

Recent progress on the molecular tetraquarks, pentaquarks and di-baryons

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2023年7月22日@日照

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Outline

✓ **Background: experimental observations**

✓ **Theoretical aspects:**

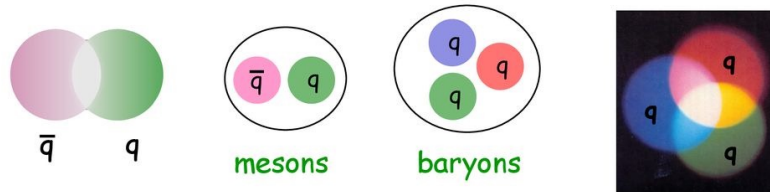
- Molecular tetraquarks
- Molecular pentaquarks
- Molecular hexaquarks (dibaryons)

✓ **Summary and outlook**

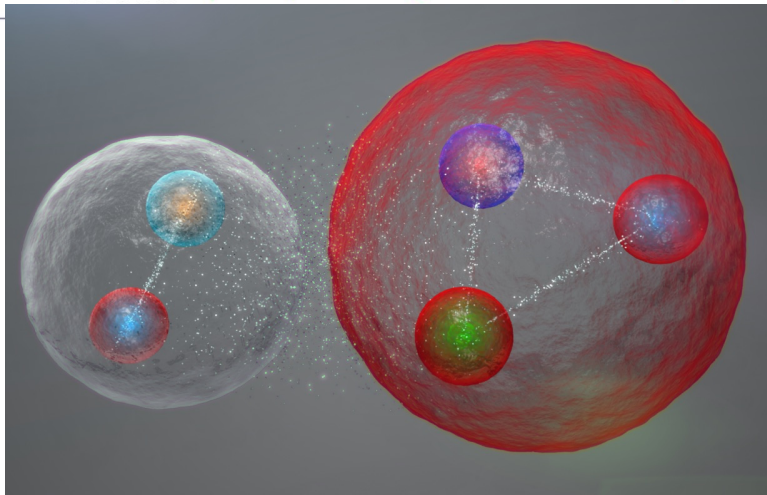
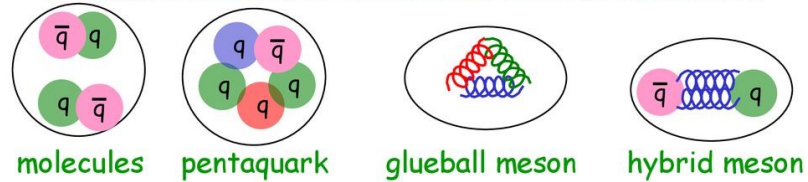
Conventional and exotic hadrons

Quarks are confined inside colorless hadrons

Quarks combine to “neutralize” color force



Configurations outside the standard quark model



5

$X(3872)$

$Z_c(3900)$

$Z_c(4020)$

$Z_b(10610)$

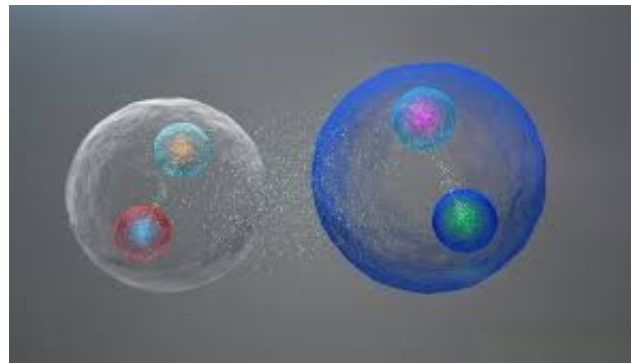
$Z_b(10650)$

$P_c(4312)$

$P_c(4440)$

$P_c(4457)$

.....



Recent reviews:

- ✓ **H.-X. Chen** et al, *Phys. Rept.* 639, 1 (2016)
- ✓ **R. Lebed** et al, *Prog. Part. Nucl. Phys.* 93, 143 (2017)
- ✓ **A. Esposito** et al, *Phys. Rept.* 668, 1(2017)
- ✓ **F.-K. Guo** et al, *Rev. Mod. Phys.* 90, 015004 (2018)
- ✓ **Y.-R. Liu** et al, *Prog. Part. Nucl. Phys.* 107, 237 (2019)
- ✓ **N. Brambilla** et al, *Phys. Rept.* 873, 1 (2020)
- ✓ **S. Chen** et al, *Front. Phys.* 18, 44601 (2023)
- ✓ **H.-X. Chen** et al, *Rept. Prog. Phys.* 86, 026201 (2023)
- ✓ **L. Meng** et al, *Phys. Rept.* 1019, 1 (2023)

Some well-known exotic states

State	Mass (MeV)	Width (MeV)	$I^{(G)}(J^{P(C)})$	Nearest threshold	Observed channels
$X(3872)$	3871.65 ± 0.06	1.19 ± 0.21	$0^+(1^{++})$	$D^0 \bar{D}^{*0}$	$J/\psi \pi \pi, D^0 \bar{D}^0 \pi^0, \chi_{c1} \pi^0, \dots$
$Z_c(3900)$	3887.1 ± 2.5	28.4 ± 2.6	$1^+(1^{+-})$	$D \bar{D}^{*-} / \bar{D} D^*$	$J/\psi \pi, D \bar{D}^{*-} / \bar{D} D^*$
$Z_c(4020)$	4024.1 ± 1.9	13 ± 5	$1^+(1^{+-})$	$D^* \bar{D}^{*-}$	$h_c \pi, D^* \bar{D}^{*-}$
$Z_{cs}(3985)$	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$	$\frac{1}{2}(1^+)$	$\bar{D}_s D^* / \bar{D}_s^* D$	$\bar{D}_s D^* / \bar{D}_s^* D$
$Z_b(10610)$	10607.2 ± 2.0	18.4 ± 2.4	$1^+(1^{+-})$	$B \bar{B}^{*-} / \bar{B} B^*$	$\Upsilon(nS) \pi, h_b(mP) \pi, B \bar{B}^{*-} / \bar{B} B^*$
$Z_b(10650)$	10652.2 ± 1.5	11.5 ± 2.2	$1^+(1^{+-})$	$B^* \bar{B}^{*-}$	$\Upsilon(nS) \pi, h_b(mP) \pi, B^* \bar{B}^{*-}$
$P_c(4312)$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\frac{1}{2}(\frac{1}{2}^-)$	$\Sigma_c \bar{D}$	$J/\psi p$
$P_c(4440)$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	$\frac{1}{2}(\frac{1}{2}^-)$	$\Sigma_c \bar{D}^{*-}$	$J/\psi p$
$P_c(4457)$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\frac{1}{2}(\frac{3}{2}^-)$	$\Sigma_c \bar{D}^{*-}$	$J/\psi p$
$P_{cs}(4338)$	$4338.3 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$0(\frac{1}{2}^-)$	$\Xi_c \bar{D}$	$J/\psi \Lambda$
$P_{cs}(4459)$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$	$0(\frac{1}{2}/\frac{3}{2}^-)$	$\Xi_c \bar{D}^{*-}$	$J/\psi \Lambda$
$T_{cc}(3875)$	3874.74 ± 0.04	$10^{-3}(47.8 \pm 1.9)$	$0(1^+)$	DD^*	$D^0 D^0 \pi^+$
$T_{cs0}(2900)$	$2866 \pm 7 \pm 2$	$57 \pm 12 \pm 4$	$0(0^+)$	$\bar{D}^{*-} K^*$	$D^- K^+$
$T_{cs0}^-(2900)$	$2908 \pm 11 \pm 20$	$136 \pm 23 \pm 11$	$1(0^+)$	$D^* K^*$	$D_s \pi$

Red: measured

Blue: preferred

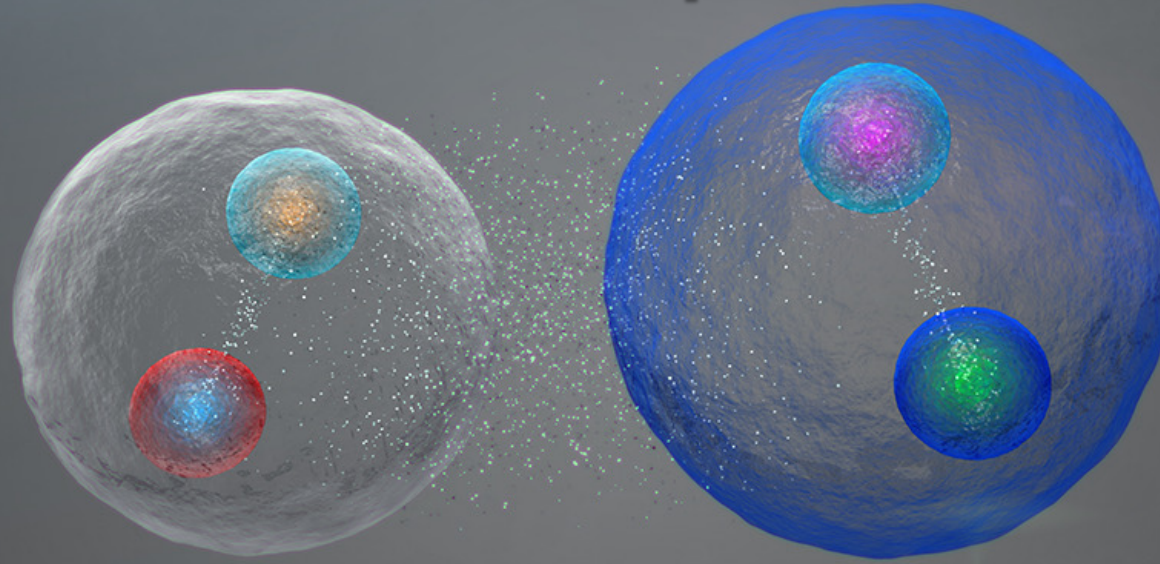
Possible combinations

$$\begin{aligned}
 & \begin{bmatrix} N \\ M_Q \\ B_Q \\ B_{QQ} \end{bmatrix} \hat{\otimes} \begin{bmatrix} N \\ M_Q \\ B_Q \\ B_{QQ} \\ \bar{N} \\ \bar{M}_Q \\ \bar{B}_Q \\ \bar{B}_{QQ} \end{bmatrix}^T \Rightarrow \begin{bmatrix} NN & NM_Q & NB_Q & NB_{QQ} & N\bar{N} & & & \\ & M_Q M_Q & M_Q B_Q & M_Q B_{QQ} & M_Q \bar{N} & M_Q \bar{M}_Q & & \\ & & B_Q B_Q & B_Q B_{QQ} & B_Q \bar{N} & B_Q \bar{M}_Q & B_Q \bar{B}_Q & \\ & & & B_{QQ} B_{QQ} & B_{QQ} \bar{N} & B_{QQ} \bar{M}_Q & B_{QQ} \bar{B}_Q & B_{QQ} \bar{B}_{QQ} \end{bmatrix} \\
 & \Rightarrow \begin{bmatrix} \text{deuteron} & \Lambda_c(2940)?, \dots & \square & \square & X(1835)? & & & \\ & T_{cc}, \dots & \square & \square & \square & X(3872), Z_{c(s)}^{(\prime)}, Z_b^{(\prime)}, \dots & & \\ & & \square & \square & \square & P_{c(s)}, \dots & \square & \\ & & & \square & \square & \square & \square & \square \end{bmatrix},
 \end{aligned}$$

- Sometimes, the K^* meson may be regarded as the heavy matter field to some extent ($m_{K^*} \sim m_N$). The $T_{cs}(2900)$ [1, 2] and $T_{cs}(2900)$ [3] observed by the LHCb are very close to the $\bar{D}^* K^*$ and $D^* K^*$ thresholds, respectively.

[1] Phys. Rev. D 102 (2020) 112003 [2] Phys. Rev. Lett. 125 (2020) 242001 [3] arXiv: 2212.02716 [hep-ex] [4] arXiv: 2212.02717 [hep-ex]

Molecular tetraquarks



1. $X(3872)$ and its possible partners

In Refs. [J. Nieves *et al*, [Phys.Rev.D 86 \(2012\) 056004](#); F. -K. Guo *et al*, [Phys.Rev.D 88 \(2013\) 054007](#); V. Baru *et al*, [Phys.Lett.B 763 \(2016\) 20-28](#)], the heavy quark spin symmetry (HQSS) partners of $X(3872)$ with the J^{PC} quantum numbers 2^{++} was proposed.

$$\begin{aligned} \left\langle \frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D}) \right\rangle_{1^{++}} &= -|1_h^{--} \otimes 1_\ell^{--}\rangle \\ |D^*\bar{D}^*\rangle_{2^{++}} &= -|1_h^{--} \otimes 1_\ell^{--}\rangle \end{aligned} \quad \xrightarrow{\hat{H}_{\text{eff}} \equiv \tilde{c}_1 + \tilde{c}_2 \boldsymbol{\ell}_1 \cdot \boldsymbol{\ell}_2} \quad \begin{aligned} V_{1^{++}}^\alpha &= C_1^\alpha = \tilde{c}_1 + \frac{1}{4}\tilde{c}_2, \\ V_{2^{++}}^\alpha &= C_1^\alpha = \tilde{c}_1 + \frac{1}{4}\tilde{c}_2. \end{aligned}$$

$SU(3)_F$ symmetry and HQSS for di-meson systems

L.M, B.W, S.-L.Z, [Sci.Bull. 66 \(2021\) 1288-1295](#)

$$\begin{aligned} D_s^{(*)} - D^{(*)} &\simeq 100 \text{ MeV}, \\ D_{(s)}^* - D_{(s)} &\simeq 140 \text{ MeV}, \end{aligned}$$

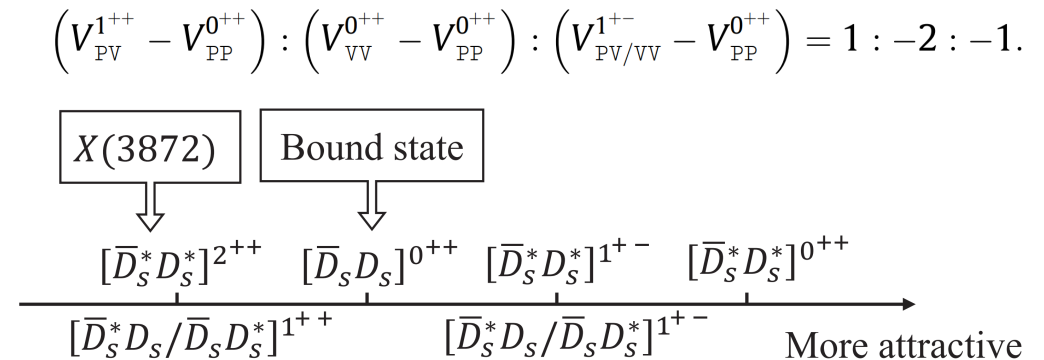
$$V_{q\bar{q}} = c_1 + c_2 \mathbf{s}_1 \cdot \mathbf{s}_2 + c_3 \mathbb{C}_2 + c_4 (\mathbf{s}_1 \cdot \mathbf{s}_2) \mathbb{C}_2$$

$m_{D^0} + m_{D^{*0}} - m_{X(3872)} = (0.00 \pm 0.18) \text{ MeV}$. It may be approximately regarded as a pure $D^0\bar{D}^{*0}/\bar{D}^0D^{*0}$ dimeson, then its flavor wave function in the light part will be $|\bar{u}u\rangle$.

$$\langle \mathbb{C}_2 \rangle_{u\bar{u}} = \langle \mathbb{C}_2 \rangle_{s\bar{s}}$$

$$V_{q\bar{q}} = \tilde{c}_1 + \tilde{c}_2 \mathbf{s}_1 \cdot \mathbf{s}_2,$$

$$\begin{aligned} \langle \mathbf{s}_1 \cdot \mathbf{s}_2 \rangle_{\{PP, VV\}}^{0^{++}} &= \begin{bmatrix} 0 & \frac{\sqrt{3}}{4} \\ \frac{\sqrt{3}}{4} & -\frac{1}{2} \end{bmatrix}, \\ \langle \mathbf{s}_1 \cdot \mathbf{s}_2 \rangle_{\{PV, VV\}}^{1^{+-}} &= \begin{bmatrix} -\frac{1}{4} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{4} \end{bmatrix}, \\ \langle \mathbf{s}_1 \cdot \mathbf{s}_2 \rangle_{\{PV\}}^{1^{++}} &= \frac{1}{4}, \quad \langle \mathbf{s}_1 \cdot \mathbf{s}_2 \rangle_{\{VV\}}^{2^{++}} = \frac{1}{4}, \end{aligned}$$



1. $X(3872)$ and its possible partners

Two prerequisites:

- The $X(3872)$ is the molecular state with its mass coinciding exactly with the $\bar{D}_0^* D_0$ threshold;
- There exist the $\bar{D}_s D_s$ bound states with $J^{PC} = 0^{++}$.

The recent lattice QCD calculation yielded a shallow $[D_s \bar{D}_s]^{0^{++}}$ bound state with $\Delta E = -6.2_{-2.0}^{+3.8}$ MeV [JHEP 06 (2021) 035]

$$T(p', p; E) = V(p', p) + \int \frac{d^3 p''}{(2\pi)^3} \frac{V(p', p'') T(p'', p; E)}{E - p^2/(2\mu) + i\epsilon},$$

$X(3872)_{\text{input}}$	$[\overline{D}_s D_s]_{\text{input}}^{0^{++}}$		$[\overline{D}_s^* D_s^*]^{0^{++}}$		$[\overline{D}_s^* D_s / \overline{D}_s D_s^*]^{1^{+-}}$		$[\overline{D}_s^* D_s^*]^{1^{+-}}$	
ΔE (MeV)	ΔE (MeV)	M (MeV)	ΔE (MeV)	M (MeV)	ΔE (MeV)	M (MeV)	ΔE (MeV)	M (MeV)
0.0	−2.4	3934.3	−20.3	4204.1	−9.5	4071.0	−11.4	4213.0
0.0	−6.2	3930.5	−45.5	4178.9	−22.5	4058.0	−25.2	4199.2
0.0	−8.2	3928.5	−57.6	4166.8	−29.0	4051.5	−32.0	4192.4
0.0	−12.9	3923.8	−84.3	4140.1	−43.7	4036.8	−47.2	4177.2
−1.0	−2.4	3934.3	−8.3	4216.1	−4.9	4075.6	−6.3	4218.1
−1.0	−6.2	3930.5	−28.9	4195.5	−15.9	4064.6	−18.2	4206.2
−1.0	−8.2	3928.5	−39.6	4184.8	−21.7	4058.8	−24.4	4200.0
−1.0	−12.9	3923.8	−64.1	4160.3	−35.2	4045.3	−38.5	4185.9
Cutoff-I [49]	−13	3924	−84	4140	−46	4035	−47	4177
Cutoff-II [49]	−9	3928	−84	4140	−41	4040	−44	4180

[49] C. Hidalgo-Duque *et al*, Phys.Rev.D 87 (2013) 7, 076006.

2. T_{cc} and its decays

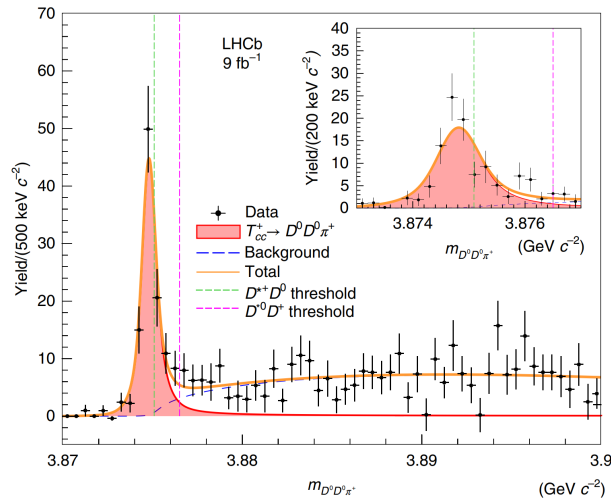


Table 1 | Parameters obtained from the fit to the $D^0 D^0 \pi^+$ mass spectrum: signal yield, N , BW mass relative to the $D^{*+} D^0$ mass threshold, δm_{BW} , and width, Γ_{BW} . The uncertainties are statistical only

Parameter	Value
N	117 ± 16
δm_{BW}	$-273 \pm 61 \text{ keV } c^{-2}$
Γ_{BW}	$410 \pm 165 \text{ keV}$

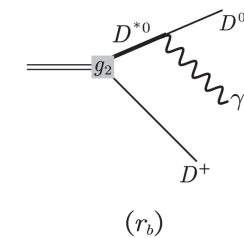
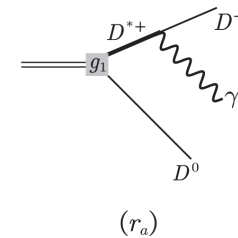
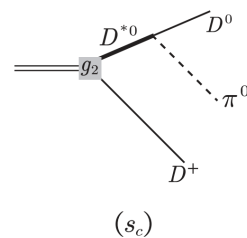
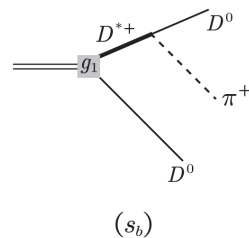
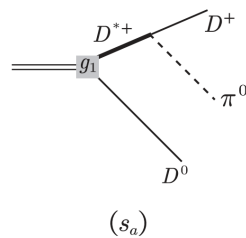
Nature Phys. 18 (2022) 7, 751-754

- ✓ Within the contact EFT [L.M, G.-J.W, B.W, S.-L.Z, [Phys.Rev.D 104 \(2021\)](#) , $|T_{cc}^+\rangle = \cos \theta |D^{*+} D^0, \phi_1\rangle + \sin \theta |D^{*0} D^+, \phi_2\rangle$, [L051502](#)]

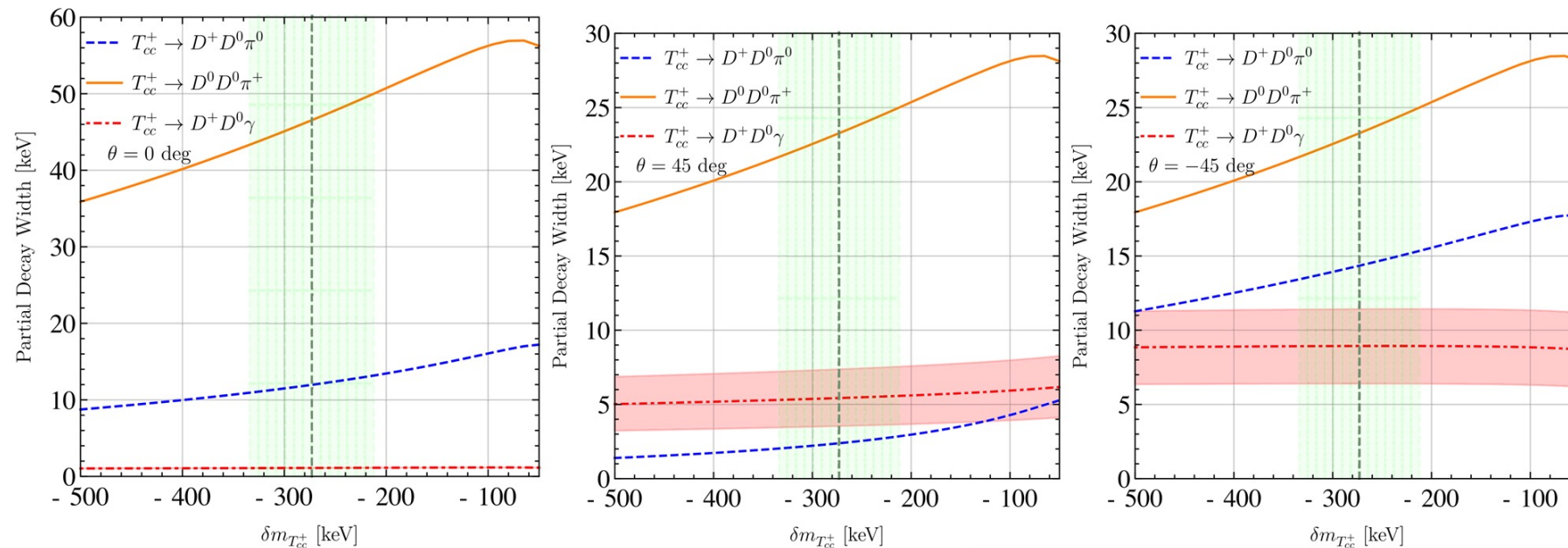
$$\lim_{E \rightarrow E_0} (E - E_0) t_{ij} = \lim_{E \rightarrow E_0} \left[\frac{d(t_{ij})^{-1}}{dE} \right]^{-1} = \frac{1}{8M_T^2 \mu} g_i g_j$$

$$g_1 = \frac{4M_T \sqrt{\pi \kappa_1}}{\sqrt{\mu}} \cos \theta, \quad g_2 = \frac{4M_T \sqrt{\pi \kappa_2}}{\sqrt{\mu}} \sin \theta.$$

$$\theta = \begin{cases} 0 & \text{pure } D^{*+} D^0 \\ \frac{\pi}{4} & I = 1, I_3 = 0. \\ -\frac{\pi}{4} & I = 0, I_3 = 0 \end{cases}$$



2. T_{cc} and its decays



Single-channel limit : $\Gamma_{\text{str}} + \Gamma_{\text{EM}} = 59.7^{+4.6}_{-4.4}$ keV.

Isospin singlet : $\Gamma_{\text{str}} + \Gamma_{\text{EM}} = 46.7^{+2.7}_{-2.9}$ keV,

Isospin triplet : $\Gamma_{\text{str}} + \Gamma_{\text{EM}} = 31.2^{+2.2}_{-2.4}$ keV.

The pole parameters are found to be

$$\delta m_{\text{pole}} = -360 \pm 40^{+4}_{-0} \text{ keV}/c^2,$$

Nature Commun. 13 (2022) 1, 3351

$$\Gamma_{\text{pole}} = 48 \pm 2^{+0}_{-14} \text{ keV},$$

isoscalar assignment for T_{cc} is supported!

The improved calculations for the DD^* interactions that based on the χ EFT up to N^2 LO was given in **B.W, L.M**, [Phys.Rev.D 107, 094002](#)

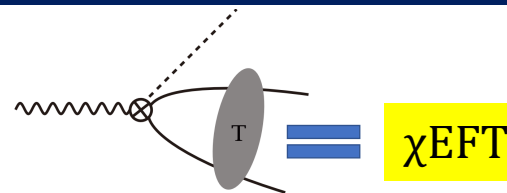
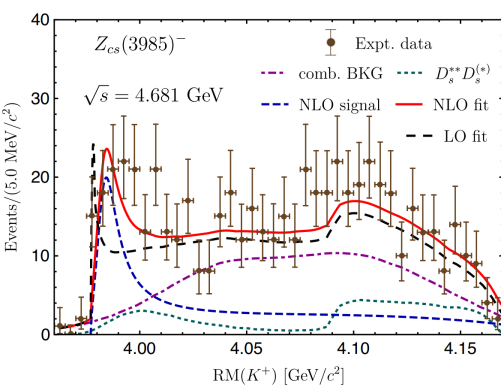
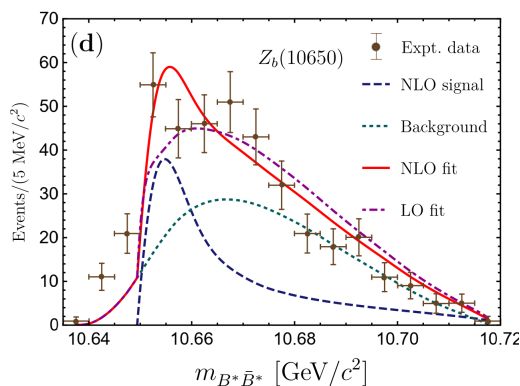
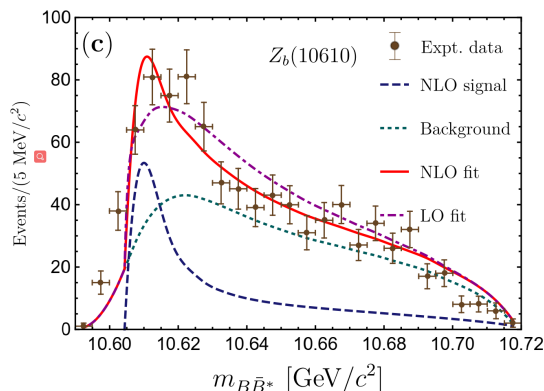
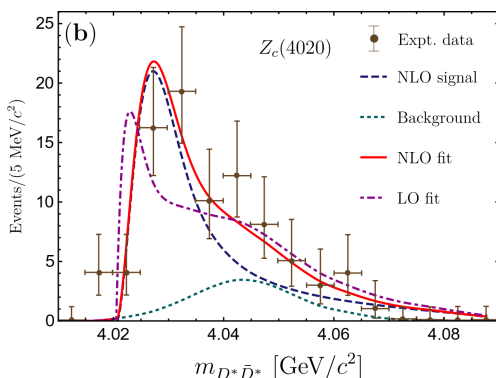
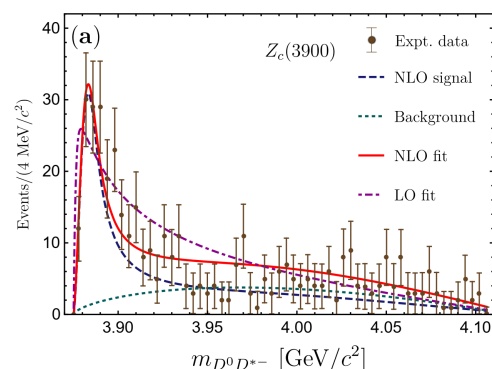
Isospin violating decays of $X(3872)$ was revisited in **L.M, G.-J.W, B.W, S.-L.Z**, [PhysRevD.104.094003](#)

3. $Z_{c,b}$ and their strange partners

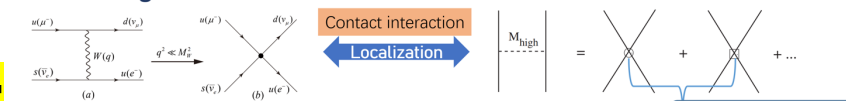
Comprehensive review on χ EFTs in heavy sectors: Phys. Rept. 1019 (2023) 1-149

1. Simulate the production process:

2. Fit the lineshapes in experiments:



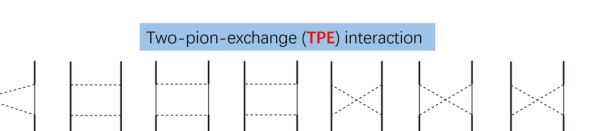
• Short-range force:



• Long-range force:

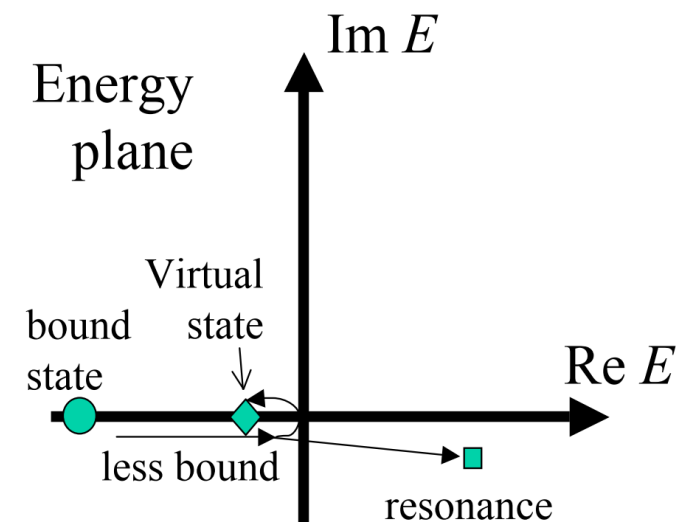


• Mid-range force:



$$\mathcal{U}(E, \mathbf{p}) = \mathcal{M}(E, \mathbf{p}) + \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \mathcal{V}(E, \mathbf{p}, \mathbf{q}) \mathcal{G}(E, \mathbf{q}) \mathcal{U}(E, \mathbf{q}),$$

Search for the poles in the Riemann sheets:



3. $Z_{c,b}$ and their strange partners

B.W, L.M, S.-L.Z, [PhysRevD.102.114019](#)
B.W, L.M, S.-L.Z, [PhysRevD.103.L021501](#)

• Fitted parameters and predicted states

Poles in the second Riemann sheet

Strangness $S = 0$

States	Thresholds	\tilde{C}_s [GeV $^{-2}$]	C_s [GeV $^{-4}$]	C_{sd} [GeV $^{-4}$]	Λ [GeV]	$[m, \Gamma]_{\text{pole}}$	$[m, \Gamma]_{\text{expt.}}$
$\frac{1}{\sqrt{2}} [D\bar{D}^* + D^*\bar{D}]$	3875.8	$3.6^{+1.2}_{-1.2}$	$-76.9^{+6.2}_{-6.2}$	$1.1^{+5.8}_{-5.8}$	$0.33^{+0.024}_{-0.024}$	$[3881.3^{+3.0}_{-3.0}, 12.4^{+5.0}_{-5.0}]$	$[3881.7^{+2.3}_{-2.3}, 26.6^{+3.0}_{-3.0}]$
$D^*\bar{D}^*$	4017.1	$4.0^{+1.6}_{-1.6}$	$-78.1^{+8.7}_{-8.7}$	$1.7^{+6.3}_{-6.3}$	$0.34^{+0.031}_{-0.031}$	$[4026.5^{+4.5}_{-4.5}, 10.1^{+7.2}_{-7.2}]$	$[4025.5^{+3.7}_{-5.6}, 26.0^{+6.0}_{-6.0}]$
$\frac{1}{\sqrt{2}} [B\bar{B}^* + B^*\bar{B}]$	10604.4	$2.2^{+0.2}_{-0.2}$	$-9.9^{+1.0}_{-1.0}$	$3.6^{+4.7}_{-4.7}$	$0.51^{+0.014}_{-0.014}$	$[10607.9^{+2.2}_{-2.2}, 10.9^{+3.0}_{-3.0}]$	$[10607.2^{+2.0}_{-2.0}, 18.4^{+2.4}_{-2.4}]$
$B^*\bar{B}^*$	10649.4	$2.2^{+0.3}_{-0.3}$	$-9.9^{+1.2}_{-1.2}$	$3.3^{+6.6}_{-6.6}$	$0.51^{+0.015}_{-0.015}$	$[10652.8^{+2.7}_{-2.7}, 10.9^{+3.4}_{-3.4}]$	$[10652.2^{+1.5}_{-1.5}, 11.5^{+2.2}_{-2.2}]$

$$(m, \Gamma) = (3982.4^{+4.8}_{-3.4}, 11.8^{+5.5}_{-5.2}) \text{ MeV,}$$

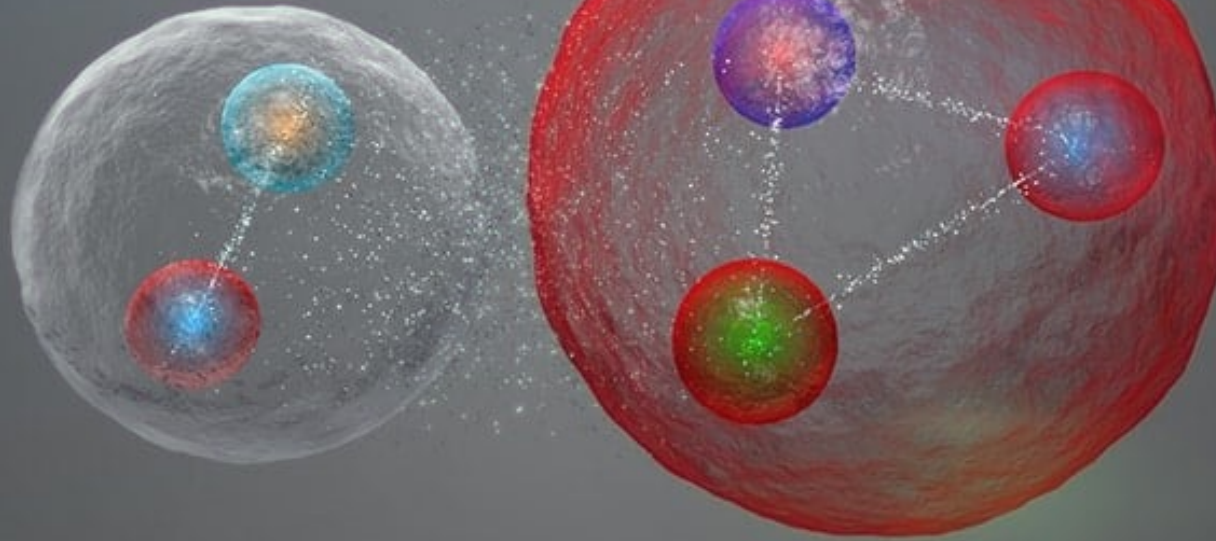
Strangness $S = -1$

Systems	$I(J^P)$	Thresholds (MeV)	Masses (MeV)	Widths (MeV)	Δm (MeV)	States
$\frac{1}{\sqrt{2}} [\bar{D}_s^* D + \bar{D}_s D^*]$	$\frac{1}{2}(1^+)$	3977.0	$3982.5^{+1.8}_{-2.6} \pm 2.1$	$12.8^{+5.3}_{-4.4} \pm 3.0$	$5.5^{+1.8}_{-2.6} \pm 2.1$	$Z_{cs}(3985)^{\dagger}$
$\bar{D}_s^* D^*$	$\frac{1}{2}(1^+)$	4119.1	$4124.2^{+5.6}_{-3.7}$	$9.8^{+5.2}_{-4.8}$	$5.1^{+5.6}_{-3.7}$	$Z_{cs}(4125)$
$\frac{1}{\sqrt{2}} [B_s^* \bar{B} + B_s \bar{B}^*]$	$\frac{1}{2}(1^+)$	10694.7	$10701.9^{+3.9}_{-2.7}$	$7.4^{+3.6}_{-4.4}$	$7.2^{+3.9}_{-2.7}$	$Z_{bs}(10700)$
$B_s^* \bar{B}^*$	$\frac{1}{2}(1^+)$	10740.1	$10747.0^{+4.3}_{-3.1}$	$7.3^{+3.7}_{-4.6}$	$6.9^{+4.3}_{-3.1}$	$Z_{bs}(10745)$

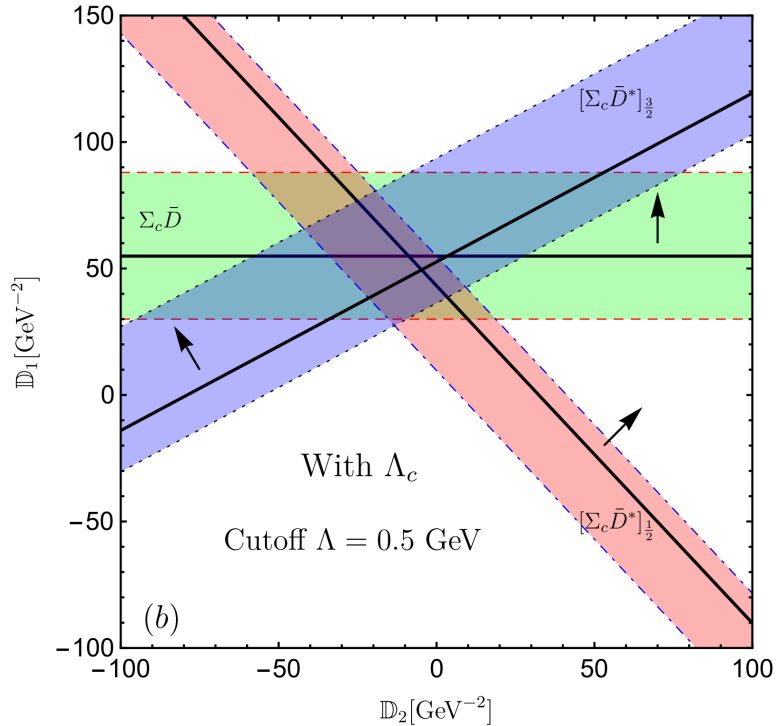
✓ New measurement from BESIII ($e^+e^- \rightarrow K^+ D_s^{*-} D^{*0} + c.c.$): Z'_{cs} , $m \sim 4123.5$ MeV, with a significance of 2.1σ . [Chin.Phys.C 47,033001 \(2023\)](#).

✓ Implications of $Z_{cs}(4000)$ and $Z_{cs}(3985)$ as two different states are given in L.M, B.W, G.-J.W, S.-L.Z, [Sci.Bull. 66, 2065 \(2021\)](#)

Molecular pentaquarks

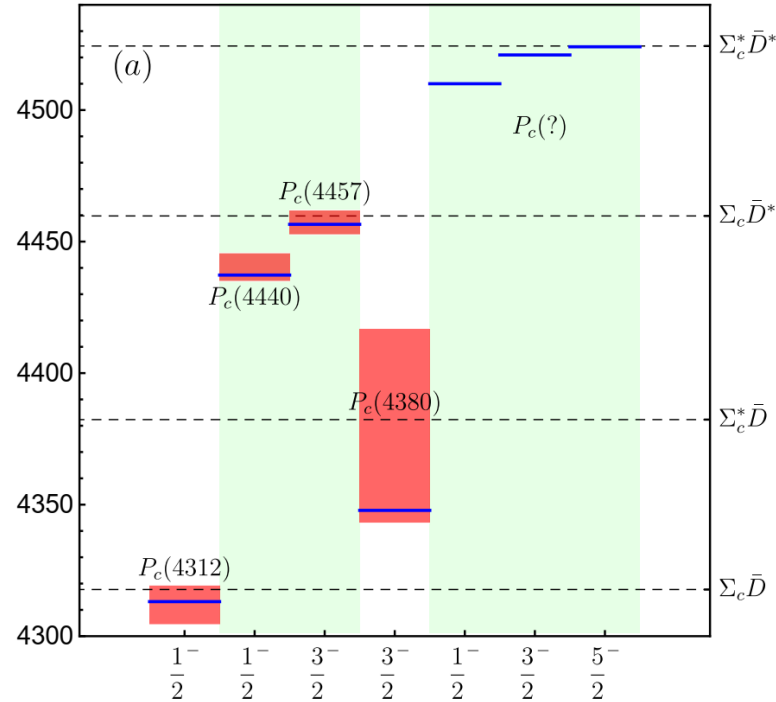


4. P_c s and their strange partners



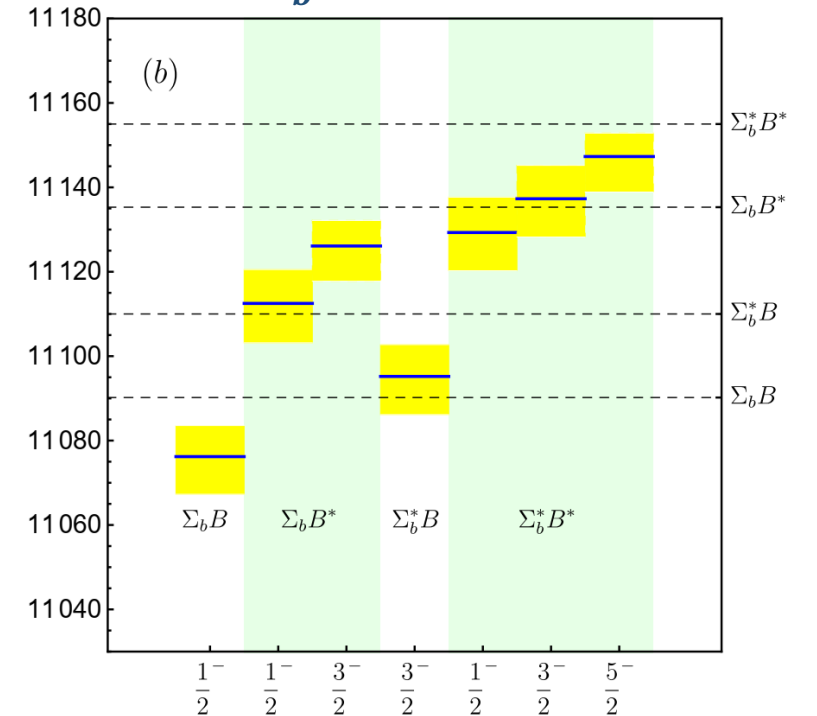
B.W, L.M, S.-L.Z, [JHEP 1911 \(2019\) 108](#)

$\Sigma_c^{(*)} \bar{D}^{(*)}$ systems



Hidden-charm spectra

$\Sigma_b^{(*)} B^{(*)}$ systems



Hidden-bottom spectra

How to observe?

- Production mechanism is unknown.
 - Reconstruction channels: $\Upsilon(nS)N$ ($n = 1, 2$), $\Lambda_b B^{(*)}$
- "There is plenty of room at the 'bottom'". - R. P. Feynman

4. P_c s and their strange partners

- $\Xi_c^{(',*)} \bar{D}^{(*)}$ systems

XYZ: $Q\bar{Q}q\bar{q}$;

P_c : $Q\bar{Q}qqq$.

1. The **heavy quark core** plays an important role in stabilizing the exotic clusters [Phys. Rev. D **84**, 014031, Phys. Rev. D **86**, 014020, Eur. Phys. J. C **74**, 3198].
2. Hydrogen molecule: two protons plus two electrons, stably exists in the nature.
3. Existence of $P_c \rightarrow$ more hadronic molecules in **SU(3) symmetry**?
4. Two heavy matter fields tend to form the bound states in the **lowest isospin channels**?

Deuteron ($I = 0$, np molecule)

$X(3872)$ ($I = 0$, $D\bar{D}^*$ molecule candidate)

P_c states ($I = \frac{1}{2}$, $\Sigma_c \bar{D}^{(*)}$ molecule candidates)

T_{cc} and T_{bb} ($I = 0$, DD^* and BB^* molecule candidates)

observed

predictions

5. Whether the $\Xi_c^{(',*)} \bar{D}^{(*)}$ systems can form bound states in the **$I = 0$** channels?
6. May be observed in $J/\psi \Lambda$ final states of the decays $\Lambda_b(\Xi_b) \rightarrow J/\psi \Lambda K(\eta)$?

4. P_c s and their strange partners

• P_{cs} spectra

B.W, L.M, S-.L.Z, Phys. RevD. 101.034018



System	$[\Xi'_c \bar{D}]_{\frac{1}{2}}$	$[\Xi'_c \bar{D}^*]_{\frac{1}{2}}$	$[\Xi'_c \bar{D}^*]_{\frac{3}{2}}$	$[\Xi_c^* \bar{D}]_{\frac{3}{2}}$	$[\Xi_c^* \bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c^* \bar{D}^*]_{\frac{3}{2}}$	$[\Xi_c^* \bar{D}^*]_{\frac{5}{2}}^{\#}$	$[\Xi_c \bar{D}]_{\frac{1}{2}}$	$[\Xi_c \bar{D}^*]_{\frac{1}{2}}$	$[\Xi_c \bar{D}^*]_{\frac{3}{2}}$
ΔE	$-18.5^{+6.4}_{-6.8}$	$-15.6^{+6.4}_{-7.2}$	$-2.0^{+1.8}_{-3.3}$	$-7.5^{+4.2}_{-5.3}$	$-17.0^{+6.7}_{-7.5}$	$-8.0^{+4.5}_{-5.6}$	$-0.7^{+0.7}_{-2.2}$	$-13.3^{+2.8}_{-3.0}$	$-17.8^{+3.2}_{-3.3}$	$-11.8^{+2.8}_{-3.0}$
M	$4423.7^{+6.4}_{-6.8}$	$4568.7^{+6.4}_{-7.2}$	$4582.3^{+1.8}_{-3.3}$	$4502.9^{+4.2}_{-5.3}$	$4635.4^{+6.7}_{-7.5}$	$4644.4^{+4.5}_{-5.6}$	$4651.7^{+0.7}_{-2.2}$	$4319.4^{+2.8}_{-3.0}$	$4456.9^{+3.2}_{-3.3}$	$4463.0^{+2.8}_{-3.0}$

1. Predicted ten P_{cs} states in the **isoscalar** channels.

Taken from Science Bulletin 66 (2021) 1278–1287

2. Three new ones in $\Xi_c \bar{D}^{(*)}$ systems.

State	M_0 (MeV)	Γ_0 (MeV)
$P_{cs}(4459)^0$	$4458.8 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{+8.0}_{-5.7}$

3. The new $[\Xi_c \bar{D}^*]_{1/2}$ state is **VERY consistent** with the newly LHCb result.

• What about the $\Lambda_c \bar{D}^{(*)}$ and other systems?

1. The $\Lambda_c \bar{D}^{(*)}$ systems: No isospin-isospin interaction, contact (repulsive)+TPE (couple-channel, attractive) $\simeq 0 \rightarrow$ **no bound states (estimation)**.

2. Other systems: $\Lambda_c \bar{D}_s^{(*)}$, $\Sigma_c \bar{D}_s^{(*)}$, $\Sigma_c^* \bar{D}_s^{(*)}$ ($s = -1$): **attractive**, but **too weak** to form bound states. $\Omega_c^{(*)} \bar{D}_s^{(*)}$ ($s = -3$): **repulsive**. **It is hard to form bound states in these systems!**

5. Doubly charmed P_{cc} states

From $\Sigma_c^{(*)} \bar{D}^{(*)}$ to $\Sigma_c^{(*)} D^*$ systems

The low energy constants of the $\Sigma_c^{(*)} D^*$ systems are estimated from the $N\bar{N}$ scattering data by introducing a quark level Lagrangian:

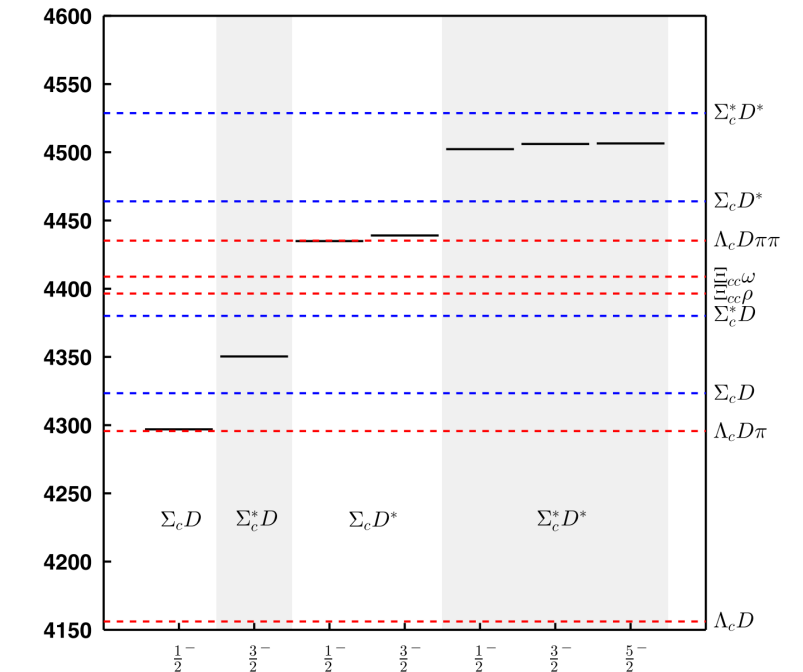
$$\mathcal{L} = g_s \bar{q} \mathcal{S} q + g_a \bar{q} \gamma_\mu \gamma^5 \mathcal{A}^\mu q,$$

$$V_{q\bar{q}} = c_s(1 - 3\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) + c_t(1 - 3\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2.$$

$$V_{\Sigma_c D^*} = 2c_s - 12c_s \mathbf{I}_1 \cdot \mathbf{I}_2 - \frac{4}{3}c_t \boldsymbol{\sigma} \cdot \mathbf{T} + 8c_t(\mathbf{I}_1 \cdot \mathbf{I}_2)(\boldsymbol{\sigma} \cdot \mathbf{T}).$$

		$[\Sigma_c D]_{\frac{1}{2}}$	$[\Sigma_c^* D]_{\frac{3}{2}}$	$[\Sigma_c D^*]_{\frac{1}{2}}$	$[\Sigma_c D^*]_{\frac{3}{2}}$	$[\Sigma_c^* D^*]_{\frac{1}{2}}$	$[\Sigma_c^* D^*]_{\frac{3}{2}}$	$[\Sigma_c^* D^*]_{\frac{5}{2}}$
Case 1	BE (MeV)	-15.4	-25.0	-31.8	-8.0	-32.8	-18.2	-3.5
	R_{rms} (fm)	1.45	1.25	1.20	1.65	1.20	1.38	1.91
Case 2	BE (MeV)	-31.3	-42.9	-30.3	-31.7	-26.6	-25.4	-29.7
	R_{rms} (fm)	1.23	1.11	1.22	1.20	1.26	1.27	1.22
Case 3	BE (MeV)	-26.5	-37.7	-29.1	-25.0	-26.4	-22.6	-22.2
	R_{rms} (fm)	1.27	1.14	1.23	1.27	1.26	1.31	1.30

K.C, B.W, S.-L.Z, [PhysRevD.103.116017](#)



All the $\Sigma_c^{(*)} D^*$ systems with isospin $I = 1/2$ can form bound states. In addition, we also investigate the interactions of the charmed-bottom $\Sigma_c^{(*)} \bar{B}^{(*)}$, $\Sigma_b^{(*)} D^{(*)}$ and $\Sigma_b^{(*)} \bar{B}^{(*)}$ systems. Among the obtained bound states, the bindings become deeper when the reduced masses of the corresponding systems are heavier.

6. More systems

Within the same framework, we also covered more systems

K.C, R.C, L.M, B.W, S.-L.Z, [Eur.Phys.J.C 82 \(2022\) 7, 581](#)

Meson-meson	$[\bar{D}\bar{D}]_0^1$	$[\bar{D}\bar{D}^*]_1^{0,1}$	$[\bar{D}^*\bar{D}^*]_{0,2}^1$
	$[\bar{D}^*\bar{D}^*]_1^0$		
Baryon-meson	$[\Lambda_c\bar{D}]_{\frac{1}{2}}^{\frac{1}{2}}$	$[\Lambda_c\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{\frac{1}{2}}$	$[\Sigma_c\bar{D}]_{\frac{1}{2}}^{\frac{1}{2},\frac{3}{2}}$
	$[\Sigma_c\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{\frac{1}{2},\frac{3}{2}}$	$[\Sigma_c^*\bar{D}]_{\frac{3}{2}}^{\frac{1}{2},\frac{3}{2}}$	$[\Sigma_c^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2},\frac{5}{2}}^{\frac{1}{2},\frac{3}{2}}$
	$[\Xi_c\bar{D}]_{\frac{1}{2}}^{0,1}$	$[\Xi_c\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{0,1}$	$[\Xi_c'\bar{D}]_{\frac{1}{2}}^{0,1}$
	$[\Xi_c'\bar{D}^*]_{\frac{1}{2},\frac{3}{2}}^{0,1}$	$[\Xi_c^*\bar{D}]_{\frac{3}{2}}^{0,1}$	$[\Xi_c^*\bar{D}^*]_{\frac{1}{2},\frac{3}{2},\frac{5}{2}}^{0,1}$
Baryon-baryon	$[\Lambda_c\Lambda_c]_0^0$	$[\Lambda_c\Sigma_c]_{0,1}^1$	$[\Sigma_c\Sigma_c]_0^{0,2}$
	$[\Sigma_c\Sigma_c]_1^1$	$[\Lambda_c\Sigma_c^*]_{1,2}^1$	$[\Sigma_c\Sigma_c^*]_{1,2}^{0,1,2}$
	$[\Sigma_c^*\Sigma_c^*]_{1,3}^1$	$[\Sigma_c^*\Sigma_c^*]_{0,2}^{0,2}$	$[\Xi_c\Xi_c]_0^1$
	$[\Xi_c\Xi_c]_1^0$	$[\Xi_c\Xi_c']_{0,1}^{0,1}$	$[\Xi_c\Xi_c^*]_{1,2}^{0,1}$
	$[\Xi_c'\Xi_c']_0^1$	$[\Xi_c'\Xi_c']_1^0$	$[\Xi_c'\Xi_c^*]_{1,2}^{0,1}$
	$[\Xi_c^*\Xi_c^*]_{1,3}^0$	$[\Xi_c^*\Xi_c^*]_{0,2}^1$	

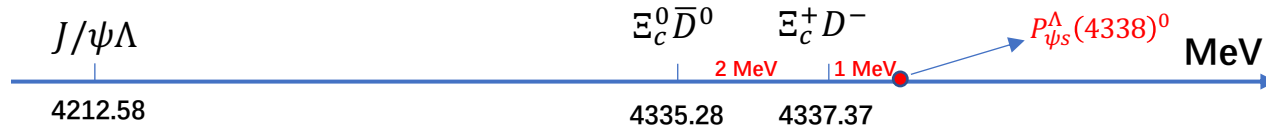
	Mass (Expt.)	BE (Expt.)	Mass (Our)	BE (Our)
$T_{cc}(3875)^+$	3874.8	-1.0	$3874.5_{-1.1}^{+1.7}$	$-1.8_{-1.1}^{+1.7}$
$P_c(4312)^+$	$4311.9 \pm 0.7_{-0.6}^{+6.8}$	$-8.9_{-0.9}^{+6.8}$ (input)	$4311.9_{-2.8}^{+6.8}$	$-8.9_{-2.8}^{+6.8}$
$P_c(4380)^+$	$4380 \pm 8 \pm 29$	-6.2 ± 30.1	$4376.2_{-2.8}^{+6.9}$	$-9.1_{-2.8}^{+6.9}$
$P_c(4440)^+$	$4440.3 \pm 1.3_{-4.7}^{+4.1}$	$-21.8_{-4.9}^{+4.3}$ (input)	$4440.2_{-5.3}^{+13.8}$	$-21.8_{-5.3}^{+13.8}$
$P_c(4457)^+$	$4457.3 \pm 0.6_{-1.7}^{+4.1}$	$-4.8_{-1.8}^{+4.1}$ (input)	$4457.3_{-1.9}^{+4.1}$	$-4.8_{-1.9}^{+4.1}$
$P_{cs}(4459)^0$	$4458.8 \pm 2.9_{-1.1}^{+4.7}$	$-19.7_{-3.1}^{+5.5}$	$4468.1_{-3.0}^{+7.3}$	$-10.0_{-3.0}^{+7.3}$

K.C, B.W, S.-L.Z, [Phys.Rev.D 105 \(2022\) 9, 096004](#)

$M-M$	DD D_sD_s	DD^* $D_sD_s^*$	D^*D^* $D_s^*D_s^*$	DD_s	$DD_s^* (D_sD^*)$	$D^*D_s^*$
$B-M$	$\Lambda_c\bar{D}$	$\Lambda_c\bar{D}^*$	$\Sigma_c\bar{D}$	$\Sigma_c\bar{D}^*$	$\Sigma_c^*\bar{D}$	$\Sigma_c^*\bar{D}^*$
	$\Lambda_c\bar{D}_s$	$\Lambda_c\bar{D}_s^*$	$\Sigma_c\bar{D}_s$	$\Sigma_c\bar{D}_s^*$	$\Sigma_c^*\bar{D}_s$	$\Sigma_c^*\bar{D}_s^*$
	$\Xi_c\bar{D}$	$\Xi_c\bar{D}^*$	$\Xi_c'\bar{D}$	$\Xi_c'\bar{D}^*$	$\Xi_c^*\bar{D}$	$\Xi_c^*\bar{D}^*$
	$\Xi_c\bar{D}_s$	$\Xi_c\bar{D}_s^*$	$\Xi_c'\bar{D}_s$	$\Xi_c'\bar{D}_s^*$	$\Xi_c^*\bar{D}_s$	$\Xi_c^*\bar{D}_s^*$
	$\Omega_c\bar{D}$	$\Omega_c\bar{D}^*$	$\Omega_c\bar{D}_s$	$\Omega_c\bar{D}_s^*$		
$B-B$	$\Lambda_c\Lambda_c$	$\Lambda_c\Sigma_c$	$\Lambda_c\Sigma_c^*$	$\Sigma_c\Sigma_c$	$\Sigma_c\Sigma_c^*$	$\Sigma_c^*\Sigma_c^*$
	$\Xi_c\Xi_c$	$\Xi_c\Xi_c'$	$\Xi_c\Xi_c^*$	$\Xi_c'\Xi_c'$	$\Xi_c'\Xi_c^*$	$\Xi_c^*\Xi_c^*$
	$\Lambda_c\Xi_c$	$\Lambda_c\Xi_c^*$	$\Lambda_c\Xi_c^*$	$\Lambda_c\Omega_c$	$\Sigma_c\Xi_c$	$\Sigma_c\Xi_c^*$
	$\Sigma_c\Xi_c^*$	$\Sigma_c\Omega_c$	$\Sigma_c^*\Xi_c$	$\Sigma_c^*\Xi_c'$	$\Sigma_c^*\Xi_c^*$	$\Sigma_c^*\Omega_c$
	$\Xi_c\Omega_c$	$\Xi_c'\Omega_c$	$\Xi_c^*\Omega_c$	$\Xi_c^*\Omega_c$	$\Omega_c\Omega_c$	

7. Lineshapes of the $P_{\psi_s}^\Lambda(4338)^0$

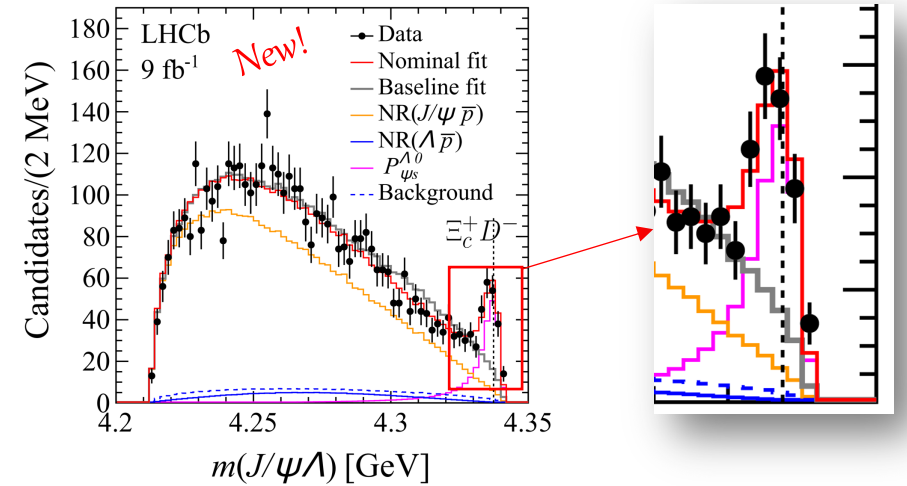
$$m = 4338.3 \pm 0.7 \pm 0.4 \text{ MeV}, \Gamma = 7.0 \pm 1.2 \pm 1.3 \text{ MeV}.$$



If a pole is close to the physical sheet and far away from the threshold,

$$\text{Breit-Weigner (BW): } T \propto 1/(E - M + i\Gamma/2)$$

The lineshape of the resonance would be **distorted from** the **conventional BW distribution** if it appears near the threshold and strongly couples to the threshold at the same time [Phys. Lett. B 63,224(1976), Phys. Rev. D 76, 034007(2007), Phys. Rev. Lett. 115, 202001(2015)].



therapies → K-matrix parameterization or LS equation

Two models for the $J/\psi\Lambda$ - $\Xi_c^0\bar{D}^0$ - $\Xi_c^+D^-$ interactions:

L.M, B.W, S.-L.Z, PhysRevD.107.014005

Model-I

$$V_I = \frac{1}{2} \begin{bmatrix} 0 & -\tilde{c} & \tilde{c} \\ -\tilde{c} & c_1 + c_0 & c_1 - c_0 \\ \tilde{c} & c_1 - c_0 & c_1 + c_0 \end{bmatrix},$$

Model-II

$$V_{II} = \frac{1}{2} \begin{bmatrix} 0 & -\tilde{c} & \tilde{c} \\ -\tilde{c} & \frac{g^2}{E^2 - m_0^2} & -\frac{g^2}{E^2 - m_0^2} \\ \tilde{c} & -\frac{g^2}{E^2 - m_0^2} & \frac{g^2}{E^2 - m_0^2} \end{bmatrix},$$

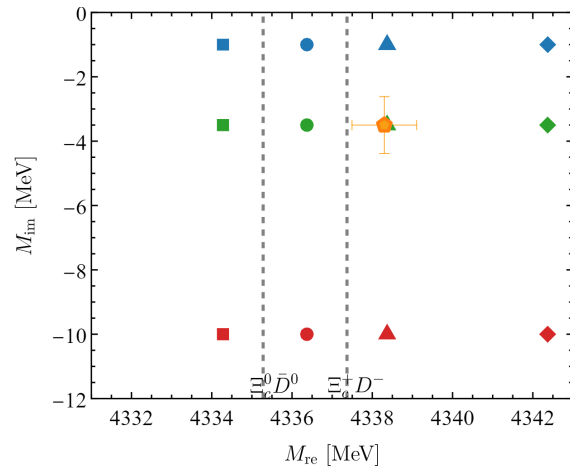
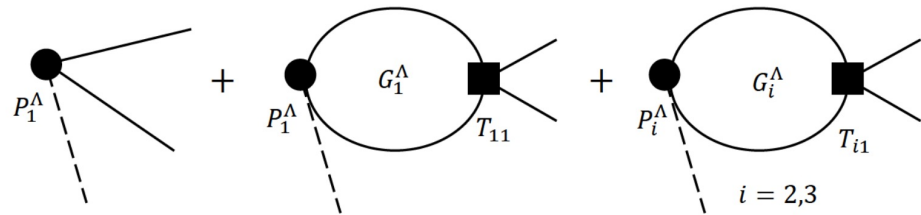
7. Lineshapes of the $P_{\psi_s}^\Lambda(4338)^0$

$$T = V + VGT, G = \text{diag}\{G_1, G_2, G_3\}$$

$$G_i(E) = \int_0^\Lambda \frac{l^2 d^2 l}{(2\pi)^2} \frac{\omega_{i1} + \omega_{i2}}{\omega_{i1} \omega_{i2} [E^2 - (\omega_{i1} + \omega_{i2})^2 + i\epsilon]}, \quad \omega_{ia} = (l^2 + m_{ia}^2)^{1/2}.$$

$$\text{Analytical continuation: } G_i \rightarrow G_i + i \frac{k_i}{4\pi E}$$

Production diagrams: L.M, B.W, S.-L.Z, [PhysRevD.107.014005](#)



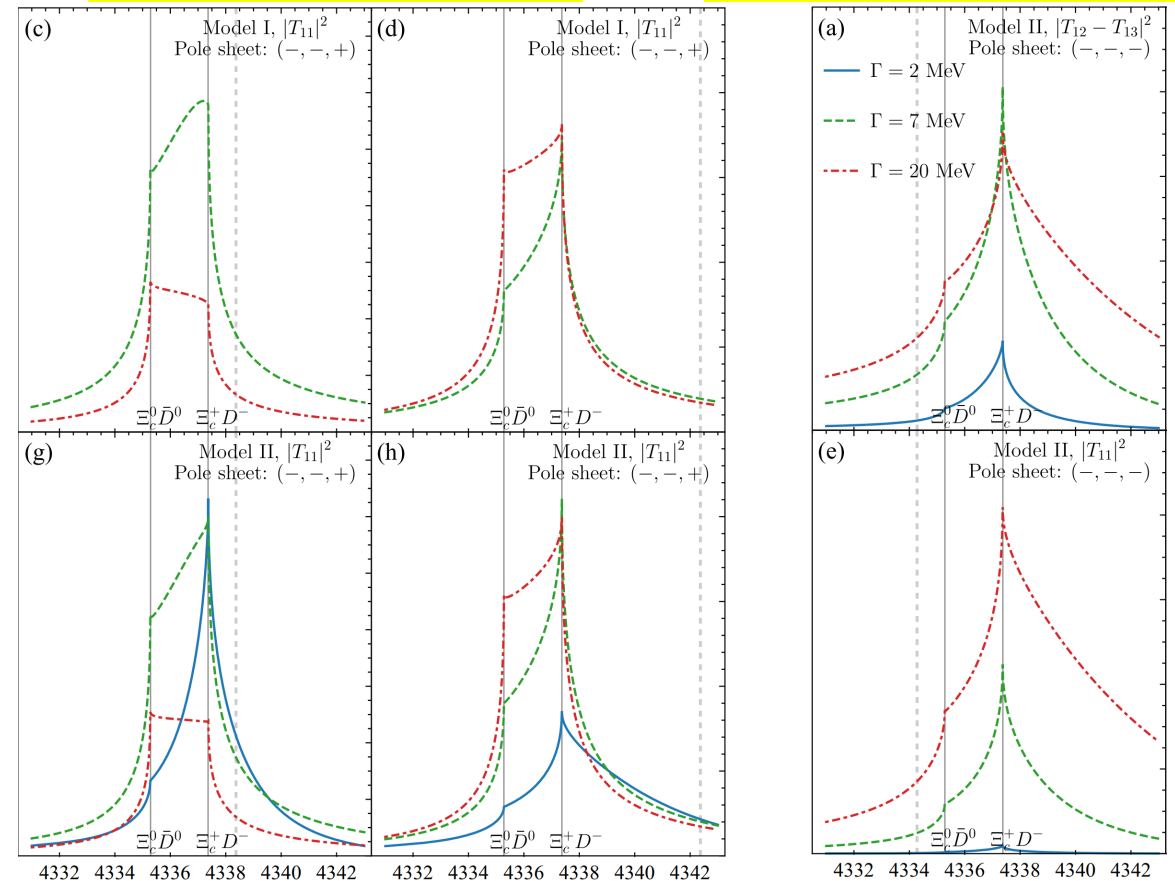
“Synthetic” poles:

$$M_{ij} = M_i - i\Gamma_j/2$$

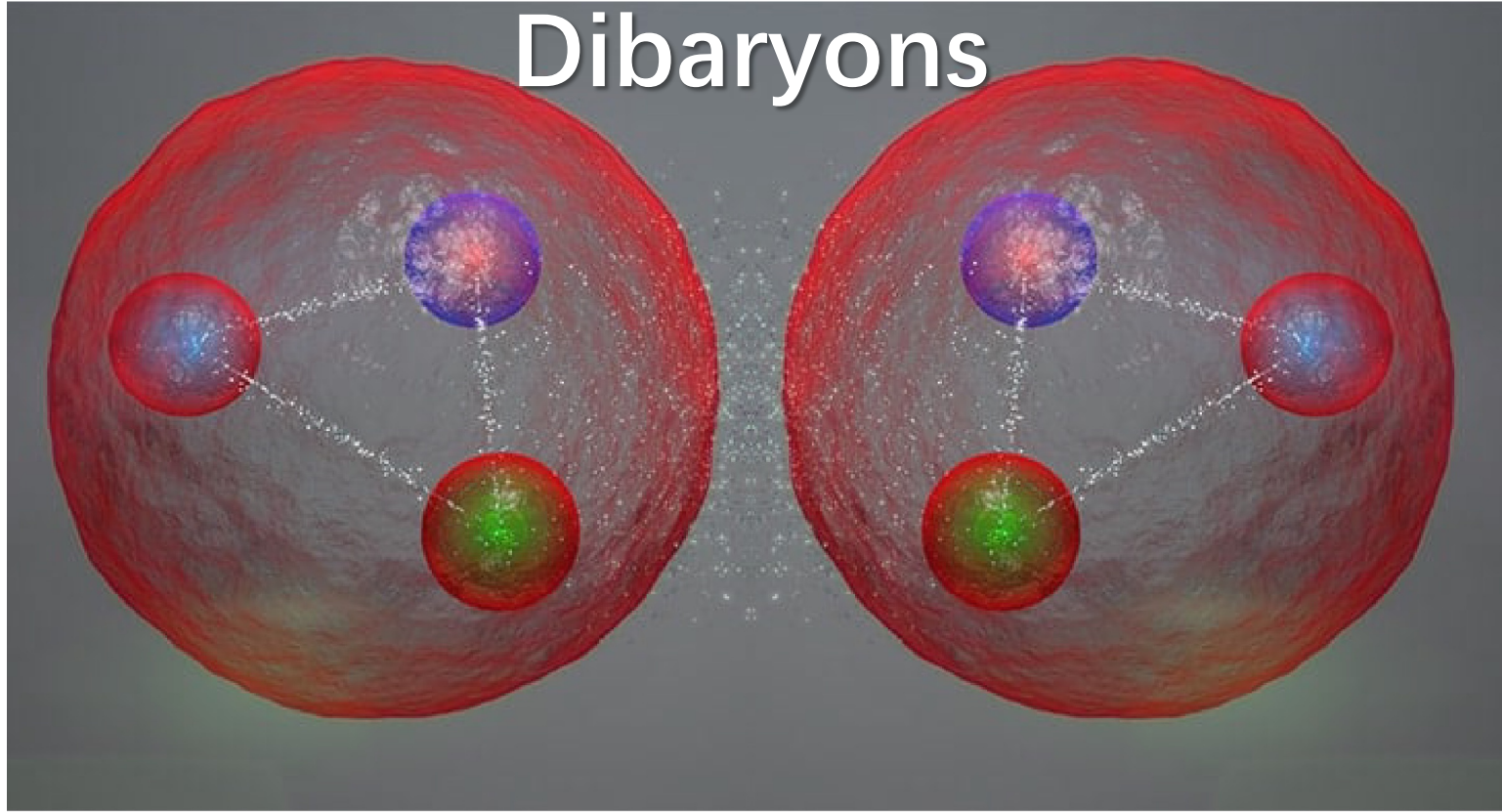
- $M_i = \{m_{\Xi_c^0 D^0} - 1, m_{\Xi_c^+ D^-} - 1, m_{\Xi_c^+ D^-} + 1, m_{\Xi_c^+ D^-} + 5\}$ MeV
- $\Gamma_i = \{2, 7, 20\}$ MeV

Poles well above the $\Xi_c^+ D^-$ threshold

Poles well below the $\Xi_c^+ D^-$ threshold



Dibaryons

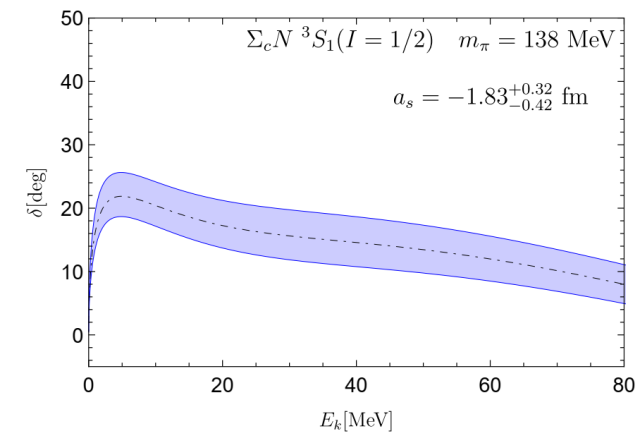
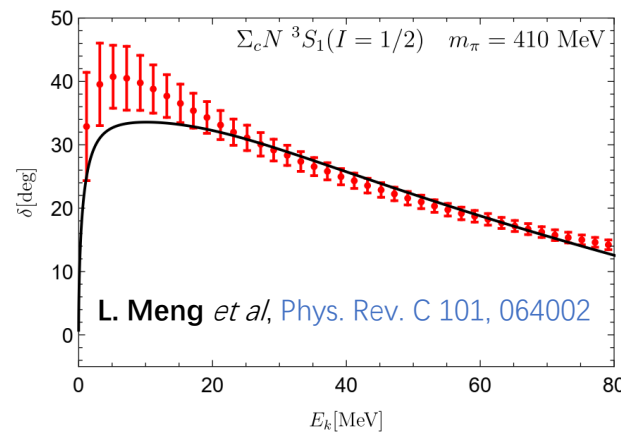
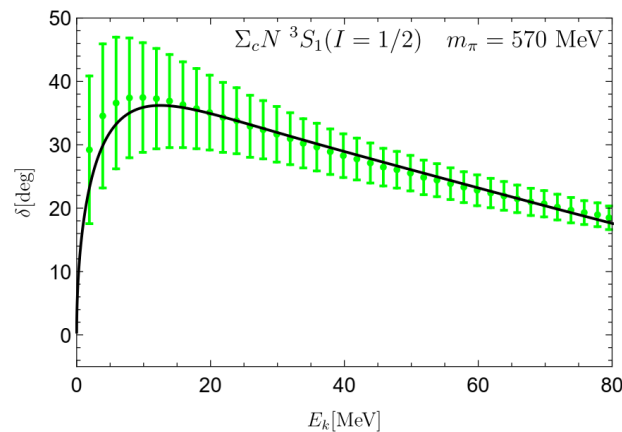


Dibaryons (molecular hexaquark)

A dibaryon is essentially a system with two baryons. There is one known dibaryon in nature- deuteron, another possible one is the $\Delta\Delta$ dibaryon- $d^*(2380)$ (disputed).

1. NB_Q and NB_{QQ} systems

- ✓ The NY_c ($Y_c = \Sigma_c, \Lambda_c$) interactions are essential for understanding the in-medium properties of the charmed baryons. The experimental proposals at the J-PARC [[arXiv:1706.07916](https://arxiv.org/abs/1706.07916)] and GIS-FAIR [[Prog. Part. Nucl. Phys. 66 \(2011\) 477–518](https://arxiv.org/abs/1101.4777)] have stimulated many investigations on the NY_c interactions.
- ✓ In Refs. [[Nucl. Phys. A 971 \(2018\) 113–129](https://arxiv.org/abs/1801.08001), [PoS Hadron2017 \(2018\) 146](https://arxiv.org/abs/1801.08001)], the HAL QCD Collaboration calculated the phase shifts of the $N\Lambda_c$ and $N\Sigma_c$ scatterings from lattice QCD at the unphysical pion mass $m_\pi = 410 - 570$ MeV.



- ✓ In Ref. [[L. Meng *et al*, Eur. Phys. J. A 54 \(9\) \(2018\) 143](https://arxiv.org/abs/1801.08001)], the authors predicted the bound states in the $N\Xi_{cc}$ and $\bar{N}\Xi_{cc}$ systems from the OBE model.

Dibaryons (molecular hexaquark)

2. $B_Q B_Q$ and $B_Q \bar{B}_Q$ systems

- ✓ In Ref. [N. Lee *et al*, [PhysRevD.84.014031](#)], the authors calculated the $\Lambda_c \Lambda_c (\bar{\Lambda}_c)$, $\Xi_c \Xi_c (\bar{\Xi}_c)$, $\Sigma_c \Sigma_c (\bar{\Sigma}_c)$, $\Xi'_c \Xi'_c (\bar{\Xi}'_c)$, $\Omega_c \Omega_c (\bar{\Omega}_c)$ systems within the OBE model, they obtained: the H-dibaryonlike state $\Lambda_c \Lambda_c$ does not exist; there may exist loosely bound deuteronlike states for the other systems
- ✓ In Ref. [J.-B. Cheng *et al*, [PhysRevD.107.054018](#)], the authors investigated the double-charm and hidden-charm hexaquarks as molecules in complex scaling method with explicit three-body effect.
- ✓ In Ref. [J.-X. Lu *et al*, [PhysRevD.99.074026](#)], the authors found that the isoscalar $\Lambda_c \bar{\Lambda}_c$, $\Sigma_c^{(*)} \bar{\Sigma}_c^{(*)}$ and isovector $\Lambda_c \bar{\Sigma}_c^{(*)}$ as well as their doubly charmed and doubly bottom counterparts are good candidates of the molecular hexaquarks.
- ✓ In Ref. [X.-Z. Ling *et al*, [Eur. Phys. J. C \(2021\) 81:1090](#)], the masses and strong decays of the $\Sigma_c^{(*)} \Sigma_c^{(*)}$ dibaryons were calculated.
- ✓ Calculations from other approaches, see [H. Huang *et al*, [PhysRevC.89.035201](#); T.-F. Carames *et al*, [PhysRevD.92.034015](#); H. Garcilazo *et al*, [Eur. Phys. J. C 80 \(8\) \(2020\) 720](#); Z. Liu *et al*, [Phys.Rev.D 105 \(2022\) 3, 034006](#); X.-K. Dong *et al*, [Commun. Theor. Phys. 73 \(12\) \(2021\) 125201](#); X.-K. Dong *et al*, [Progr. Phys. 41 \(2021\) 65–93](#)].

3. $B_{QQ} B_Q$ and $B_{QQ} \bar{B}_{QQ}$ systems

The $\Xi_{cc}^{(*)} [\bar{\Xi}_{cc}^{(*)}]$ can be related to the $\bar{D}^{(*)} [D^{(*)}]$ with the heavy diquark-antiquark symmetry (HDAS),

$$\Xi_{cc}^{(*)} \xrightarrow{\text{HDAS}} \bar{D}^{(*)} \quad \bar{\Xi}_{cc}^{(*)} \xrightarrow{\text{HDAS}} D^{(*)}$$

Dibaryons (molecular hexaquark)

3. $B_{QQ}B_Q$ and $B_{QQ}\bar{B}_{QQ}$ systems

As a consequence of the HDAS, the $\Xi_{cc}^{(*)}D^{(*)}$, $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$ and $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^{(*)}$ systems can be related to the $\bar{D}^{(*)}D^{(*)}$, $\bar{D}^{(*)}\Sigma_c^{(*)}$ and $\bar{D}^{(*)}D^{(*)}$ systems, respectively.

Thus, the existence of the molecular states in the $\bar{D}^{(*)}D^{(*)}$ and $\bar{D}^{(*)}\Sigma_c^{(*)}$ systems should also imply the existence of the molecular states in the $\Xi_{cc}^{(*)}D^{(*)}$, $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$ and $\Xi_{cc}^{(*)}\bar{\Xi}_{cc}^{(*)}$ systems.

- ✓ In Ref. [B. Yang *et al*, [Eur. Phys. J. A56 \(2\) \(2020\) 67](#)], Yang et al investigated the possible bound states in the $\Xi_{cc}^{(*)}\Xi_{cc}^{(*)}(\bar{\Xi}_{cc}^{(*)})$ systems, and predicted the molecular candidates in the isoscalar and isovector channels.
- ✓ In Ref. [F.-K. Guo *et al*, [PhysRevD.88.054014](#)], the authors predicted the triply heavy pentaquarks with $I(J^P) = 0(3/2^-)$, $0(5/2^-)$ with the $X(3872)$ as input, as well as the $1(1/2^-)$ and $1(3/2^-)$ ones with the $Z_b(10650)$ as input. see also R. Chen *et al*, [PhysRevD.96.114030](#).
- ✓ In Ref. [Y.-W. Pan *et al*, [PhysRevD.102.011504](#)], the authors proposed an alternative way to determine the spins of the $P_c(4440)$ and $P_c(4457)$ from the spectrum of the $\Xi_{cc}^{(*)}\Sigma_c^{(*)}$ systems with the help of lattice QCD.

4. $B_{QQQ}B_{QQQ}$ systems

- ✓ **Lattice:** [Phys. Rev. Lett.127.072003](#); [Phys. Rev. Lett.130.111901](#)
- ✓ **Models:** [Chin. Phys. Lett. 38, 101201](#); [Eur. Phys. J. C 82, 805](#); [Int. J. Mod. Phys. A 37, 2250166](#); [arXiv: 2207.05505](#); [arXiv: 2208.03041](#)

Summary and outlook

1. Many near-threshold states have been observed in experiments.
2. Their spectra and decays were intensively studied within various models.
3. Most of the nowadays observed exotic states have the same origin? —The dynamically generated resonances (bound states) from the analogue of nuclear forces in different sectors.
4. What forces govern the formations of these states—the “general nuclear forces”?
5. Weak(er) model-dependent approaches need to be developed.

Thank you!