Threshold effects as the origin of some exotic phenomena

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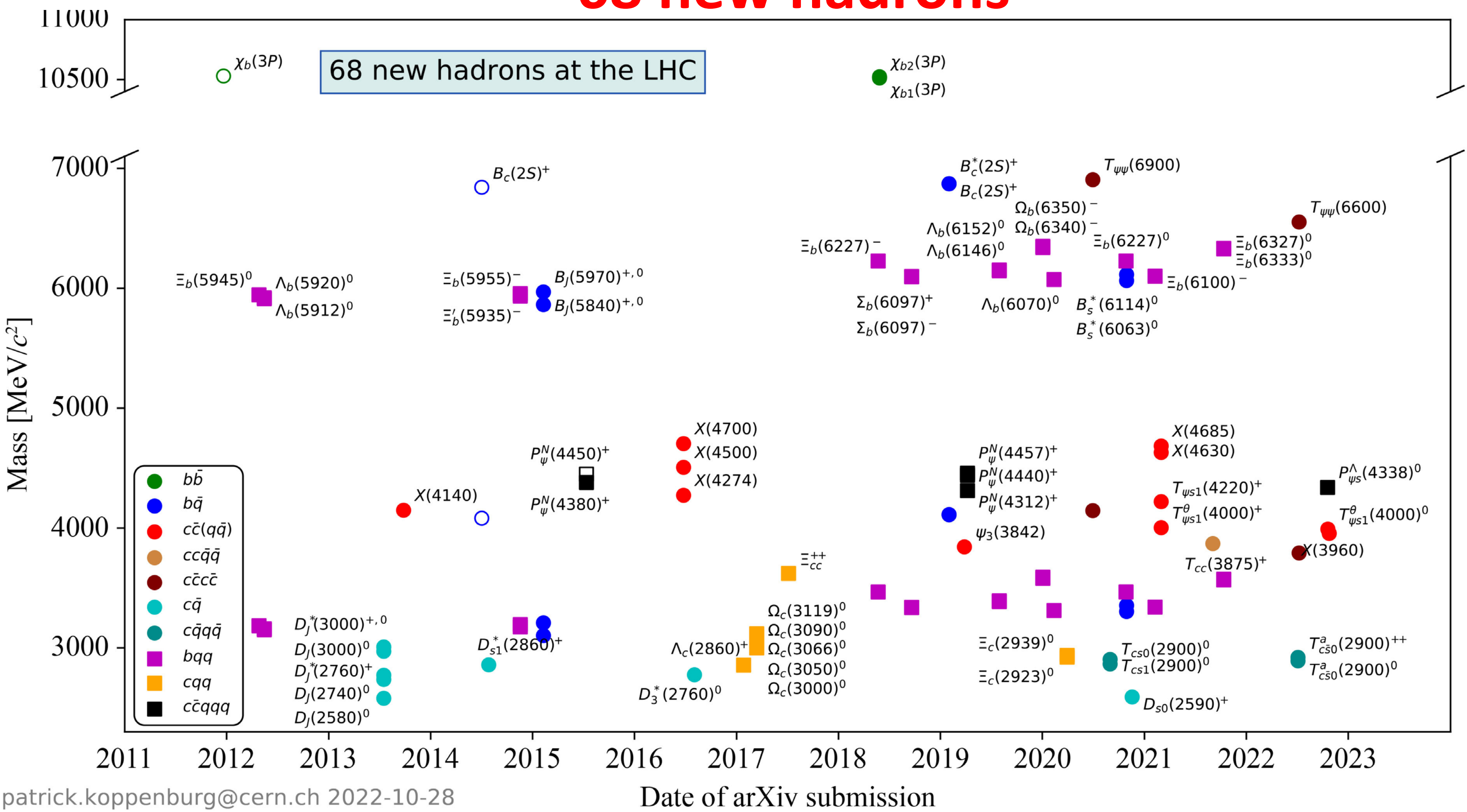
CRC110 Workshop, PKU, Rizhao, July 22, 2023

Outline

- Brief introduction to exotic hadrons
- Cusp effect
- Triangle singularity (TS) phenomena
- Threshold effects and some newly observed XYZ states
- Summary

New particles discovered at the LHC

68 new hadrons



S. Chen *et al.*, Frontiers of Physics 18,44601(2023)

Renaissance of Hadron Spectroscopy!

Theoretical Interpretation

- ✓Hadronic molecule
- ✓ Compact multiquark state
- ✓Hybrid
- ✓Hadrocharmonium

} Genuine resonance interpretations

- ✓Threshold effect (cusp, triangle singularity, …) (Non-resonance interpretation)

XYZ particle
Near threshold characteristic

Hadronic molecule

The "molecular state" concept is not exotic， the most exotic thing is that nearly all of the XYZ particles could be interpreted as molecular states

Threshold effect

Contributions cannot be ignored

Theoretical Interpretation

- ✓Hadronic molecule
- ✓ Compact multiquark state
- ✓Hybrid
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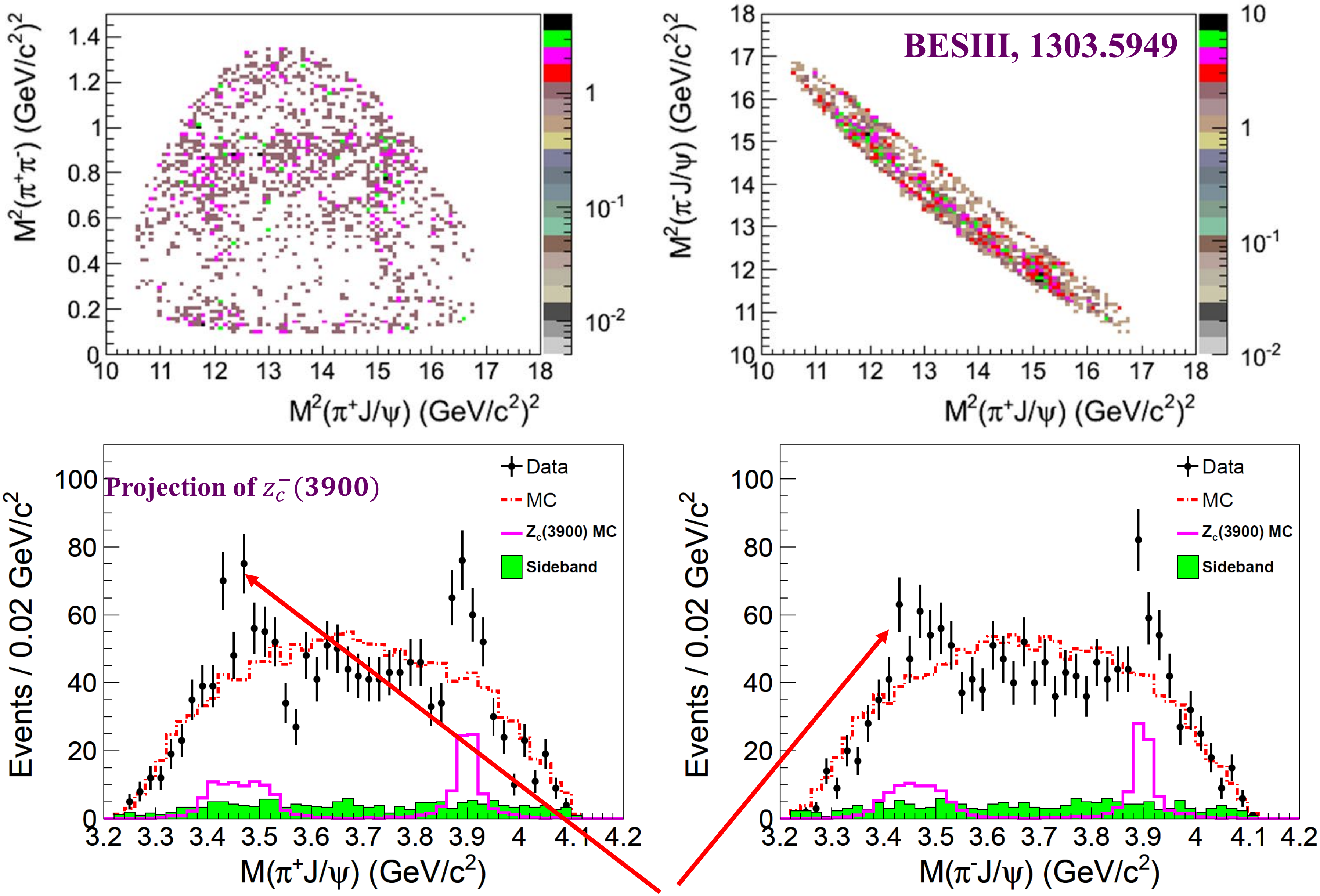
} Genuine resonance interpretations

- ✓Threshold effect (cusp, triangle singularity, …) (Non-resonance interpretation)

"Resonance-like" structure $\overset{?}{=}$ Genuine particle

“Resonance-like” structure $\stackrel{?}{\neq}$ Genuine particle

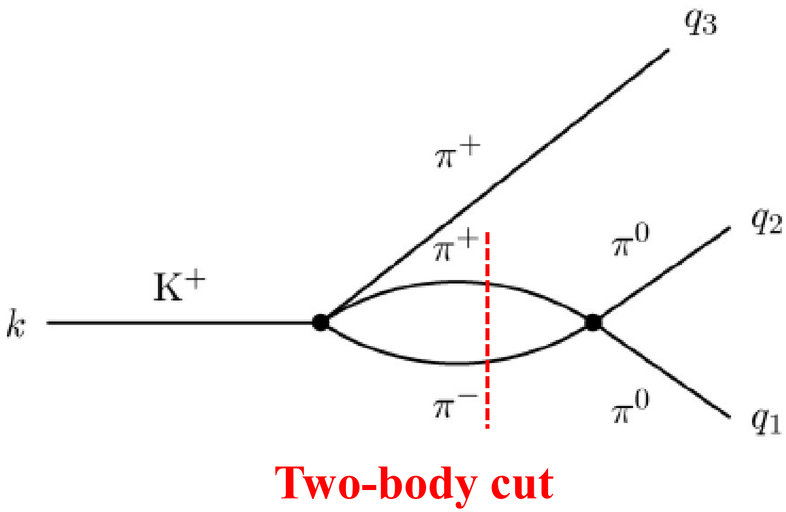


“Reflection” in Dalitz plot

Cusp effect

E.P. Wigner, *"On the Behavior of Cross Sections Near Thresholds"*, PR73, 1002 (1948)

Induced by the charge-exchange rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$



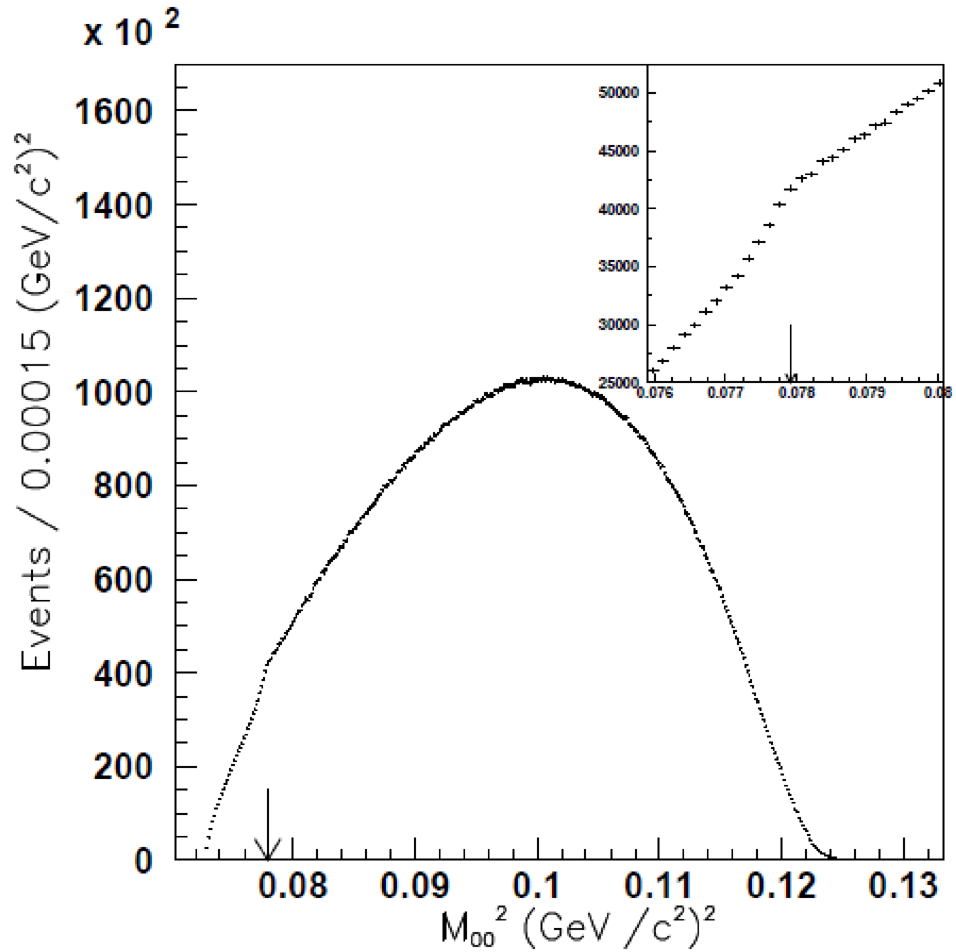
Budini & Fonda, PRL6,419(1961);
Cabibbo, PRL93,121801(2004);

Branching ratio

$K^+ \rightarrow \pi^+\pi^-\pi^+$ $((5.59 \pm 0.04)\%)$

much larger than

$K^+ \rightarrow \pi^0\pi^0\pi^+$ $((1.761 \pm 0.022)\%)$

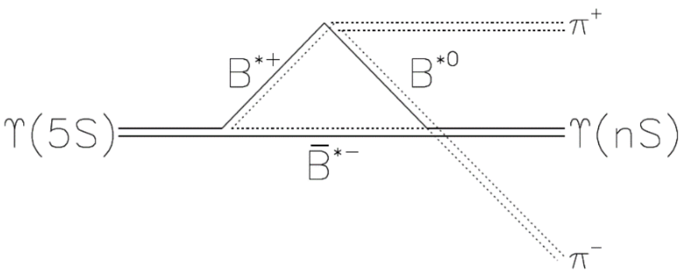


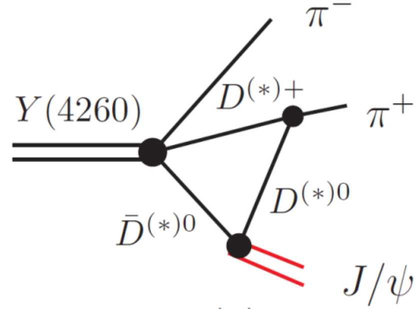
NA48/2, PLB633,173 (2006)
6×10^7 events

Cusp effect

F.K. Guo, XHL, S. Sakai, PPNP 112, 103757 (2020)

- **Possible correlation with some XYZ states: Z_b(10610/10650), Z_c(3900), Z_c(4020)**





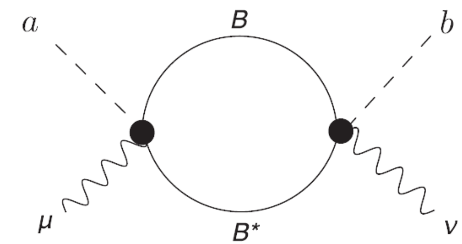


FIG. 1. Coupled channels in $\Upsilon\pi$ scattering.

D.V. Bugg, EPL96, 11002(2011)

D.Y. Chen, X. Liu, PRD88, 11002(2013)

E. Swanson, PRD91, 034009(2015)

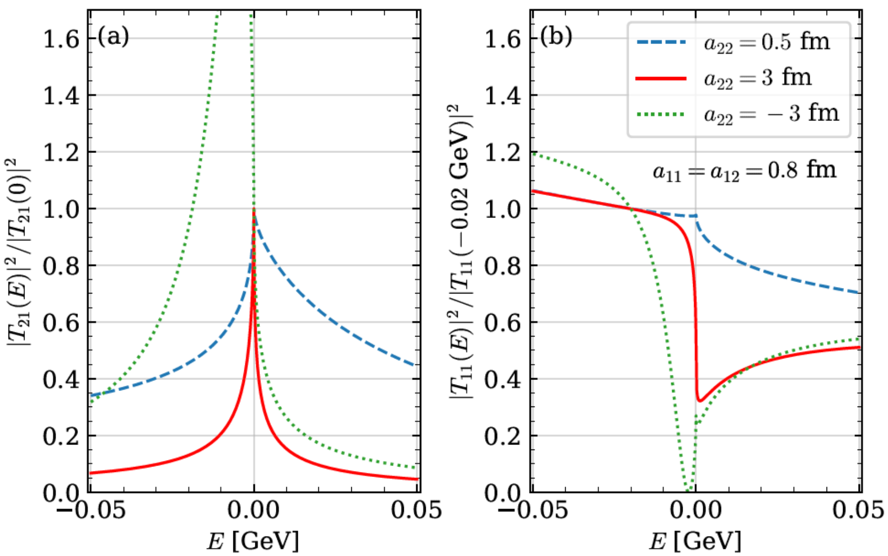


FIG. 2. Illustration of threshold behaviors. Here we use the masses of the π^- and J/ψ for channel-1 and those of the D^0 and D^{*-} for channel-2, and the values of used a_{ij} parameters

X.K. Dong, F.K. Guo, B.S. Zou, PRL126, 152001(2021)

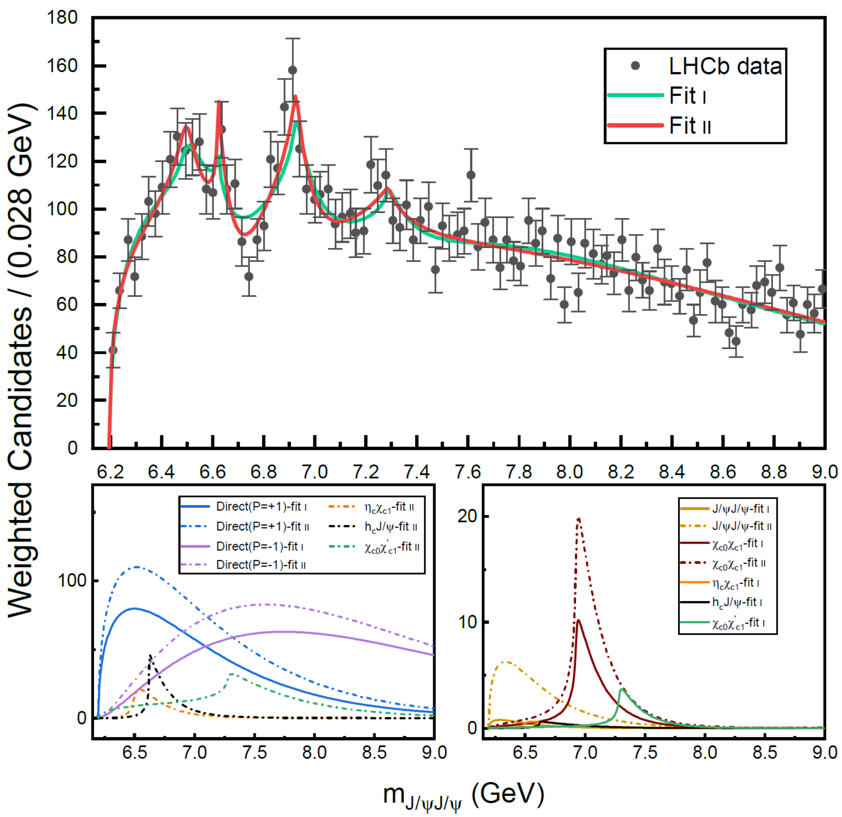
J/ψπ-DD* interaction

Threshold cusp and X(6900)

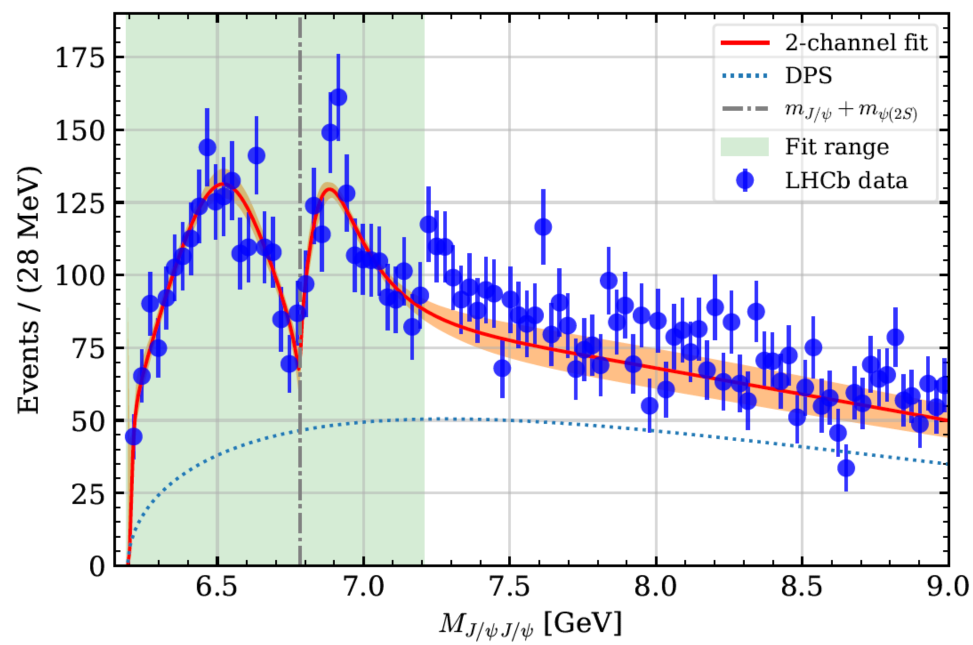
$X(6900): M = 6905 \pm 11 \pm 7$ MeV **LHCb, Sci. Bull. 65, 1983-1993(2020)**

$\Gamma = 80 \pm 19 \pm 33$ MeV . **Molecule, compact state, or ……?**

Two charmonia rescattering into di-J/psi



J.Z. Wang, D.Y. Chen, X. Liu, T. Matsuki, PRD103, L071503(2021)



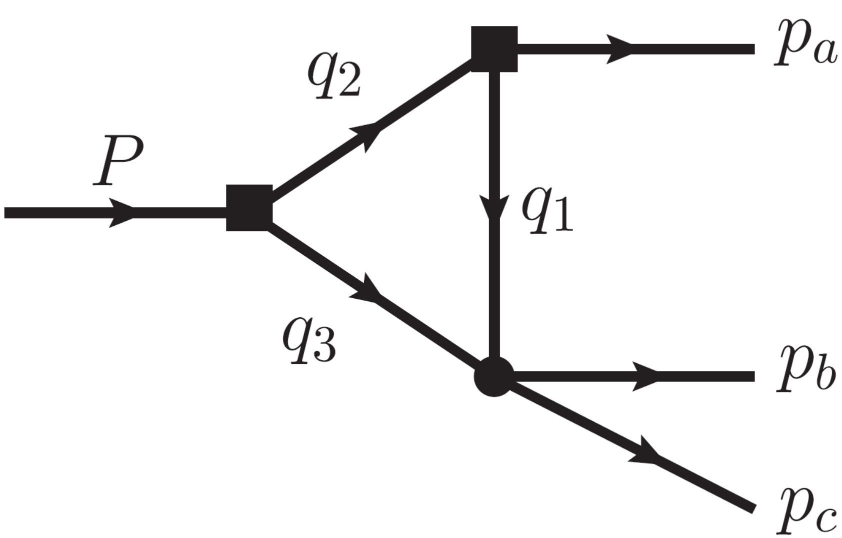
X.K. Dong, V. Baru, F.K. Guo, C. Hanhart, A. Nefediev, PRL126, 132001(2021)

Triangle Singularity Mechanism

"The kinematic conditions for the existence of singularities on the physical boundary are equivalent to the condition that the relevant Feynman diagram be interpretable as a picture of an energy and momentum-conserving process occurring in space-time, with all internal particles real, on the mass shell and moving forward in time." –Coleman-Norton theorem

Coleman&Norton, Nuovo Cimento 38,5018 (1965)

Fronsdal&Norton,J.Math.Phys.5, 100(1964)

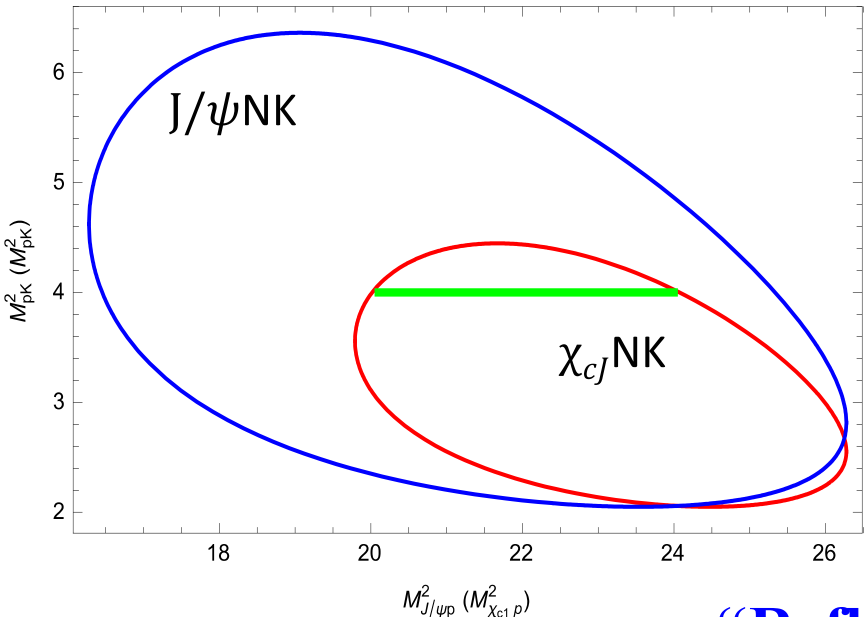


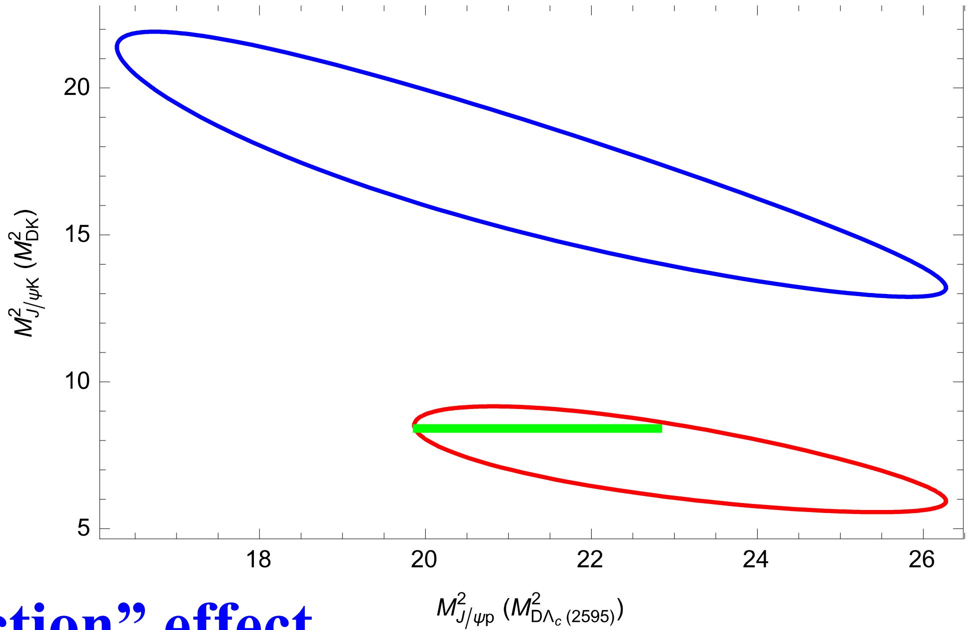
Also see Jia-Jun and Feng-Kun's talk

Width effect

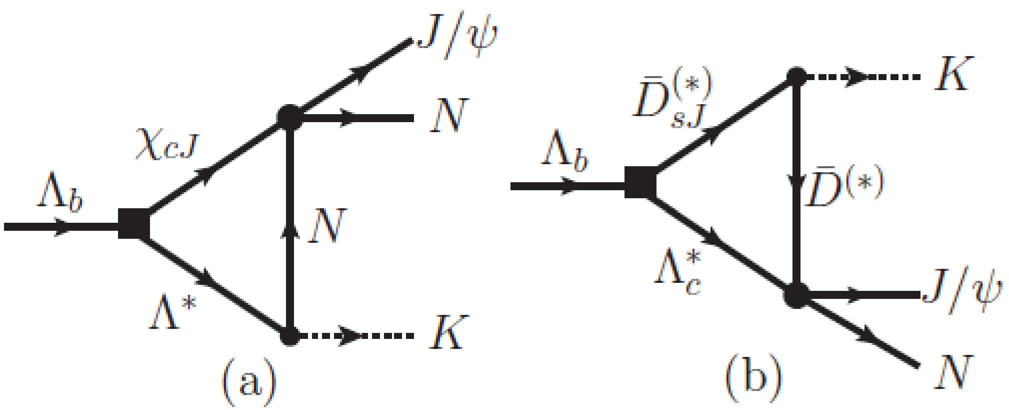
Complex mass scheme: m_2=m-iΓ/2

TS mechanism

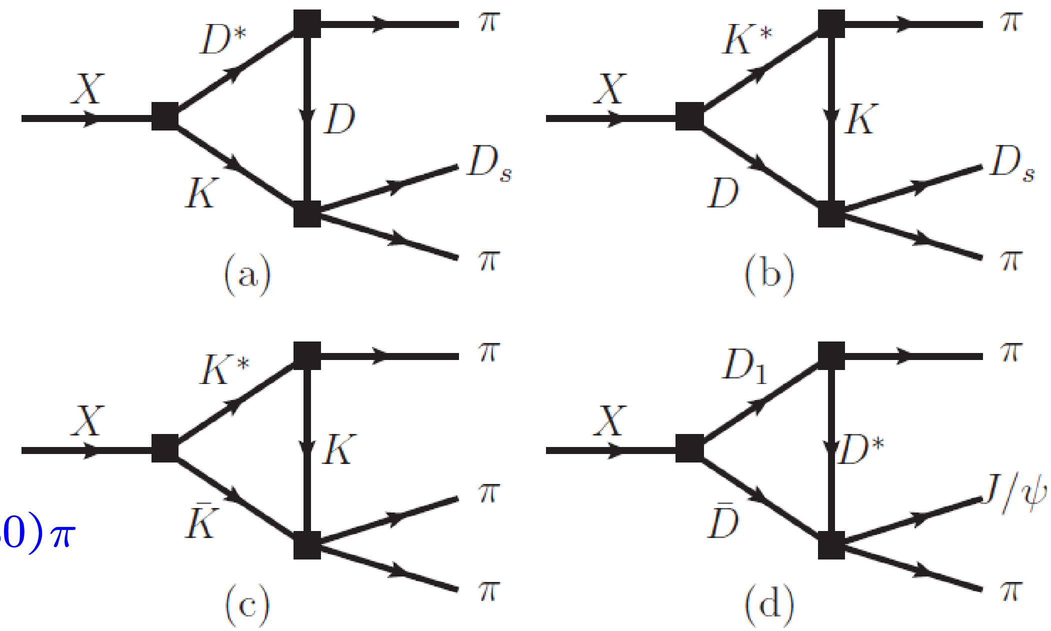




"Reflection" effect



Triangle Singularity Phenomena



$\eta(1405) \to f_0(980)\pi$

$Y(4260) \to Z_c(3900)\pi$

Wu, Liu, Zhao & Zou, PRL108,081803(2012)

Wang,Hanhart,Zhao,PRL111,132003(2013)

Kinematic region of ATS

[MeV]	Fig. 3(a)	Fig. 3(b)	Fig. 3(c)	Fig. 3(d)
$\Delta_{s_1}^{\max}$	0.089	96	49	16
$\Delta_{s_2}^{\max}$	0.087	62	38	15

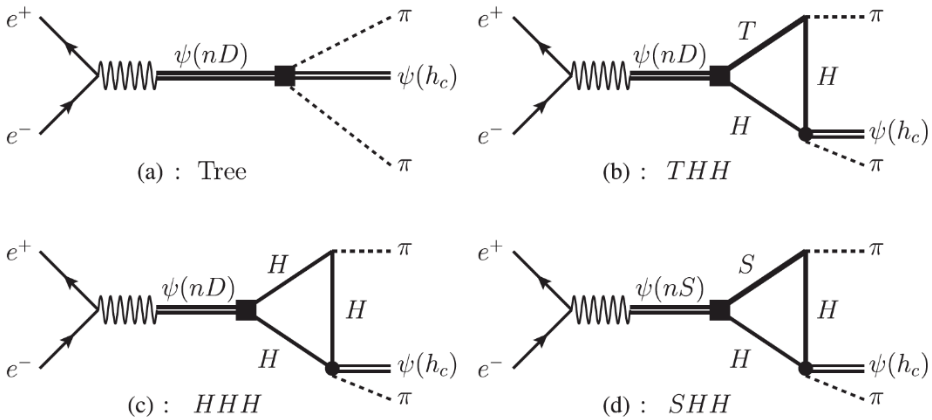
The gap between the anomalous and normal threshold

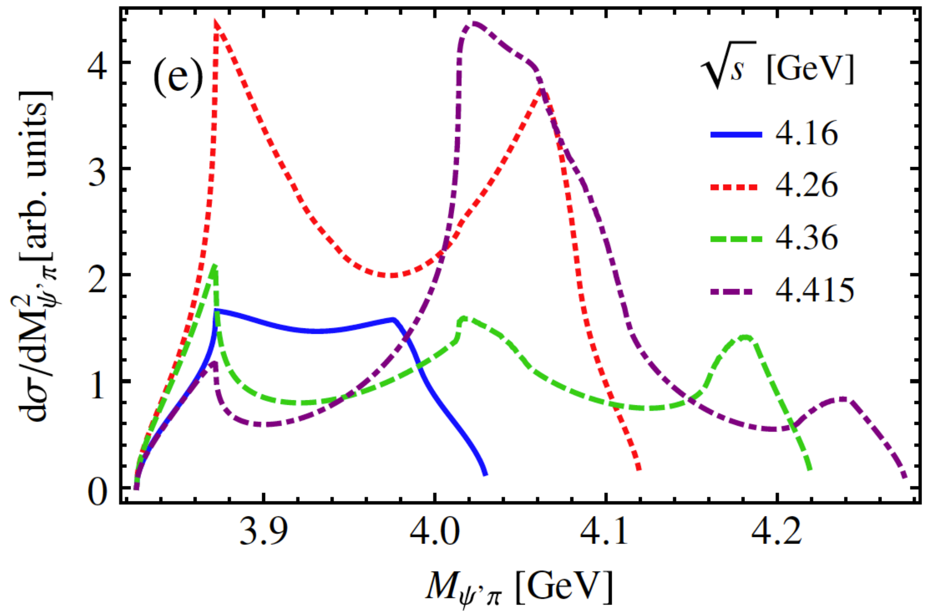
$$\Delta_{s_1} = \sqrt{s_1^-} - \sqrt{s_{1N}},$$

$$\Delta_{s_2} = \sqrt{s_2^-} - \sqrt{s_{2N}}.$$

Liu, Oka, Zhao, PLB753, 297(2016)

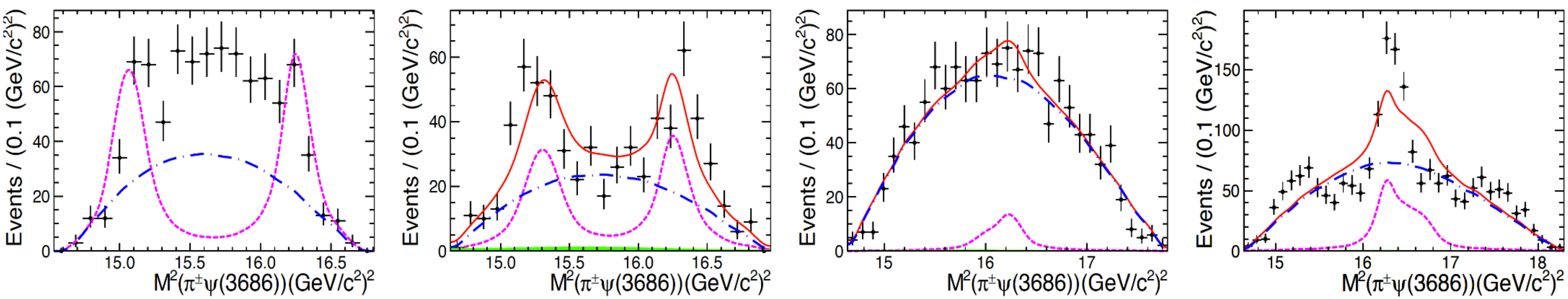
TS mechanism and structures in $e^+e^- \rightarrow \psi(3686)\pi\pi$





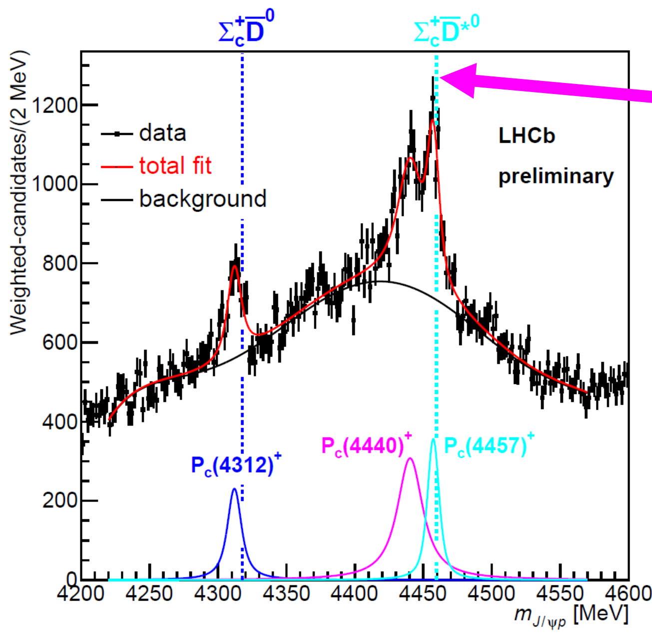
X.H. Liu, PRD90,074004(2014)

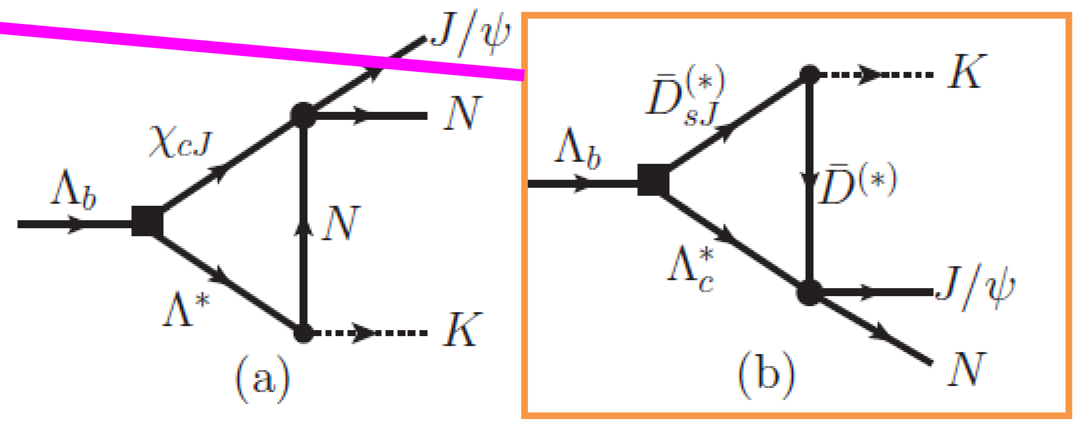
BESIII, arXiv:1703.08787



Theoretical predictions are consistent with the observed ψ(3686)pi invariant mass distributions at various CM energies

TS mechanism and the heavy pentaquark "Pc"





Liu, Wang, Zhao, arXiv:1507.05359

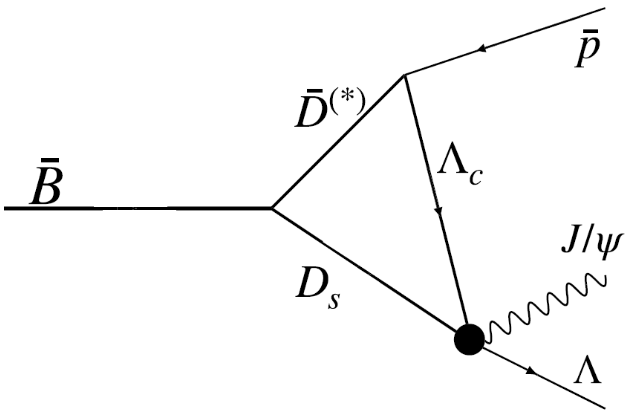
LHCb, arXiv: 1904.03947

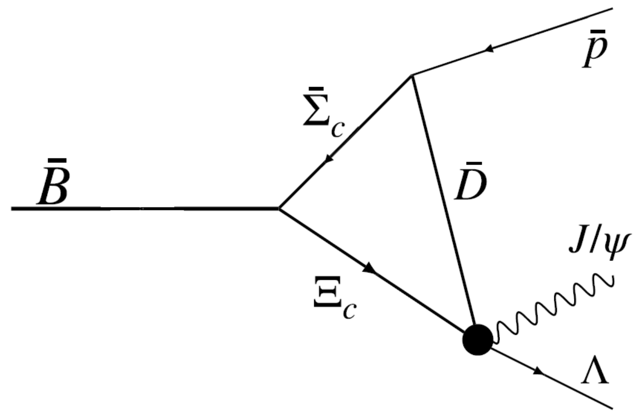
State	M [MeV]	Γ [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+\ 3.7}_{-\ 4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+\ 8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+\ 5.7}_{-\ 1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$

Thresholds [GeV]	$\Lambda_c(2286)\ 1/2^+$	$\Lambda_c(2595)\ 1/2^-$	$\Lambda_c(2625)\ 3/2^-$
$\bar{D}(1865)\ 0^-$	4.151	4.457	4.493
$\bar{D}^*(2007)\ 1^-$	4.293	4.599	4.635

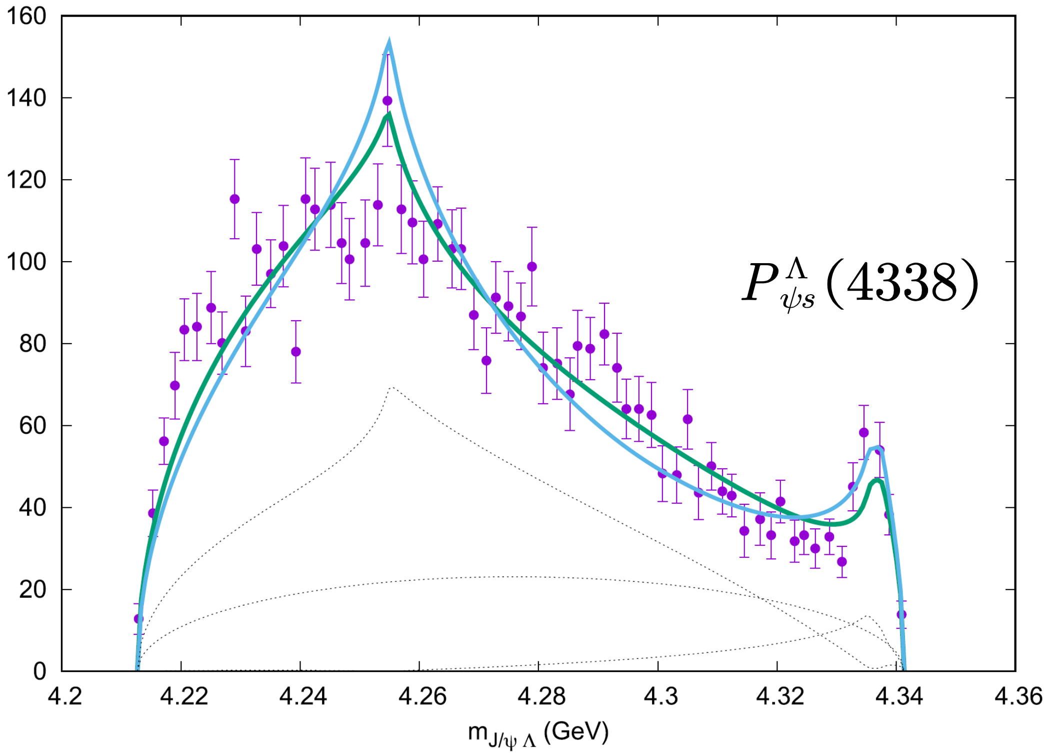
The possibility of TS has not been completely ruled out

TS mechanism and "Pcs"





T.J. Burns & E.S. Swanson, PLB838 (2023) 137715



$$\Xi_c^0 \bar{D}^0 = 4335.28 \pm 0.33 \text{ MeV}$$

$$\Xi_c^+ D^- = 4337.37 \pm 0.28 \text{ MeV}$$

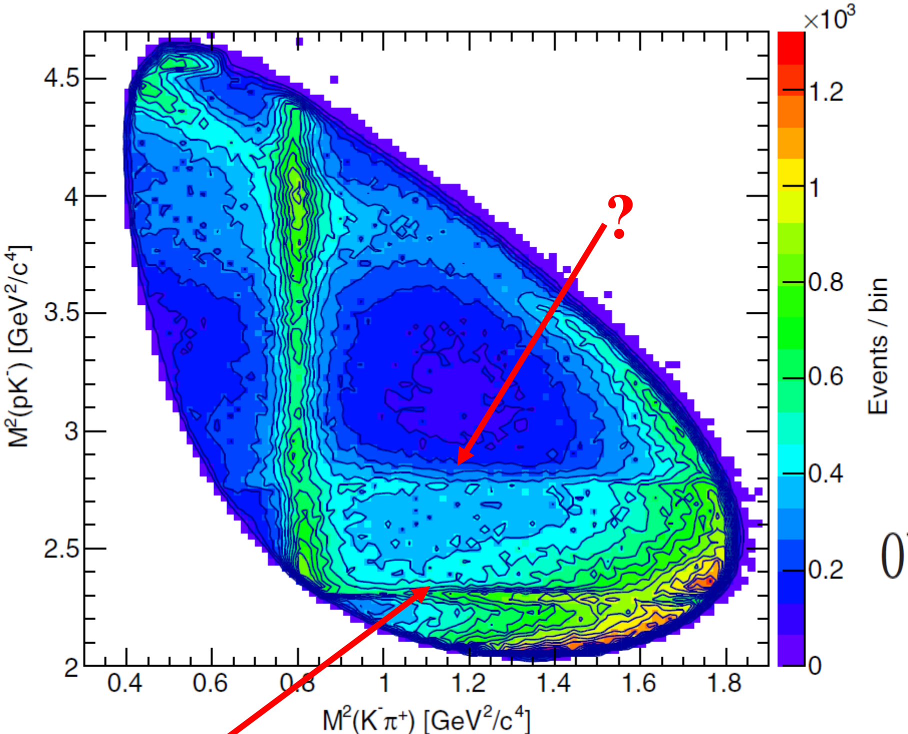
Data from LHCb

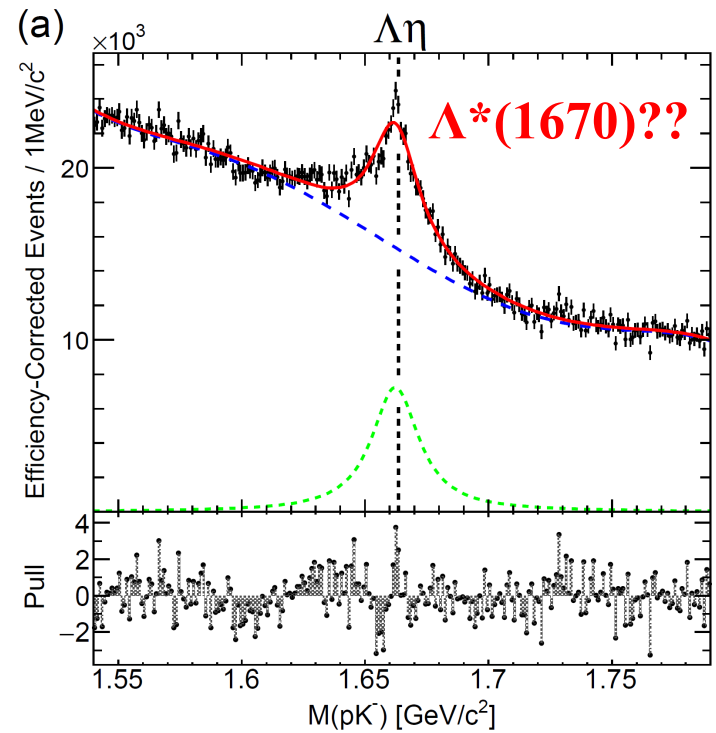
Observation of a "cusp"

Dalitz plot for $\Lambda_c \to pK^- \pi^+$

Belle, PRL117,011801(2016)

1.452 $\times 10^6$ events





Fit with BW

Observation of a Threshold Cusp at the $\Lambda\eta$ Threshold in the pK^- Mass Spectrum with $\Lambda_c^+ \to pK^-\pi^+$ Decays

Belle, arXiv: 2209.00050

- ✓ **Bin width: 1 MeV**
- ✓ **M≈1663 MeV   Γ ≈10 MeV**
- ✓ **$\Lambda\eta$ threshold: 1663.545 MeV**

Observation of a "cusp"

Hyperons around 1663 MeV [PDG]

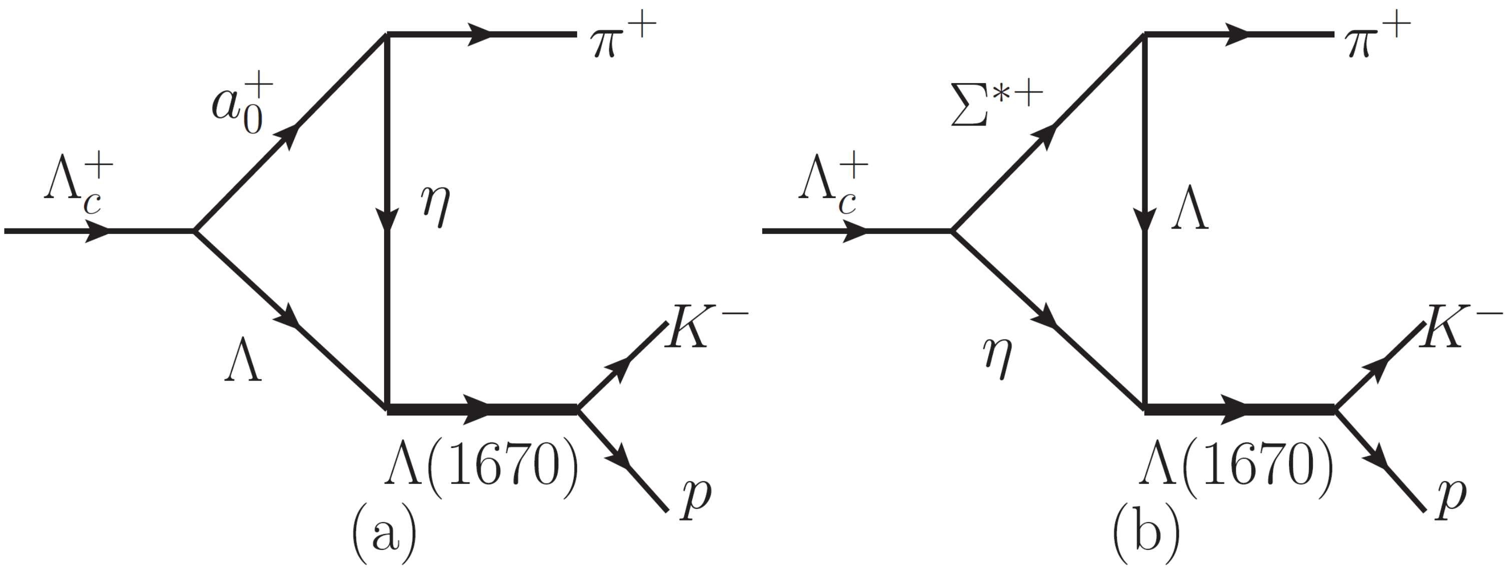
[MeV]	Mass	Width	J^P
X(1663)	1663	~10	?
Λ*(1670)	1660 to 1680 ≈1670	25 to 50 ≈35	1/2-
Λ*(1690)	1685 to 1695 ≈1690	50 to 70 ≈60	3/2-
Σ*(1660)	1630 to 1690 ≈1660	40 to 200 ≈100	1/2+
Σ*(1670)	1665 to 1685 ≈1670	40 to 80 ≈60	3/2-

No established hyperons correspond to this "X(1663)"

Two groups claim there is a narrow Λ* with J=3/2:

- **Liu & Xie [PRC85, 038201; PRC86,055202]**
  J^P=3/2-(D_{03}), M=1668.5±0.5 MeV, Γ=1.5±0.5 MeV
- **Kamano *et al.* [PRC90, 065204; PRC92, 025205]**
  J^P=3/2+(P_{03}), M=1671+2-8 MeV, Γ=10+22-4 MeV

Contributions from rescattering processes



- ✓ **Cabibbo-favored process**
- ✓ **Strong couplings**
- ✓ **Exp. value:** $Br(\Lambda_c \to \Lambda\eta\pi^+) \sim (2.2 \pm 0.5)\%$

  $Br(\Lambda_c \to \Sigma(1385)\,\eta \to \Lambda\eta\pi^+) \sim (1.06 \pm 0.32)\%$

X.H. Liu, G. Li, J.J. Xie, Q. Zhao, PRD100 (2019) 054006

TS location

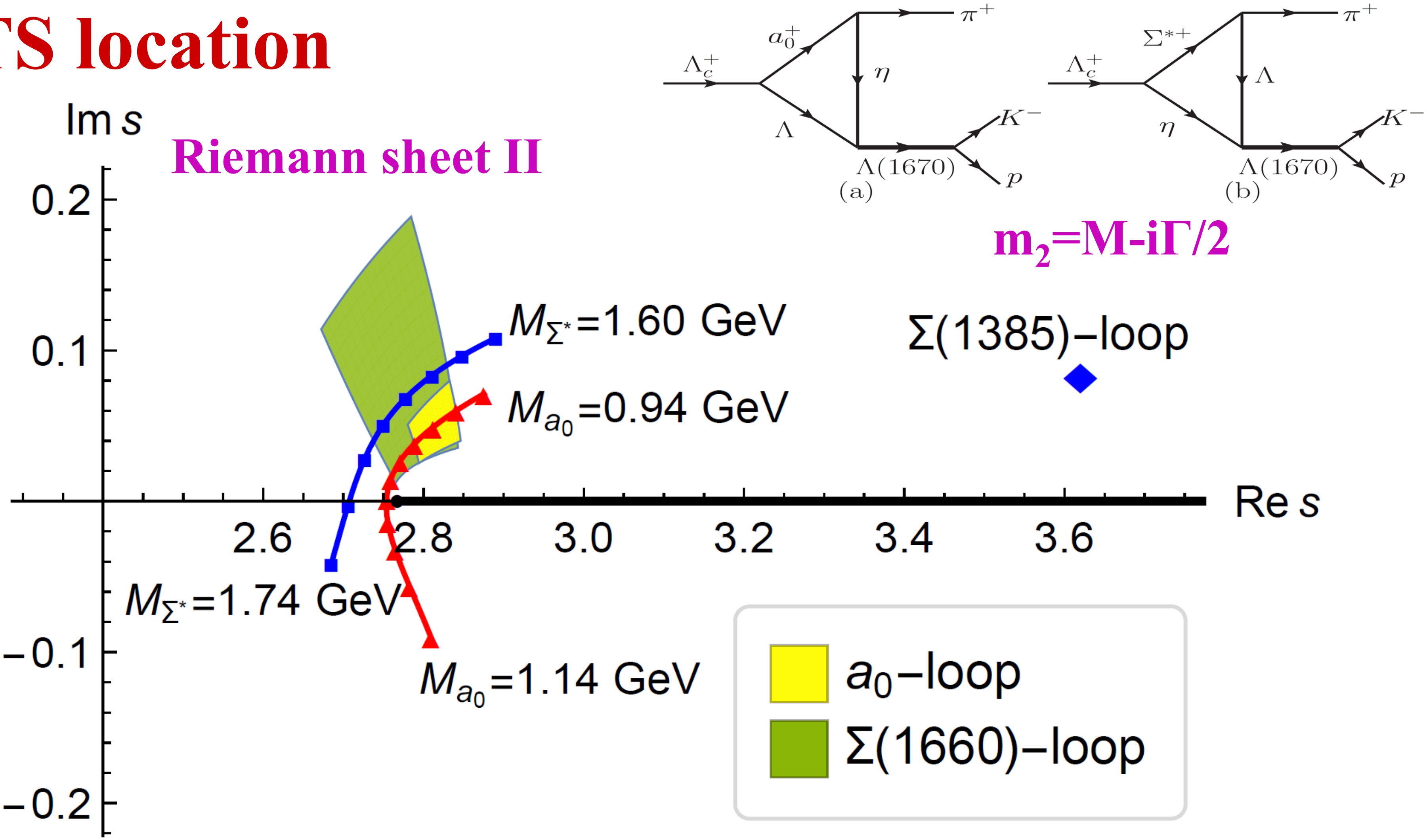
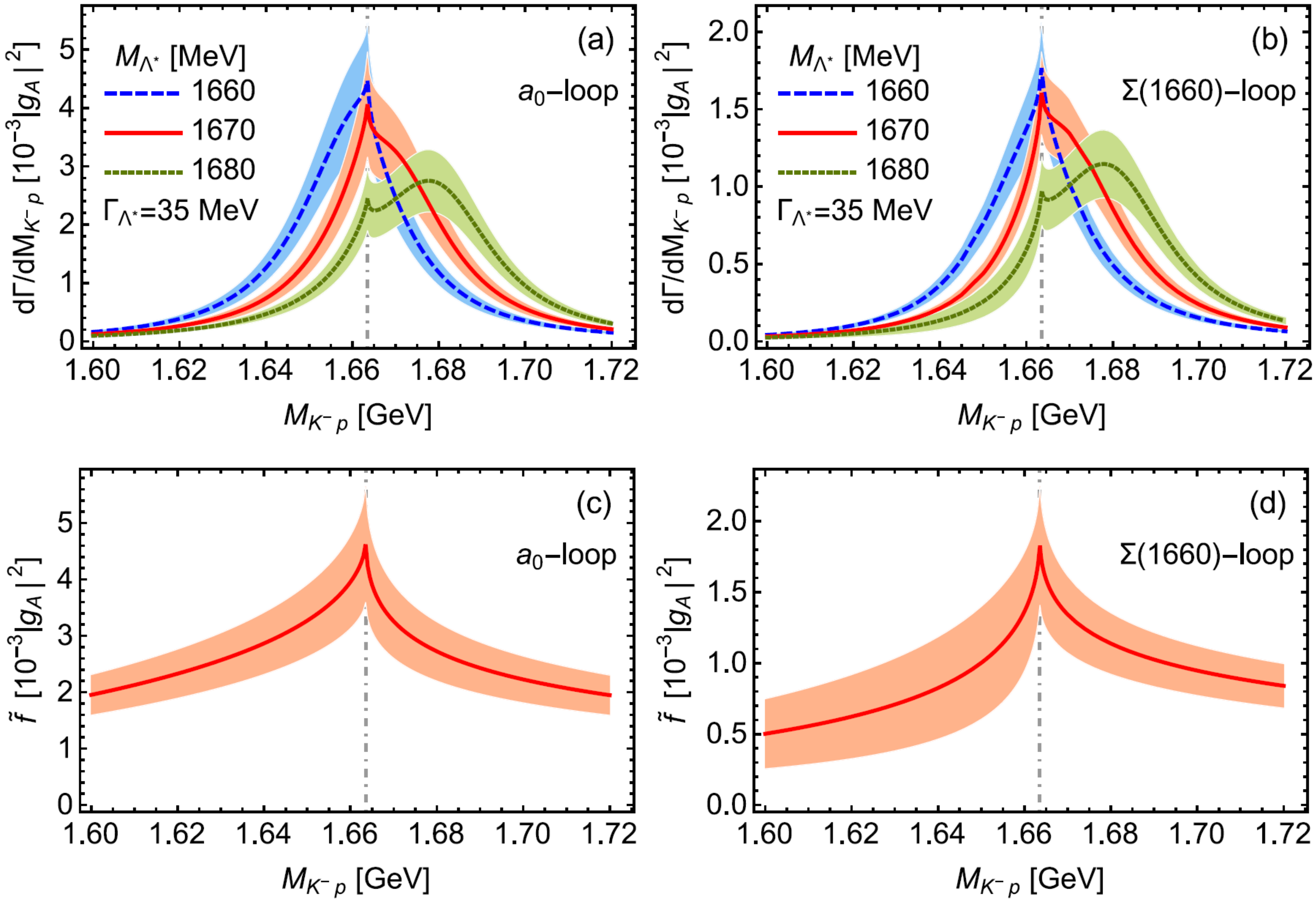


FIG. 2: The TS location of $\mathcal{T}(s, m_2^2)$ in the complex s-plane. The thick line on the real axis represents the unitary cut starting from $s_{\rm th}$. The trajectory marked with triangle (box) is obtained by varying M_{a_0} (M_{Σ^*}) and fixing $\Gamma_{a_0} = 75$ MeV ($\Gamma_{\Sigma^*} = 100$ MeV).

Invariant Mass Distributions



$$\tilde{f}(M_{K^-p}) = \left| \frac{s - M_{\Lambda^*}^2 + iM_{\Lambda^*}\Gamma_{\Lambda^*}}{M_{\Lambda^*}\Gamma_{\Lambda^*}} \right|^2 \times \frac{d\Gamma}{dM_{K^-p}}$$

Liu, Li, Xie, Zhao, PRD100 (2019) 054006

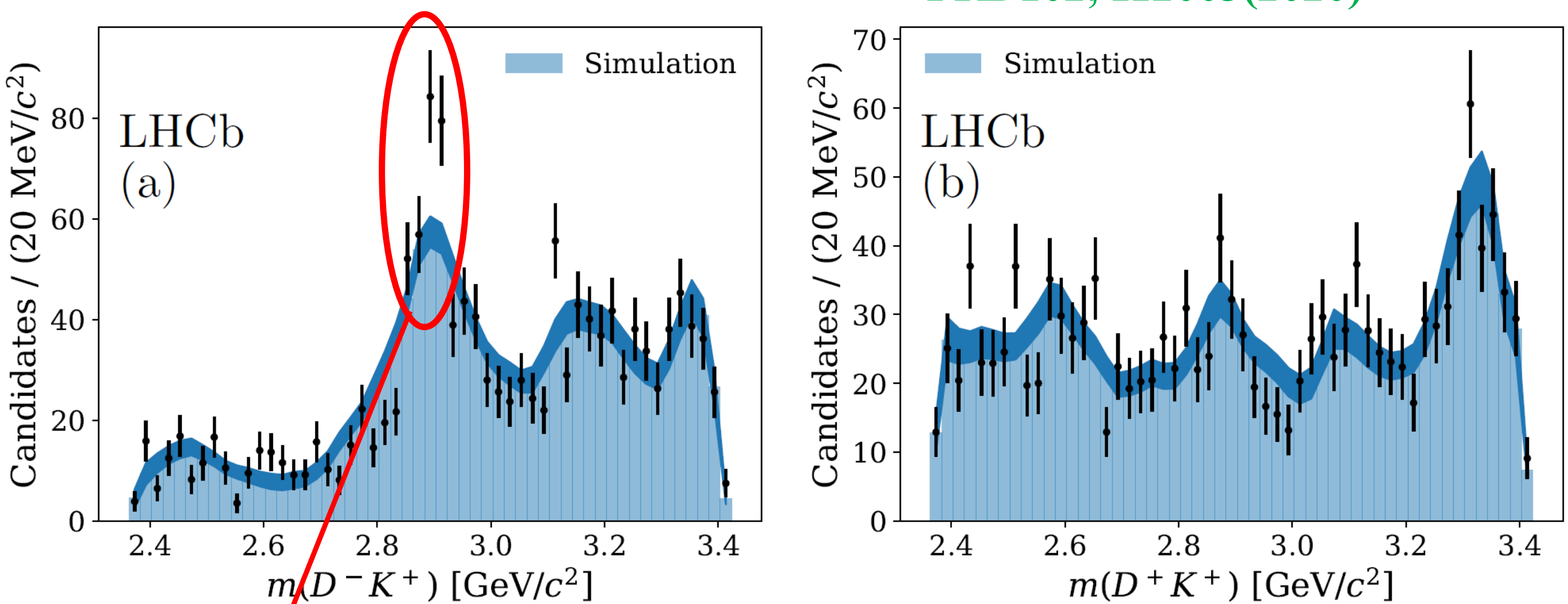
Threshold effects and newly observed

X(2900), Tcs(2900)

Observation of D^-K^+ ($\bar{c}\bar{s}ud$) structure

$$B^+ \to D^+ D^- K^+$$

LHCb, PRL125, 242001(2020); PRD102, 112003(2020)



States	Mass/MeV	Width/MeV	Fraction/%	J^P
$X_0(2900)$	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$5.6 \pm 1.4 \pm 0.5$	0^+
$X_1(2900)$	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$30.6 \pm 2.4 \pm 2.1$	1^-

Observation of D^-K^+ ($\bar{c}\bar{s}ud$) structure

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$X_1(2900)$	$2904 \pm 5 \pm 1$	$110 \pm 11 \pm 4$	$30.6 \pm 2.4 \pm 2.1$	1^-

Two close thresholds :

D^*K^* ~2902 MeV

D_1K ~2914 MeV

Interpretations :

J. He, D.Y. Chen, 2020

- **$\bar{D}^*K^*$, $\bar{D}_1K$ molecular state**
- **Tightly bound tetraquark state**

Predictions: an excited 0^+ tetraquark with mass 2850 MeV, and a 1^+ state with mass 2902 MeV are predicted. Many other states are also predicted

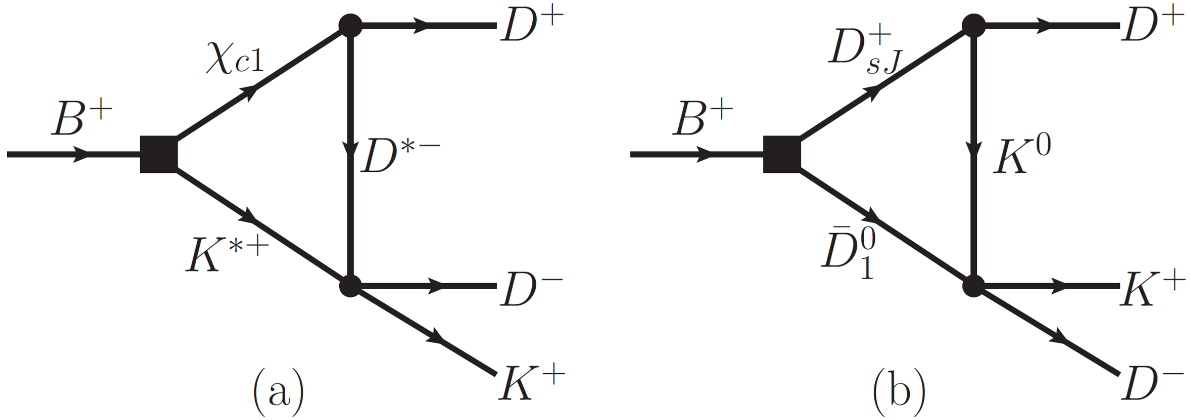
Y.R. Liu et al, PRD101, 114017(2020)

Puzzle

Chiral symmetry implies:

$\bar{D}_1K$ molecule ⟺ $\bar{D}K$ molecule

Threshold effects and $X_{0,1}(2900)$

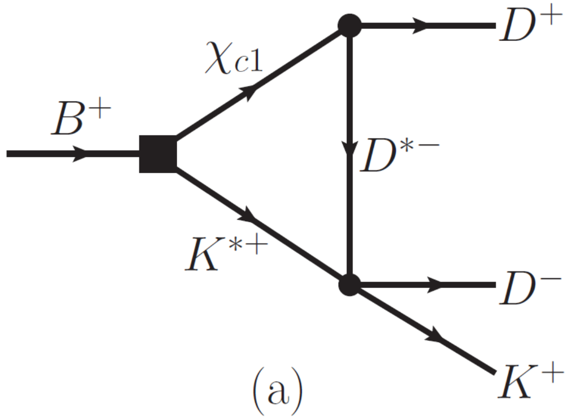


$$\mathcal{A}^{[\chi_{c1}K^{*+}D^{*-}]}_{B^+\to D^+D^-K^+} = -i\int \frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to\chi_{c1}K^{*+})}{(q_1^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*})}$$
$$\times\frac{\mathcal{A}(\chi_{c1}\to D^+D^{*-})\mathcal{A}(D^{*-}K^{*+}\to D^-K^+)}{(q_2^2-m_{\chi_{c1}}^2+im_{\chi_{c1}}\Gamma_{\chi_{c1}})(q_3^2-m_{D^*}^2)}, \quad (7)$$

$$\mathcal{A}^{[D^+_{sJ}\bar{D}_1^0K^0]}_{B^+\to D^+D^-K^+} = -i\int \frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to D^+_{sJ}\bar{D}_1^0)}{(q_1^2-m_{\bar{D}_1}^2+im_{\bar{D}_1}\Gamma_{\bar{D}_1})}$$
$$\times\frac{\mathcal{A}(D^+_{sJ}\to D^+K^0)\mathcal{A}(\bar{D}_1^0K^0\to D^-K^+)}{(q_2^2-m_{D_{sJ}}^2+im_{D_{sJ}}\Gamma_{D_{sJ}})(q_3^2-m_K^2)}. \quad (8)$$

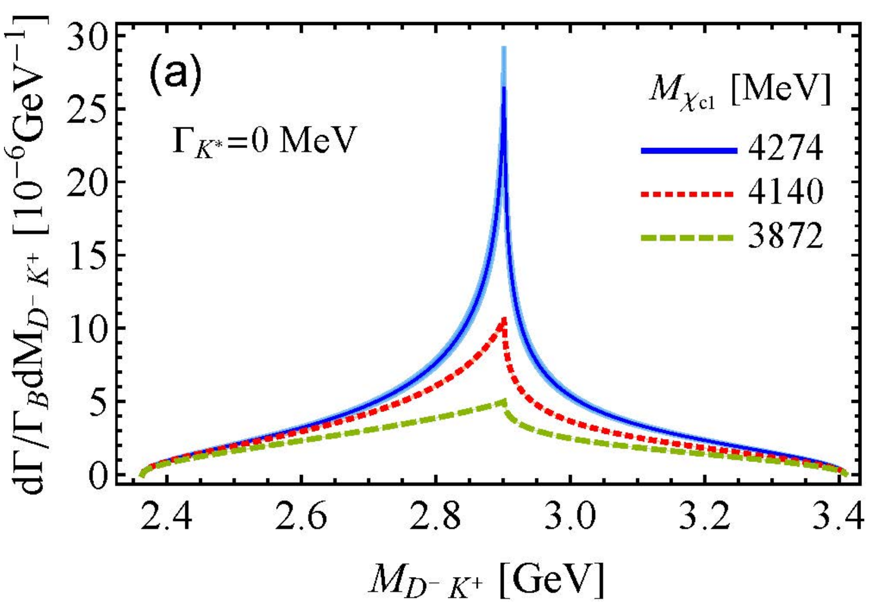
XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

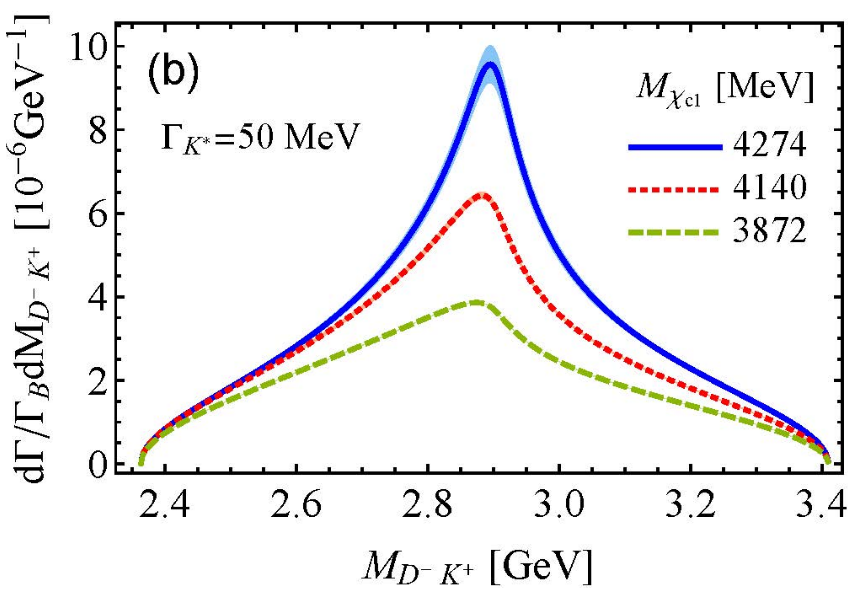
Threshold effects and $X_0(2900)$



$$\mathcal{A}(D^{*-}K^{*+} \to D^-K^+) = C_a \epsilon_{D^{*-}} \cdot \epsilon_{K^{*+}}$$

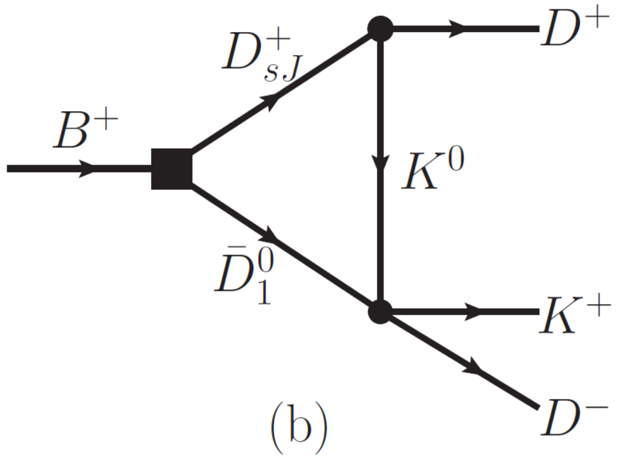
$J^P=0^+$





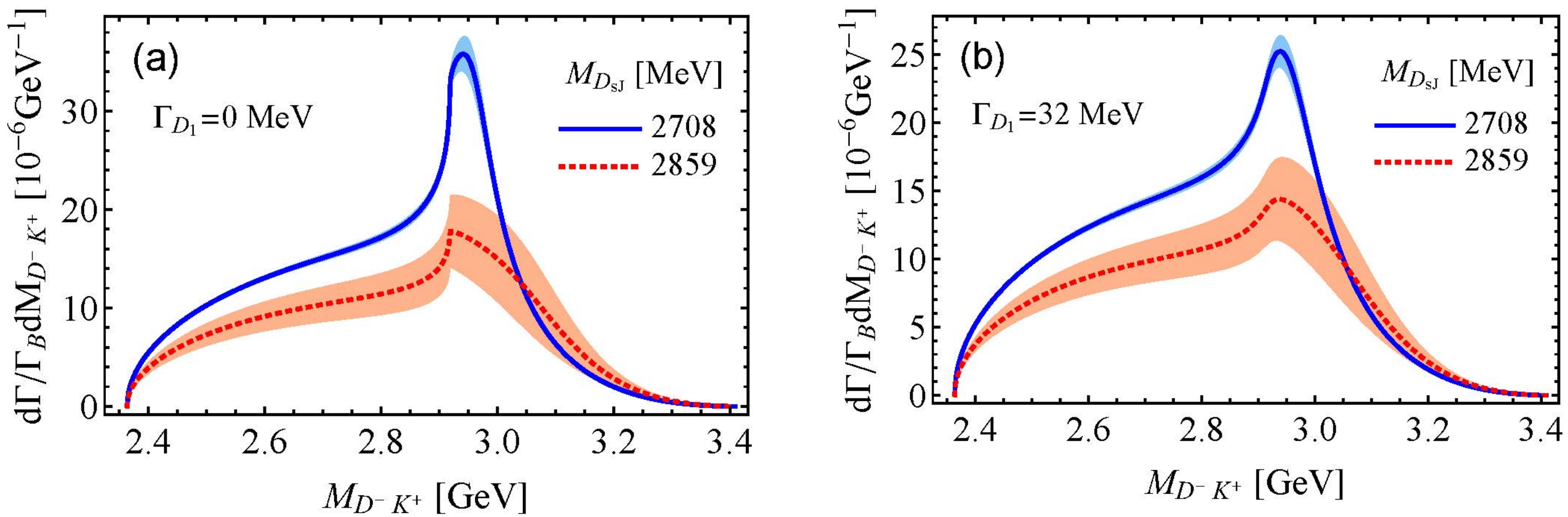
XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

Threshold effects and $X_1(2900)$



$$\mathcal{A}(\bar{D}_1^0 K^0 \to D^- K^+) = C_b(p_{K^0} + p_{K^+}) \cdot \epsilon_{\bar{D}_1^0}$$

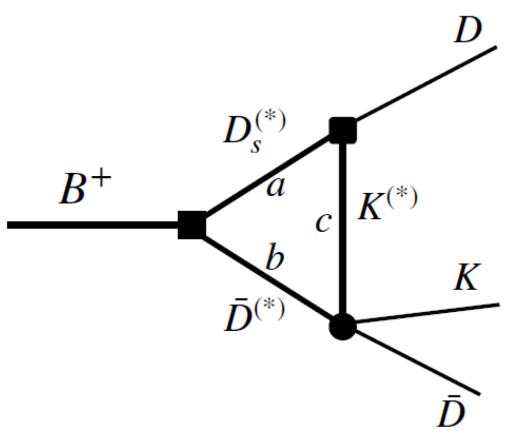
$J^P = 1^-$

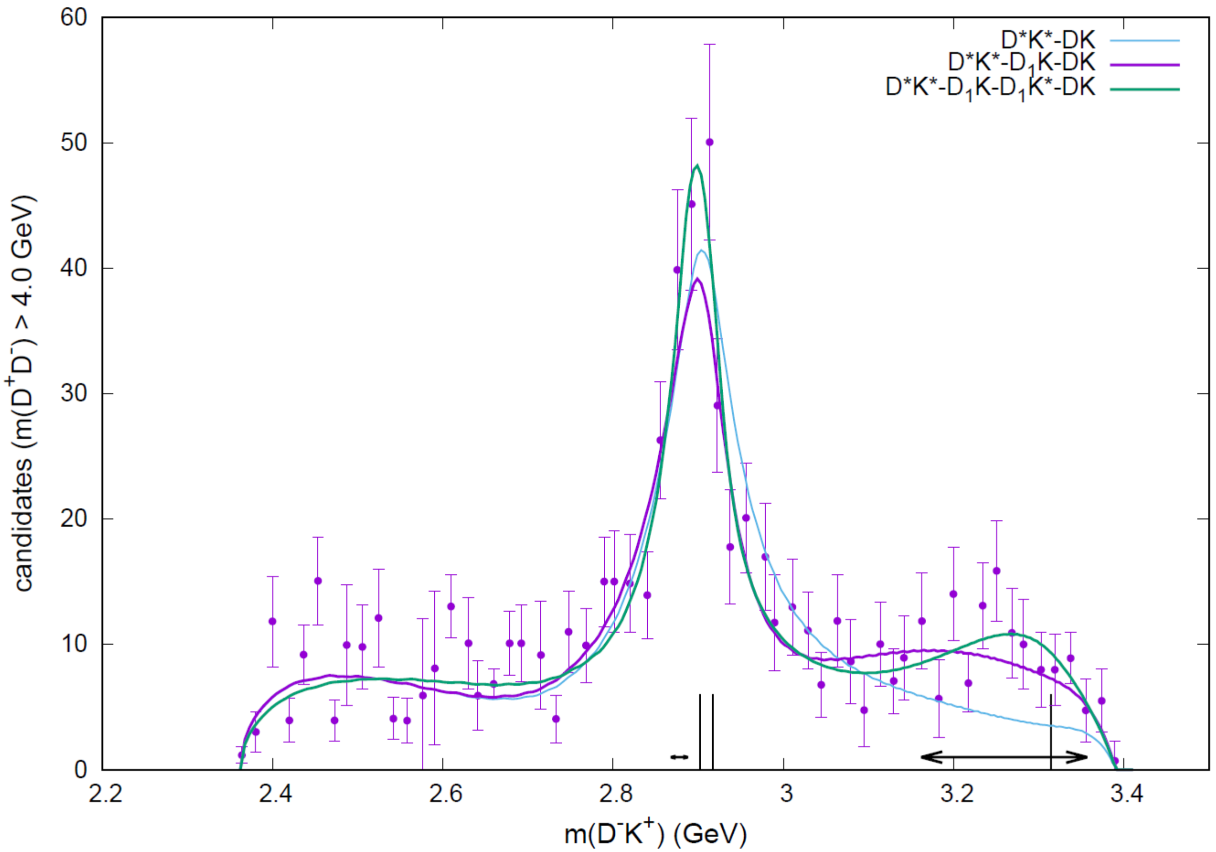


XHL, M.J. Yan, H.W. Ke, G. Li, J.J. Xie, arXiv:2008.07190

Threshold effects and $X_1(2900)$

T.J. Burns, E.S. Swanson, PLB813, 106057(2021)

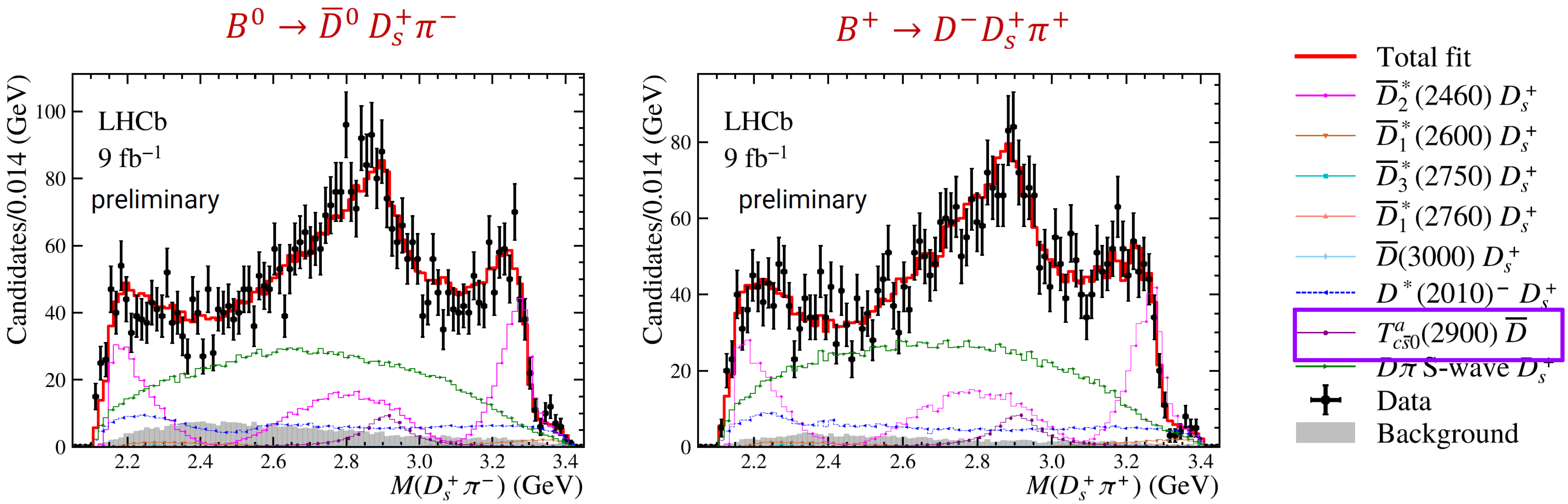




$$\triangle_{a\alpha}(s_{\bar{D}K}) = \int \frac{d^3q}{(2\pi)^3} F_{\rm ew}(\boldsymbol{q}+\boldsymbol{k}/2)\, F_{\rm 3P0}(3\boldsymbol{k}/4-\boldsymbol{q}/2)\, F_{L_\alpha}(q) Y_{L_\alpha M_\alpha}(\hat{q}) \cdot$$
$$[m_B - m_a^\alpha - m_b^\alpha - (\boldsymbol{q}+\boldsymbol{k}/2)^2/(2\mu_{ab}) + i\Gamma_a^\alpha/2 + i\Gamma_b^\alpha/2]^{-1} \cdot$$
$$[m_B - E_D - m_b^\alpha - m_c^\alpha - (\boldsymbol{q}+\boldsymbol{k}/2)^2/(2m_b^\alpha) - (\boldsymbol{q}-\boldsymbol{k}/2)^2/(2m_c^\alpha) + i\Gamma_b^\alpha/2 + i\Gamma_c^\alpha/2]^{-1}.$$

$$F_L(x) = \frac{x^L}{1+x^2}, \quad x = \frac{p}{\beta},$$

Threshold effects and $T_{cs}(2900)$

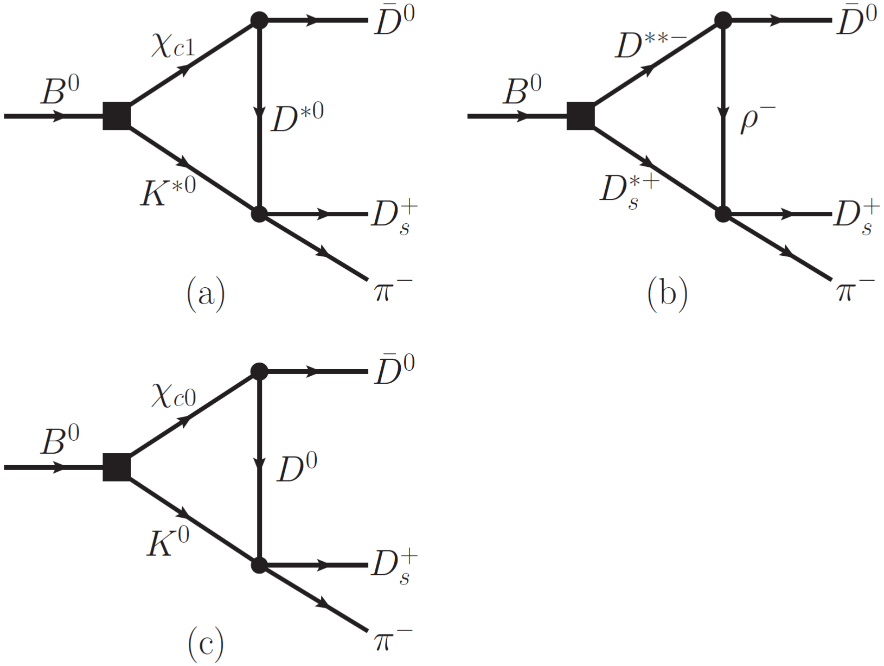


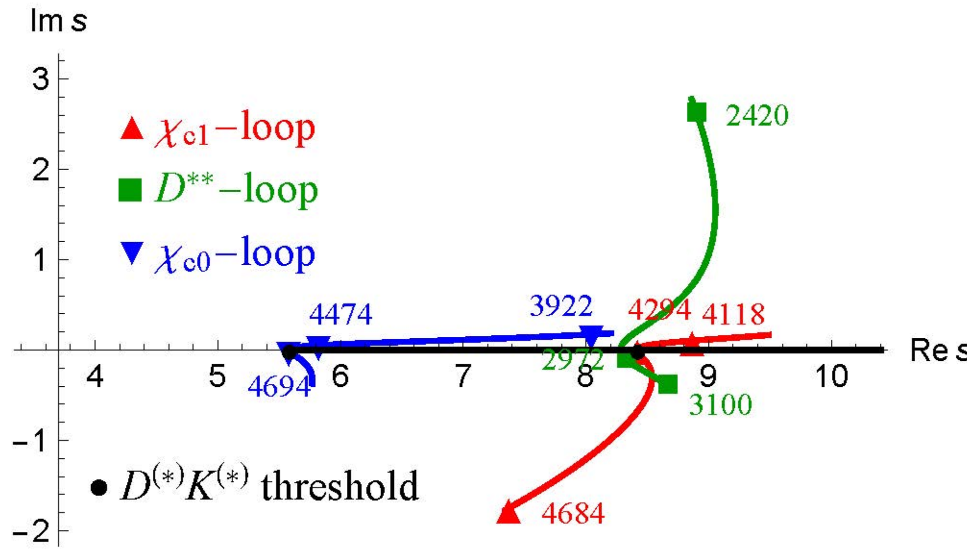
Spin-parity: $J^P = 0^+$

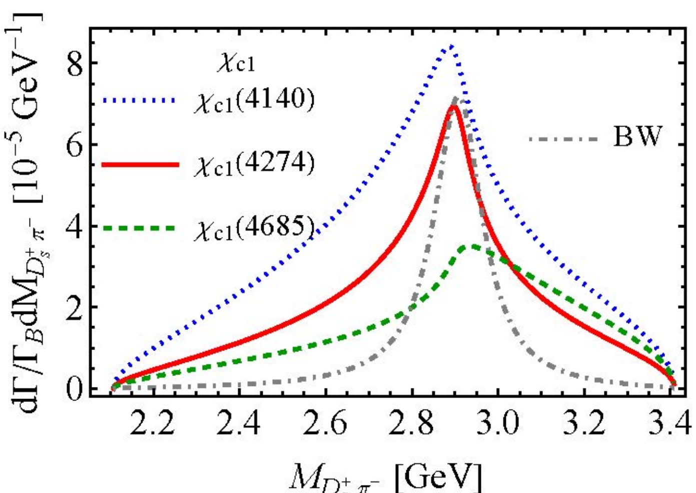
$$M = 2.908 \pm 0.011 \pm 0.020\,\mathrm{GeV}$$

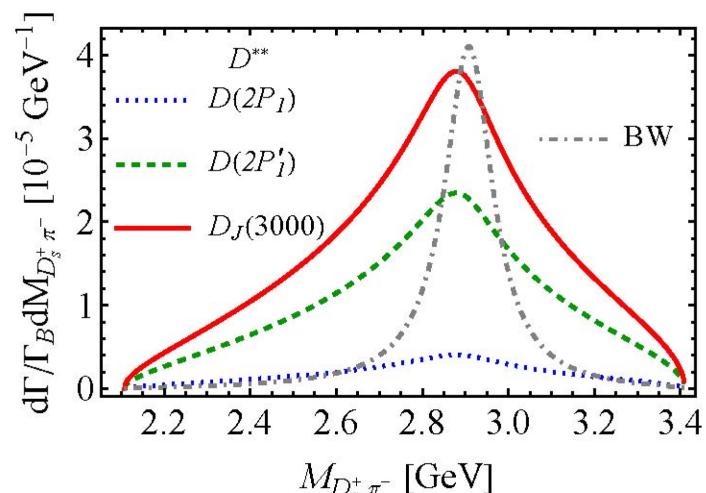
$$\Gamma = 0.136 \pm 0.023 \pm 0.011\,\mathrm{GeV}$$

Threshold effects and $T_{cs}(2900)$



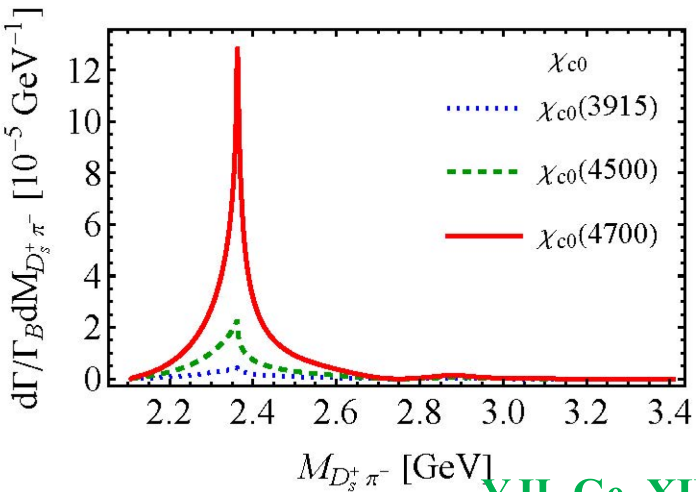


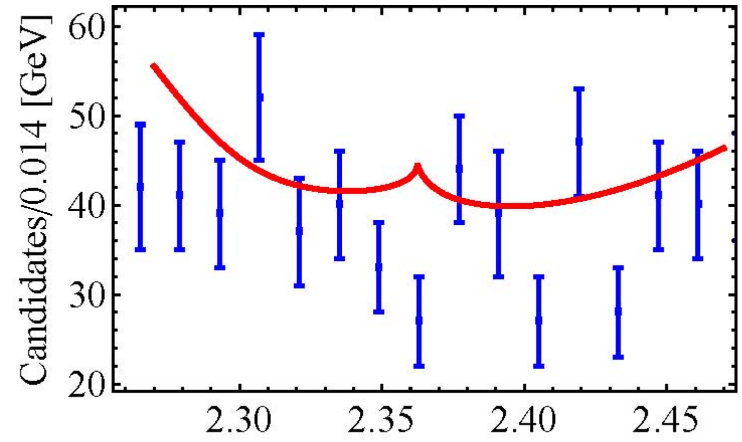




Y.H. Ge, XHL, H.W. Ke, arXiv:2207.09900

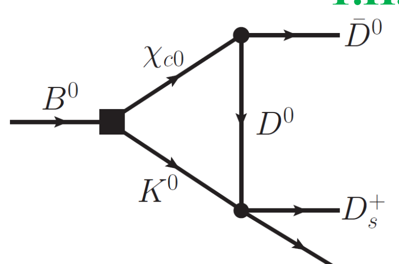
Non-resonant structure at DK threshold





Y.H. Ge, XHL, H.W. Ke, 2207.09900

$M_{D_s^+\pi^-}$ [GeV]



(c)

$$\mathcal{A} = \mathcal{A}_{bk} + e^{i\phi}\mathcal{A}^{loop}$$

$$\mathcal{A}_{bk} = b_0 e^{i\theta_0} T_{S-wave} + b_1 e^{i\theta_1} T_{D^*} + b_2 e^{i\theta_2} T_{D_2}$$

LHCb, arXiv:2212.02716

Particle	Amplitude	Phase	B^0 Fraction (%)	B^+ Fraction (%)
$T^a_{c\bar{s}0}(2900)$	$0.149 \pm 0.031 \pm 0.024$	$-1.26 \pm 0.22 \pm 0.34$	$2.45 \pm 0.65 \pm 0.69$	$2.55 \pm 0.64 \pm 0.68$
$D^*(2007)^0$	$2.58 \pm 0.11 \pm 1.07$	$-3.01 \pm 0.06 \pm 0.31$	–	$14.0 \pm 1.1 \pm 2.7$
$D^*(2010)^-$	$3.05 \pm 0.11 \pm 0.48$	$-2.91 \pm 0.06 \pm 0.28$	$17.0 \pm 1.0 \pm 2.4$	–
$D_2^*(2460)$	1	0	$22.35 \pm 0.76 \pm 0.72$	$22.53 \pm 0.74 \pm 0.51$
$D_1^*(2600)$	$0.218 \pm 0.030 \pm 0.051$	$0.13 \pm 0.16 \pm 0.22$	$1.28 \pm 0.39 \pm 0.60$	$1.32 \pm 0.38 \pm 0.59$
$D_3^*(2750)$	$0.153 \pm 0.032 \pm 0.039$	$-2.80 \pm 0.19 \pm 0.59$	$0.32 \pm 0.15 \pm 0.21$	$0.33 \pm 0.14 \pm 0.20$
$D_1^*(2760)$	$0.12 \pm 0.04 \pm 0.15$	$-0.18 \pm 0.34 \pm 1.01$	$0.26 \pm 0.27 \pm 1.37$	$0.28 \pm 0.26 \pm 1.35$
$D_J^*(3000)$	$1.44 \pm 0.23 \pm 1.14$	$1.40 \pm 0.23 \pm 1.33$	$0.45 \pm 0.16 \pm 0.33$	$0.46 \pm 0.15 \pm 0.32$
$D\pi$ S-wave	$1.142 \pm 0.045 \pm 0.074$	$-0.972 \pm 0.045 \pm 0.084$	$45.0 \pm 1.9 \pm 3.1$	$48.3 \pm 1.8 \pm 3.0$

Summary

- **Kinematic singularities can simulate resonance-like peaks in the invariant mass distribution, which implies that non-resonance interpretation for some exotic hadron candidates is possible.**
- **Being different from the genuine resonances, the TS mechanism is a process-dependent mechanism, and sensitive to the kinematic configurations.**

Model independent but Process dependent.

- **Study on threshold effects is necessary before claiming that a resonance-like structure is a genuine particle.**

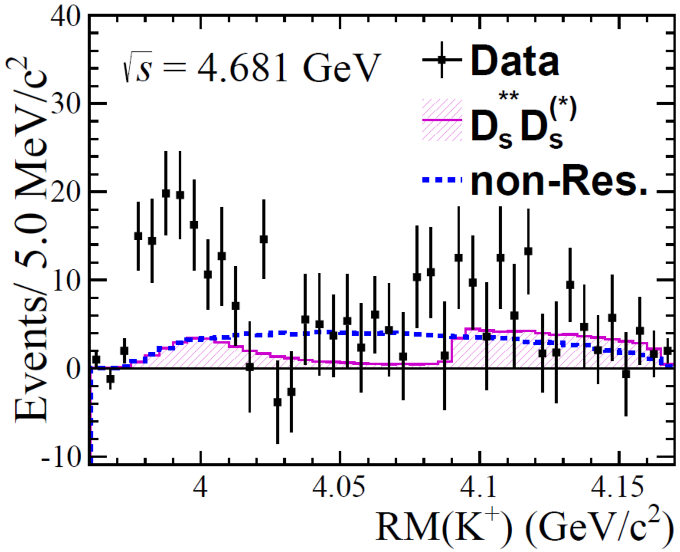
Thanks!

Backup

$Z_{cs}(3985)$ and $Z_{cs}(4000)$

BESIII, 2011.07855

$$e^+e^- \to K^+(D_s^- D^{*0} + D_s^{*-} D^0)$$

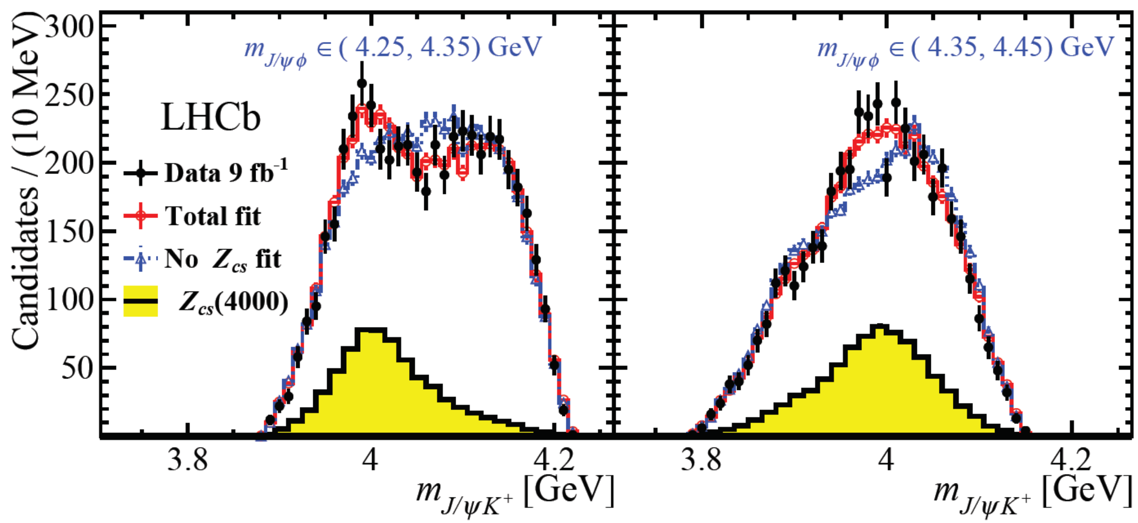


$$m_{\rm pole}(Z_{cs}(3985)^-) = (3982.5^{+1.8}_{-2.6} \pm 2.1)\,\text{MeV}/c^2,$$

$$\Gamma_{\rm pole}(Z_{cs}(3985)^-) = (12.8^{+5.3}_{-4.4} \pm 3.0)\,\text{MeV}.$$

LHCb, 2103.01803

$$B^+ \to J/\psi \phi K^+$$



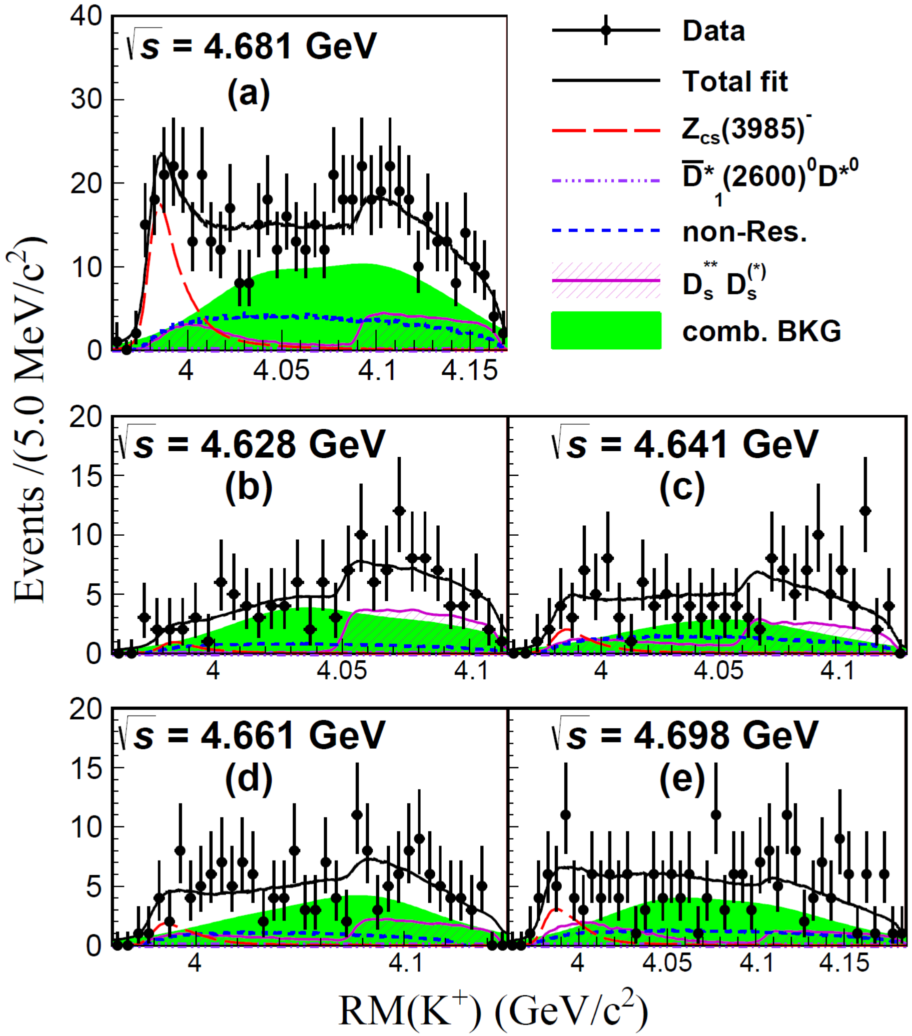
$$Z_{cs}(4000)$$

$$M = 4003 \pm 6^{+4}_{-14}\ \text{MeV}, \quad \Gamma = 131 \pm 15 \pm 26\ \text{MeV}$$

Widths are quite different

Different origin?

$Z_{cs}(3985)$ and $Z_{cs}(4000)$



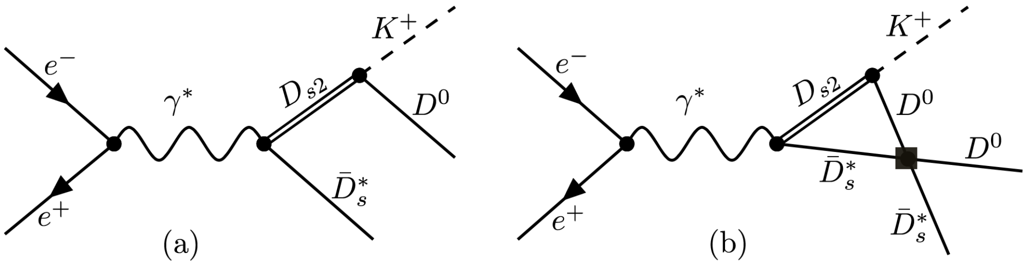
Theoretical interpretations:

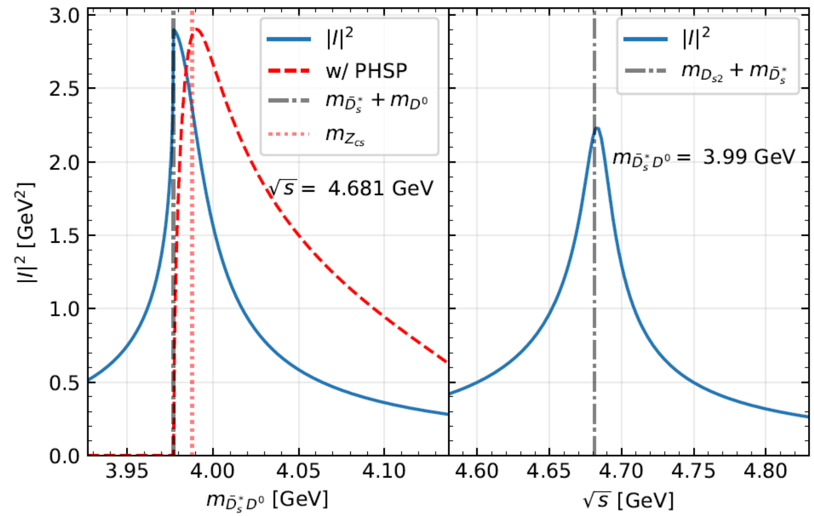
- Molecular partner of Zc(3900), DsD*+Ds*D interactions via exchanging σ, f_0, η, J/ψ …
- Virtual state — Ortega, Entem, Fernandez, 2103.07781
- Compact tetraquark — Yang, Cao, Guo, Nieves, Valderrama 2011.08725
- Threshold effects
- Reflection effects

Wang, Zhou, Liu, Matsuki, 2011.08628

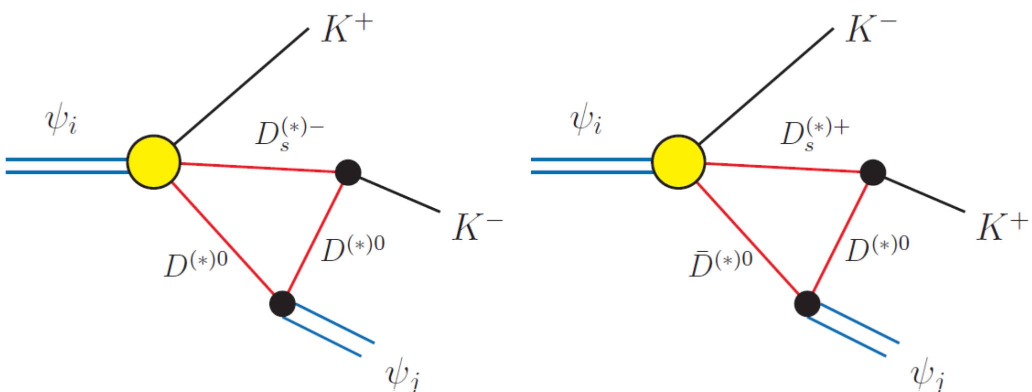
$D_s^{*-} D_{s2}^*(2573)^+$ threshold~ 4681 MeV

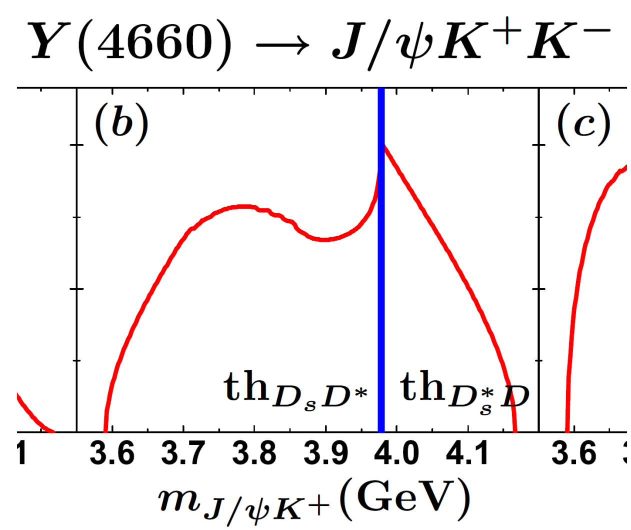
Z_{cs}(3985) and Z_{cs}(4000)





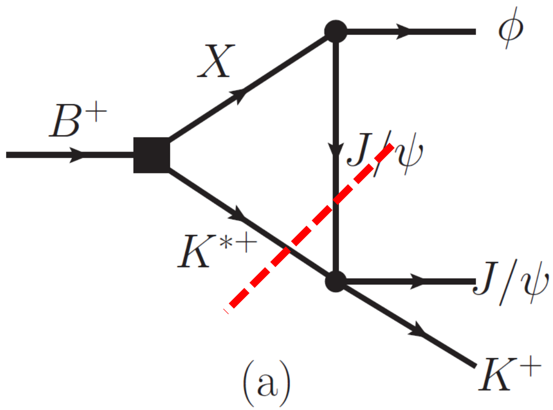
Yang, Cao, Guo, Nieves, Valderrama, 2011.08725





D.Y. Chen, X. Liu, T. Matsuki, PRL110, 232001(2013)

Threshold effects and $Z_{cs}(4000)$



J/ψK* threshold~ 3989 MeV

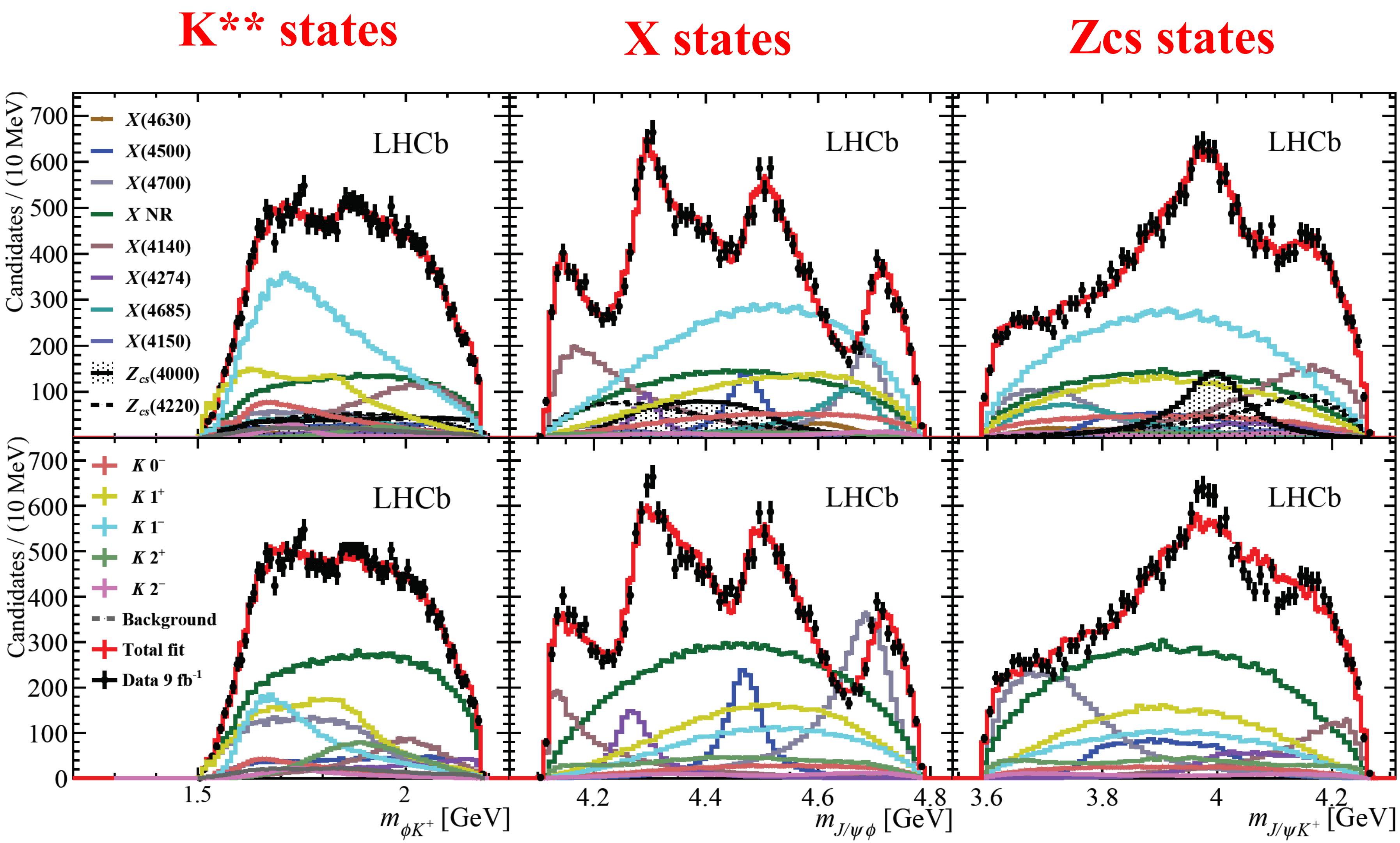
TS kinematic region

Diagram	$M_X/M_{K^{**}}$	$M_{J/\psi K^+}$
Fig. 1(a)	M_X: 4372∼4388	3989∼4005
Fig. 1(b)	$M_{K^{**}}$: 2068∼2182	3989∼4099

Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

$B^+ \to J/\psi \phi K^+$

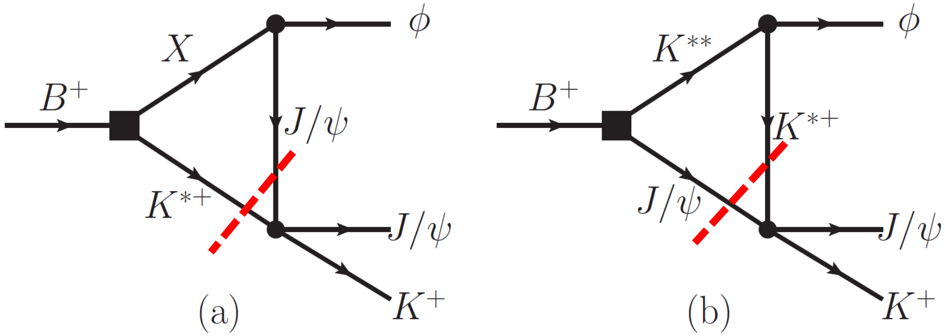
LHCb, 2103.01803



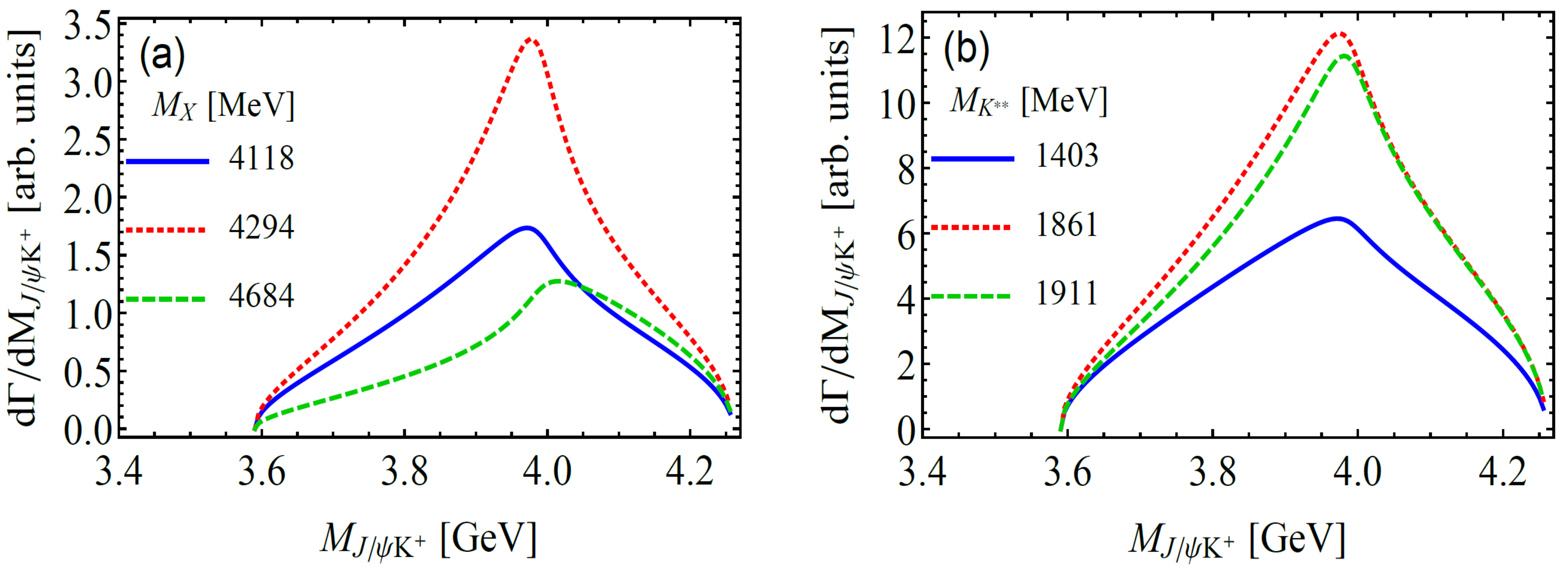
LHCb, 2103.01803

Contribution		Significance $[\times\sigma]$	M_0 [MeV]	Γ_0 [MeV]	FF [%]
	All $K(1^+)$				$25 \pm 4 ^{+\ 6}_{-\ 15}$
$2^1\mathrm{P}_1$	$K(1^+)$	4.5 (4.5)	$1861 \pm 10 ^{+\ 16}_{-\ 46}$	$149 \pm 41 ^{+\ 231}_{-\ 23}$[illegible]	
$2^3\mathrm{P}_1$	$K'(1^+)$	4.5 (4.5)	$1911 \pm 37 ^{+\ 124}_{-\ 48}$	$276 \pm 50 ^{+\ 319}_{-\ 159}$	
$1^3\mathrm{P}_1$	$K_1(1400)$	9.2 (11)	1403	174	$15 \pm 3 ^{+\ 3}_{-\ 11}$
	All $K(2^-)$				$2.1 \pm 0.4 ^{+\ 2.0}_{-\ 1.1}$
$1^1\mathrm{D}_2$	$K_2(1770)$	7.9 (8.0)	1773	186	
$1^3\mathrm{D}_2$	$K_2(1820)$	5.8 (5.8)	1816	276	
	All $K(1^-)$				$50 \pm 4 ^{+\ 10}_{-\ 19}$
$1^3\mathrm{D}_1$	$K^*(1680)$	4.7 (13)	1717	322	$14 \pm 2 ^{+\ 35}_{-\ 8}$
$2^3\mathrm{S}_1$	$K^*(1410)$	7.7 (15)	1414	232	$38 \pm 5 ^{+\ 11}_{-\ 17}$
	$K(2^+)$				
$2^3\mathrm{P}_2$	$K_2^*(1980)$	1.6 (7.4)	$1988 \pm 22 ^{+\ 194}_{-\ 31}$	$318 \pm 82 ^{+481}_{-\ 101}$	$2.3 \pm 0.5 \pm 0.7$
	$K(0^-)$				
$2^1\mathrm{S}_0$	$K(1460)$	12 (13)	1483	336	$10.2 \pm 1.2 ^{+\ 1.0}_{-\ 3.8}$
	$X(2^-)$				
	$X(4150)$	4.8 (8.7)	$4146 \pm 18 \pm 33$	$135 \pm 28 ^{+\ 59}_{-\ 30}$	$2.0 \pm 0.5 ^{+\ 0.8}_{-\ 1.0}$
	$X(1^-)$				
	$X(4630)$	5.5 (5.7)	$4626 \pm 16 ^{+\ 18}_{-\ 110}$	$174 \pm 27 ^{+\ 134}_{-\ 73}$	$2.6 \pm 0.5 ^{+\ 2.9}_{-\ 1.5}$
	All $X(0^+)$				$20 \pm 5 ^{+\ 14}_{-\ 7}$
	$X(4500)$	20 (20)	$4474 \pm 3 \pm 3$	$77 \pm 6 ^{+\ 10}_{-\ 8}$	$5.6 \pm 0.7 ^{+\ 2.4}_{-\ 0.6}$
	$X(4700)$	17 (18)	$4694 \pm 4 ^{+\ 16}_{-\ 3}$	87 ± 8 +1[illegible] −[illegible]	$8.9 \pm 1.2 ^{+\ 4.9}_{-\ 1.4}$
	$\mathrm{NR}_{J/\psi\phi}$	4.8 (5.7)			$28 \pm 8 ^{+\ 19}_{-\ 11}$
	All $X(1^+)$				$26 \pm 3 ^{+\ 8}_{-\ 10}$
	$X(4140)$	13 (16)	$4118 \pm 11 ^{+\ 19}_{-\ 36}$	$162 \pm 21 ^{+\ 24}_{-\ 49}$	$17 \pm 3 ^{+\ 19}_{-\ 6}$
	$X(4274)$	18 (18)	$4294 \pm 4 ^{+\ 3}_{-\ 6}$	$53 \pm 5 \pm 5$	$2.8 \pm 0.5 ^{+\ 0.8}_{-\ 0.4}$
	$X(4685)$	15 (15)	$4684 \pm 7 ^{+\ 13}_{-\ 16}$	$126 \pm 15 ^{+\ 37}_{-\ 41}$	$7.2 \pm 1.0 ^{+\ 4.0}_{-\ 2.0}$
	All $Z_{cs}(1^+)$				$25 \pm 5 ^{+\ 11}_{-\ 12}$
	$Z_{cs}(4000)$	15 (16)	$4003 \pm 6 ^{+\ 4}_{-\ 14}$	$131 \pm 15 \pm 26$	$9.4 \pm 2.1 \pm 3.4$
	$Z_{cs}(4220)$	5.9 (8.4)	$4216 \pm 24 ^{+\ 43}_{-\ 30}$	$233 \pm 52 ^{+\ 97}_{-\ 73}$	$10 \pm 4 ^{+\ 10}_{-\ 7}$

Threshold effects and $Z_{cs}(4000)$

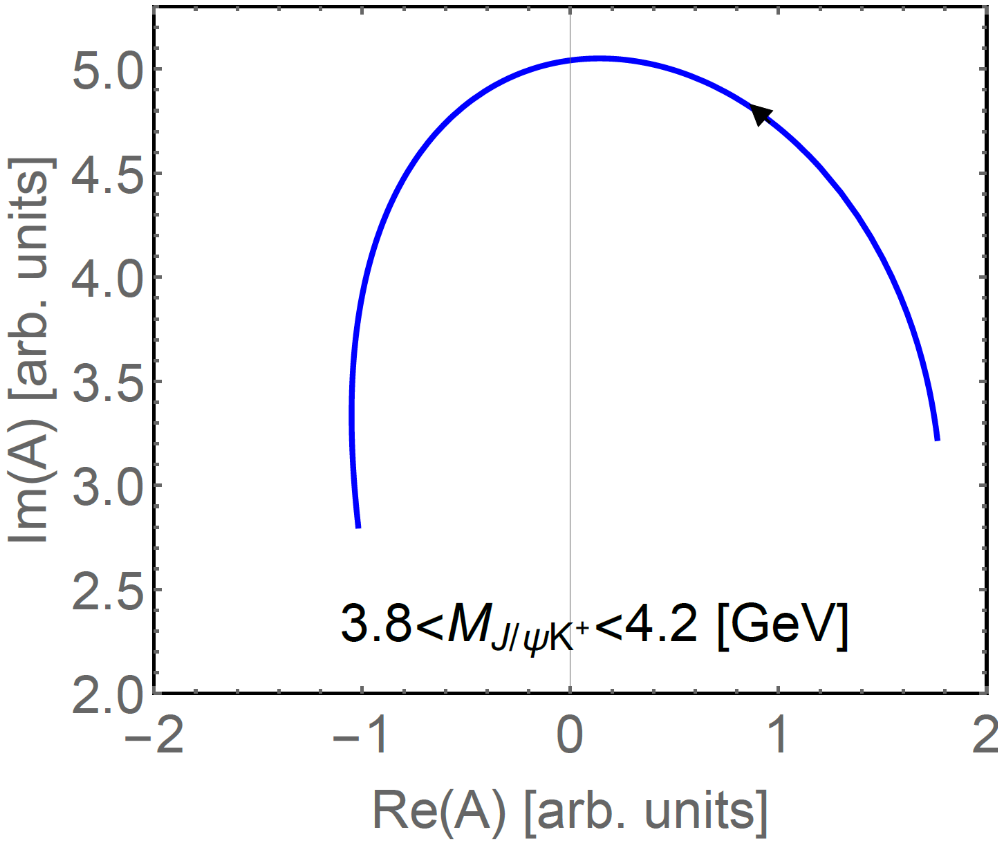


$$\mathcal{A}^{[K^{**}\psi K^*]}_{B^+\to J/\psi\phi K^+} = -i\int \frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to J/\psi K^{**})}{(q_1^2-M_{J/\psi}^2)} \times \frac{\mathcal{A}(K^{**}\to\phi K^{*+})\mathcal{A}(J/\psi K^{*+}\to J/\psi K^+)}{(q_2^2-M_{K^{**}}^2+iM_{K^{**}}\Gamma_{K^{**}})(q_3^2-M_{K^*}^2+iM_{K^*}\Gamma_{K^*})}$$



Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

Threshold effects and $Z_{cs}(4000)$

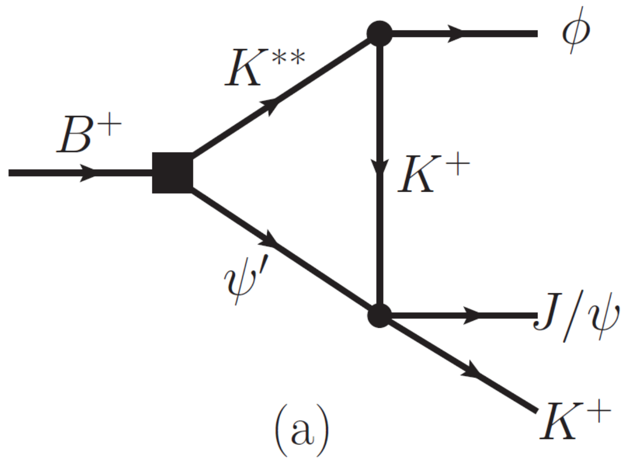


Argand plot

$$\mathcal{A}^{[K^{**}\psi K^{*}]}_{B^+\to J/\psi\phi K^+} = -i\int \frac{d^4q_1}{(2\pi)^4}\frac{\mathcal{A}(B^+\to J/\psi K^{**})}{(q_1^2-M_{J/\psi}^2)}$$

$$\times\frac{\mathcal{A}(K^{**}\to\phi K^{*+})\mathcal{A}(J/\psi K^{*+}\to J/\psi K^+)}{(q_2^2-M_{K^{**}}^2+iM_{K^{**}}\Gamma_{K^{**}})(q_3^2-M_{K^*}^2+iM_{K^*}\Gamma_{K^*})}$$

Threshold effects and $Z_{cs}(4220)$, X(4700)



X.H. Liu, PLB766, 117 (2017)

ψ(2S)K threshold~ 4180 MeV

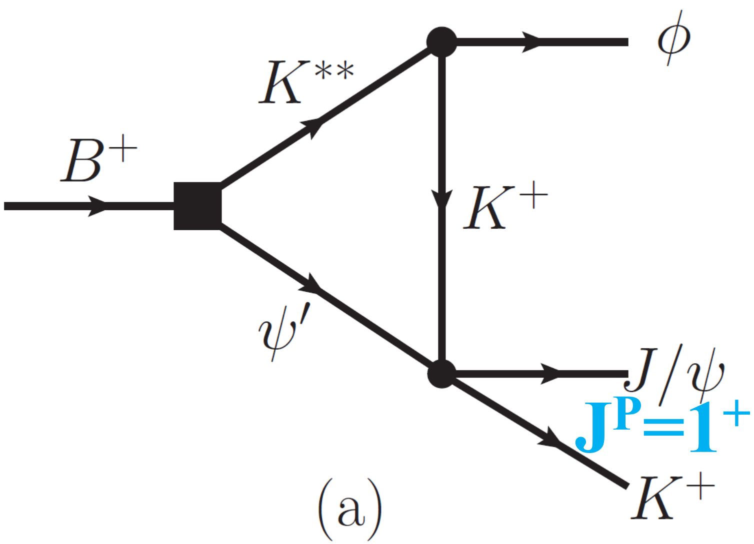
ψ(2S)ϕ threshold~ 4706 MeV

TS kinematic region

Diagram	$M_{K^{**}}$	$M_{J/\psi K^+}/M_{J/\psi\phi}$
Fig. 4(a)	1546∼1593	$M_{J/\psi K^+}$: 4180∼4226
Fig. 4(b)	1572∼1593	$M_{J/\psi\phi}$: 4706∼4727

Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

Threshold effects and $Z_{cs}(4220)$, X(4700)



$$\mathcal{A}(B^+ \to \psi' K^{**}) = a' \, \epsilon^*(\psi') \cdot \epsilon^*(K^{**})$$

$K^{**}(1^+) \to \phi K^+$

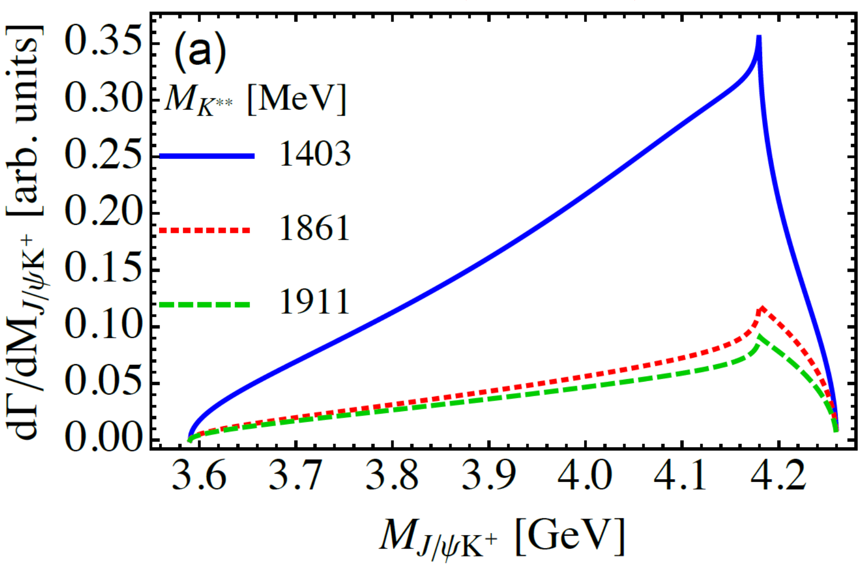
$$\mathcal{A}(K^{**} \to \phi K^+) = g_A \epsilon(K^{**}) \cdot \epsilon^*(\phi)$$

$K^{**}(1^-) \to \phi K^+$

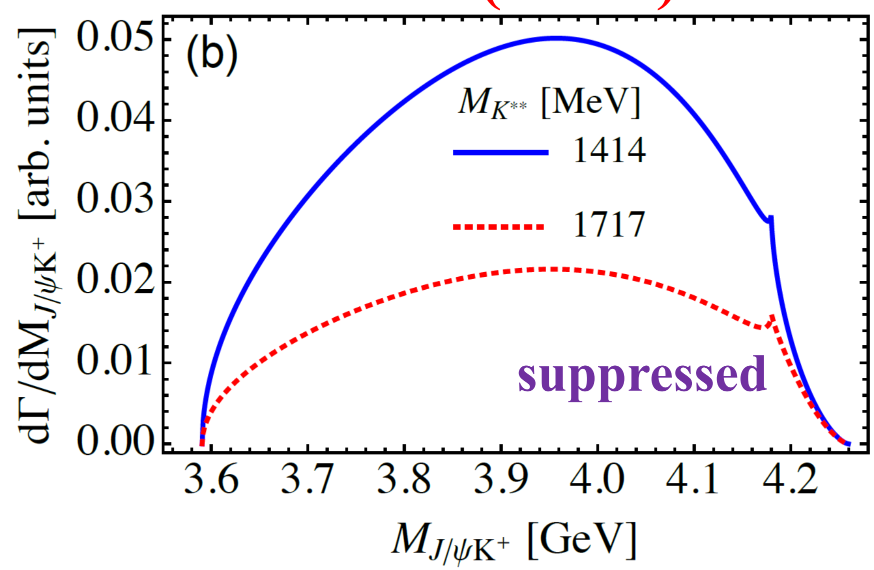
$$\mathcal{A}(K^{**} \to \phi K^+) = g_V \varepsilon_{\mu\nu\alpha\beta} p^{\mu}_{K^{**}} p^{\nu}_{\phi} \epsilon^{\alpha}(K^{**}) \epsilon^{*\beta}(\phi)$$

$$\mathcal{A}(\psi' K^+ \to J/\psi K^+) = g_{\psi' K} \, \epsilon(\psi') \cdot \epsilon(J/\psi)$$

$K^{**}(J^P=1^+)$

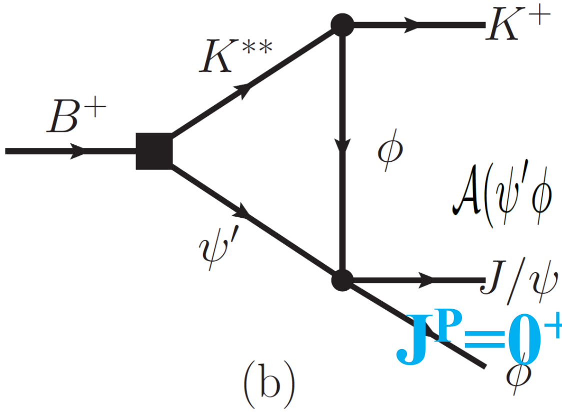


$K^{**}(J^P=1^-)$



Y.H. Ge, XHL, H.W. Ke, arXiv:2103.05282

Threshold effects and $Z_{cs}(4220)$, X(4700)



$J^P=0^+$, 1^+

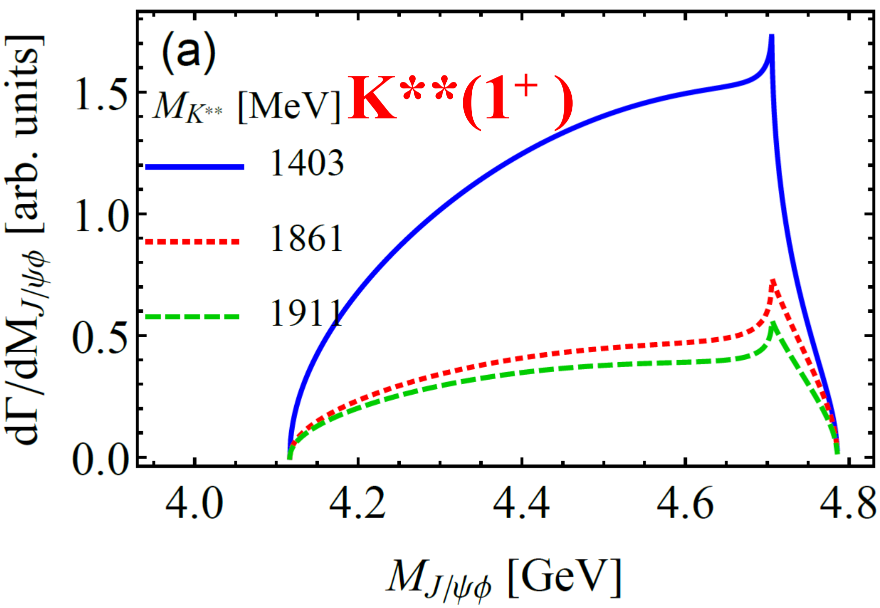
$J/\psi\phi$ ($J^P=0^+$)

$$\mathcal{A}(\psi'\phi \to J/\psi\phi) = g_{\psi\phi}\ \epsilon(\psi')\cdot\epsilon(\phi)\ \epsilon^*(J/\psi)\cdot\epsilon^*(\phi)$$

$$\mathcal{A}(\psi'\phi \to J/\psi\phi) = \tilde{g}_{\psi\phi}\ \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\gamma\delta\lambda\rho}(p^{\mu}_{J/\psi}+p^{\mu}_{\phi})(p^{\gamma}_{J/\psi}+p^{\gamma}_{\phi})g^{\nu\delta}\epsilon^{\alpha}(\psi')\epsilon^{\beta}(\phi)\epsilon^{*\lambda}(J/\psi)\epsilon^{*\rho}(\phi)$$

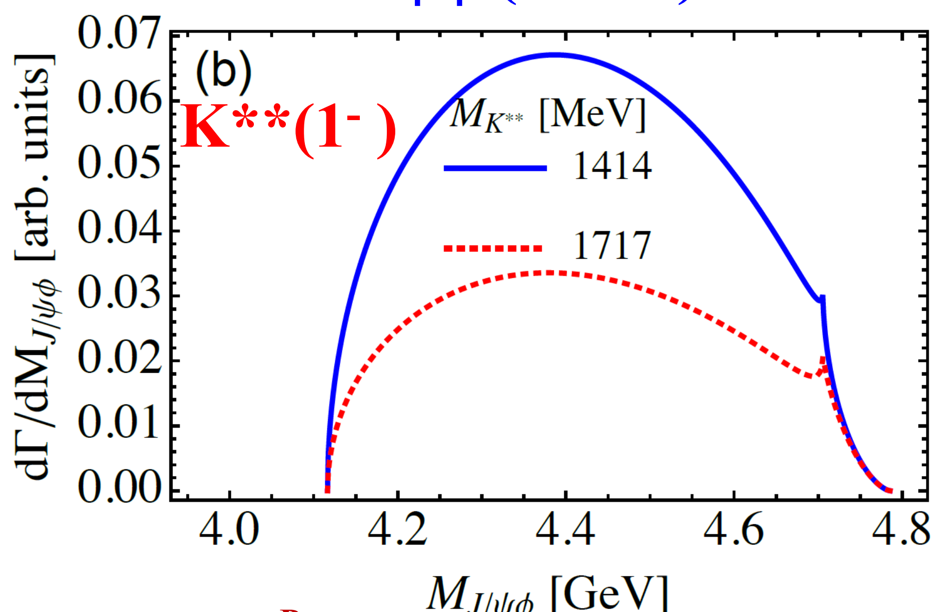
$J/\psi\phi$ ($J^P=1^+$)

$J/\psi\phi$ ($J^P=0^+$)



X(4700) $J^P=0^+$

$J/\psi\phi$ ($J^P=1^+$)



X(4685) $J^P=1^+$, cannot be simulated by the threshold effect

Distinguish Kinematic Singularities from Dynamic Poles

Cusp effect

➢ **A sharp peak cannot be resulted by a pure threshold cusp in the elastic channel [Guo, Hanhart, Wang, Zhao, PRD91, 051504(2015)]: *Z_c(3900)* was also observed in the DD* invariant mass distributions**