The 7th Symposium on “Symmetries and the emergence of Structure in QCD”

Partial width from the analytical extension of the wave function: P_c states

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OUTLINE

- Wave function on the 2nd Riemann sheet
- Partial width formula
- Three-body unitary cut
- Application to the T_{cc}^{+} and P_c states

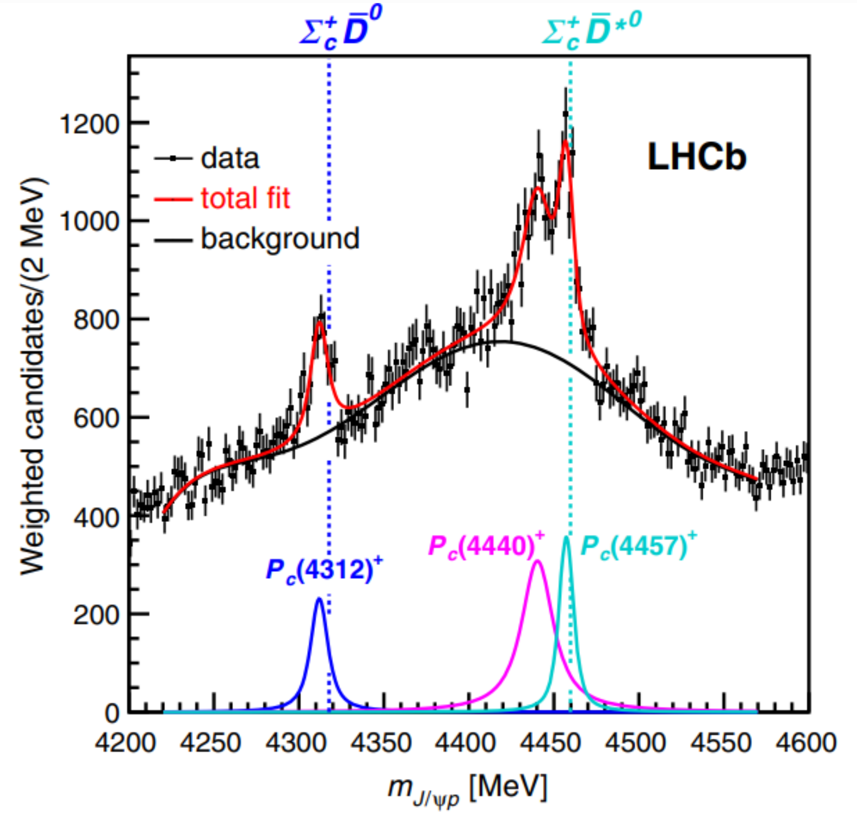
01 Introduction —— P_c states

- Pentaquark candidates $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ discovered by the LHCb Collaboration in the $\Lambda_b^0 \to J/\psi p K^-$ decay

- Molecular states of $\Sigma_c \bar{D}^{(*)}$

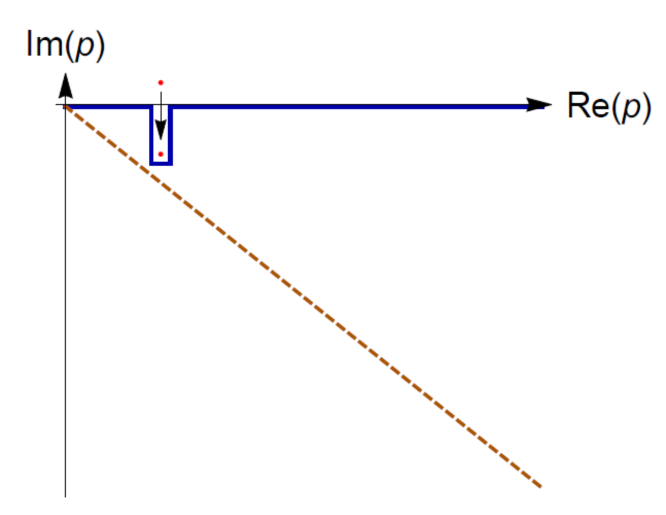
- Coupled to $\Lambda_c \bar{D}^{(*)}$ channels via the one-pion exchange (OPE)

State	M [MeV]	Γ [MeV]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$



R. Aaij *et al*. Phys. Rev. Lett. 122, 222001(2019)

01 Introduction —— Schrödinger equation & LS equation



- Schrödinger equation

$$\frac{\boldsymbol{k}^2}{2m}\phi(\boldsymbol{k}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} V(\boldsymbol{k},\boldsymbol{p})\phi(\boldsymbol{p}) = E\phi(\boldsymbol{k})$$

- Lippmann-Schwinger equation

$$T(\boldsymbol{k}',\boldsymbol{k};E) = V(\boldsymbol{k}',\boldsymbol{k}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{V(\boldsymbol{k}',\boldsymbol{p})T(\boldsymbol{p},\boldsymbol{k};E)}{E-\boldsymbol{p}^2/2\mu+i\epsilon}$$

- Analytical continuity applies for both equations

- How can we learn about the poles of the T matrix from the Schrödinger equation?

02 Formalism —— Schrödinger equation on the 2nd Riemann sheet

- Schrödinger equation $\langle k|\hat{T}+\hat{V}|\phi\rangle = E_R\langle k|\phi\rangle$
- Analytical extension of the wave function

$$\underline{\phi(\boldsymbol{k})} = \frac{1}{E_R - \frac{\boldsymbol{k}^2}{2m}} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} V(\boldsymbol{k},\boldsymbol{p})\underline{\phi(\boldsymbol{p})}$$

k can be anywhere on the complex plane

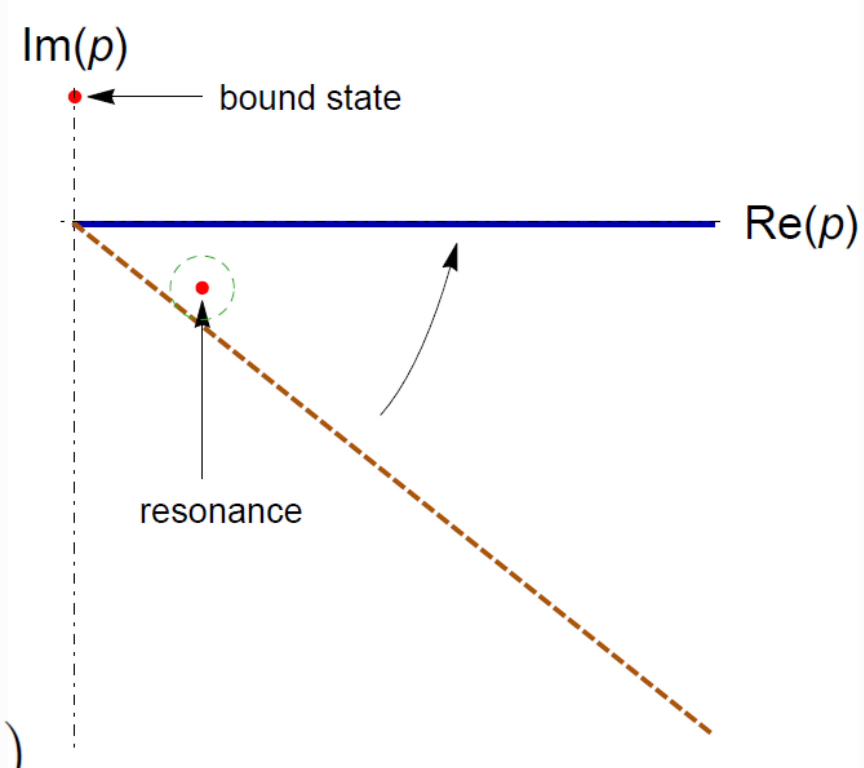
p is on the integral path

- $\phi(\boldsymbol{k})$ has two poles $k = \pm\sqrt{2mE_R}$

→ discontinuity (unitary cut)

02 Formalism —— Schrödinger equation on the 2nd Riemann sheet

- 1st Riemann sheet: integrate along the real axis
- 2nd Riemann sheet: the residue of the pole must be include
- Or change the integral path



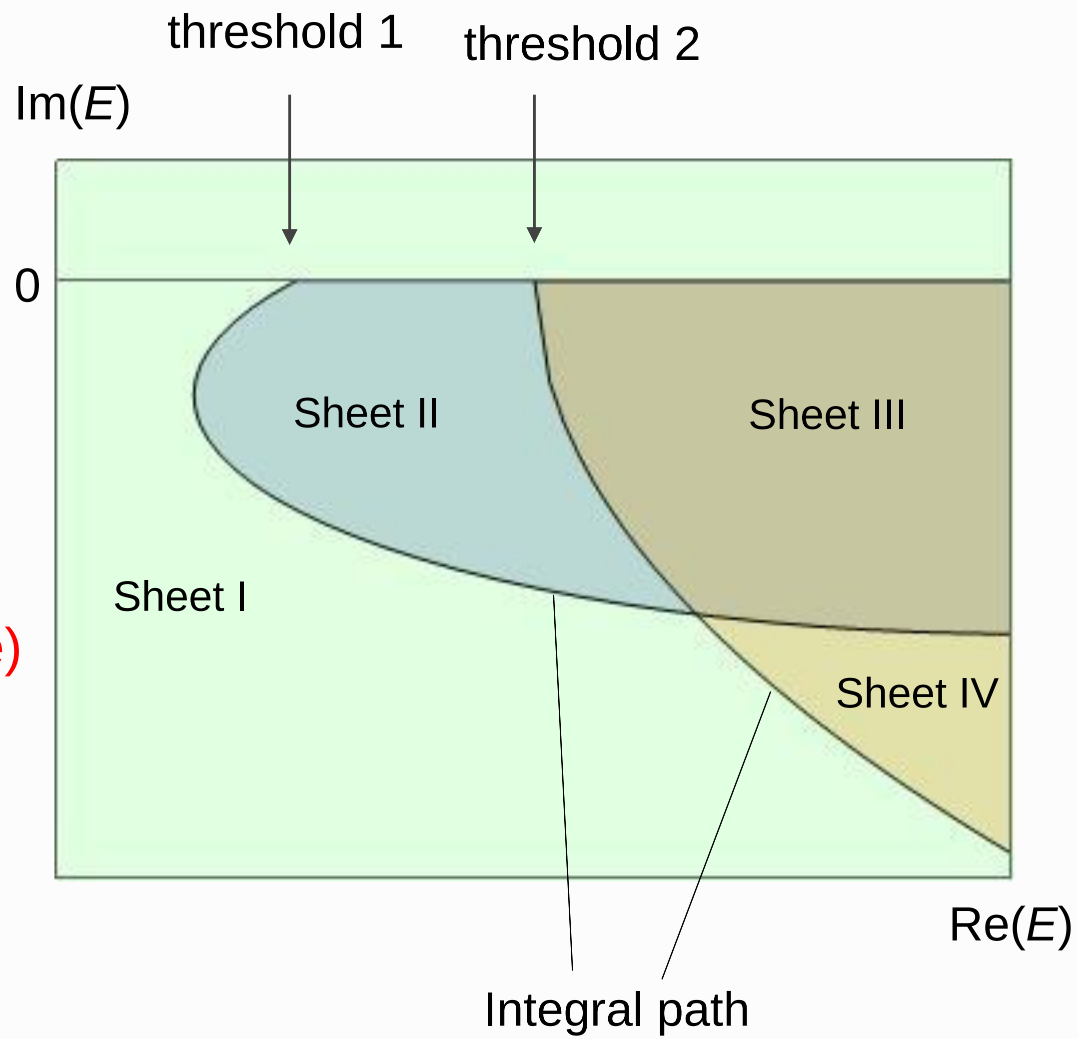
- E.g. complex scaling method:

$$E\tilde{\phi}_l(p) = \frac{p^2 e^{-2i\theta}}{2m}\tilde{\phi}_l(p) + \int \frac{p'^2 e^{-3i\theta} dp'}{(2\pi)^3} V_{l,l'}(pe^{-i\theta}, p'e^{-i\theta})\tilde{\phi}_{l'}(p')$$

- Avoid the branch cut in the potential

02 Formalism —— Schrödinger equation on the 2nd Riemann sheet

- 2-channel example
- $k_1 = \sqrt{2\mu_1(E - E_{th1})}$ $\quad k_2 = \sqrt{2\mu_2(E - E_{th2})}$
- Sheet I: $\mathrm{Im}(k_1) > 0 \quad \mathrm{Im}(k_2) > 0$
  bound state
- Sheet II: $\mathrm{Im}(k_1) < 0 \quad \mathrm{Im}(k_2) < 0$
  quasibound state (Feshbach-type resonance)
- Sheet III: $\mathrm{Im}(k_1) < 0 \quad \mathrm{Im}(k_2) < 0$
  resonance
- Sheet IV: $\mathrm{Im}(k_1) < 0 \quad \mathrm{Im}(k_2) < 0$
  "threshold cusp"



03 Formalism —— partial width

- Integrals concerning the wave function must include the residue
- Partial-wave Fourier transformation (along the real axis)

$$\begin{aligned}\psi_l(r) &= \int_0^\infty \frac{4\pi p^2}{(2\pi)^3}\phi_l(p) i^l j_l(pr)dp + 2\pi i \mathrm{Res}\{\frac{4\pi p^2}{(2\pi)^3}\phi_l(p) i^l j_l(pr)\}\Big|_{p=k_R} \\ &= \underbrace{\int_0^\infty \frac{4\pi p^2}{(2\pi)^3}\phi_l(p) i^l j_l(pr)dp}_{\text{convergent at } r\to\infty} + i^{l+1}\frac{k_R^2}{\pi}\underbrace{j_l(k_R r)}_{\text{divergent at } r\to\infty}\boxed{\lim_{p\to k_R}(p-k_R)\phi_l(p)}\end{aligned}$$

- j_l stands for the l-th spherical Bessel function
- Partial widths are related to the residues of the wave function

04 Formalism —— chiral effective field theory

- Leading-order pion-exchange and contact terms in

$$\mathcal{L}_{H\phi}^{(1)} = -\langle (iv\cdot\partial H)\bar{H}\rangle + \langle Hv\cdot\Gamma\bar{H}\rangle + g\langle H\not{u}\gamma_5\bar{H}\rangle - \tfrac{1}{8}\delta\langle H\sigma^{\mu\nu}\bar{H}\sigma_{\mu\nu}\rangle$$

$$H = \frac{1+\not{v}}{2}(P_\mu^*\gamma^\mu + iP\gamma_5), \quad P = (D^0, D^+), \quad P_\mu^* = (D^{*0}, D^{*+})$$

$$\mathcal{L}_{\mathcal{B}\phi} = -\mathrm{Tr}(\bar{\psi}^\mu iv\cdot D\psi^\mu) + \frac{i\delta_a}{2}\mathrm{Tr}(\bar{\psi}^\mu\sigma_{\mu\nu}\psi^\nu) + ig_1\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left(\bar{\psi}^\mu u^\rho v^\sigma\psi_\nu\right) + g_2\mathrm{Tr}\left(\bar{\psi}^\mu u_\mu\mathcal{B}_1 + \mathrm{h.c.}\right)$$

$$\psi_1 = \begin{bmatrix} 0 & \Lambda_c^+ \\ -\Lambda_c^+ & 0 \end{bmatrix}, \quad \psi_3 = \begin{bmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 \end{bmatrix}, \quad \psi_{3^*}^\mu = \begin{bmatrix} \Sigma_c^{*++} & \frac{\Sigma_c^{*+}}{\sqrt{2}} \\ \frac{\Sigma_c^{*+}}{\sqrt{2}} & \Sigma_c^{*0} \end{bmatrix}^\mu$$

$$\psi^\mu = \mathcal{B}_{3^*}^\mu - \frac{1}{\sqrt{3}}\left(\gamma^\mu + v^\mu\right)\gamma^5\mathcal{B}_3$$

$$\mathcal{B}_i = e^{iM_i v\cdot x}\frac{1+\not{v}}{2}\psi_i$$

- large mass difference

$$m_{D^*} - m_D > m_\pi$$

$$m_{\Sigma_c} - m_{\Lambda_c} > m_\pi$$

$\Rightarrow$ pion can be on-shell

04 Formalism —— three-body effect in OPE (T_{cc}^{+})

- Optical theorem: on-shell $DD\pi$ intermediate states

  ⇨ negative imaginary part of the potential

- Instantaneous approximation

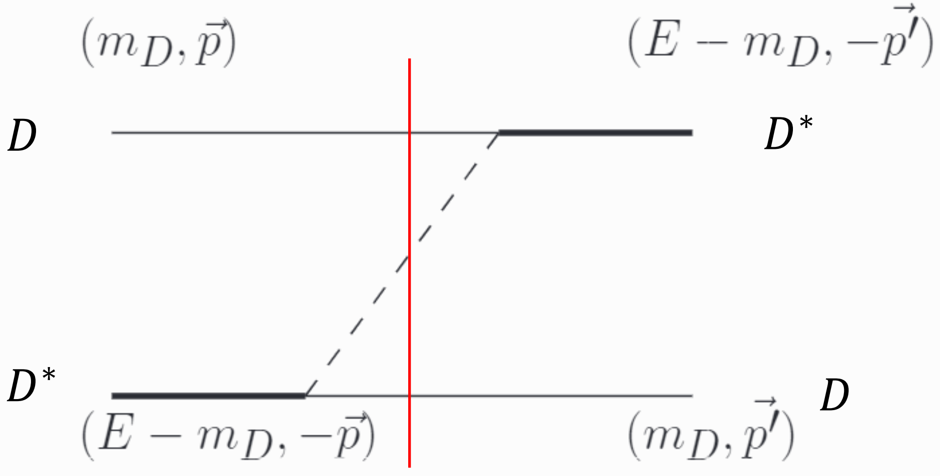
$$\frac{i}{q^2 - m_\pi^2 + i\epsilon} \longrightarrow -\frac{i}{\boldsymbol{q}^2 + m_\pi^2 - i\epsilon}$$

- Nonzero q_0 ($\delta = m_{D^*} - m_D$)

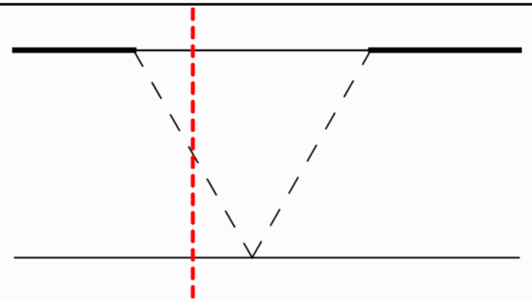
$$\frac{i}{q^2 - m_\pi^2 + i\epsilon} \longrightarrow -\frac{i}{\boldsymbol{q}^2 - (\delta^2 - m_\pi^2) - i\epsilon}$$

- Three-body effect: energy-dependent potential

$$\frac{i}{q^2 - m_\pi^2 + i\epsilon} \longrightarrow \frac{i}{(E+\delta)^2 - \boldsymbol{q}^2 - m_\pi^2 + i\epsilon}$$



$DD\pi$



In TPE, an imaginary part shows up naturally in the loop integral.

05 Results —— T_{cc}^{+} and X(3872) states （DD^*）

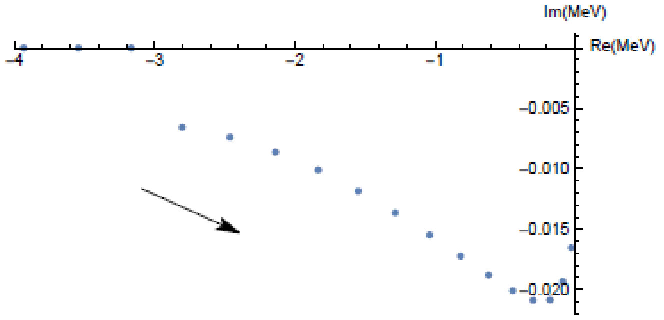


Figure 5. Bound states in the $I = 0$ S-wave DD^* system with the OPE and contact terms only. When we shift C_s from -35 GeV^{-2} to -19 GeV^{-2}, the pole moves in the positive real axis direction. With the parameters chosen, the $DD\pi$ threshold is located at -3 MeV. The cutoff Λ is fixed to be 0.5 GeV.

- The width goes to zero when the mass is lower than the $DD\pi$ threshold
- Two-pion-exchange potential may break the unitarity

- The width of X(3872) is of the order of 10 keV
- Annihilation processes can be included by introducing imaginary contact terms

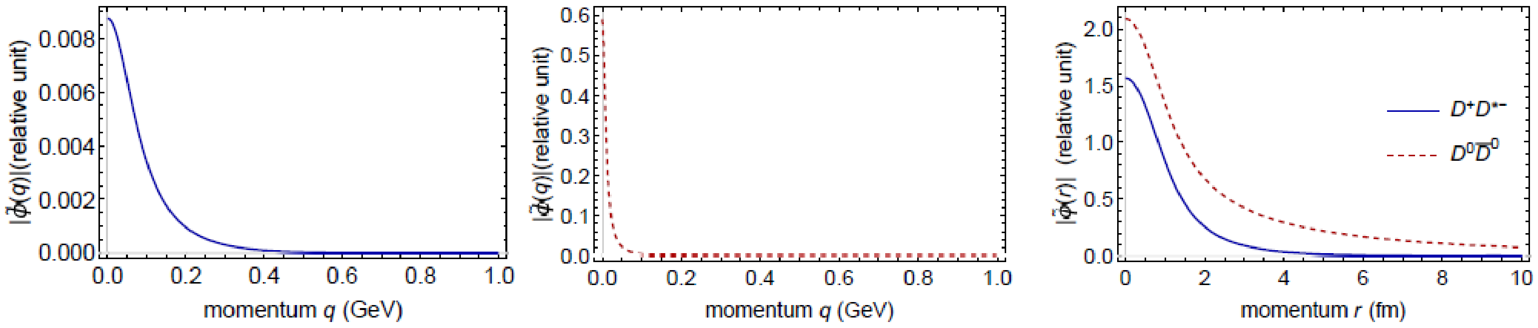


Figure 6. The complex scaled wavefunction $\tilde{\phi}_l(\boldsymbol{q}) = \phi_l(\boldsymbol{q}e^{-i\theta})$ solved in Eq. (2.10). The first two graphs stand for the $D^0\bar{D}^{*0}$ (red dashed curve), $D^{\pm}\bar{D}^{*\mp}$ (blue solid curve) channels, respectively. The third graph shows the complex scaled wavefunction $\tilde{\phi}_l(\boldsymbol{r}) = \phi_l(\boldsymbol{r}e^{-i\theta})$ in coordinate space.

ZYL, J.–B. Cheng, S.- L. Zhu, 2305.19073

05 Results —— P_c states $(\Sigma_c\bar{D}^{(*)})$

$\frac{1}{2}^-$	$P_c(4312)$	$P_c(4440)$	$P_c(4504)$
M	$4309.4^{+2.7}_{-2.5}$	$4443.5^{+3.7}_{-3.5}$	$4504.0^{+6.1}_{-4.7}$
Γ	$7.8^{+6.6}_{-6.6}$	$3.1^{+0.8}_{-1.4}$	$1.5^{+0.4}_{-1.4}$
$M_{\rm exp}$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	
$\Gamma_{\rm exp}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	
$\sqrt{\langle\phi_i\|r^2\|\phi_i\rangle}$	$0.63 - 0.11i^{+0.07+0.09i}_{-0.07-0.09i}$	$0.60 - 0.01i^{+0.03+0.01i}_{-0.01-0.00i}$	$0.58 + 0.00i^{+0.03+0.00i}_{-0.01i-0.01i}$
$\Lambda_c\bar{D}$	$\boxed{0.04^{+0.01}_{-0.02}}$	$10.8^{+8.0}_{-2.7}$	$8.7^{+7.0}_{-6.6}$
$\Lambda_c\bar{D}^*$	$99.96^{+0.02}_{-0.01}$	$38.4^{+24.9}_{-30.6}$	$24.6^{+17.1}_{-18.3}$
$\Sigma_c\bar{D}$	–	$50.9^{+38.6}_{-27.4}$	$31.6^{+16.2}_{-14.4}$
$\Sigma_c\bar{D}^*$	–	–	$35.2^{+8.7}_{-9.7}$
$\Sigma_c^*\bar{D}^*$	–	–	–
$\frac{3}{2}^-$	$P_c(4380)$	$P_c(4457)$	$P_c(4516)$
M	$4377.9^{+2.3}_{-3.0}$	$4458.6^{+1.4}_{-2.5}$	$4516.0^{+2.1}_{-2.5}$
Γ	$3.2^{+1.7}_{-3.1}$	$1.0^{+0.3}_{-0.4}$	$3.2^{+1.4}_{-1.7}$
$M_{\rm exp}$		$4457.3 \pm 0.6^{+4.1}_{-1.7}$	
$\Gamma_{\rm exp}$		$6.4 \pm 2.0^{+5.7}_{-1.9}$	
$\sqrt{\langle\phi_i\|r^2\|\phi_i\rangle}$	$0.74 - 0.03i^{+0.06+0.02i}_{-0.06-0.02i}$	$0.84 + 0.01i^{+0.08+0.01i}_{-0.08-0.01i}$	$0.67 - 0.01i^{+0.03+0.01i}_{-0.02-0.01i}$
$\Lambda_c\bar{D}^*$	100	$26.9^{+30.0}_{-22.5}$	$18.1^{+23.7}_{-14.5}$
$\Sigma_c^*\bar{D}$	–	$73.1^{+22.5}_{-30.0}$	$45.6^{+7.9}_{-13.1}$
$\Sigma_c\bar{D}^*$	–	–	$36.2^{+6.6}_{-10.6}$
$\Sigma_c^*\bar{D}^*$	–	–	–

- Masses and widths of the $P_c(4312)$, $P_c(4440)$, $P_c(4457)$ used as inputs
- J^P assigned: $P_c(4312)$, $P_c(4440)$: $\frac{1}{2}^-$ $P_c(4457)$: $\frac{3}{2}^-$

ZYL, J.–B. Cheng, B.-L, Huang, S.- L. Zhu, 2305.19073

06

Summary

- The Schrödinger equation and the wave function are extended to the 2nd Riemann sheet

  The Schrödinger equation is good at pole search, while the LS equation is good at line shape.

- The partial width (residue of T matrix) is related to the residue of the wave function

- Three-body effects in the OPE potential

  T_{cc}^{+} : important P_c: not significant

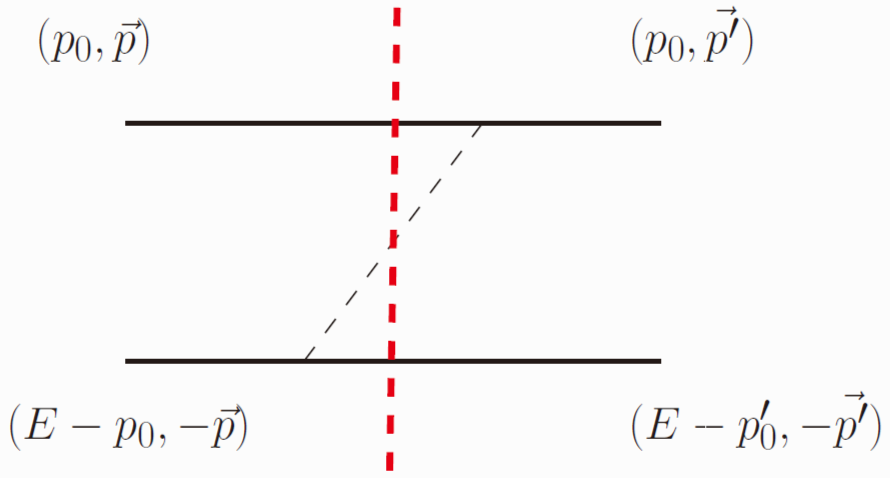
- Branching ratios of P_c states are obtained

Thank you for your attention!

Backups —— three-body effect in OPE

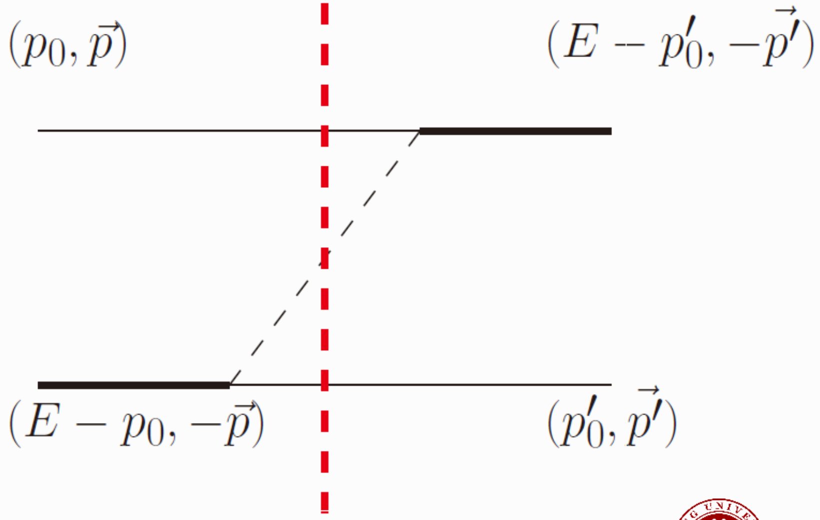
- Different choice of q_0 in the denominator

$$\frac{i}{q_0^2 - \boldsymbol{q}^2 - m_\pi^2}$$



- Direct diagram (P_c)

$$q_0 = p_0 - p'_0$$



- Cross diagram (T_{cc}^{+})

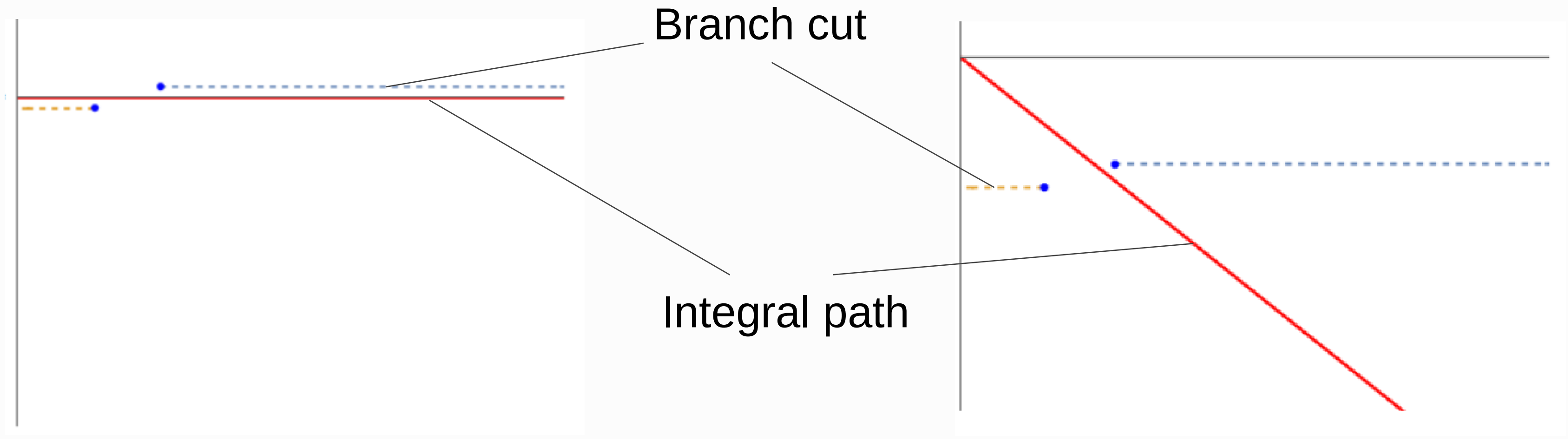
$$q_0 = E - p_0 - p'_0$$

Backups —— singularity of OPE

- Non-analytical term in S-wave OPE

$$T(\boldsymbol{k}',\boldsymbol{k};E)=V(\boldsymbol{k}',\boldsymbol{k})+\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{V(\boldsymbol{k}',\boldsymbol{p})T(\boldsymbol{p},\boldsymbol{k};E)}{E-\boldsymbol{p}^2/2\mu+i\epsilon}$$

$$V_{OPE}\propto\ln\frac{(p+p')^2-m_{eff}^2-i\epsilon}{(p-p')^2-m_{eff}^2-i\epsilon}\qquad m_{eff}^2=q_0^2-m_\pi^2$$



Im(E)=0

Im(E)<0