Lattice QCD calculation of the charmonium radiative decay

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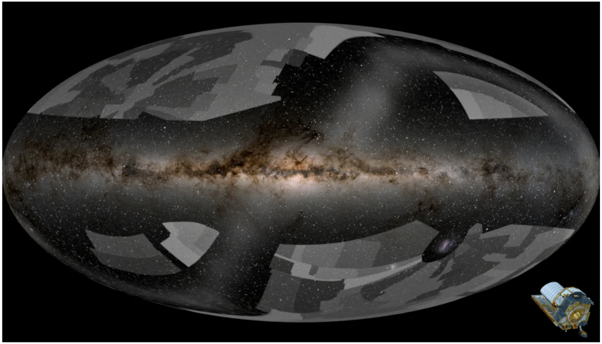
Outline

- Introduction
- $\eta_c \to 2\gamma$
- $J/\psi \to \gamma\eta_c$
- $J/\psi \to \gamma\nu\bar{\nu}$
- Summary and outlook

Euclid space telescope



ESA

- Launching: 11:12 a.m,Sat July 1, 2023
- Mission: 3D map of the universe, with billions of galaxies that stretch 10 billion light-years away

Light Dark Matter

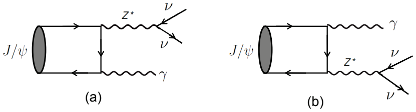
- Dark matter from collider:

$$J/\psi/\Upsilon \to \gamma + \text{invisible}$$

CLEO(2010) BaBar(2011) Belle(2019)

BESIII,PRD,**101**,112005(2020) $\mathcal{B}(\gamma + \text{invisible}) < 7 \times 10^{-7}$
BESIII,PRD,**105**,012008(2022) $\mathcal{B}(\gamma A_0)\mathcal{B}(\mu^+\mu^-) < 1 \times 10^{-9}$
BESIII,PLB,**838**,137698(2023) $\mathcal{B}(\gamma a) < 8.3 \times 10^{-8}$

- SM background: $J/\psi \to \gamma + \nu\bar{\nu}$



We provide an exact theoretical predication for the invisible decay.

Introduction: $\eta_c \to 2\gamma$

- Experiments

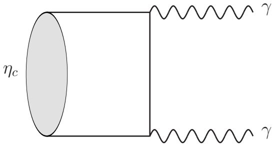
	Br $\times 10^5$	Note
CLEO	$0.7^{+1.6}_{-0.7} \pm 0.2$	PRL 101,101801(2008)
BESIII	$2.7 \pm 0.8 \pm 0.6$	PRD 87,032003(2013)
World average	$2.2^{+0.9}_{-0.6}$	PDG-aver
Global fit	1.68 ± 0.12	PDG-fit

- Theories: perturbative+nonperturbative

	$\Gamma(\mathrm{keV})$	Note
NRQCD	$9.9 \sim 10.6$	PRL**119**,252001(2017)
DSE	$6.32 \sim 6.39$	PRD**95**,016010(2017)
Lattice	$6.04(68)$	CLQCD(2020)
PDG-fit	$5.4(4)$	PDG-fit

- High precision lattice simulation $\Leftarrow$ new method+systematic analysis

$\eta_c \to 2\gamma$



- Amplitude:

$$\mathcal{M} = e^2 \epsilon_\mu(p)\epsilon_\nu(p')H_{\mu\nu}(p,q)$$

$$H_{\mu\nu}(p,q) = \int d^4x e^{-ipx}\mathcal{H}_{\mu\nu}(x,q), \quad \mathcal{H}_{\mu\nu} = \langle 0|\mathrm{Tr}[J_\mu(x)J_\nu(0)]|\eta_c(q)\rangle$$

- Form factor: $$H_{\mu\nu}(p,q) = \epsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta F(p^2)$$

- Decay width:

$$\Gamma_{\eta_c\gamma\gamma} = \alpha_{\mathrm{em}}^2 \frac{\pi}{4} m_{\eta_c}^3 |F_{\eta_c\gamma\gamma}|^2, \quad F_{\eta_c\gamma\gamma} = F(0)$$

Methodology

- Off-shell form factors by projecting discrete momentum $\vec{p} = \frac{2\pi\vec{n}}{L}$

$$F(p^2) \xrightarrow{\text{Cont.Limit}} F(0), \quad |\vec{p}| = \frac{m_{\eta_c}}{2}$$

Y.Chen et al(CLQCD), CPC**44**,083108(2020)

  - Systematics by momentum extraplation
  - Additional computation costs

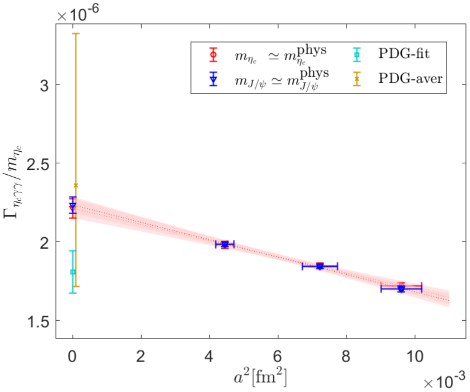
- Scalar function method: calculate on-shell form factor directly

$$F_{\eta_c\gamma\gamma}(\Delta t) = -\frac{1}{2m_{\eta_c}} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{x} \frac{j_1(|\vec{p}||\vec{\boldsymbol{x}}|)}{|\vec{p}||\vec{x}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x, \Delta t)$$

  - Finite-volume effect
  - Excited-state contamination

Results: $\eta_c \to 2\gamma$

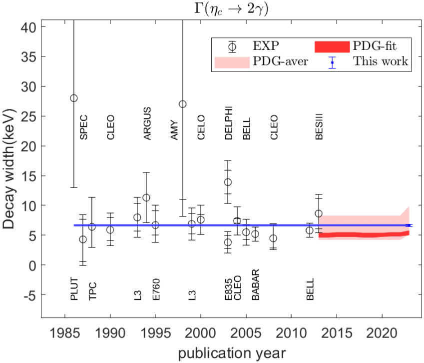
- NRQCD(NLO) ~ 6.2
  K.-T.Chao et al, PRD**56**,368(1997)
- NRQCD(NNLO) ~ 10
  F.Feng et al, PRL**119**,252001(2017)
- Lattice QCD $\sim 6.0(7)$
  CLQCD(2020)
- DSE ~ 6.4
  J.Chen et al, PRD**95**,016010(2017)

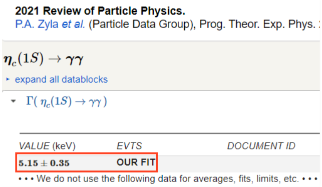


$$\Gamma(\eta_c \to 2\gamma) = \begin{cases} 6.67(16)(6)\,\text{keV} \\ 5.4(4)\,\text{keV} \quad \text{PDG-fit} \\ 7.04^{+2.9}_{-1.9}\,\text{keV} \quad \text{PDG-aver} \end{cases}$$

YM et al, accepted by Sci Bull

- The tension with PDG-fit is 2.9 σ





2021 Review of Particle Physics.
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys.

$\eta_c(1S) \to \gamma\gamma$

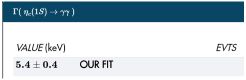
expand all datablocks

$\Gamma(\eta_c(1S) \to \gamma\gamma)$

VALUE (keV)	*EVTS*	*DOCUMENT ID*
$\mathbf{5.15 \pm 0.35}$	OUR FIT	

• • • We do not use the following data for averages, fits, limits, etc. • • •

PDG-update(2023)



$\Gamma(\eta_c(1S) \to \gamma\gamma)$

VALUE (keV)	*EVTS*
$\mathbf{5.4 \pm 0.4}$	OUR FIT

- H-P.Wang and C-Z.Yuan,New puzzle in charmonium decays, 2112.08545, CPC**46**,071001(2022) $5.15(35) \Rightarrow 5.43\left(^{+0.41}_{-0.38}\right)$
- HPQCD, $\Gamma_{\eta_c\gamma\gamma} = 6.788(45)_{\rm fit}(41)_{\rm syst}$ keV, 2305.06231

PDG global fit

An overall fit to total width, 10 combinations of particle width obtained from integrated cross section, 21 branching ratios uses 97 measurements and one constraint to determine 15 parameters. The overall fit has a $\chi^2 = 120.8$ for 83 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $\langle\delta x_i \delta x_j\rangle / (\delta x_i \cdot \delta x_j)$, in percent, from the fit to parameters p_i, including the branching fractions, $x_i = \Gamma_i / \Gamma_{total}$. The fit constrains the x_i whose labels appear in this array to sum to one.

x_1	100														
x_5	13	100													
x_8	25	14	100												
x_{15}	10	5	10	100											
x_{17}	5	3	6	2	100										
x_{34}	30	17	35	13	7	100									
x_{35}	15	8	17	6	3	48	100								
x_{38}	16	9	17	7	3	21	10	100							
x_{42}	12	7	13	5	3	21	10	8	100						
x_{45}	18	10	20	8	4	24	12	13	10	100					
x_{48}	17	10	19	7	4	26	13	12	9	14	100				
x_{50}	4	2	5	2	1	6	3	3	2	3	24	100			
x_{56}	-45	-25	-50	-19	-10	-60	-29	-31	-24	-36	-36	-9	100		
Γ	-2	-1	-2	-1	0	-3	-1	-2	-1	-2	6	1	-28	100	
x_{999}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	100
	x_1	x_5	x_8	x_{15}	x_{17}	x_{34}	x_{35}	x_{38}	x_{42}	x_{45}	x_{48}	x_{50}	x_{56}	Γ	x_{999}

	Mode	Rate (MeV)	Scale factor
Γ_1	$\eta_c(1S) \to \eta'(958)\pi\pi$	0.60 ± 0.08	
Γ_5	$\eta_c(1S) \to K^*(892)\overline{K}^*(892)$	0.20 ± 0.04	
Γ_8	$\eta_c(1S) \to \phi\phi$	0.051 ± 0.006	
Γ_{15}	$\eta_c(1S) \to \omega\omega$	0.066 ± 0.015	
Γ_{17}	$\eta_c(1S) \to f_2(1270) f_2(1270)$	0.31 ± 0.08	
Γ_{34}	$\eta_c(1S) \to K\overline{K}\pi$	2.25 ± 0.14	
Γ_{35}	$\eta_c(1S) \to K\overline{K}\eta$	0.42 ± 0.05	
Γ_{38}	$\eta_c(1S) \to K^+K^-\pi^+\pi^-$	0.206 ± 0.032	
Γ_{42}	$\eta_c(1S) \to 2(K^+K^-)$	0.044 ± 0.009	
Γ_{45}	$\eta_c(1S) \to 2(\pi^+\pi^-)$	0.277 ± 0.035	
Γ_{48}	$\eta_c(1S) \to p\overline{p}$	0.043 ± 0.004	
Γ_{50}	$\eta_c(1S) \to \Lambda\overline{\Lambda}$	0.033 ± 0.007	
Γ_{56}	$\eta_c(1S) \to \gamma\gamma$	0.0054 ± 0.0004	

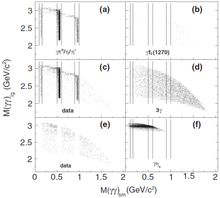
- The smaller errros of fitting may due to: (i)large uncertainties of $\eta_c \to 2\gamma$; (ii)Highly correlated with other decay channels ;

Discussion

- CLEO(08) and BESIII(13) extract the branching fraction of $\eta_c \to 2\gamma$ by

$$J/\psi \to \gamma\eta_c \to 3\gamma$$

- Using 1.06×10^8 $\psi(3686) \to \pi^+\pi^- J/\psi$



VALUE (10^{-4})	CL%	EVTS	DOCUMENT ID	TECN	COMMENT
1.68 ± 0.12 OUR FIT					
$2.2^{+0.9}_{-0.6}$ OUR AVERAGE					
$2.7 \pm 0.8 \pm 0.6$			[1] ABLIKIM 2013I	BES3	
$0.7\ ^{+1.6}_{-0.7} \pm 0.2$		$1.2\ ^{+2.8}_{-1.1}$	[2] ADAMS 2008	CLEO	$\psi(2S) \to \pi^+\pi^- J/\psi$
• • We do not use the following data for averages, fits, limits, etc. • •					
$2.0\ ^{+0.9}_{-0.7} \pm 0.2$		13	[3] WICHT 2008	BELL	$B^\pm \to K^\pm\gamma\gamma$
$2.80\ ^{+0.67}_{-0.58} \pm 1.0$			[4] ARMSTRONG 1995F	E760	$\overline{p}\,p \to \gamma\gamma$
< 9	90		[5] BISELLO 1991	DM2	$J/\psi \to \gamma\gamma\gamma$
$6\ ^{+4}_{-3} \pm 4$			[4] BAGLIN 1987B	SPEC	$\overline{p}\,p \to \gamma\gamma$
< 18	90		[6] BLOOM 1983	CBAL	$J/\psi \to \eta_c\gamma$

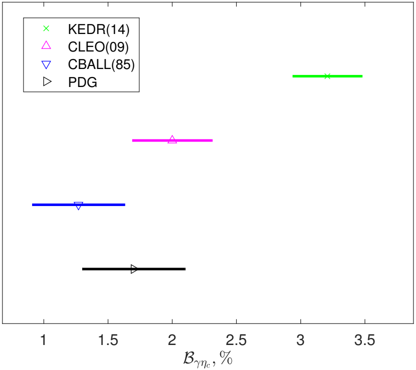
[1] ABLIKIM 2013I reports $[\Gamma(\eta_c(1S) \to \gamma\gamma)/\Gamma_{\text{total}}] \times [B(J/\psi(1S) \to \gamma\eta_c(1S))] = (4.5 \pm 1.2 \pm 0.6) \times 10^{-6}$ which we divide by our best value $B(J/\psi(1S) \to \gamma\eta_c(1S)) = 0.017 \pm 0.004$. Our first error is their experiment's error and our second error is the systematic error from using our best value.

[2] ADAMS 2008 reports $[\Gamma(\eta_c(1S) \to \gamma\gamma)/\Gamma_{\text{total}}] \times [B(J/\psi(1S) \to \gamma\eta_c(1S))] = (1.2\ ^{+2.7}_{-1.1} \pm 0.3) \times 10^{-6}$ which we divide by our best value $B(J/\psi(1S) \to \gamma\eta_c(1S)) = 0.017 \pm 0.004$. Our first error is their experiment's error and our second error is the systematic error from using our best value.

The contribution of $J/\psi \to \gamma\eta_c$ is crucial

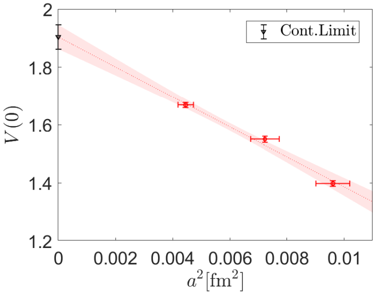
$J/\psi \to \gamma\eta_c$ radiative transition from lattice QCD

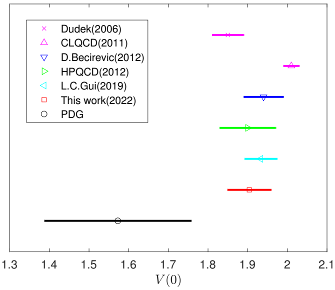
Experiments: $J/\psi \to \gamma\eta_c$



- KEDR(14), $\Gamma_{\eta_c} = 27.2 \pm 3.1 \left({}^{+5.4}_{-2.6} \right)$ MeV
- PDG(23), $\mathcal{B}_{\gamma\eta_c} = 1.7(4)\%, \Gamma_{\eta_c} = 32.0(0.7)$ MeV

Lattice results



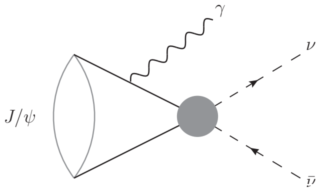


- Our result $\mathrm{Br}(J/\psi \to \gamma\eta_c) = 2.31(13)\%$ [To appear] $\Leftarrow 1.7(4)\%$ PDG
- Combining the BESIII(13), it gives

$$\Gamma(\eta_c \to 2\gamma) = 6.2(1.7)(0.9)\mathrm{keV} \Leftarrow 8.64(2.56)(1.92)\mathrm{keV} \quad \text{PDG-BES}$$

$J/\psi \to \gamma\nu\bar{\nu}$ decay width from lattice QCD

Decay width



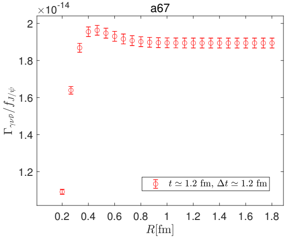
$$
\begin{aligned}
i\mathcal{M} &= -i\frac{(q_c e)G_F}{\sqrt{2}}[H_{\mu\nu\alpha}(q,p)\epsilon^{\alpha}_{J/\psi}(p)\epsilon^{\nu *}(q)\bar{u}(q_1)\gamma^{\mu}(1-\gamma_5)v(q_2)] \\
H_{\mu\nu\alpha}(q,p) &= \int d^4x e^{-iqx}\mathcal{H}_{\mu\nu\alpha}(x,p), \quad \mathcal{H}_{\mu\nu\alpha}(x,p) = \langle 0|T\{J^{\mathrm{em}}_{\mu}(x)J^{Z}_{\nu}(0)\}|J/\psi(p)_{\alpha}\rangle \\
&\equiv \epsilon_{\mu\nu\alpha\beta}q_{\beta}F_{\gamma\nu\bar{\nu}} \\
\Gamma(J/\psi\to\gamma\nu\bar{\nu}) &= \frac{q_c^2\alpha G_F^2}{3\pi^2}\int_0^{\frac{m_{J/\psi}}{2}} |\vec{q}|^3(m_{J/\psi}-|\vec{q}|)|F_{\gamma\nu\bar{\nu}}|^2 d|\vec{q}| \\
&\doteq \frac{q_c^2\alpha G_F^2}{3\pi^2}\sum_{i=1}^{N_{MC}}\left(E_{\gamma}^3(m_{J/\psi}-E_{\gamma})|F_{\gamma\nu\bar{\nu}}(E_{\gamma})|^2\right)_i, E_{\gamma}\equiv|\vec{q}|
\end{aligned}
$$

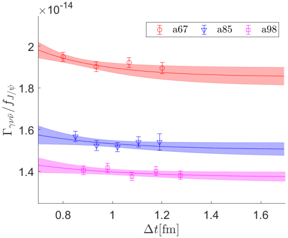
Methodology

- Scalar function method

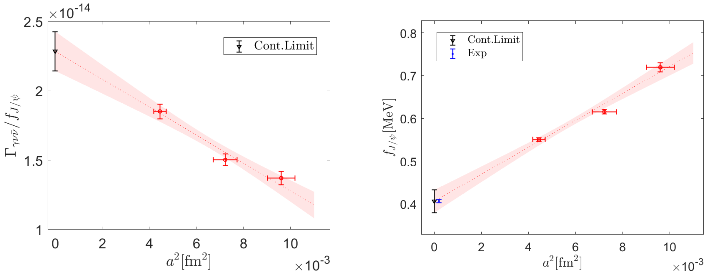
$$F_{\gamma\nu\bar{\nu}}(E_\gamma,\Delta t) = \frac{1}{6p\cdot q}\epsilon_{\mu\nu\alpha\beta}p_\beta H_{\mu\nu\alpha}(q,p)$$
$$= -\frac{i}{6E_\gamma}\int e^{E_\gamma t}dt\int d^3\vec{x}j_0(E_\gamma|\vec{x}|)\epsilon_{\mu\nu\alpha 0}\mathcal{H}_{\mu\nu\alpha}(x,\Delta t)$$

- Examining the finite-volume effects($R \equiv |\vec{x}|$)
- Removing the excited-state contamination





Preliminary results



- The first lattice QCD calculation [to appear]

$$\mathrm{Br}[J/\psi \to \gamma\nu\bar{\nu}] = 1.00(9)(7) \times 10^{-10}$$

- Phenomenological estimation $\sim 0.7 \times 10^{-10}$
  Dao-Neng Gao,PRD **90**,077501(2014)

Summary and outlook

- Summary
  - Lattice studies on $\eta_c \to 2\gamma, J/\psi \to \gamma\eta_c$ and $J/\psi \to \gamma\nu\bar{\nu}$ with the novel method
  - New puzzle in $\eta_c \to 2\gamma$, 2.9σ tension with the PDG value
  - Direct measurement with no-dependence on $J/\psi \to \gamma\eta_c$ is necessary
- Outlook
  - Toward precision test + new physics in charmonium sector

    BESIII,STCF,etc & Lattice QCD & Phenomenological

Thank you !