Motivation of longitudinal polarization

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Historical summary

- Froissart and Store calculated the depolarization on spin resonances tor protons [1]
- Bargmann, Michel and Telegdi [2] reformulated the Thomas equation for the spin procession in arbitrary electron magnetic fields
- Ternov et.al. [3] discovered theoretically the spontaneous polarization of electron in a magnetic field, which is analysed in detail by Skolov and Ternov [4].
- In 1970, radiative polarization was indeed observed and studied at ACO(Orsay) [5] and at VEPP-2 at about 0.5 GeV [6]
- Soon after, polarized protons were accelerated up to 12 GeV in the ZGS (Argonne) [7]
- The development of techniques to accelerate polarized beams at higher and higher energies: AGS 22 GeV, RHIC and SSC have been considered, SLC (46 GeV).
- And in-situ polarization build-up of electron beams circulating in HERA, LEP…

Ref:

[1] M.Froissart and R.Stora, Nucl. Instr. Meth., 7 (1960) 297.

[2] V. Bargmann at.al., PRL 2 (1959) 435.

[3] Ternov and et.al Phys. JETP, 14 (1962), 921.

[4] Ternov and Sokolov . Phys.-Doklady, 8,(1964) 1203

[5] The Orsay Storage ring group, in Proc. 8th Int High Energy Acclerates CERN,1971. P.127

[6] V.N.Baier, Sov.phys.-USPEKHI,14 (1976) 1063.

[7] T.Khoe et al, Part.Accel., 6 (1975) 213.

Summary Table

表 1.1 国内外部分涉及极化束流的加速器。

类别	机器	机构	位置	极化束流
环形加速器	HERA	DESY	Hamburg, Germany	Longitudinal polarized e^-/e^+
	VEPP-(2,2M,3,4)	BINP	Novosibrisk, Russia	Transversal polarized e^-/e^+
	ACO	LAL	Orsay, France	Transversal polarized e^-/e^+
	SPEAR	SLAC	CA, USA	Transversal polarized e^-/e^+
	LEP	CREN	Geneva, Switzerland	Transversal polarized e^-/e^+
	AGS	BNL	Upton, NY,USA	Transversal polarized protons
	RHIC	BNL	Upton, NY,USA	Longitudinal polarized protons
	IUCF Cooler	IUCF	Bloomingtion, USA	Transversal polarized protons/deuterons
	AmPS	NIKHEF	Amsterdam, Netherlands	Transversal//longitudinal polarized e^-
	SHR	MIT-Bates	Middleton, MA, USA	Transversal/longitudinal polarized e^-
	KEK-PS	KEK	Tsukuba, Japan	Transversal polarized protons/deuterons
	SuperKEKB	**KEK**	**Tsukuba, Japan**	**Longitudinal polarized** e^-/e^+
	FCC-ee	**CREN**	**Geneva, Switzerland**	**Transversal polarized** e^-/e^+
	EIC	**BNL**	**Upton,NY,USA**	**Longitudinal polarized** e^-/e^+**, protons,deuterons**
	CEPC	**IHEP**	**China**	**Transversal/longitudinal polarized** e^-/e^+
直线加速器	SLC	SLAC	CA, USA	Longitudinal polarized e^-
	CEBAF	Jlab	Newport News, VA, USA	Longitudinal polarized e^-
	ILC	**KEK**	**Japan**	**Longitudinal polarized** e^-/e^+
	CLIC	**CREN**	**Switzerland**	**Longitudinal polarized** e^-/e^+

Ref: 夏文昊博士毕业论文：《CEPC 加速器极化束关键物理研究与设计》

Why has any effort been expended on the polarized beams?

- The answer, at least in the case of polarized electron beams, is that electron acceleraters and storage rings have been in recent years achieved sufficient energy to begin to probe the weak interaction directly.

- The weak interaction distinguishes between left- and right-handed fermionic currents.

- left-handed particles interact in a fundamentally different way than their right-handed counterparts.

- If the experimenter wishes to explore or exploit this difference, he (or she) must either prepare the spin state of the incident particles or analyze the spin state of outgoing particles.

Polarization

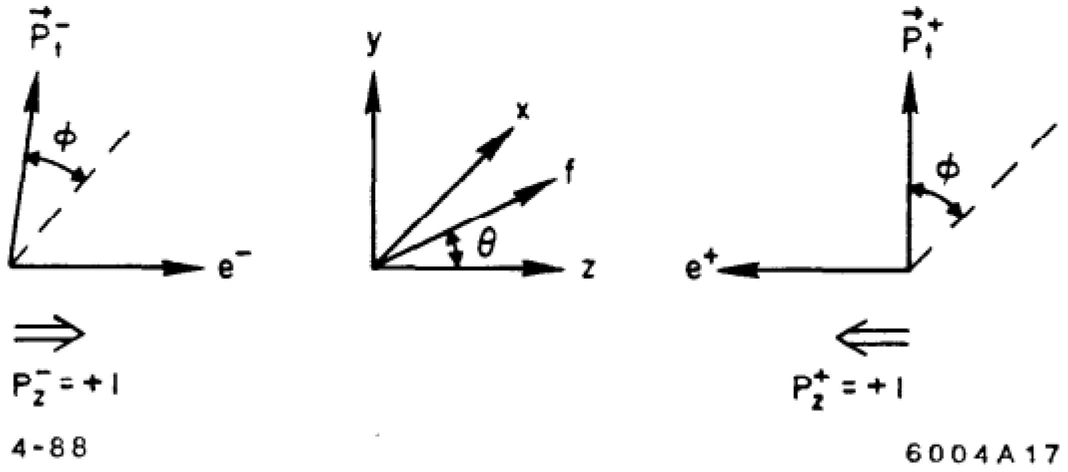

Fig. 17. The electron-positron coordinate system. The electron beam is moving in the +z direction. The x-axis points in the horizontal direction and the y-axis in the vertical direction. The electron and positron longitudinal polarizations are described in terms of a helicity basis. Right-handed particles (and antiparticles) have $\mathcal{P}_z = +1$ and left-handed particles have $\mathcal{P}_z = -1$.

Ref: https://www.slac.stanford.edu/pubs/slacpubs/4500/slac-pub-4656.pdf

The cross section for $e^+e^- \to f\bar{f}$

- 在SLC上工作的Mark-Ⅱ合作组在1989年8月首次报告了他们在e^+e^-对撞实验中测量到Z^0玻色子，1992年实现了e^-束流的极化， Mark-Ⅱ探测器也更换为SLD探测器。
- 到1998年SLC停止运行，SLD合作组采集了大约6e5的Z^0产生和衰变事例。
- 尽管SLD的数据量远小于LEP-Ⅰ的数据量，但由于使用了纵向极化束流，在测量Z^0矢量玻色子与其他粒子的耦合强度和特性时有其独特的优势，所以与高数量量但无极化束流的LEP-Ⅰ实验形成竞争与互补

The cross section for $e^+e^- \to f\bar{f}$

费曼图 → 振幅 → 截面

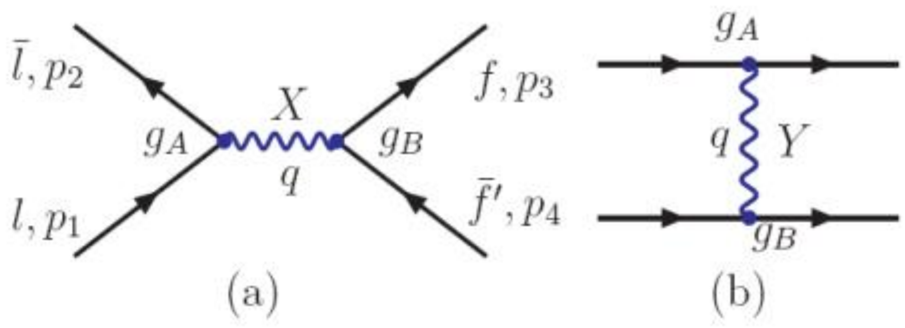

图 8.4: $e^+e^- \to f\bar{f}$ 散射过程的"S-道"和"t-道"树图贡献。

- 假设左手的电子和左手的正电子 和 右手的电子和右手的正电子的湮灭矩阵均为0，那么对于$e^+e^- \to f\bar{f}$ 振幅的模方可表示为：

$$\sum_{\substack{\text{final}\\\text{spins}}} |\mathcal{M}|^2 \sim (1-P_z^-)(1+P_z^+)|\mathcal{M}_L|^2 + (1+P_z^-)(1-P_z^+)|\mathcal{M}_R|^2 + 2P_t^- P_t^+\left[\mathrm{Re}(\mathcal{M}_L\mathcal{M}_R^*)\cos\Phi + \mathrm{Im}(\mathcal{M}_L\mathcal{M}_R^*)\sin\Phi\right]$$

$P_Z^{\pm}$：代表正电子和电子束的纵向极化	$P_t^{\pm}$:：代表正电子和电子束的横向极化
M_L：代表左手电子和右手正电子的反应矩阵	M_R：代表右手电子和左手正电子的反应矩阵
$\Phi = 2\phi - \phi^- - \phi^+$	
Φ：是outgoing fermion 的azimuth	$\Phi^{\pm}$：是正电子和电子的横向极化矢量

The cross section for $e^+e^- \to f\bar{f}$

Using Equation (7.1), we can write the cross section for $e^+e^- \to f\bar{f}$ in the center-of-mass frame as [25]

$$\frac{4s}{\alpha^2}\frac{d\sigma}{d\Omega} = (1 - \mathcal{P}_z^+\mathcal{P}_z^-)\sigma_u + (\mathcal{P}_z^+ - \mathcal{P}_z^-)\sigma_z + \mathcal{P}_t^+\mathcal{P}_t^-\left[\sigma_t \cos\Phi + \tilde{\sigma}_t \sin\Phi\right] \qquad (7.3)$$

s:是质心系的能量

The Helicity Cross Sections for the Process $e^+e^- \to f\bar{f}$

Symbol	Helicity Cross Section	Matrix Element Structure
σ_u	Unpolarized	$\sigma_u \sim (\lvert\mathcal{M}_L\rvert^2 + \lvert\mathcal{M}_R\rvert^2)$
σ_z	Longitudinal	$\sigma_z \sim (\lvert\mathcal{M}_L\rvert^2 - \lvert\mathcal{M}_R\rvert^2)$
σ_t	Real Transverse	$\sigma_t \sim \mathrm{Re}(\mathcal{M}_L\mathcal{M}_R^*)$
$\tilde{\sigma}_t$	Imaginary Transverse	$\tilde{\sigma}_t \sim \mathrm{Im}(\mathcal{M}_L\mathcal{M}_R^*)$.

- 几个结论:
  - 只要左-右手截面是不同的，那么纵向极化的截面一定 non-zero
  - 单独的电子束/正电子束或者both beams纵向极化将影响 σz的微分截面
  - 只有both beams are polarized， 才会有 Transverse polarization effect

The cross section for $e^+e^- \to f\bar{f}$

- 假设$e^+e^- \to f\bar{f}$仅仅由photon and Z^0 exchange only
  - 然后，each of the four helicity cross sections 包括:
    - γ-交换项
    - Z^0-交换项
    - $\gamma - Z^0$ 两者的干涉项

- 通常，螺旋度截面依赖于:

In general, the helicity cross sections depend upon: v and a, the vector and axial vector couplings of the Z° to the electron; v_f and a_f, the vector and axial vector couplings of the Z° to the fermionic current; Q_f, the electric charge of the fermion (in units of e); $\Gamma(s)$, the normalized Z° propagator ($\Gamma(s) = s/[s - M_Z^2 + iM_Z\Gamma_Z]$); and c, the cosine of the polar angle of the outgoing fermion. We have

chosen the following definition for the vector and axial vector coupling constants,

$$v_f = (I_3^f - 2Q_f \sin^2\theta_w)/\sin 2\theta_w$$
$$a_f = -I_3^f/\sin 2\theta_w \qquad (7.4)$$

弱同位旋 电弱混合相角

where I_3^f is the third component of fermion weak isospin and $\sin^2\theta_w$ is the well-known electroweak mixing parameter. The following section lists the tree-level

The tree-level terms of the four helicity cross section

以最低阶/树图计算散射截面

1. The unpolarized cross section can be decomposed as follows:

$$\sigma_u = \sigma_u^\gamma + \sigma_u^{\gamma Z} + \sigma_u^Z,$$

$$\sigma_u^\gamma = Q_f^2(1+c^2)$$

$$\sigma_u^{\gamma Z} = -2Q_f \mathrm{Re}\{\Gamma(s)\}\left[(1+c^2)vv_f + 2caa_f\right]$$

$$\sigma_u^Z = |\Gamma(s)|^2\left[(1+c^2)(v^2+a^2)(v_f^2+a_f^2) + 8cvav_fa_f\right].$$

2. The longitudinal cross section can also be decomposed into three parts:

$$\sigma_z = \sigma_z^\gamma + \sigma_z^{\gamma Z} + \sigma_z^Z, \quad (7.6)$$

$$\sigma_z^\gamma = 0$$

$$\sigma_z^{\gamma Z} = 2Q_f \mathrm{Re}\{\Gamma(s)\}\left[(1+c^2)av_f + 2cva_f\right]$$

$$\sigma_z^Z = -|\Gamma(s)|^2\left[2(1+c^2)va(v_f^2+a_f^2) + 4c(v^2+a^2)v_fa_f\right].$$

3. The real transverse cross section can be decomposed as follows:

$$\sigma_t = \sigma_t^\gamma + \sigma_t^{\gamma Z} + \sigma_t^Z,$$

$$\sigma_t^\gamma = Q_f^2(1-c^2)$$

$$\sigma_t^{\gamma Z} = -2Q_f \mathrm{Re}\{\Gamma(s)\}(1-c^2)vv_f$$

$$\sigma_t^Z = -|\Gamma(s)|^2(1-c^2)(v^2-a^2)(v_f^2+a_f^2).$$

4. The imaginary transverse cross section can be decomposed as follows:

$$\tilde{\sigma}_t = \tilde{\sigma}_t^\gamma + \tilde{\sigma}_t^{\gamma Z} + \tilde{\sigma}_t^Z,$$

$$\tilde{\sigma}_t^\gamma = 0$$

$$\tilde{\sigma}_t^{\gamma Z} = 2\mathrm{Im}\{\Gamma(s)\}(1-c^2)av_f$$

$$\tilde{\sigma}_t^Z = 0.$$

Polarization-dependent cross section for $e^+e^- \to f\bar{f}$ at the Z^0 pole

Most of our discussion will involve the remarkable properties of polarized Z° production. At the Z° pole, the various helicity cross sections can be simplified considerably. The pure photon exchange terms (denoted by σ^γ) are quite small as compared with the pure Z° exchange terms (denoted by σ^Z). Most of the electroweak interference terms (denoted by $\sigma^{\gamma Z}$) are proportional to the real part of the Z° propagator

mass. This term is therefore very small. The complete, polarization-dependent cross section for $e^+e^- \to f\bar{f}$ at the Z° pole can thus be written as

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4}\frac{s}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\times \\ \Big\{(1-\mathcal{P}_z^+\mathcal{P}_z^-)\left[(1+c^2)(v^2+a^2)(v_f^2+a_f^2)+8cvav_fa_f\right] \\ -(\mathcal{P}_z^+-\mathcal{P}_z^-)\left[2(1+c^2)va(v_f^2+a_f^2)+4c(v^2+a^2)v_fa_f\right] \\ +\mathcal{P}_t^+\mathcal{P}_t^-\cos\Phi(1-c^2)(v^2-a^2)(v_f^2+a_f^2)\Big\}.\end{aligned} \tag{7.9}$$

Coupling Constants

Coupling constant

It is often quite convenient to transform the vector and axial vector coupling constants to a left- and right-handed helicity basis.* The definitions of the left- and right-handed couplings to the $Z°$ current are

$$g_L^f = v_f - a_f = (2I_3^f - 2Q_f \sin^2\theta_w)/\sin 2\theta_w$$
$$g_R^f = v_f + a_f = -2Q_f \sin^2\theta_w/\sin 2\theta_w. \quad (7.10)$$

- To get a feeling for the magnitude of the couplings of the various fermions to the Z^0 , Equations (7.4) and (7.10) are evaluated in Table III for a complete quark and lepton generation.

- 在各个能区的正负电子对撞机实验中，均可以对$e^+e^- \to f\bar{f}$ 的产生截面作精确测量。这里的末态费米子对可以是轻子对和中微子对，也可以是夸克对。

The coupling constants of various fermions to the $Z°$. The value of $\sin^2\theta_w$ is assumed to be 0.230. All coupling constants are listed in units of $[\sin 2\theta_w]^{-1}$.

Fermion	a	v	g_L	g_R
Neutrino	-0.5	0.5	1.0	0
Charged Lepton	0.5	-0.04	-0.54	0.46
u Type Quark	-0.5	0.19	0.69	-0.31
d Type Quark	0.5	-0.35	-0.85	0.15

Asymmetry

Asymmetry

- 低能情况下， $e^+e^- \to f\bar{f}$ 过程的微分截面由光子交换主导，所以截面角分布对称，无极化依赖性。
- 在Z^0 处的微分截面被期望有近乎对称的轻子角分布和在束流螺旋度上也近乎对称
- 因此 对称性的偏离是 电弱混合相角($sin^2\theta_\omega$)的敏感函数
- 测量不同的asymmetry是检验标准模型的一种手段，寻找新物理的一种手段

8.1 The unpolarized forward-backward asymmetry

不含极化的前后的不对称性（forward-backward asymmetry）可反映电弱干扰效应。对于$e^+e^- \to f\bar{f}$过程，forward-backward asymmetry定义为A_{FB}^f

$$A_{FB}^f(x) \equiv \frac{\int_0^x dc \frac{d\sigma}{dc} - \int_{-x}^0 dc \frac{d\sigma}{dc}}{\int_{-x}^x dc \frac{d\sigma}{dc}}$$

x 是探测器接受度所给的积分限值；

又因为截面定义为：

$$\begin{aligned}\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4}\frac{s}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\times \\ \Big\{(1-\mathcal{P}_z^+\mathcal{P}_z^-)\left[(1+c^2)(v^2+a^2)(v_f^2+a_f^2)+8cvav_fa_f\right] \\ -(\mathcal{P}_z^+-\mathcal{P}_z^-)\left[2(1+c^2)va(v_f^2+a_f^2)+4c(v^2+a^2)v_fa_f\right] \\ +\mathcal{P}_t^+\mathcal{P}_t^-\cos\Phi(1-c^2)(v^2-a^2)(v_f^2+a_f^2)\Big\}.\end{aligned} \quad (7.9)$$

8.1 The unpolarized forward-backward asymmetry

可得:

$$A_{FB}^{f}(x) = F(x) \cdot \frac{3ava_f v_f}{(a^2+v^2)(a_f^2+v_f^2)}$$

$$= F(x) \cdot \frac{3}{4} \frac{(g_L^2 - g_R^2)(g_L^{f^2} - g_R^{f^2})}{(g_L^2 + g_R^2)(g_L^{f^2} + g_R^{f^2})}$$

where the function $F(x)$ is

$$F(x) = \frac{4x}{3+x^2}.$$

Since $F(1) = 1$, we choose to define the symbol $A_{FB}^{f}(1) = A_{FB}^{f}$.

We can make several observations about the forward-backward asymmetry.

- forward-backward asymmetry同时取决于电子和费米子与Z^0的耦合；
- 由于强子jet的味很难精确测量，因此不可避免地要用μ子（或者也许用τ轻子）对A_{FB}进行精确测量。
- 对于轻子末态的forward-backward asymmetry是非常小的，假设轻子是普适性的，那么：

$$A_{FB}^{l} = \frac{3a^2v^2}{(a^2+v^2)^2} \simeq 2\% \quad (\text{at } \sin^2\theta_w = 0.230).$$

8.2 The left-right polarization asymmetry

- The longitudinal helicity cross section is violates parity

◆ 有效极化(或说是 归一化的极化度)定义为: $P_g = \frac{P_Z^+ - P_Z^-}{1 - P_Z^+ P_Z^-}$

◆ 同时可被表示为:

$$A_{LR}^f(x) = \frac{\int_{-x}^{x} dc \frac{d\sigma}{dc}(P_g = 1) - \int_{-x}^{x} dc \frac{d\sigma}{dc}(P_g = -1)}{\int_{-x}^{x} dc \frac{d\sigma}{dc}(P_g = 1) + \int_{-x}^{x} dc \frac{d\sigma}{dc}(P_g = -1)}.$$

◆ 以归一化极化P_g表示，忽略横向极化的部分和归一化参数，此时截面可被表示为:

$$\frac{d\sigma}{dc} \propto (1+c^2)\left[(v^2+a^2)(v_f^2+a_f^2) - P_g 2va(v_f^2+a_f^2)\right] + c\left[8vav_fa_f - P_g 4v_fa_f(v^2+a^2)\right].$$

$$A_{LR}^f(x) = \frac{-2va}{(v^2+a^2)} = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}.$$

We can make several observations about the left-right asymmetry.

$$A^f_{LR}(x) = \frac{-2va}{(v^2 + a^2)} = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2}.$$

1. Left-right asymmetry不依赖于末态费米子的耦合，仅仅与初态电子的耦合有关;
2. 也不依赖于探测器的接受度;
3. 电子矢量的耦合是线性的，即left-right asymmetry比forward-backward asymmetry大很多。

forward-backward asymmetry. At $\sin^2\theta_w = 0.230$, the left-right asymmetry is 0.16 which is about eight times larger than A^l_{FB}.

Sensitivity to 电弱混合相角$sin^2\theta_\omega$

- 左-右不对称性(left-right asymmetry， A_{LR})和电弱混合相角($sin^2\theta_\omega$)有一个非常简单的依赖关系：
- 所以不对称性是电弱混合相角($sin^2\theta_\omega$)的一个非常敏感的函数，在$sin^2\theta_\omega$一个很小的变化会导致在A_{LR}上的巨大变化

The left-right asymmetry has a very simple dependence on the electroweak mixing parameter $\sin^2\theta_w$,

$$A_{LR} = \frac{2(1-4\sin^2\theta_w)}{1+(1-4\sin^2\theta_w)^2}. \quad (8.10)$$

The asymmetry is a very sensitive function of $\sin^2\theta_w$. Small changes in $\sin^2\theta_w$ lead to very large changes in A_{LR},

$$\delta A_{LR} \simeq 8\delta\sin^2\theta_w.$$