The 7th Symposium on "Symmetries and the emergence of Structure in QCD"

Chromopolarizability of fully heavy hadrons

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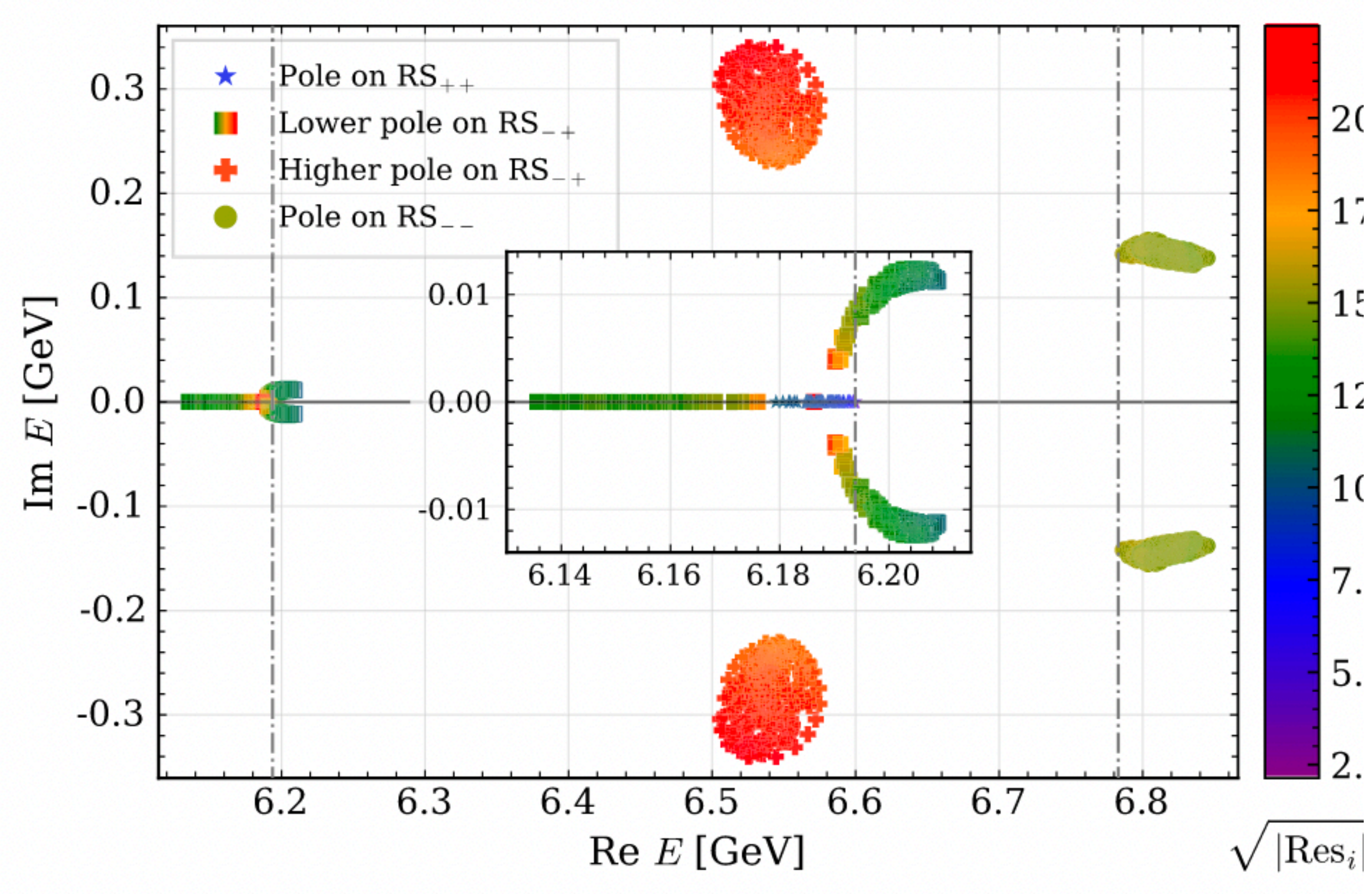
XKD, F.K. Guo, A. Nevediev & J. Tarrús-Castellà, *Phys.Rev.D* 107 (2023) 3, 034020

Bound states of fully-heavy hadrons

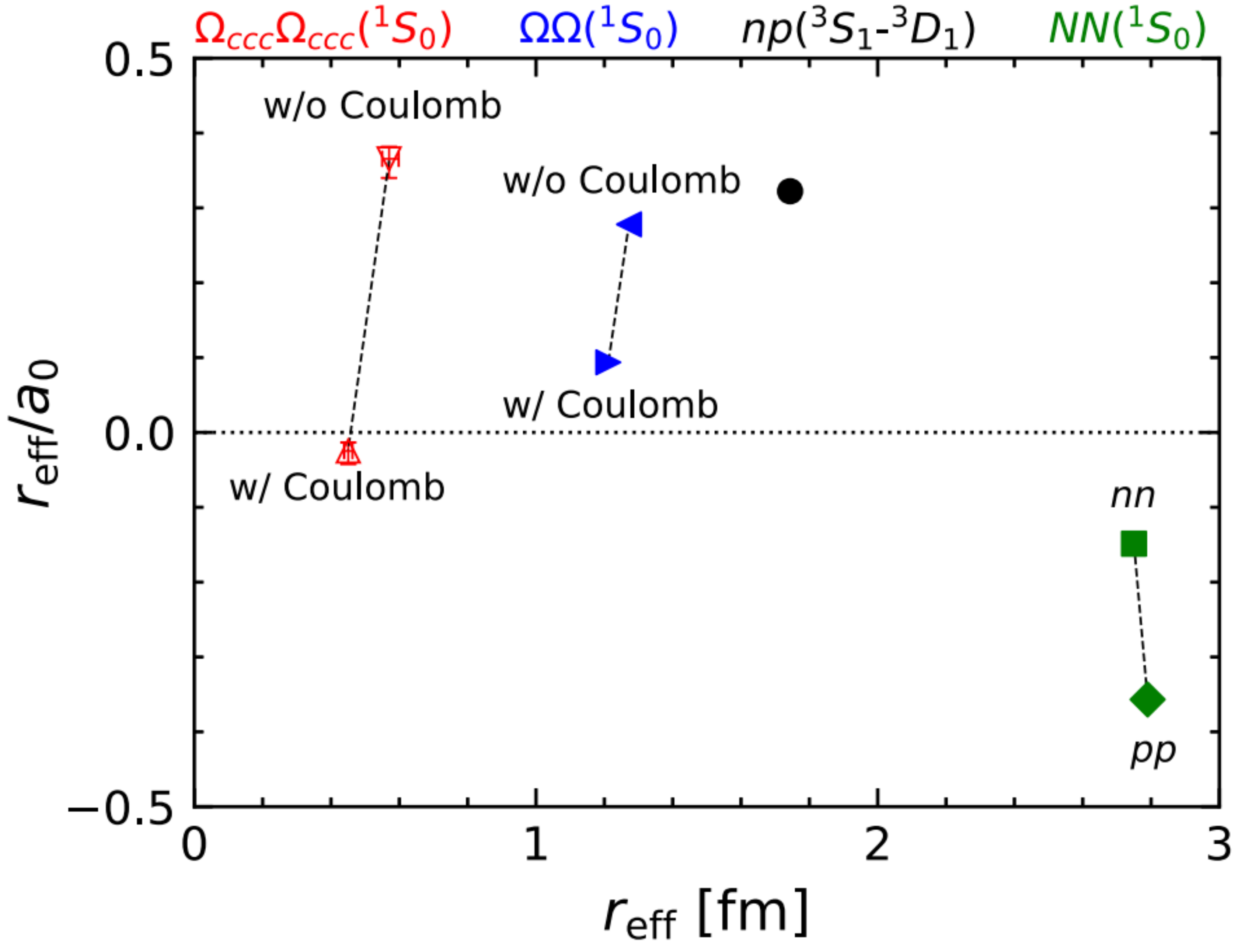


$J/\psi J/\psi$

X.-K. Dong, et al.,
Phys.Rev.Lett. 126 (2021) 13, 132001

LHCb, Sci.Bull. 65 (2020) 23, 1983-1993

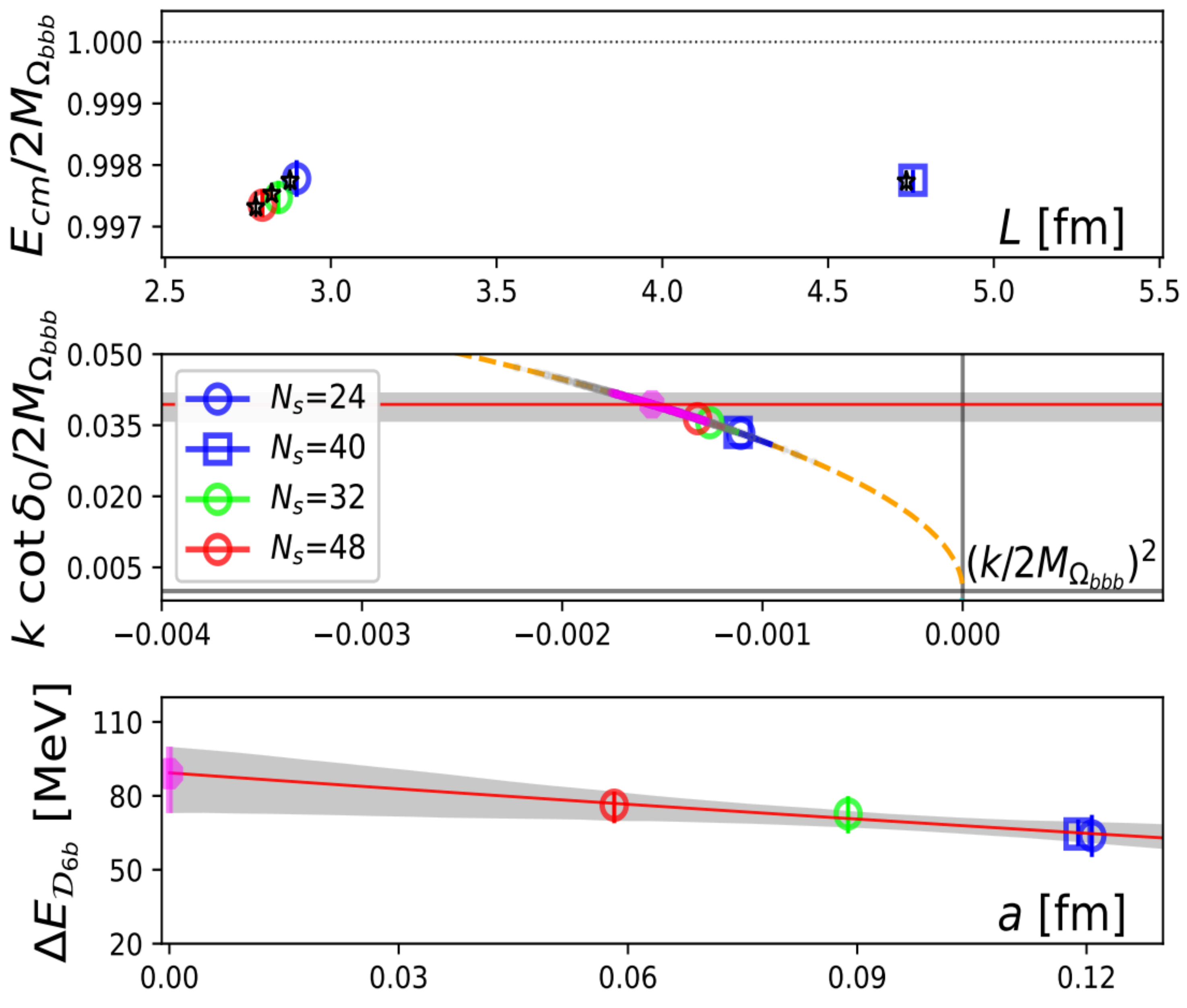
Virtual or bound



$\Omega_{ccc}\Omega_{ccc}$

Y. Lyu, et al.,
Phys.Rev.Lett. 127 (2021) 7, 072003

-5.7 MeV



$\Omega_{bbb}\Omega_{bbb}$

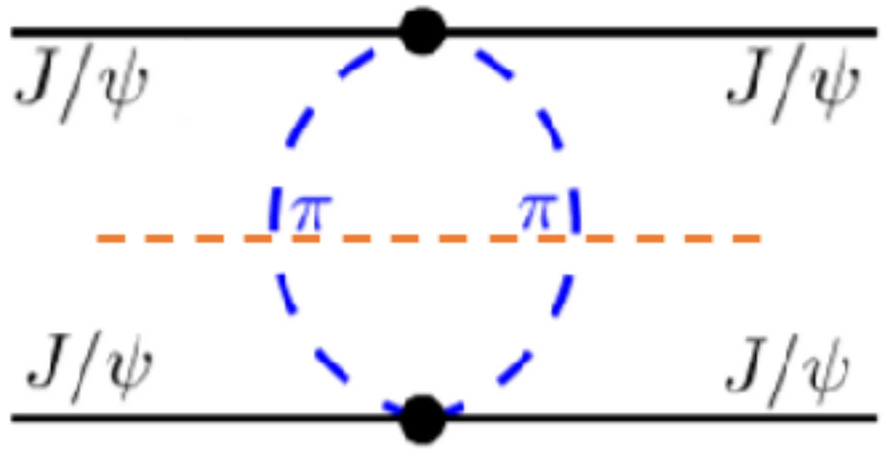
N. Mathur, et al.,
Phys.Rev.Lett. 130 (2023) 11, 111901

-89^{+16}_{-12} MeV

Interactions of fully-heavy hadrons

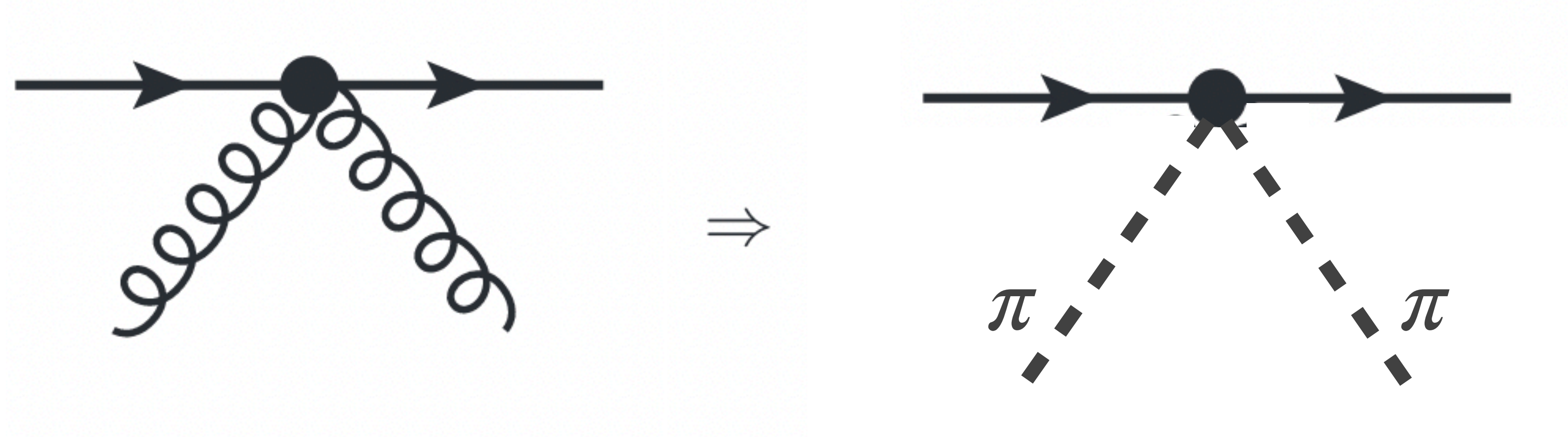
- Long-range potential from two pion exchange between 2 S-wave bottomonia

N. Brambilla et al, Phys.Rev.D 93, 054002 (2016)

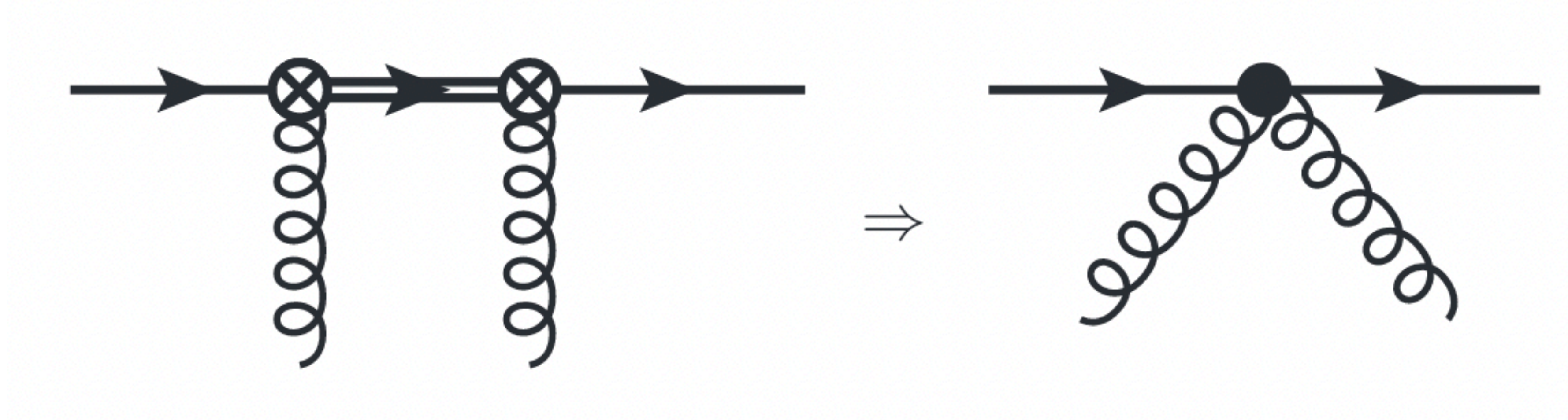


- At long-range, soft gluon exchange → two pion exchange + heavier…

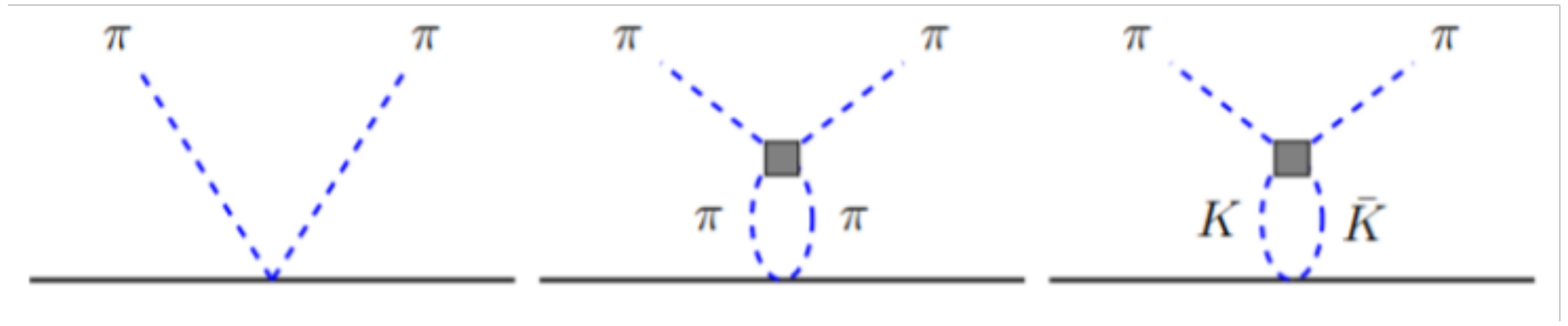
OPE highly suppressed by isospin



- Gluon emission from fully-heavy hadrons: **chromopolarizability**



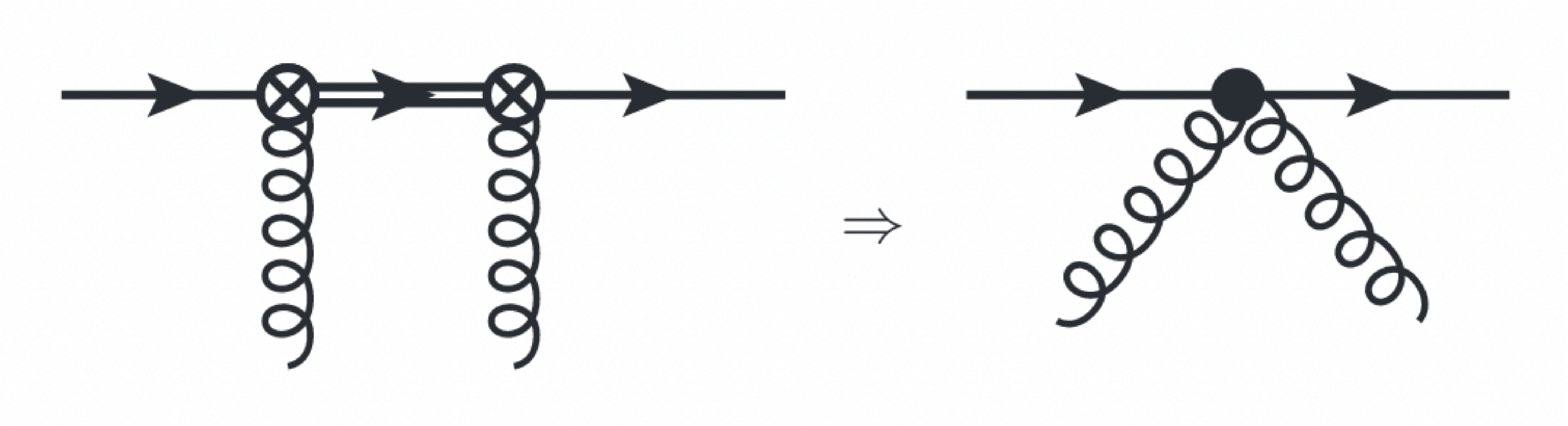
- $\pi\pi$ FSI and $\pi\pi - K\bar{K}$ coupling



- Binding of $J/\psi J/\psi$

X.K. Dong, et al., Sci.Bull. 66 (2021) 24, 2462-2470

Chromopolarizability of fully heavy hadrons



N. Brambilla et al, Phys.Rev.D 93, 054002 (2016)

Chromopolarizability of heavy quarkonium

- EFT for heavy hadron and gluon coupling

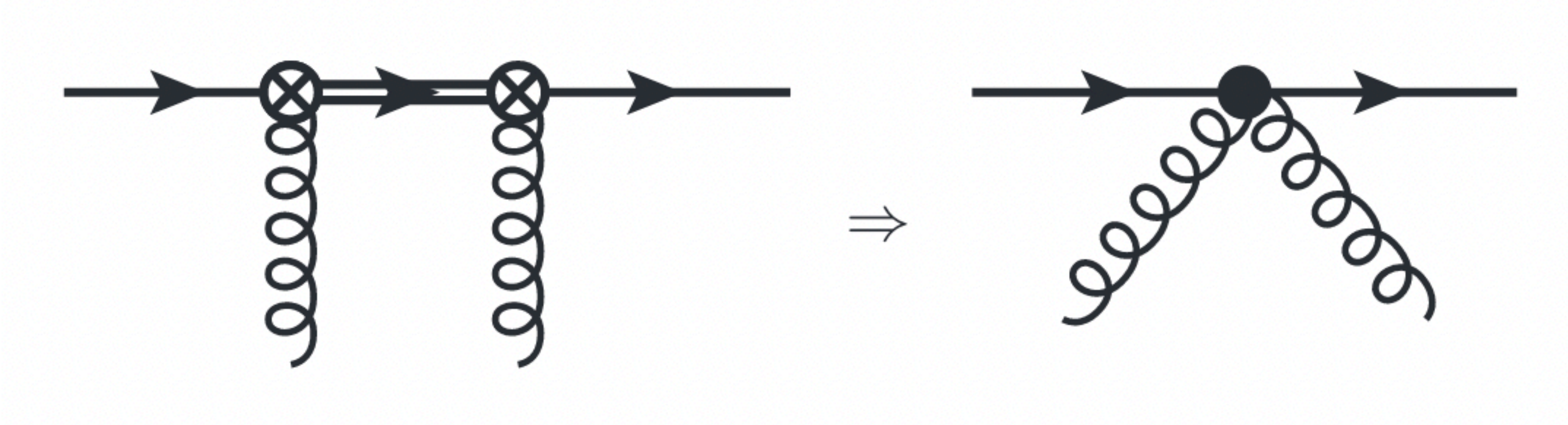
$$L_{\rm EFT}^{H} = \int d^3X H^\dagger(t,\boldsymbol{X})\left\{ i\partial_0 + \frac{\nabla_X^2}{2m_H} + \frac{1}{2}\beta_H g^2 \boldsymbol{E}^{a2} + \dots \right\} H(t,\boldsymbol{X}),$$

- pNRQCD N. Brambilla et al, Phys.Rev.D 93, 054002 (2016)

$$\mathscr{L}^{(0)}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\left[S^\dagger \left(i\partial_0 - \hat{h}_S \right) S + O^{a\dagger} \left(i\partial_0 - \hat{h}_O \right) O^a + \left(S^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}^a O^a + \text{ H.c. } \right)\right],$$

- Expression of chromopolarizability

$$\beta_\psi = -\frac{1}{9}\langle \phi | \mathbf{r} \frac{1}{E_\phi - h_o} \mathbf{r} | \phi \rangle = -\frac{1}{9} \sum_{p,l,l_z} \sum_{r_i = x,y,z} \langle \phi | r_i | p, l, l_z \rangle \frac{1}{E_\phi - p^2/2\mu} \langle p, l, l_z | r_i | \phi \rangle$$

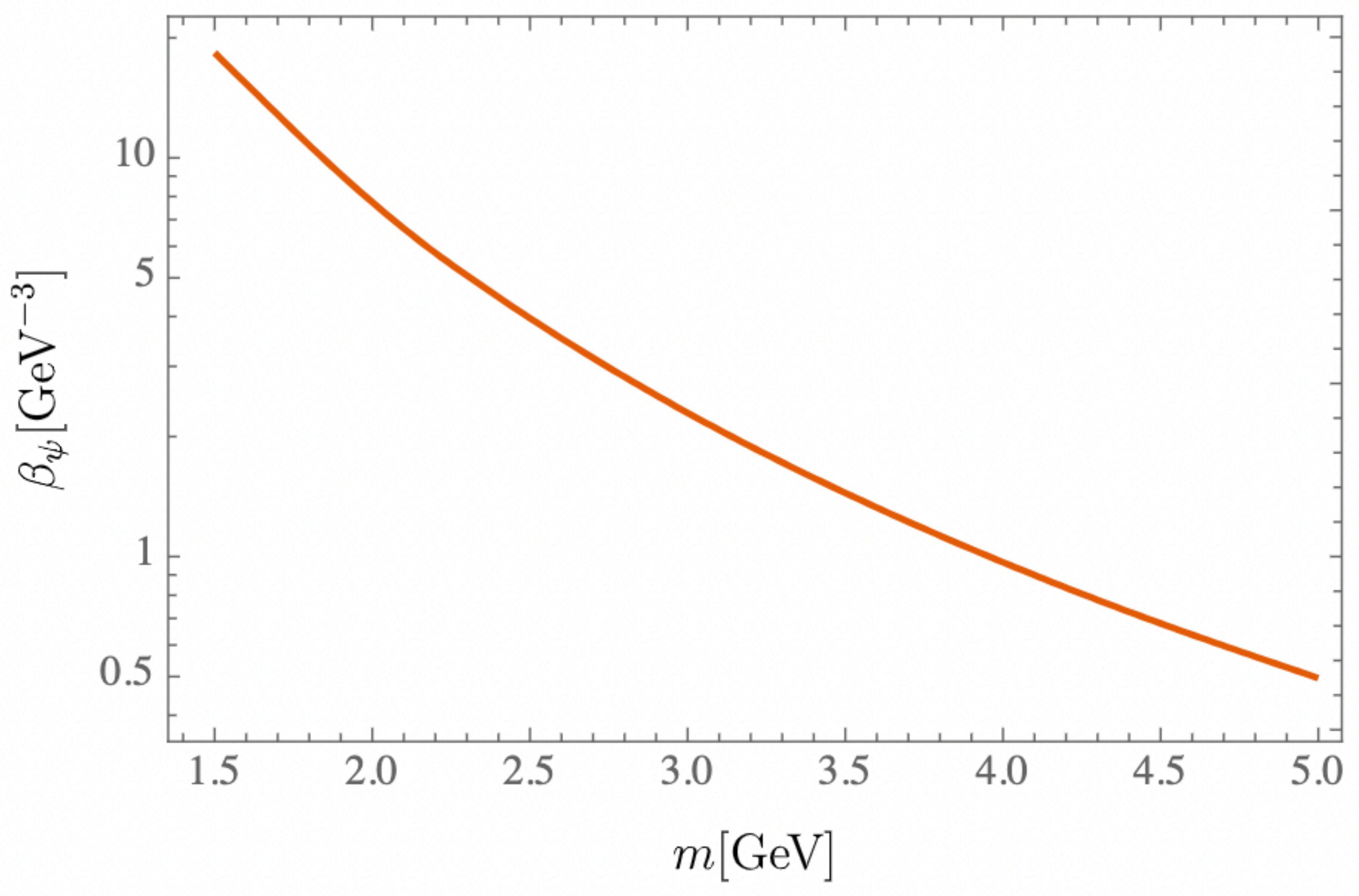


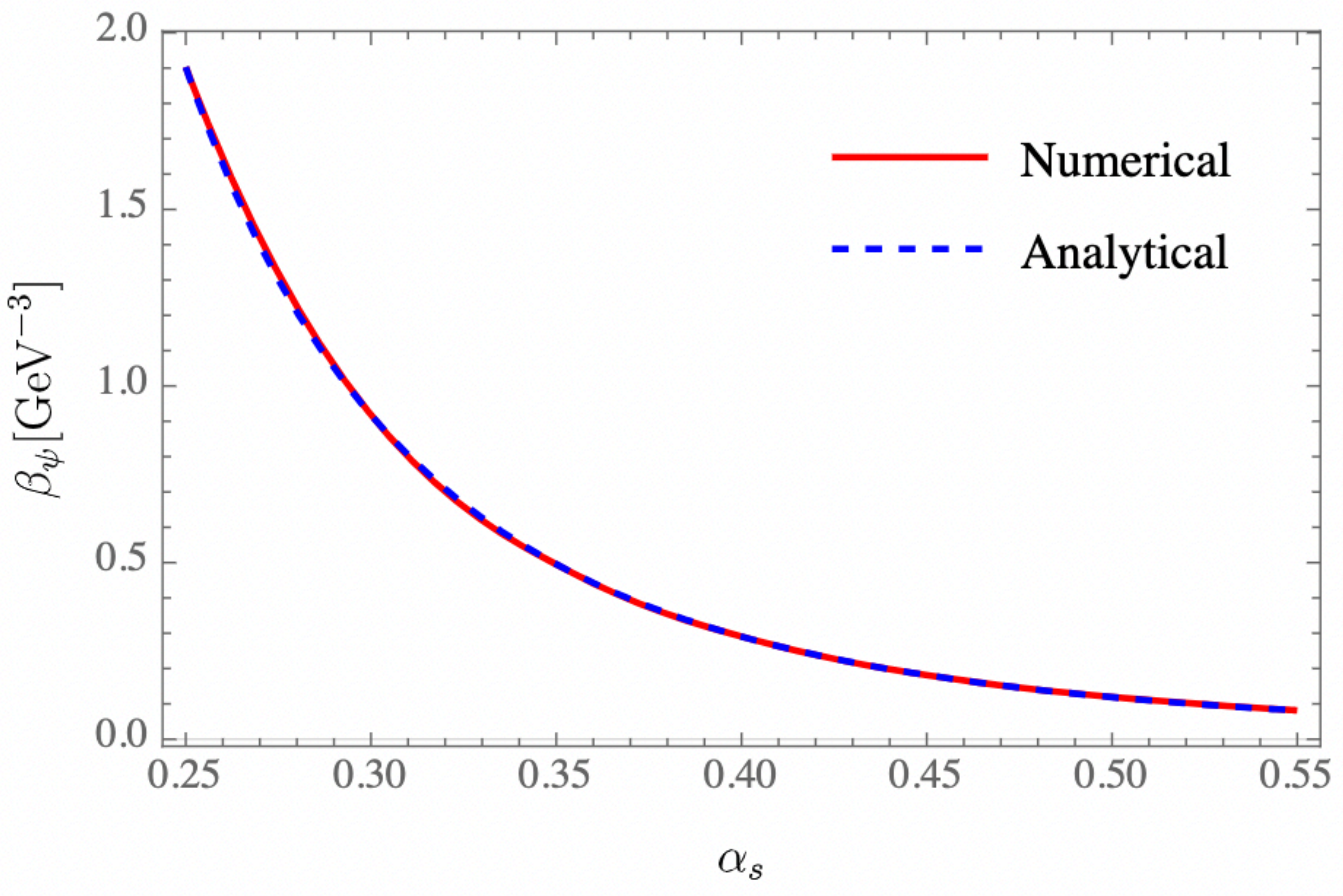
Chromopolarizability of heavy quarkonium

- Analytical N. Brambilla et al, Phys.Rev.D 93, 054002 (2016)

$$\beta_\psi = 256\frac{\rho(\rho+2)^2}{3N_c}\frac{1}{mE_\phi^2}I = C_\psi m^{-3}\alpha_s^{-4}, \quad C_\psi \approx 0.93$$

- Numerical
  - Put both color singlet and octet $Q\bar{Q}$ in a infinite well with length L_{box}.





Chromopolarizability of $\Omega_{QQQ'}$

- Jacobi coordinates, $m = m_Q, m_3 = m_{Q'}$

$$\boldsymbol{X} = \frac{m(\boldsymbol{x}_1 + \boldsymbol{x}_2) + m_3 \boldsymbol{x}_3}{M}, \quad \boldsymbol{\rho} = \boldsymbol{x}_1 - \boldsymbol{x}_2, \quad \boldsymbol{\lambda} = \frac{2}{\zeta}\left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2} - \boldsymbol{x}_3\right)$$

- The emission of a gluon from a singlet baryon is described by

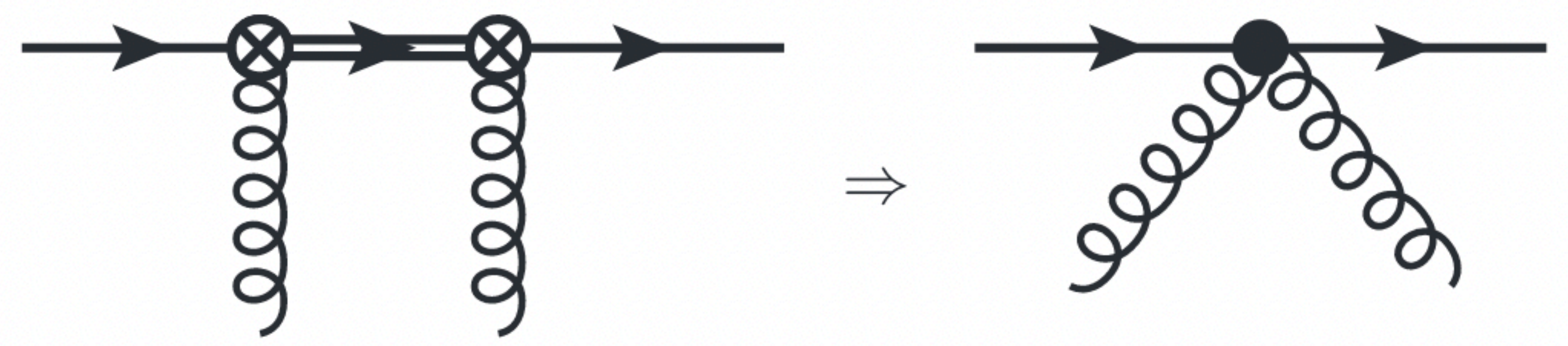
$$\mathscr{L}^{S-O}_{\text{pNRQCD}} = \int d^3\rho d^3\lambda \left\{ \frac{1}{2\sqrt{2}} \left[S^\dagger \boldsymbol{\rho} \cdot g\boldsymbol{E}^a O^{Sa} + \text{ H.c.} \right] - \frac{\zeta}{2\sqrt{6}} \left[S^\dagger \lambda \cdot g\boldsymbol{E}^a O^{Aa} + \text{ H.c.} \right] \right\},$$

- The emission of two gluons from a singlet baryon is described by

$$L_{LEFT} = \int d^3X\, \Omega^\dagger_{ccc}(t, \boldsymbol{X}) \left\{ i\partial_0 + \frac{\boldsymbol{\nabla}^2_{\boldsymbol{X}}}{2m_\Omega} + \frac{1}{2}\beta g^2 \boldsymbol{E}^2_a + \dots \right\} \Omega_{ccc}(t, \boldsymbol{X}) + \mathcal{L}_{\text{light d.o.f}}$$

- Matching the above two we have

$$\beta = \beta_\rho + \beta_\lambda = -\frac{1}{12}\langle \Omega | \boldsymbol{\rho} \frac{1}{E_B - h_{O^S}} \boldsymbol{\rho} | \Omega \rangle - \frac{\zeta^2}{36} \langle \Omega | \boldsymbol{\lambda} \frac{1}{E_B - h_{O^A}} \boldsymbol{\lambda} | \Omega \rangle$$



Evaluation of β_ρ

- Hamiltonians of color singlet and octet baryons (only Coulomb-type interactions considered)

$$h_S = -\frac{1}{2\mu}\left(\boldsymbol{\nabla}_\rho^2 + \boldsymbol{\nabla}_\lambda^2\right) - \frac{2\alpha_s}{3}\left(\frac{1}{|\boldsymbol{\rho}|} + \frac{2}{|\zeta\boldsymbol{\lambda}+\boldsymbol{\rho}|} + \frac{2}{|\zeta\boldsymbol{\lambda}-\boldsymbol{\rho}|}\right),$$

$$h_{O^S} = -\frac{1}{2\mu}\left(\boldsymbol{\nabla}_\rho^2 + \boldsymbol{\nabla}_\lambda^2\right) + \frac{\alpha_s}{6}\left(\frac{2}{|\boldsymbol{\rho}|} - \frac{5}{|\zeta\boldsymbol{\lambda}+\boldsymbol{\rho}|} - \frac{5}{|\zeta\boldsymbol{\lambda}-\boldsymbol{\rho}|}\right),$$

$$h_{O^A} = -\frac{1}{2\mu}\left(\boldsymbol{\nabla}_\rho^2 + \boldsymbol{\nabla}_\lambda^2\right) - \frac{\alpha_s}{6}\left(\frac{4}{|\boldsymbol{\rho}|} - \frac{1}{|\zeta\boldsymbol{\lambda}+\boldsymbol{\rho}|} - \frac{1}{|\zeta\boldsymbol{\lambda}-\boldsymbol{\rho}|}\right),$$

- Completeness of the continuum eigenstates the operator h_{O^S}

$$\langle\Omega|\boldsymbol{\rho}\frac{1}{E_B - h_{O^S}}\boldsymbol{\rho}|\Omega\rangle = \sum_{i=x,y,z}\langle\Omega|\rho_i\frac{1}{E_B - h_{O^S}}\rho_i|\Omega\rangle = 3\langle\Omega|\rho_z\frac{1}{E_B - h_{O^S}}\rho_z|\Omega\rangle$$

$$= 3\sum_{\nu,\nu'}\langle\Omega|\rho_z|\Psi_\nu^{(c)}\rangle\langle\Psi_\nu^{(c)}|\frac{1}{E_B - h_{O^S}}|\Psi_{\nu'}^{(c)}\rangle\langle\Psi_{\nu'}^{(c)}|\rho_z|\Omega\rangle$$

$$= 3\sum_\nu\frac{|\langle\Omega|\rho_z|\Psi_\nu^{(c)}\rangle|^2}{E_B - \varepsilon_\nu} = 3\sum_\nu\frac{|\langle\Psi_0^{(b)}|\rho_z|\Psi_\nu^{(c)}\rangle|^2}{E_B - \varepsilon_\nu},$$

with $h_S|\Omega\rangle = E_B|\Omega\rangle$ and $h_{O^S}|\Psi_\nu^{(c)}\rangle = \varepsilon_\nu|\Psi_\nu^{(c)}\rangle$

- Solve the Schroedinger equation to find eigen energies and wave functions

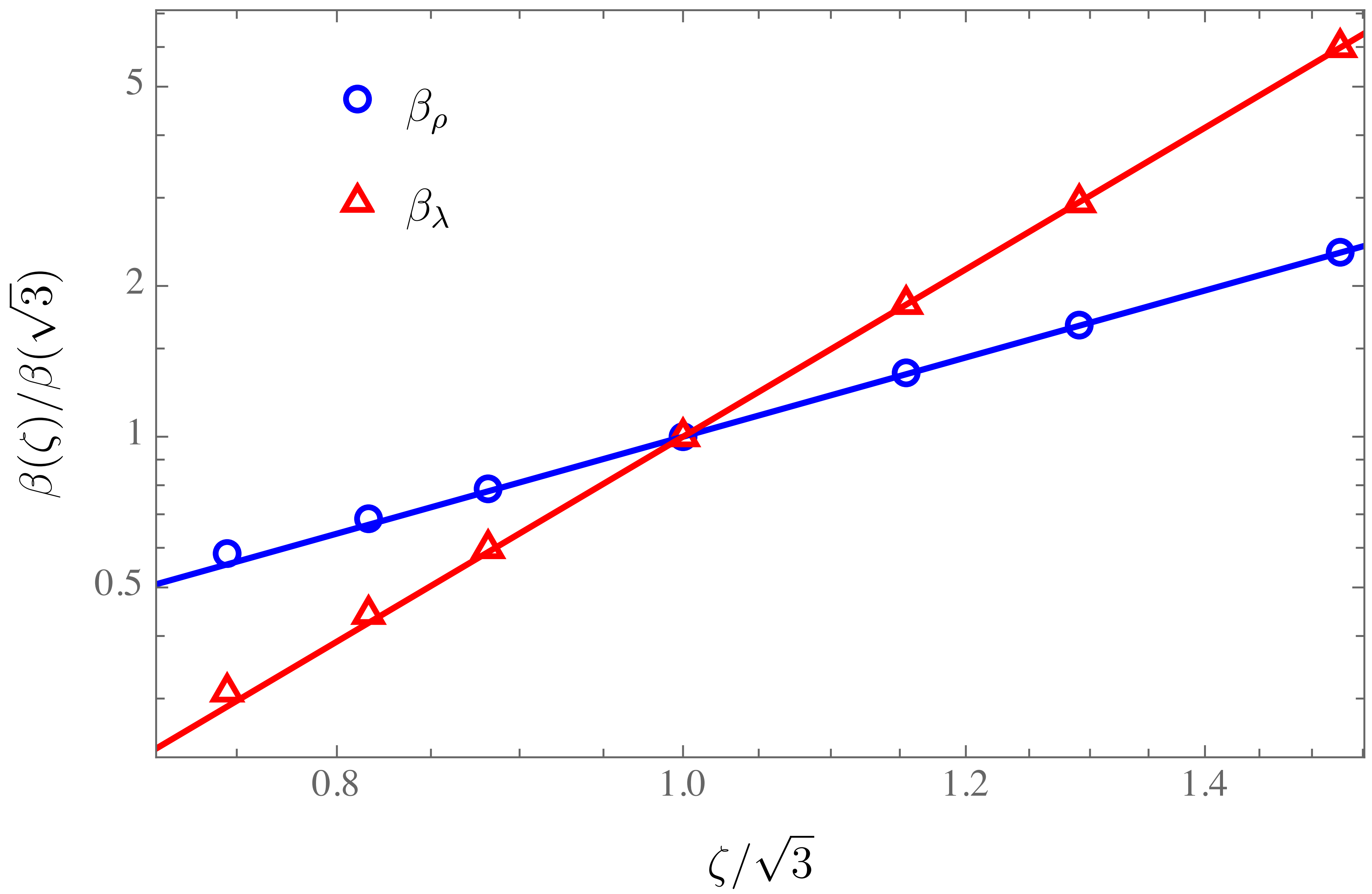
Numerical results

- It finally turns out that, if using the same quark mass,

$$\beta_\Omega = C_\Omega m^{-3}\alpha_s^{-4},\ C_\Omega = C_\Omega^{(\rho)} + C_\Omega^{(\lambda)} \approx 2.4 \approx 2.6 C_\psi .$$

- For the case where quarks in a baryon have different masses, i.e., Ω_{bcc} or Ω_{bbc}, define $\zeta = \sqrt{1 + 2m_Q/m_{Q'}}$

$$\beta_\rho(\zeta) = \beta_\rho(\sqrt{3})\left(\frac{\zeta}{\sqrt{3}}\right)^{1.95},\ \ \beta_\lambda(\zeta) = \beta_\lambda(\sqrt{3})\left(\frac{\zeta}{\sqrt{3}}\right)^{4.15}.$$



Summary

- The chromopolarizabilities of QQQ baryon and $Q\bar{Q}$ quarkonium are estimated,

  $\beta_H = C_H m^{-3} \alpha_s^{-4}, \; C_\Omega \approx 2.6 C_\psi .$

- Uncertainties:
  - Treated as purely color-Coulombic systems: corrections from nonperturbative interaction: 10% for bottom while $\mathcal{O}(1)$ for charm.
  - Ignore the mixing of O^S and O^A, correction around 7%.
  - Large uncertainties from α_s,

$$\alpha_s\left(\nu_r = 1\text{GeV}\right) = 0.4798$$
$$\alpha_s\left(\nu_r = 1.5\text{GeV}\right) = 0.3485$$
$$\alpha_s\left(\nu_r = 2\text{GeV}\right) = 0.3015$$

- For bottom, $C_{\Omega_{bbb}}/C_{\bar{b}b} \approx 2.6 \pm 0.3$; For charm, large uncertainty.

TABLE I. The mean radii and chromopolarizabilities of the ground-state heavy $\bar{Q}Q$ mesons with $Q = c, b$ evaluated in this work. See the main text for the discussion of the uncertainties.

State	$\bar{c}c$	$\bar{b}b$
$\langle r \rangle$ [fm]	$0.85^{+0.13}_{-0.23}$	$0.26^{+0.04}_{-0.07}$
β_ψ [GeV^{-3}]	19^{+15}_{-14}	$0.54^{+0.43}_{-0.39}$

TABLE II. The mean radii and chromopolarizabilities of the fully heavy QQQ' baryons with $Q, Q' = c, b$ evaluated in this work. See Sec. II B for the discussion of the uncertainties.

State	ccc	ccb	cbb	bbb
$\langle R \rangle$ [fm]	$1.66^{+0.26}_{-0.46}$	$1.44^{+0.23}_{-0.40}$	$0.65^{+0.10}_{-0.18}$	$0.51^{+0.08}_{-0.14}$
β_Ω [GeV^{-3}]	49^{+38}_{-35}	19^{+15}_{-14}	$6.7^{+5.3}_{-4.8}$	$1.4^{+1.1}_{-1.0}$

- Interactions from soft gluon exchange for di-QQQ are considerably stronger than those for di-$Q\bar{Q}$.

Backup slides

Wave function of singlet baryon

- The wave function of the singlet state can be expressed as

$$\Psi^{(b)}(R,\Omega_5) = \frac{1}{R^{5/2}} \sum_{K,\alpha} \psi^{(b)}_{K,\alpha}(R) \mathscr{Y}_{K,\alpha}(\Omega_5)$$

where we have introduced a hyper-spherical basis

$$\{R, \theta, \hat{\boldsymbol{\rho}}, \hat{\boldsymbol{\lambda}}\} \equiv \{R, \Omega_5\},$$

with $\rho = R\cos\theta, \lambda = R\sin\theta$

- For a ground state

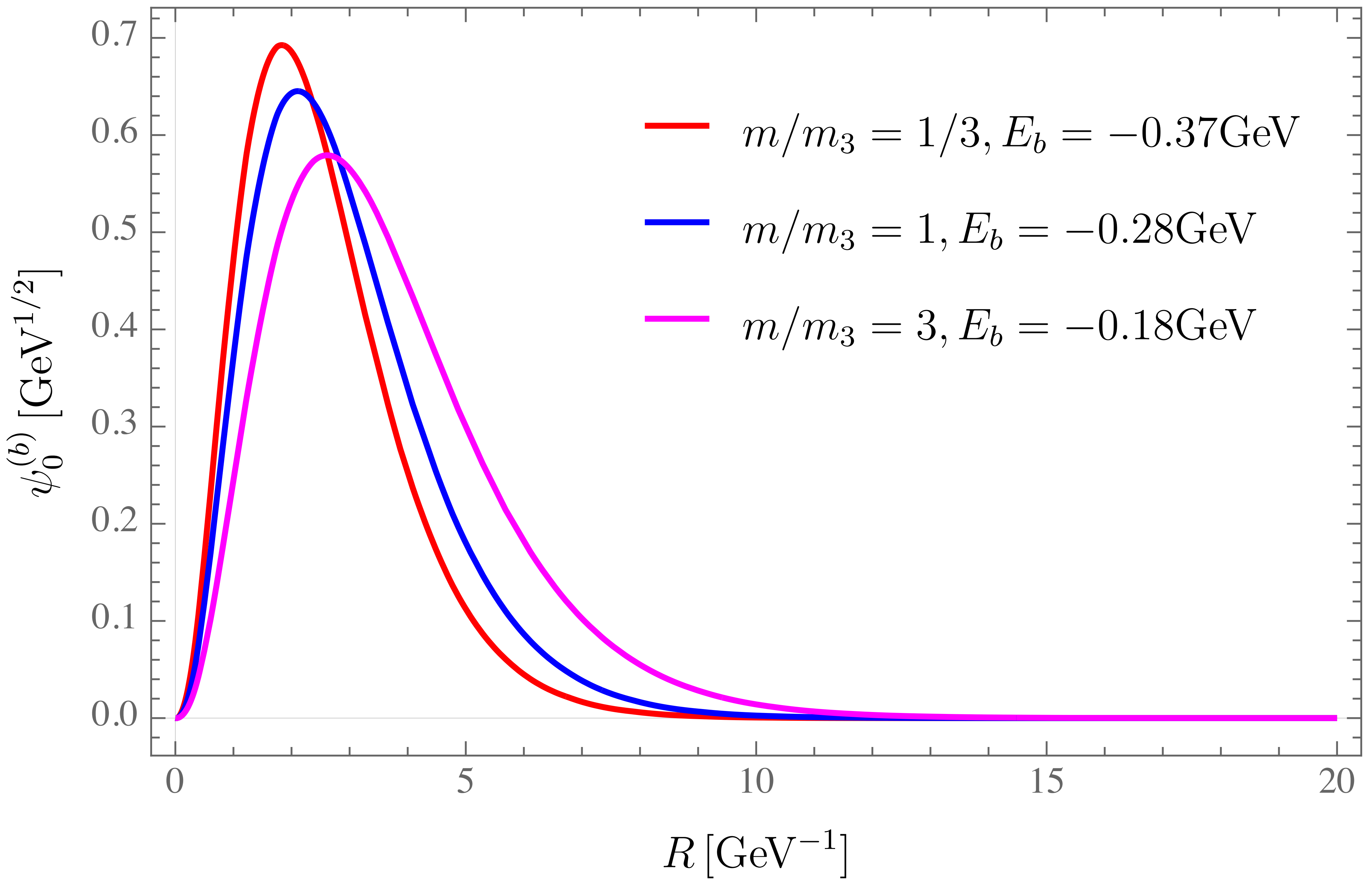
$$\Psi^{(b)}_0 = \frac{1}{\pi^{3/2} R^{5/2}} \psi^{(b)}_0(R)$$

and satisfis

$$\left(-\frac{1}{2\mu}\frac{d^2}{dR^2} + \frac{L_0(L_0+1)}{2\mu R^2} + \langle 0 | V_S | 0 \rangle \right) \psi^{(b)}_0(R) = E_B \psi^{(b)}_0(R)$$

with

$$\langle 0 | V_S | 0 \rangle = -\frac{32\alpha_s}{9\pi R}\left(1 + \frac{4}{\sqrt{1+\zeta^2}}\right)$$



Wave function of octet state

- The wave function of the octet state can be expressed as

$$\Psi_{\nu}^{(c)} = \psi_{\nu}^{(c)}(\rho, \lambda)\mathcal{Y}_{\nu}(\hat{\rho}, \hat{\lambda})$$

with

$$\mathcal{Y}_{\nu}(\hat{\rho}, \hat{\lambda}) = \sum_{m_\rho + m_\lambda = L_z} C^{LL_z}_{l_\rho m_\rho j_\lambda m_\lambda} Y_{l_\rho m_\rho}(\hat{\rho}) Y_{l_\lambda m_\lambda}(\hat{\lambda})$$

- The radial wave function satisfies

$$\langle h_{O^S} \rangle \psi_{p,q}^{(c)}(\rho, \lambda) = \varepsilon_{p,q} \psi_{p,q}^{(c)}(\rho, \lambda), \quad \langle h_{O^A} \rangle \tilde{\psi}_{p,q}^{(c)}(\rho, \lambda) = \varepsilon_{p,q} \tilde{\psi}_{p,q}^{(c)}(\rho, \lambda)$$

with

$$\langle h_{O^S} \rangle = -\frac{1}{2\mu\rho}\frac{\partial^2}{\partial\rho^2}\rho - \frac{1}{2\mu\lambda}\frac{\partial^2}{\partial\lambda^2}\lambda + \frac{1}{\mu\rho^2} + \frac{\alpha_s}{3}\left(\frac{1}{\rho} - \frac{5}{\max(\rho, \zeta\lambda)}\right)$$

$$\langle h_{O^A} \rangle = -\frac{1}{2\mu\rho}\frac{\partial^2}{\partial\rho^2}\rho - \frac{1}{2\mu\lambda}\frac{\partial^2}{\partial\lambda^2}\lambda + \frac{1}{\mu\lambda^2} - \frac{2\alpha_s}{3}\left(\frac{1}{\rho} - \frac{1}{2\max(\rho, \zeta\lambda)}\right)$$

and

$$\varepsilon_{p,q} = \frac{p^2 + q^2}{2\mu}.$$

Expression of β

- The chromopolarizability can be evaluated by

$$\beta_\rho = \frac{1}{4}\sum_{p,q}\frac{|f_1(p,q)|^2}{\varepsilon_{p,q}-E_B}, \quad \beta_\lambda = \frac{\zeta^2}{12}\sum_{p,q}\frac{|f_2(p,q)|^2}{\varepsilon_{p,q}-E_B}$$

with $$f_{1,2}(p,q) \equiv \langle\Psi_0^{(b)}|\rho_z|\Psi_\nu^{(c)}\rangle = \frac{4}{\sqrt{3\pi}}\int \rho^2 d\rho \lambda^2 d\lambda \frac{\rho,\lambda}{(\rho^2+\lambda^2)^{5/4}}\psi_0^{(b)}\left(\sqrt{\rho^2+\lambda^2}\right)\psi_\nu^{(c)}(\rho,\lambda)$$

- The wave functions of octet states are obtained by putting them into a 2D (ρ,λ) infinite well.

$$V_{O^S}(\rho,\lambda) = \begin{cases} \frac{1}{\mu\rho^2}+\frac{\alpha_s}{3}\left(\frac{1}{\rho}-\frac{5}{\max(\rho,\zeta\lambda)}\right) & 0<\rho,\lambda<L_{\text{box}} \\ +\infty & \text{otherwise.}\end{cases}$$

$$V_{O^A}(\rho,\lambda) = \begin{cases} \frac{1}{\mu\lambda^2}-\frac{2\alpha_s}{3}\left(\frac{1}{\rho}-\frac{1}{2\max(\rho,\zeta\lambda)}\right) & 0<\rho,\lambda<L_{\text{box}} \\ +\infty & \text{otherwise.}\end{cases}$$

Solving PDE

- Using the following substitutions

$$\psi = \frac{\chi}{\rho\lambda}, \rho = \tilde{\rho}L_0, \lambda = \tilde{\lambda}L_0, L_{\text{box}} = \tilde{L}_{\text{box}}L_0, \mu = \tilde{\mu}/L_0, E = \tilde{E}/L_0$$

with $L_0 = 0.197$ fm and $1/L_0 = 1$ GeV, the eom turns to

$$\frac{1}{2\tilde{\mu}}\left(-\frac{\partial^2}{\partial\tilde{\rho}^2} - \frac{\partial^2}{\partial\tilde{\lambda}^2} + \tilde{V}(\tilde{\rho},\tilde{\lambda})\right)\tilde{\chi}(\tilde{\rho},\tilde{\lambda}) = \tilde{E}\tilde{\chi}(\tilde{\rho},\tilde{\lambda})$$

with

$$\tilde{V}(\tilde{\rho},\tilde{\lambda}) = \frac{2}{\tilde{\rho}^2} + \frac{\alpha_s}{3}2\tilde{\mu}\left(\frac{1}{\tilde{\rho}} - \frac{5}{\max(\tilde{\rho},\zeta\tilde{\lambda})}\right) \quad \text{or} \quad \tilde{V}(\tilde{\rho},\tilde{\lambda}) = \frac{2}{\tilde{\lambda}^2} - \frac{2\alpha_s}{3}2\tilde{\mu}\left(\frac{1}{\tilde{\rho}} - \frac{1}{2\max(\tilde{\rho},\zeta\tilde{\lambda})}\right)$$

The boundary conditions read

$$\tilde{\chi}(\tilde{\rho},\tilde{\lambda})\Big|_{\tilde{\rho}=0,\tilde{L}_{\text{box}}} = 0,\ \tilde{\chi}(\tilde{\rho},\tilde{\lambda})\Big|_{\tilde{\lambda}=0,\tilde{L}_{\text{box}}} = 0.$$

They are normalized as

$$\int \rho^2 d\rho \lambda^2 d\lambda\ \psi^{(c)\dagger}_{p,q}(\rho,\lambda)\psi^{(c)}_{p',q'}(\rho,\lambda) = \delta_{p,p'}\delta_{q,q'}.$$