Lattice QCD Calculation of $0\nu2\beta$ Decay

Xin-Yu Tuo (脱心宇)

School of physics, Peking University

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In collaboration with Xu Feng (冯旭), Luchang Jin (靳路昶),

Zi-Yu Wang (王子毓) and Teng Wang (王腾)

Plan

1. Background: Lattice QCD and $0\nu2\beta$ decay
2. Lattice work 1: pionic $0\nu2\beta$ decay
3. Lattice work 2: sterile neutrino contribution
4. Outlook for g_ν^{NN}

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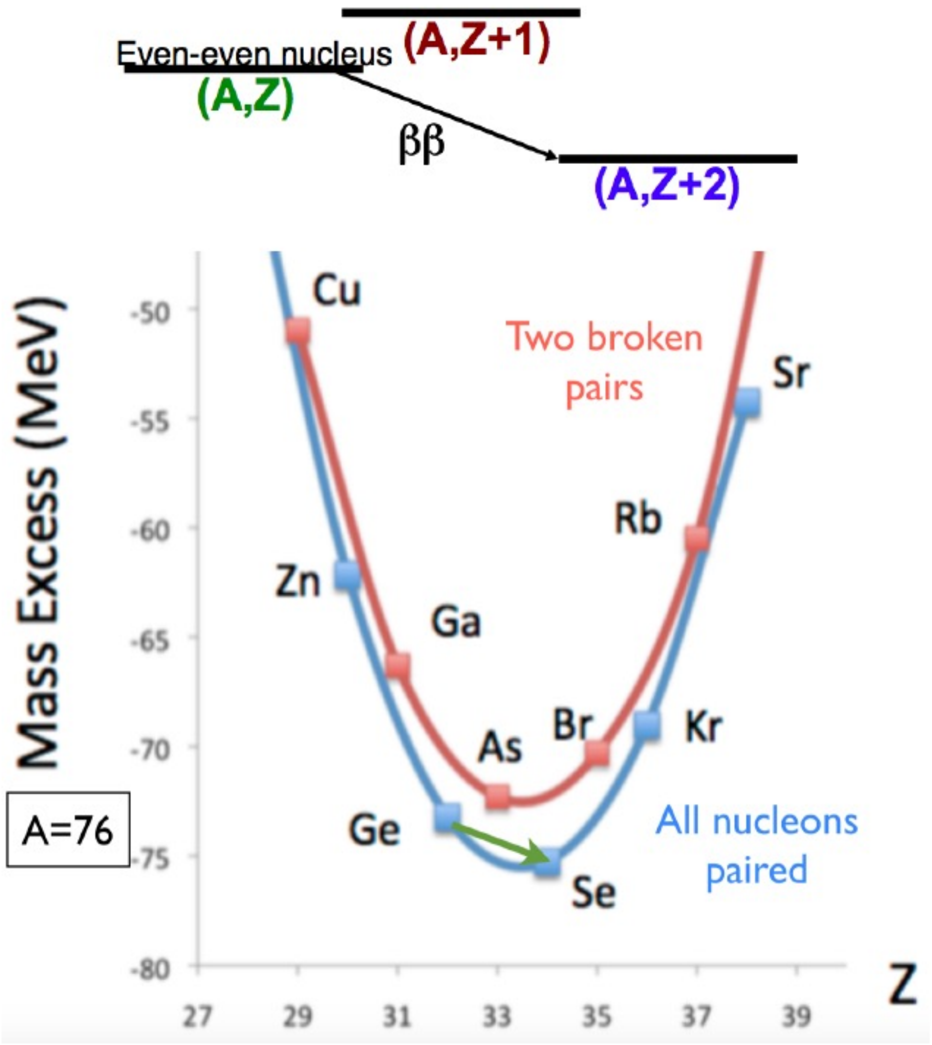
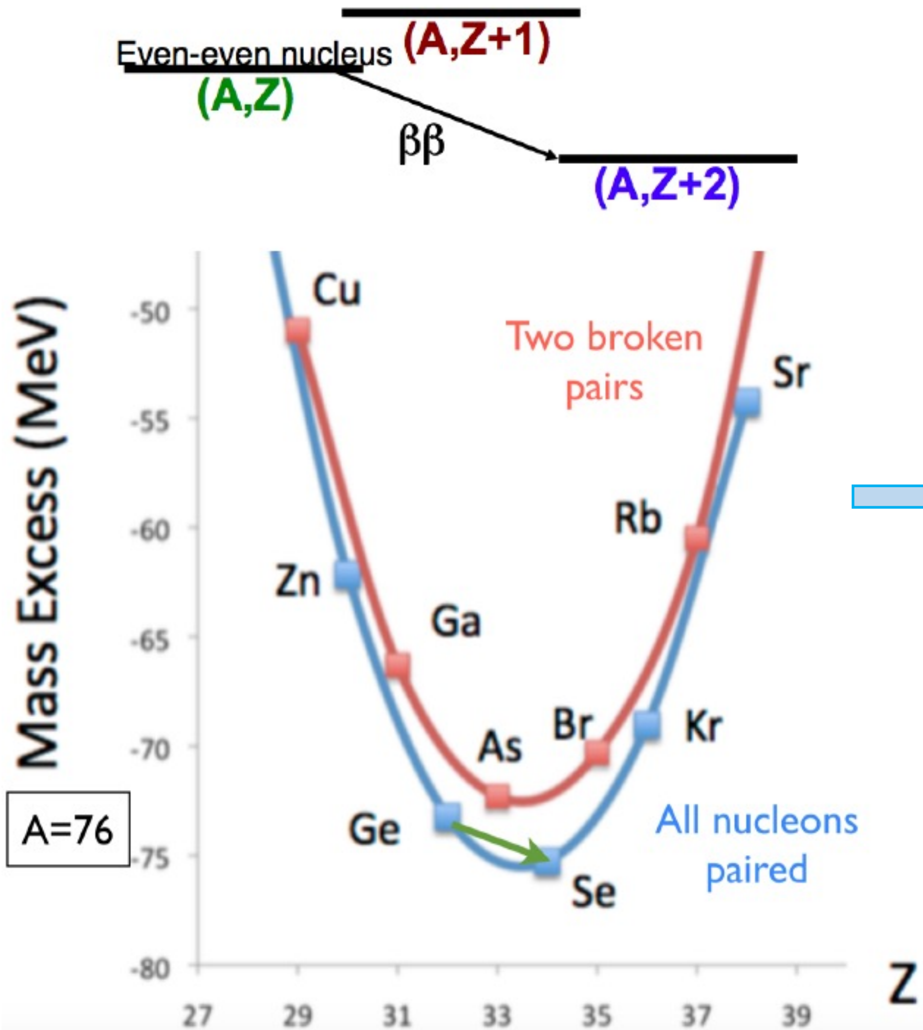
$0\nu2\beta$ decay



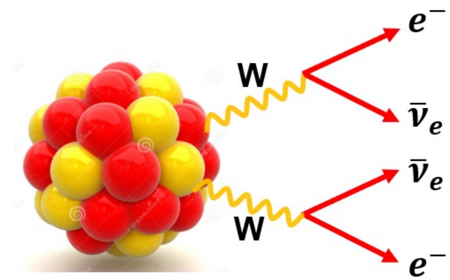
[1] Maria Goeppert-Mayer. Physical Review. 1935, 48(6):512

[2] Wendell H Furry. Physical Review. 1939, 56(12):1184

$0\nu2\beta$ decay

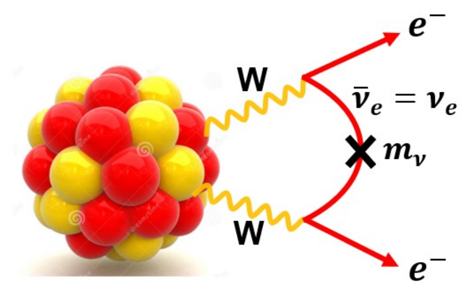


$2\nu2\beta$



1935： $2\nu2\beta$ decay [1]

$0\nu2\beta$



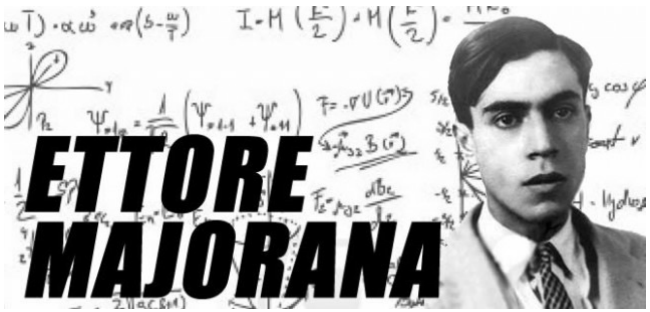
1939： $0\nu2\beta$ decay [2]

[1] Maria Goeppert-Mayer. Physical Review. 1935, 48(6):512

[2] Wendell H Furry. Physical Review. 1939, 56(12):1184

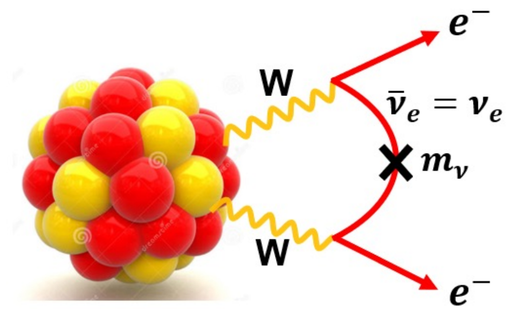
Why are $0\nu2\beta$ decays so important?

➢ Test the nature of neutrino: Dirac fermion? Majorana fermion?



1937: Majorana fermion [1]

$$\nu = \bar{\nu}\,?$$



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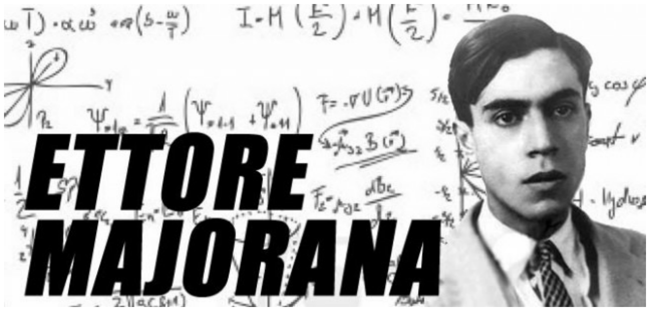

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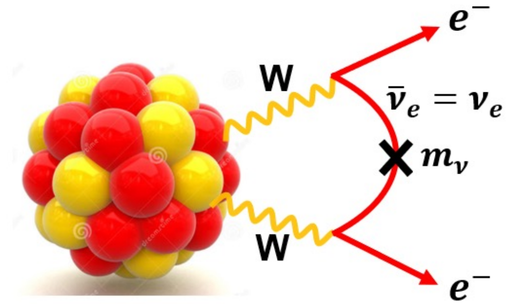
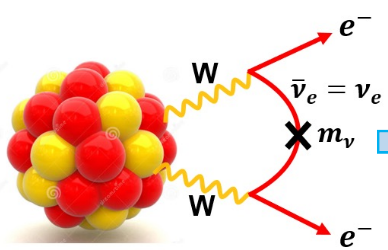
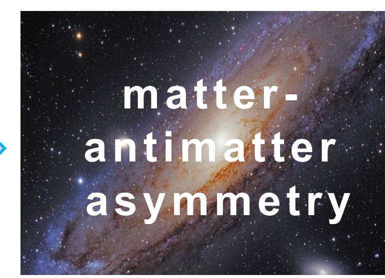


[1] Ettore Majorana. Nuovo Cim. 1937, 14:171–184

- Lepton-number violation: BSM

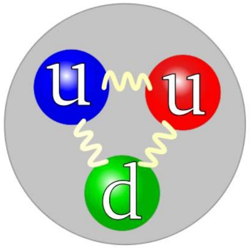


Lepton-number violation → Baryon-number violation →



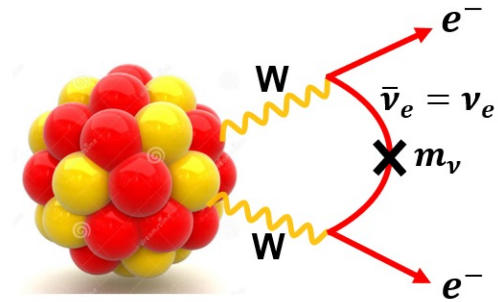
[2] M. A. Luty. Phys Rev. 1992, D45:455–465

Cooperation between EFTs and LQCD



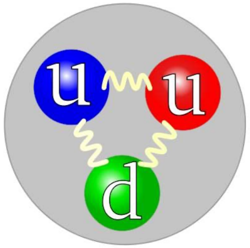
Particle physics

wide energy scales



Nuclear physics

Cooperation between EFTs and LQCD



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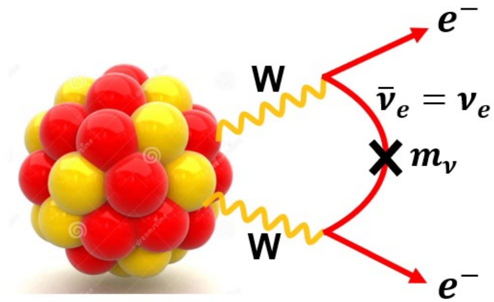
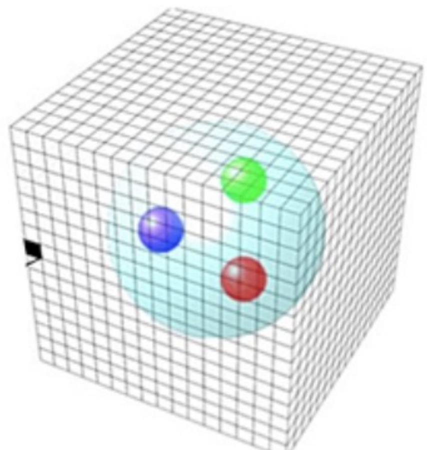


Nuclear physics

First principle
QCD calculation



Lattice QCD

Provide low energy
constants (LECs)
for ChPT/ChEFT

Effective Field Theories (EFTs)

Lattice QCD

nuclear many-body theory

} Cooperation

Vincenzo Cirigliano, et al. Snowmass 2021. arxiv:2203.12169

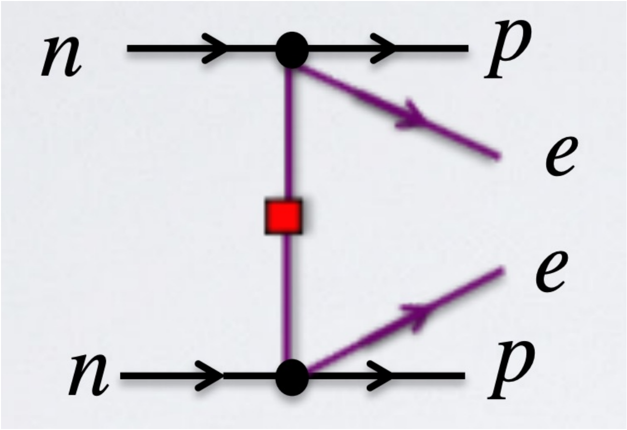
Lattice QCD inputs for $0\nu2\beta$ decay

- Long-range contribution:

  hadronic inputs: single-nucleon g_A

  ⇨ Extracted from lattice QCD

  Y. Aoki, et al. Eur Phys J C. 2022, 82(10):869



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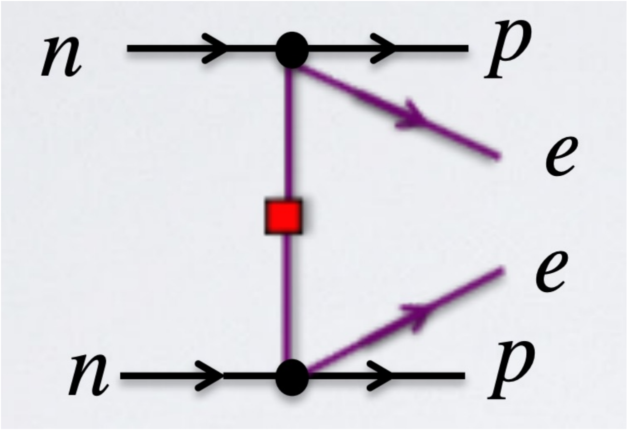
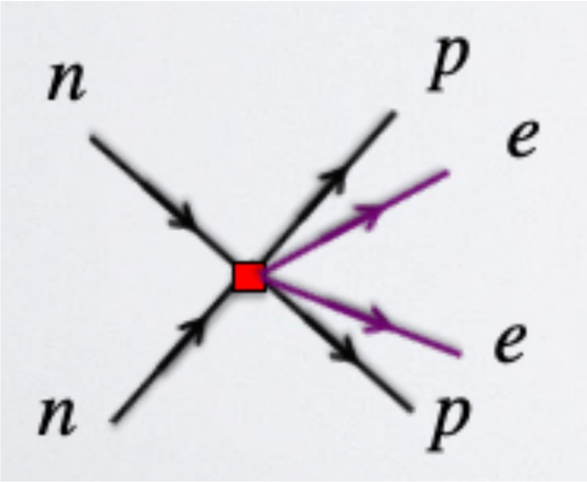


- Short-range contribution:

  contact term from hard neutrino exchange



$\sim g_\nu^{NN}$

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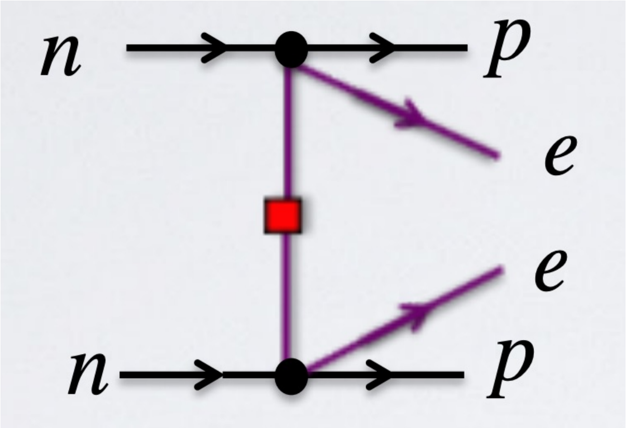
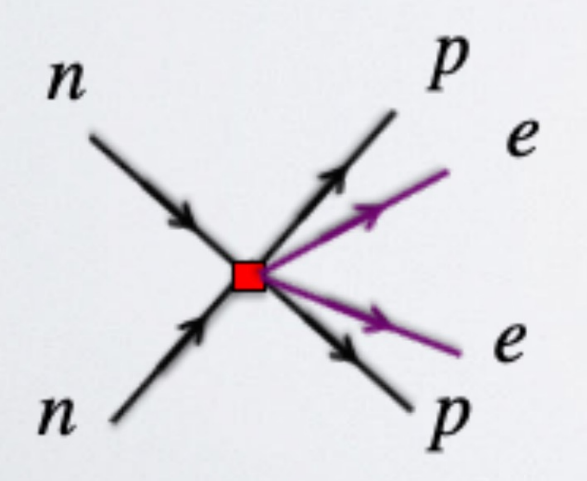


- Short-range contribution:

  contact term from hard neutrino exchange

- Naive power-counting:

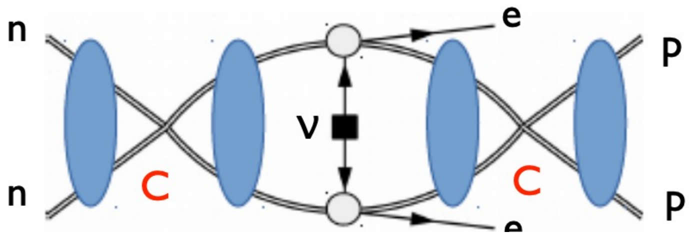
  g_ν^{NN} appears at next-to-leading order



$\sim \boldsymbol{g_\nu^{NN}}$

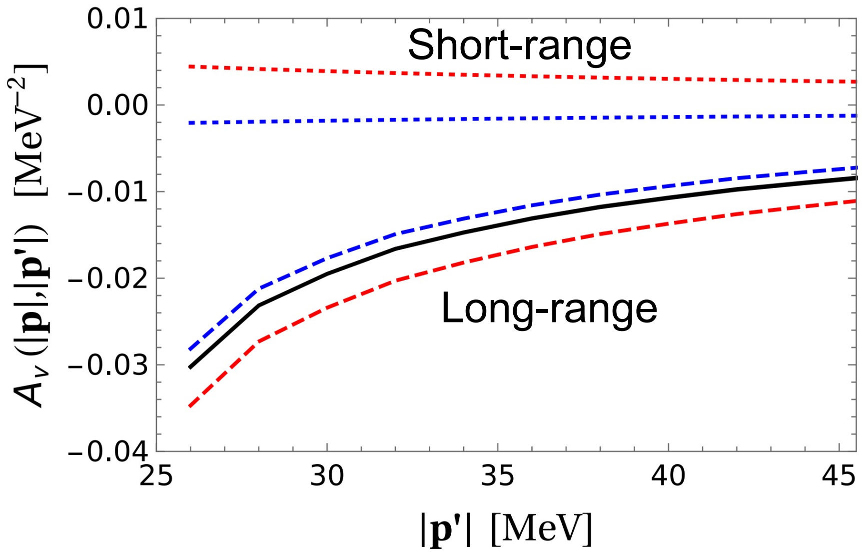
Lattice QCD and $0\nu2\beta$ decay

- Divergence term at LO:



$\sim log\,\frac{\Lambda^2}{p^2}$

- Strong dependence on cutoff Λ: sensitive to short-range physics



Blue: $\Lambda \sim 2\ fm^{-1}$

Red: $\Lambda \sim 20\ fm^{-1}$

Lattice QCD and $0\nu 2\beta$ decay

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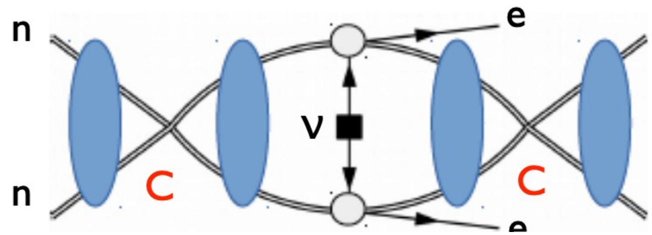

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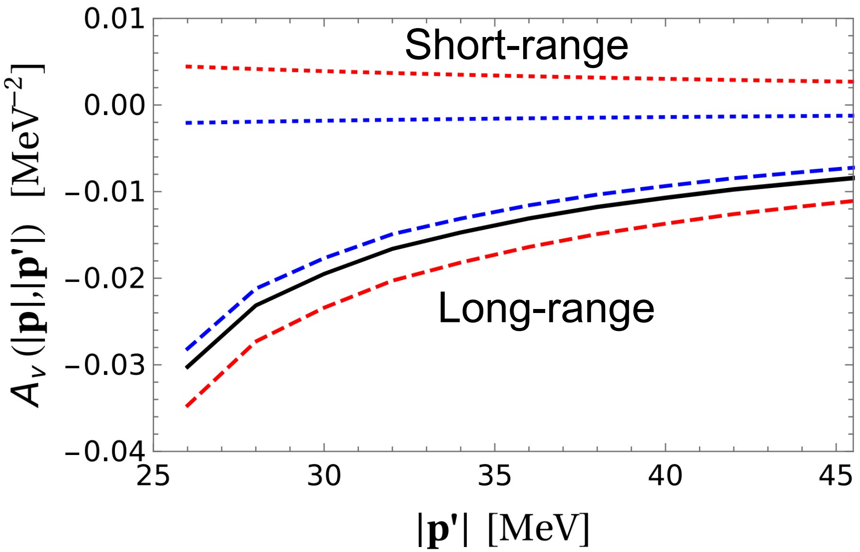

Blue: $\Lambda \sim 2\ fm^{-1}$

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PHYSICAL REVIEW LETTERS **120,** 202001 (2018)

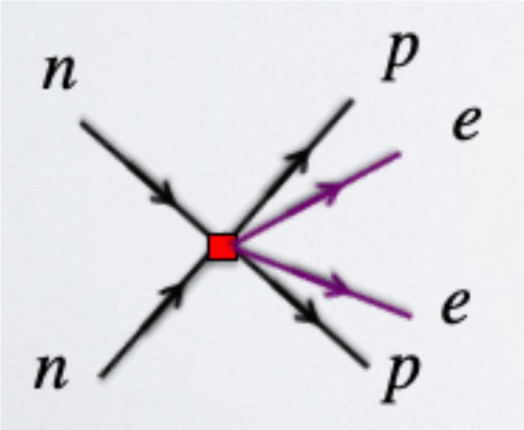
New Leading Contribution to Neutrinoless Double-β Decay

Vincenzo Cirigliano,[1] Wouter Dekens,[1] Jordy de Vries,[2] Michael L. Graesser,[1] Emanuele Mereghetti,[1] Saori Pastore,[1] and Ubirajara van Kolck[3,4]

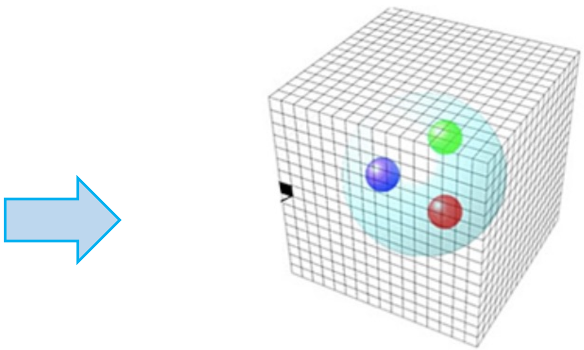
g_ν^{NN} should be considered at LO

Lattice QCD and $0\nu2\beta$ decay

- Determination of hard neutrino exchange contribution



Non-trivial task



Lattice calculation

Lattice QCD and $0\nu2\beta$ decay

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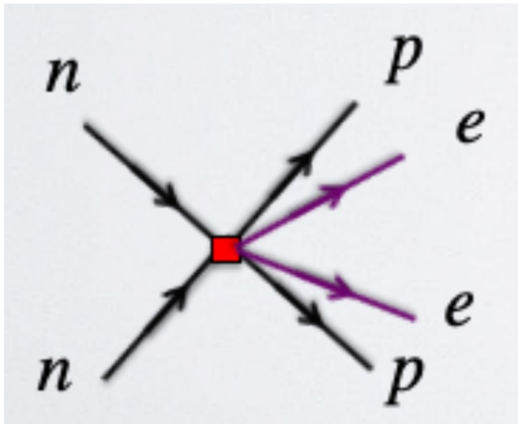

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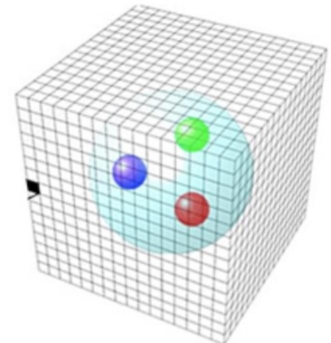
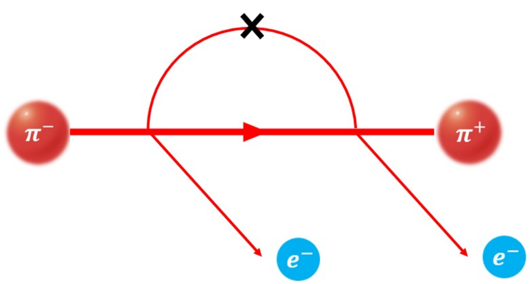


Lattice calculation

- Start with the simpler case: pionic $0\nu2\beta$ decay

hard neutrino



$$\sim g_\nu^{\pi\pi}$$

[1] **X. Tuo**, X. Feng, L. Jin, PRD100 (2019) 094511
[2] X. Feng, L. Jin, **X. Tuo**, S. Xia, PRL122 (2019) 022001
[3] W. Detmold, D. Murphy (2020), arxiv:2208.05322

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1 Background: Lattice QCD and $0\nu2\beta$ decay

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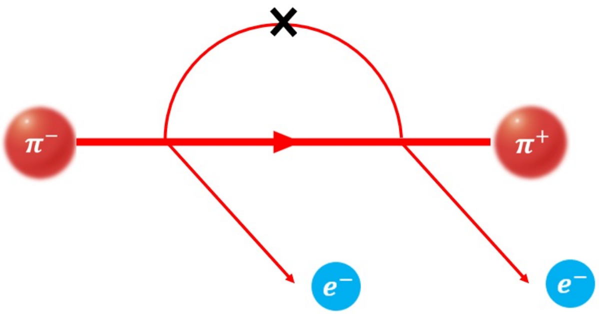
Challenge: massless neutrino



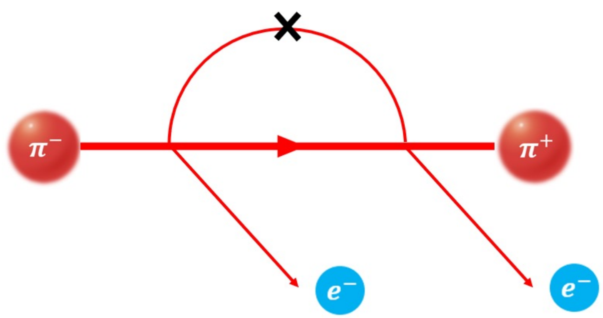
$$\mathcal{A} = -2T_{lept} \int d^4x \, H(x) S_0(x)$$

hadronic part from lattice

massless neutrino

How to combine massless propagator into lattice calculation?

Challenge: massless neutrino



$$\mathcal{A} = -2T_{lept} \int d^4x\, H(x) S_0(x)$$

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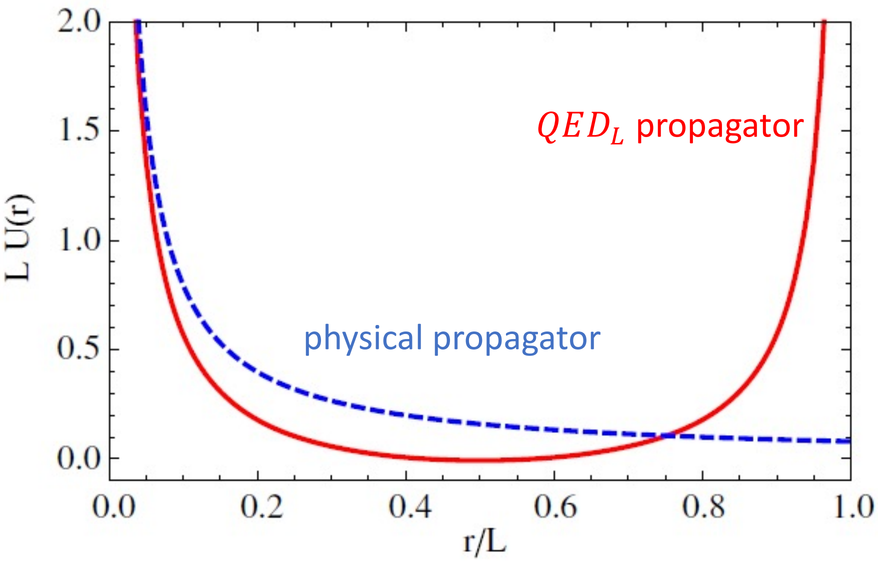
How to combine massless propagator into lattice calculation?

- Traditional method QED_L: subtract zero mode of neutrino



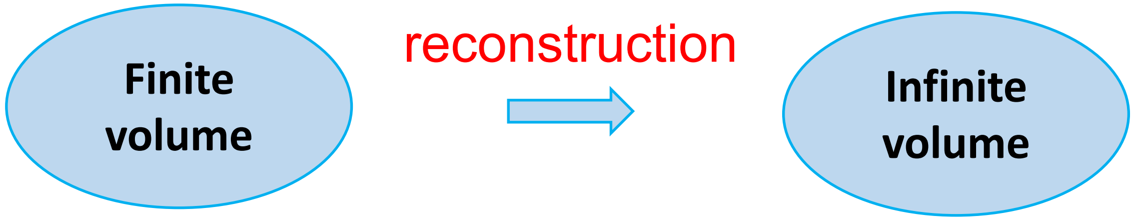
large $O(1/L)$ finite volume errors

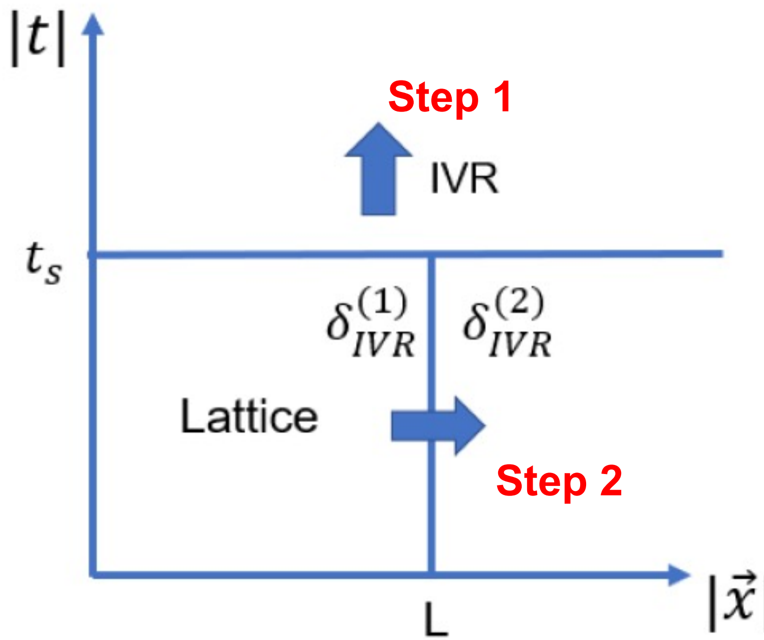
New method: Infinite volume reconstruction

$$\mathcal{A} = -2T_{lept}\int d^4x\, H(x) S_0(x)$$

$S_0(x)$: keep the infinite volume version

$H(x)$:

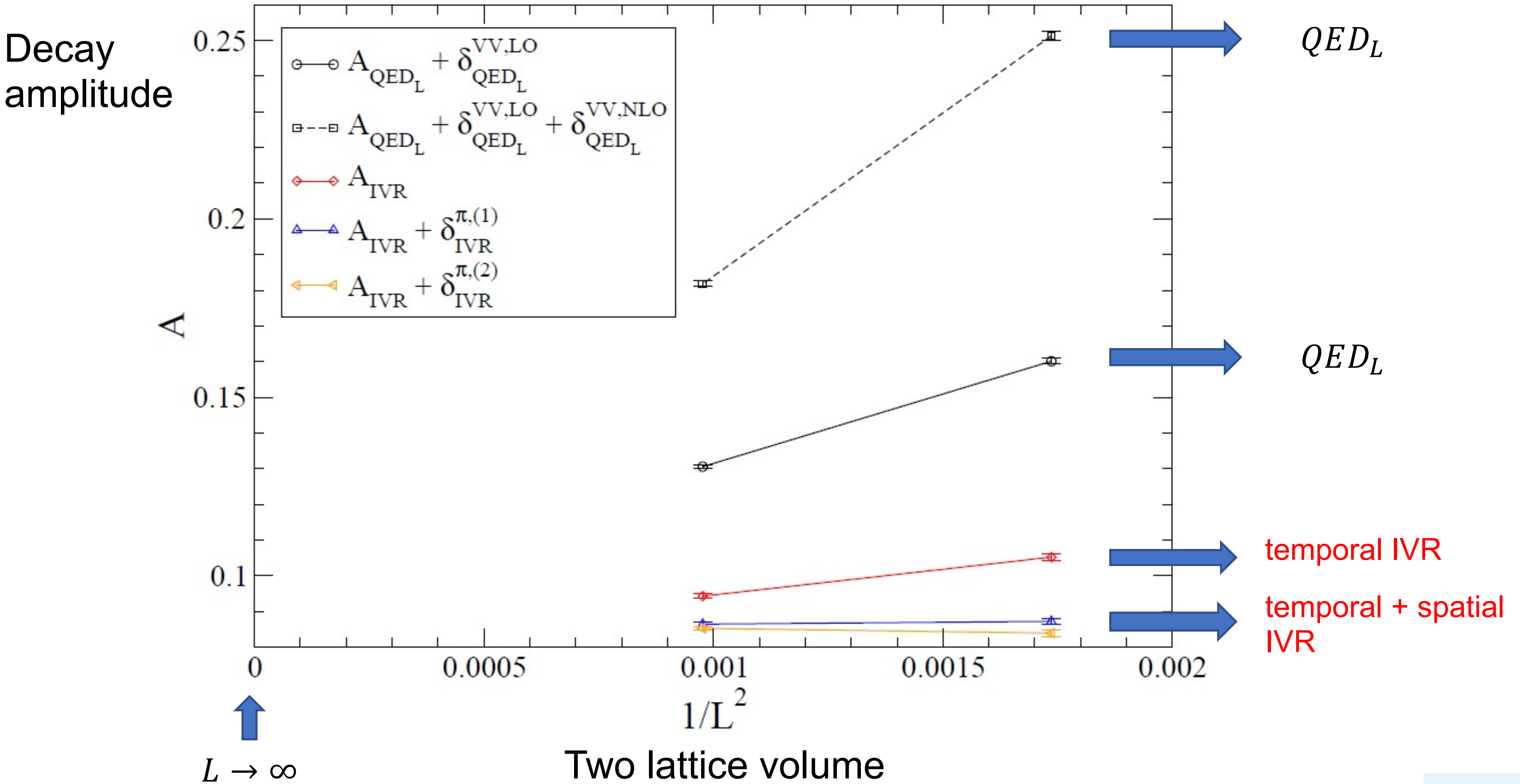




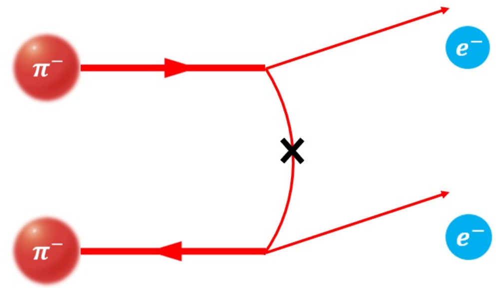
[1] X. Feng, L. Jin, PRD100 (2019) 094509, arXiv:1809.10511
[2] **X. Tuo**, X. Feng, L. Jin, PRD100 (2019) 094511, arXiv:1909.13525

Improving finite volume errors

Benefit of new method: $O(e^{-mL})$ FV errors

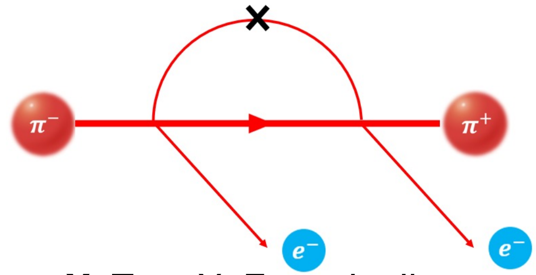


Provide LEC $g_\nu^{\pi\pi}$



X. Feng, L. Jin, **X. Tuo**, S. Xia,
PRL122 (2019) 022001

$$g_\nu^{\pi\pi} = -12.0(3)_{stat}$$

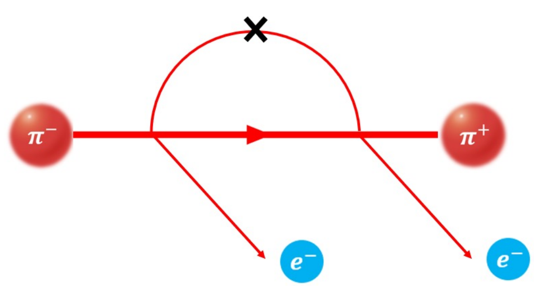


X. Tuo, X. Feng, L. Jin,
PRD100 (2019) 094511

$$g_\nu^{\pi\pi} = -10.9(3)_{stat}(7)_{sys}$$

Our work

NPLQCD



$$g_\nu^{\pi\pi} = -10.8(1)_{stat}(5)_{sys}$$

W. Detmold, D. Murphy (2020)
arxiv:2208.05322

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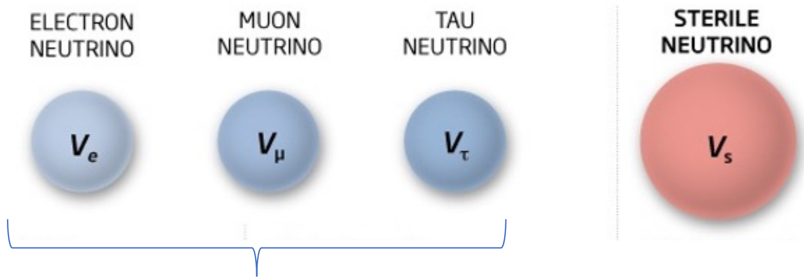
Lattice work 2: sterile neutrino contribution

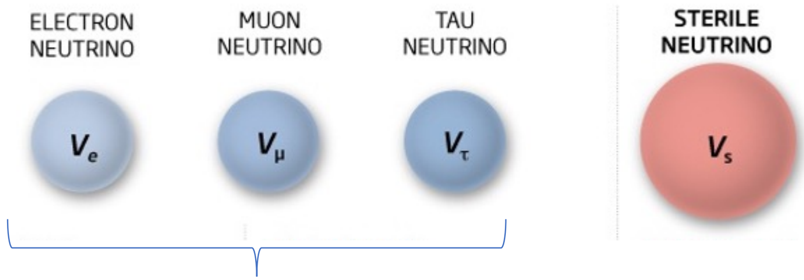


SM neutrinos

Sterile neutrinos

Sterile neutrino: explain the source of tiny mass of neutrino through the seesaw mechanism, the hypothesis of many BSM models

Lattice work 2: sterile neutrino contribution



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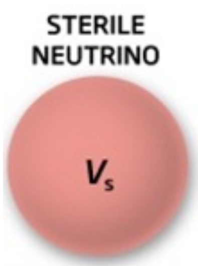
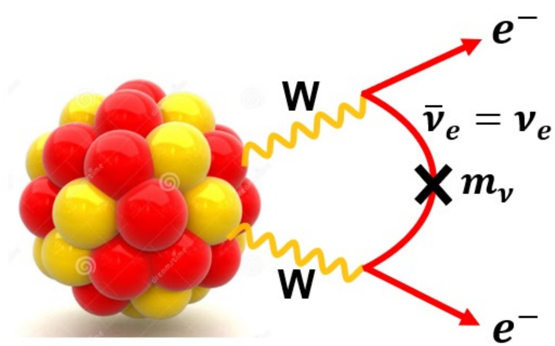


influence $0\nu 2\beta$ decay

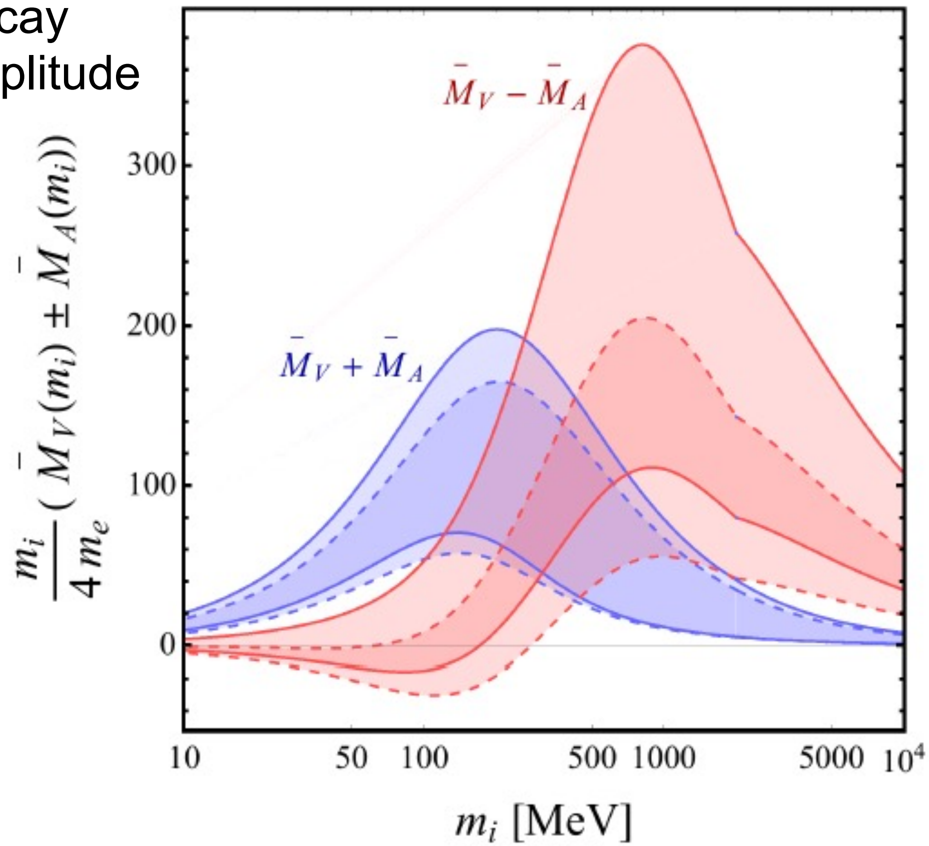


Enhancement due to sterile neutrino

W. Dekens, J. de Vries, K. Fuyuto, E. Mereghetti, and G. Zhou, JHEP 06, 097 (2020)

$0\nu2\beta$ decay can be enhanced by sterile neutrino contribution in pion exchange diagram

Decay amplitude

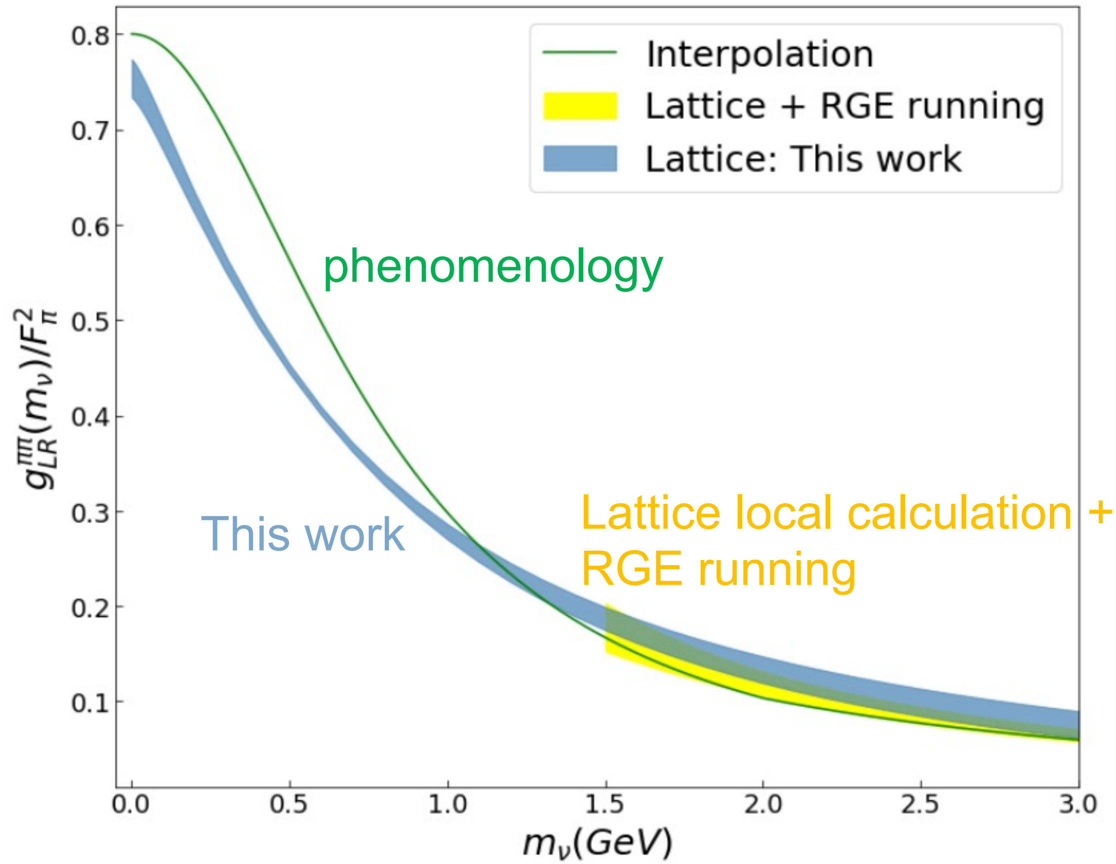


Big error band：poor knowledge of LECs $g_\nu^{NN}, g_{LR}^{NN}, g_{LR}^{\pi\pi}$, ...

Lattice QCD can help to reduce uncertainties in LECs

Lattice calculation of $g_{LR}^{\pi\pi}(m_\nu)$

X. Tuo, X. Feng, L. Jin, PRD106 (2022) 074510, arXiv:2206.00879



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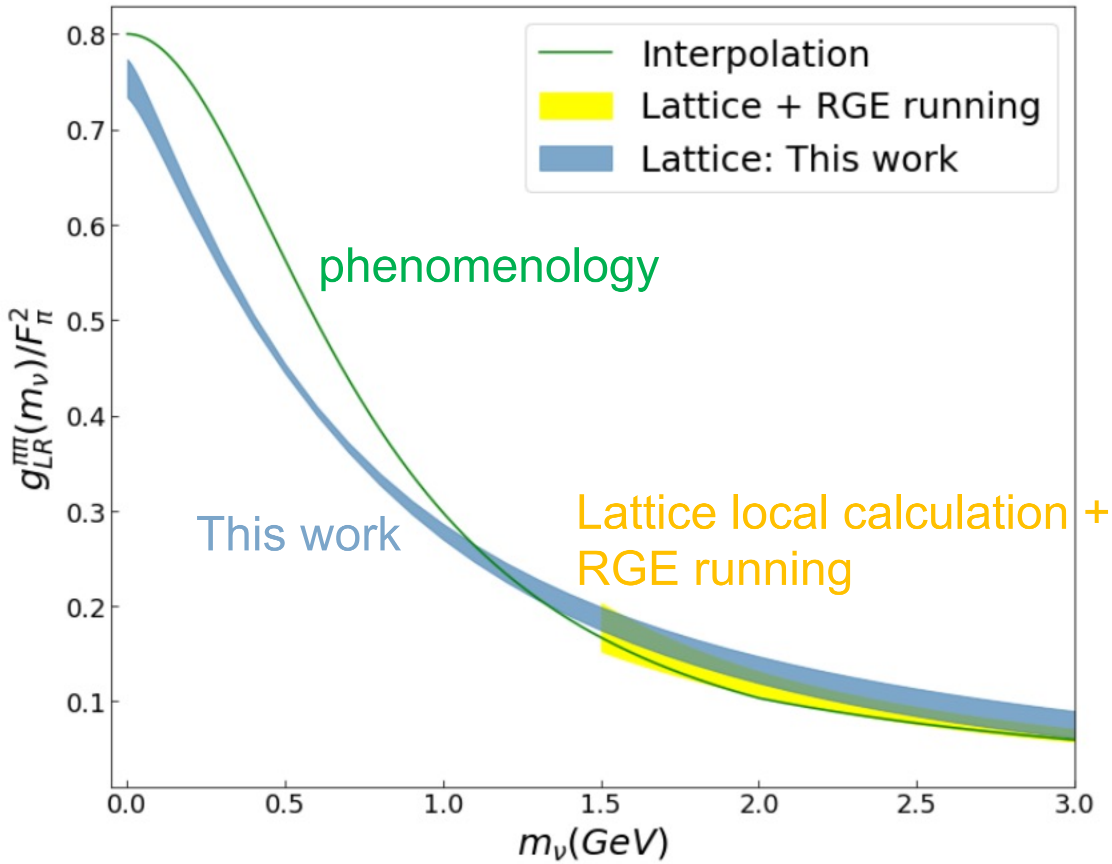

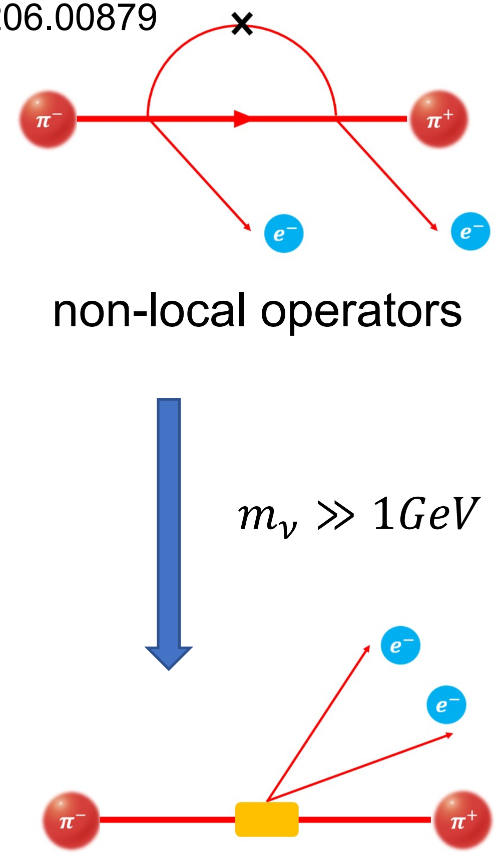

local operators
A. Nicholson, et al., PRL121 (2018) 172501

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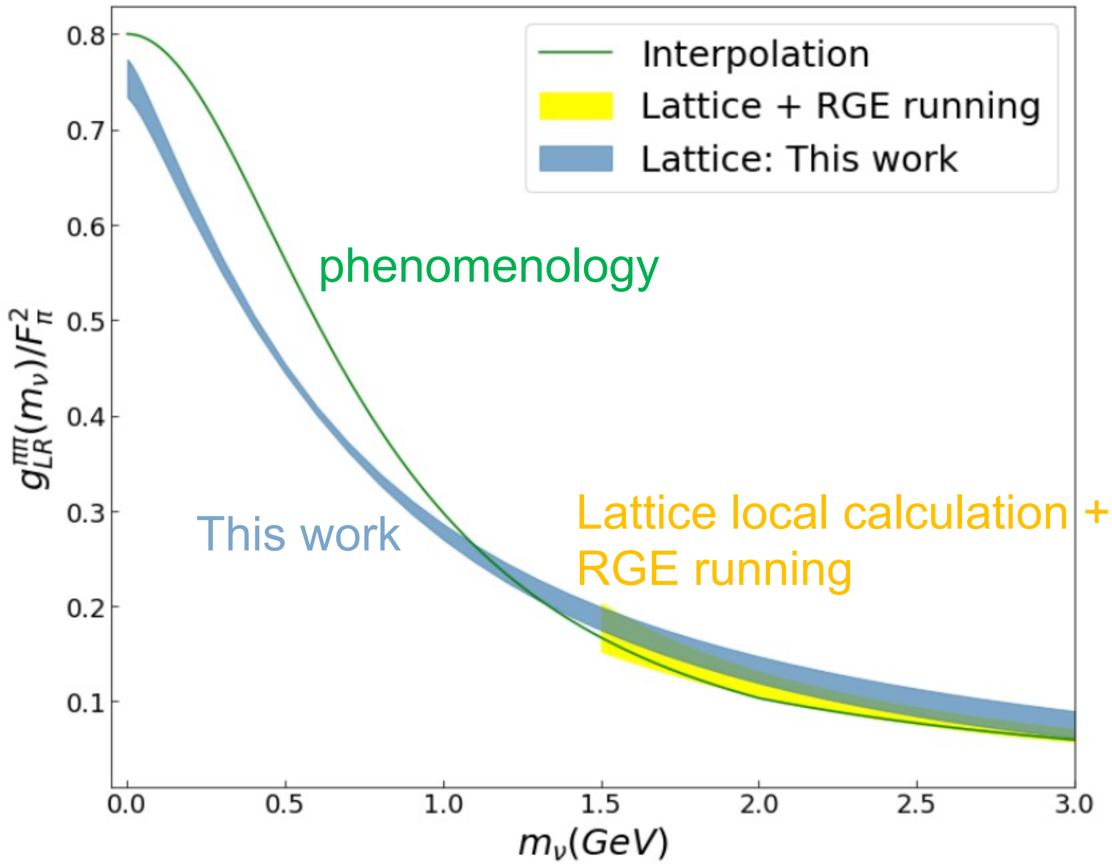
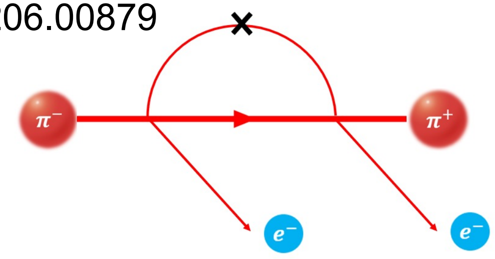
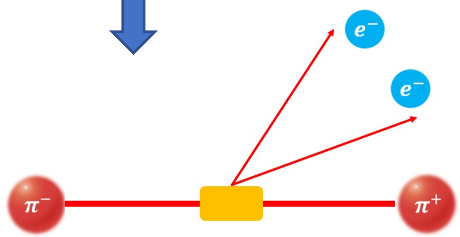




non-local operators

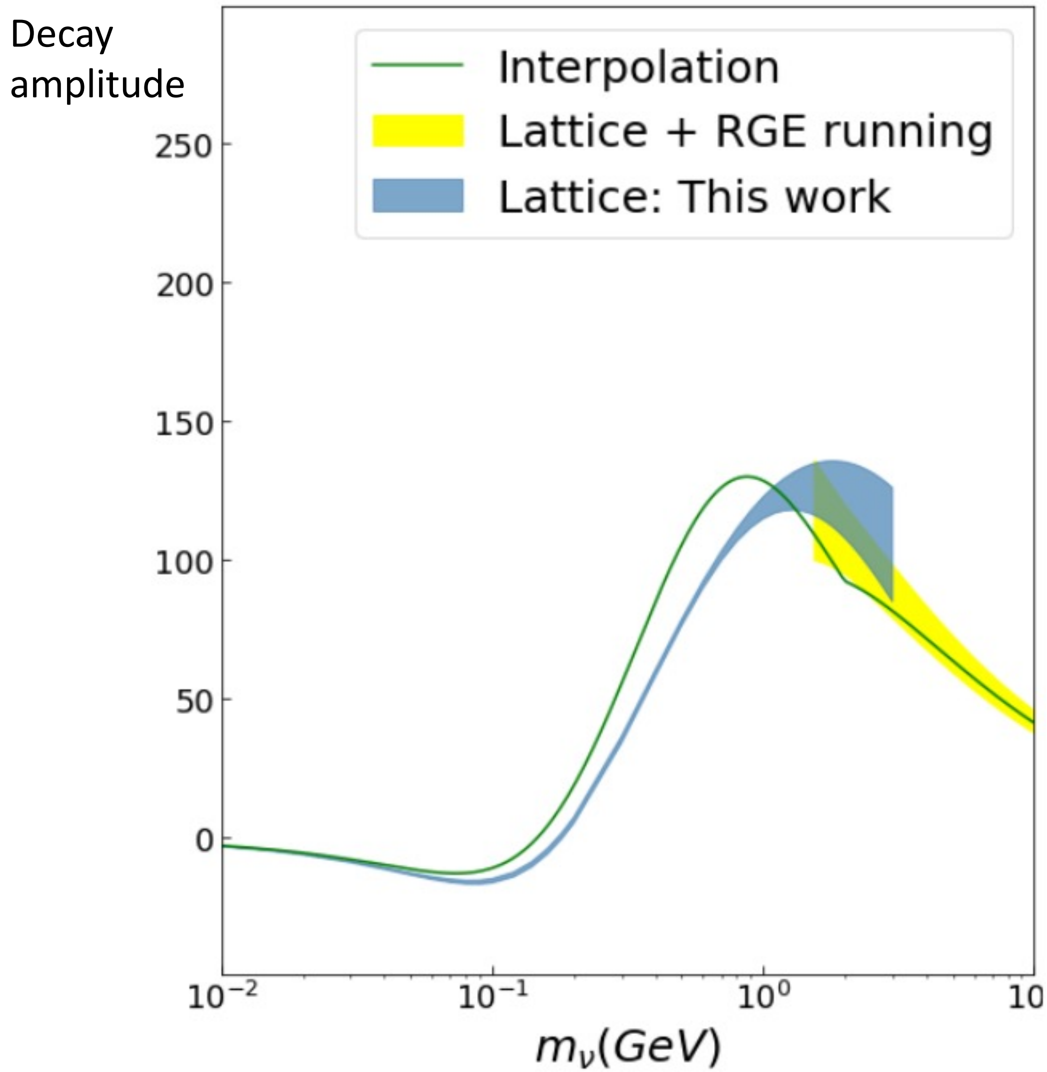
$m_\nu \gg 1GeV$



local operators
A. Nicholson, et al., PRL121 (2018) 172501

Nontrivial consistency check !

Enhancement due to $g_{LR}^{\pi\pi}(m_\nu)$



Help to reduce the uncertainties from LEC $g_{LR}^{\pi\pi}(m_\nu)$ and determine the peak shape

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1 Background: Lattice QCD and $0\nu2\beta$ decay

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4 Outlook for g_ν^{NN}

Outlook: nucleon sector g_ν^{NN}

[1] Zohreh Davoudi, et al. Report of the Snowmass 2021 Topical Group on Lattice Gauge Theory[C]. Snowmass 2021.

Three stages:

1. Calculation of two-nucleon spectra and elastic scattering

2. Calculation of two-nucleon $0\nu2\beta$ matrix elements

3. Relating lattice quantities to physical g_ν^{NN}

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Challenging due to signal-to-noise problem, main goal of future lattice QCD study

[2] Xu Feng, Lu-Chang Jin, Zi-Yu Wang, Zheng Zhang. Phys Rev D. 2021, 103(3):034508
[3] Zohreh Davoudi, Saurabh V. Kadam. Phys Rev Lett. 2021, 126(15):152003

In progress …

Outlook: nucleon sector g_ν^{NN}

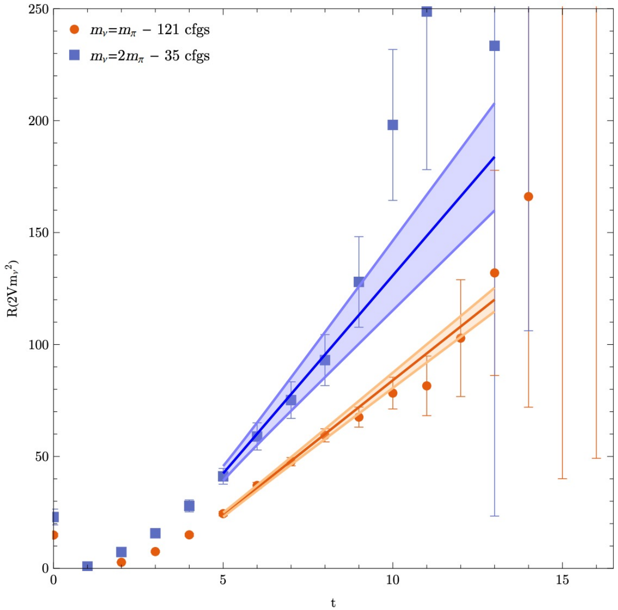
[1] Zohreh Davoudi, et al. Report of the Snowmass 2021 Topical Group on Lattice Gauge Theory[C]. Snowmass 2021.

Three stages:

1. Calculation of two-nucleon spectra and elastic scattering

2. Calculation of two-nucleon $0\nu2\beta$ matrix elements

3. Relating lattice quantities to physical g_ν^{NN}



Zi-Yu Wang (王子毓)

Matching, finite-volume effects:

Teng Wang (王腾)

In progress …