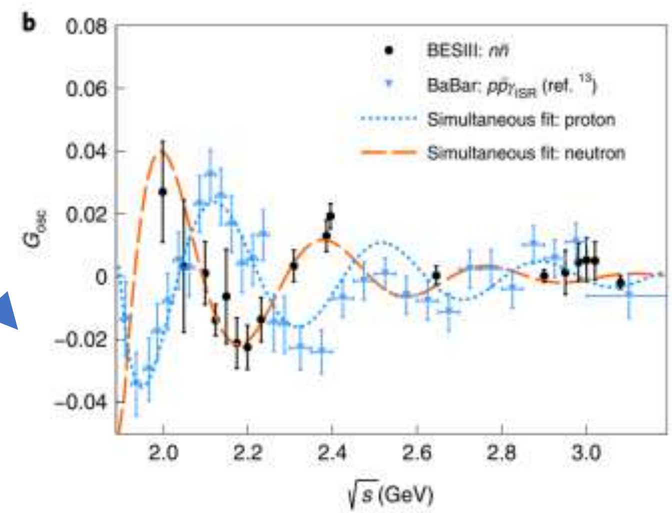


$$F_{3p}(s) = \frac{F_0}{\left(1+\frac{s}{m_a^2}\right)\left(1-\frac{s}{m_0^2}\right)^2},$$

$$F_{osc}(p(s)) = Ae^{-Bp}\cos(Cp+D).$$

2021, BESIII Collaboration,Nature Phys., 2021, 17(11): 1200-1204.

$$F_{osc}^{n,p} = A^{n,p}\exp\left(-B^{n,p}p\right)\cos\left(Cp+D^{n,p}\right)$$



New parametrization

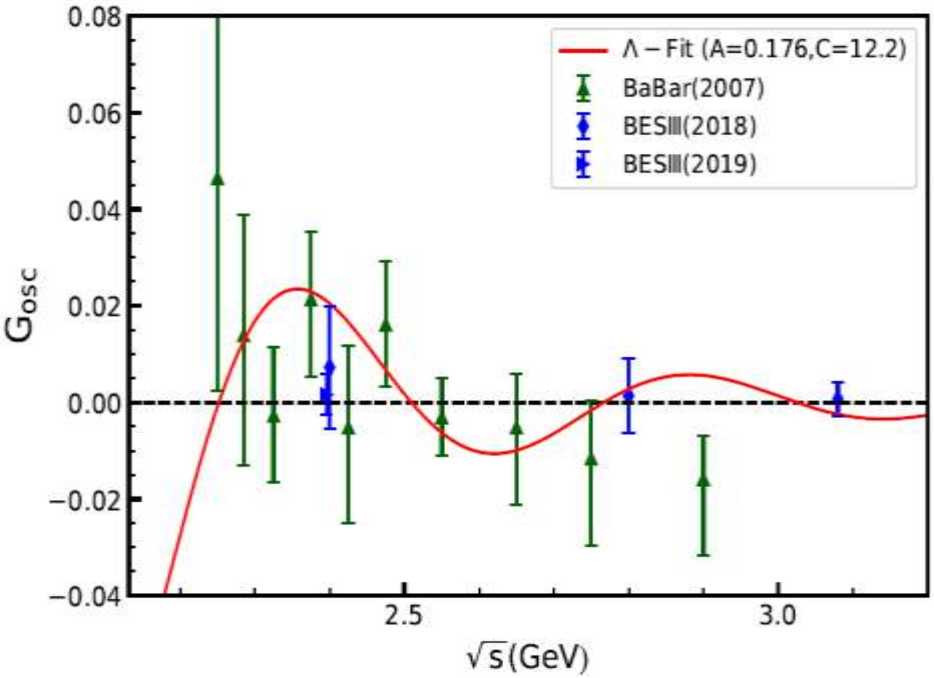
$$G_{osc} = A \cdot \frac{c_0}{(1-\gamma \cdot s)^2} \cdot \cos\left(C \cdot \sqrt{s} + D\right)$$

$$G_D(q^2) = \frac{c_0}{(1 - \gamma q^2)^2}$$

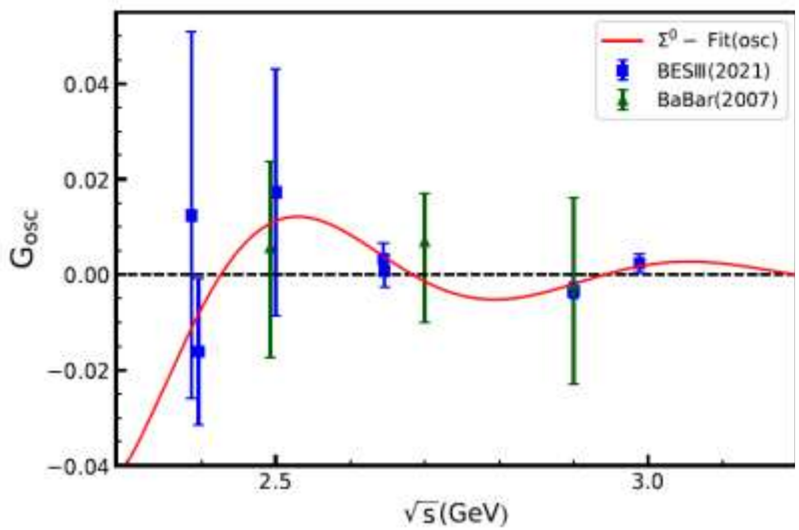
$$\begin{aligned} G_{\mathrm{eff}}(s) &= G_{\mathrm{D}}(s) + G_{\mathrm{osc}}(s) \\ &= \frac{c_0}{(1-\gamma s)^2}\left(1 + A\cos(C\sqrt{s} + D)\right) \end{aligned}$$

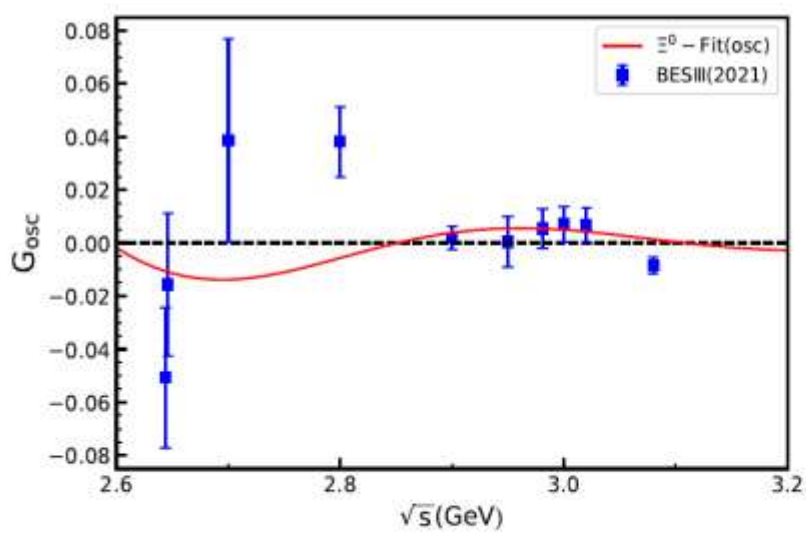
$$data = G_{eff} = G_D + G_{osc}$$

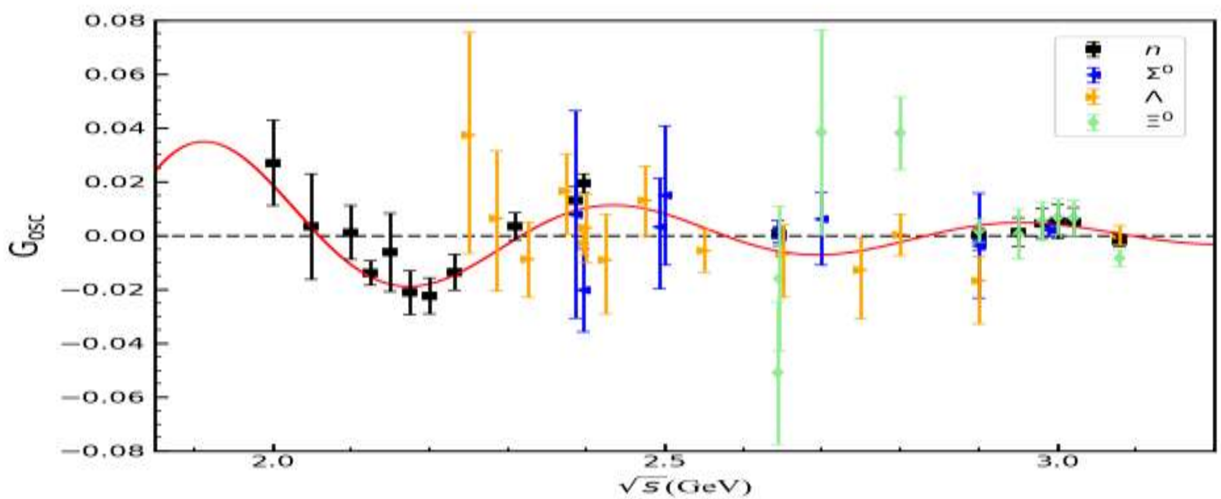
$$\rightarrow \quad G_{osc} = data - G_D$$



Numerical results







A.X. Dai, Z.Y. Li, L. Chang and J.J. Xie, Chin. Phys. C 46, 073104 (2022).

Summary

1、 Threshold enhancement

a), Final state interaction   b), Flatte （strong coupling）

2、 Oscillation of baryon effective form factors

Vector mesons

a), Phenomenology   b), Mechanism unknown

$$g(q^2) = \frac{1}{(1-\gamma q^2)^2}$$

Thank you very much for your attention!

e-Print: 2206.01494 [nucl-th]

New insights into the oscillation of the nucleon electromagnetic form factors

Qin-He Yang[1,2], Ling-Yun Dai[1,2,*] Di Guo[1,2], Johann Haidenbauer[3], Xian-Wei Kang[4,5], and Ulf-G. Meißner[6,3,7]

PHYSICAL REVIEW D **105**, L071503 (2022)

Letter

Timelike nucleon electromagnetic form factors: All about interference of isospin amplitudes

Xu Cao,[1,2,*] Jian-Ping Dai,[3,†] and Horst Lenske[4,‡]

PHYSICAL REVIEW D **107**, L091502 (2023)

Letter

Toy model to understand the oscillatory behavior in timelike nucleon form factors

Ri-Qing Qian,[1,2,3,4,*] Zhan-Wei Liu,[1,2,3,4,†] Xu Cao,[2,3,5,6,‡] and Xiang Liu[1,2,3,4,§]

PHYSICAL REVIEW LETTERS **128**, 052002 (2022)

New Insights into the Nucleon's Electromagnetic Structure

Yong-Hui Lin,[1] Hans-Werner Hammer,[2,3] and Ulf-G. Meißner[1,4,5]