Doubly-heavy tetraquark bound states in quark potential models

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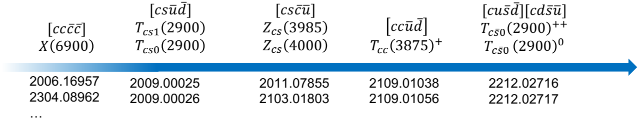
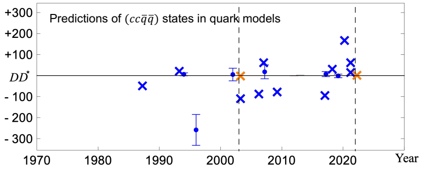
Background

Recently, more and more hadrons composed of at least four quarks were observed



Different quark models predicted different results[Chen:2022asf]



Reasons for variations: Interactions + few-body methods

- Benchmark calculations : (AL1,AP1,SLM) $\otimes$ (GEM,RGM,DMC)

DMC: *Baryons and Tetraquark States with Diffusion Monte Carlo Method*, Y. Ma.

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Quark potential models

- Cornell model: One-gluon-exchange+Confinement

$$V_{ij}(r) = \left[\frac{\alpha_s}{r} + \left(-\frac{3b}{4}r + V_c\right) - \frac{8\pi\alpha_s}{3m_i m_j}\frac{\tau^3}{\pi^{3/2}}e^{-\tau^2 r^2}\boldsymbol{s}_i\cdot\boldsymbol{s}_j\right]\frac{\lambda_i\cdot\lambda_j}{4}$$

- Semay-Silvestre-Brac Models [Silvestre-Brac:1996myf]

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j}\kappa' \frac{1}{\pi^{3/2} r_0^3} e^{\left(-r^2/r_0^2\right)}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right]\lambda_i\cdot\lambda_j$$

AL1: $p = 1$ and AP1: $p = 2/3$

- Chiral quark models (e.g Salamanca model (SLM)) [Gonzalez:2012gka]

$$V_{ij}(r) = \left[\frac{\alpha_s}{4}\left(\frac{1}{r} - \frac{1}{6m_i m_j}\frac{e^{-r/r_0}}{r_0^2 r}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\right) + \left(-a_c\left(1 - e^{-\mu_c r}\right) + \Delta\right)\right]\lambda_i\cdot\lambda_j$$
$$+V_\pi + V_K + V_\eta + V_\sigma$$

- In this work, we use AL1, AP1 and SLM

Tetraquark systems

Over 150 states: $q = u, d, s,\ \ Q = b, c$

- Fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}$)
- Triply heavy tetraquark states ($QQ\bar{Q}\bar{q}$)
- Doubly heavy tetraquarks states ($QQ\bar{q}\bar{q}$)
- Single heavy strange states ($Qs\bar{q}\bar{q}$, $Q\bar{s}q\bar{q}$)
- Only S-wave are considered : $J^P = (0, 1, 2)^+$

In this work, we only focus on bound states

	$QQ\bar{Q}\bar{Q}$	$QQ\bar{Q}\bar{q}$	$QQ\bar{q}\bar{q}$	$Qs\bar{q}\bar{q}$	$Q\bar{s}q\bar{q}$
$J^P = 0^+$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$
$J^P = 1^+$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$
$J^P = 2^+$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$

Masses are shifted to align the theoretical thresholds with the physical ones.

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$J^P = 0^+$	×	×	✓	✓	×
$J^P = 1^+$	×	×	✓	✓	×
$J^P = 2^+$	×	×	✓	✓	×

Masses are shifted to align the theoretical thresholds with the physical ones.

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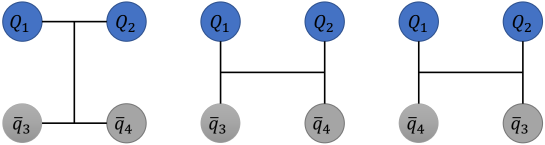
Gaussian Expansion Method

- Color-spin wave functions (richer color-structure in multiquark systems)

$$\left[\{Q_1 Q_2\}^{S_{12}}_{\bar{3}_c}\ \{\bar{q}_3\bar{q}_4\}^{S_{34}}_{3_c}\right]^{S_{tot}}_{1_c},\quad \left[\{Q_1 Q_2\}^{S_{12}}_{6_c}\ \{\bar{q}_3\bar{q}_4\}^{S_{34}}_{\bar{6}_c}\right]^{S_{tot}}_{1_c}$$

$$\left[\{Q_1\bar{q}_3\}^{S_{13}}_{1_c}\ \{Q_2\bar{q}_4\}^{S_{24}}_{1_c}\right]^{S_{tot}}_{1_c},\quad \left[\{Q_1\bar{q}_3\}^{S_{13}}_{8_c}\ \{Q_2\bar{q}_4\}^{S_{24}}_{8_c}\right]^{S_{tot}}_{1_c}$$

- Spatial wave functions : diquark-antidiquark and di-meson correlations



$$\phi_{nlm}(\boldsymbol{r}) = N_{lm} r^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r}),\quad r_n = r_0 a^{n-1}\quad \text{[Hiyama:2003cu]}$$

Embed both long- and short-range correlations

- Antisymmetrization (e.a. $Q_1 = Q_2$ and $q_3 = q_4$) :

$$\psi = \mathcal{A}\left[\psi_{\text{color}} \otimes \psi_{\text{spin}} \otimes \psi_{\text{spatial}} \otimes \psi_{\text{flavor}}\right],\quad \mathcal{A} = (1 - P_{12})(1 - P_{34})$$

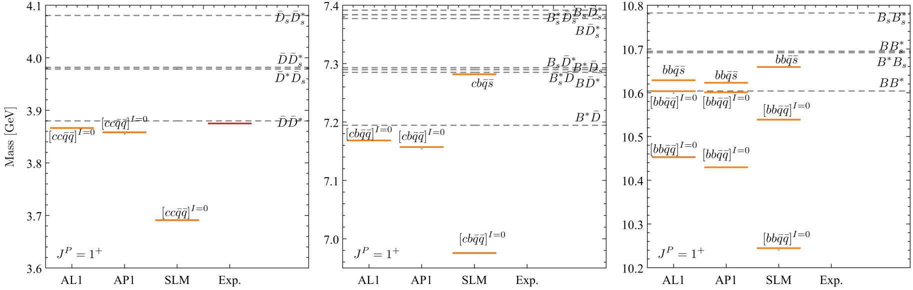
$QQ\bar{q}\bar{q}$ systems with $J^P = 1^+$

Points of agreement:

- $[QQ\bar{n}\bar{n}]^{I=0}$($QQ = cc$ or bb or bc) bound states; $[bb\bar{n}\bar{s}]$ bound states .
- For $[bb\bar{n}\bar{n}]^{I=0}$ systems, the 1st excited states are bound states.
- No $[QQ\bar{n}\bar{n}]^{I=1}$ bound states.

SLM:

- $[cc\bar{n}\bar{n}]^{I=0}$ are too deep compared with ex. (200MeV VS 200keV)
- $[cb\bar{n}\bar{s}]$ bound state



$QQ\bar{q}\bar{q}$ systems with $J^P = 0^+, 2^+$

Points of agreement:

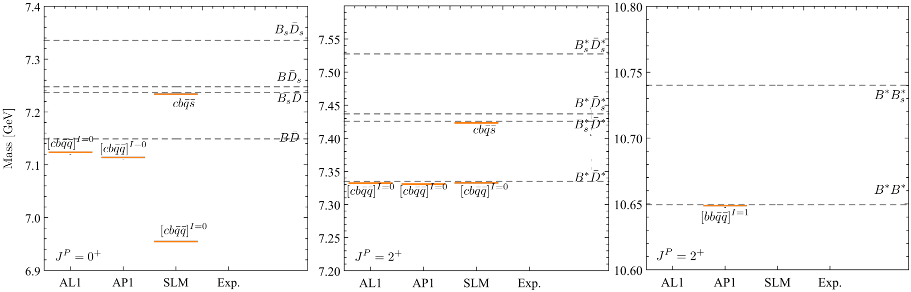
- $[cb\bar{n}\bar{n}]^{I=0}$ bound states for $J^P = 0^+, 2^+$

SLM:

- $[cb\bar{n}\bar{s}]$ bound state for $J^P = 0^+, 2^+$

AP1:

- $[bb\bar{n}\bar{n}]^{I=1}$ bound state for $J^P = 2^+$



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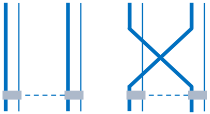
Resonating Group Method

Dimeson-wave function

$$\psi_{AB}(\boldsymbol{P}) = \mathcal{A}\left[\phi_A\left(\boldsymbol{p}_A\right)\phi_B\left(\boldsymbol{p}_B\right)\chi(\boldsymbol{P})\chi_{AB}^{CST}\right]$$



- ϕ_A and ϕ_B are meson wave functions
- We use GEM to get the meson wave functions

Schrödinger equation of RGM [Entem:2000mq, Ortega:2022efc]

$$\left(\frac{\boldsymbol{P}'^2}{2\mu} - E\right)\chi\left(\boldsymbol{P}'\right) + \int d^3\boldsymbol{P}\left(V_D\left(\boldsymbol{P}', \boldsymbol{P}\right) + K_{Ex}\left(\boldsymbol{P}', \boldsymbol{P}\right)\right)\chi(\boldsymbol{P}) = 0$$

- V_D direct interaction, K_{Ex} the exchange kernel

Compared with GEM

- The spin-color-flavor wave functions are complete as well
- The RGM neglecting the distortion of the meson wave functions in the tetraquark system
- Only the di-meson-type spatial correlations are included
- The trial functions are not as general as GEM : $E_{RGM} \gtrsim E_{GEM}$

Results from RGM

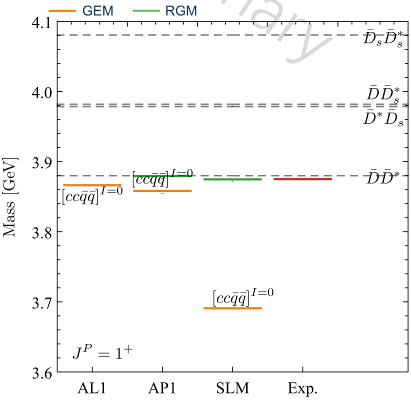
The RGM gives the smaller binding energies.

The RGM results agree with the GEM neglecting diquark-antidiquark correlation

- Not general enough trail wave function
- Cannot get the ground state accurately
- A drawback as a few-body method

However...

- Some quark models (e.g. SLM) constraining the para. using NN phase shifts with RGM
- The spatial correlations other than di-hadron types are neglected from birth
- Perhaps, it is misleading to use diquark-antidiquark type trial functions for these models.[Entem:2000mq]



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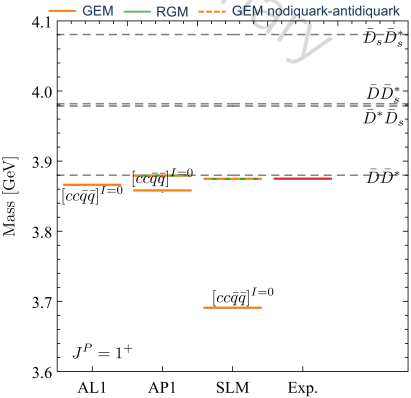


Summary and outlook

Investigate the T_{QQ} tetraquark bound states with (AL1,AP1,SLM) $\otimes$ (GEM,RGM)

- $(QQ\bar{Q}\bar{Q}), (QQ\bar{Q}\bar{q}), (QQ\bar{q}\bar{q}), (Qs\bar{q}\bar{q}), \qquad q = u, d, s$

Recommended tetraquark bound states (consistent predictions of 3 models)

- $J^P = 1^+ : [cc\bar{n}\bar{n}]^{I=0}, [bb\bar{n}\bar{n}]^{I=0}, [bc\bar{n}\bar{n}]^{I=0}, [bb\bar{n}\bar{s}], \quad [bs\bar{n}\bar{n}]^{I=0}$
- $J^P = 0^+ : [cb\bar{n}\bar{n}]^{I=0}, [cs\bar{n}\bar{n}]^{I=0}, [bs\bar{n}\bar{n}]^{I=0}$
- $J^P = 2^+ : [cb\bar{n}\bar{n}]^{I=0}$

The trial functions of RGM are not general enough to give the ground state

- For quark models born with RGM, it is inconsistent to include diquark-antidiquark correlations

Outlook:

- Resonances and virtual states (on-going)

Thanks for your attention

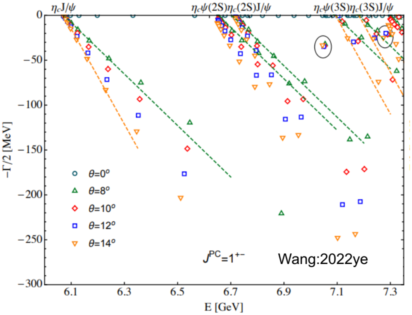
Methods to obtain resonance and virtual states

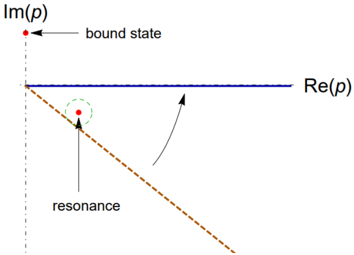
Complex scaling methods with GEM

- It is hard to detect the higher states
- The unclear relation with Riemann surface
- The tetraquark resonance: two-body scattering problems (confinement)

RGM + Complex Scaling in coupled-channel two-body problem

- Solving Freedholm determinant ⇒ Eigenvalue problem





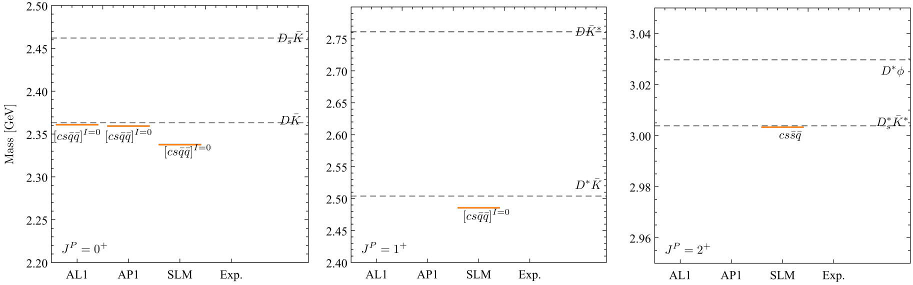
$cs\bar{q}\bar{q}$ systems with $J^P = 0^+, 1^+, 2^+$

Points of agreement

- $[cs\bar{n}\bar{n}]^{I=0}$ bound states for $J^P = 0^+$

SLM:

- $[cs\bar{n}\bar{n}]^{I=0}$ for $J^P = 1^+$and $[cs\bar{s}\bar{n}]$ for $J^P = 2^+$ bound states



$bs\bar{q}\bar{q}$ systems with $J^P = 0^+, 1^+, 2^+$

Points of agreement

- $[bs\bar{n}\bar{n}]^{I=0}$ bound states for $J^P = 0^+, 1^+$

SLM:

- $[bs\bar{n}\bar{n}]^{I=0}$ and $[bs\bar{s}\bar{n}]$ for $J^P = 2^+$bound states