Dense nuclear matter equation of state from heavy-ion collisions

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INSTITUTE for NUCLEAR THEORY

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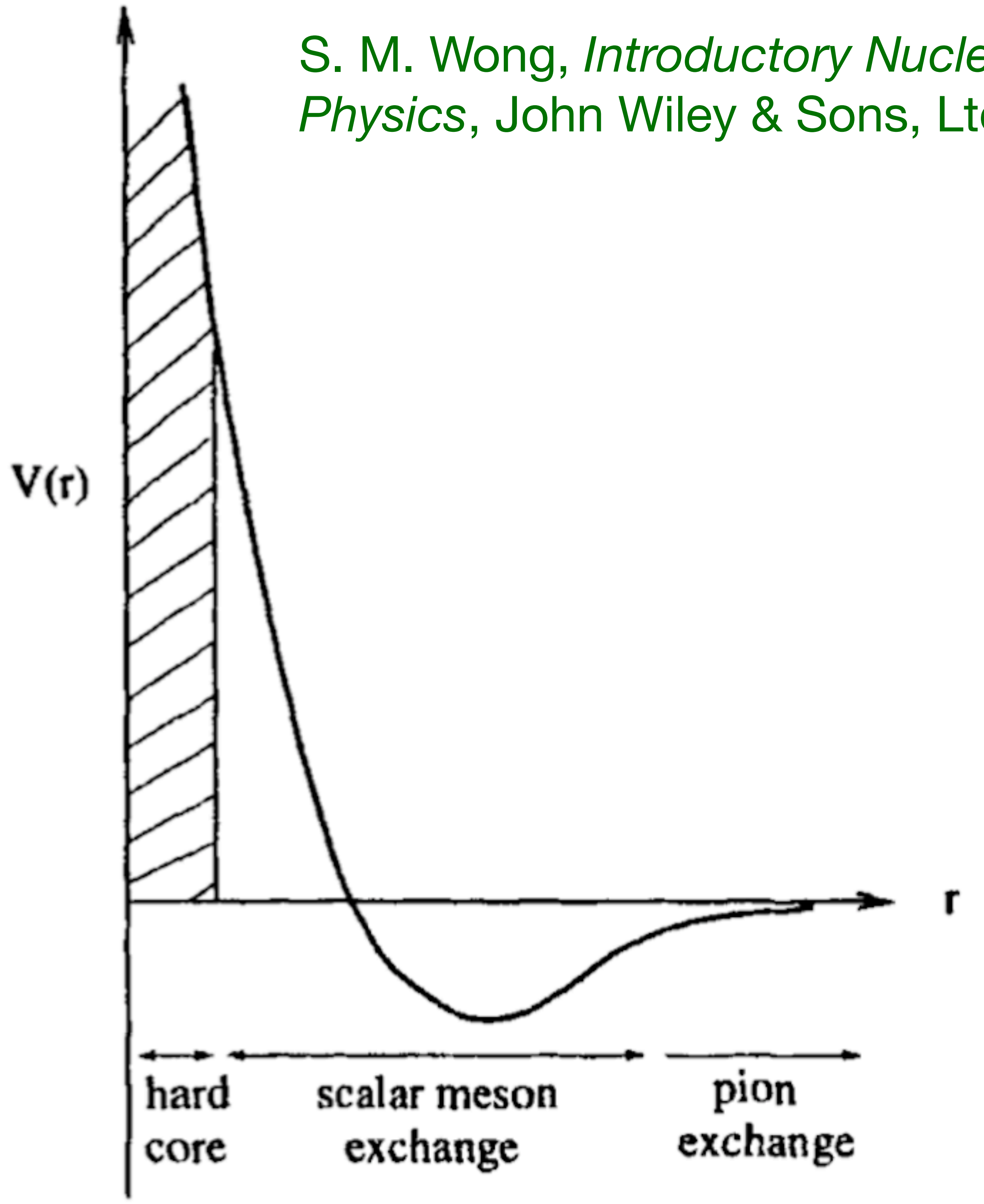
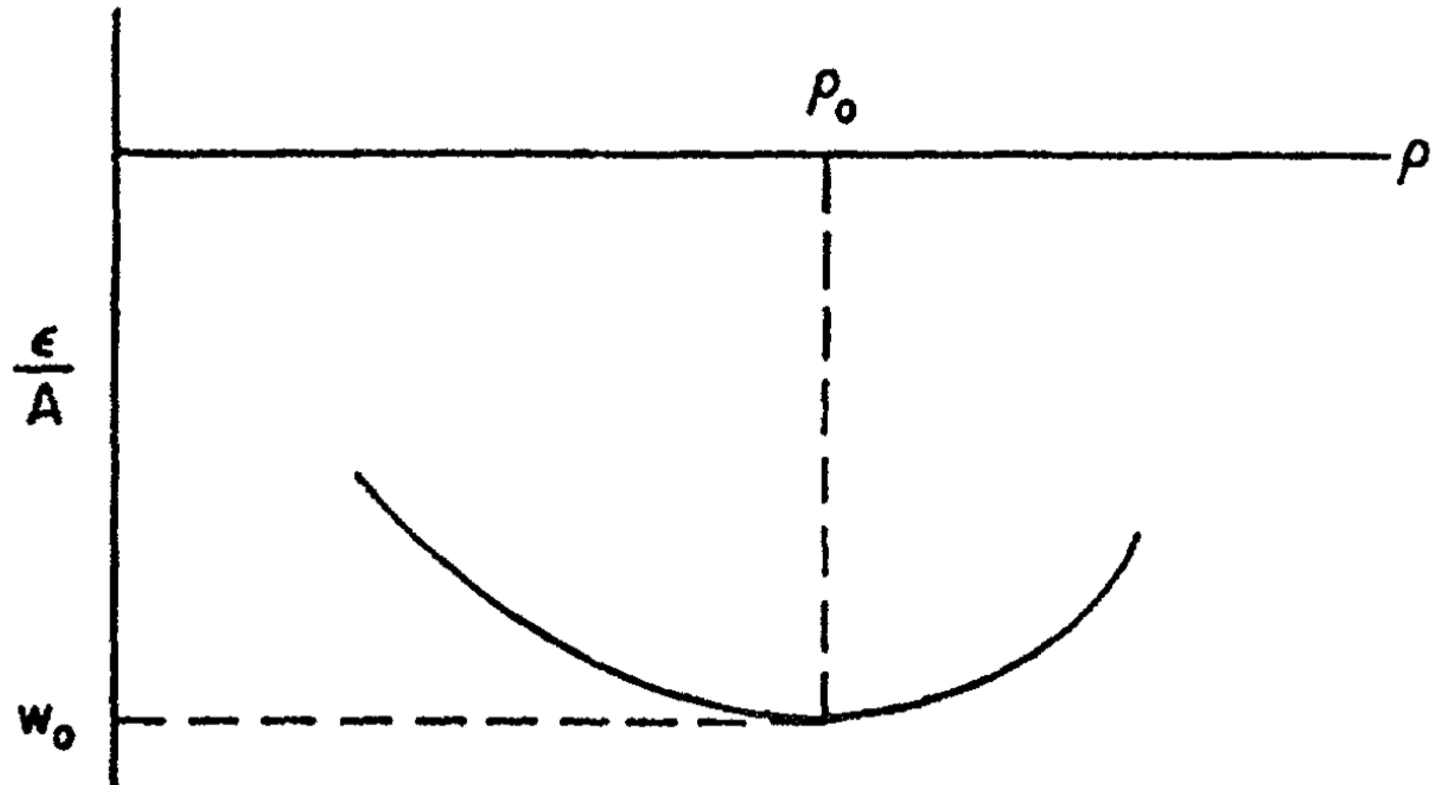
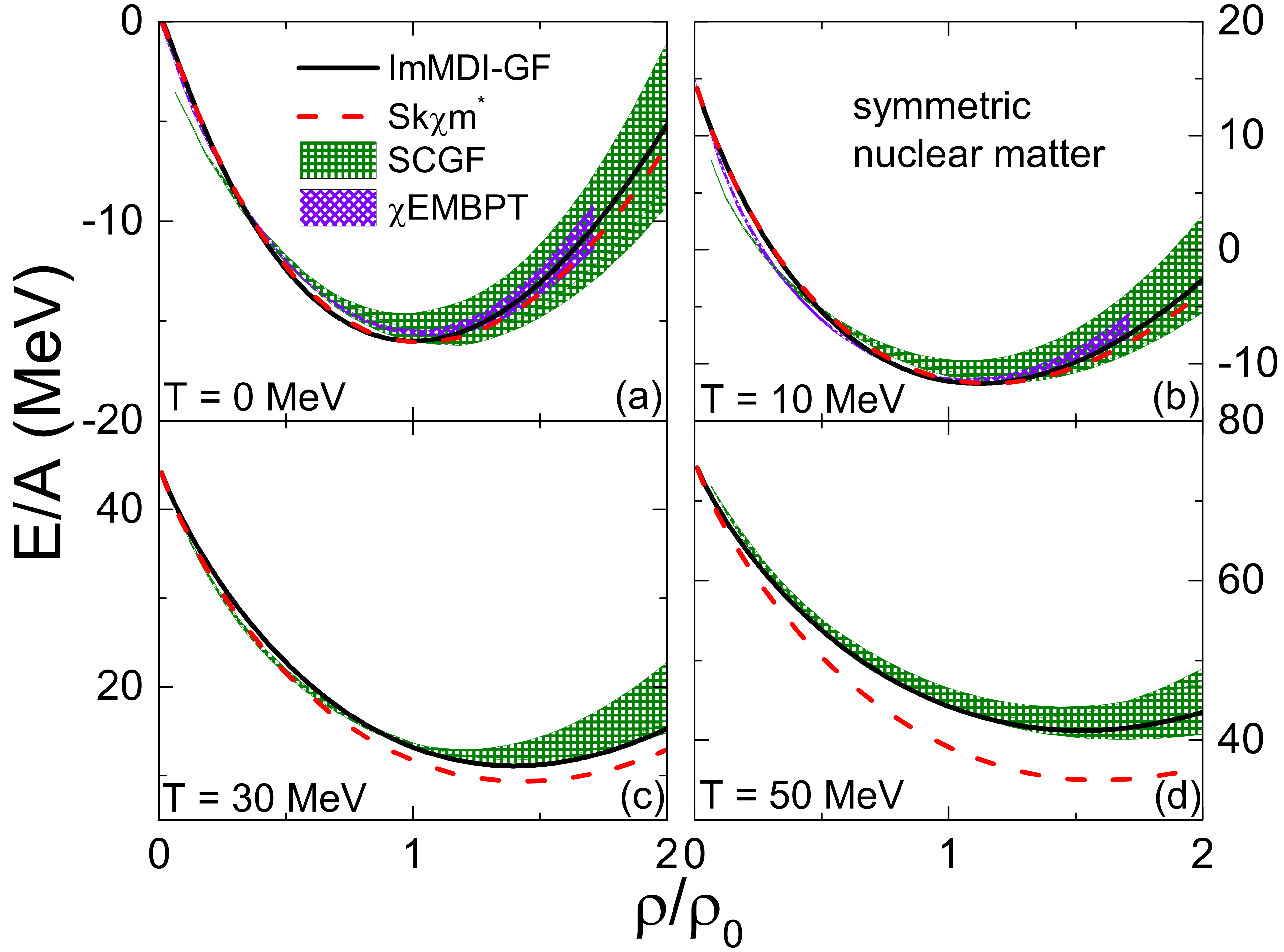
Properties of nuclear matter are reflected in the EOS



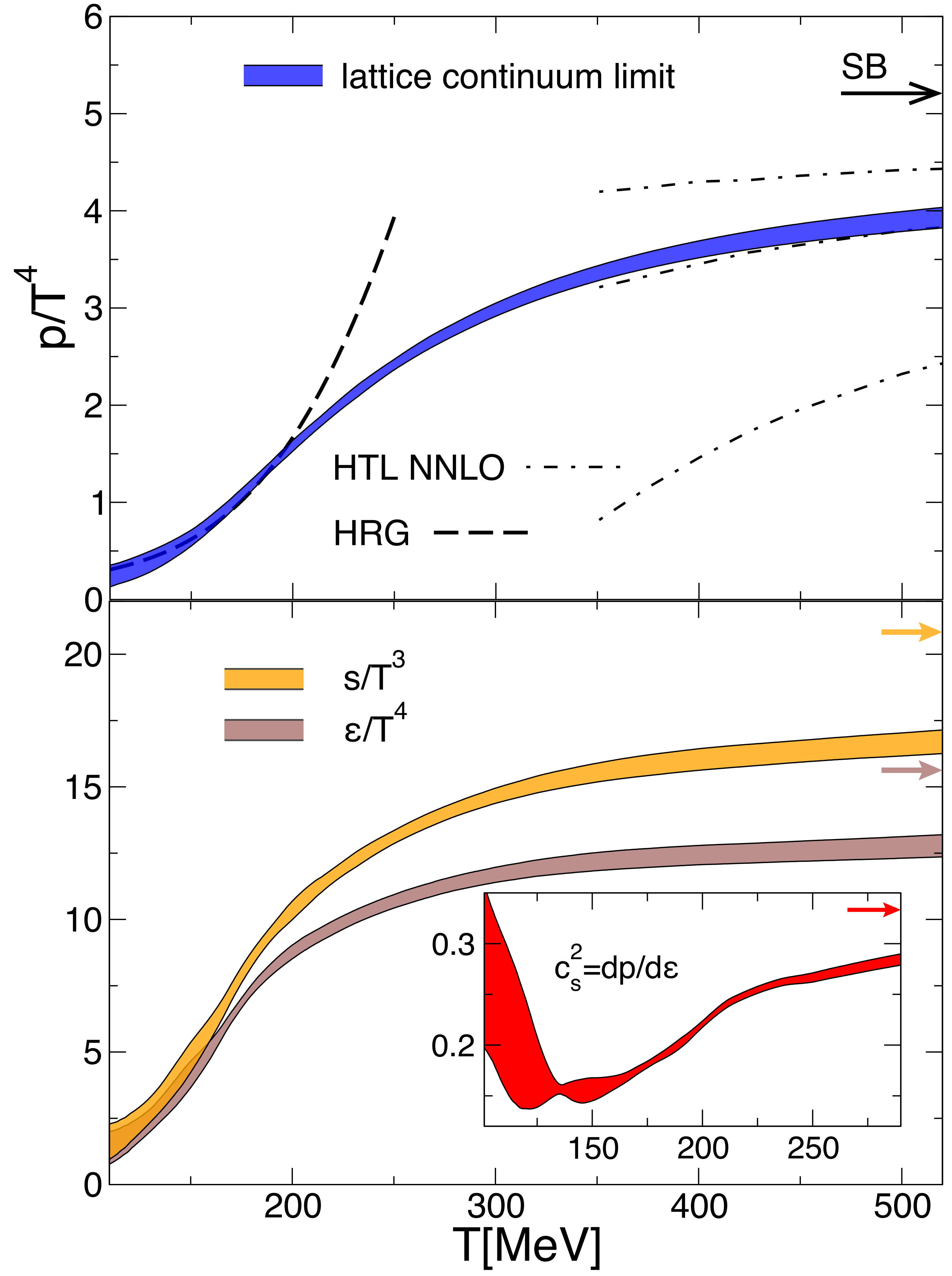
S. M. Wong, *Introductory Nuclear Physics*, John Wiley & Sons, Ltd, 1998



B. D. Day, Rev. Mod. Phys. **39**,719–744 (1967)



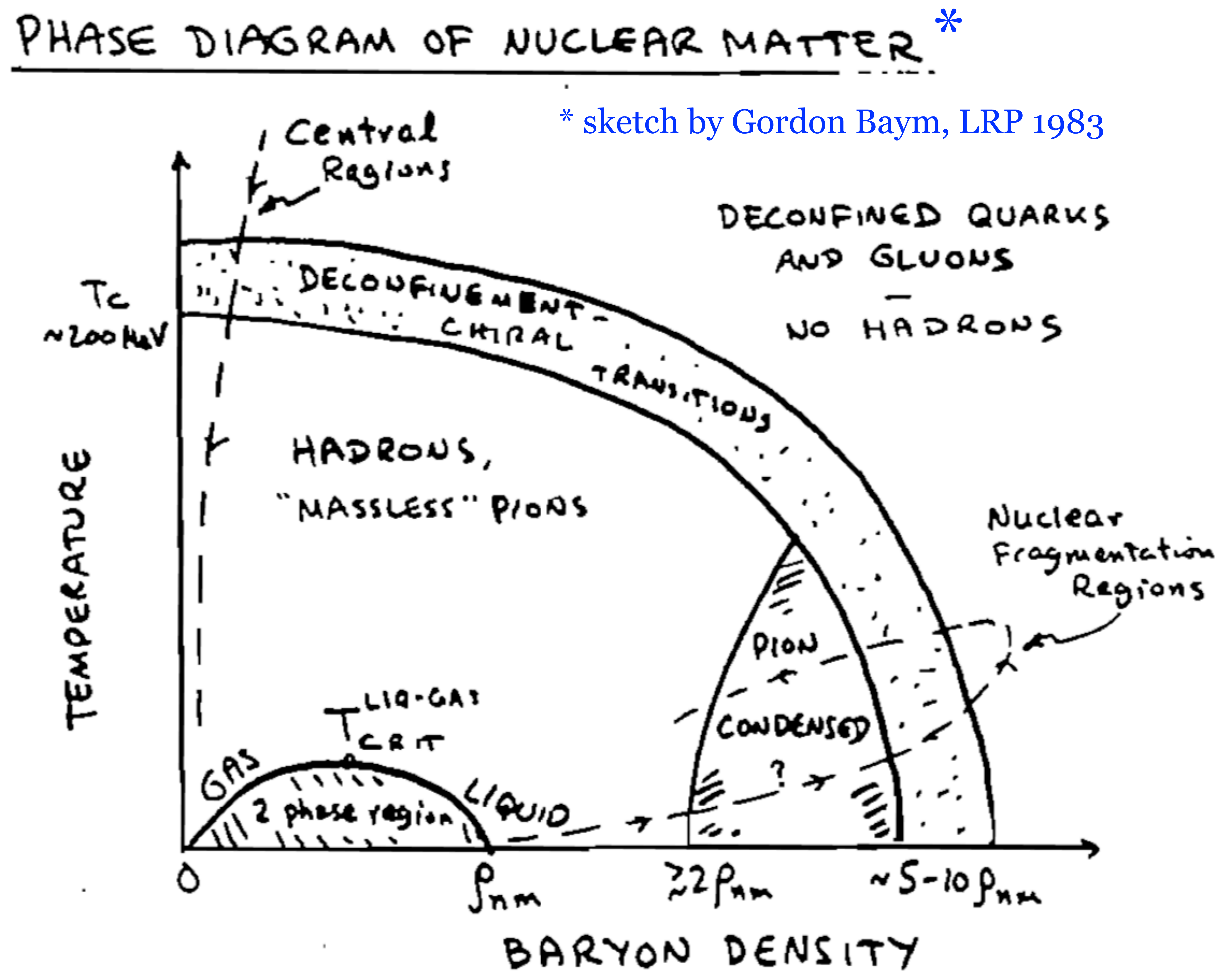
J. Xu, A. Carbone, Z. Zhang, C.-M. Ko,
Phys. Rev. C **100**, 2, 024618 (2019) arXiv:1904.09669



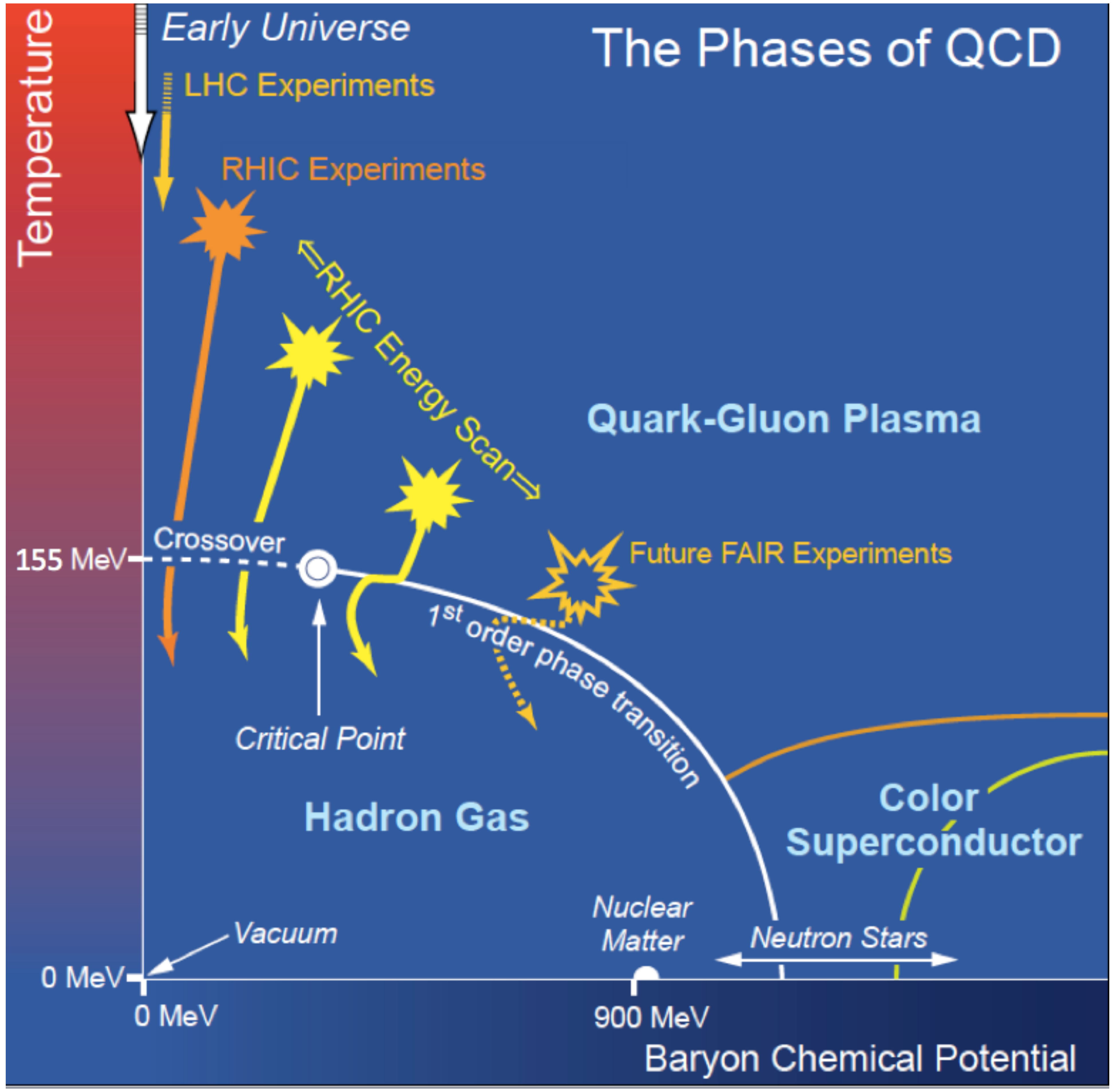
S. Borsanyi, Z. Fodor, C. Hoelbling, S. Katz, S. Krieg, K. K. Szabo,
Phys. Lett. B **730** 99–104 (2014)
arXiv:1309.5258

The EOS = key to understanding fundamental properties of QCD matter

1) Uncovering the phase diagram of QCD matter



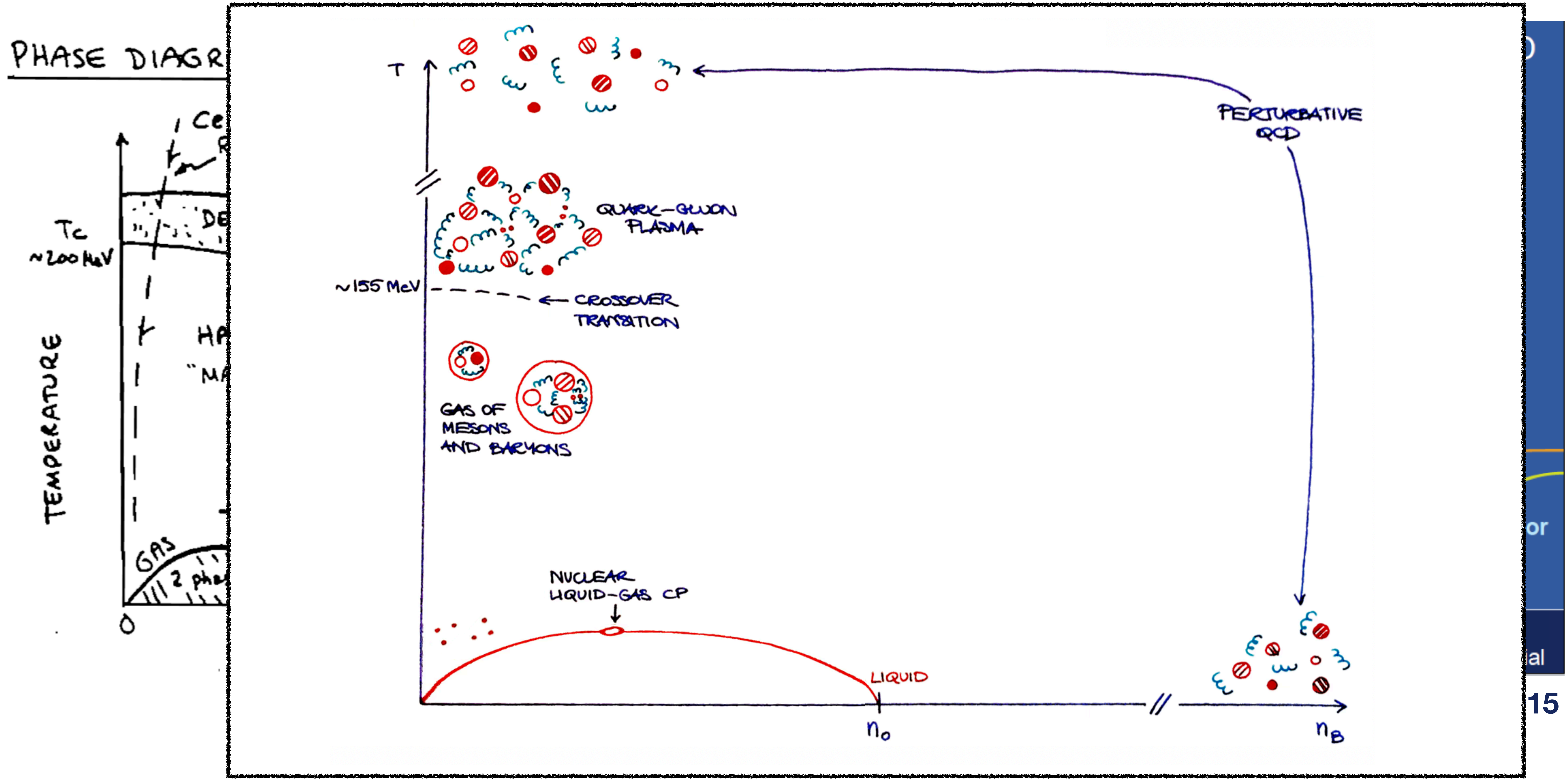
* sketch by Gordon Baym, LRP 1983



The Hot QCD White Paper for LRP 2015

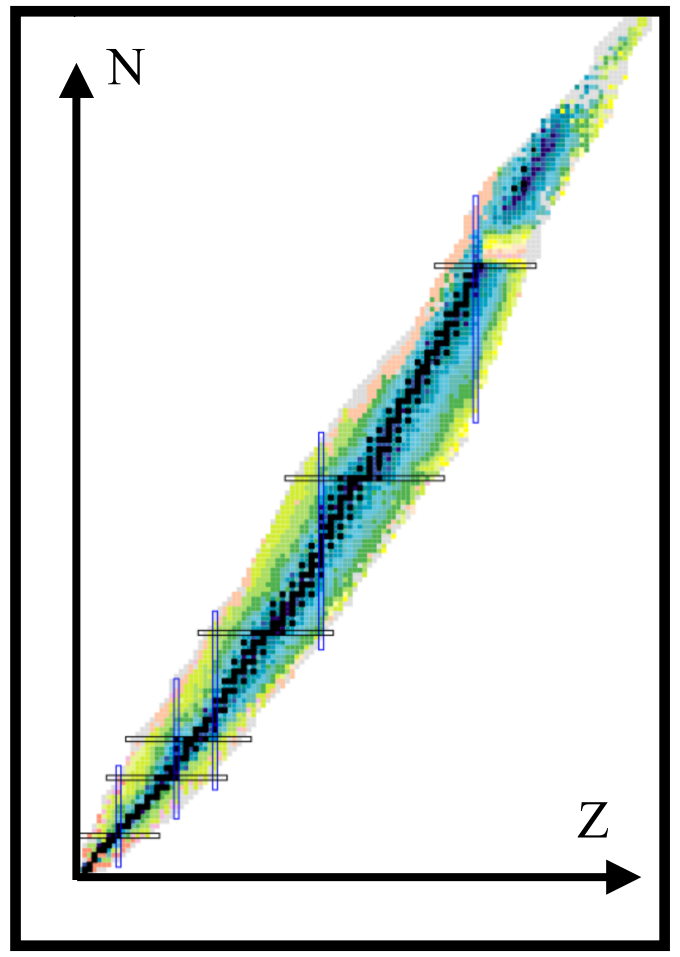
The EOS = key to understanding fundamental properties of QCD matter

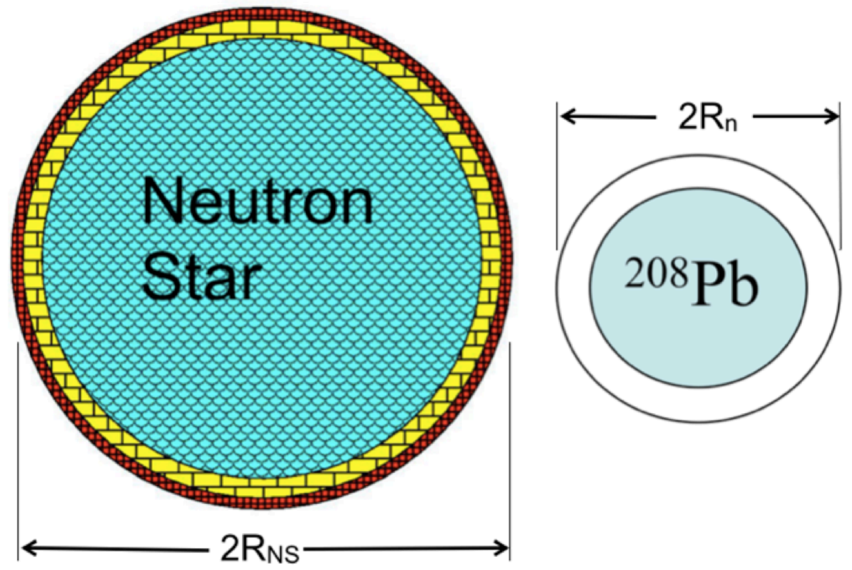
1) Uncovering the phase diagram of QCD matter



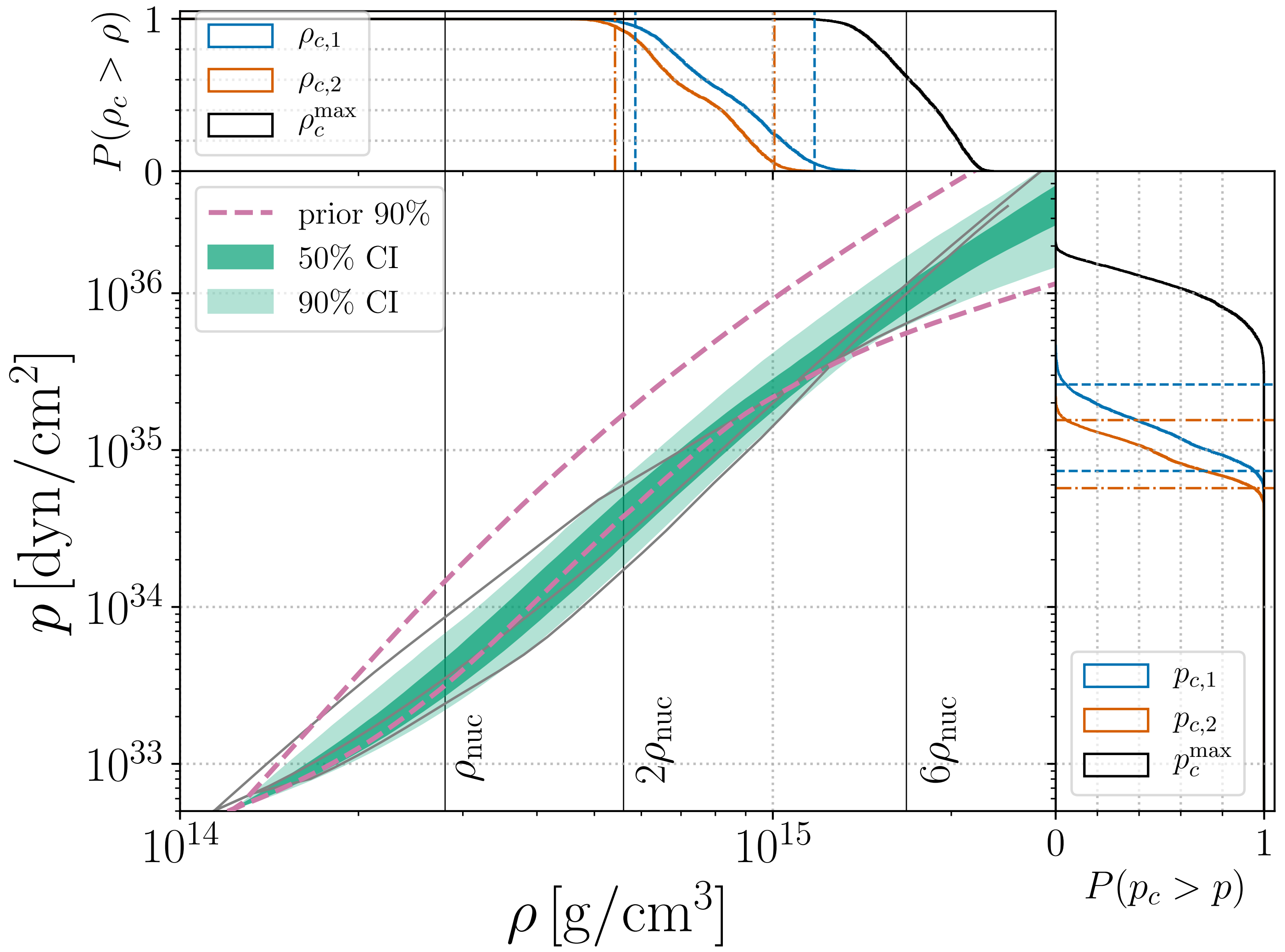
The EOS = key to understanding fundamental properties of QCD matter

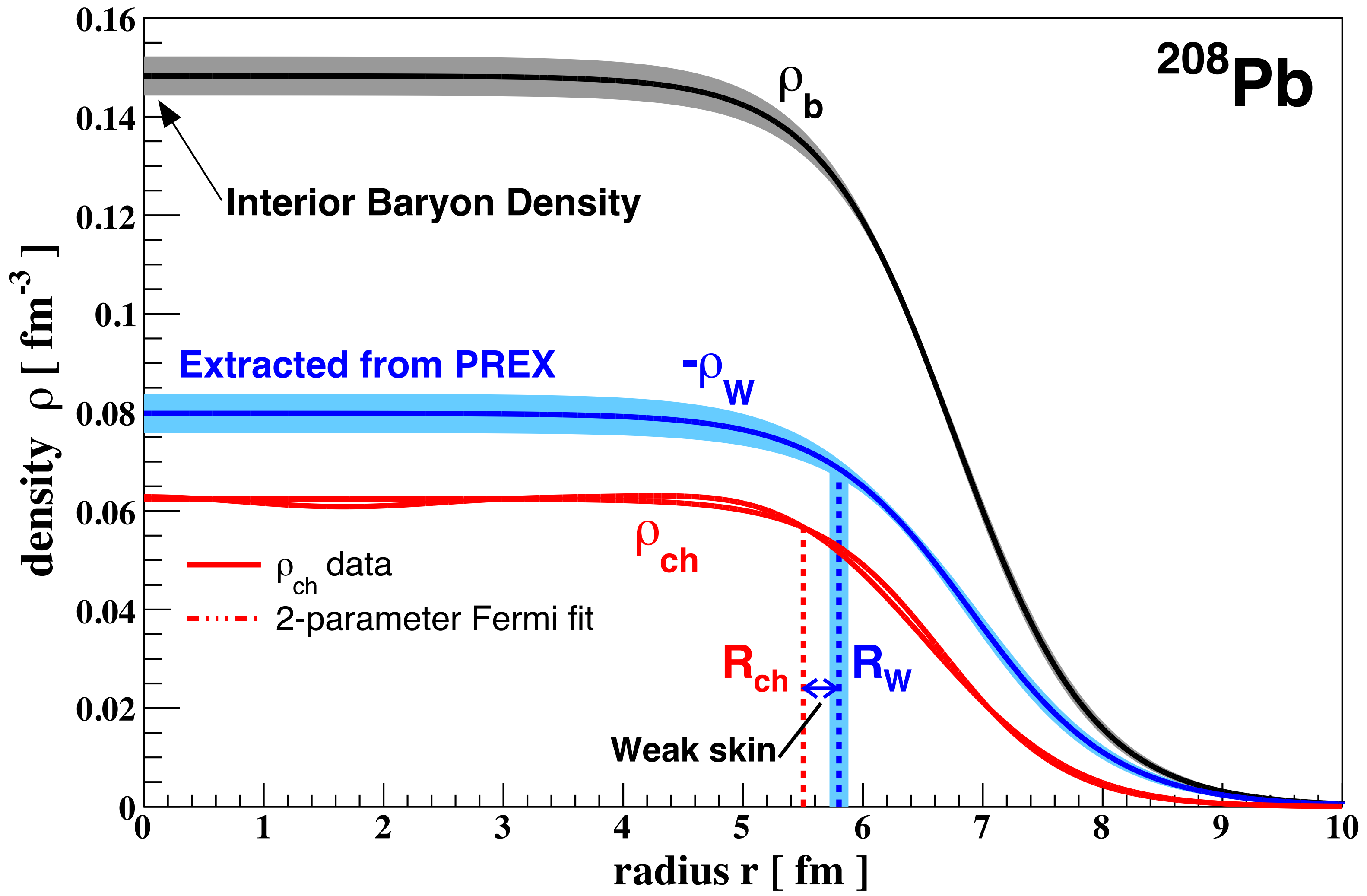
2) Uncovering the isospin-dependence of strong interactions



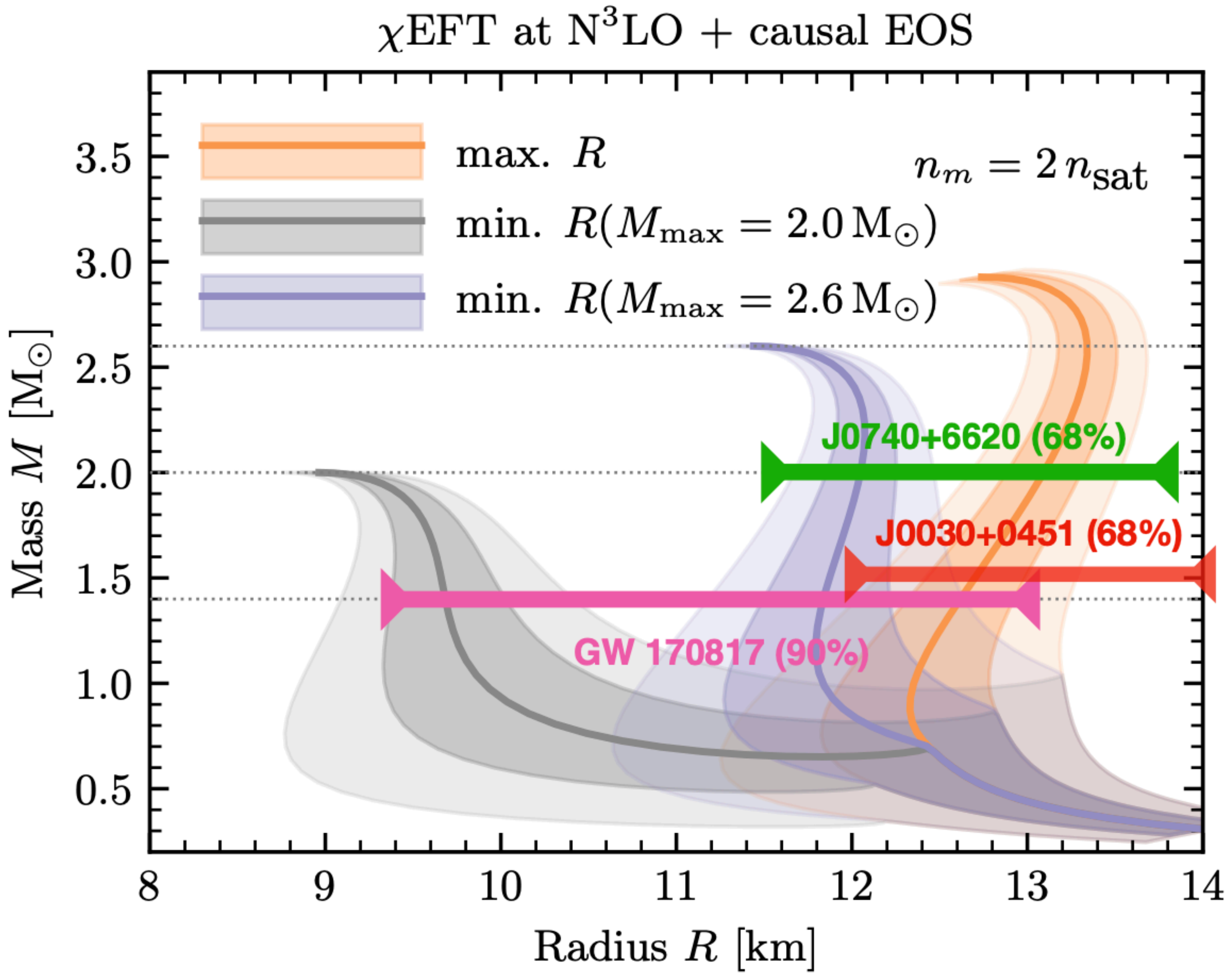


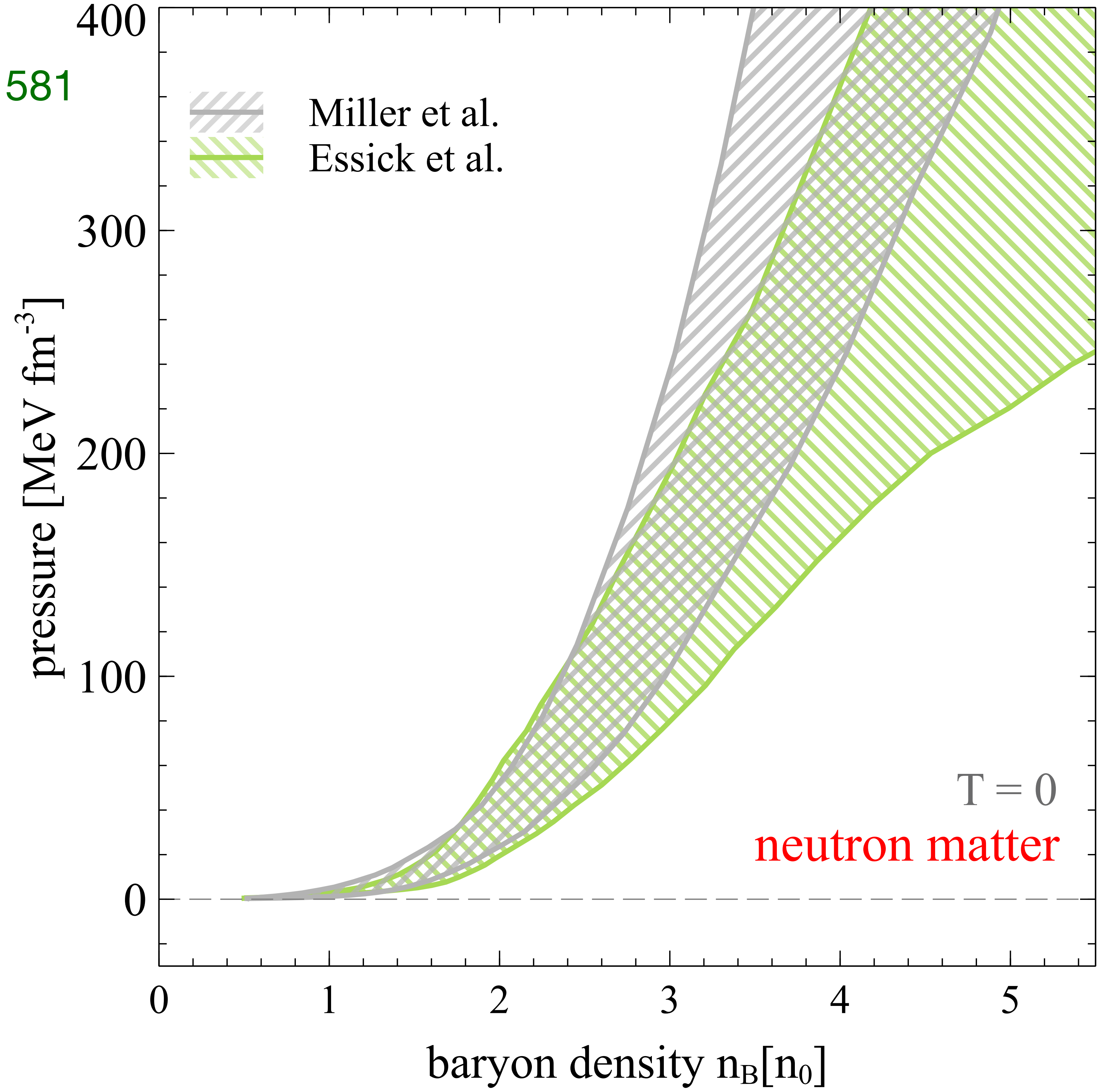
LIGO, Phys. Rev. Lett. **121** 16, 161101 (2018), arXiv:1805.11581





D. Adhikari *et al*. (PREX Collaboration),
Phys. Rev. Lett. **126** 17, 172502 (2021), arXiv:2102.10767





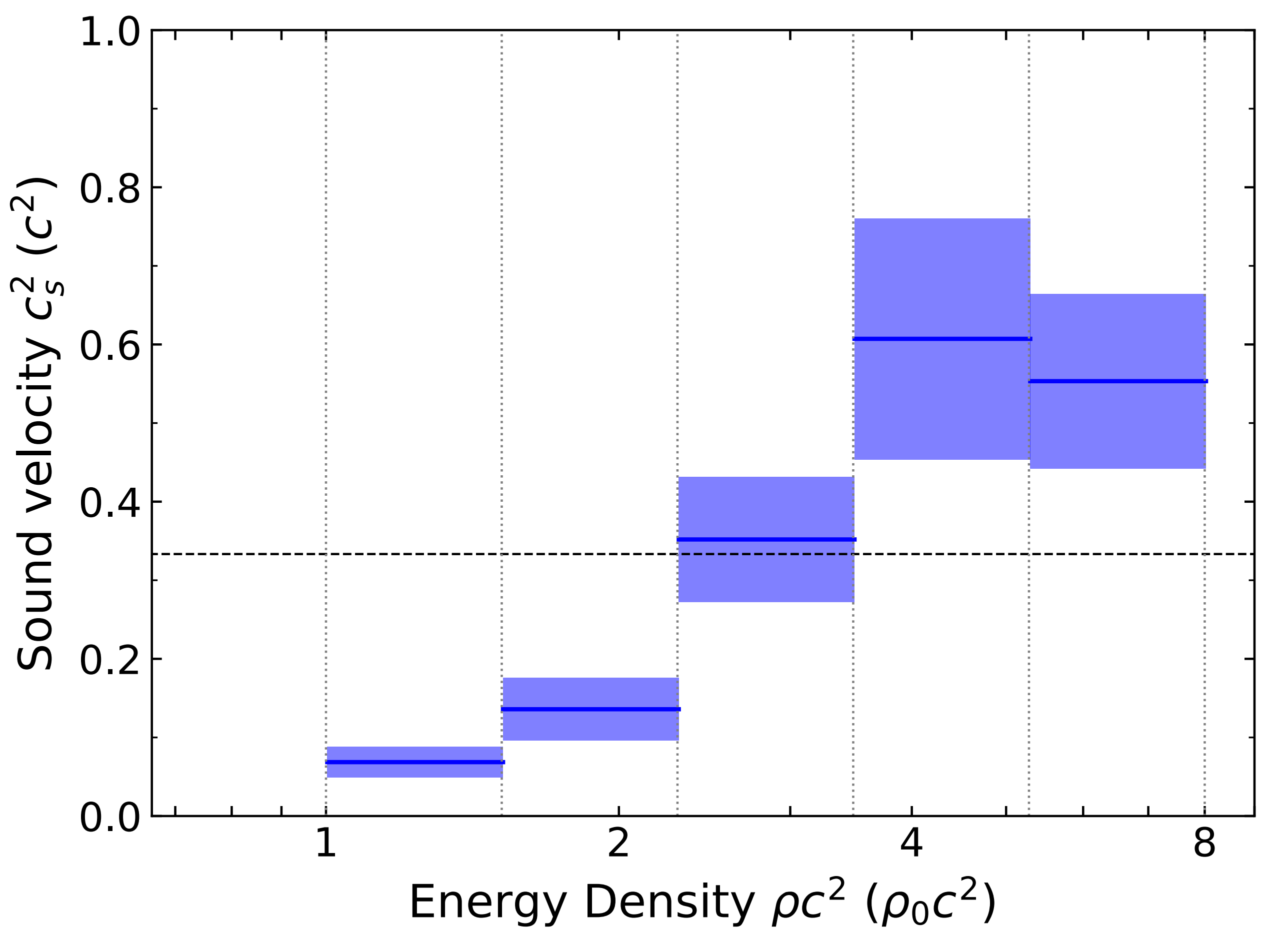
R. Essick, I. Tews, P. Landry, S. Reddy, D. E. Holz, Phys. Rev. C **102**, 055803 (2020), arXiv:2004.07744

* from S. Reddy's slides; M-R results:
C. Drischler, S. Han, J. M. Lattimer, M. Prakash, S. Reddy, T. Zhao, Phys. Rev. C **103** 4, 045808 (2021), arXiv:2009.06441

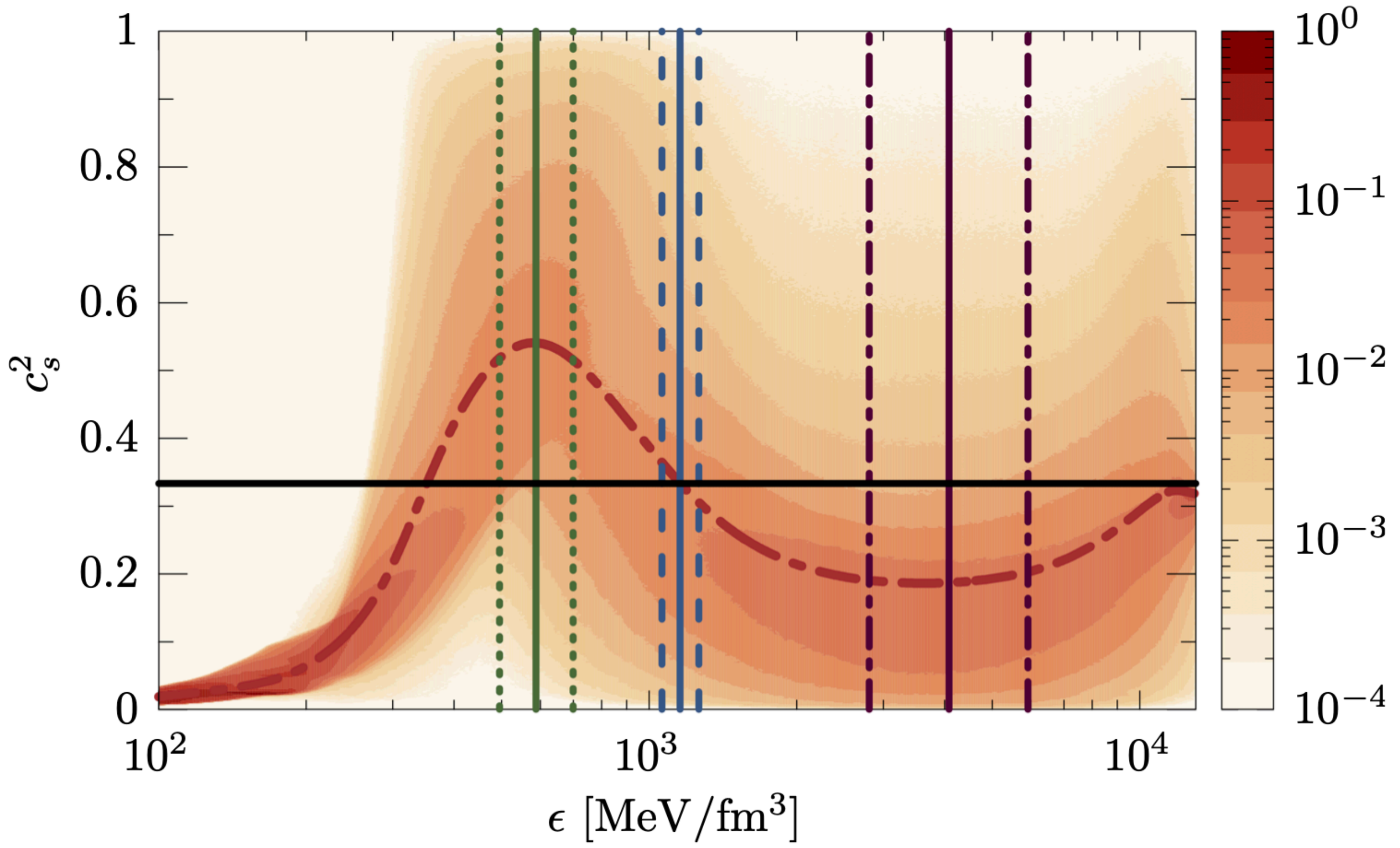
The EOS = key to understanding fundamental properties of QCD matter

3) Understanding extreme behavior at high baryon densities: is $c_s^2 > 1/3$ for symmetric matter?

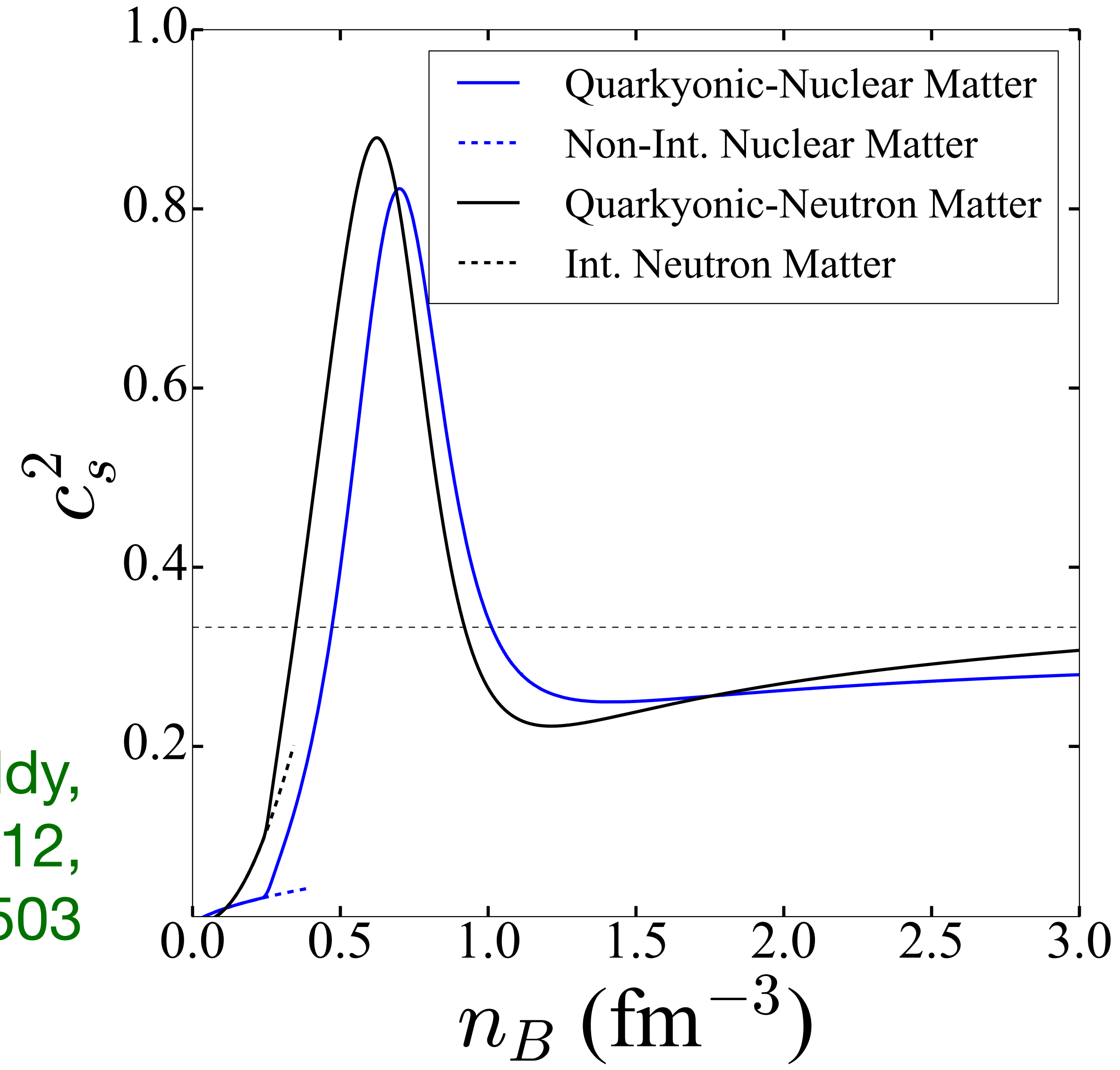
P. Bedaque and A. W. Steiner, Phys. Rev. Lett. **114**, no.3, 031103 (2015), arXiv: 1408.5116
I. Tews, J. Carlson, S. Gandolfi and S. Reddy, Astrophys. J. **860**, no.2, 149 (2018), arXiv:1801.01923



Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D **101**, 5, 054016 (2020), arXiv:1903.03400



M. Marczenko, L. McLerran, K. Redlich, C. Sasaki, Phys. Rev. C **107**, 2, 025802 (2023) arXiv:2207.13059



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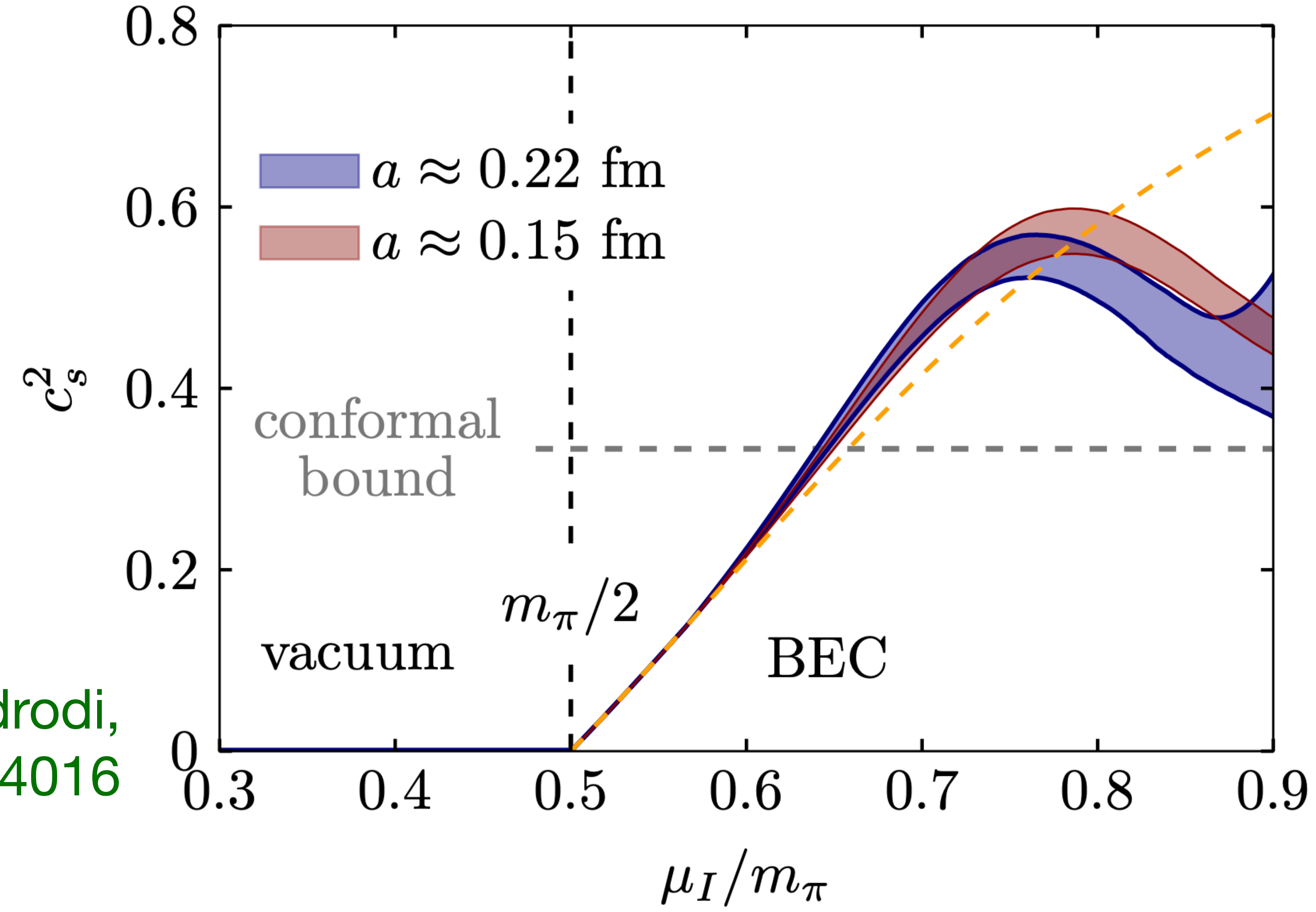

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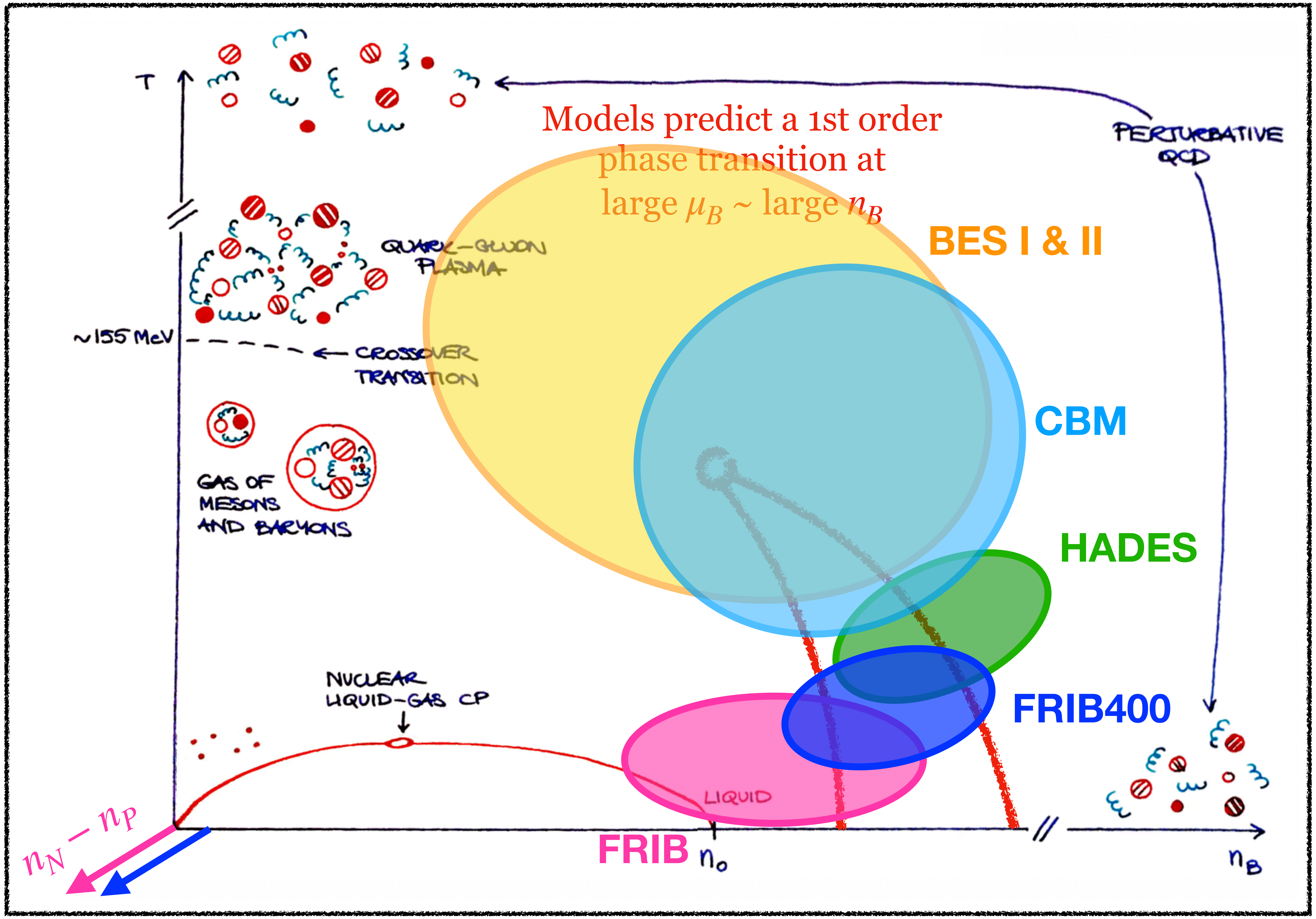
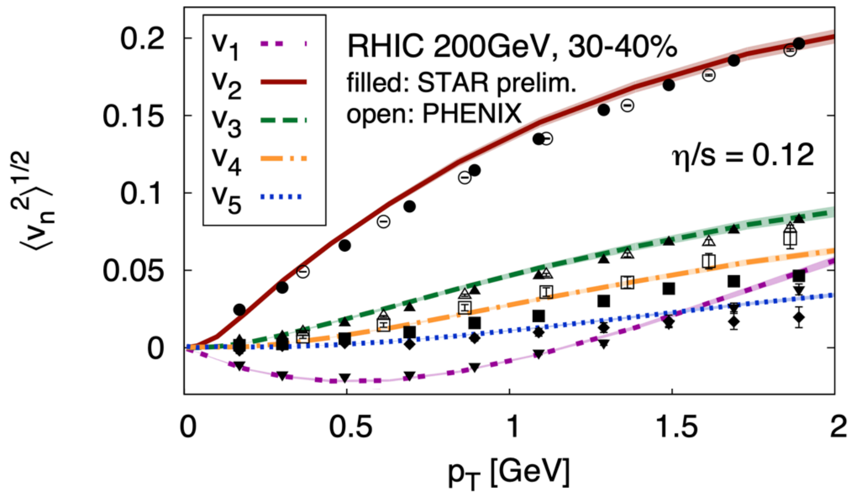

T
~155 MeV
QUARK-GLUON PLASMA
CROSSOVER TRANSITION
GAS OF MESONS AND BARYONS
NUCLEAR LIQUID-GAS CP
LIQUID
PERTURBATIVE QCD
Models predict a 1st order phase transition at large μ_B ~ large n_B
BES I & II
CBM
HADES
FRIB400
FRIB
n_0
n_B
$n_N - n_P$

The QCD phase diagram: enormous interest in behavior at high n_B

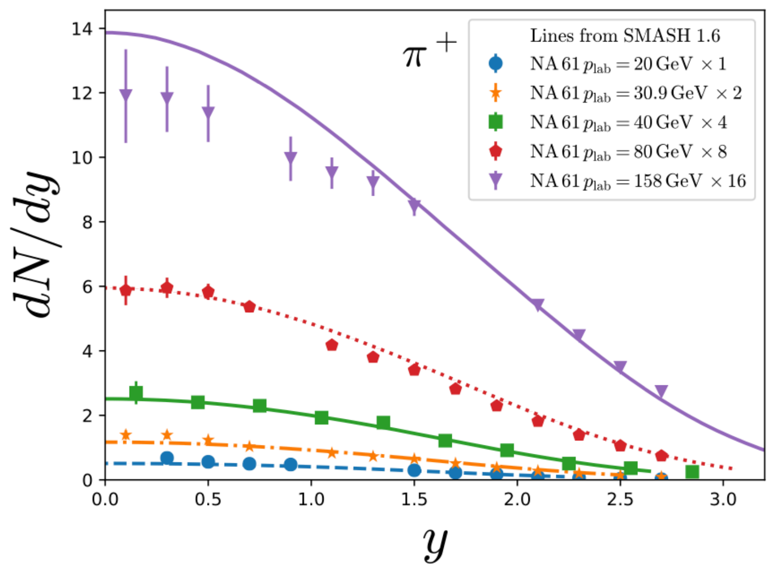
Relativistic viscous hydrodynamic simulations with LQCD EOS: amazing agreement with data from high-energy collisions



C. Gale, S. Jeon, B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. **110** (2013) 1, 012302, arXiv:1209.6330

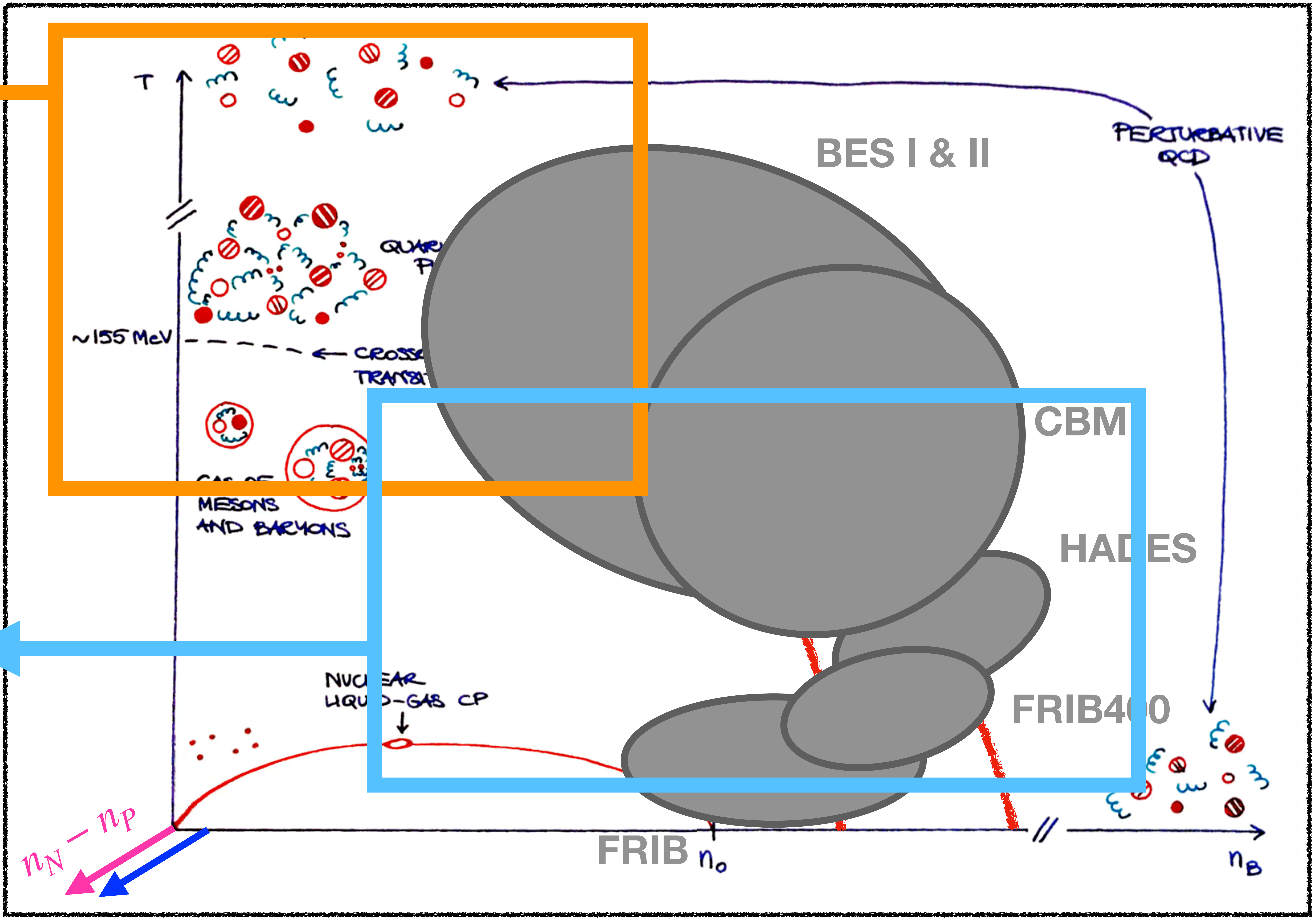
fast equilibration = hydro applies

Hadronic transport simulations:

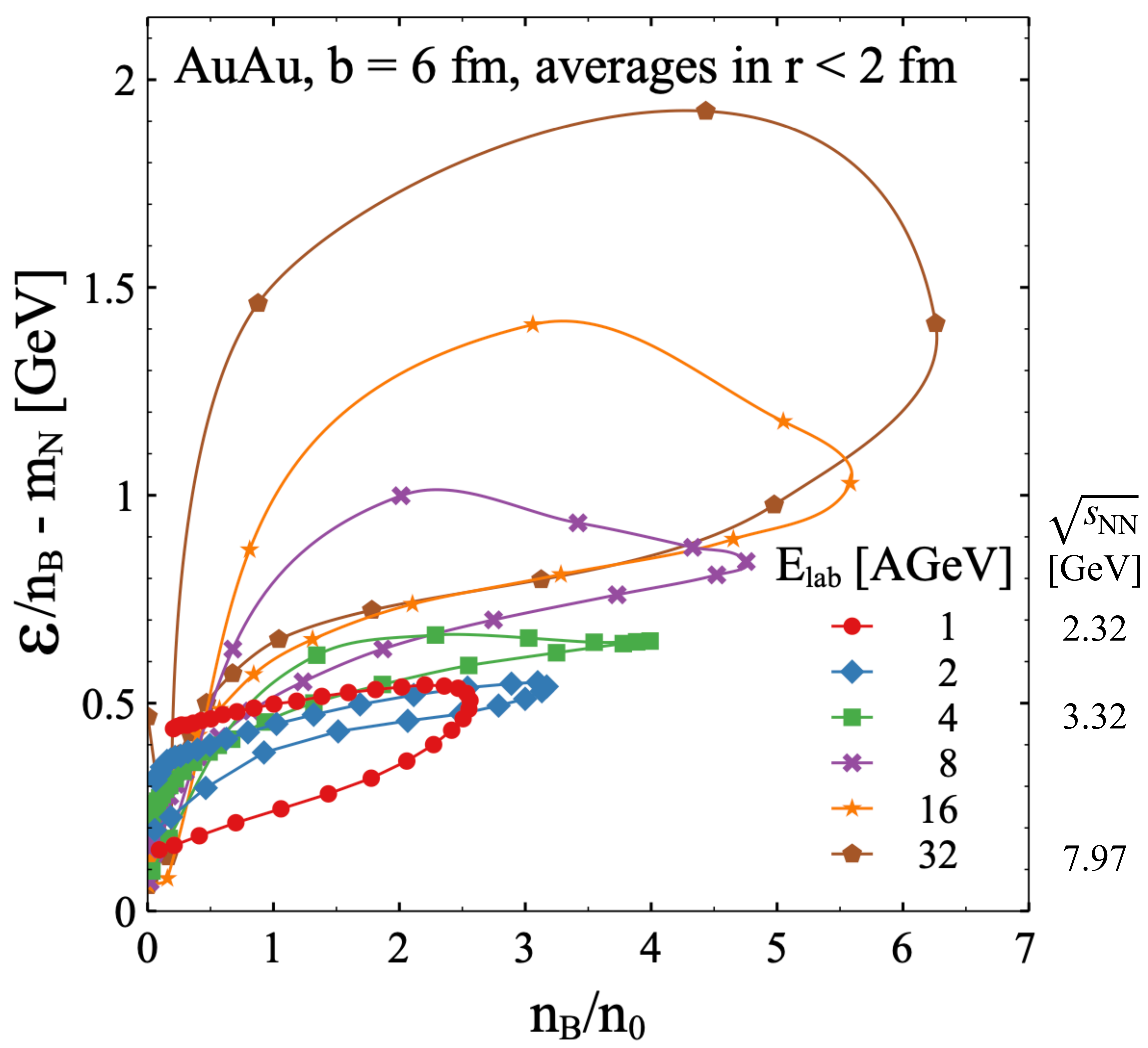


systems out of equilibrium = microscopic approach needed

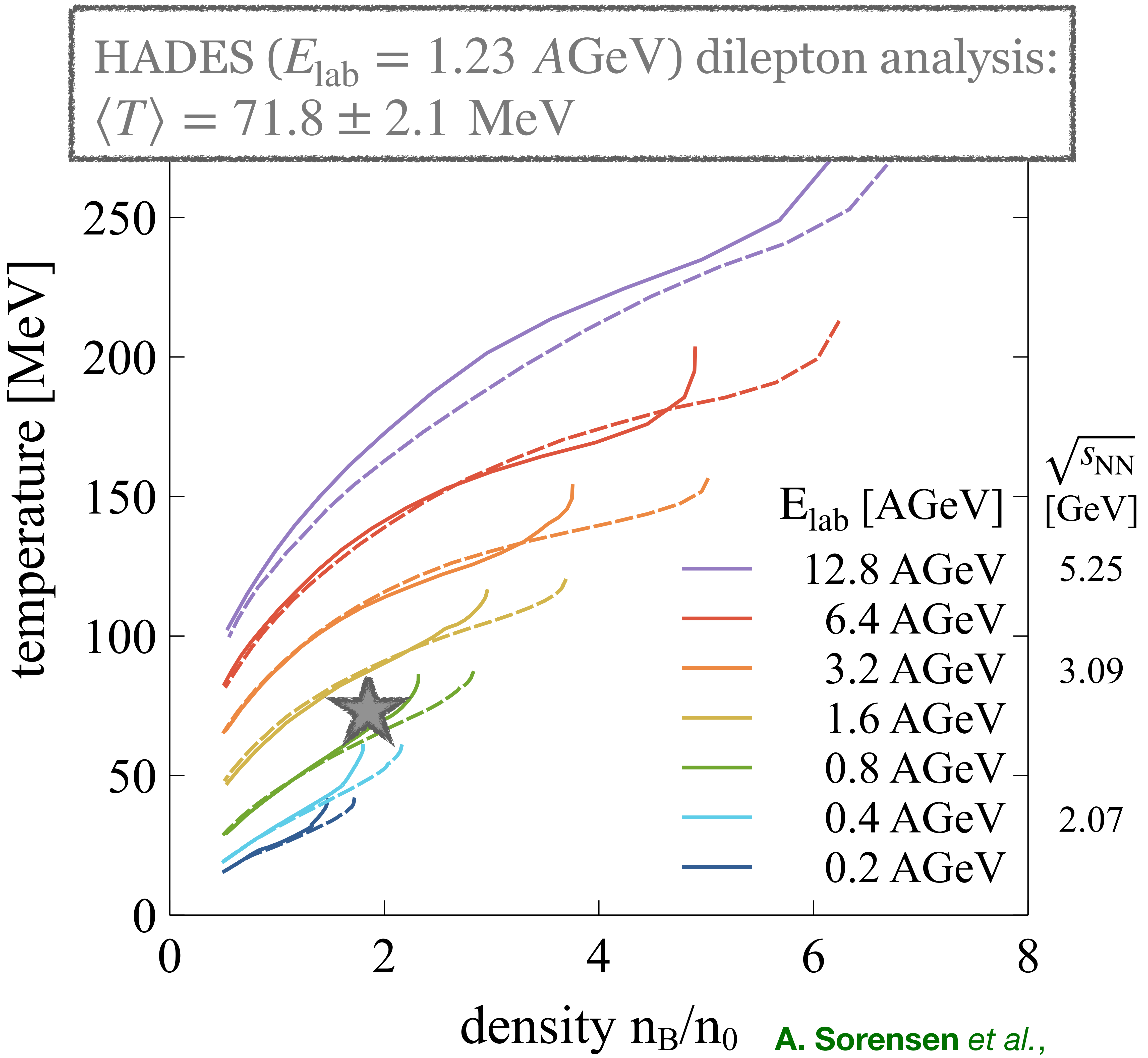
J. Mohs, S. Ryu, H. Elfner, J. Phys. G **47** (2020) 6, 065101 arXiv:1909.05586



Intermediate-energy heavy-ion collisions probe wide ranges of density and temperature

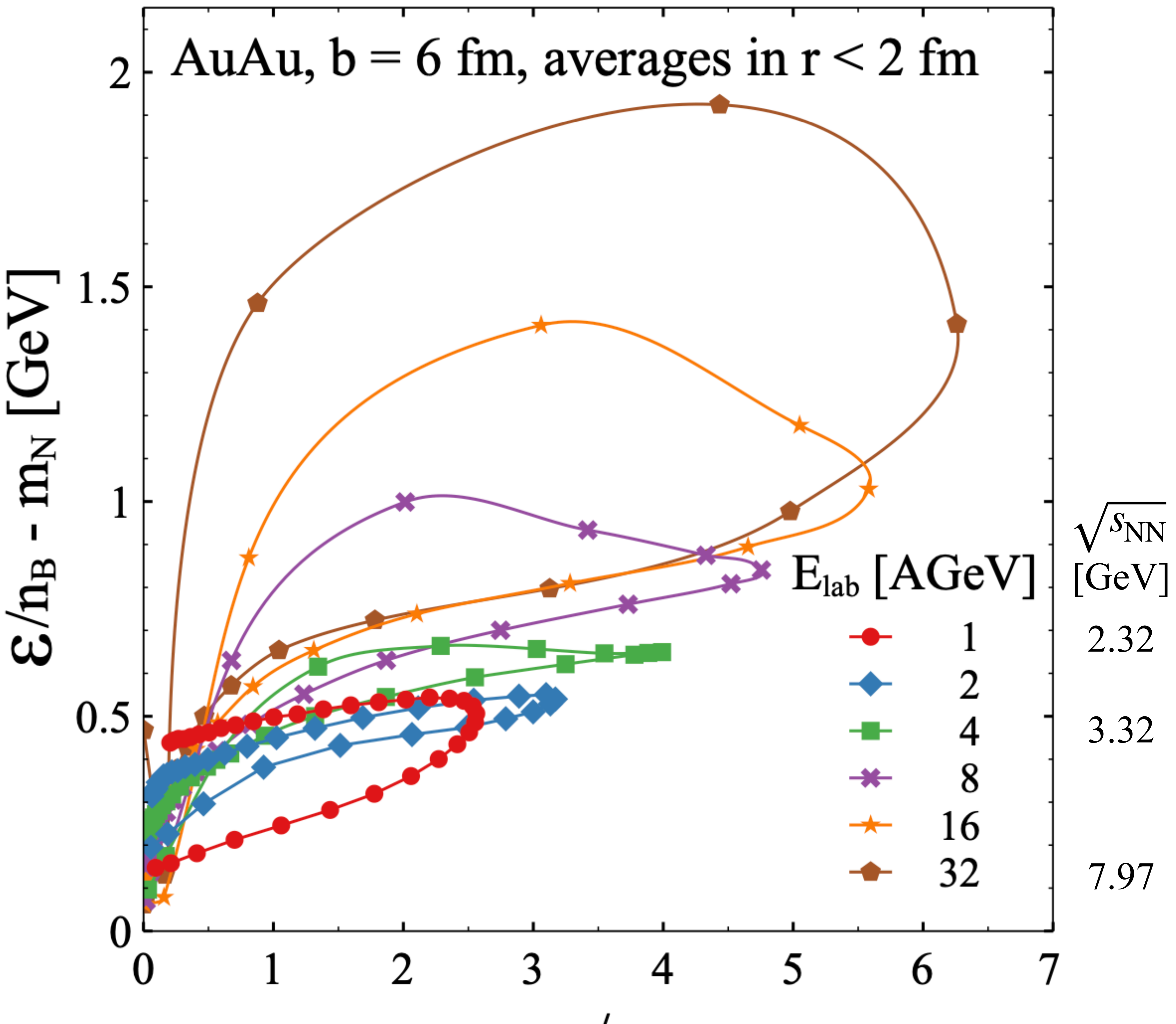


D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

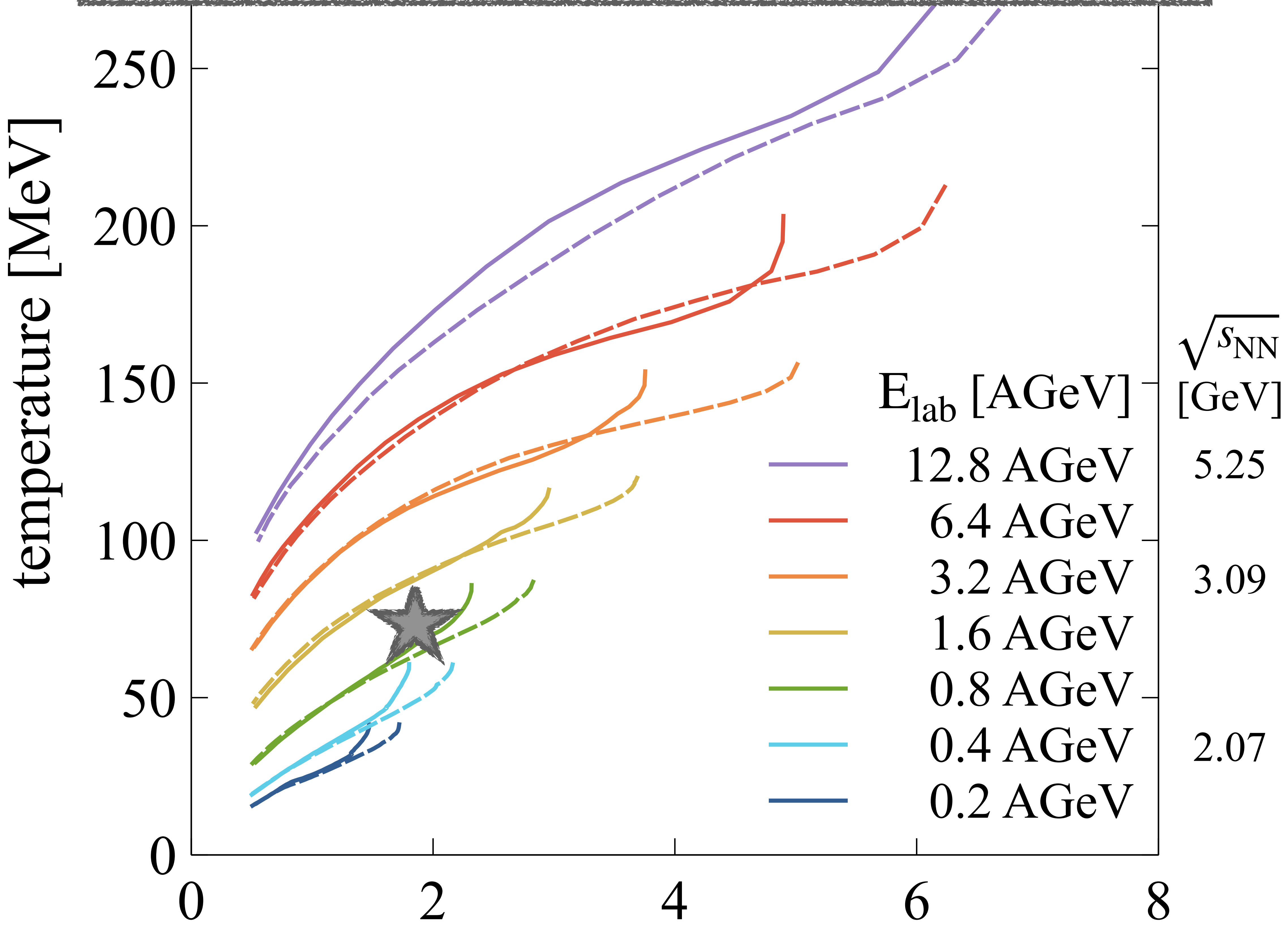


A. Sorensen *et al.*, arXiv:2301.13253

Intermediate-energy heavy-ion collisions probe wide ranges of density and temperature



HADES ($E_{lab} = 1.23$ AGeV) dilepton analysis:
$\langle T \rangle = 71.8 \pm 2.1$ MeV



HICs = the only means to probe densities away from n_0 in controlled terrestrial experiments

Hadronic transport is necessary to interpret the results: BES FXT, HADES, CBM, FRIB, FRIB400

The EOS is a common effort within the nuclear physics community

A. Sorensen *et al.*, arXiv:2301.13253

Dense Nuclear Matter Equation of State from Heavy-Ion Collisions *

Agnieszka Sorensen[1], Kshitij Agarwal[2], Kyle W. Brown[3,4], Zbigniew Chajecki[5], Paweł Danielewicz[3,6], Christian Drischler[7], Stefano Gandolfi[8], Jeremy W. Holt[9,10], Matthias Kaminski[11], Che-Ming Ko[9,10], Rohit Kumar[3], Bao-An Li[12], William G. Lynch[3,6], Alan B. McIntosh[10], William G. Newton[12], Scott Pratt[3,6], Oleh Savchuk[3,13], Maria Stefaniak[14], Ingo Tews[8], ManYee Betty Tsang[3,6], Ramona Vogt[15,16], Hermann Wolter[17], Hanna Zbroszczyk[18]

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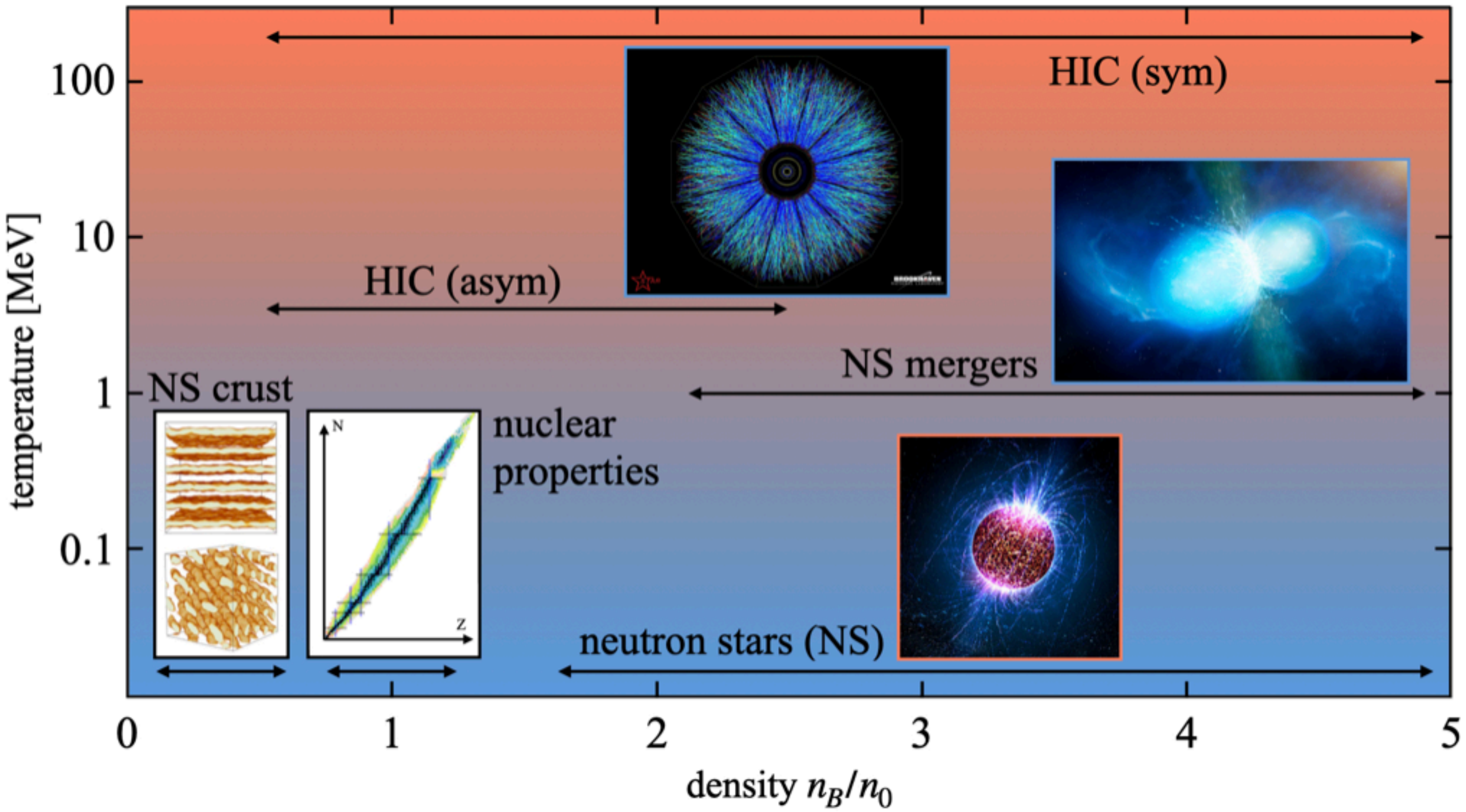
Hot QCD

Low energy + Astro



Transport model simulations of heavy-ion collisions

- Boltzmann-Uehling-Uhlenbeck (BUU)-type codes:

  - solve coupled Boltzmann equations $\forall i: \quad \frac{\partial f_i}{\partial t} + \frac{d\mathbf{x}_i}{dt}\frac{\partial f_i}{\partial \mathbf{x}_i} + \frac{d\mathbf{p}_i}{dt}\frac{\partial f_i}{\partial \mathbf{p}_i} = I_{\text{coll}}^{(i)}$

    with the method of test particles: the distribution is *over*sampled with a *large* number of discrete test-particles, which are evolved according to the single-particle EOMs
    (test particles probe the evolution in the phase space)

  - forces from gradients of single-particle energies (mean-fields: needs a robust density calculation!)

  - collision term based on measured cross-sections for scatterings and decays

- Quantum Molecular Dynamics (QMD)-type codes
  - solve molecular dynamics problem (evolve nucleons according to their EOMs)

  - forces: in principle distance-dependent particle-particle interactions, in practice: often mean-fields!

  - collisions based on measured cross-sections for scatterings and decays

Transport model simulations of heavy-ion collisions

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  (test particles pro

  - forces from gradi nsity calculation!)

  - collision term bas

Transport ***automatically*** includes:

- non-equilibrium evolution, including triggered by probing unstable regions of the phase diagram
- effects due to the interplay between participants and spectators
- baryon, strangeness, charge transport/diffusion

- Quantum Molecular Dynamics (QMD)-type codes
  - solve molecular dynamics problem (evolve nucleons according to their EOMs)

  - forces: in principle distance-dependent particle-particle interactions, in practice: often mean-fields!

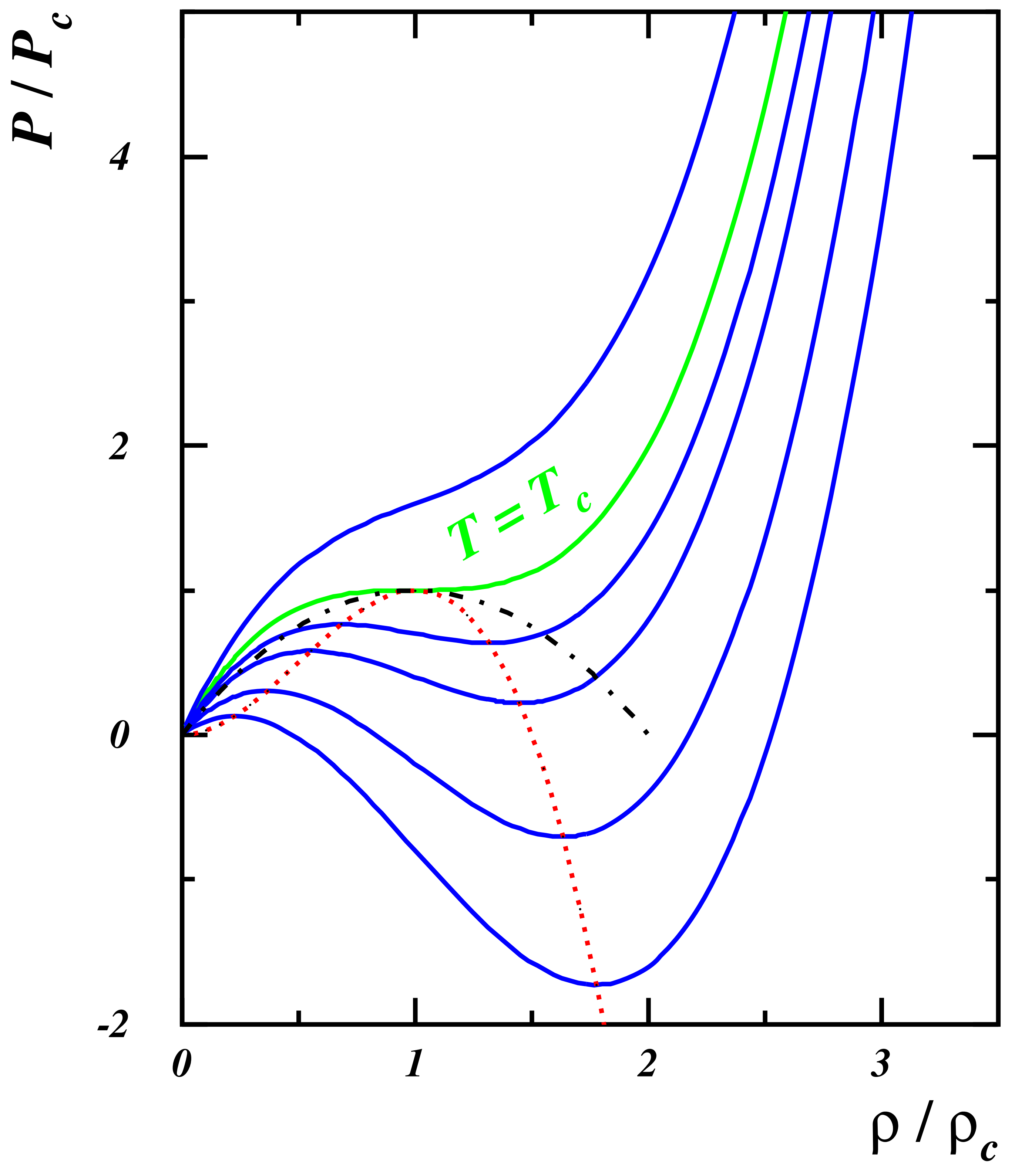
  - collisions based on measured cross-sections for scatterings and decays

Two ways of using hadronic transport

1) Use it as a "non-critical baseline"

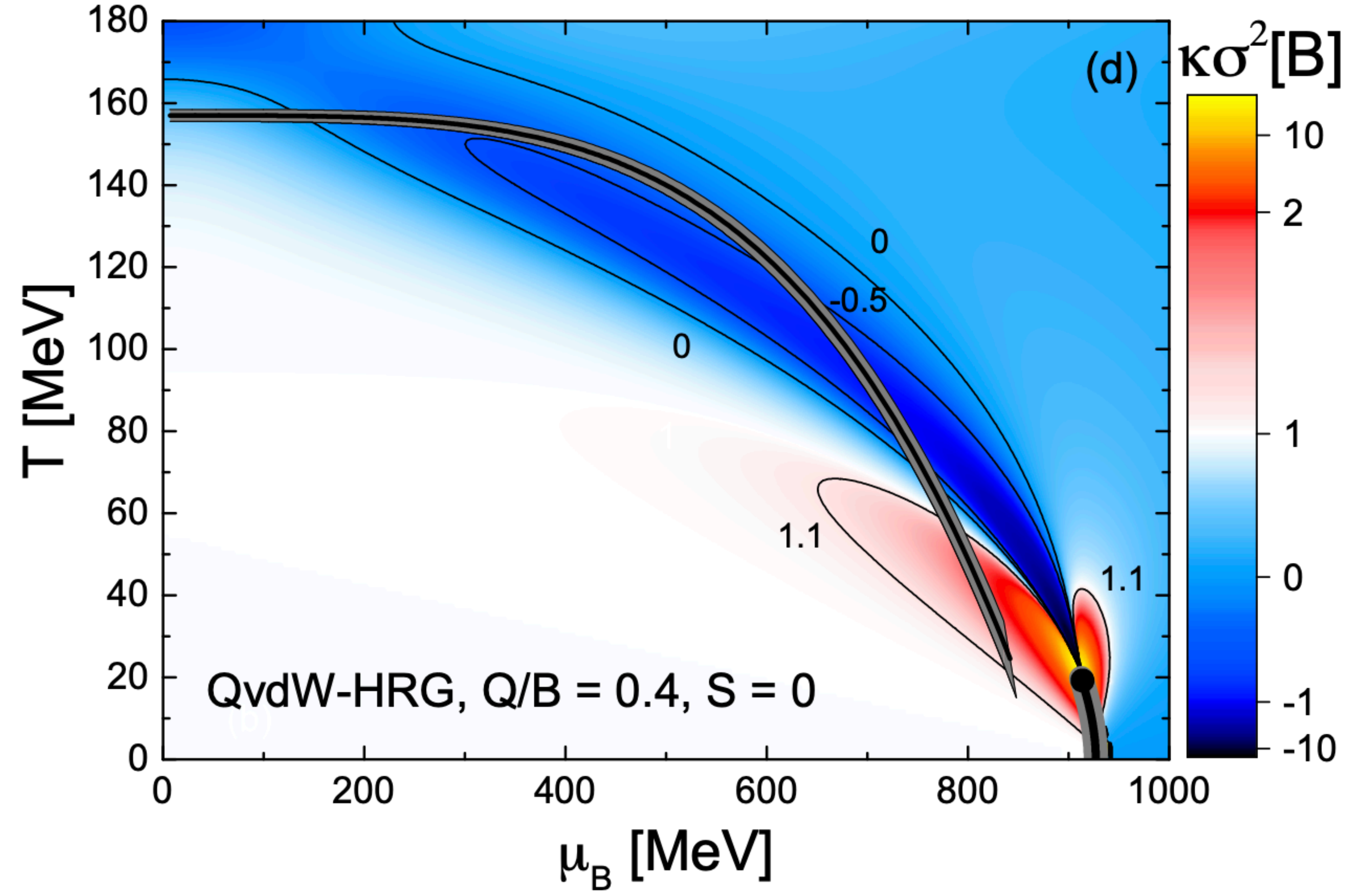
Most would agree this means "perform simulations as if there is no hadron-QGP transition".
BUT that doesn't mean there are no interactions.

Consequences of these interactions may be significant over vast ranges of the QCD phase diagram



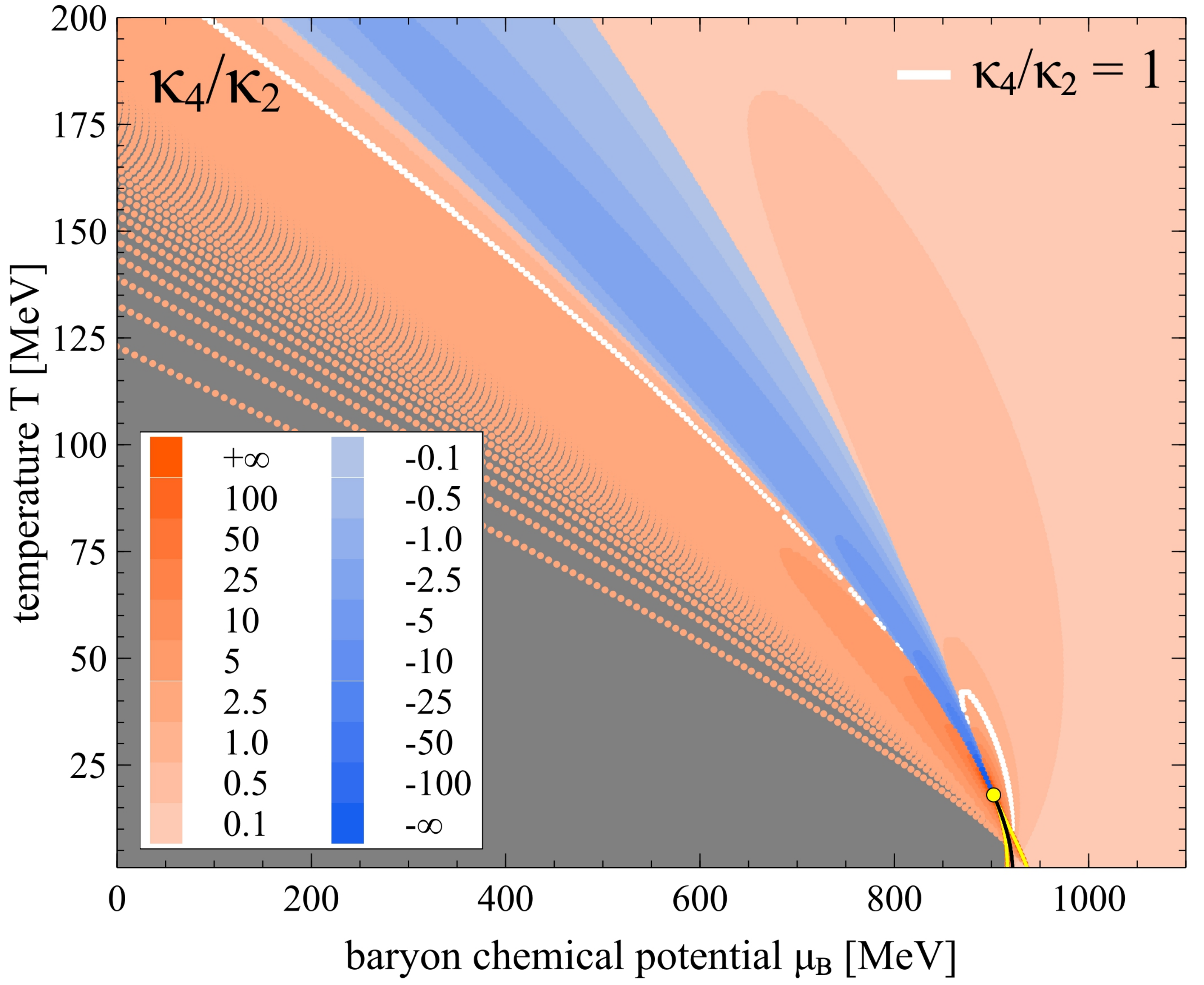
B. Borderie, J.D. Frankland, Prog. Part. Nucl. Phys. **105**, 82-138 (2019), arXiv:1903.02881

QvdW-HRG model



R. Poberezhnyuk, R. V. Vovchenko, A. Motornenko, M. I. Gorenstein, H. Stoecker, Phys. Rev. C **100** 5, 054904 (2019) arXiv:1906.01954

VDF model

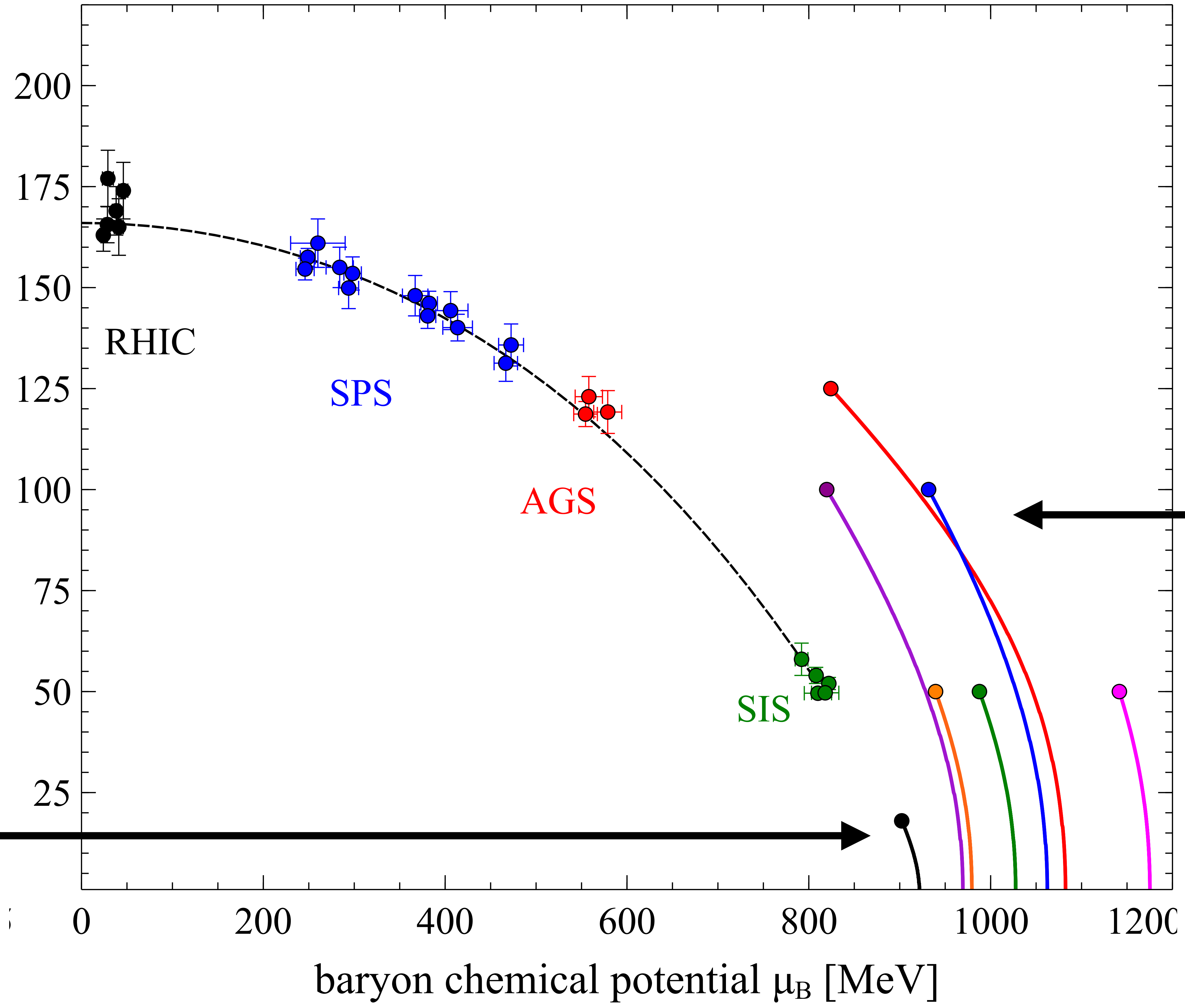


A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

Two ways of using hadronic transport

2) Use it to map out the QGP-hadron phase transition

e.g., use parametrizable interactions to search for the softening of the EOS

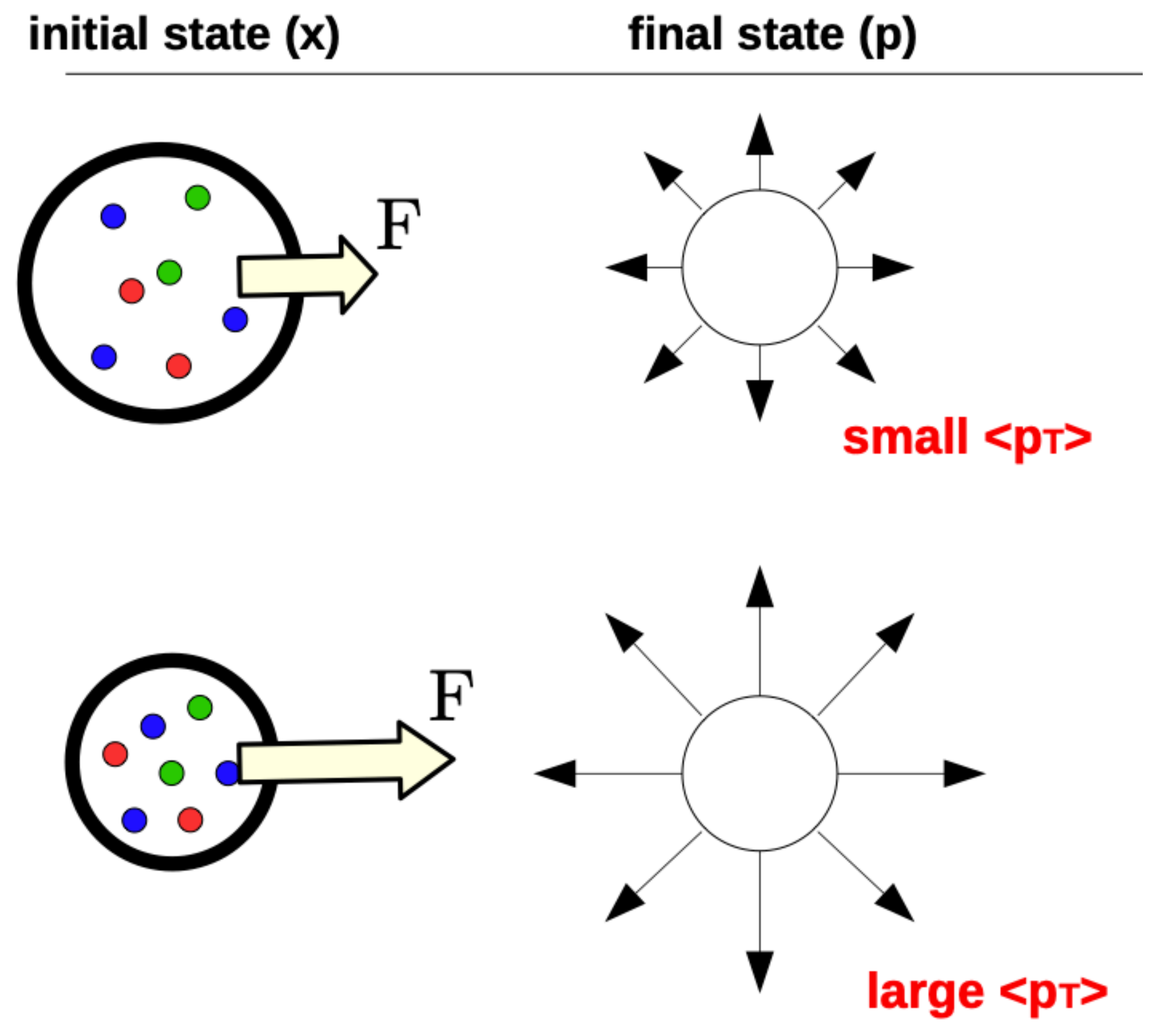


various possible "hadron-QGP" phase transitions

nuclear liquid-gas phase transition

Flow observables in heavy-ion collisions

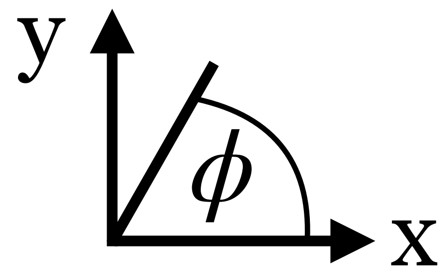
Mean transverse momentum p_T



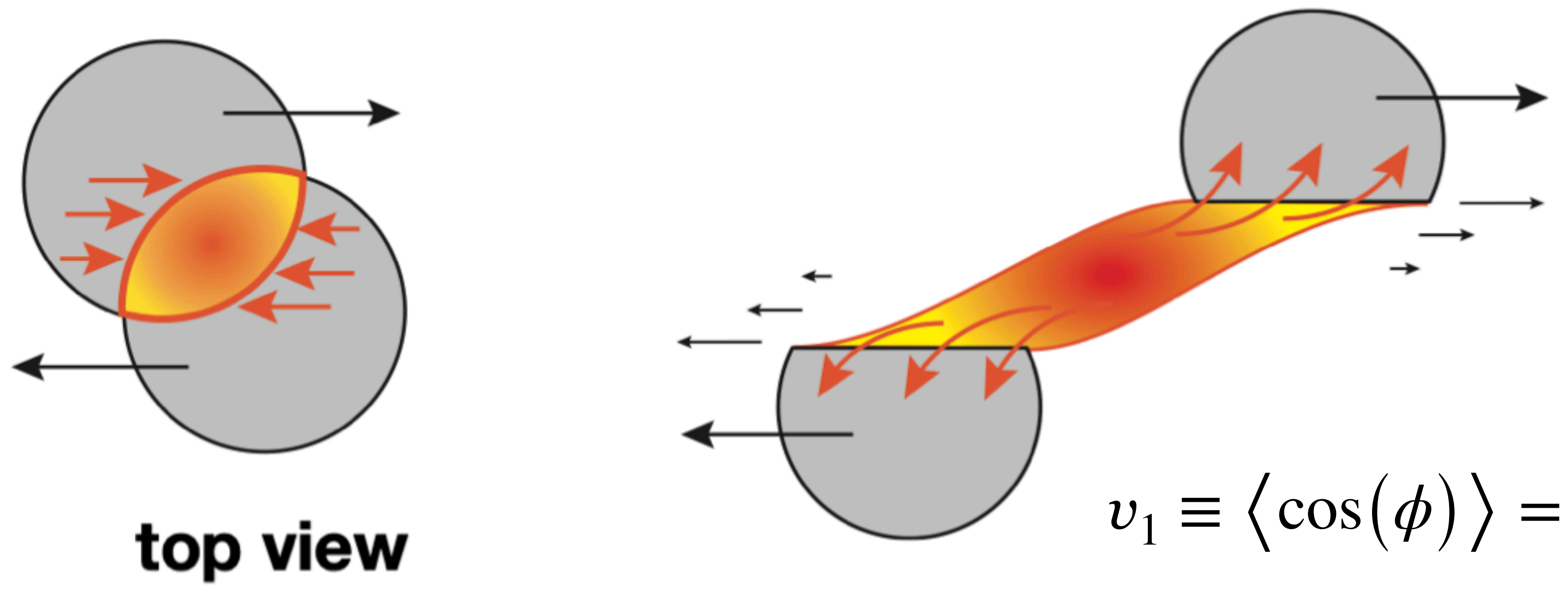
illustrations from a presentation by G. Giacalone

These observables are extremely sensitive to the EOS

Flow $v_n \equiv \langle \cos(n\phi) \rangle$



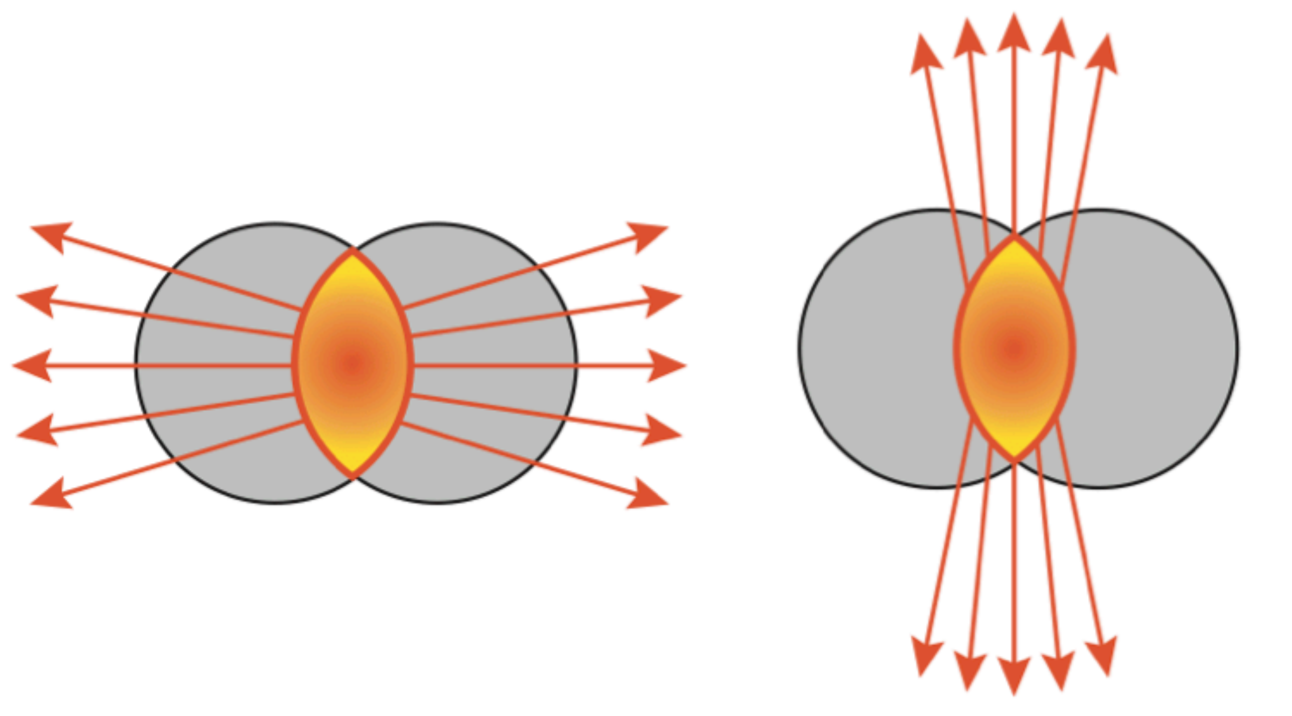
directed flow v_1 (dv_1/dy ~ longitudinal expansion)



$v_1 \equiv \langle \cos(\phi) \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$

elliptic flow v_2 ($v_2(y \approx 0)$ ~ midrapidity)

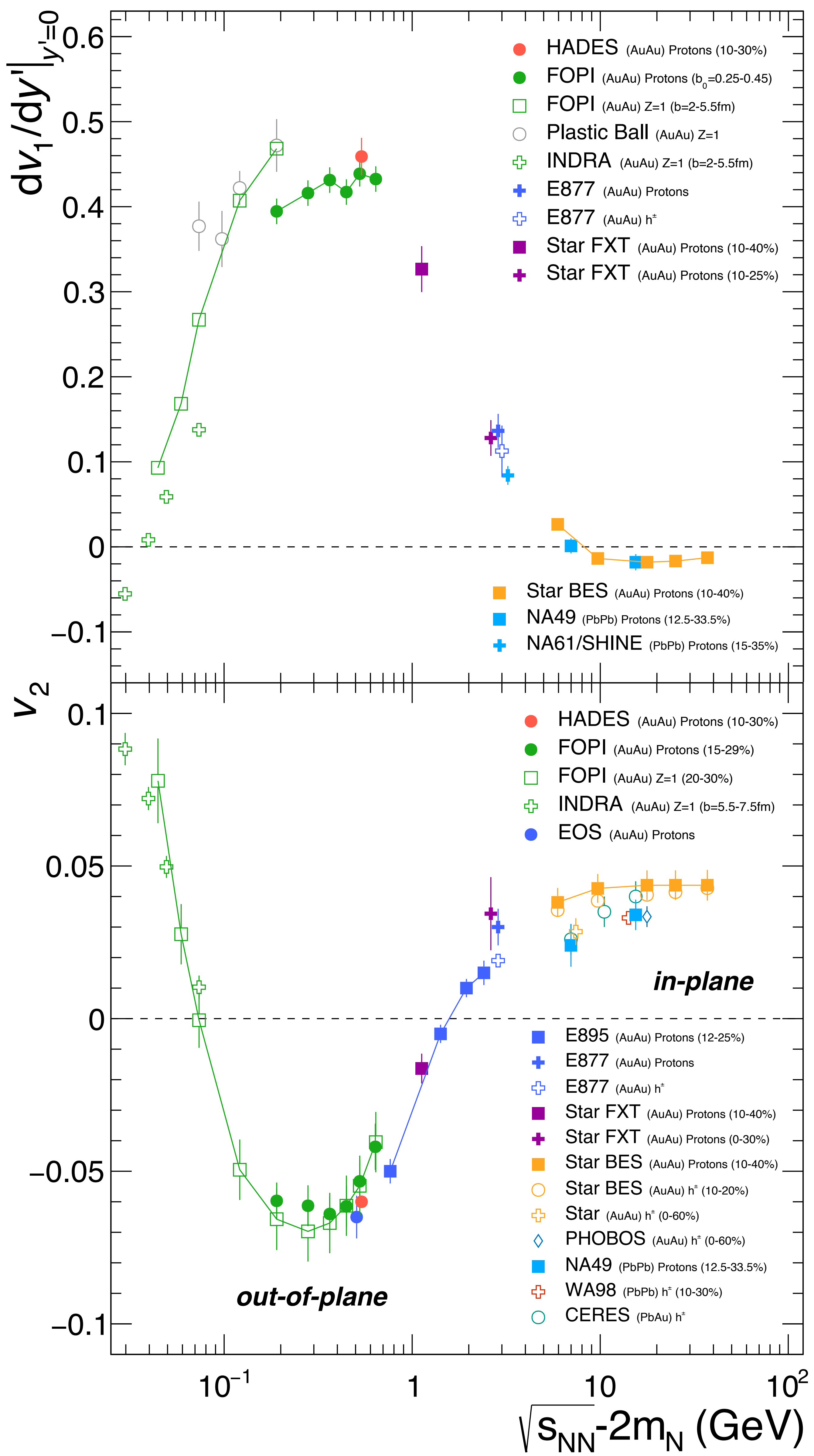
front view



$v_2 \equiv \langle \cos(2\phi) \rangle$

in-plane **out-of-plane**

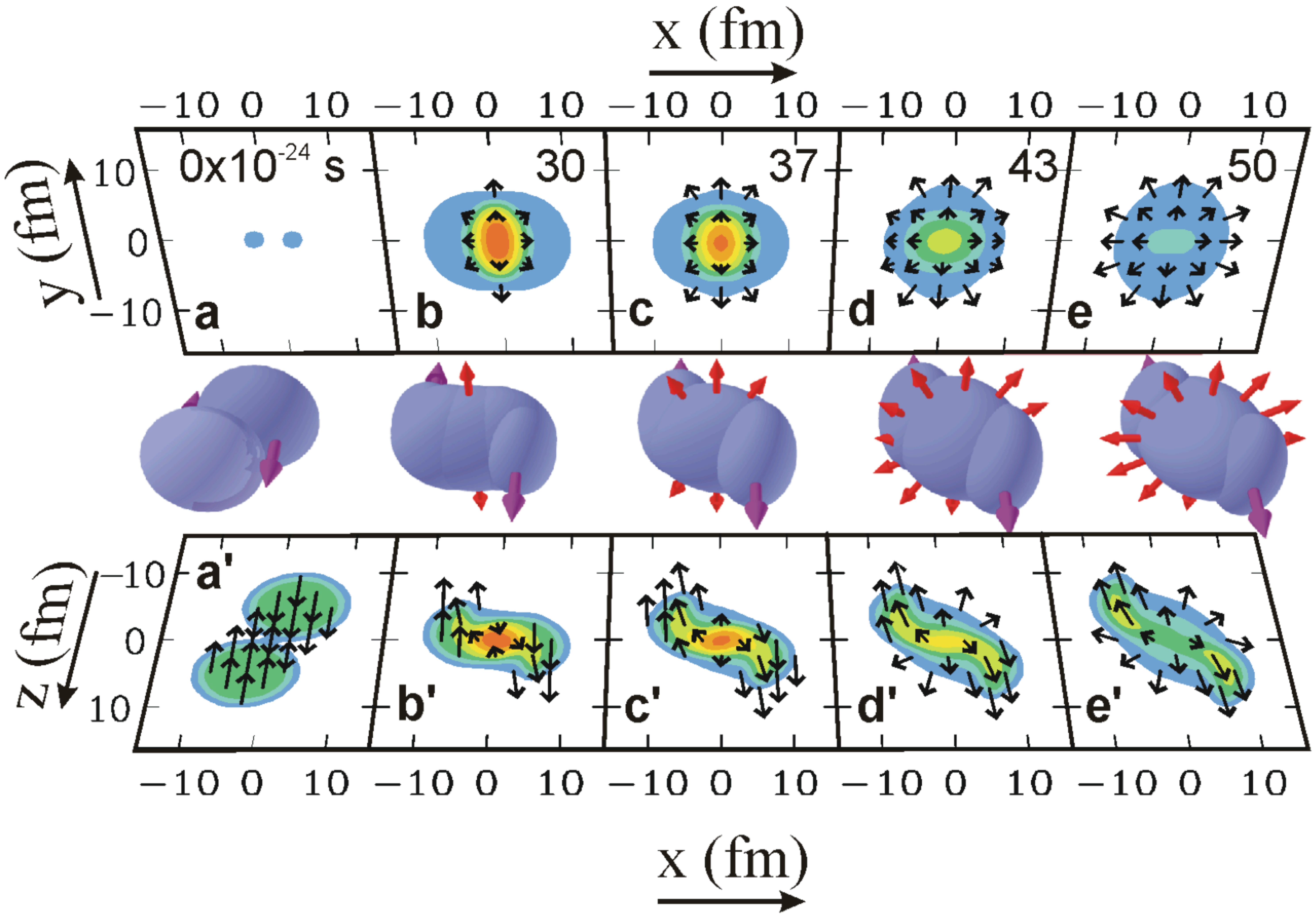
illustrations from a presentation by B. Kardan (HADES)



J. Adamczewski-Musch *et al.* (HADES), arXiv:2208.02740

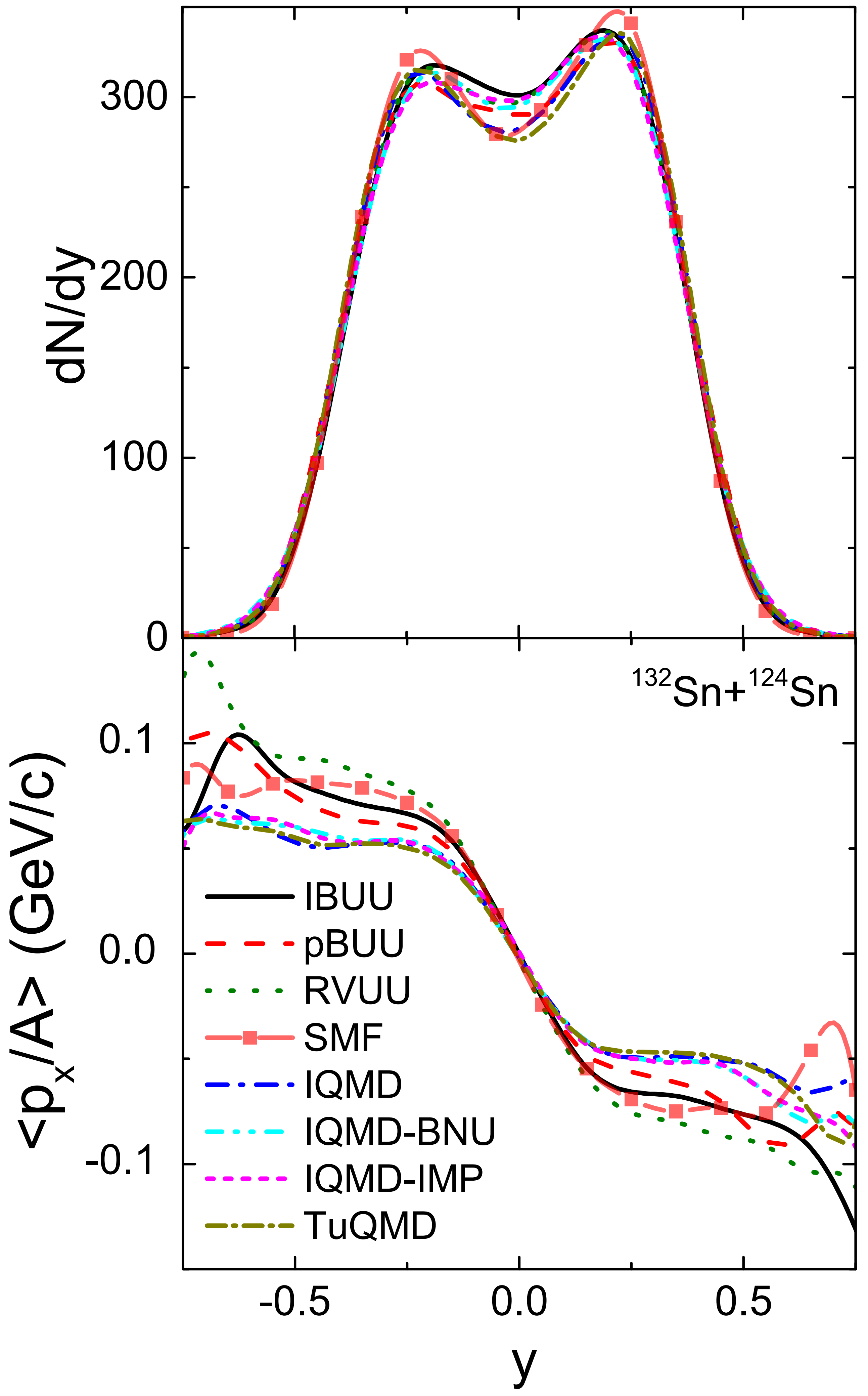
Flow observables in heavy-ion collisions

Flow observables are the canonical observables for extracting the EOS



P. Danielewicz, R. Lacey, W. G. Lynch,
Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

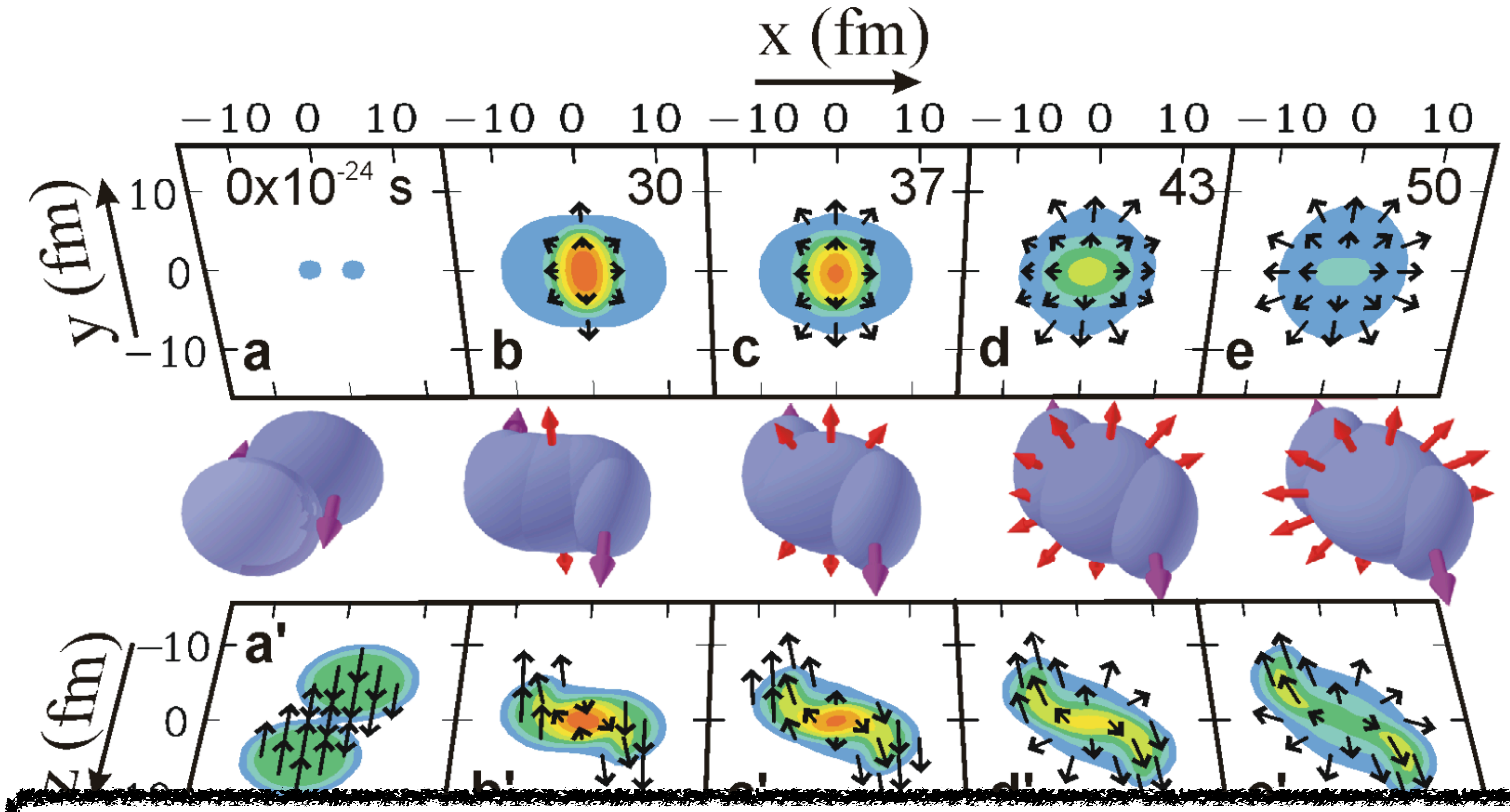
There is code-dependence:



J. Xu *et al.* (TMEP Collaboration), *in preparation*

Flow observables in heavy-ion collisions

Flow observables are the canonical observables for extracting the EOS



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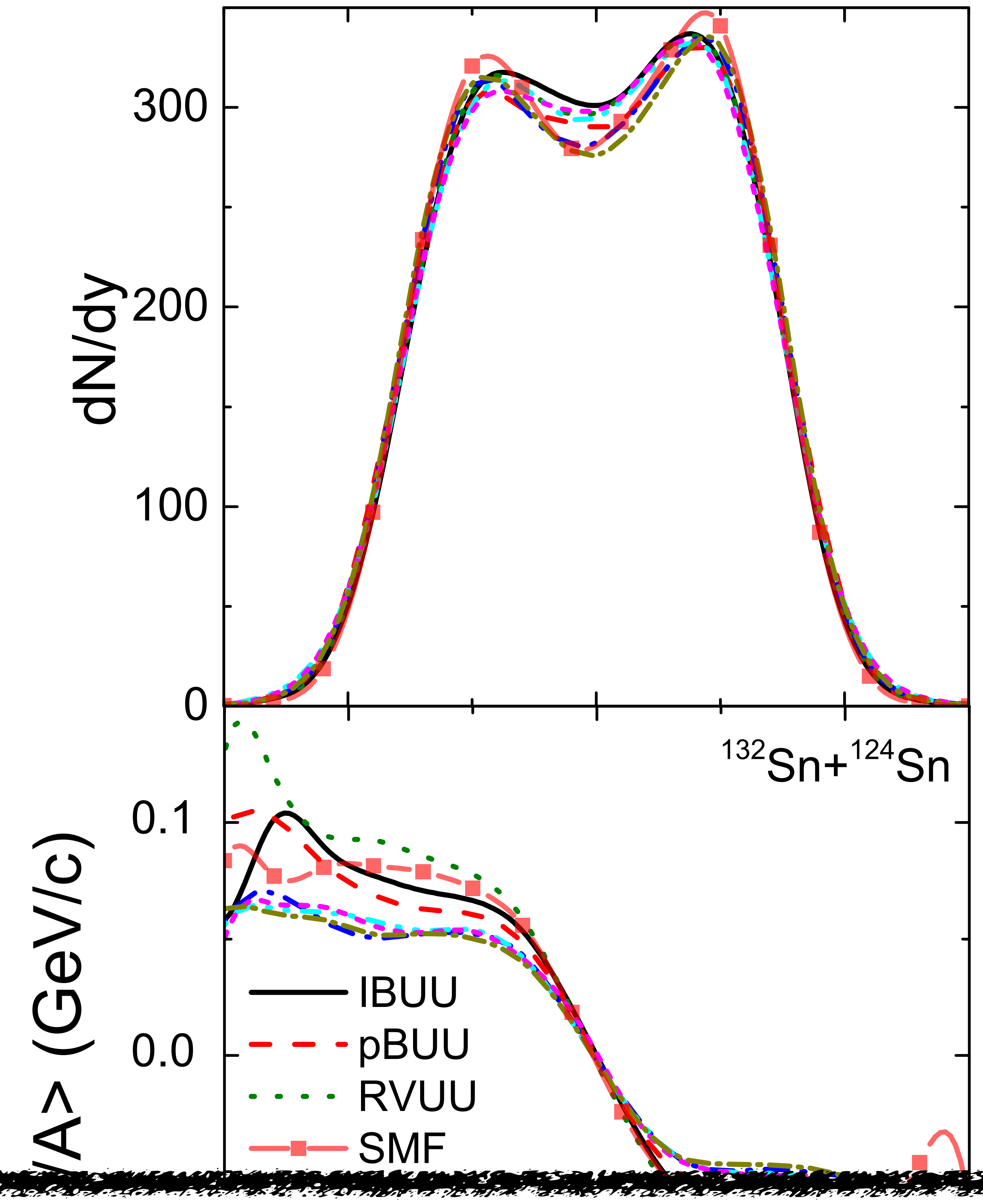
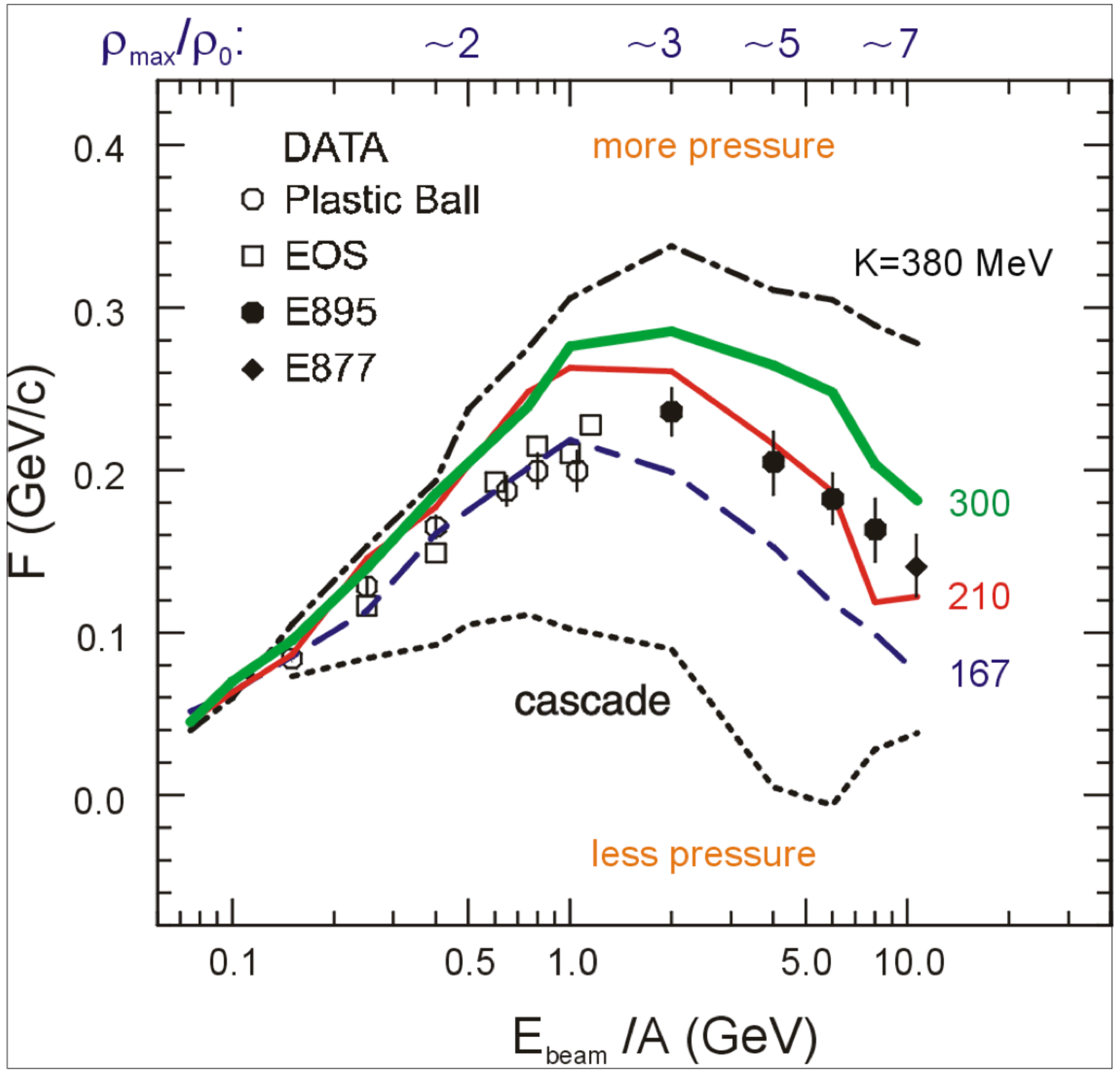


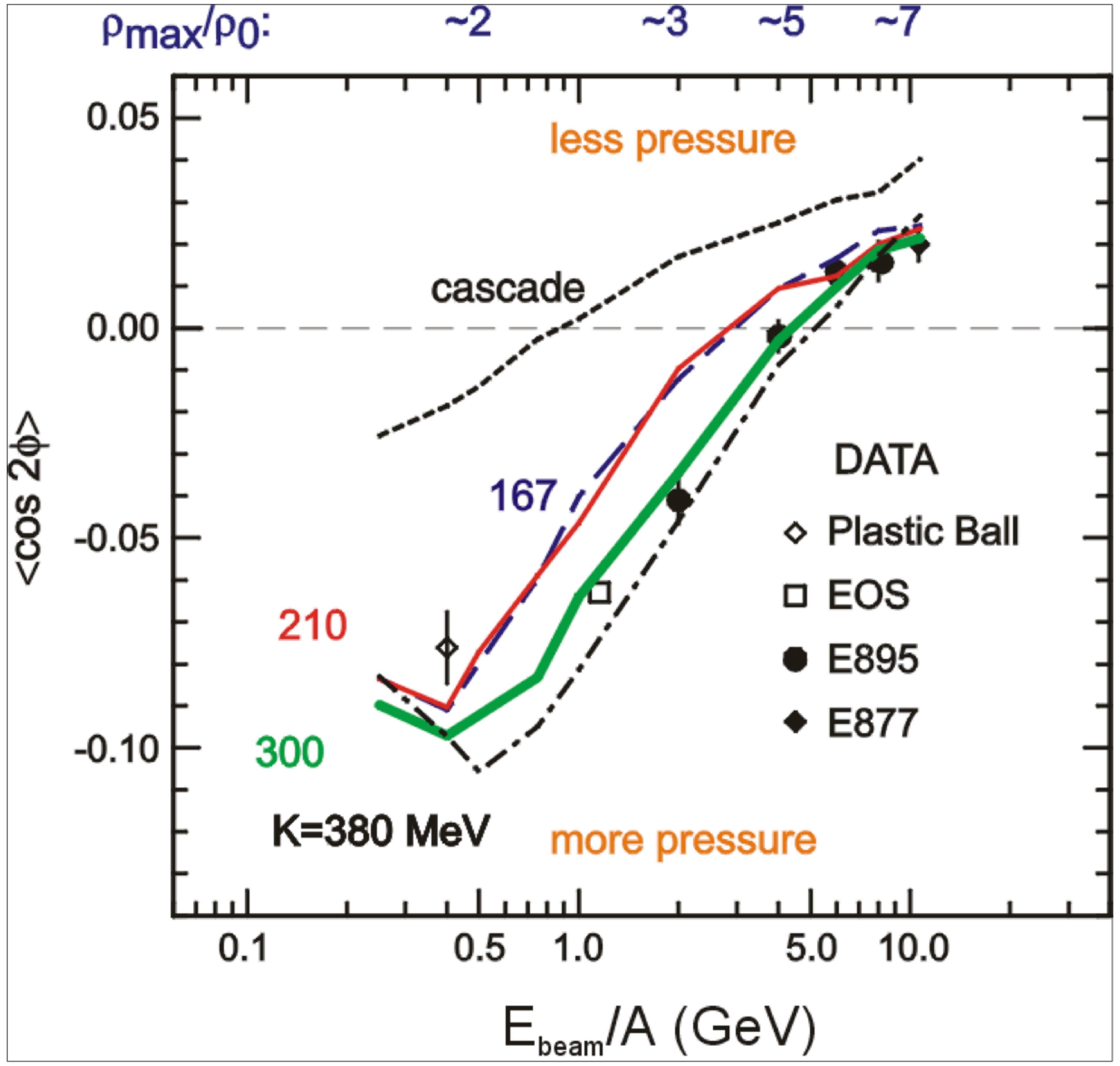
Comparisons between different codes are needed to understand the dependence on:
1) different physics assumptions 2) different implementation solutions
See efforts by, e.g., TMEP collaboration

Standard way of modeling the EOS: Skyrme potential

The most common form of the EOS is the "Skyrme potential": $U(n_B) = A\left(\frac{n_B}{n_0}\right) + B\left(\frac{n_B}{n_0}\right)^{\tau}$

DLL used something a bit more sophisticated: $U(n_B) = \left(an_B + bn_B^{\tau}\right)/\left[1 + (0.4n_B/n_0)^{\tau-1}\right] + U_p$





$$F = \left.\frac{d\langle p_x/A\rangle}{d(y/y_{cm})}\right|_{y/y_{cm}=1}$$

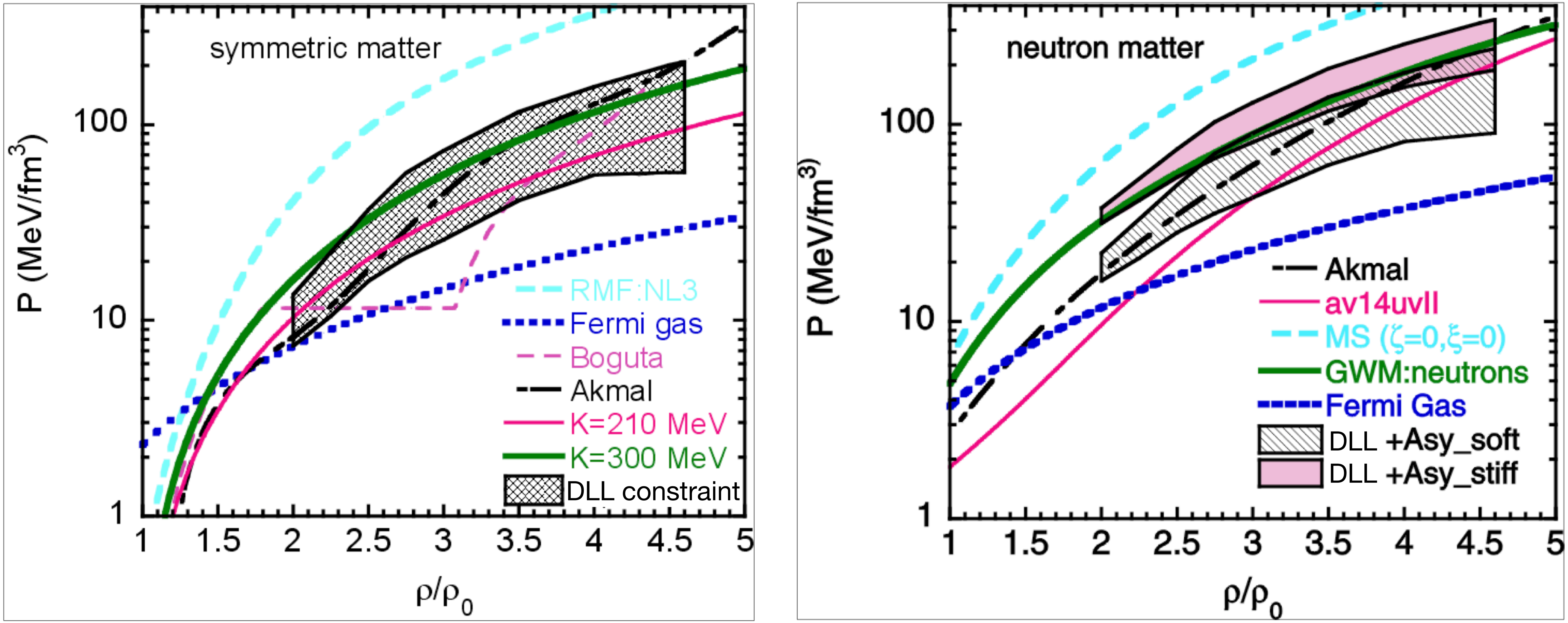
P. Danielewicz, R. Lacey, W. G. Lynch, Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

Standard way of modeling the EOS: Skyrme potential

T

D

“the heavy-ion constraint”



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Standard way of modeling the EOS: Skyrme potential

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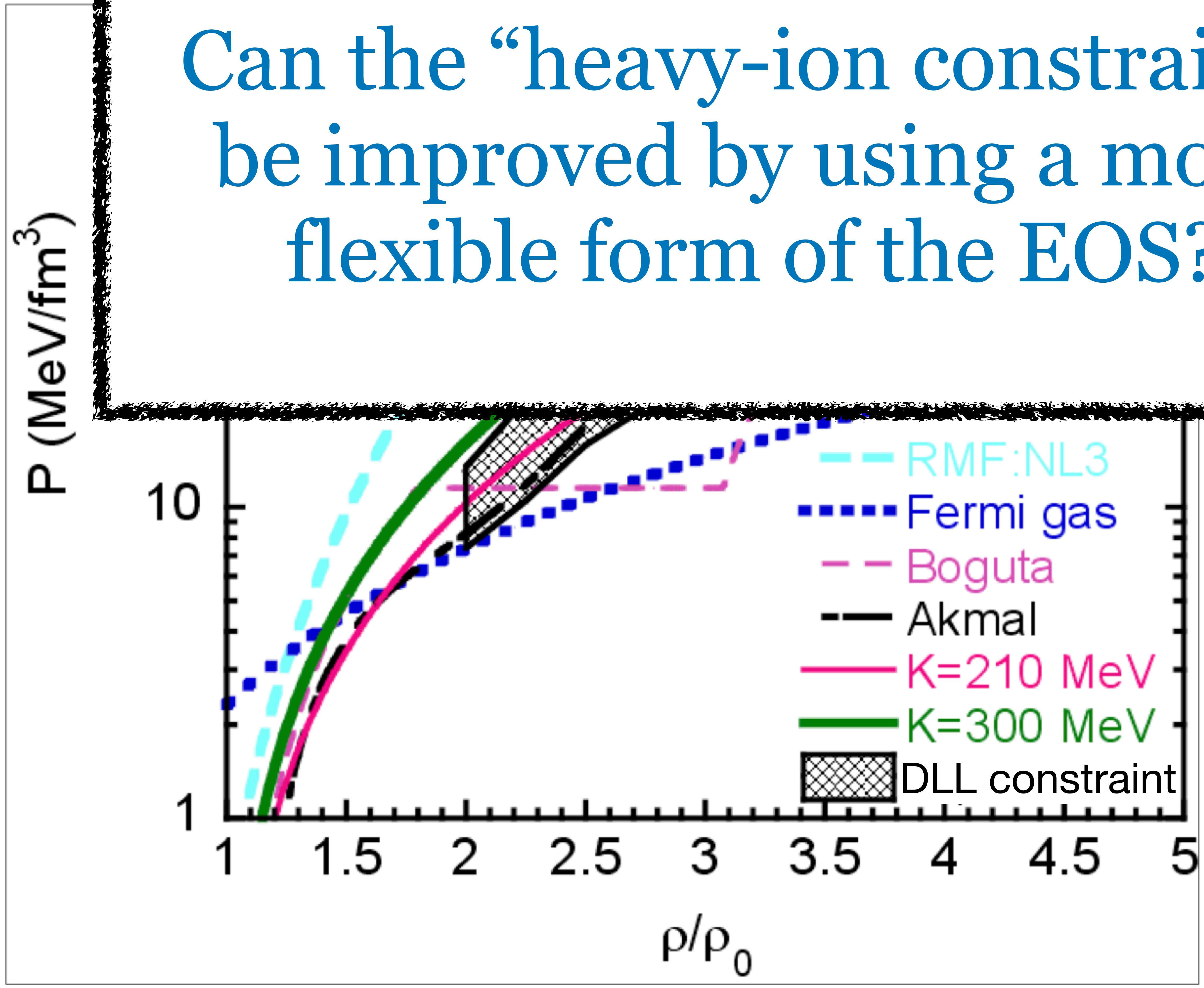
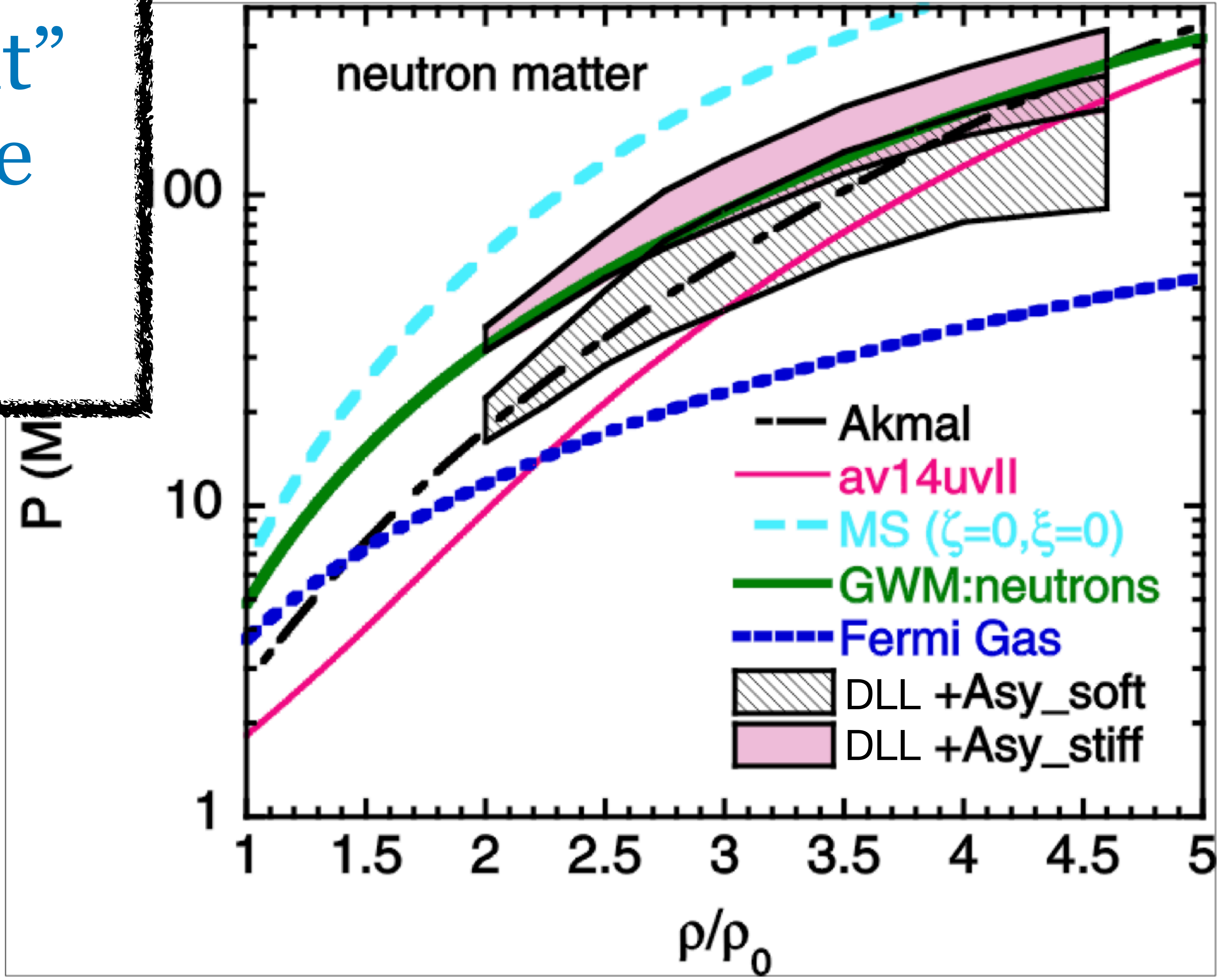
Can the "heavy-ion constraint" be improved by using a more flexible form of the EOS?

"the heavy-ion constraint"





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P. Danielewicz, R. Lacey, W. G. Lynch, Science **298**, 1592–1596 (2002), arXiv:nucl-th/0208016

Relativistic vector density functional (VDF) model

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

inspired by relativistic Landau Fermi-liquid theory:
G. Baym, S. A. Chin, Nucl. Phys. A **262**, 527 (1976)

$j_\mu j^\mu = n_B^2$

1) Postulate the energy density of the system:

$$\mathcal{E}_N = \mathcal{E}_N[f_\mathbf{p}] = g\int \frac{d^3p}{(2\pi)^3}\,\epsilon_{\text{kin}}\, f_\mathbf{p} + \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1}\left[j^0 j^0 - g^{00}\left(\frac{b_i-1}{b_i}\right) j_\lambda j^\lambda\right]$$

← Lorentz covariant

$$\epsilon_{\text{kin}} = \sqrt{\left(\vec{p} - \sum_{i=1}^{N} C_i (j_\mu j^\mu)^{\frac{b_i}{2}-1}\vec{j}\right)^2 + m^2}$$

$$\mathcal{E}_N\Big|_{\text{rest frame}} = g\int \frac{d^3p}{(2\pi)^3}\sqrt{\vec{p}^{\,2}+m^2}\, f_\mathbf{p} + \sum_{i=1}^{N}\frac{C_i}{b_i} n_B^{b_i}$$

← mean-field interactions parameterized by C_i and b_i

2) Quasiparticle energy:

$$\varepsilon_\mathbf{p} \equiv \frac{\delta \mathcal{E}[f_\mathbf{p}]}{\delta f_\mathbf{p}} = \epsilon_{\text{kin}} + \sum_{i=1}^{N} C_i \left(j_\mu j^\mu\right)^{\frac{b_i}{2}-1} j^0$$

thermodynamic consistency!

3) Get EOMs

$$\frac{dx^i}{dt} \equiv -\frac{\partial \varepsilon_\mathbf{p}}{\partial p_i}, \qquad \frac{dp^i}{dt} \equiv \frac{\partial \varepsilon_\mathbf{p}}{\partial x_i}$$

← input to transport code; use in Boltzmann eq. to obtain $T^{\mu\nu}$

4) Use $T^{\mu\nu}$ to get the pressure:

$$P_N = \frac{1}{3}\sum_k T^{kk}\Big|_{\text{rest frame}} = g\int \frac{d^3p}{(2\pi)^3}\, T\, \ln\left[1 + e^{-\beta(\varepsilon_\mathbf{p}-\mu_B)}\right] + \sum_{i=1}^{N} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$

VDF model: two 1st order phase transitions

A. Sorensen, V. Koch, Phys. Rev. C **104** (2021) 3, 034904, arXiv:2011.06635

Systems with two 1st order phase transitions: nuclear and "quark/hadron", or "QGP-like"

- degrees of freedom: nucleons
- "QGP-like" PT: "more dense" matter coexists with "less dense" matter
- minimal model: 4 interactions terms = 8 parameters to fix:

$$P = g\int \frac{d^3p}{(2\pi)^3}\, T\, \ln\left[1 + e^{-\beta(\epsilon_{\mathbf{p}} - \mu_B)}\right] + \sum_{i=1}^{N=4} C_i \frac{b_i - 1}{b_i} n_B^{b_i}$$

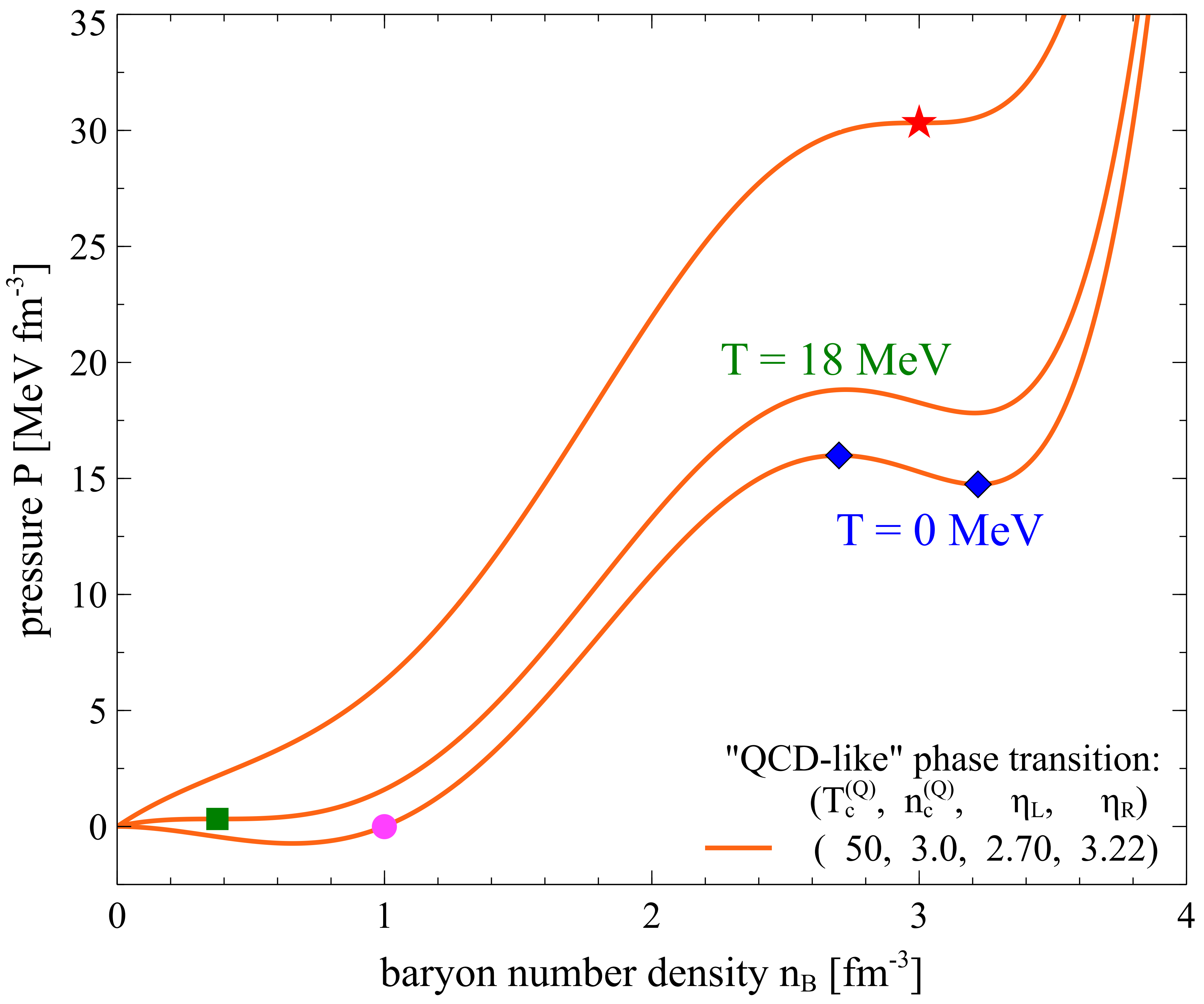
C_i and b_i are fitted to reproduce:

$n_0 = 0.160\ \mathrm{fm}^{-3}$, $E_B = -16.3$ MeV

$T_c^{(N)} = 18$ MeV, $n_c^{(N)} = 0.375 n_0$

$T_c^{(Q)} = ?$, $n_c^{(Q)} = ?$

$\eta_L = ?$, $\eta_R = ?$



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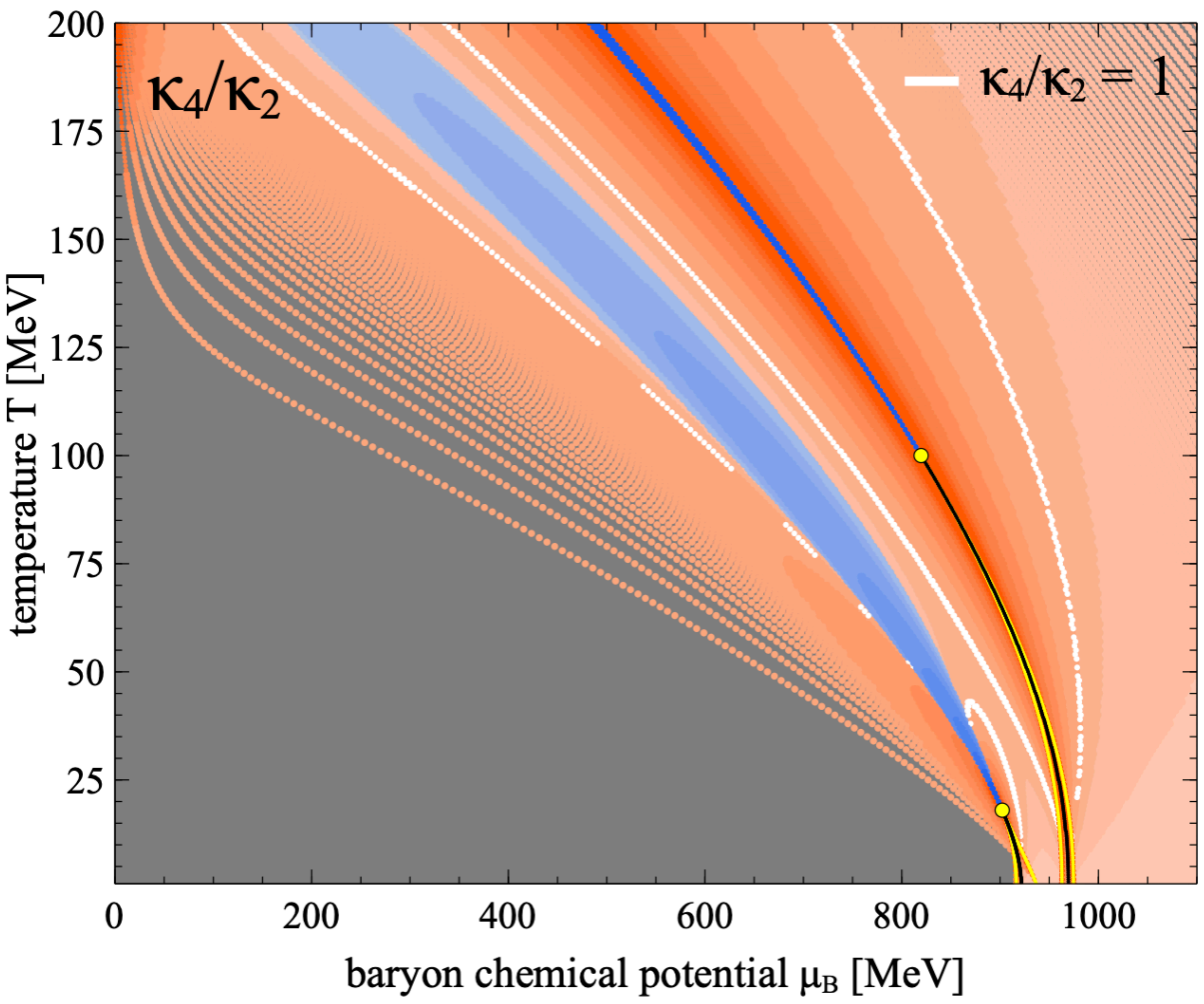

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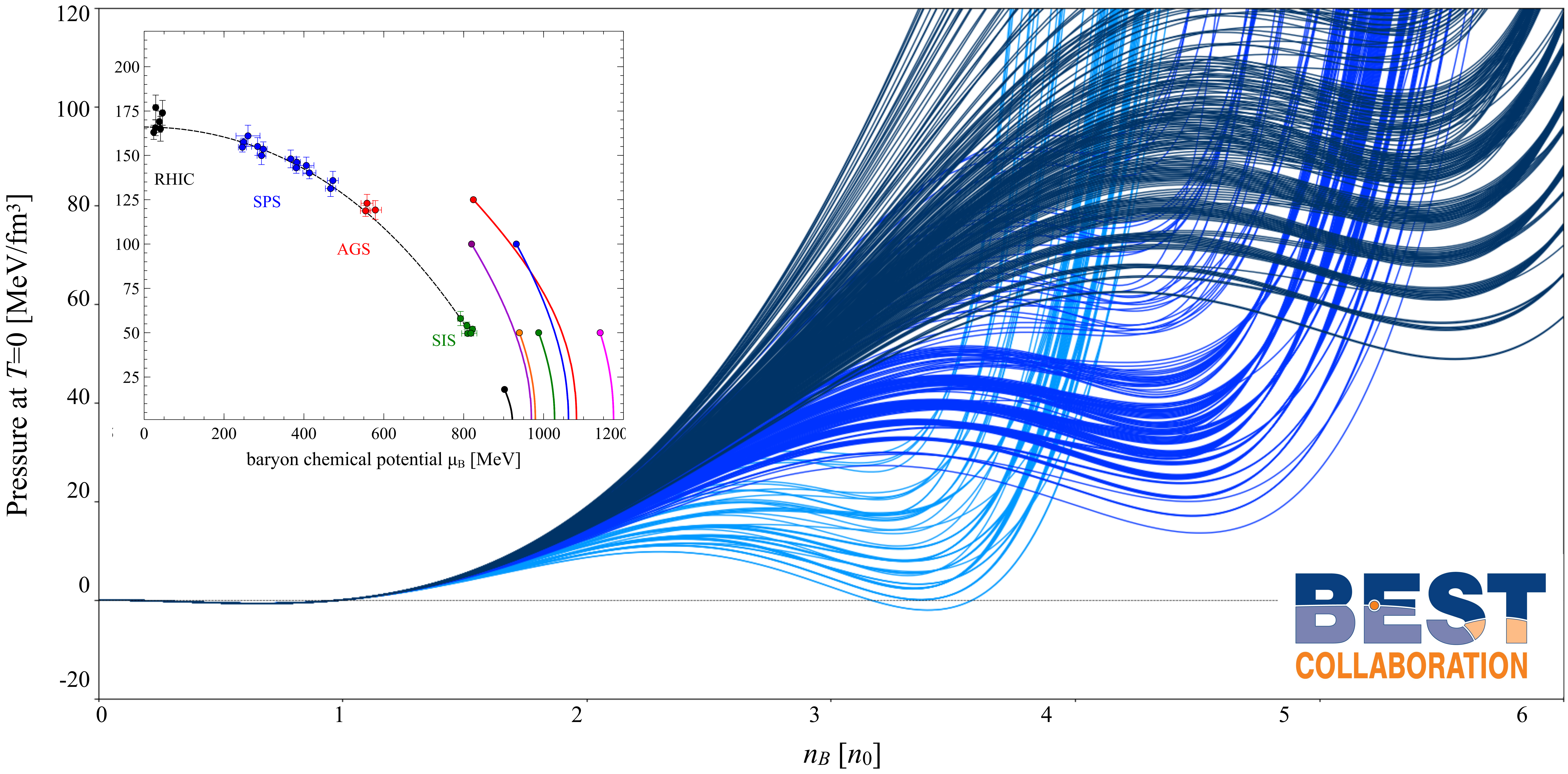
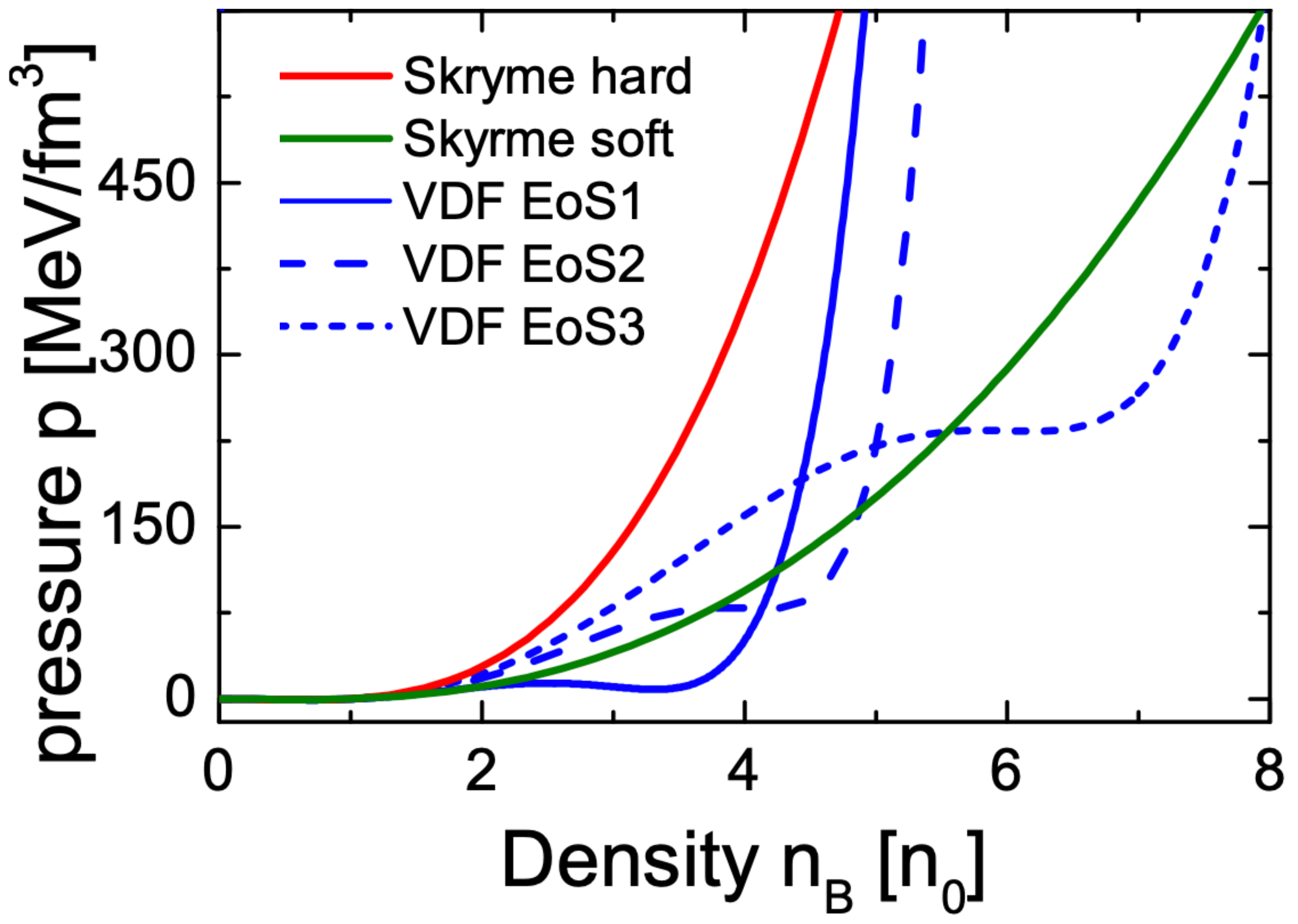


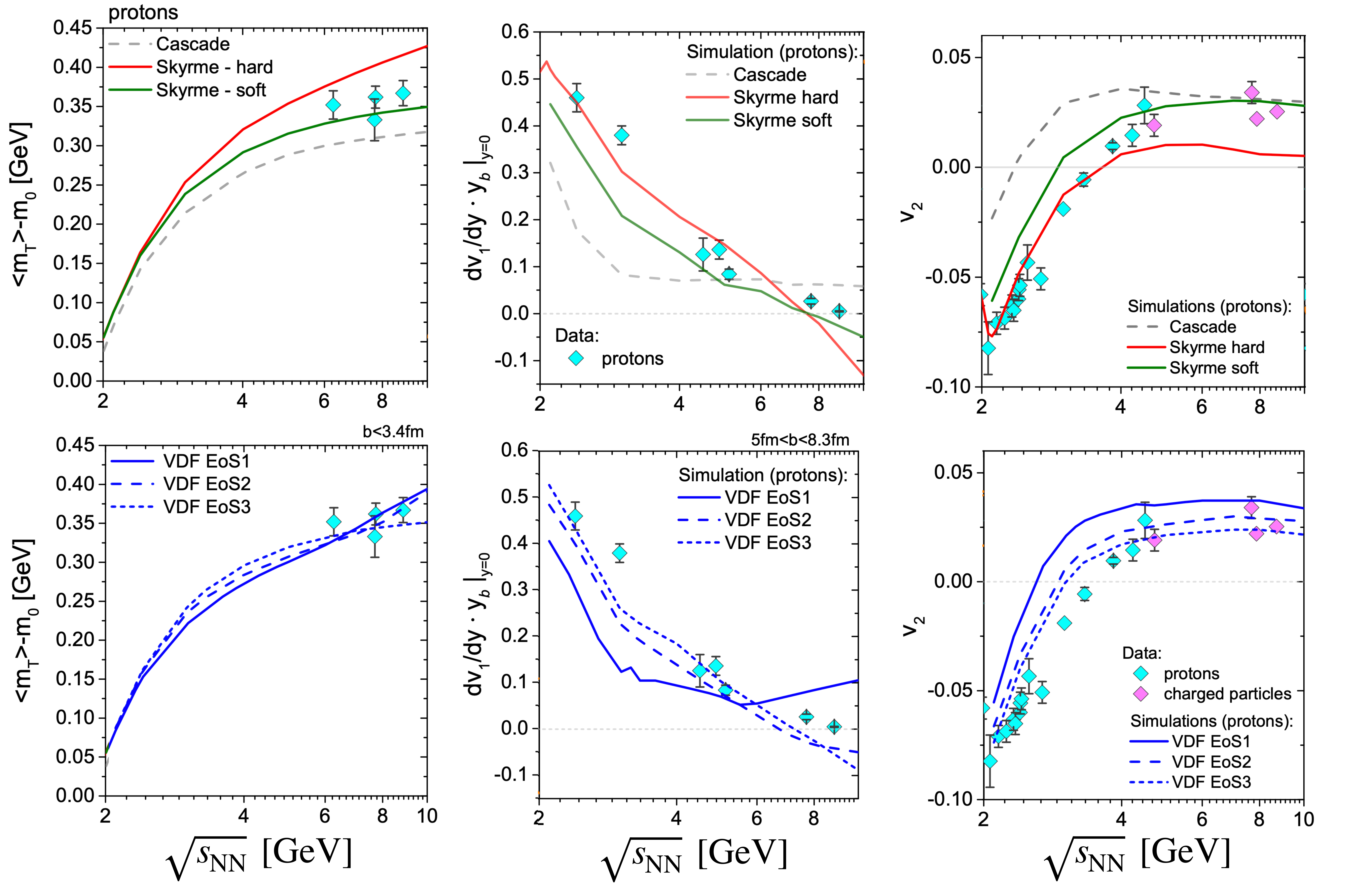
Results from UrQMD with (non-relativistic) VDF

J. Steinheimer, A. Motornenko, **A. Sorensen**, Y. Nara, V. Koch,
M. Bleicher, Eur. Phys. J. C **82**, 10, 911 (2022) arXiv:2208.12091



EoS	$T_c^{(N)}$[MeV]	$n_c^{(Q)}[n_0]$	$T_c^{(Q)}$[MeV]	K_0[MeV]
VDF1	18	3.0	100	261
VDF2	18	4.0	50	279
VDF3	22	6.0	50	356

Very soft EOS at $n_B \in (2,3)n_0$
not supported in VDF+UrQMD



Results from UrQMD with (non-relativistic) CMF

J. Steinheimer, A. Motornenko, **A. Sorensen**, Y. Nara, V. Koch,
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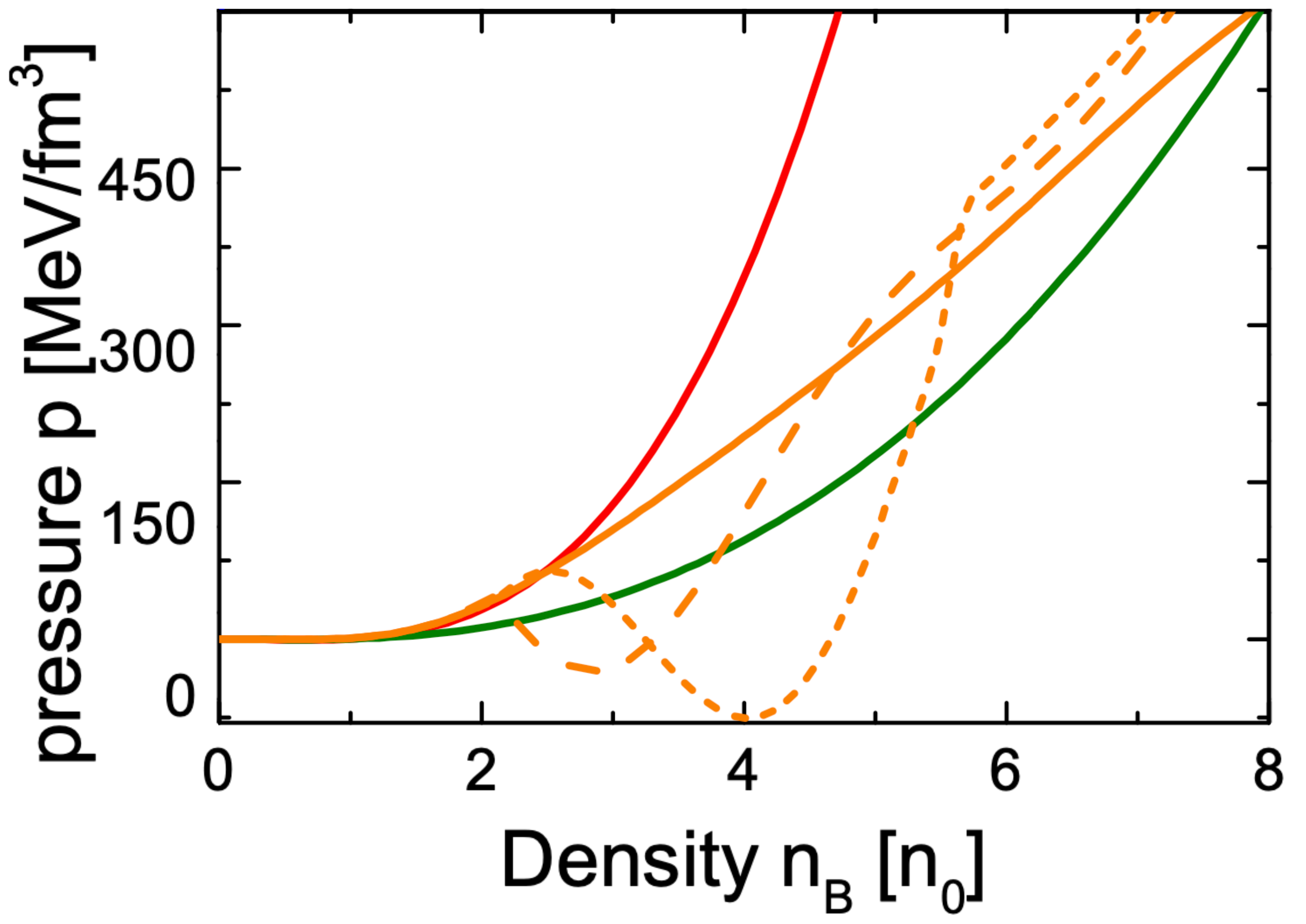

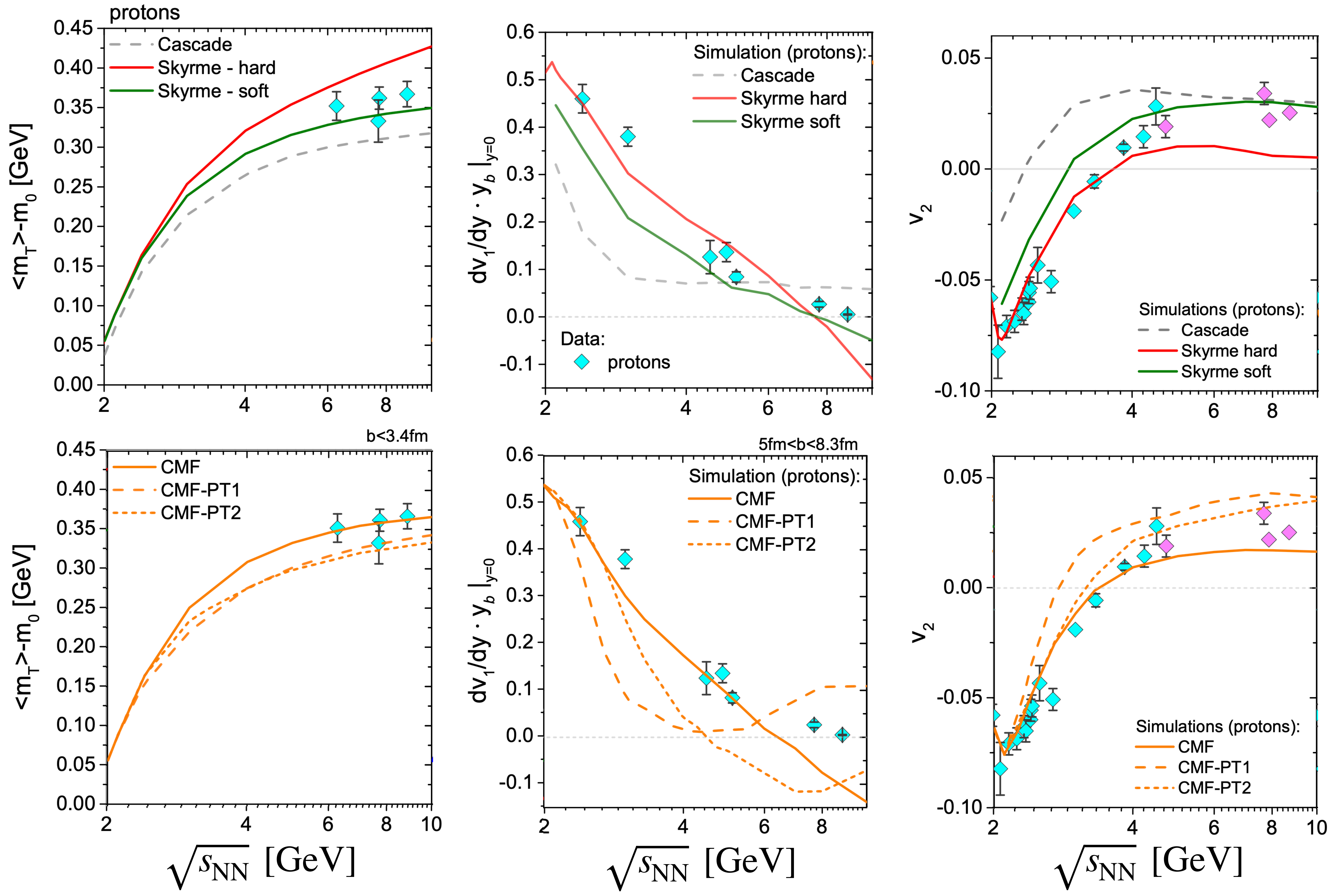

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not supported in CMF+UrQMD

Generalized VDF model: custom c_s^2

VDF model: $$\mathcal{E}_N = g\int \frac{d^3p}{(2\pi)^3}\, \epsilon^*_{\text{kin}}\, f_{\mathbf{p}} \;+\; \sum_{i=1}^{N} A_k^0 j_0 - g^{00}\sum_{i=1}^{N}\left(\frac{b_i-1}{b_i}\right)A_k^{\lambda} j_{\lambda}$$ $$j_\mu j^\mu = n_B^2$$

$$\varepsilon_{\mathbf{p}} = \epsilon_{\text{kin}} \;+\; \sum_{i=1}^{N} A_i^0$$ $$A_k^\mu = C_k\left(j_\lambda j^\lambda\right)^{\frac{b_k}{2}-1} j^\mu$$

The distribution (Fermi or Boltzmann) will have factors of $e^{\beta(\varepsilon_{\mathbf{p}}-\mu_B)} = e^{\beta(\varepsilon_{\text{kin}}+\sum_{i=1}^{N}A_i^0-\mu_B)} = e^{\beta(\varepsilon_{\text{kin}}-\mu^*)}$

Assume arbitrary vector interactions: $A^\mu = \alpha(n_B)j^\mu$

The effective chemical potential is $\mu^* = \mu_B - \alpha(n_B)n_B$

At $T=0$, $\epsilon_F = \mu^*$ and the density is given by $n_B = \frac{g}{6\pi^2}p_F^3 = \frac{g}{6\pi^2}\left(\mu^{*2} - m^2\right)^{3/2}$

Combining the two allows one to solve for $\mu_B(n_B) = \alpha(n_B)n_B + \sqrt{m^2 + \left(\frac{6\pi n_B}{g}\right)^{2/3}}$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

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Combining the two allows one to solve for $\mu_B(n_B) = \alpha(n_B) n_B + \sqrt{m^2 + \left(\frac{6\pi n_B}{g}\right)^{2/3}}$

On the other hand, $c_s^2\Big|_{T=0} = \frac{d \ln \mu_B}{d \ln n_B}$, and solving for μ_B: $\mu_B(n_B) = \mu_B\left(n_B^{(0)}\right) \exp\left(\int_{n_B^{(0)}}^{n_B} d\ln n \; c_s^2(n) \right)$

Solve for vector interactions: $\alpha(n_B) = \frac{1}{n_B}\left[\mu_B\left(n_B^{(0)}\right) \exp\left(\int_{n_B^{(0)}}^{n_B} d\ln n \; c_s^2(n) \right) - \sqrt{m^2 + \left(\frac{6\pi n_B}{g}\right)^{2/3}} \right]$

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A

C

These interactions, parametrized with a chosen shape of c_s^2 as a function of n_B, can be used in hadronic transport simulations!

C

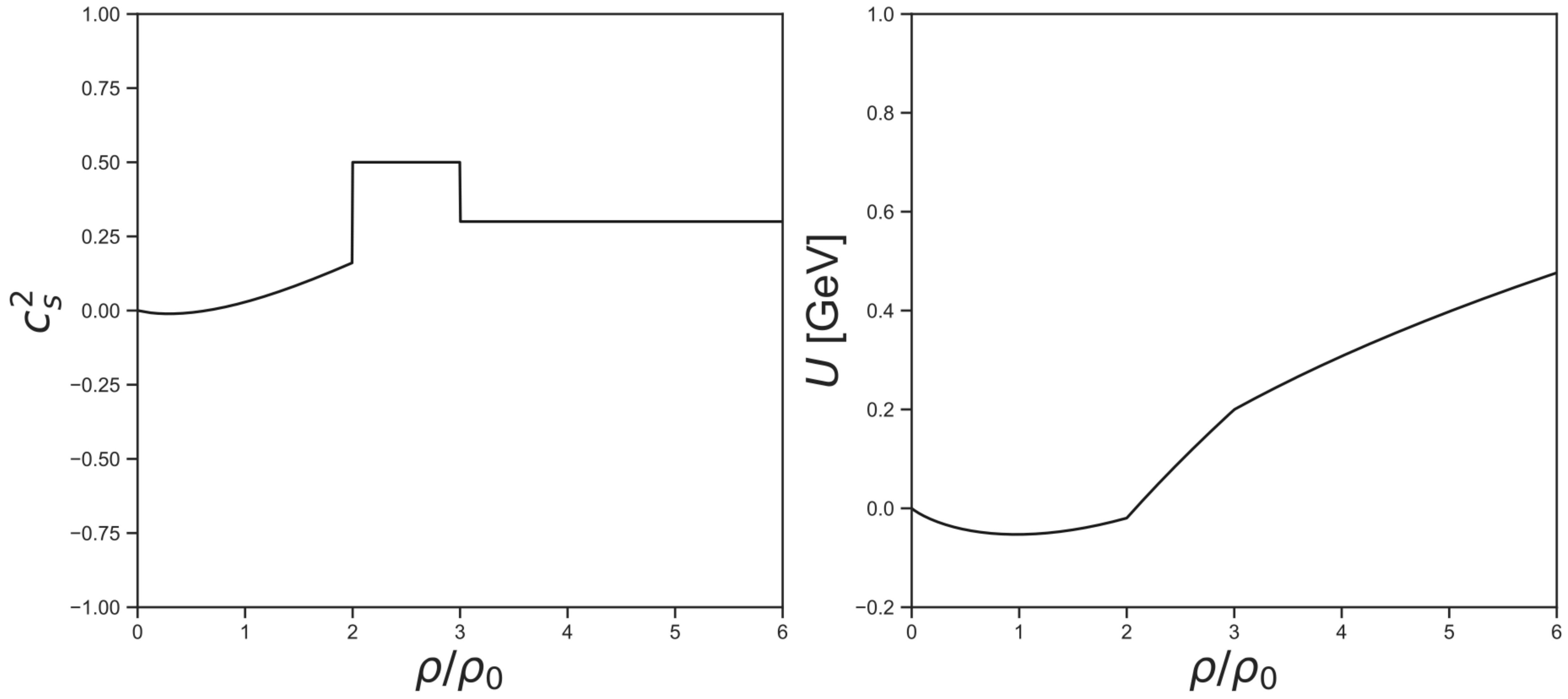
Solve for vector interactions: $$\alpha(n_B) = \frac{1}{n_B}\left[\mu_B\left(n_B^{(0)}\right)\exp\left(\int_{n_B^{(0)}}^{n_B} d\ln n\ c_s^2(n)\right) - \sqrt{m^2 + \left(\frac{6\pi n_B}{g}\right)^{2/3}}\right]$$

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

Better suited for detailed studies: piecewise parametrization of c_s^2

Piecewise parametrization of $c_s^2(n_B)$:

$$c_s^2(n_B) = \begin{cases} c_s^2(\text{Skyrme}), & n_B < n_1 = 2n_0 \\ c_1^2, & n_1 < n_B < n_2 \\ c_2^2, & n_2 < n_B < n_3 \\ \dots & \\ c_m^2, & n_m < n_B \end{cases}$$



Single-particle potential $U(n_B) = \alpha(n_B)n_B$:

$$U(n_B) = \begin{cases} U_{\mathrm{Sk}}(n_B)\ , & n_B < n_1 = 2n_0 \\ \Big[U_{\mathrm{Sk}}(n_1) + \mu^*(\rho_1)\Big]\left(\frac{\rho}{n_1}\right)^{c_1^2} - \mu^*(n_B)\ , & n_1 < n_B < n_2 \\ \Big[U_{\mathrm{Sk}}(n_1) + \mu^*(n_1)\Big]\left(\frac{n_B}{n_k}\right)^{c_k^2} \prod_{i=2}^{k}\left(\frac{n_i}{n_{i-1}}\right)^{c_{i-1}^2} - \mu^*(n_B)\ , & n_k < n_B < n_{k+1} \end{cases}$$

Gradients of $U(n_B)$ enter the EOMs!

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

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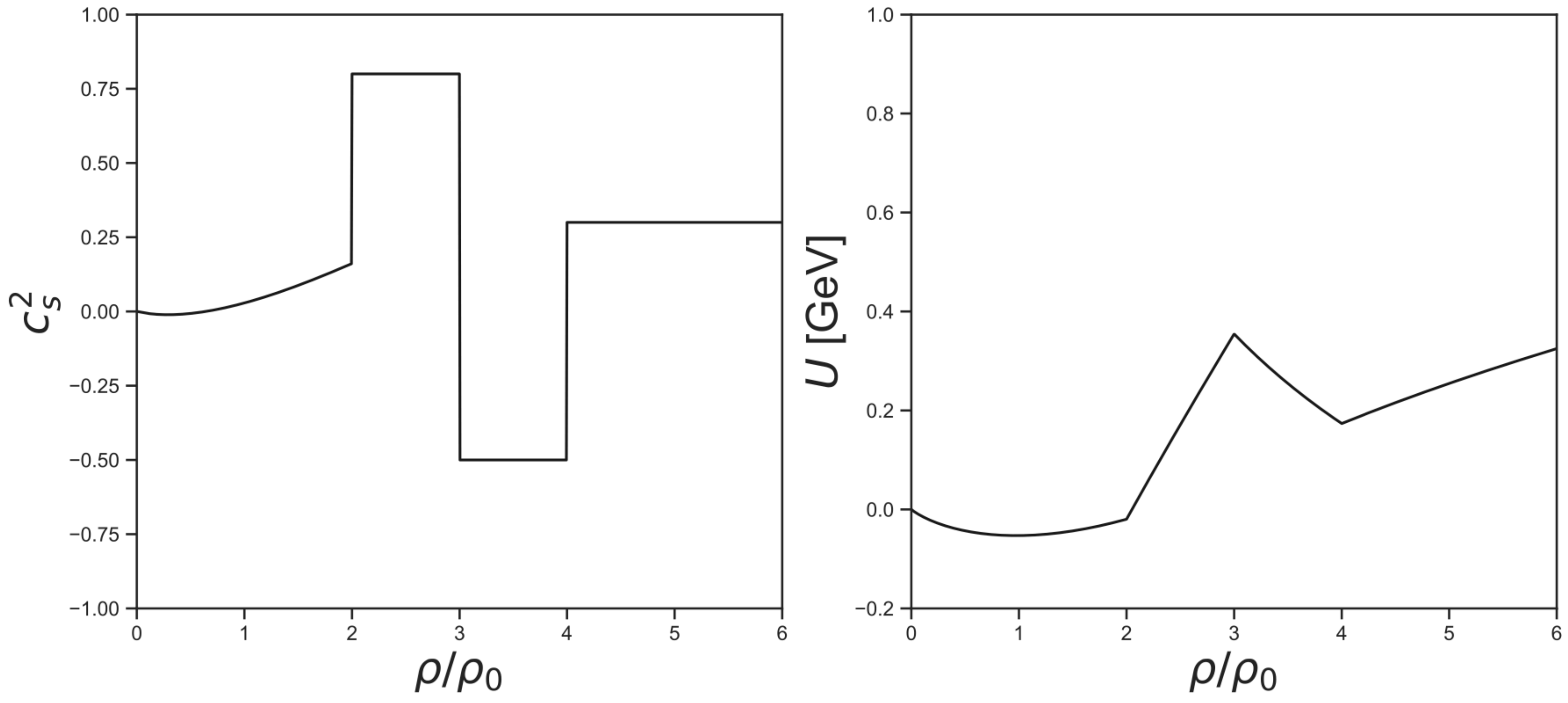

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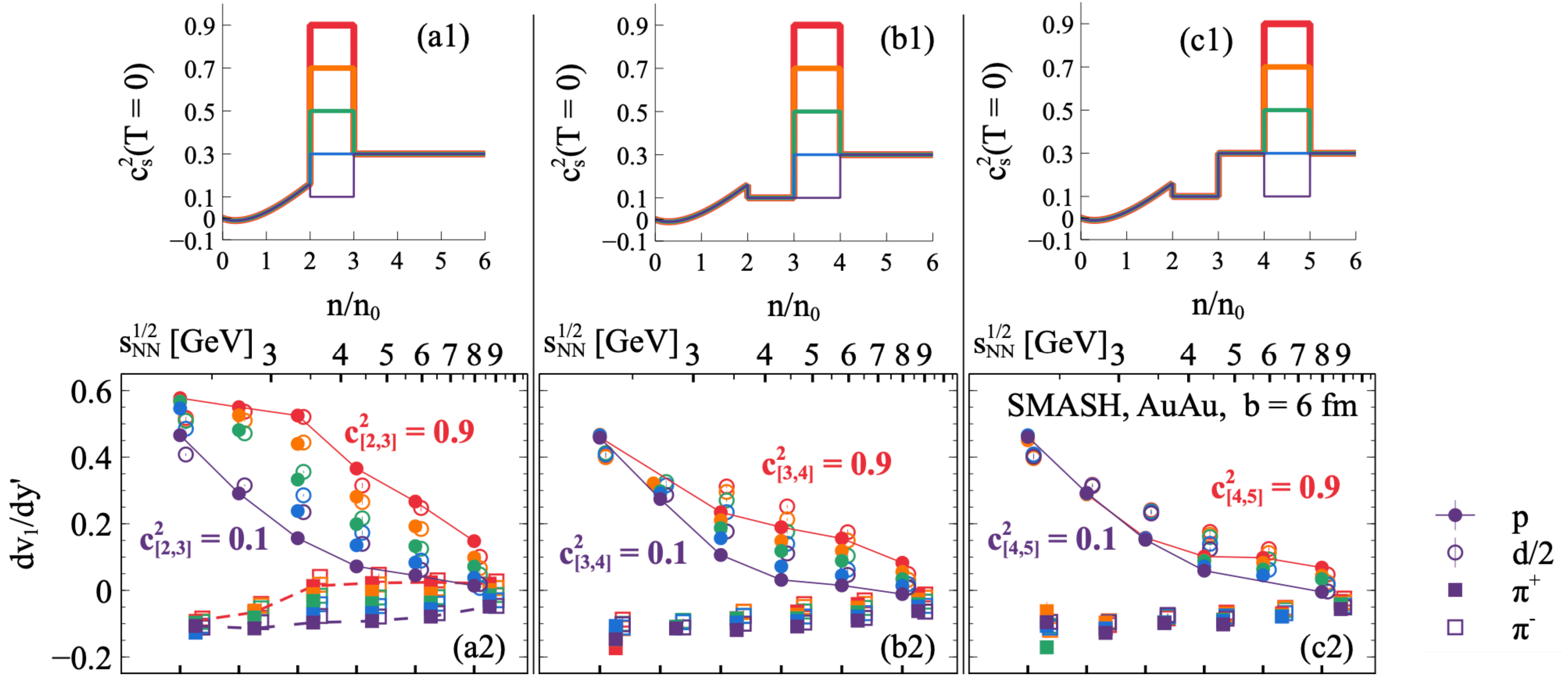
Hadronic transport with c_s^2-parametrized mean-fields

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran,
arXiv:2208.11996

Generalized VDF (n_B-dependent interaction coefficients):

mean-field potential piecewise parametrized by (constant) values of c_s^2 for $n_i < n_B < n_j$



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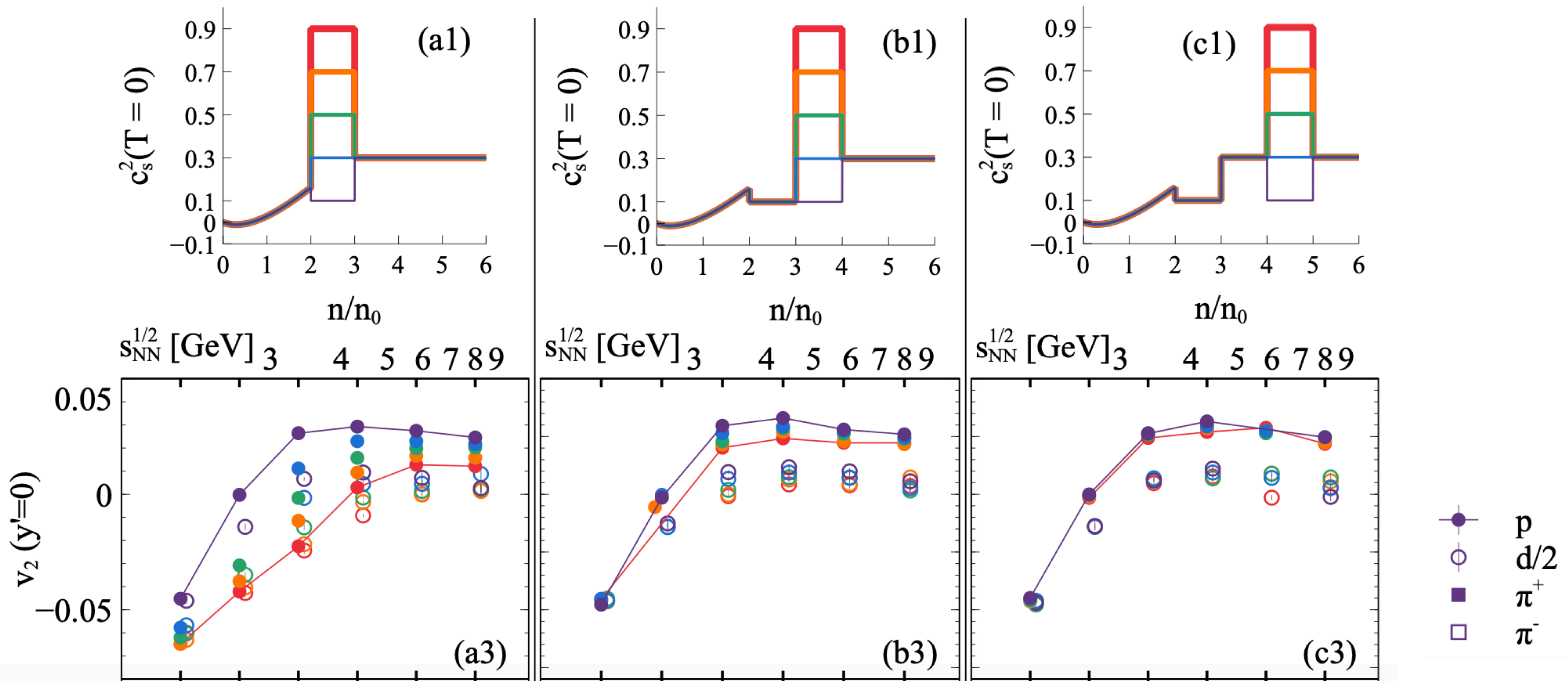
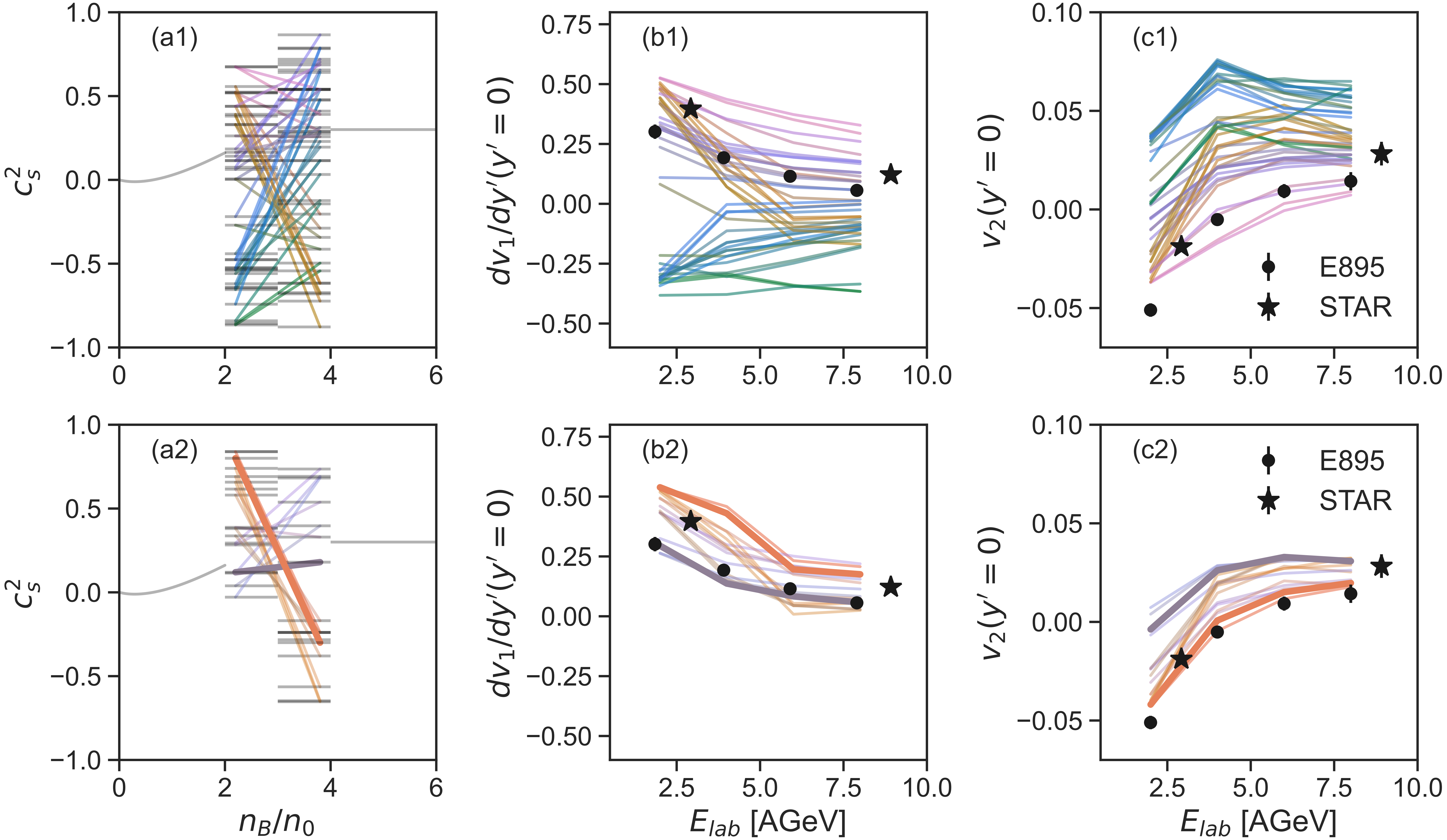
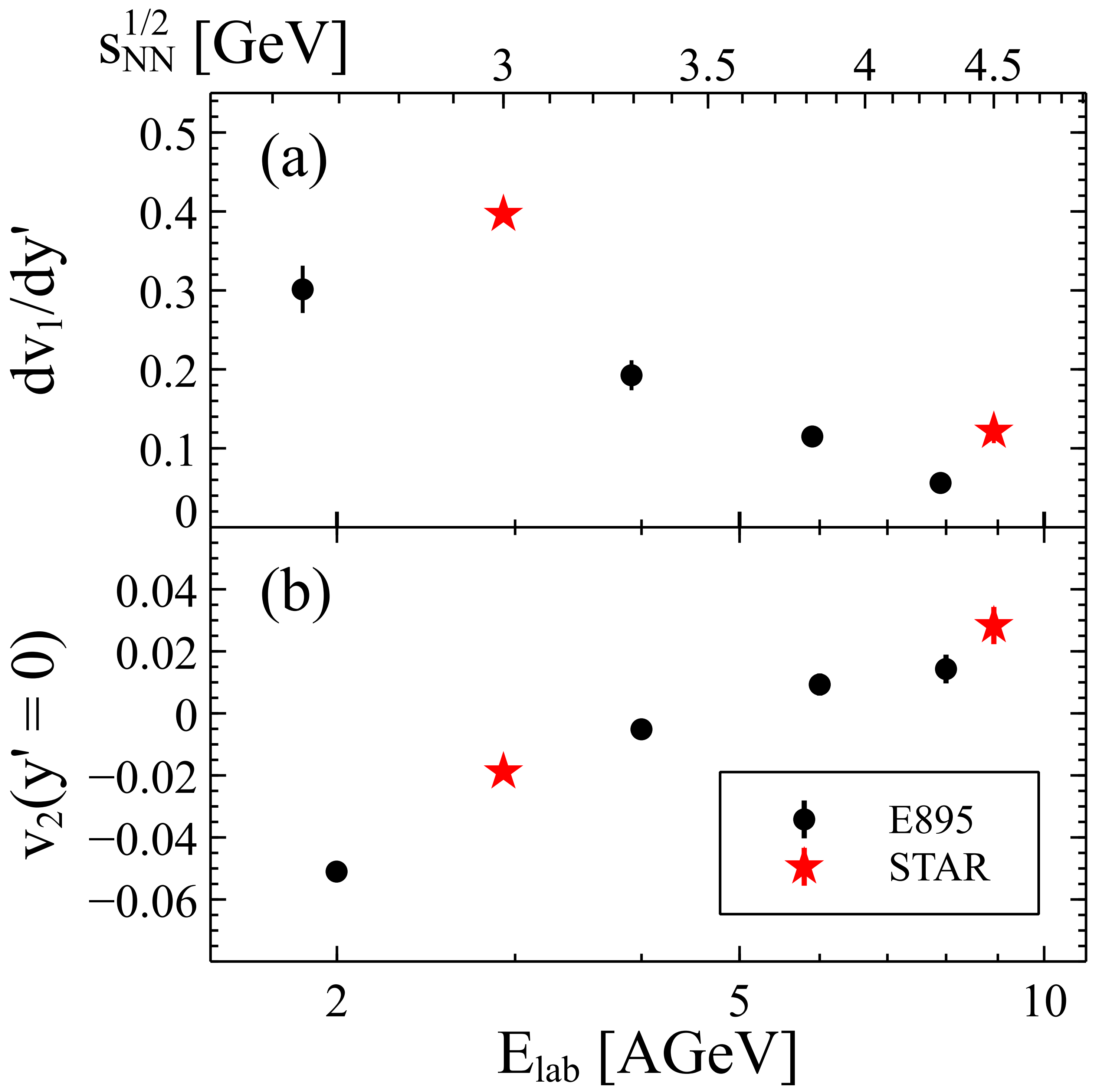


STAR and E895 data cannot be simultaneously described





tension between the data sets

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

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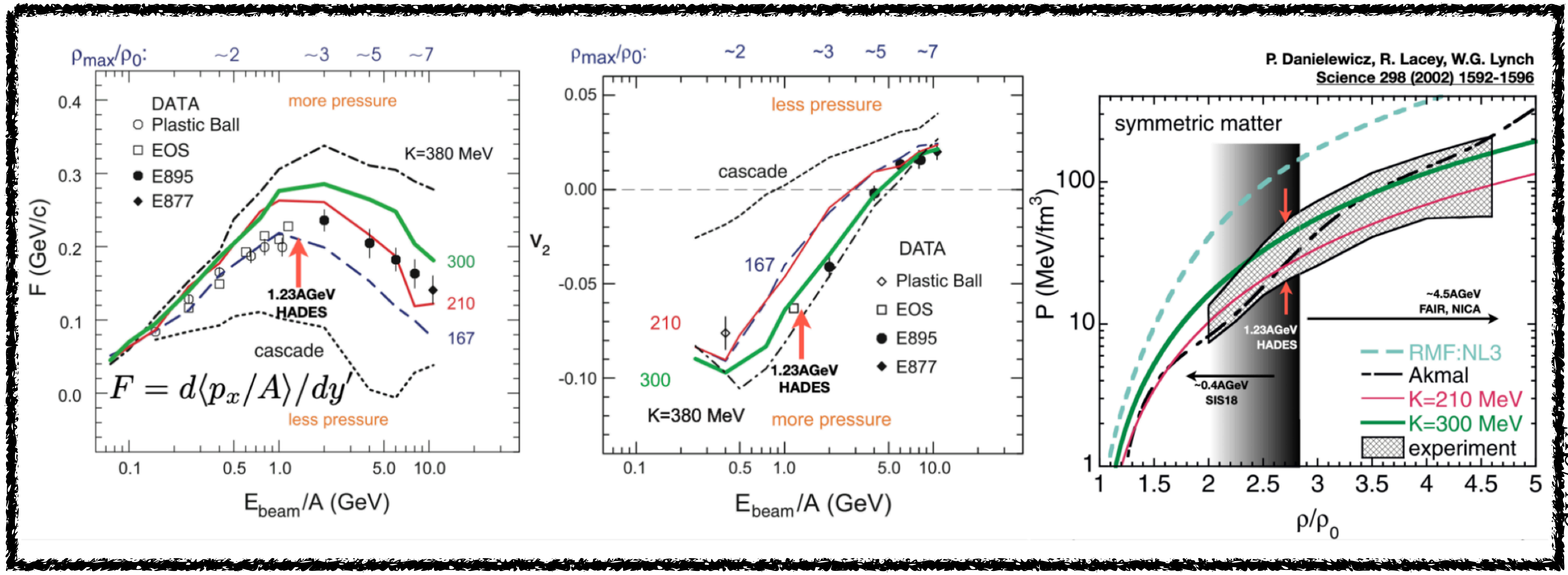
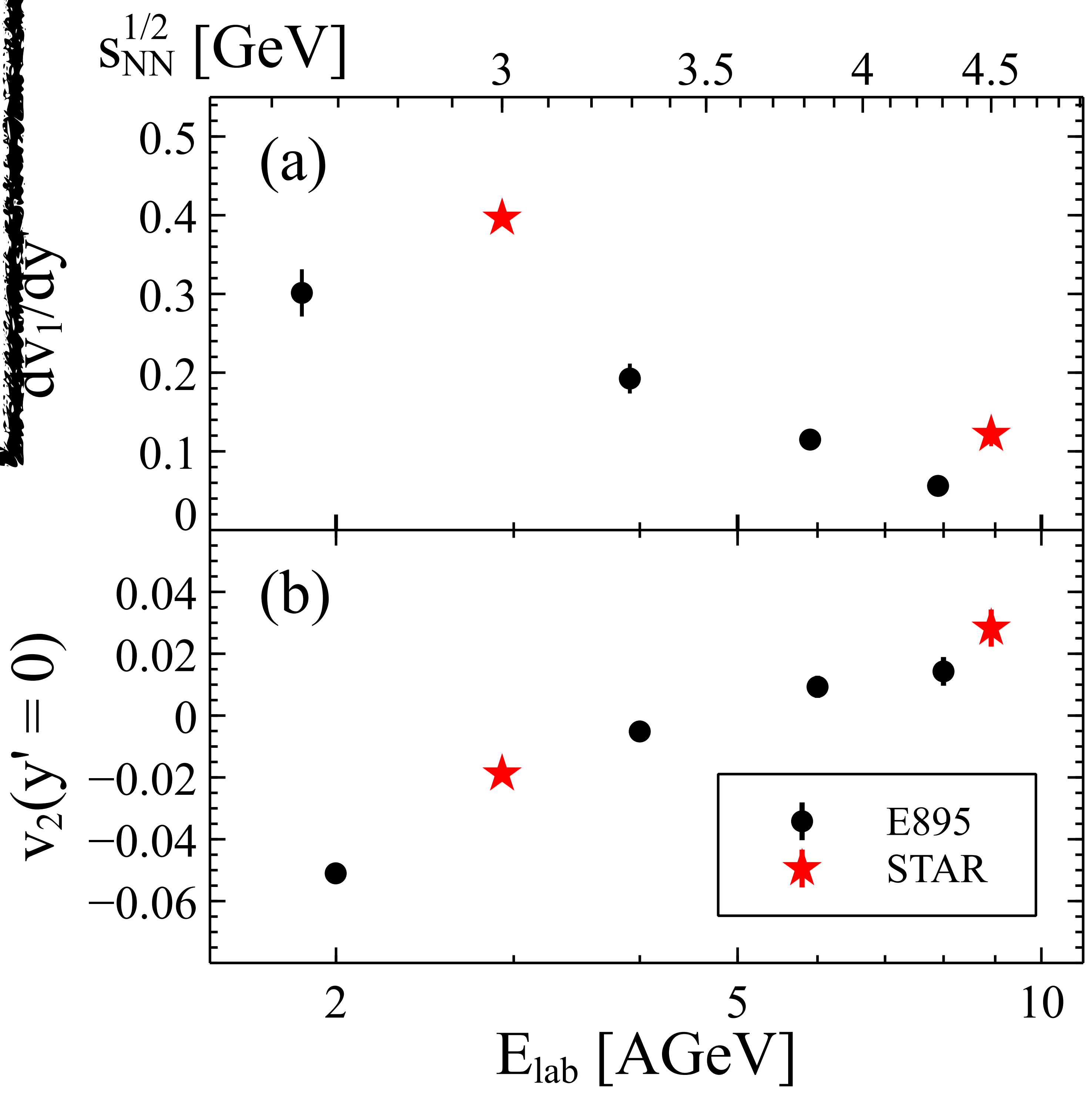
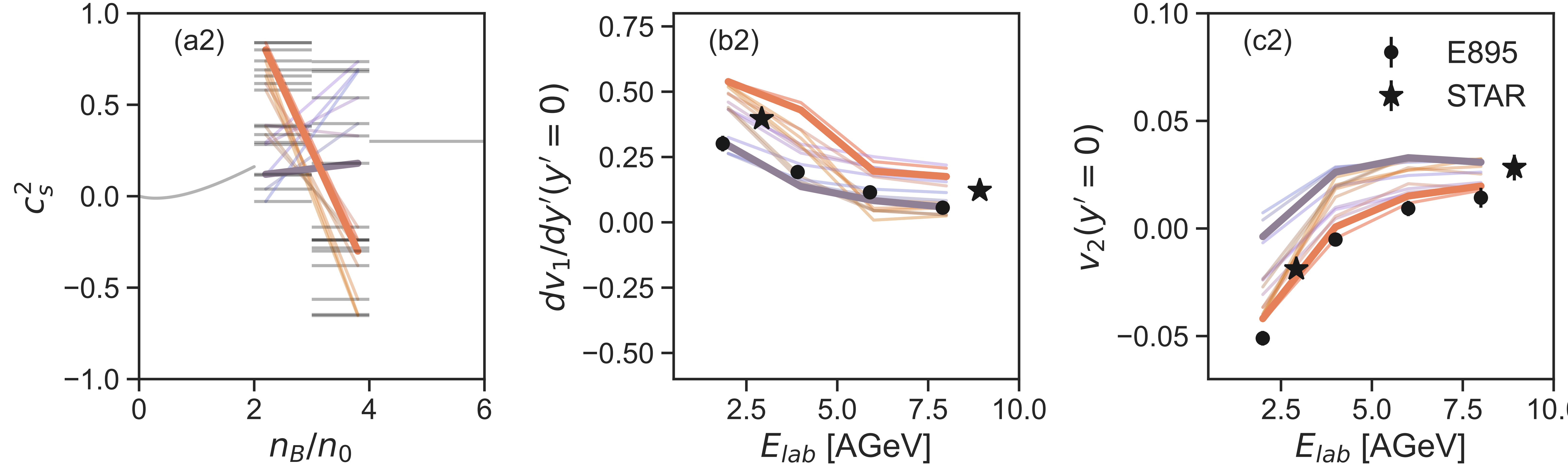


Same problem as in the DLL constraint!

Danielewicz, Lacey, Lynch, Science **298**,1592–1596 (2002)





tension between the data sets

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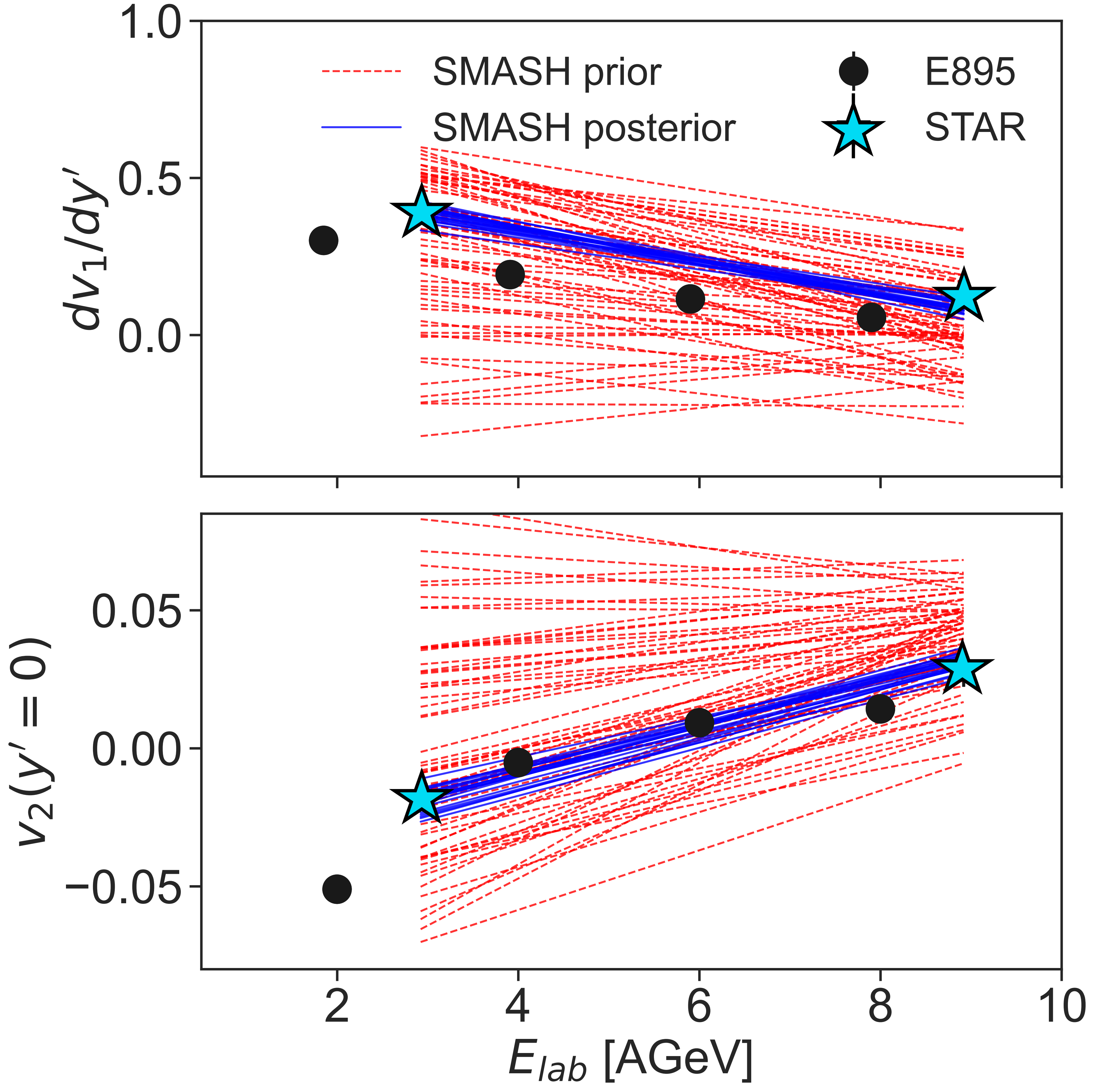
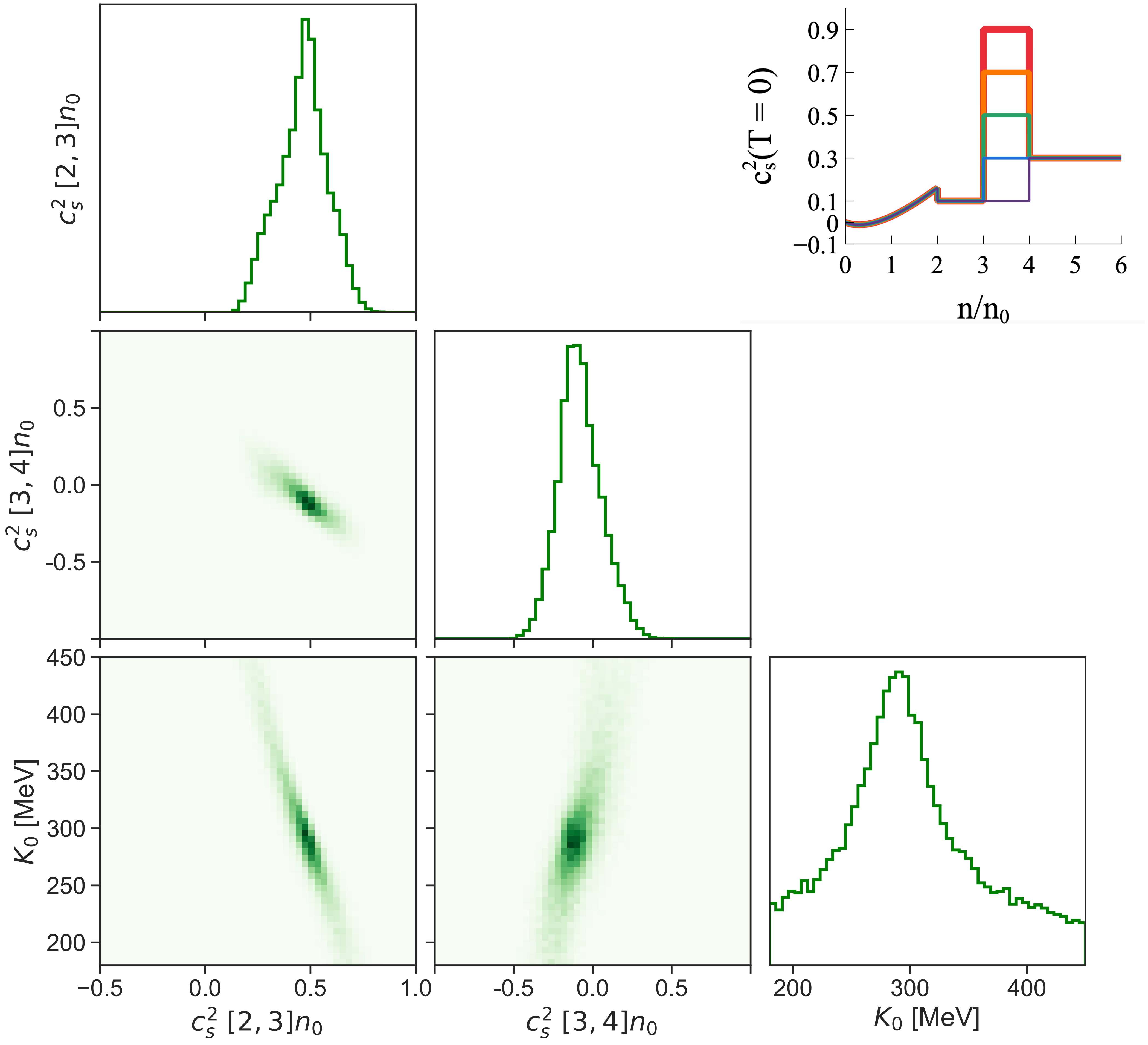
Bayesian analysis of STAR flow data with varying K_0, $c^2_{[2,3]n_0}$, $c^2_{[3,4]n_0}$





The maximum a posteriori probability (MAP) parameters are

$K_0 = 300 \pm 60$MeV, $\quad c^2_{[2,3]n_0} = 0.47 \pm 0.12, \quad c^2_{[3,4]n_0} = -0.08 \pm 0.14$

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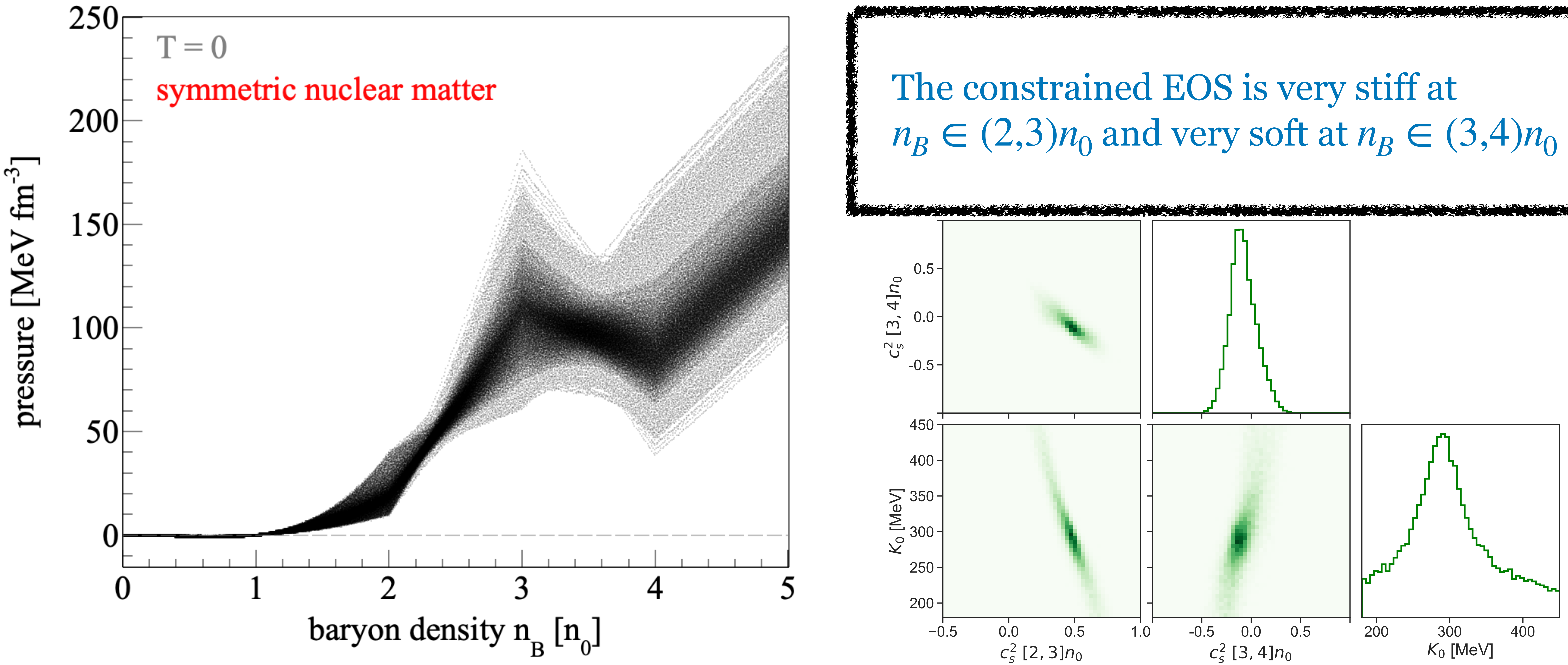

The constrained EOS is very stiff at $n_B \in (2,3)n_0$ and very soft at $n_B \in (3,4)n_0$!

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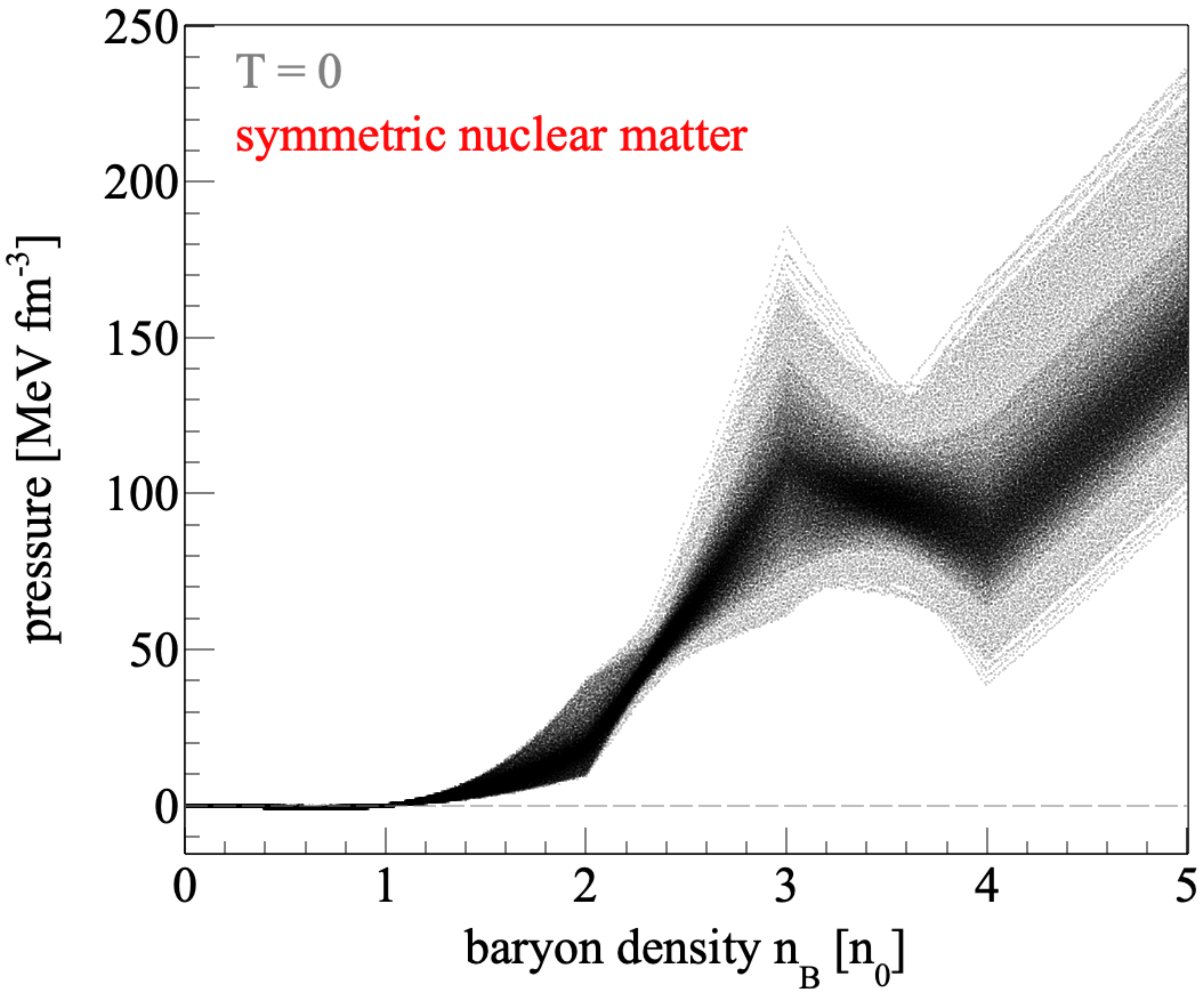

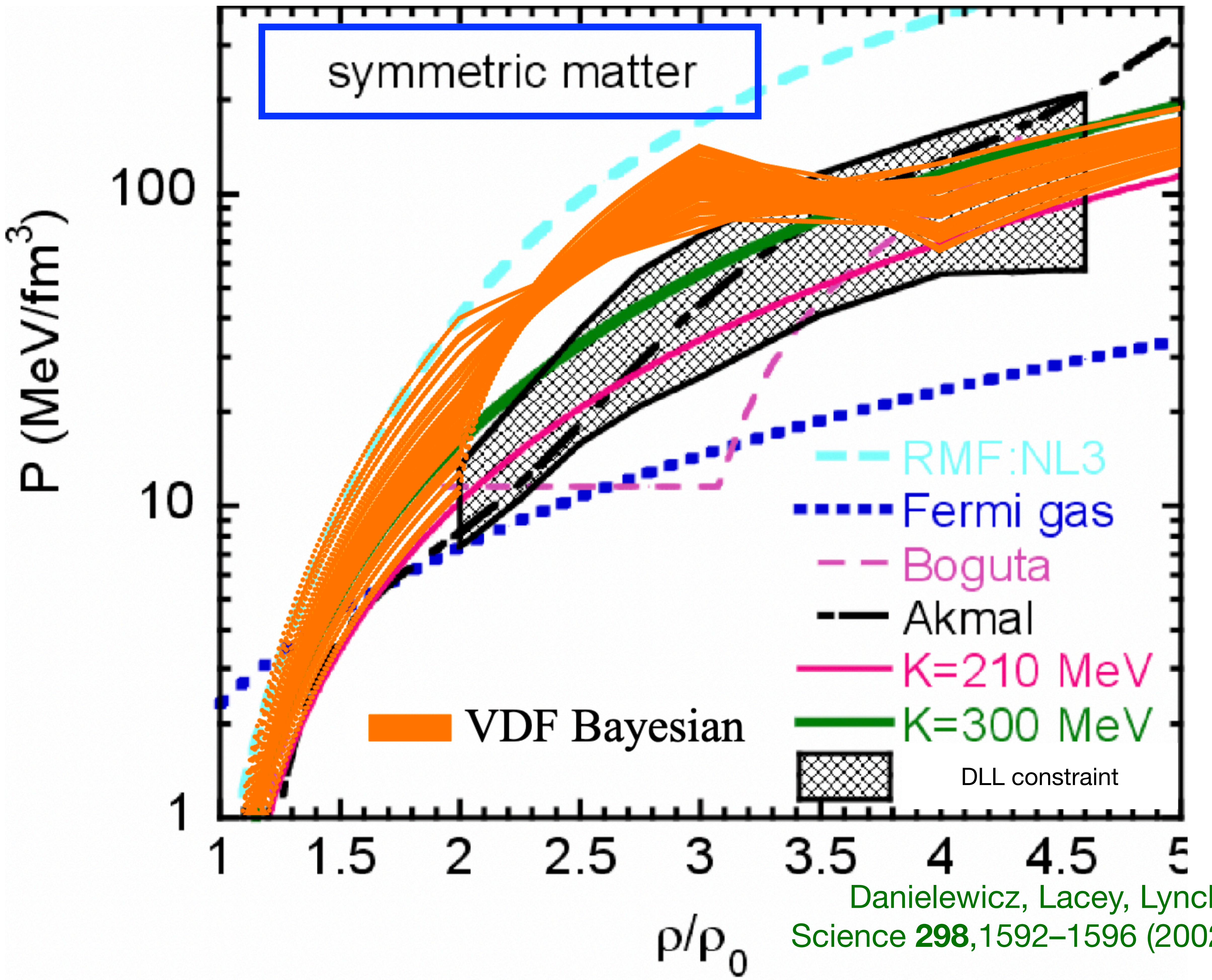

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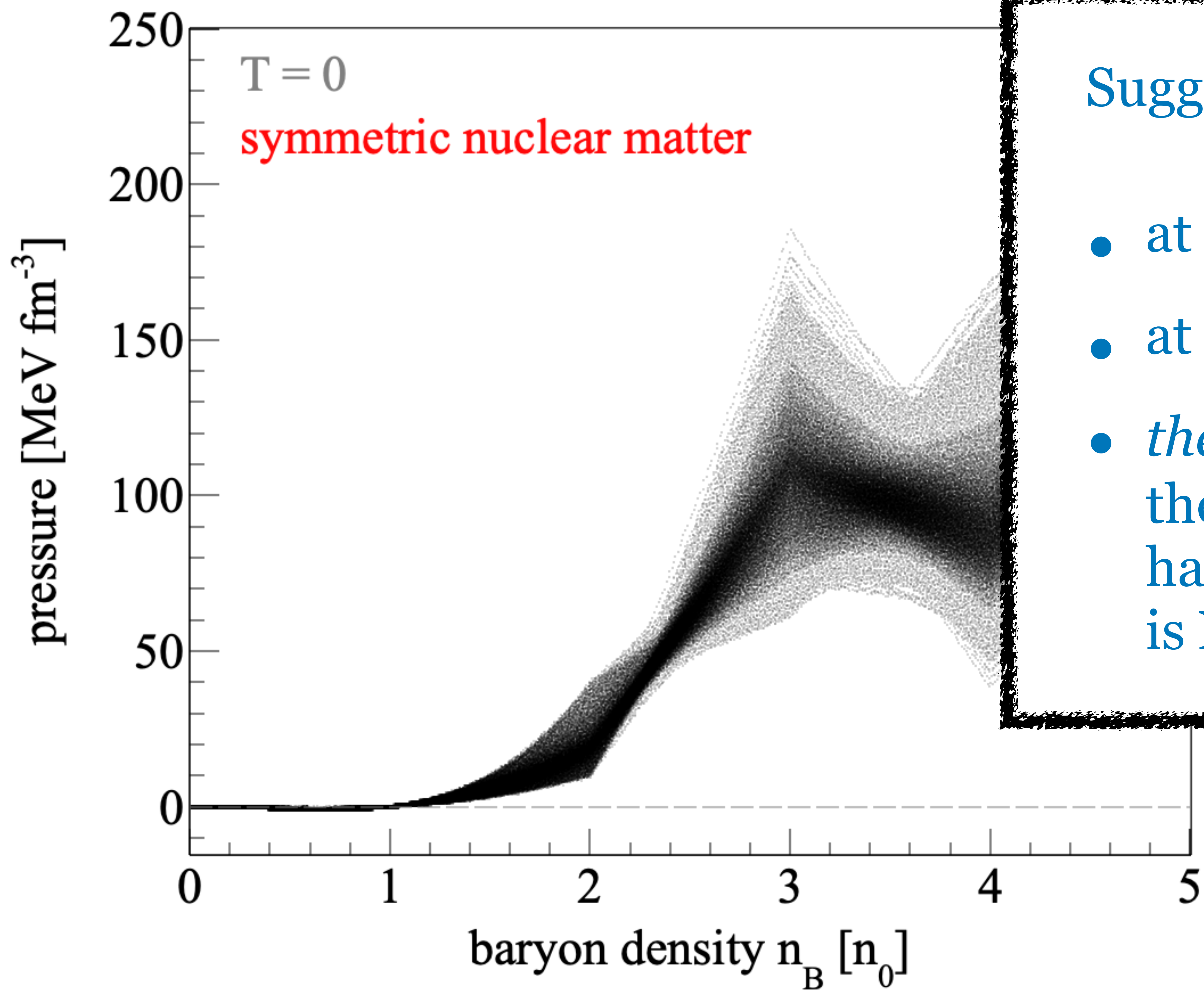
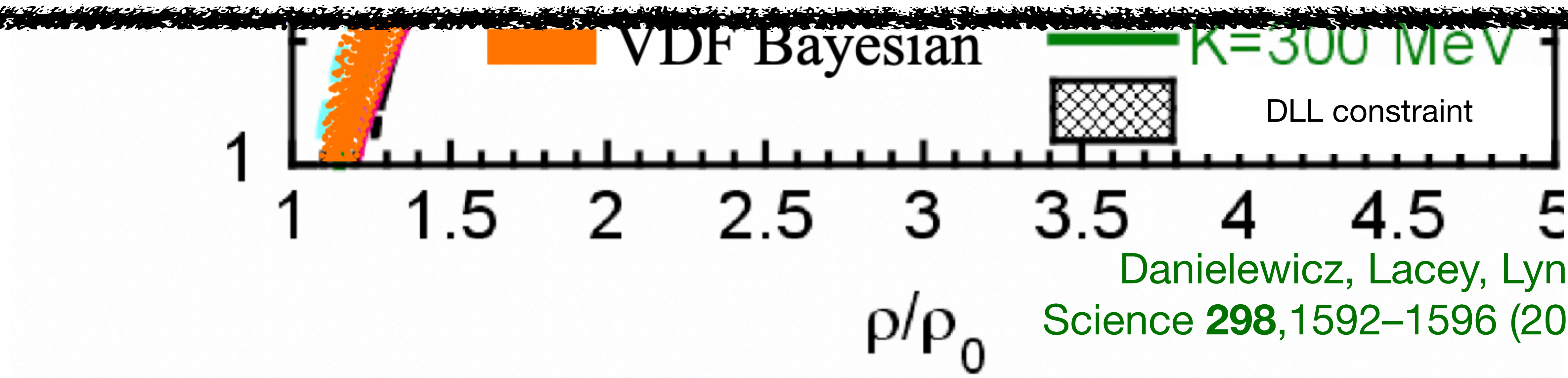


Suggests that:

- at $\sqrt{s_{NN}} = 4.5$ GeV, collisions probe QGP
- at $\sqrt{s_{NN}} = 3.0$ GeV, *some* QGP probed?
- *therefore*, using EOSs parametrized only by K_0 (like the canonical Skyrme soft/hard EOS which doesn't have a QGP-like phase transition) is **NOT ENOUGH** to describe this region



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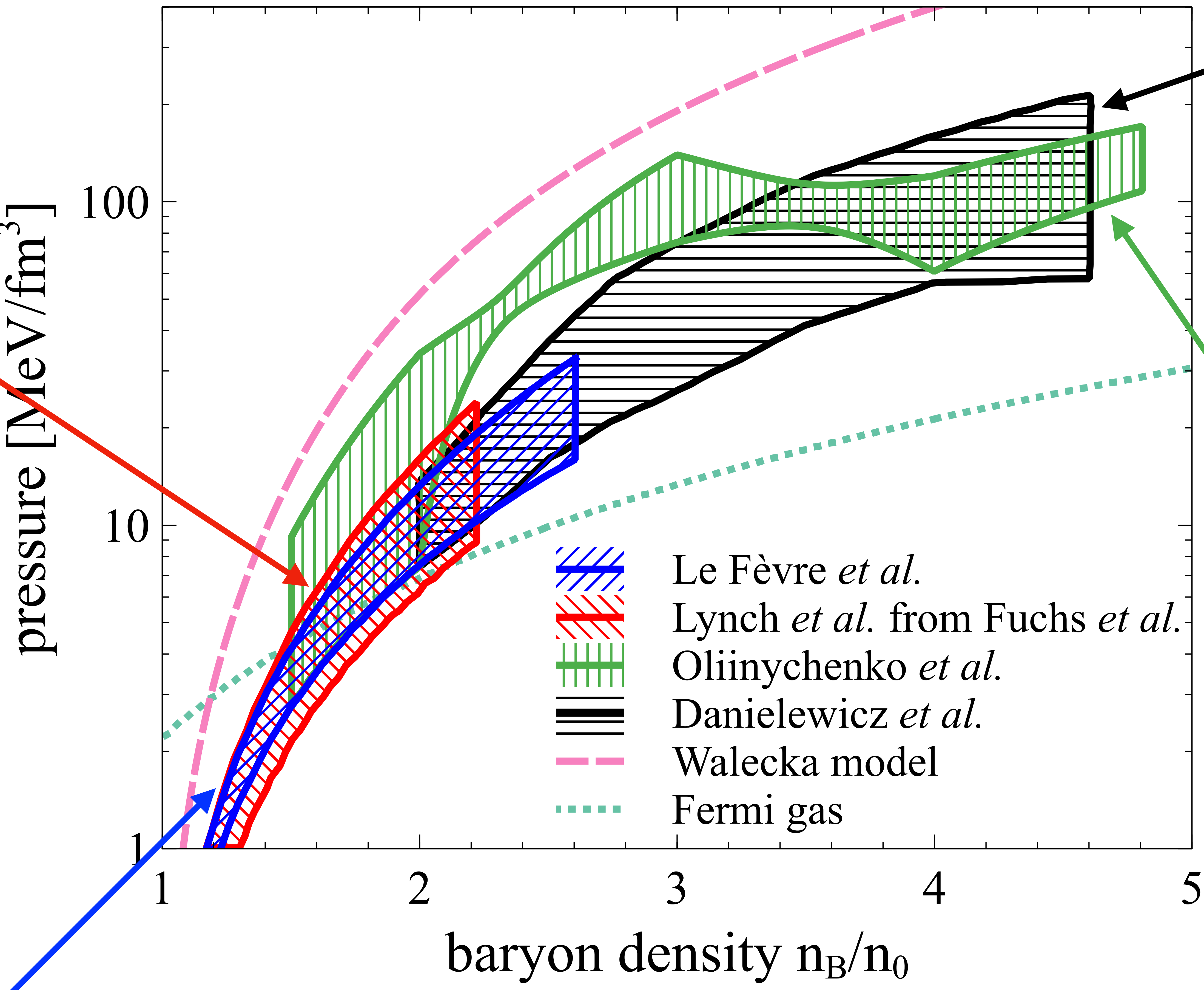
EOS of symmetric nuclear matter: selected results

Symmetric nuclear matter

197Au+197Au & 12C+12C @ < 1.5 GeV/u
($\sqrt{s_{NN}}$ < 2.5 GeV)
observables: subthreshold kaon production (KaoS)
model used: QMD w/ nucleons, Δ, N*(1440), pions, kaons;
EOS parametrized by K_0;
kaon potentials, momentum dependence
C. Fuchs *et al.*, Prog. Part. Nucl. Phys. **53**, 113–124 (2004) arXiv:nucl-th/0312052

197Au+197Au @ 0.4—1.5 GeV/u
($\sqrt{s_{NN}}$ = 2.07 — 2.52 GeV)
observables: proton flow (FOPI)
model used: isospin QMD (IQMD) w/ nucleons, Δ, N*(1440), deuterons, tritons;
EOS parametrized by K_0;
momentum dependence
A. Le Fèvre, Y. Leifels, W. Reisdorf, J. Aichelin, C. Hartnack, Nucl. Phys. A 945, 112 (2016), arXiv:1501.05246



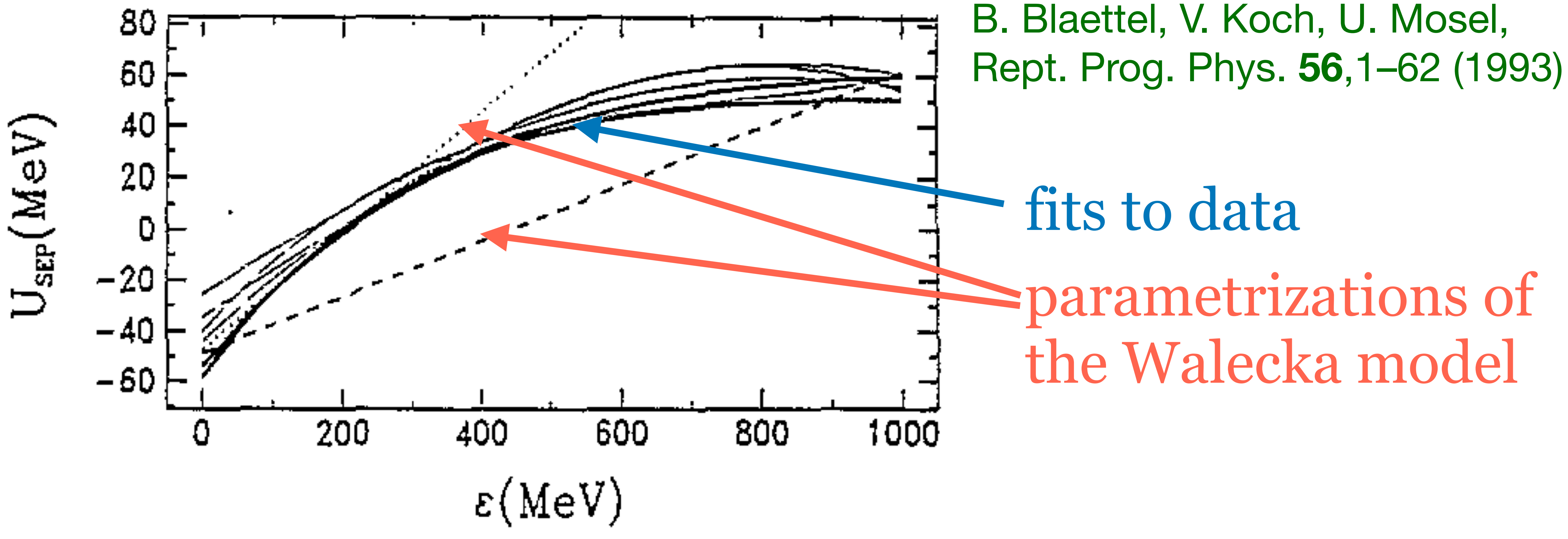
A. Sorensen *et al.*, arXiv:2301.13253

197Au+197Au @ 0.15—10 GeV/u
($\sqrt{s_{NN}}$ = 1.95 — 4.72 GeV)
observables: proton flow (Plastic Ball, EOS, E877, E895)
model used: pBUU w/ nucleons, Δ, N*(1440), pions;
EOS parametrized by K_0;
momentum dependence
Danielewicz, Lacey, Lynch, Science **298**,1592–1596 (2002)

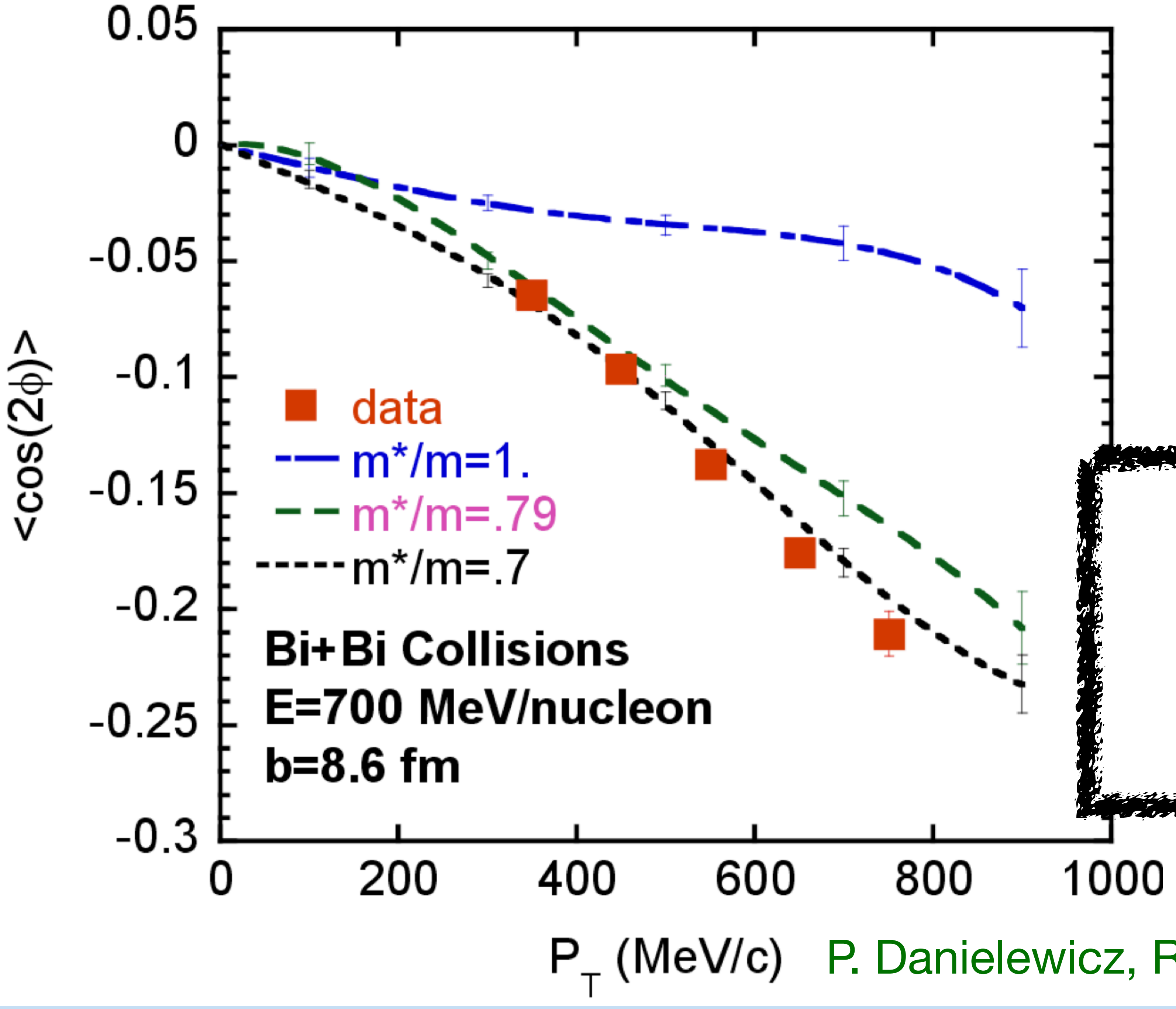
197Au+197Au @ 2.9—9 GeV/u
($\sqrt{s_{NN}}$ = 3 — 4.5 GeV)
observables: proton flow (STAR)
model used: SMASH w/ over 120 hadronic species, including deuterons;
relativistic EOS parametrized independently in different density regions;
NO momentum dependence
D. Oliinychenko, AS, V. Koch, L. McLerran, arXiv:2208.11996

Momentum-dependent mean-fields are a necessary component

Measured in scattering experiments:



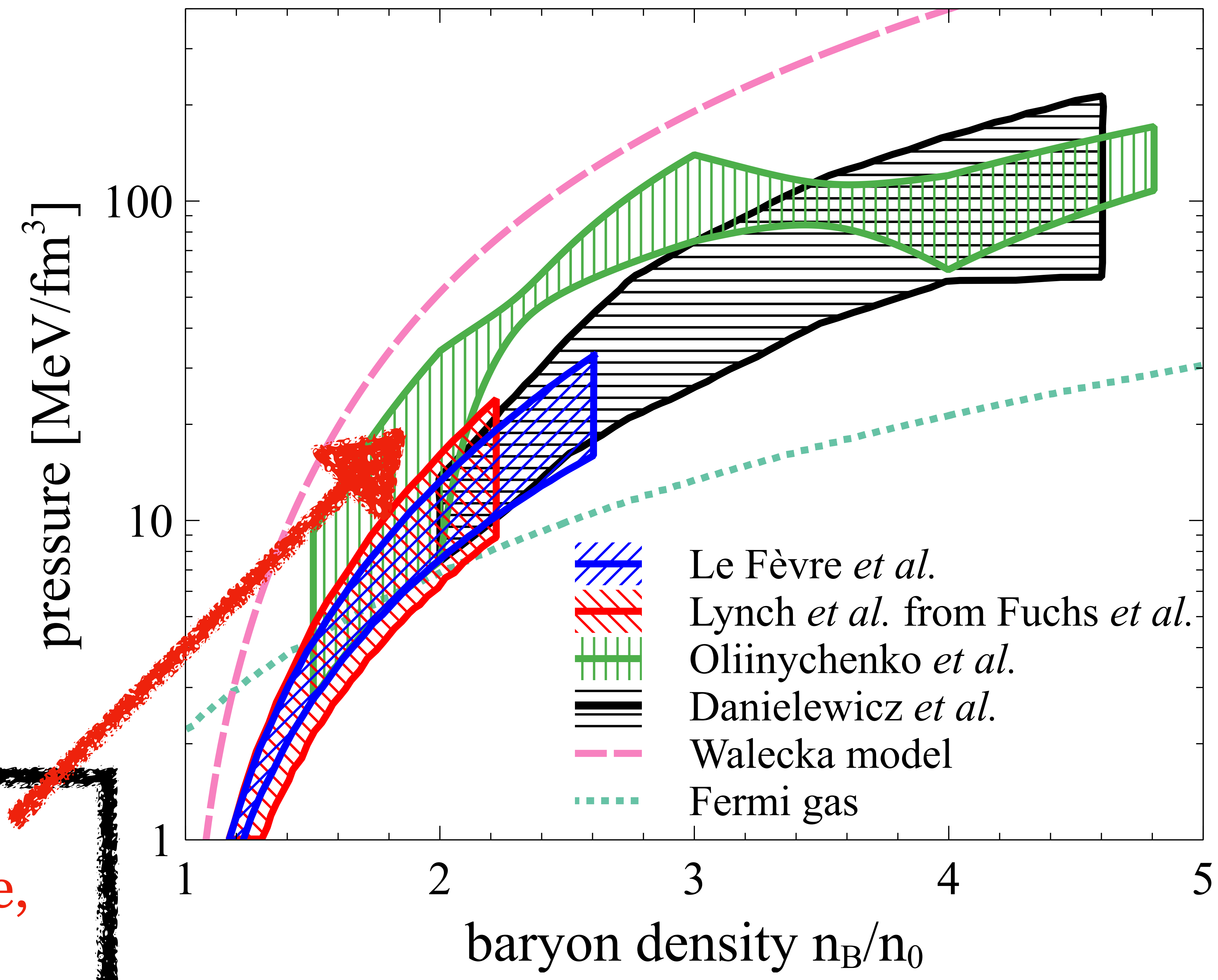
B. Blaettel, V. Koch, U. Mosel, Rept. Prog. Phys. **56**,1–62 (1993)



P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:nucl-th/0208016

Affects the p_T-dependence of the elliptic flow

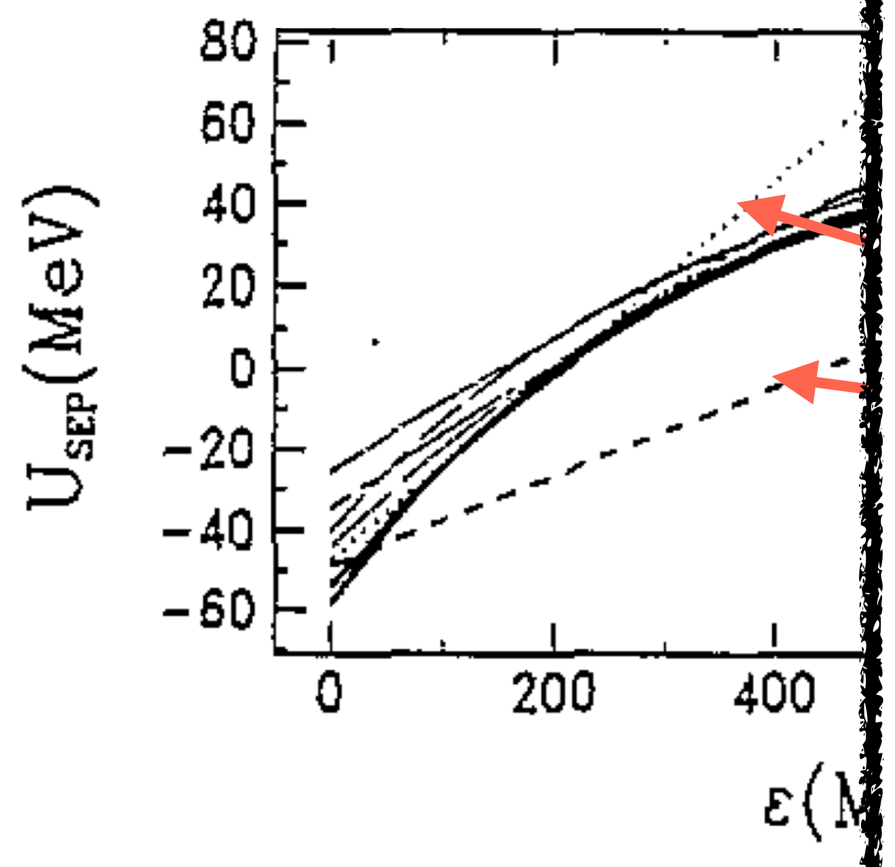
Without momentum dependence, the extracted EOS is too stiff!

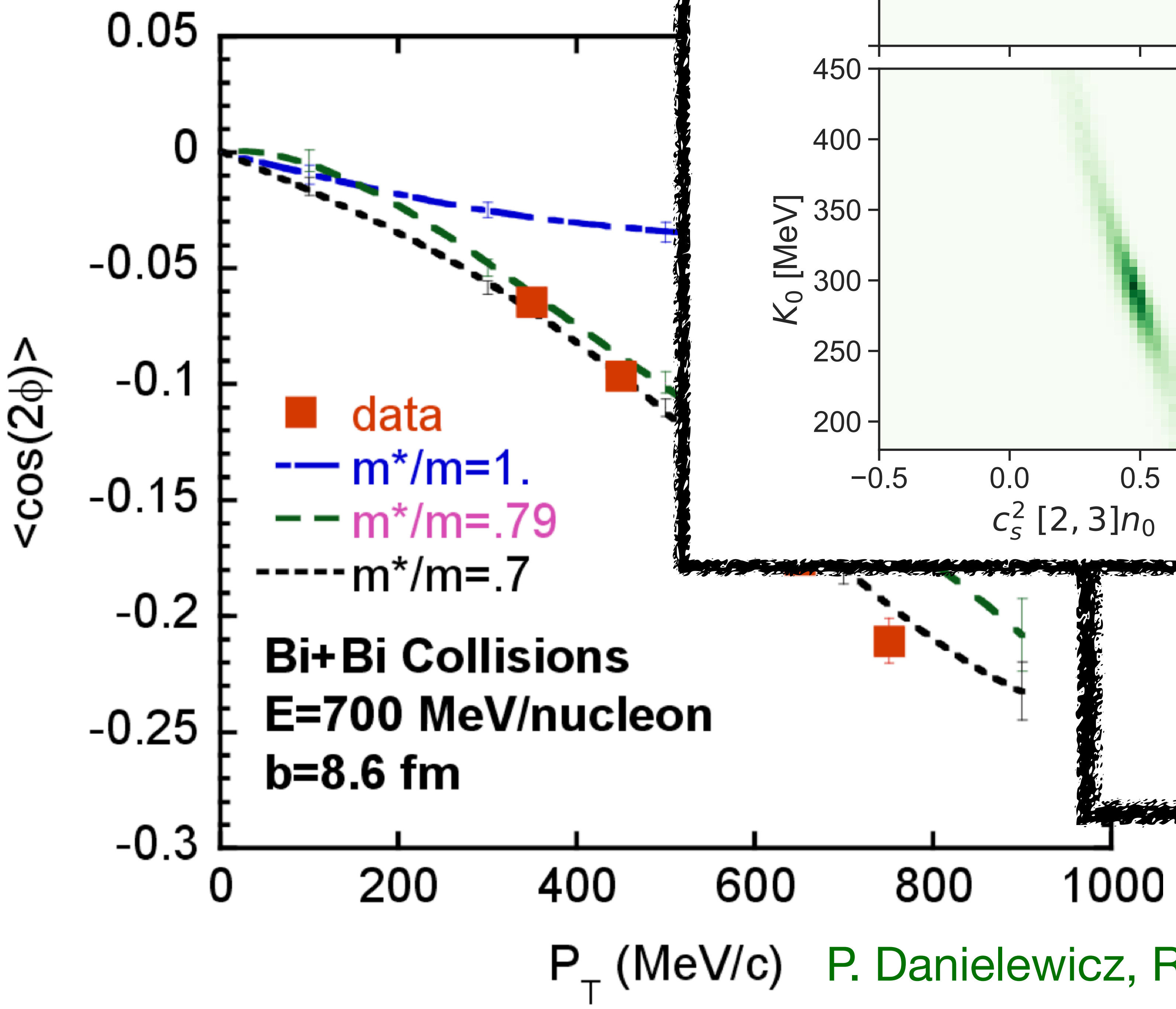


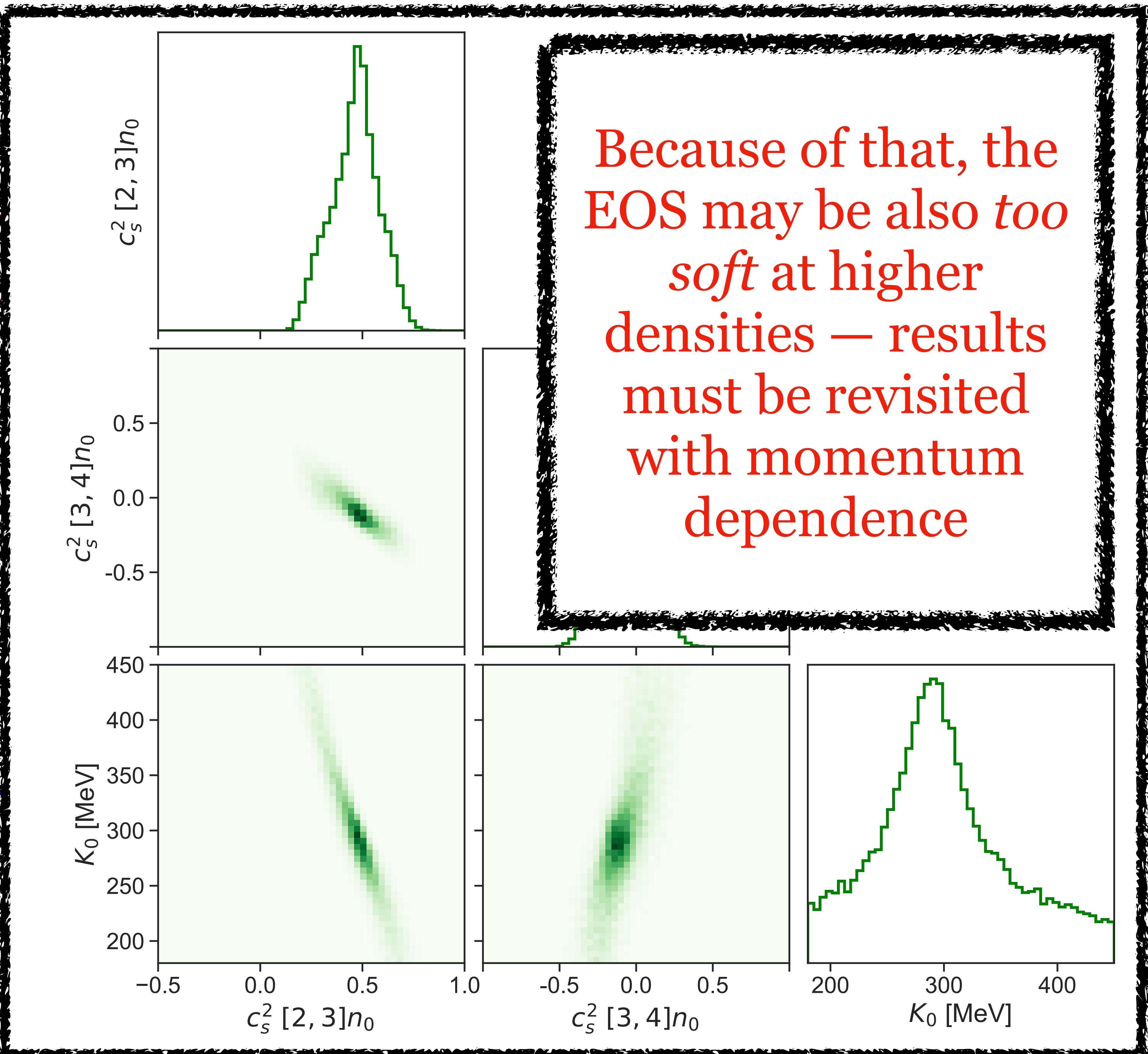
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

Momentum-dependent mean-fields are a necessary component

Measured in scatteri

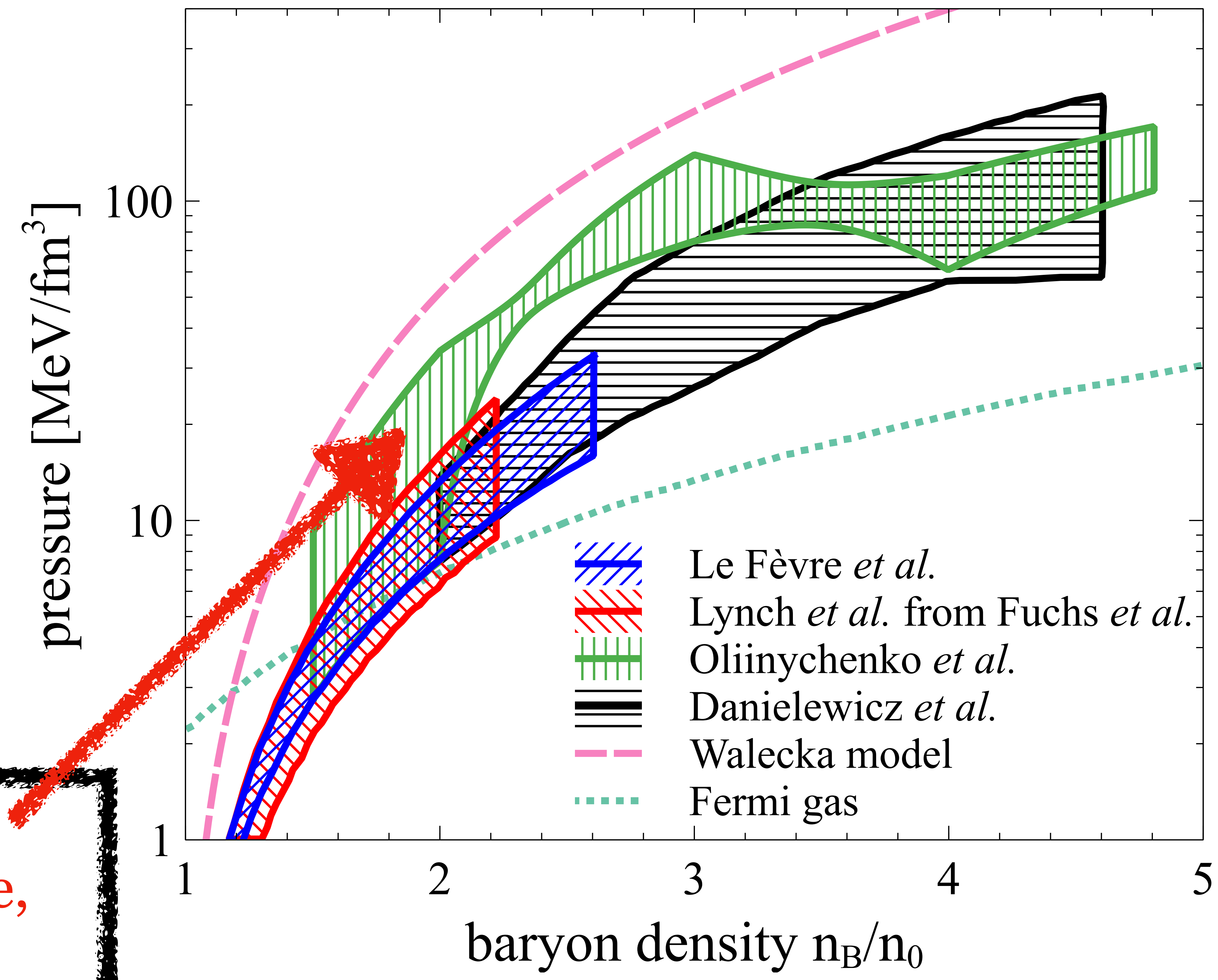






Because of that, the EOS may be also *too soft* at higher densities — results must be revisited with momentum dependence

Without momentum dependence, the extracted EOS is too stiff!

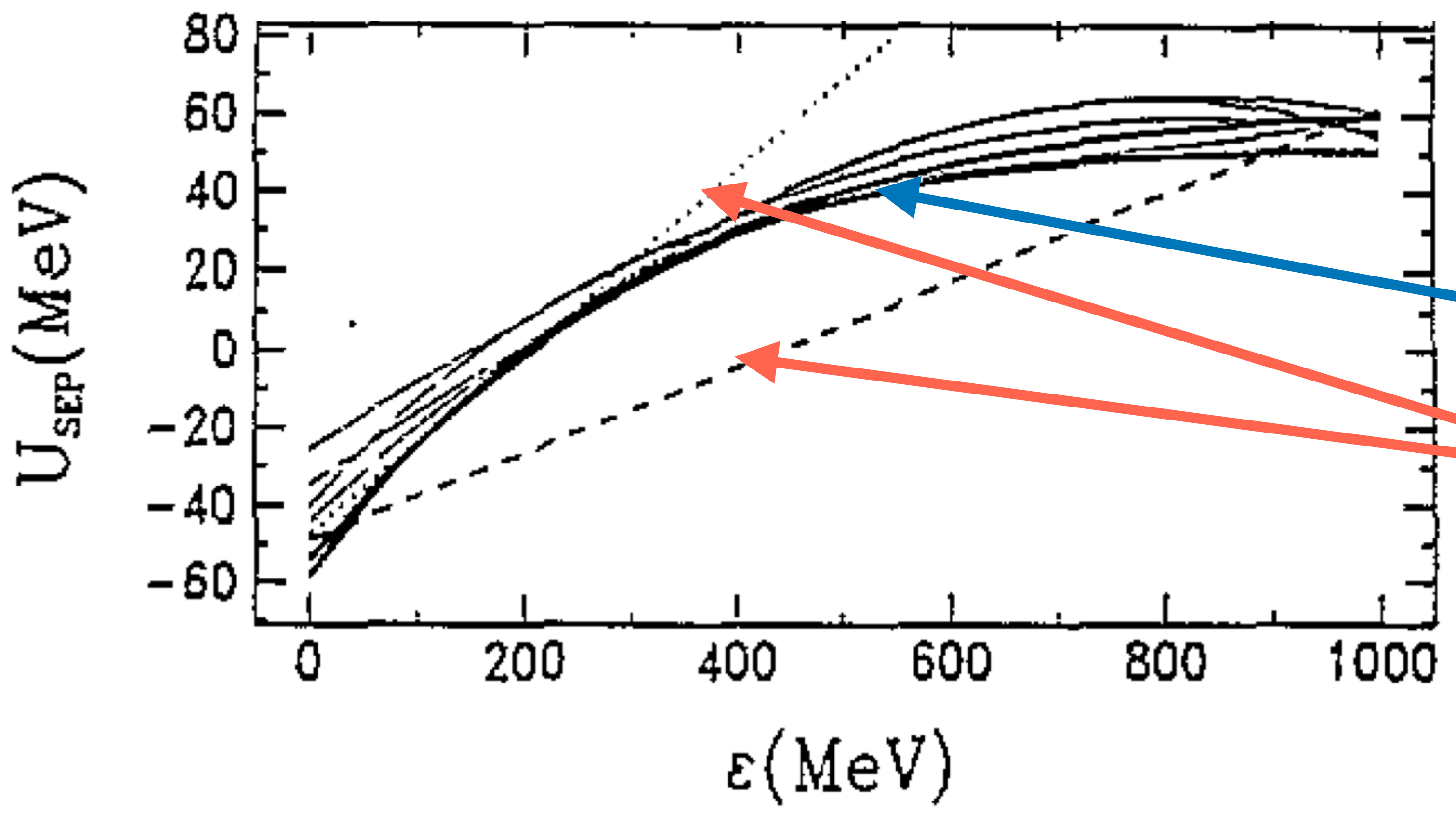


D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

P. Danielewicz, R. Lacey, and W. G. Lynch, Science **298**, 1592 (2002), arXiv:nucl-th/0208016

Work in progress: Flexible momentum-dependent mean-fields

Measured in scattering experiments:



B. Blaettel, V. Koch, U. Mosel, Rept. Prog. Phys. **56**,1–62 (1993)

Solution:
vector+scalar density functional model (VSDF)

Challenge: scalar fields are costly to compute

VDF model:

$$\mathcal{E}_N = g\int \frac{d^3p}{(2\pi)^3}\,\epsilon_{\text{kin}}\, f_{\mathbf{p}} + \sum_{i=1}^{N} A_k^0 j_0 - g^{00}\sum_{i=1}^{N}\left(\frac{b_i-1}{b_i}\right)A_k^\lambda j_\lambda$$

$$A_k^\mu = C_k\big(j_\lambda j^\lambda\big)^{\frac{b_k}{2}-1} j^\mu\ , \qquad j_\mu j^\mu = n_B^2\ , \qquad j^\mu = g\int \frac{d^3p}{(2\pi)^3}\,\frac{p^\mu - A^\mu}{\epsilon^*_{kin}}\, f_{\mathbf{p}}$$

VSDF model:

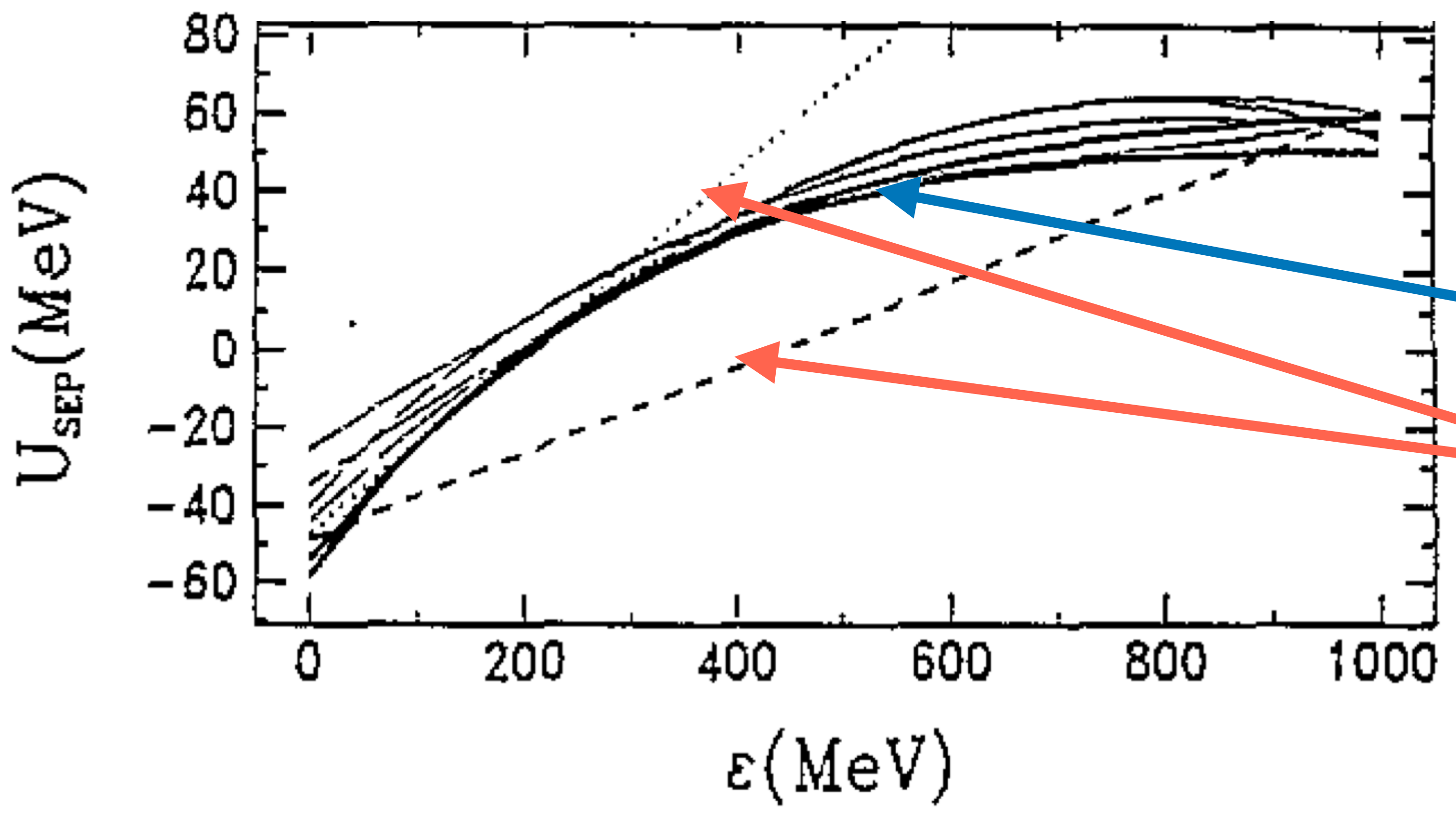
$$\mathcal{E}_{N,\,M} = g\int \frac{d^3p}{(2\pi)^3}\,\epsilon^*_{\text{kin}}\, f_{\mathbf{p}} + \sum_{i=1}^{N} A_k^0 j_0 - g^{00}\sum_{i=1}^{N}\left(\frac{b_i-1}{b_i}\right)A_k^\lambda j_\lambda + g^{00}\sum_{m=1}^{M} G_m\left(\frac{d_m-1}{d_m}\right) n_s^{d_m}$$

$$m^* = m_0 - \sum_{m=1}^{M} G_M n_s^{d_m-1} \qquad n_s = g\int \frac{d^3p}{(2\pi)^3}\,\frac{m^*}{\epsilon^*_{\text{kin}}}\, f_{\mathbf{p}}$$

A. Sorensen, "Density Functional Equation of State and Its Application to the Phenomenology of Heavy-Ion Collisions," arXiv:2109.08105, Sorensen:2021zxd

Work in progress: Flexible momentum-dependent mean-fields

Measured in scattering experiments:



B. Blaettel, V. Koch, U. Mosel, Rept. Prog. Phys. **56**,1–62 (1993)

Solution:
vector+scalar density functional model (VSDF)

Challenge: scalar fields are costly to compute

VDF mod

Justin Mohs is currently implementing standard momentum dependence in SMASH:

$$U_{\mathbf{p}} = C \int \frac{d^3p'}{(2\pi)^3} \frac{f(\mathbf{r}, \mathbf{p}')}{1 + \left(\frac{\mathbf{p} - \mathbf{p}'}{\Lambda}\right)^2}$$

$\frac{p^\mu - A^\mu}{\epsilon^*_{kin}} f_{\mathbf{p}}$

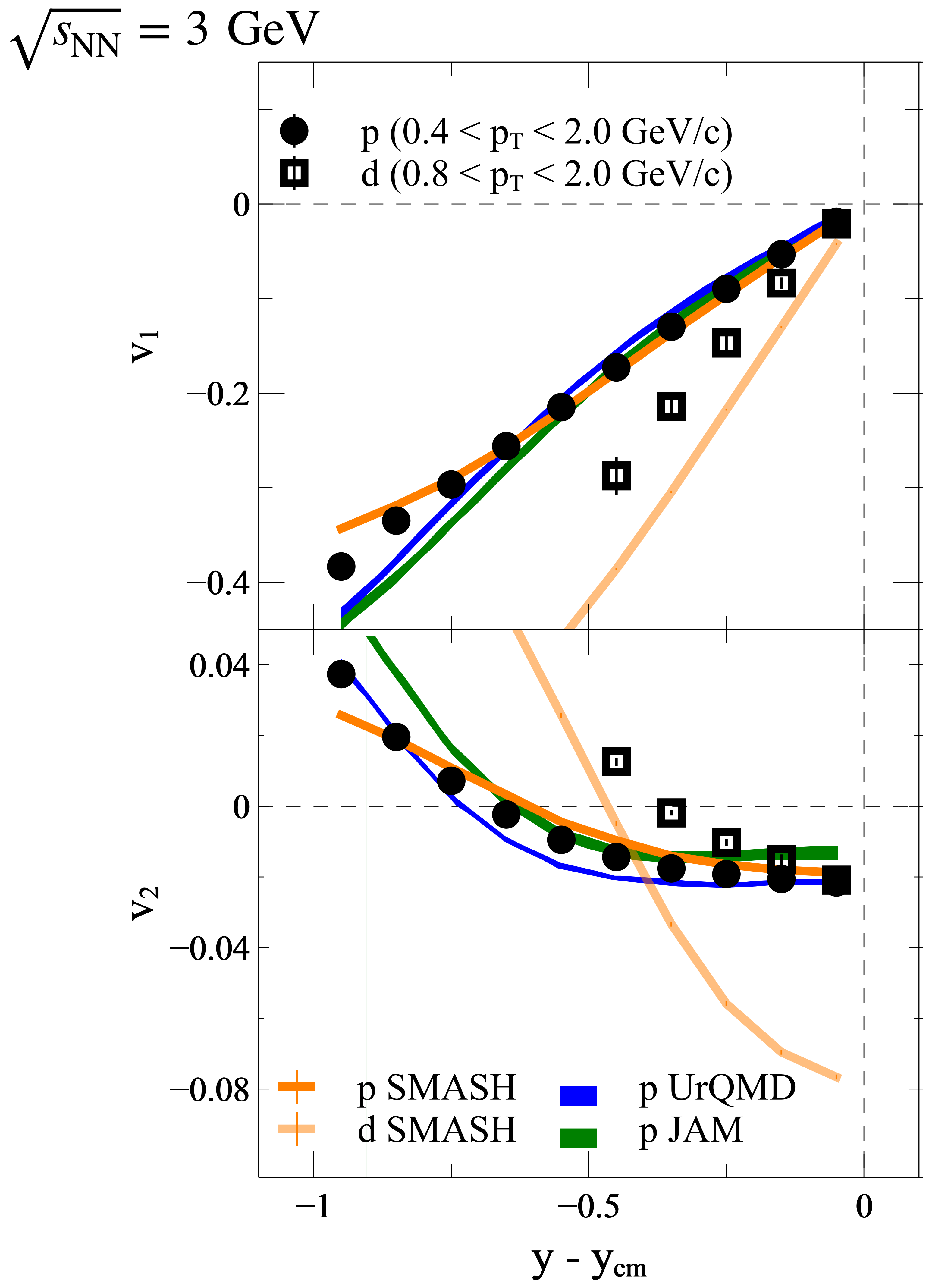
VSDF mo

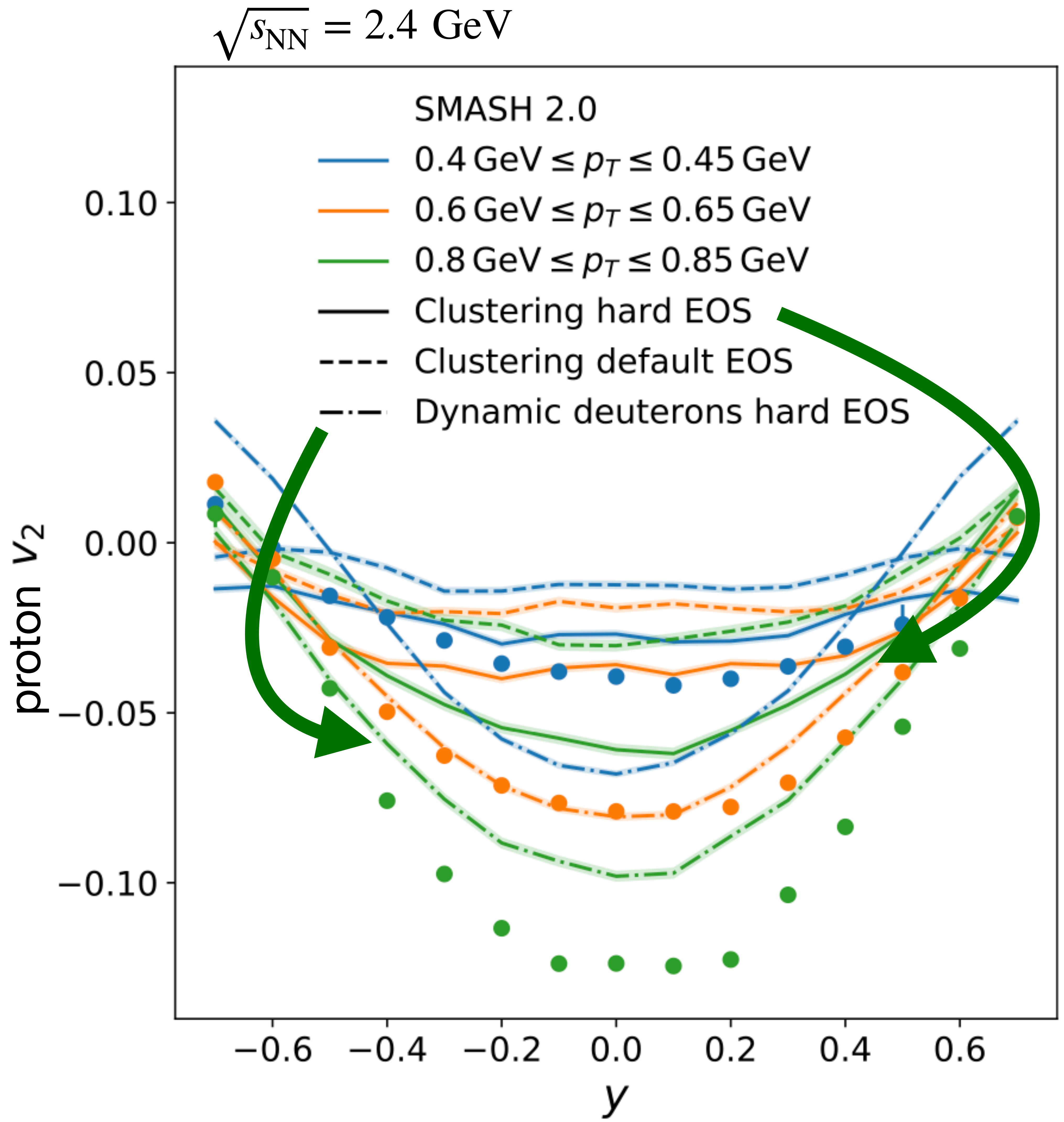
$i=1$ $i=1$ $m=1$

$$m^* = m_0 - \sum_{m=1}^{M} G_M n_s^{d_m - 1} \qquad n_s = g \int \frac{d^3p}{(2\pi)^3} \frac{m^*}{\epsilon^*_{\text{kin}}} f_{\mathbf{p}}$$

A. Sorensen, "Density Functional Equation of State and Its Application to the Phenomenology of Heavy-Ion Collisions," arXiv:2109.08105, Sorensen:2021zxd

Describing proton flow is not enough





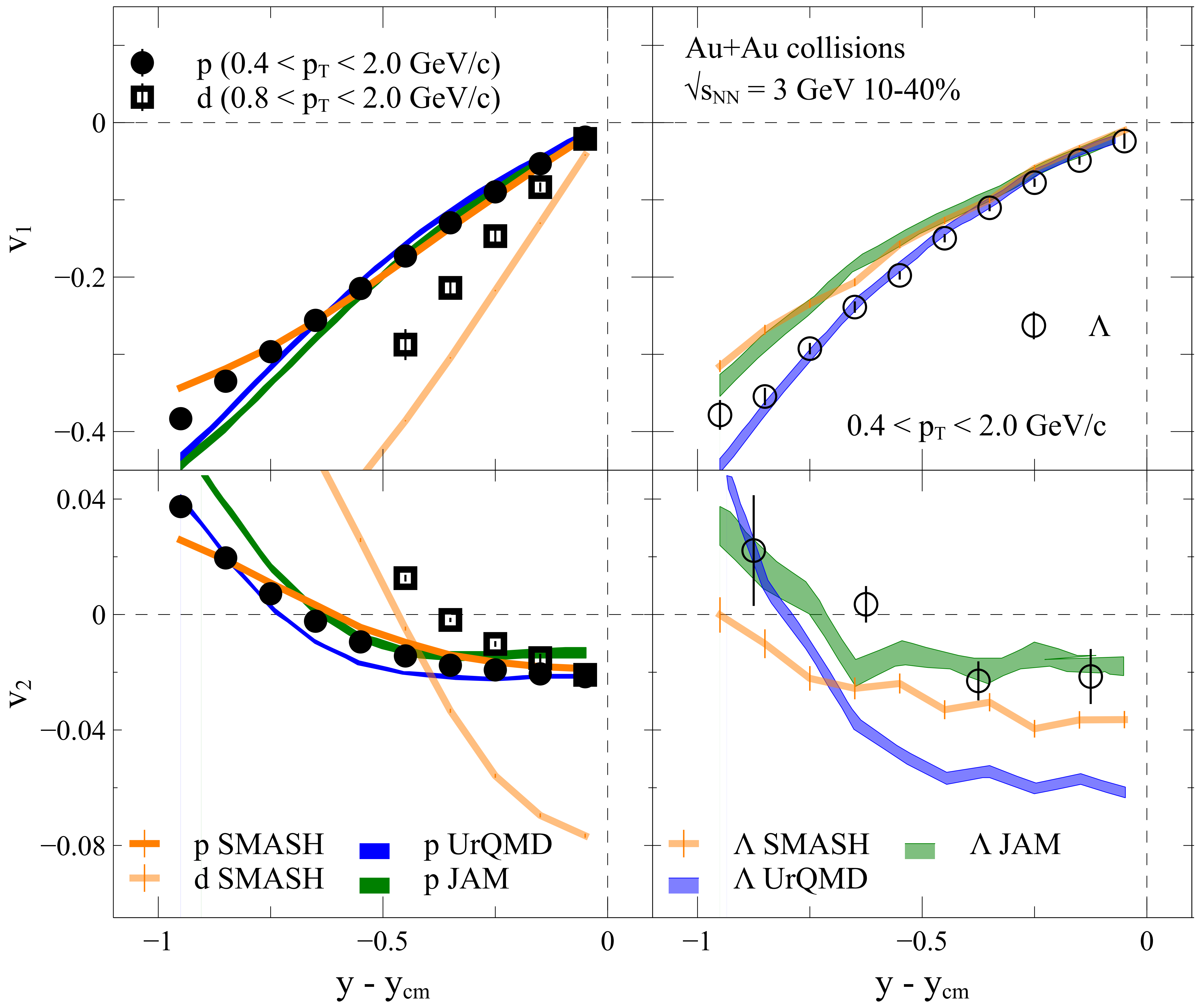
Realistic description of light cluster production needed:

- coalescence: doesn't take into account the dynamic role of light clusters throughout the evolution
- nucleon/pion catalysis: consider as separate degrees of freedom (pBUU, SMASH), produced through N or π collisions
- the Holy Grail: dynamical production through potentials

STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
A. Sorensen *et al.*, arXiv:2301.13253

J. Mohs, M. Ege, H. Elfner, M. Mayer, Phys. Rev. C **105** 3, 034906 (2022), arXiv:2012.11454

Describing proton flow is not enough



Strange baryons are not well described
- the results may depend on:

- nucleon-hyperon and hyperon-hyperon interactions
- in-medium modifications of interactions

Models of interactions exists and could be tested; interactions could be based on those obtained within first-principle calculations (e.g., HALQCD collaboration HAL QCD, Nucl. Phys. A **998** 121737 (2020), arXiv:1912.08630)

STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
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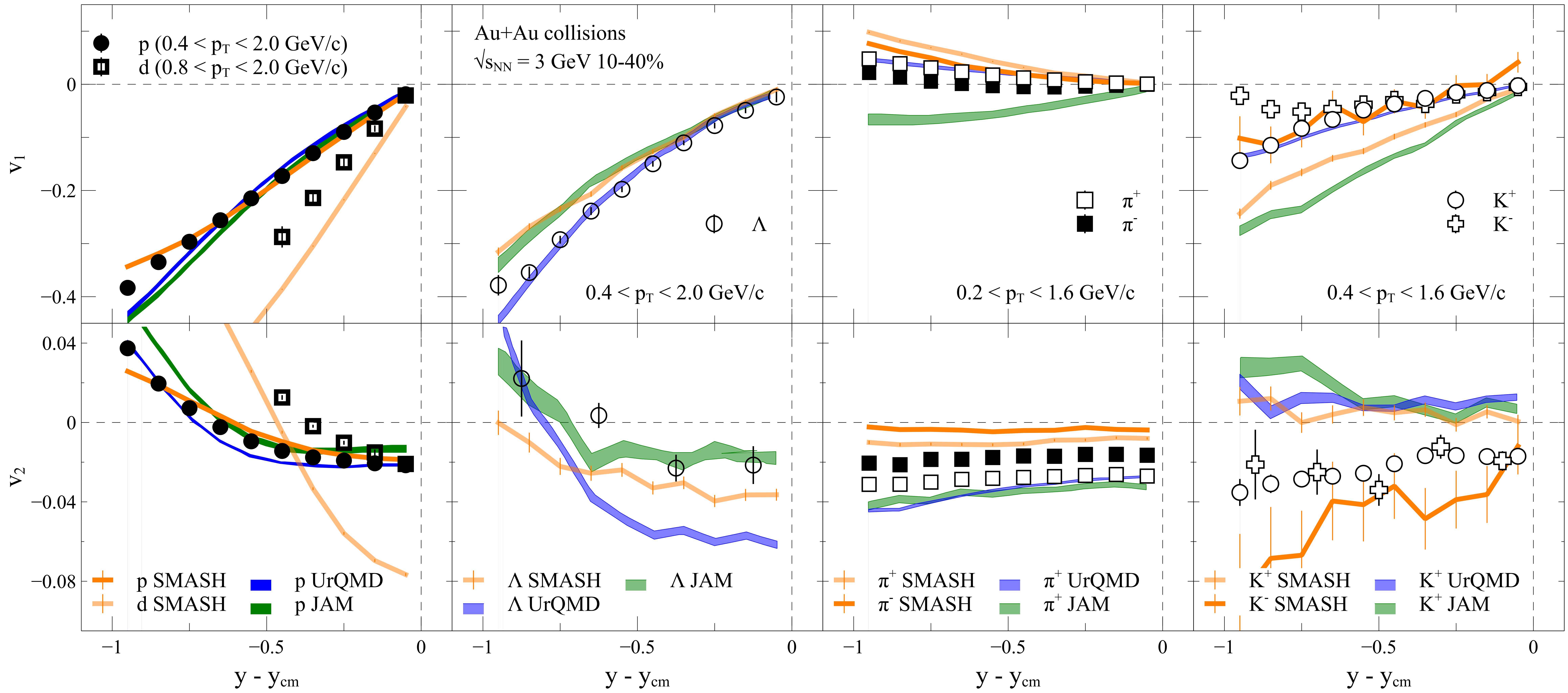


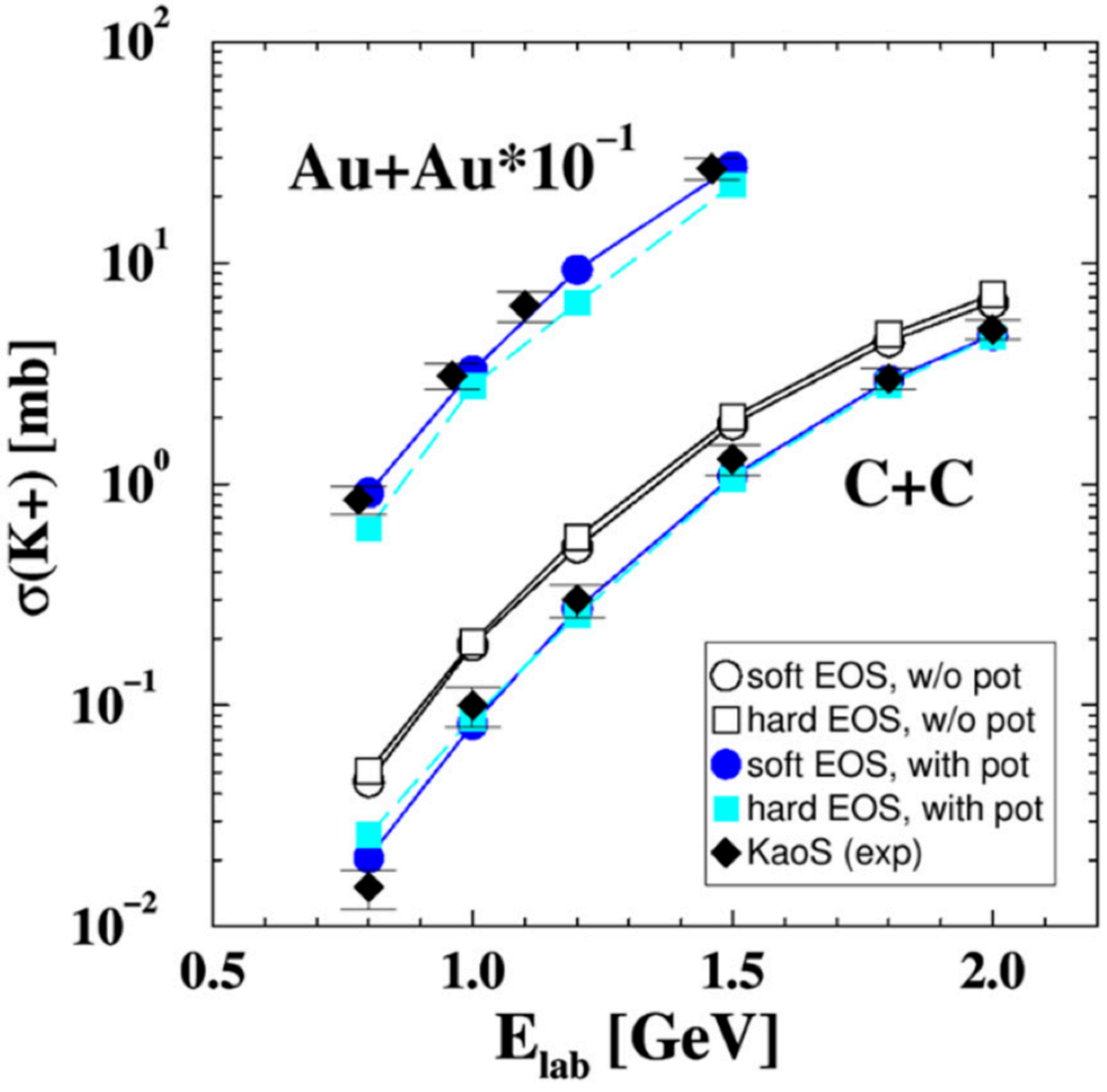
STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908

D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996

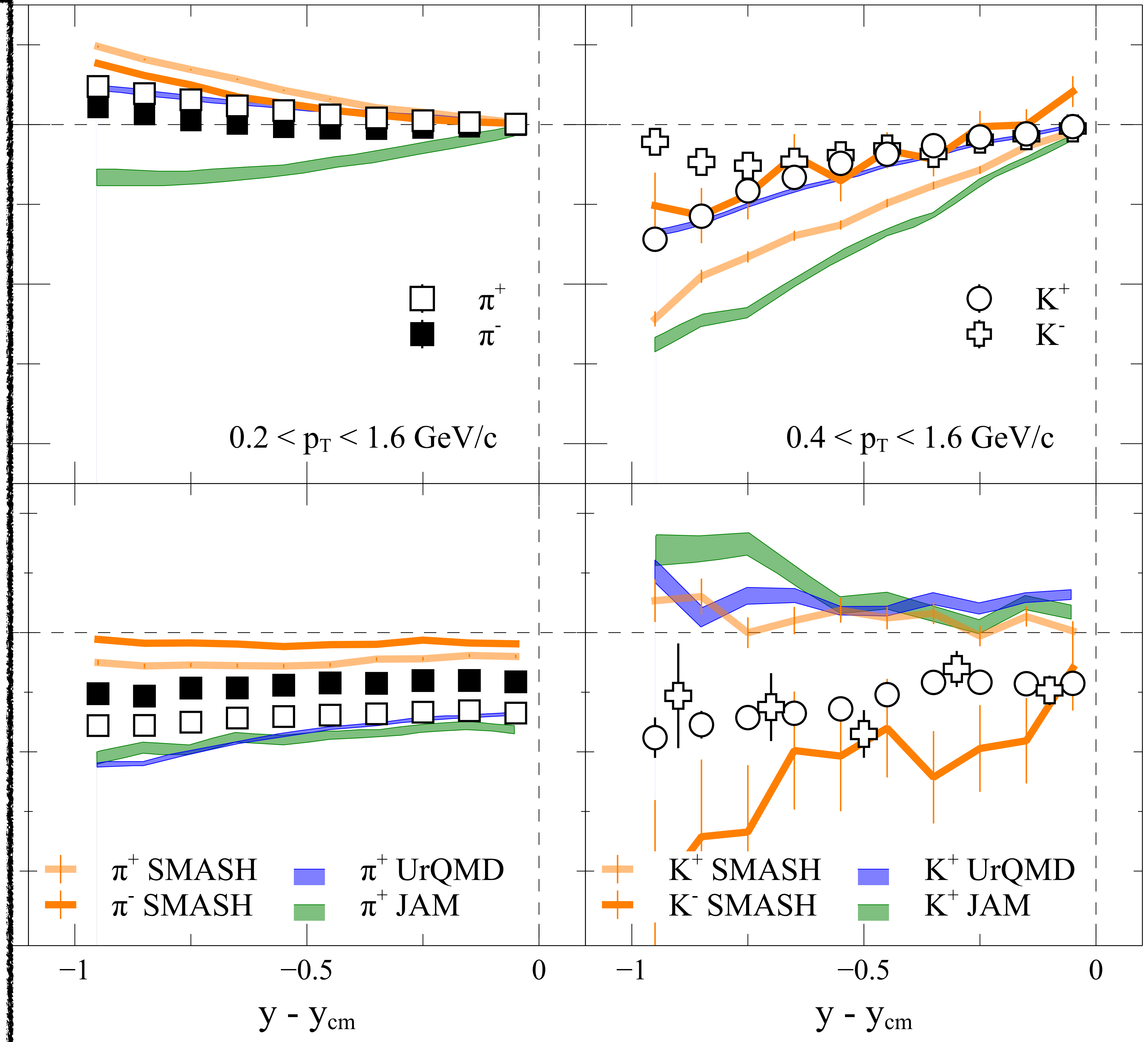
A. Sorensen *et al.*, arXiv:2301.13253

Describing proton flow is not enough

Pions and kaons NOT described!
Not very surprising: UrQMD, JAM, and SMASH don't have mean-fields for mesons

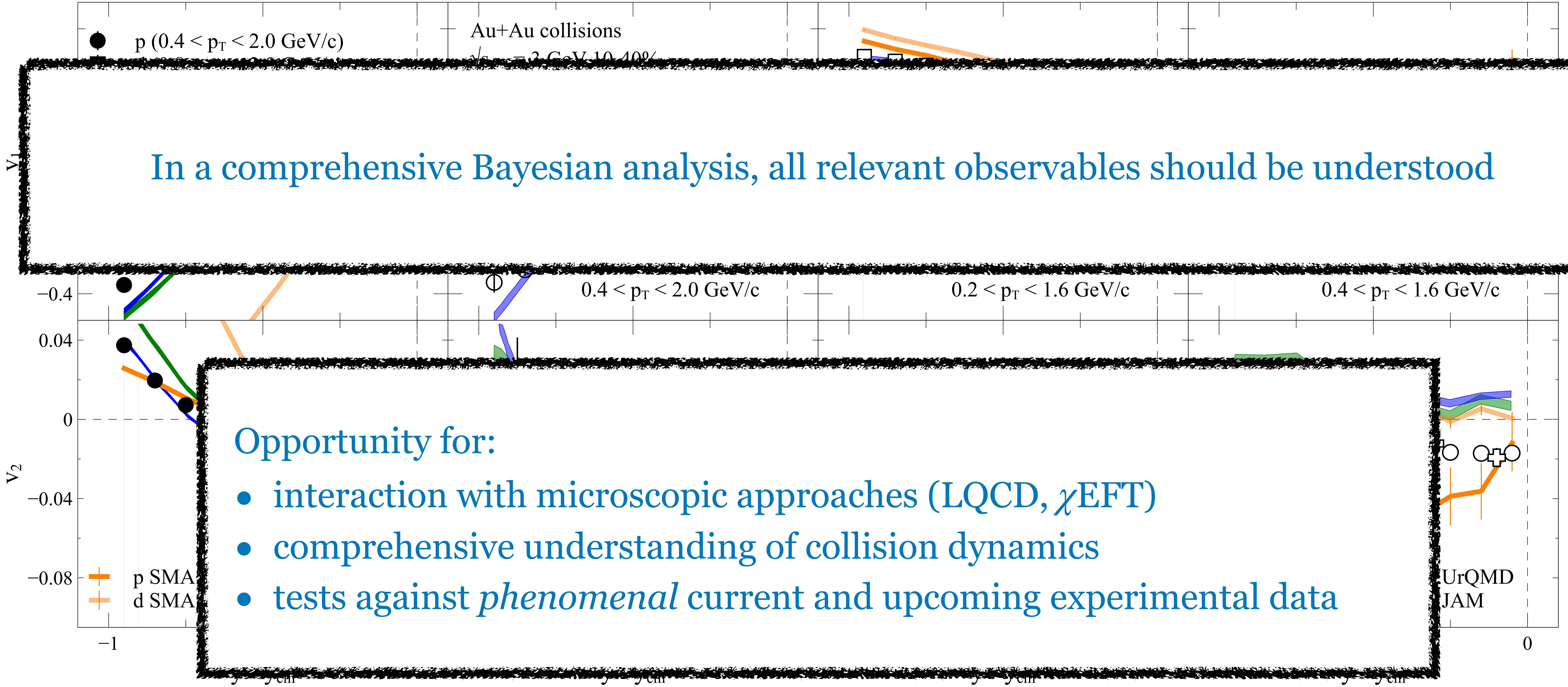


C. Fuchs, PoS CPOD07 060 (2007) arXiv:0711.3367



STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
A. Sorensen *et al.*, arXiv:2301.13253

Describing proton flow is not enough



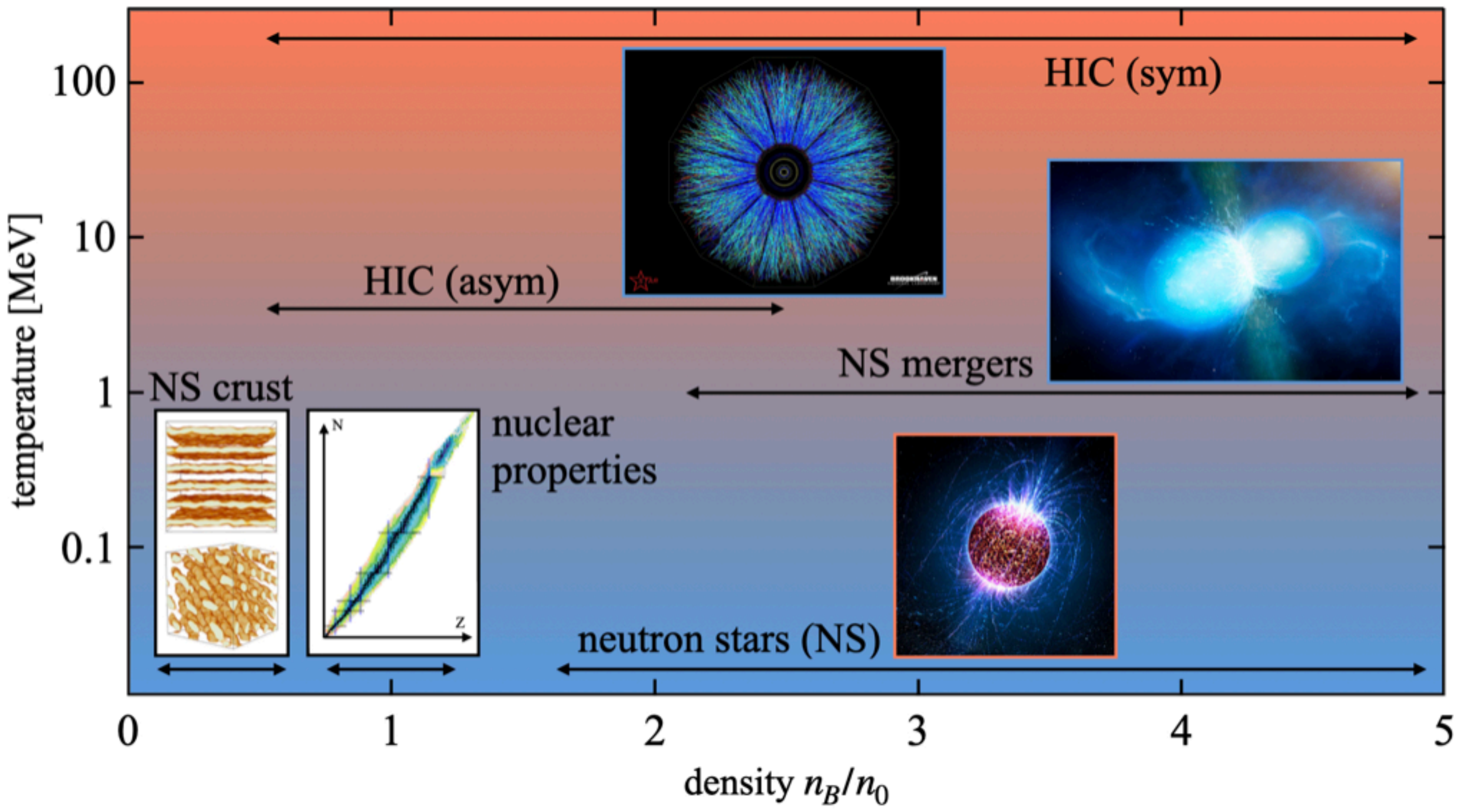
In a comprehensive Bayesian analysis, all relevant observables should be understood

Opportunity for:

- interaction with microscopic approaches (LQCD, χEFT)
- comprehensive understanding of collision dynamics
- tests against *phenomenal* current and upcoming experimental data

STAR, Phys. Lett. B **827**, 137003 (2022) arXiv:2108.00908
D. Oliinychenko, **A. Sorensen**, V. Koch, L. McLerran, arXiv:2208.11996
A. Sorensen *et al.*, arXiv:2301.13253

Precision era of heavy-ion collisions needs precision simulations



A. Sorensen *et al.*, arXiv:2301.13253

Dense Nuclear Matter Equation of State from Heavy-Ion Collisions *

Agnieszka Sorensen[1], Kshitij Agarwal[2], Kyle W. Brown[3,4], Zbigniew Chajecki[5], Paweł Danielewicz[3,6], Christian Drischler[7], Stefano Gandolfi[8], Jeremy W. Holt[9,10], Matthias Kaminski[11], Che-Ming Ko[9,10], Rohit Kumar[3], Bao-An Li[12], William G. Lynch[3,6], Alan B. McIntosh[10], William G. Newton[12], Scott Pratt[3,6], Oleh Savchuk[3,13], Maria Stefaniak[14], Ingo Tews[8], ManYee Betty Tsang[3,6], Ramona Vogt[15,16], Hermann Wolter[17], Hanna Zbroszczyk[18]

Endorsing authors:

Navid Abbasi[19], Jörg Aichelin[20,21], Anton Andronic[22], Steffen A. Bass[23], Francesco Becattini[24,25], David Blaschke[26,27,28], Marcus Bleicher[29,30], Christoph Blume[31], Elena Bratkovskaya[14,29,30], B. Alex Brown[3,6], David A. Brown[32], Alberto Camaiani[33], Giovanni Casini[25], Katerina Chatziioannou[34,35], Abdelouahad Chbihi[36], Maria Colonna[37], Mircea Dan Cozma[38],

2023

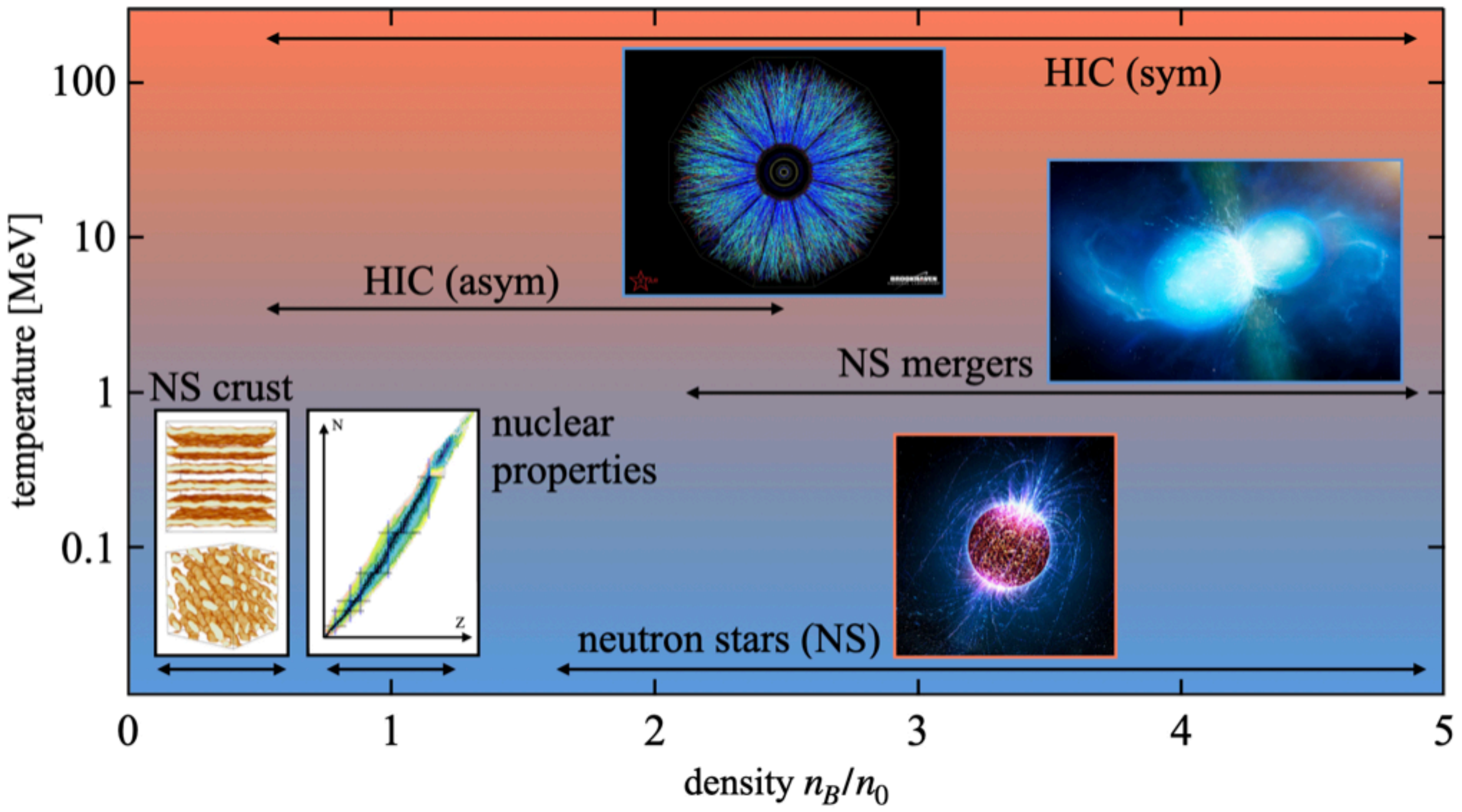
II. THE EQUATION OF STATE FROM 0 TO $5n_0$

A. Transport model simulations of heavy-ion collisions

3. Challenges and opportunities

Selected results presented in Fig. 9 showcase significant achievements in determining the EOS and, simultaneously, the need to develop improved transport models to obtain tighter and more reliable constraints. Answering this need will require support for a sustained collaborative effort within the community to address remaining challenges in modeling collisions, in particular in the intermediate energy range ($E_{\text{lab}} \approx 0.05$–$25$ AGeV, or $\sqrt{s_{NN}} \approx 1.9$–$7.1$ GeV). In the following, we will address selected areas where we see the need for such developments: (1) comprehensive treatment of both mean-field potentials and the collision term in transport codes, (2) use of microscopic information on mean fields and in-medium cross sections, such as discussed in Section II B, in transport, (3) better description of the initial state of heavy-ion collisions in hadronic transport codes, (4) deeper understanding of fluctuations in transport approaches, which affect many aspects of simulations, (5) inclusion of correlations beyond the mean field into transport, which is crucial for a realistic description of, e.g., light-cluster production, (6) treatment of short-range-correlations in transport, which are tightly connected to multi-particle collisions as well as off-shell transport, (7) sub-threshold particle production, (8) connections between quantum many-body theory and semiclassical transport theory, (9) investigations focused on extending the limits of applicability of hadronic transport approaches, (10) studies of new observables, e.g., azimuthally resolved spectra, to obtain tighter constraints on the EOS, (11) the question of quantifying the uncertainty of results obtained in transport simulations, and (12) the use of emulators and flexible parametrizations for wide-ranging explorations of all possible EOSs. Fortunately, advances in transport theory as well as the greater availability of high-performance computing make many of these improvements possible. Support for these developments will lead to a firm control and greater understanding of multiple complex aspects of the collision dynamics, allowing comparisons of transport model calculations and heavy-ion experiment measurements to provide an important contribution to the determination of the EOS of dense nuclear matter, which, in particular, cannot be determined by any other method at intermediate densities $(1$–$5)n_0$.

Precision era of heavy-ion collisions needs precision simulations



A. Sorensen *et al.*, arXiv:2301.13253

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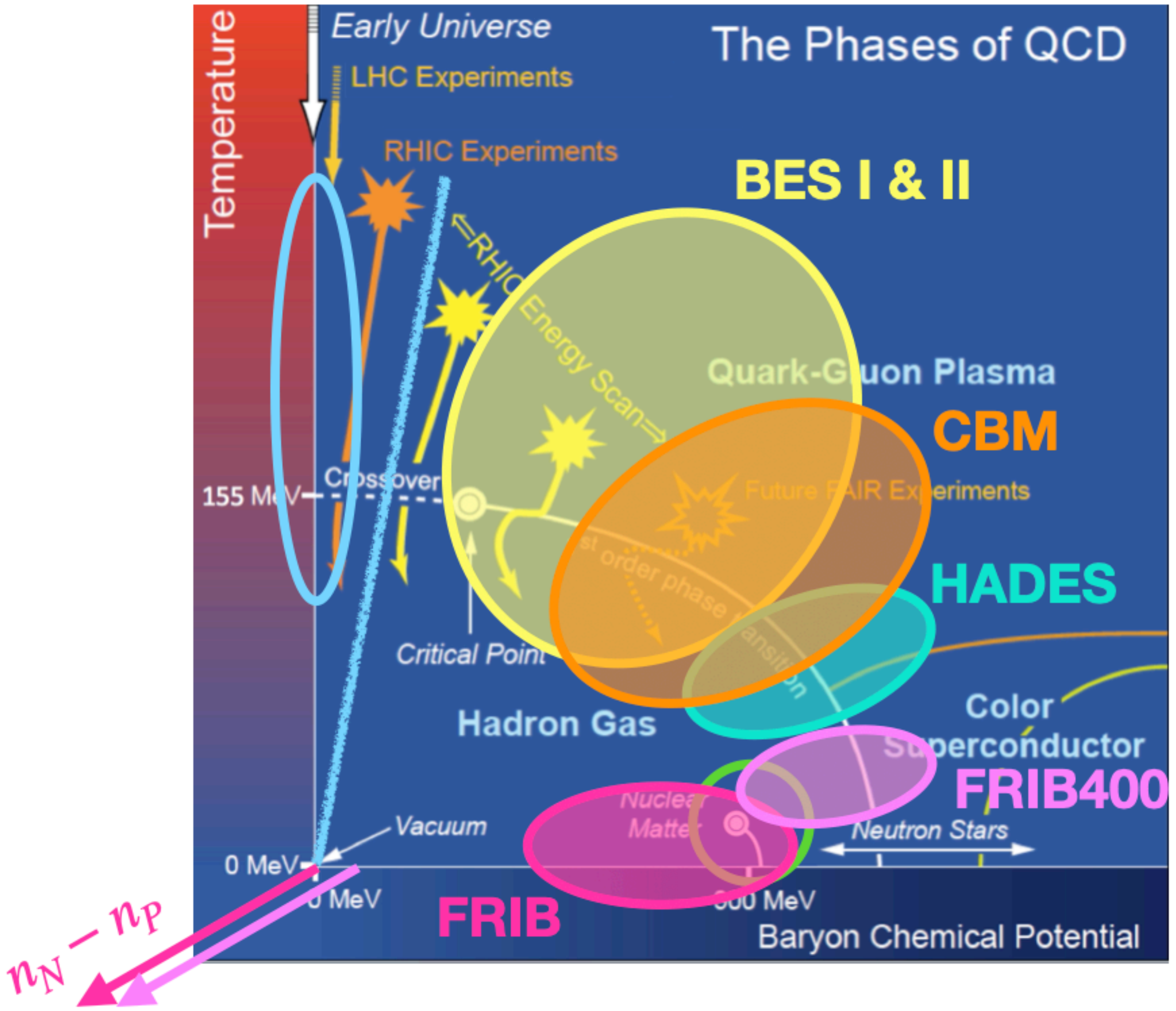
2023

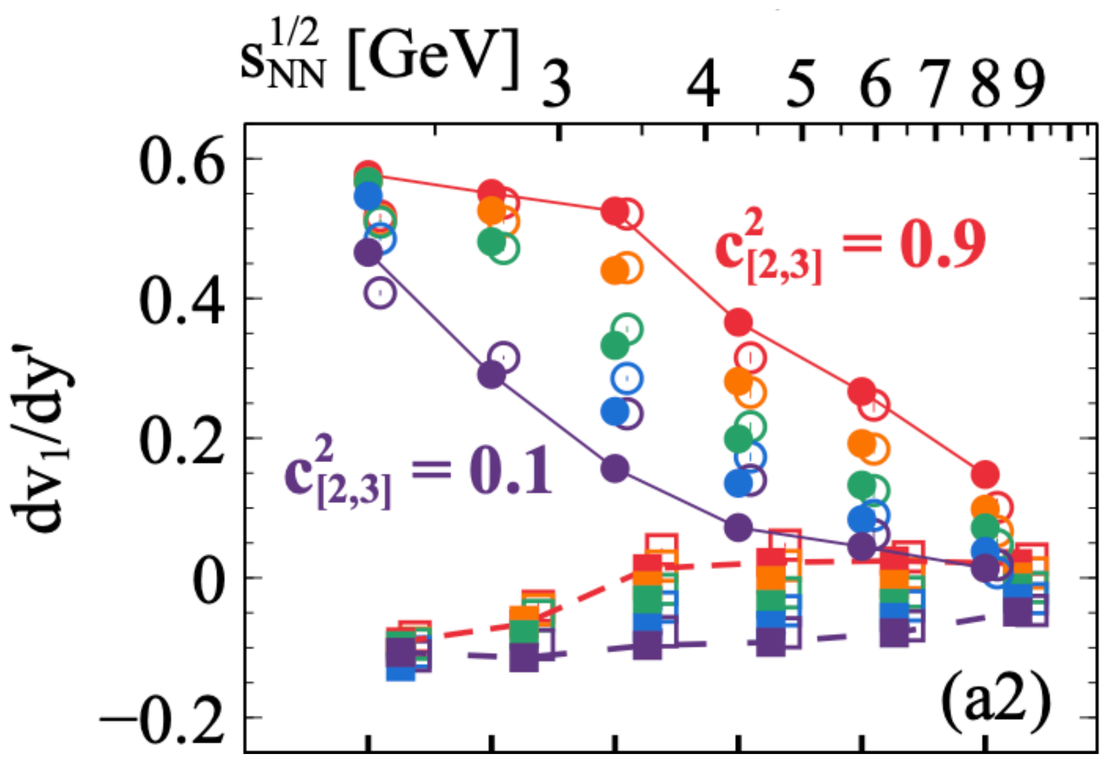
CONTENTS

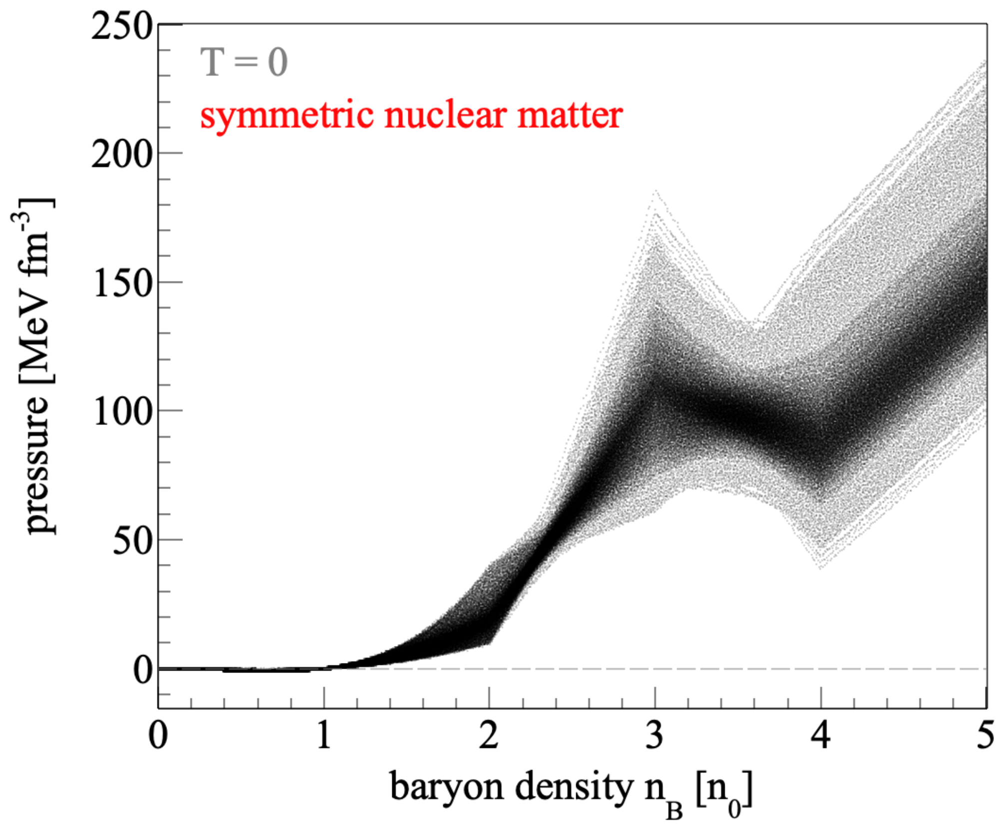
Summary

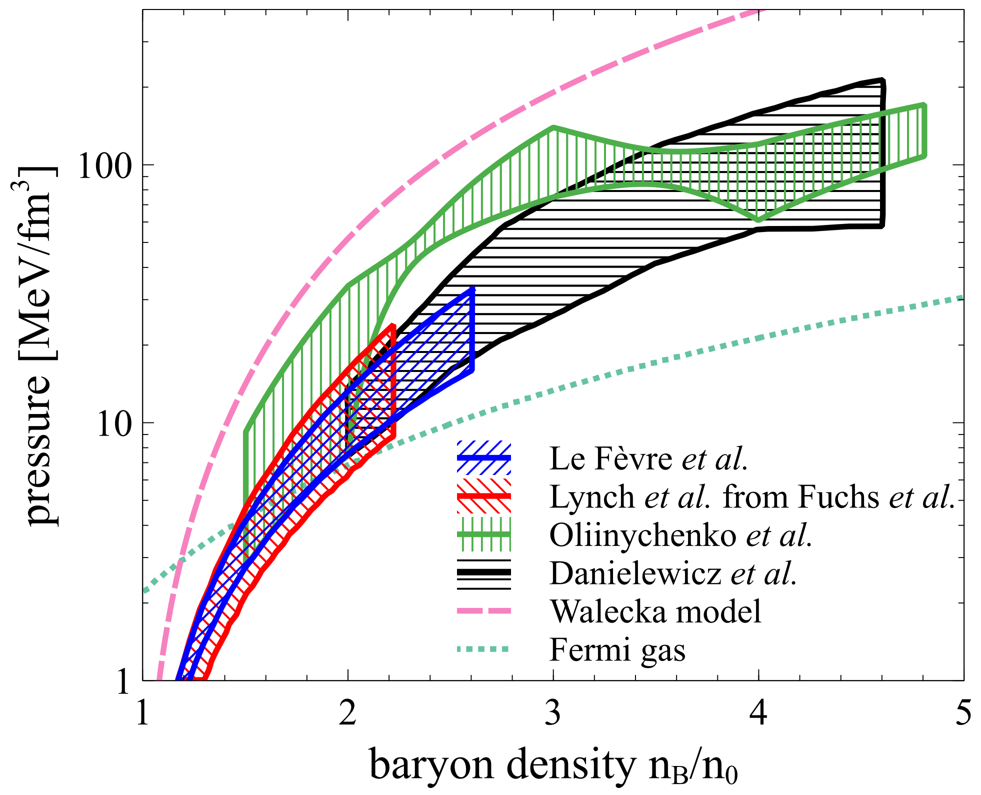
What's different, new, exciting about *now*?

- New analyses, new understanding: e.g., triangular flow, quark number scaling, cumulants
- New detectors, new data: unprecedented measurements, from ultra-precise triple-differential flow observables to hyperon-hyperon interactions
- New computing capabilities: large-scale simulations possible with state-of-the-art, benchmarked hadronic transport codes
- New approach to constraining the EOS: Bayesian analyses using flexible parametrizations of the EOS





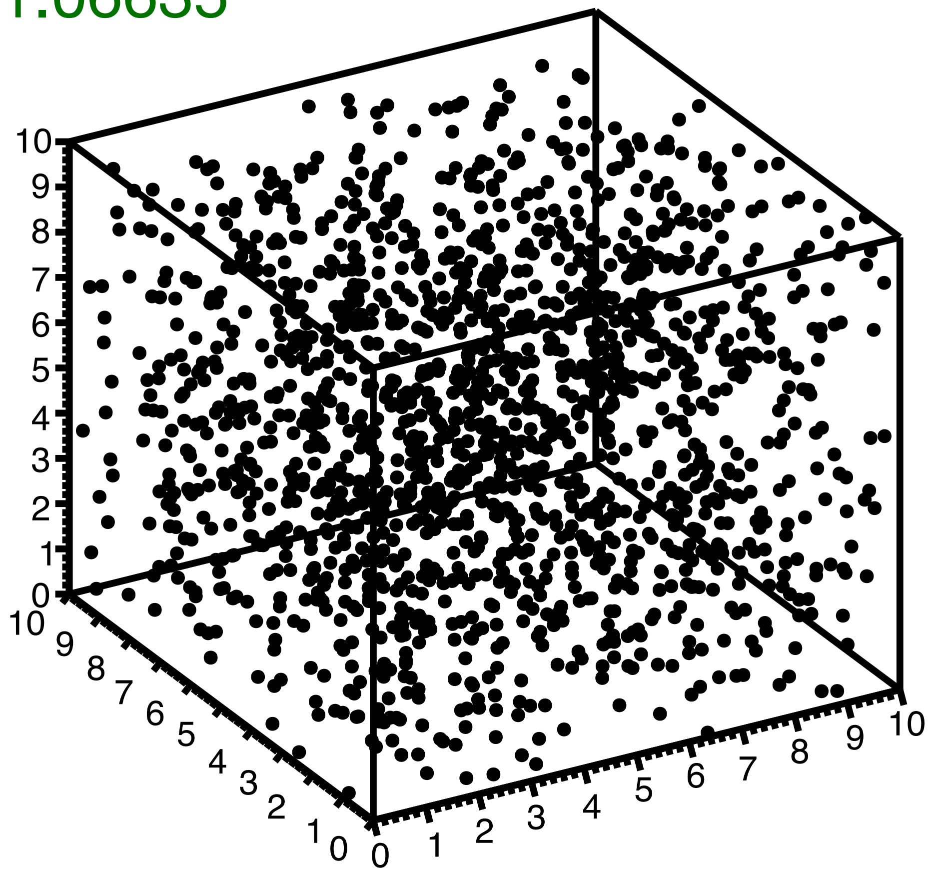




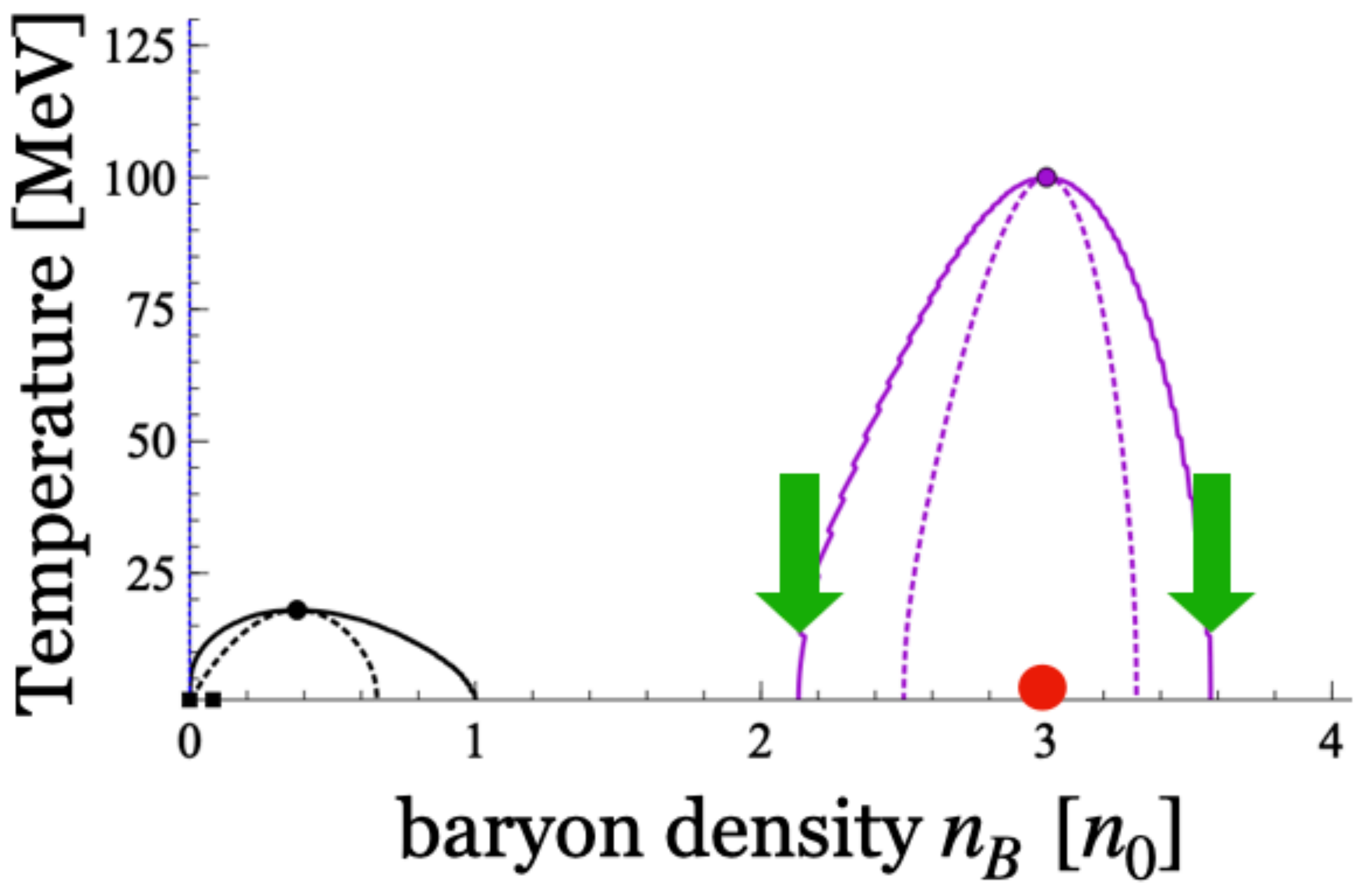
Thank you for your attention

VDF in SMASH: tests in the spinodal region

A. Sorensen, V. Koch, Phys. Rev. C **104**, 3, 034904 (2021)
arXiv:2011.06635



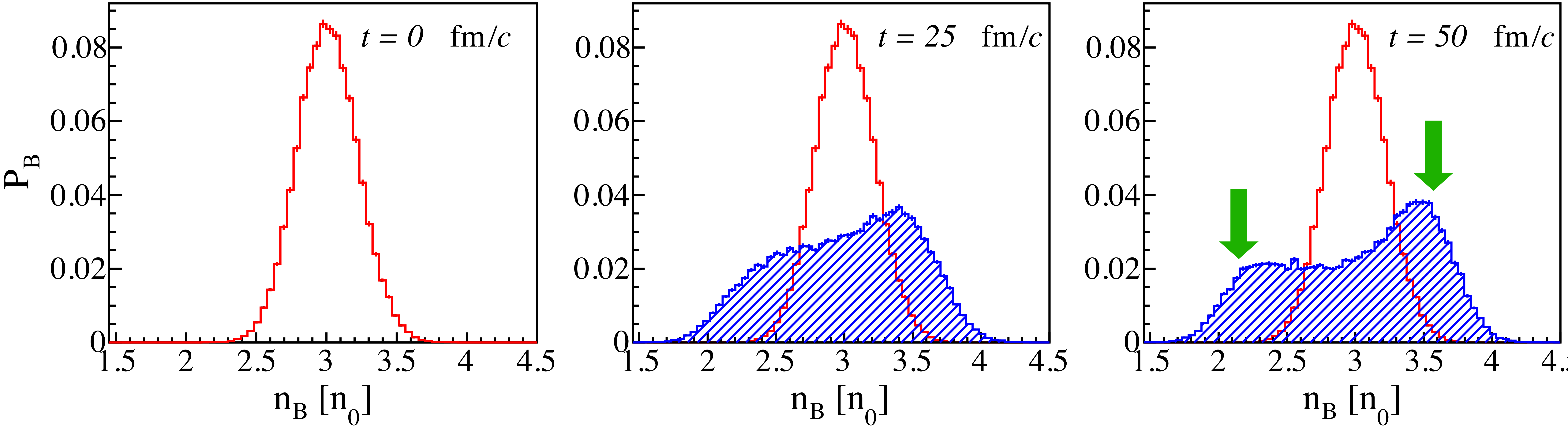
t = 0 fm/c



Simulation info for practitioners:

time step: 0.1 fm/c
smearing: triangular with range 2 fm
lattice: cubic cells with 1 fm on a side
collisions: off

500 events
bin width = 2 fm



The distribution becomes bimodal as the system separates!