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University of Stavanger

Heavy flavor in QCD matter and the lattice

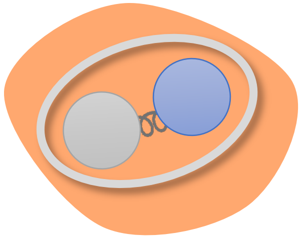
Alexander Rothkopf
Faculty of Science and Technology
Department of Mathematics and Physics
University of Stavanger

Norwegian Particle, Astroparticle & Cosmology Theory network

RHIC Beam Energy Scan Virtual Seminar Series – May 2nd 2023 – Online

Heavy Quarkonium

a clean
QCD laboratory



a precision probe
in HIC studies

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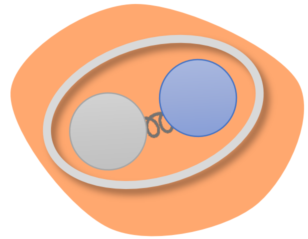
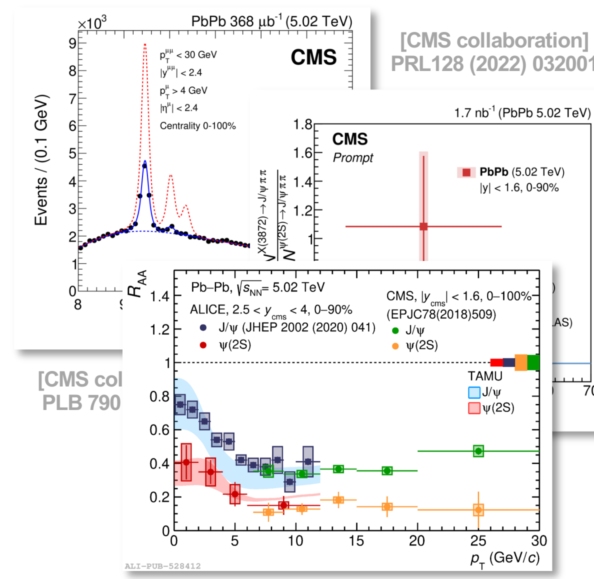


a precision probe in HIC studies

Experiment advantage: clean signals via enhanced dilepton decay channels



[CMS collaboration] PRL128 (2022) 032001

[CMS col PLB 790

[ALICE collaboration] arXiv:2210.08893

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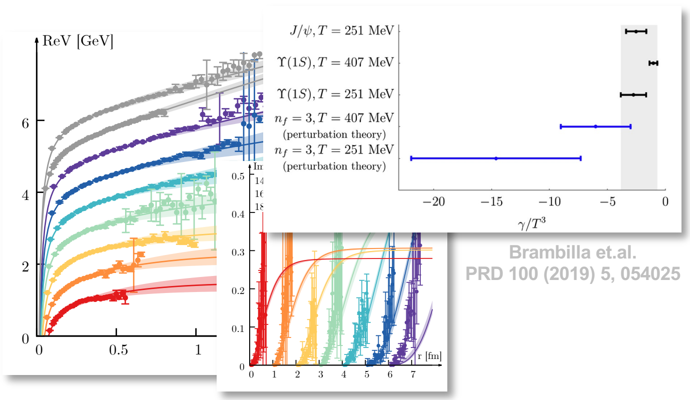
a clean QCD laboratory

Theory advantage: separation of scales enables powerful effective field theory tools



Brambilla et.al. PRD 100 (2019) 5, 054025

Y.Burnier, O.Kaczmarek, A.R. PRL114 (2015) 082001
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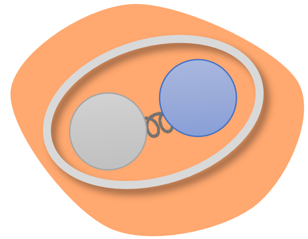
1st principles + intuitive non-relativistic language (e.g. potential and transport coefficients)

see also A.R. Phys.Rept. 858 (2020) 1-117



$$\frac{\Lambda_{QCD}}{m_Q} \ll 1 \qquad \frac{\varepsilon_{env}}{m_Q} \ll 1$$

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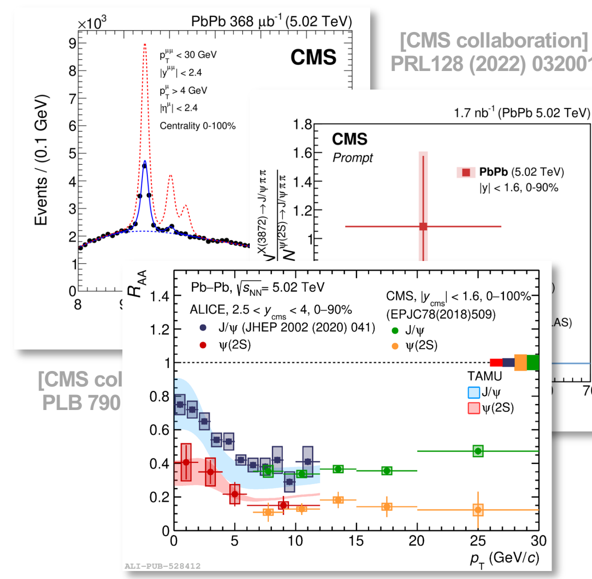

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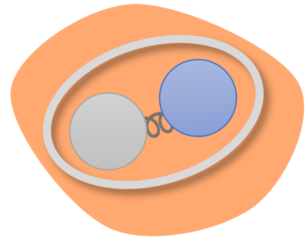

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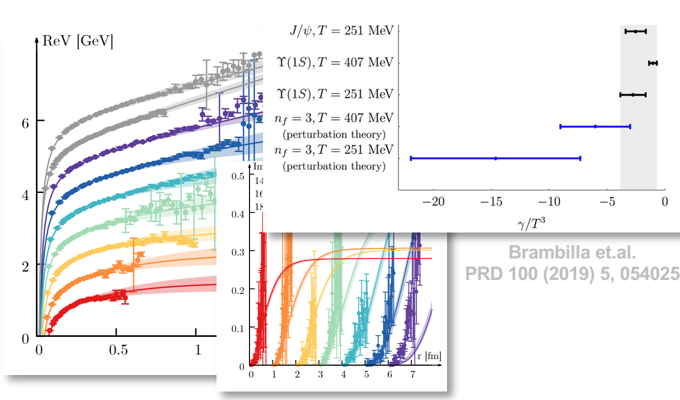

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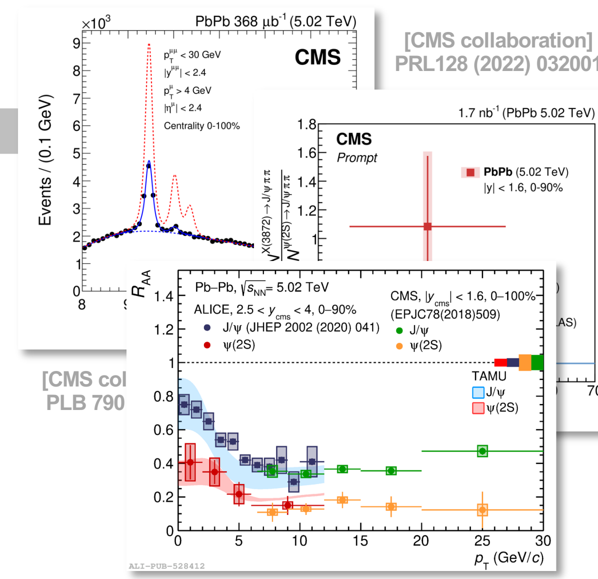

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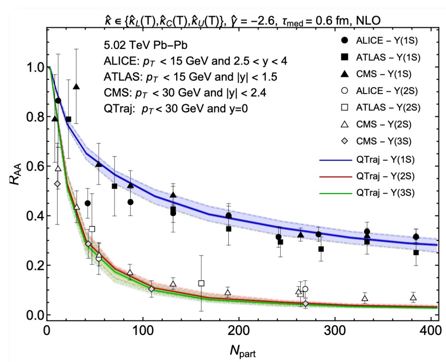
Quarkonium as Open Quantum System



see e.g. TUM-Kent State study in JHEP 08 (2022) 303

gain a **quantitative** under-standing of QCD in-medium bound states & QCD medium properties

Heavy quarkonium and RHICs

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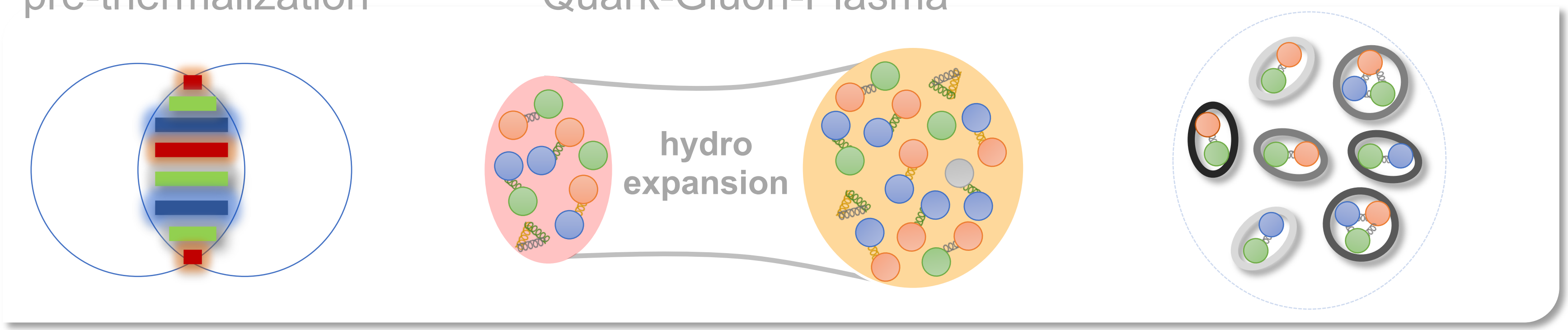
bulk: pre-thermalization Quark-Gluon-Plasma hadronization



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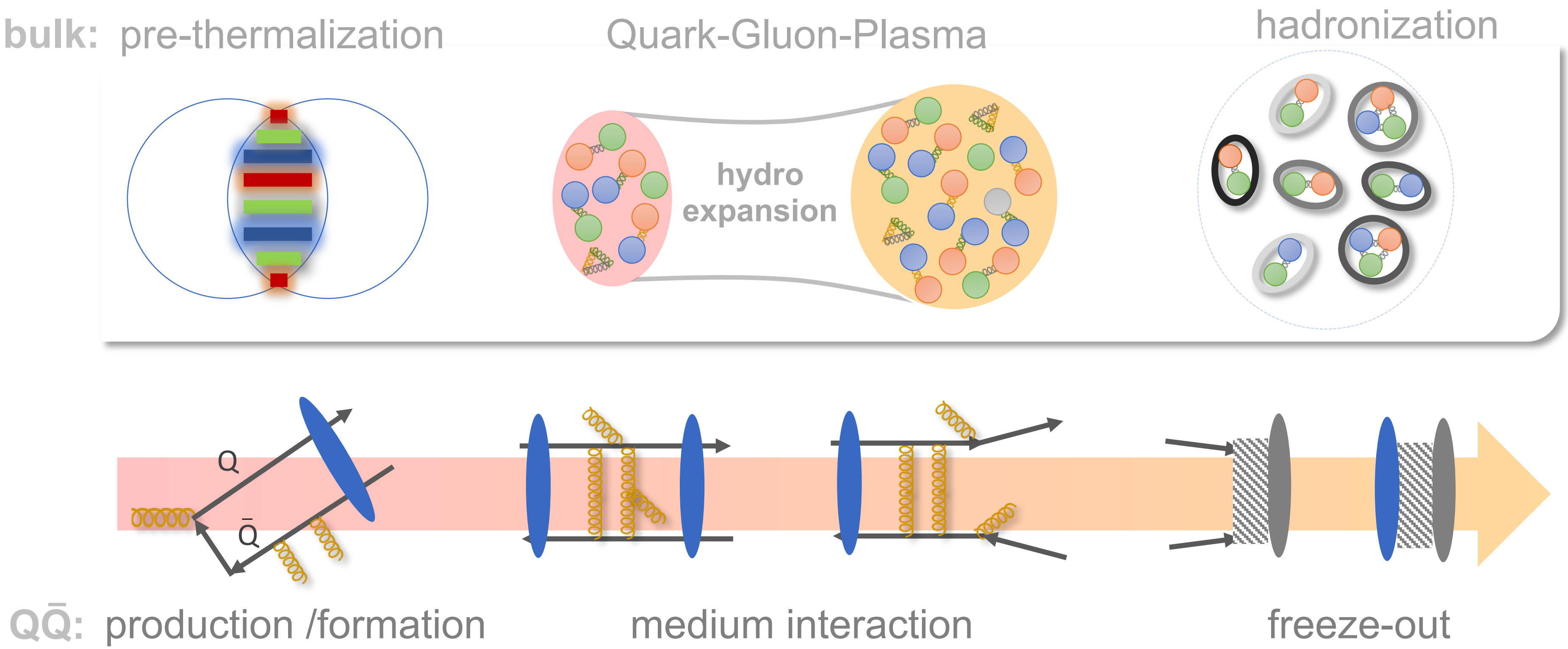

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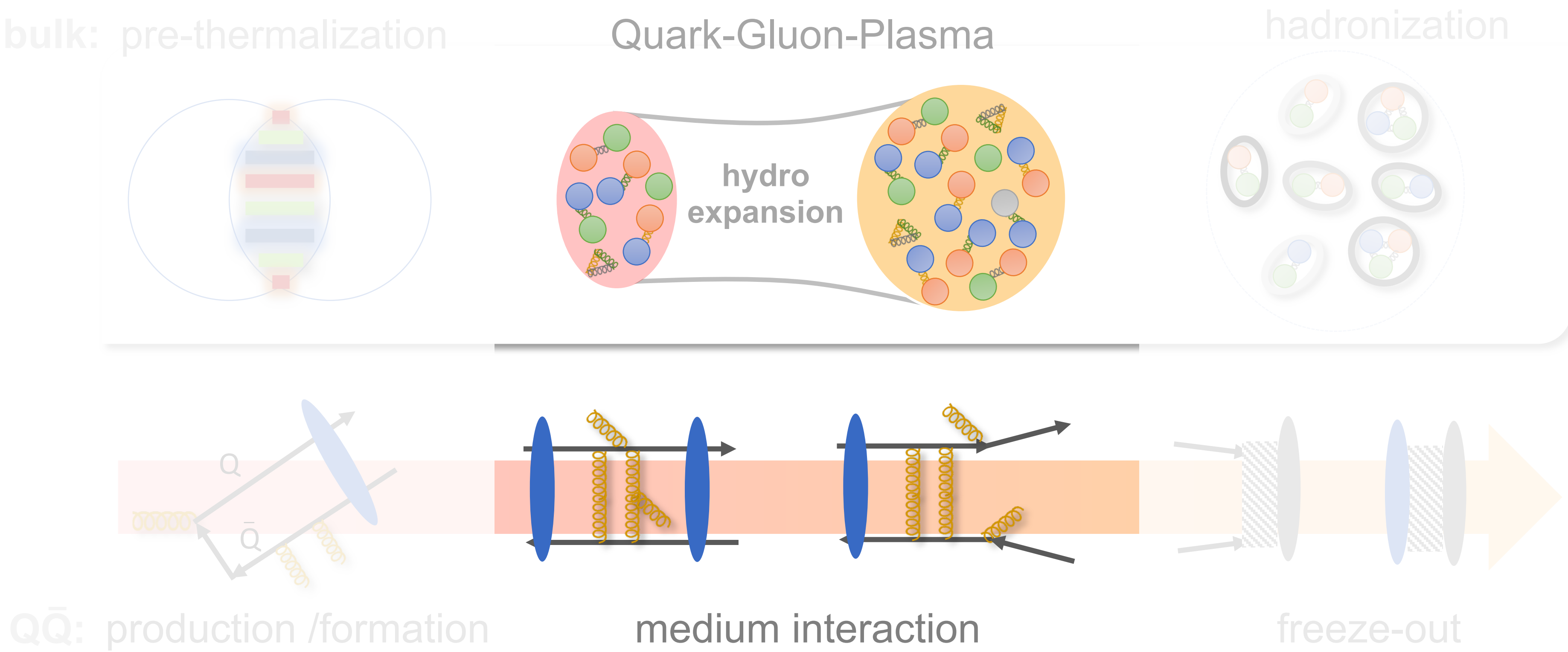

Physics motivation

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Open physics questions

(Thermal) bound states

idealized heavy-quarkonium

T. Matsui and H. Satz, Phys.Lett.B 178 (1986)

Kinetic equilibration

heavy-quarkonium as OQS

Stavanger/Osaka: PRD 101 (2020) 3, 034011
Kent State: JHEP 03 (2021) 235
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Chemical equilibration

heavy quarks at the FCC

Bodecker, Laine JHEP 07 (2012) 130

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select recent Lattice insights on T>0 quarkonium

T>0 static potential

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Transport coefficients

heavy quark diffusion

N. Brambilla et.al. PRD 102 (2020) 7, 074503
L. Altenkort et.al. PRD 103 (2021) 1, 014511
TUMQCD PRD 107 (2023) 5, 054508

Sommerfeld enhancement

Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

The various faces of lattice QCD

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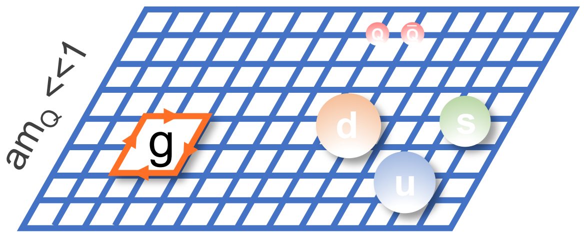
Lattice discretization

- Gauge fields as links: $U_\mu(x)=\exp[\, i\, g\, a_\mu\, A_\mu(x)\,]$
- Discretized N_f flavors of light fermions on the nodes



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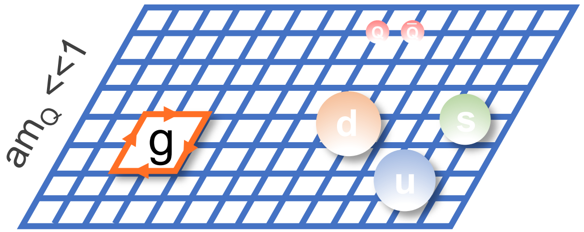

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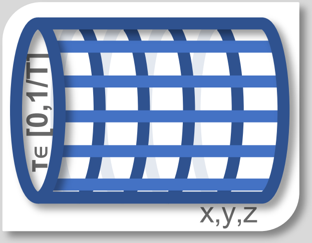

Formulated in compact **imaginary time** for MC

Gattringer, Lang, QCD on a lattice
10.1007/978-3-642-01850-3

$$\langle O(\tau)\rangle = \int \mathcal{D}U O(U) e^{-S_E^{QCD}[U]}$$

ab-initio sim. of a quantum path integral

$$P[U] \propto e^{-S_E[U,\psi,\bar{\psi}]}$$

$$\langle O\rangle = \frac{1}{N} \lim_{N\to\infty} \sum_{k=1}^{N} O(U^k)$$

return to real-time very costly

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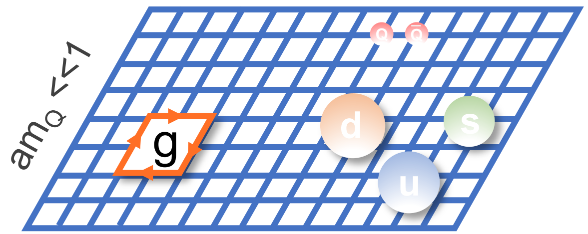

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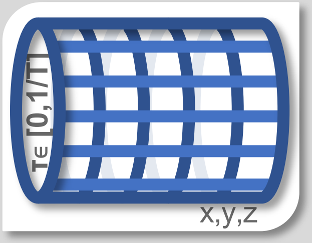

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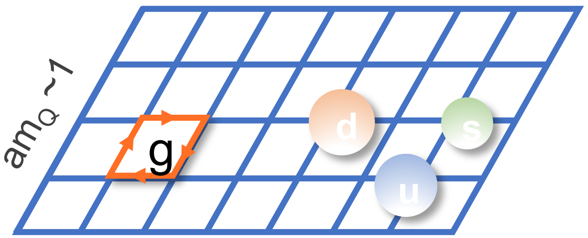

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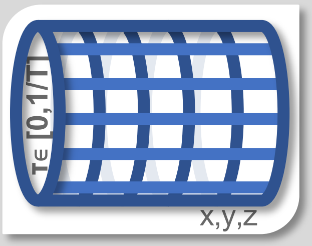

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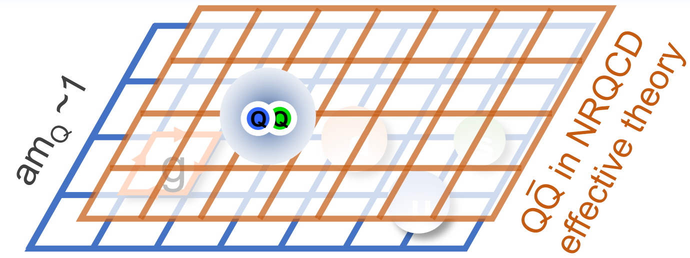

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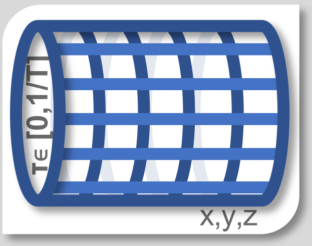

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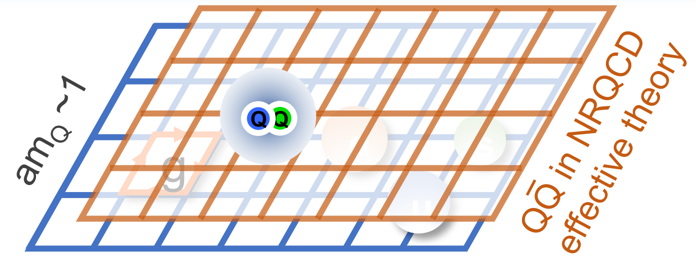

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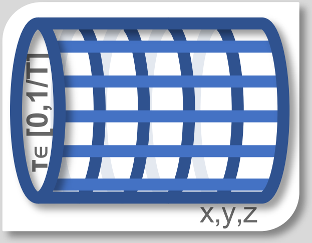

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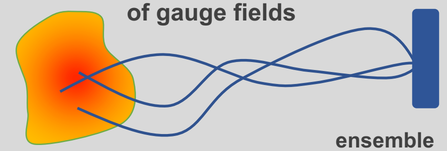

Formulated in **Minkowski time** directly

V. Kasper et. al.
PRD 90, 025016 (2014)

$$\langle O(t)\rangle = \int dE_0 dU_0\, P[U_0,E_0] O(U(t),E(t))$$

valid at highly occupancy: glasma
or deep in the IR: sphaleron transitions

$$P[U_i,E_i]|_{t=0} \sim e^{-H/T}$$

$$\partial_\mu F^a_{\mu\nu}[U,E] = j^a_\nu[\psi]$$

continuum limit intricate & no confinement

Spectral functions on Euclidean lattices

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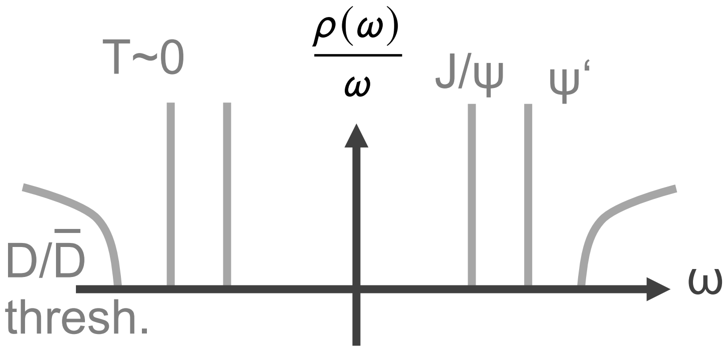
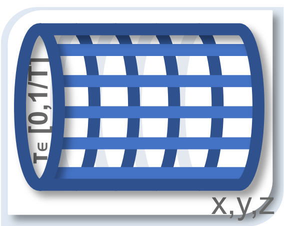
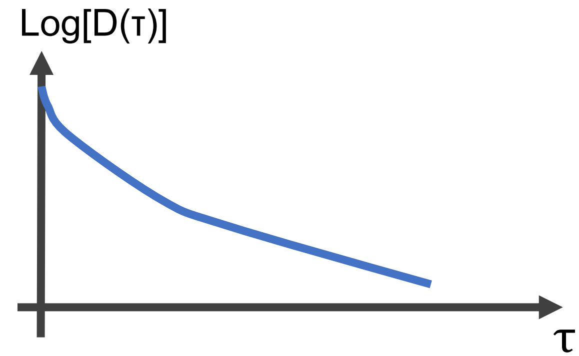
- **Euclidean lattice QCD** simulations are similar to a (very) **imperfect detector**

Relativistic formulation







Quarkonium spectral function

$$D(\tau) = \int d\omega K(\omega,\tau)\,\rho(\omega)$$

Euclidean time correlation function

Ill-posed inverse problem

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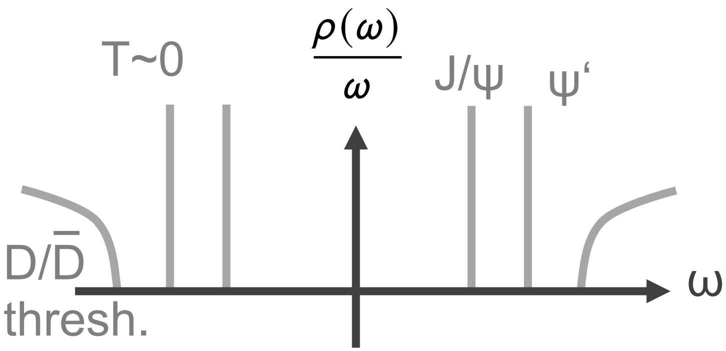

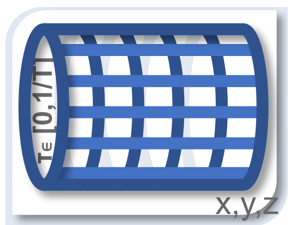

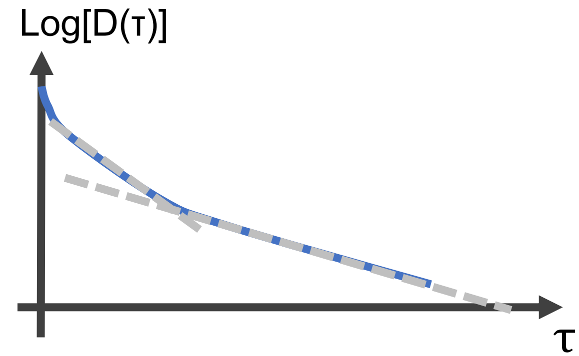

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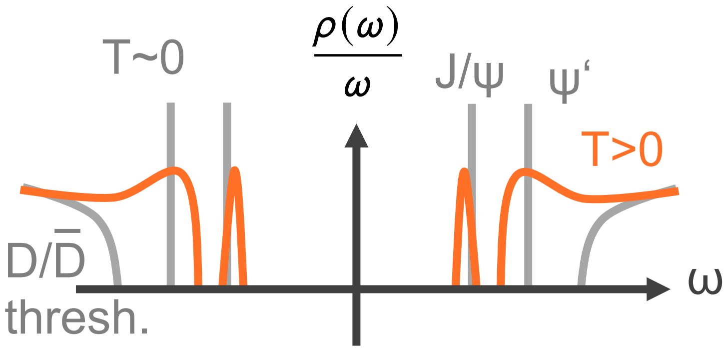

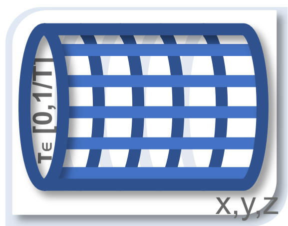

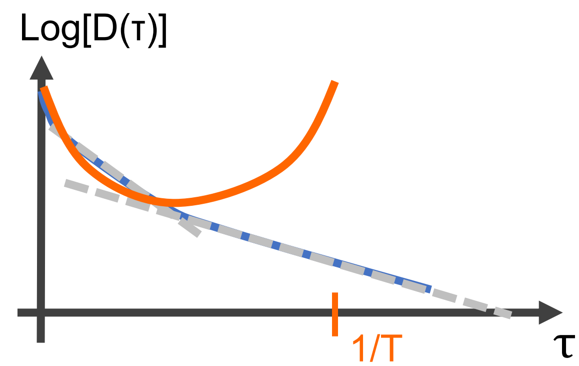

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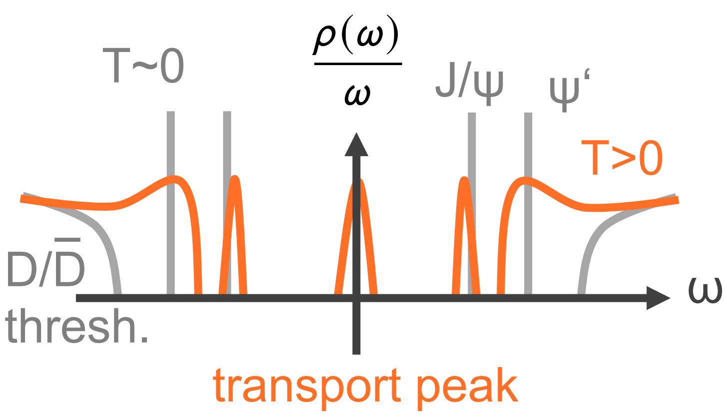

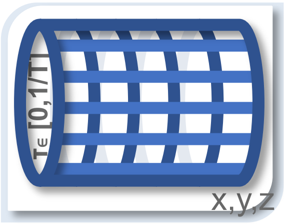

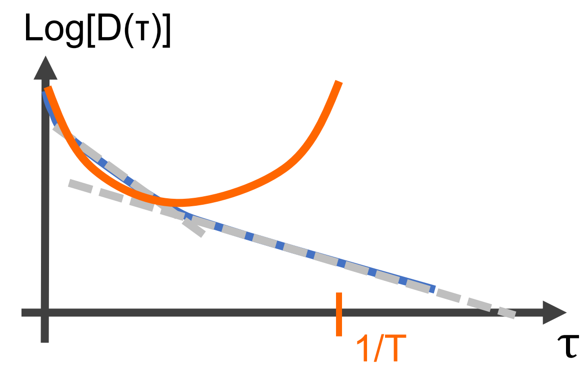

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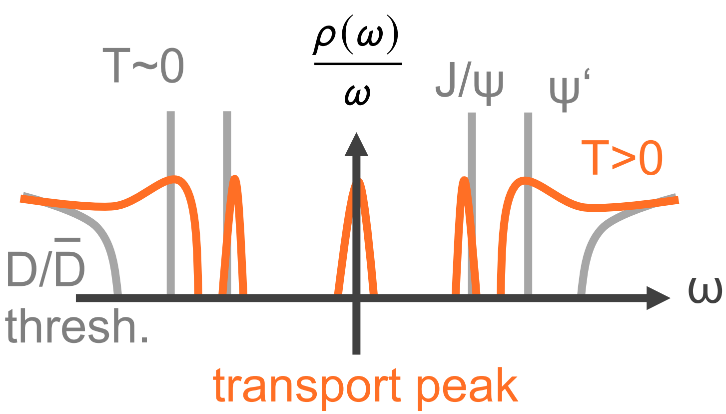
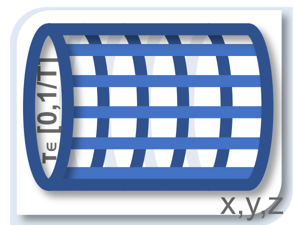
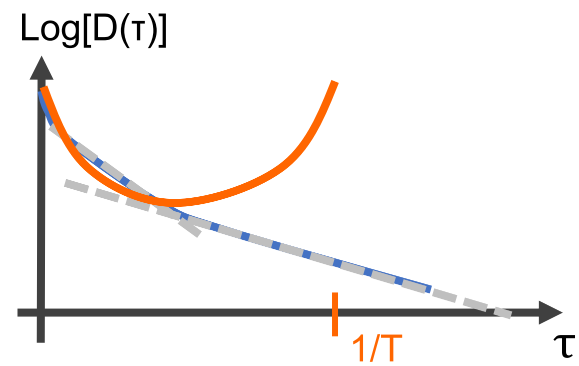




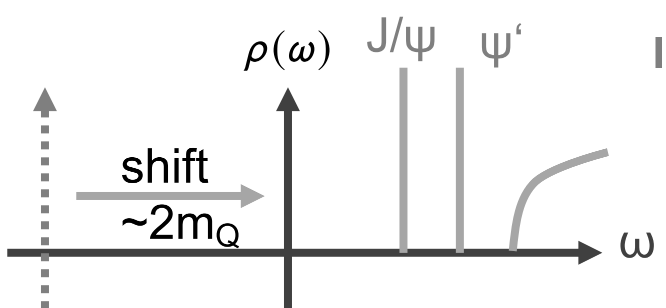


Quarkonium spectral function

$$D(\tau) = \int d\omega K(\omega,\tau)\rho(\omega)$$

Euclidean time correlation function

Ill-posed inverse problem



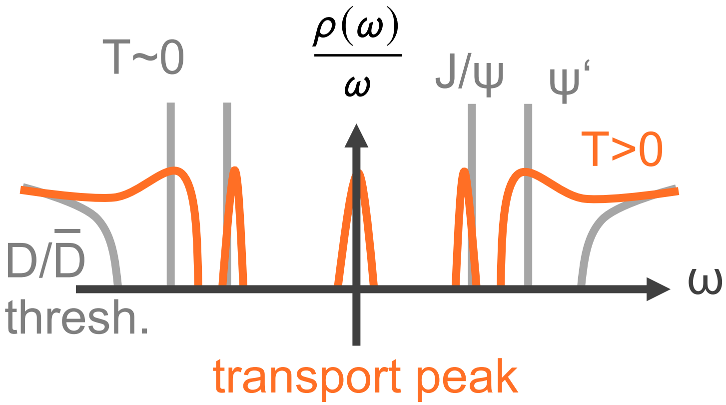
Non-relativistic formulation (NRQCD)

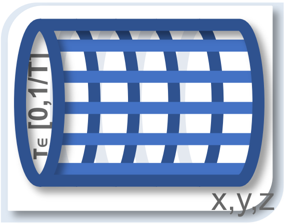
Spectral functions on Euclidean lattices

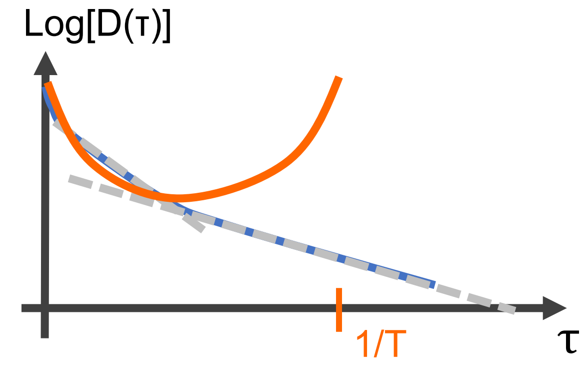
University of Stavanger

- **Euclidean lattice QCD** simulations are similar to a (very) **imperfect detector**

Relativistic formulation







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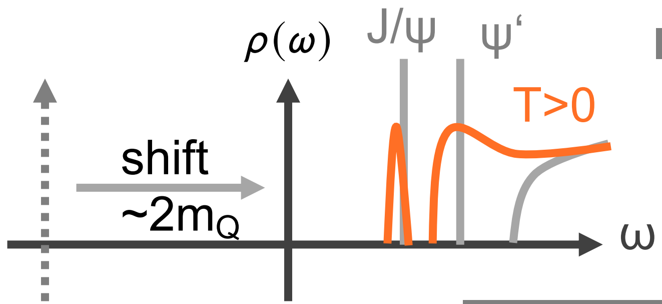

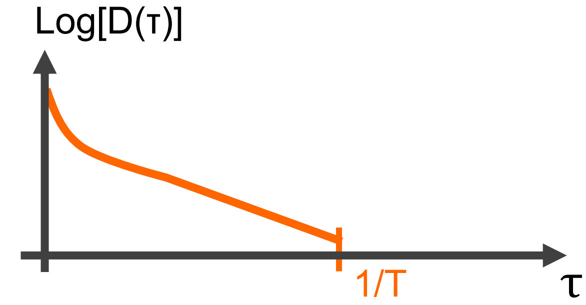

Non-relativistic formulation (NRQCD)

Towards spectral functions

University of Stavanger

Bayesian Spectral reconstruction

Supply prior information to regularize the inverse problem

Maximum Entropy Method
(positivity +
do not introduce correlations
where there are none in the data)
M. Asakawa, T. Hatsuda, Y. Nakahara
Prog.Part.Nucl.Phys. 46 (2001) 459

BR method
(positivity + 2 x differentiability +
result independent of units used)
Y. Burnier, A.R., PRL 111 (2013) 182003

careful analysis of **regularization artifacts** necessary, often combined with mock-data analyses

see A.R., Front. Phys. 10:1028995 (2022) for a review

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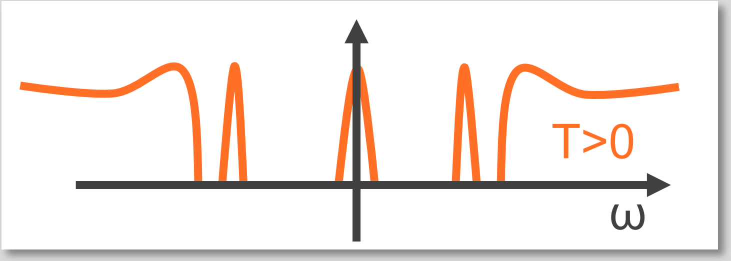
Model fits

Realistic spectral forms **with few parameters from continuum comp.**



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University of Stavanger

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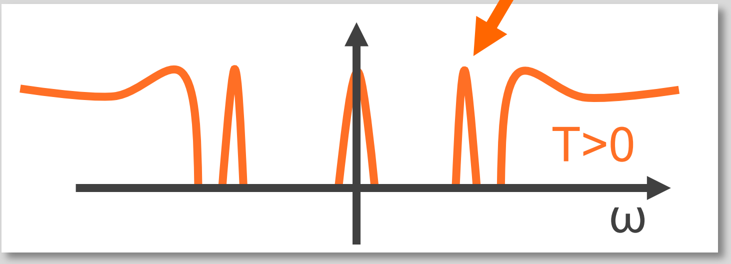

Towards spectral functions

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transport peak form (IR effective theory)
S. C.-Huot et.al. JHEP 04 (2009) 053
P. Petreczky et.al. PRD73 (2006) 014508

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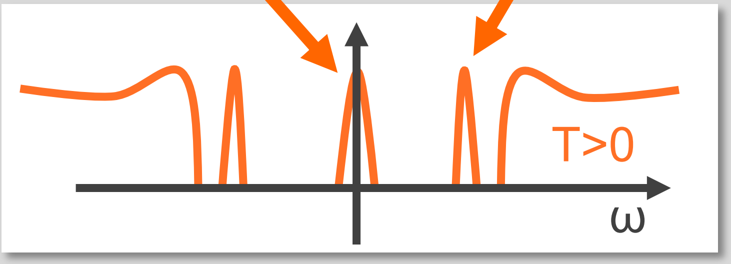

Towards spectral functions

University of Stavanger

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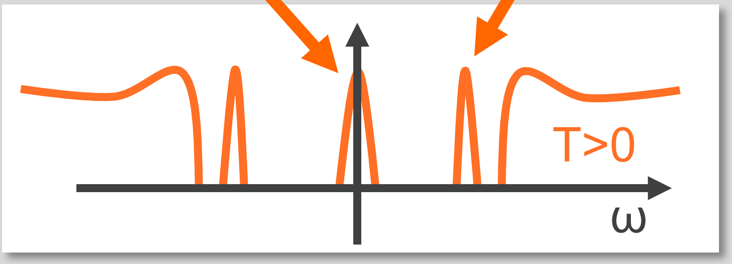
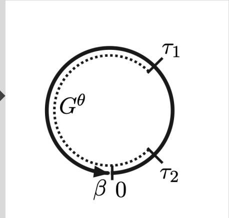


Reformualtion strategies

relate transport peak physics to a Euclidean correlators

$$\Omega_{\text{chem}} \equiv \lim_{\omega \ll T} 2T\omega\rho_\Delta(\omega)$$



S. Kim, M. Laine, JHEP 1607 (2016) 143
A. Eller et.al. PRD 99, 094042 (2019)

Part I

University of Stavanger

select recent Lattice insights on T>0 quarkonium

T>0 static potential

Euclidean quantum

Y. Burnier, O. Kaczmarek, A.R.
JHEP 12 (2015) 101
Y. Burnier, A.R. PRD 95 (2017) 5, 054511
HotQCD et.al. PRD 105 (2022) 5, 054513

classical statistical

A. Lehmann, A.R. JHEP 07 (2021) 067
K. Boguslavski, B. Kasmai, M. Strickland
JHEP 10 (2021) 083

T>0 quarkonium spectra

relativistic formulation

Y. Burnier et. al. JHEP 11 (2017) 206

using lattice EFT (NRQCD)

S. Kim, P. Petreczky, A.R. JHEP 11 (2018) 088
FASTSUM PoS LATTICE2019 (2019) 076
R. Larsen et. al. Phys.Lett.B 800 (2020) 135119

Transport coefficients

heavy quark diffusion

N. Brambilla et.al. PRD 102 (2020) 7, 074503
L. Altenkort et.al. PRD 103 (2021) 1, 014511
TUMQCD PRD 107 (2023) 5, 054508

Sommerfeld enhancement

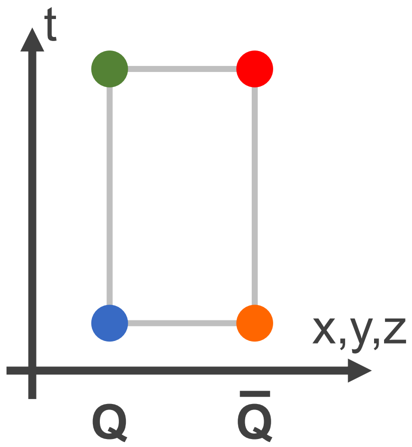
Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

Static quark potential at T>0

University of Stavanger

- Simplest model system: **infinitely heavy** color sources

$$\langle(\bar{Q}Q)(\bar{Q}Q)^\dagger\rangle \overset{m_Q\to\infty}{=} W_\Box(r,t) = exp\Big[ig\int_\Box dz^\mu A_\mu\Big]$$



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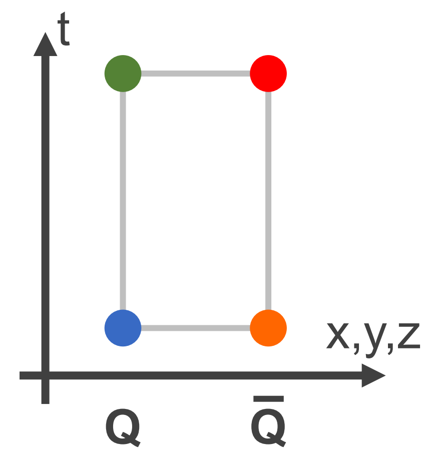

Central question: can its real-time evolution be understood via a **Schrödinger equation** i.e. by a **potential**?

$$i\partial_t W_\square(r,t) \overset{?}{=} V(r) W_\square(r,t)$$

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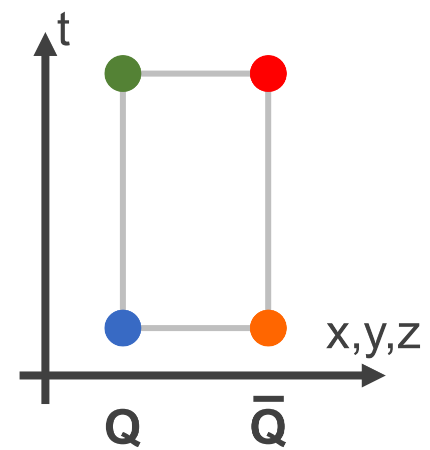

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For weak coupling results see
M. Laine et.al. JHEP 03 (2007) 054,
N. Brambilla et.al. PRD 78 (2008) 014017

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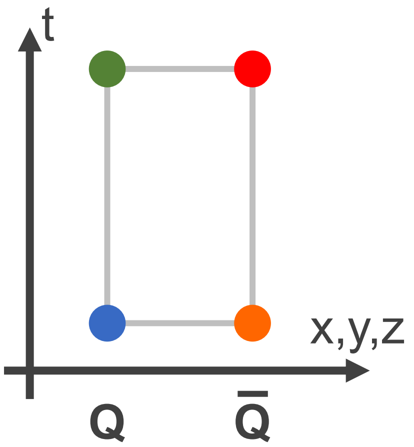

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A.R., T. Hatsuda, S. Sasaki PRL 108 (2012) 162001, Y. Burnier, A.R. PRD 86 (2012) 051503

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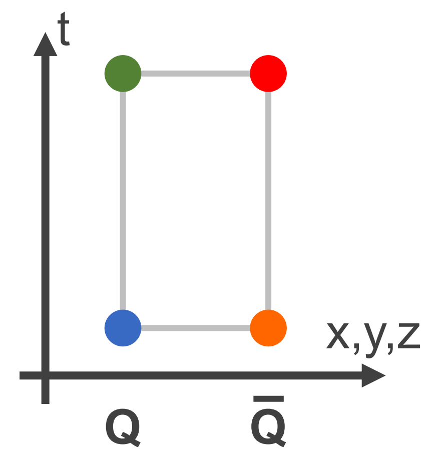

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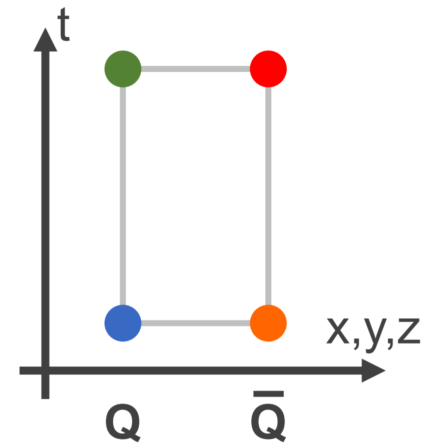

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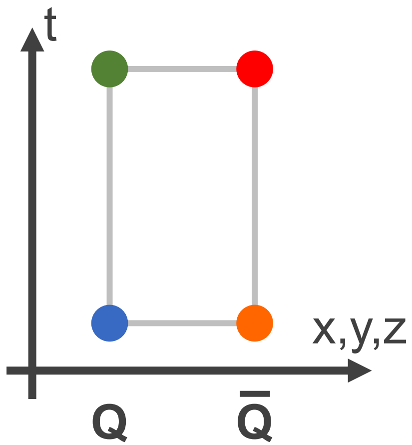

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Lowest lying spectral peak dominates late real-time behavior Re[V]=ω_0 Im[V]=Γ_0

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University of Stavanger

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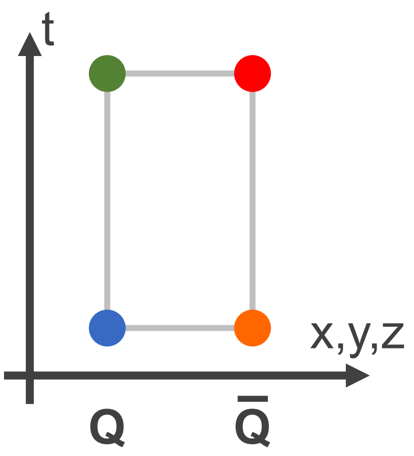

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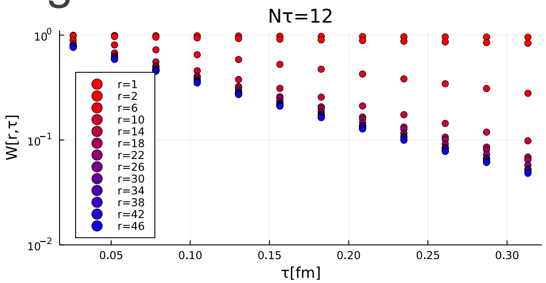
Static quark potential I (Euclidean)

University of Stavanger

- **Bayesian reconstruction** of Wilson correlator spectral functions from nontrivial HTL correlators as **testing ground**

Y. Burnier, A.R. PRD 86 (2012) 051503 & PRD 87 (2013) 114019 & PRL 111 (2013) 182003

HTL Wilson line correlator
in Coulomb gauge to avoid cusp divergencies



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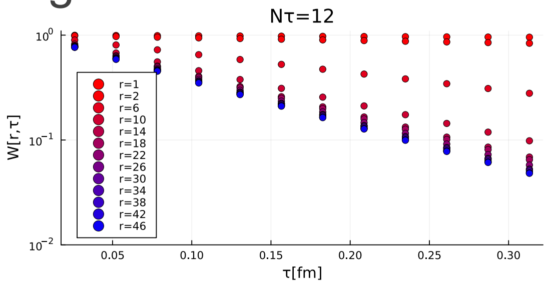

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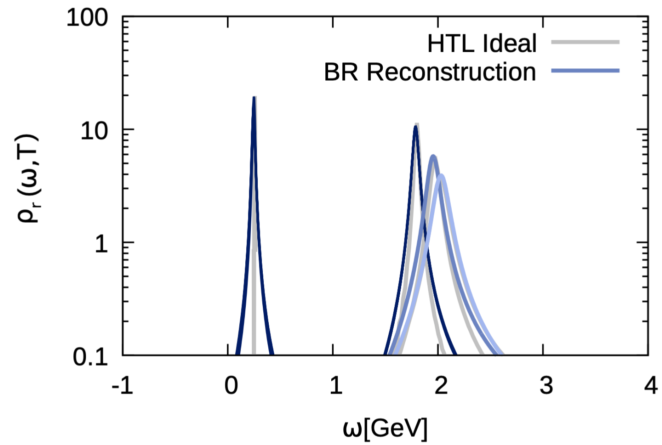

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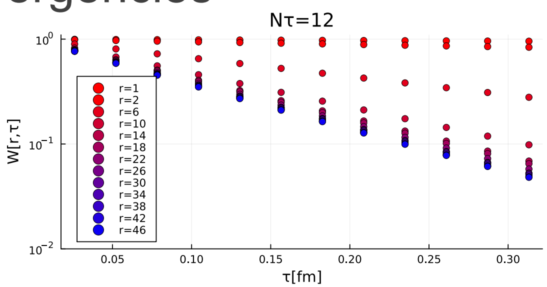

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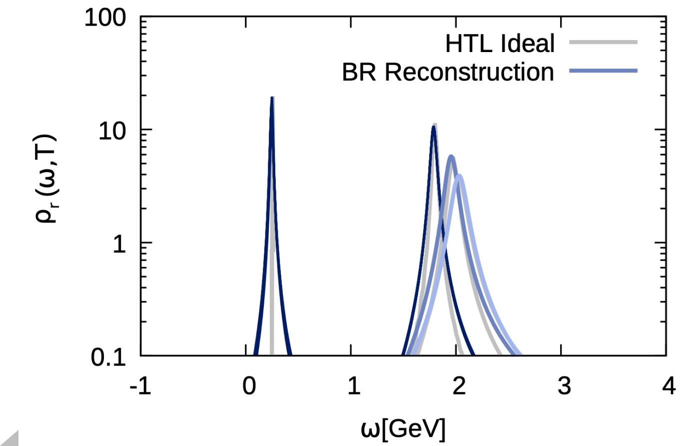
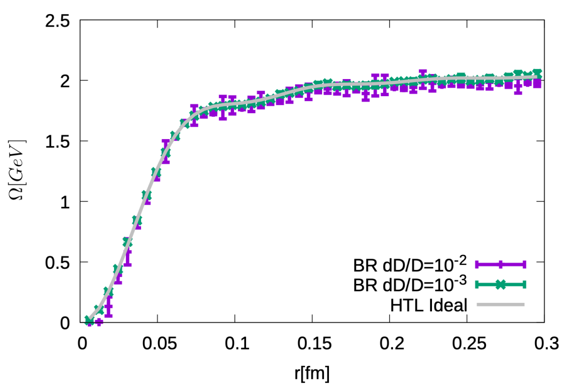


Re[V] from lowest peak position



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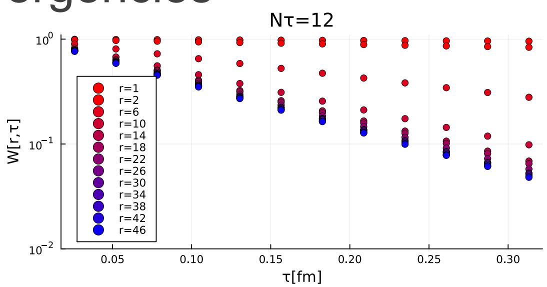

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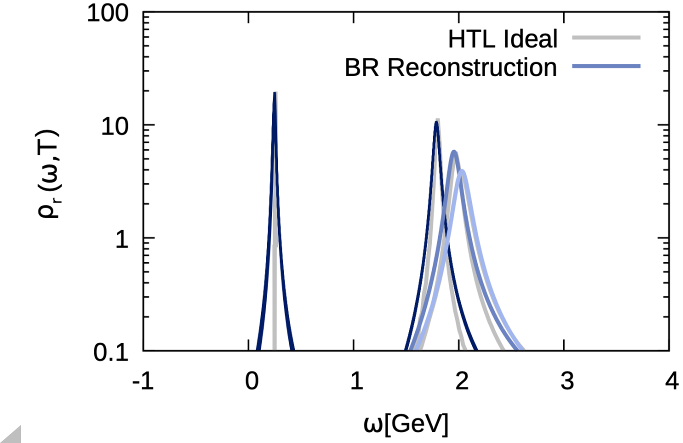

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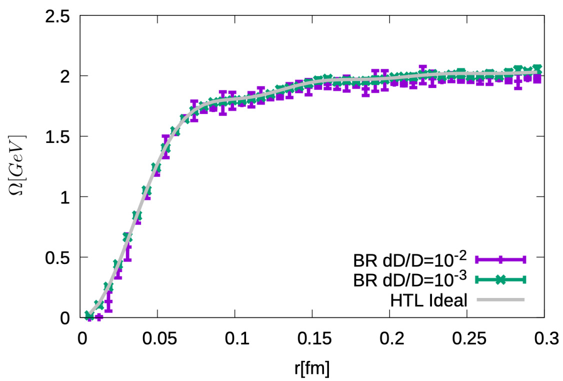


- Already with 10^{-2} relative errors on input data – excellent reproduction of Re[V]

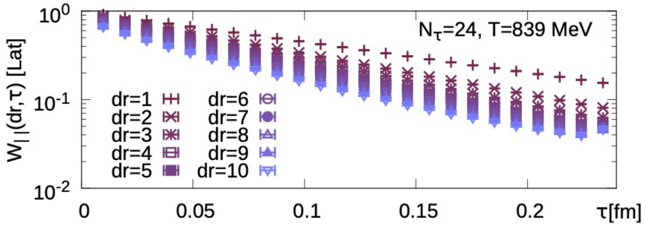
Static quark potential II (Euclidean)

University of Stavanger

- **Bayesian reconstruction** of Wilson correlator spectral functions from Euclidean lattices with **heavier than physical** quarks (N_f=2+1)

Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101

Lattice Wilson line correlator
in Coulomb gauge



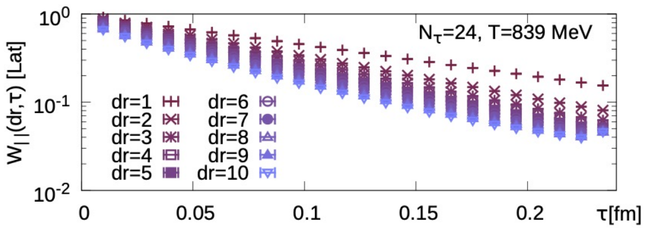
Static quark potential II (Euclidean)

University of Stavanger

- **Bayesian reconstruction** of Wilson correlator spectral functions from Euclidean lattices with **heavier than physical** quarks (N_f=2+1)

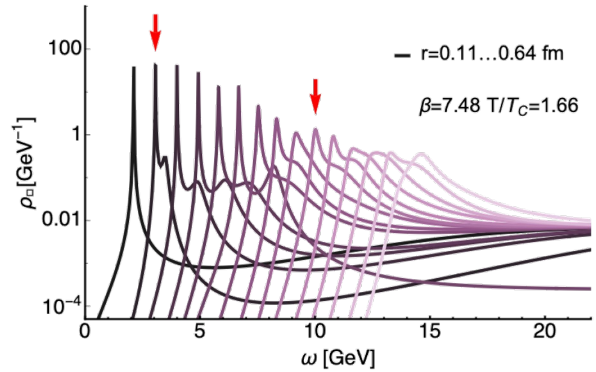
Y. Burnier, O. Kaczmarek, A.R. JHEP 12 (2015) 101

Lattice Wilson line correlator in Coulomb gauge



Identification of a potential peak in spectra

150MeV < T< 300MeV



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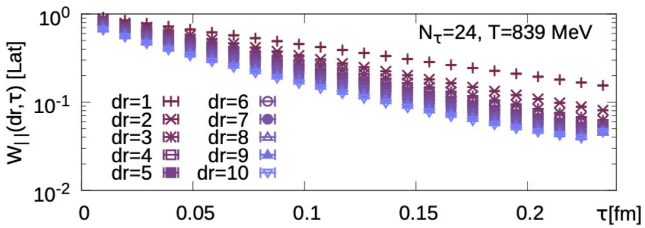

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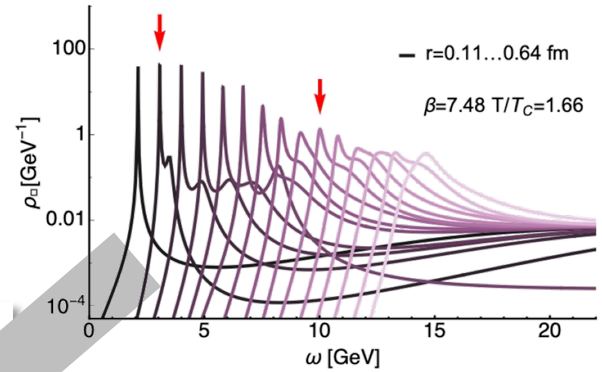

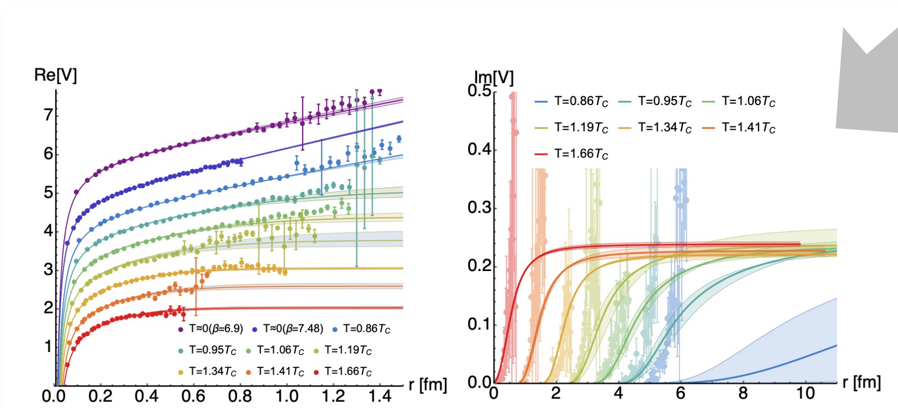

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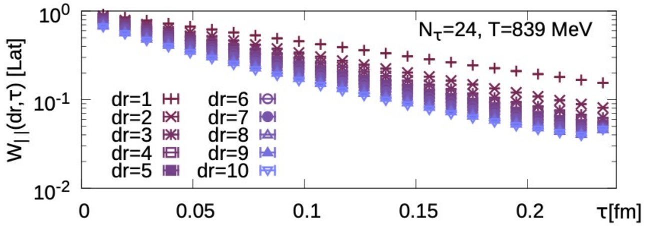

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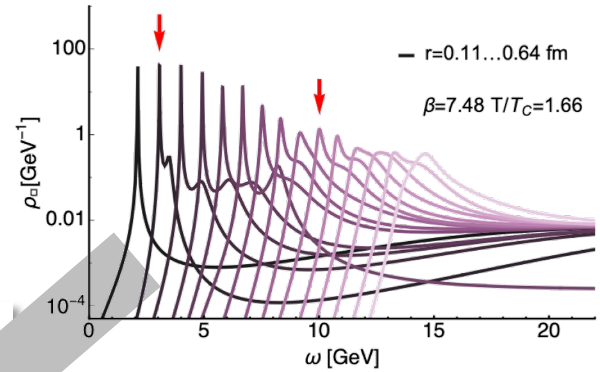

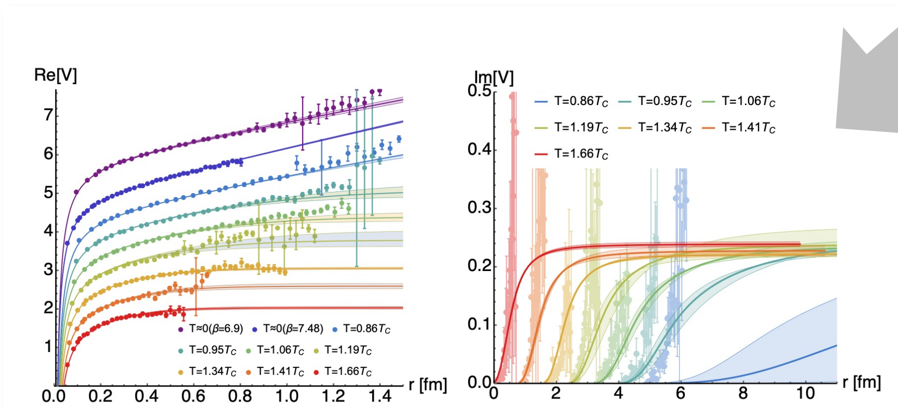

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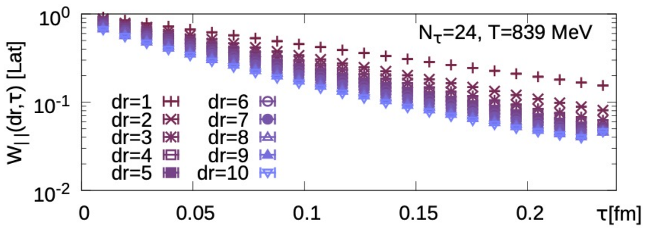

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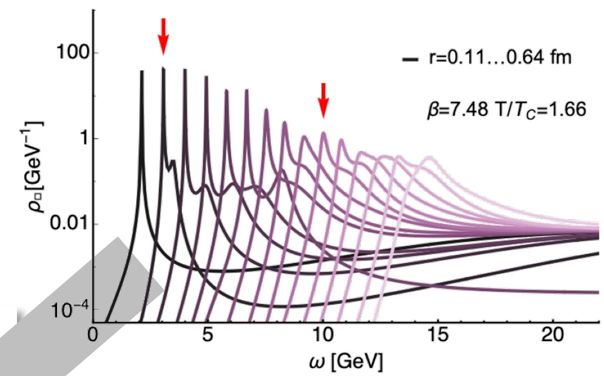

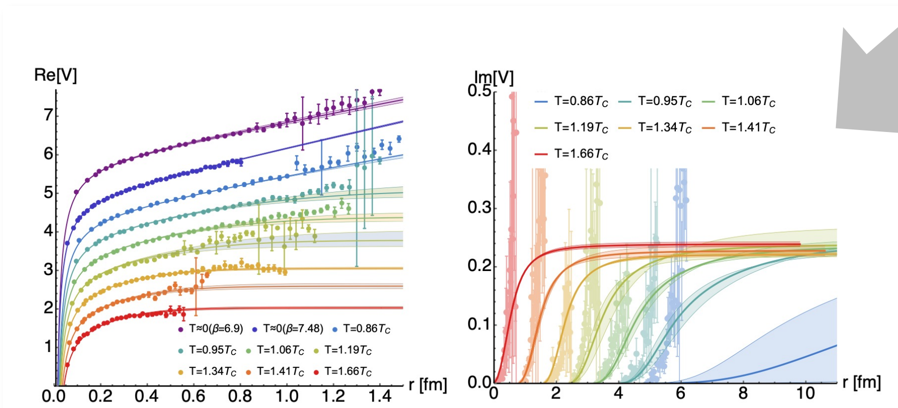

Past results in quenched QCD or full QCD with legacy discretization.

- Extracted values of Re[V] show gradual screening, Im[V]>0 in QGP phase

Static quark potential III (& a mystery)

- Latest update from **state-of-the-art lattice ensembles** with physical pion mass (N_f=2+1). [caveat: improved actions induce non-postitive spectra]
  see e.g. HotQCD PRD 97 (2018) 1, 014510

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Pade: interpolate Wilson line data and explicit analytic continuation

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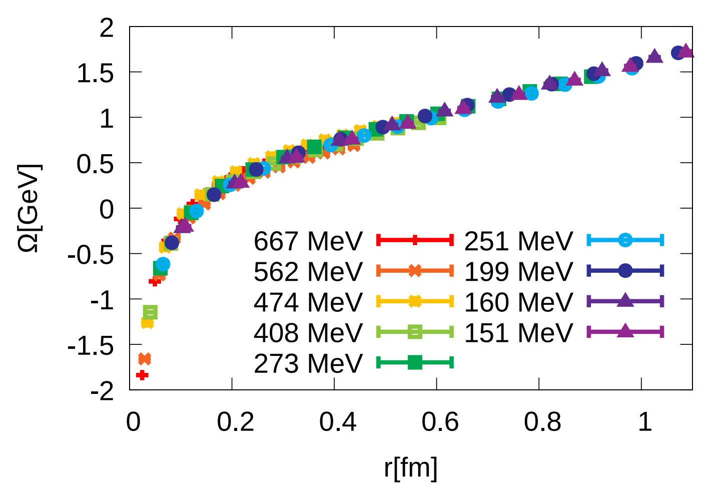
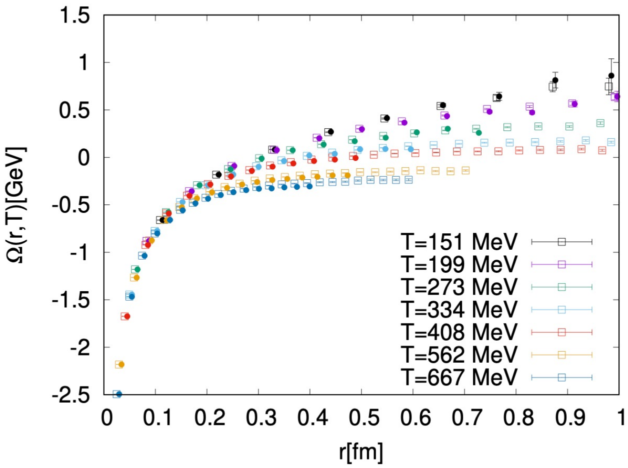


HotQCD et.al. (incl. A.R., R. Larsen, G. Parkar) PRD 105 (2022) 5, 054513



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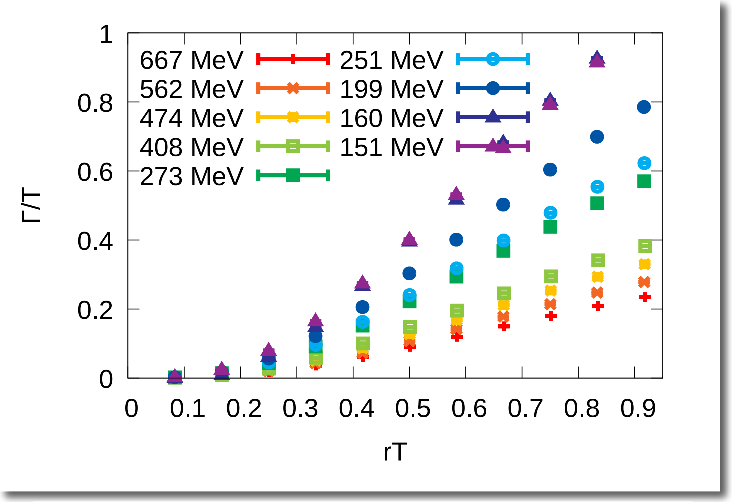

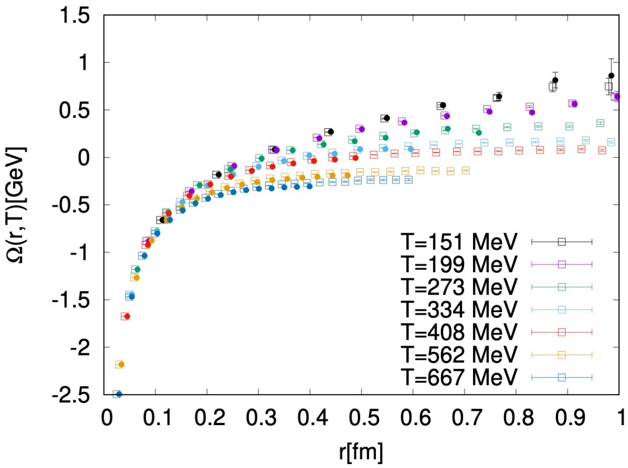

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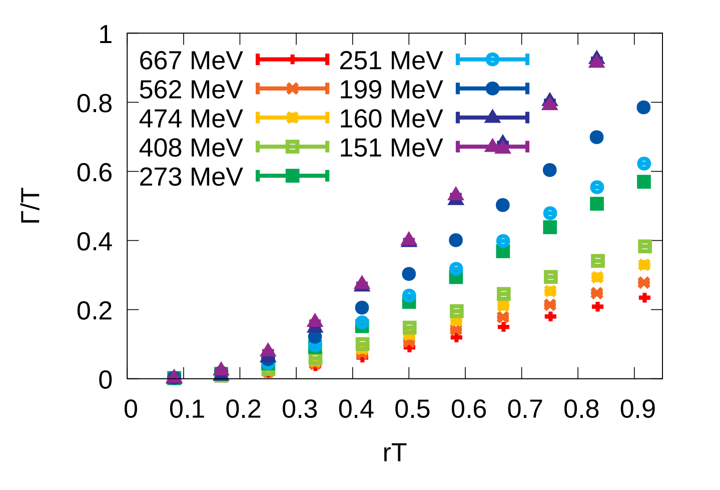

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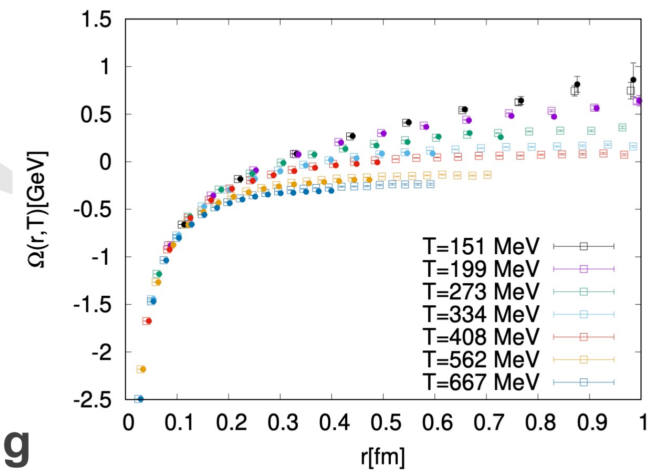

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see e.g. Bazavov et.al. PRD 98 (2018) 5, 054511

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Need to clarify this situation:

- **Rechecking results on finer lattices:** so far exactly the same outcome observed (in progress, see e.g. R. Larsen at Hard Probes 2023)
- **Comparing results using different lattice discretization:** community effort among different T>0 lattice collaborations (in progress)
- **Revisiting quenched QCD**, deploying all different methods (Bayesian, Pade, model spectral fit, HTL inspired fits) (in preparation)

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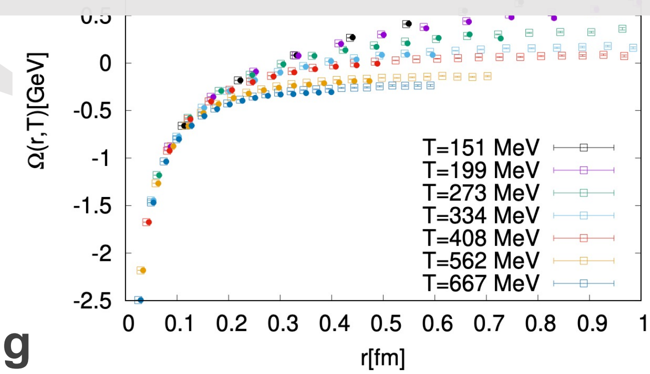

Static quark potential (class. stat.)

University of Stavanger

- **Fourier transform** of real-time Wilson loop from classical statistical simulations with a purely **gluonic medium**

M. Laine, M. Tassler JHEP 09 (2007) 066
A. Lehmann, A.R. JHEP 07 (2021) 067
K. Boguslavski, B. Kasmei, M. Strickland JHEP 10 (2021) 083

In axial gauge A_0=0 quarks only **modify Gauss' law &** affect sampling of physical **init. cond.**

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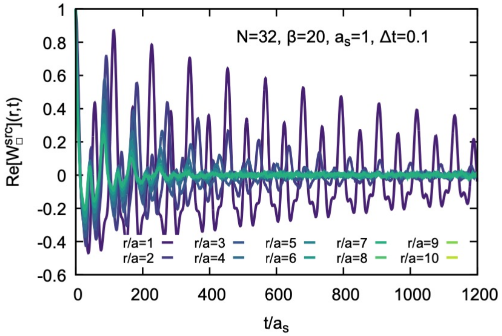
Wilson loop w/ **damped osc. behavior**: spectral peaks at ω_0>0 and width Γ_0>0



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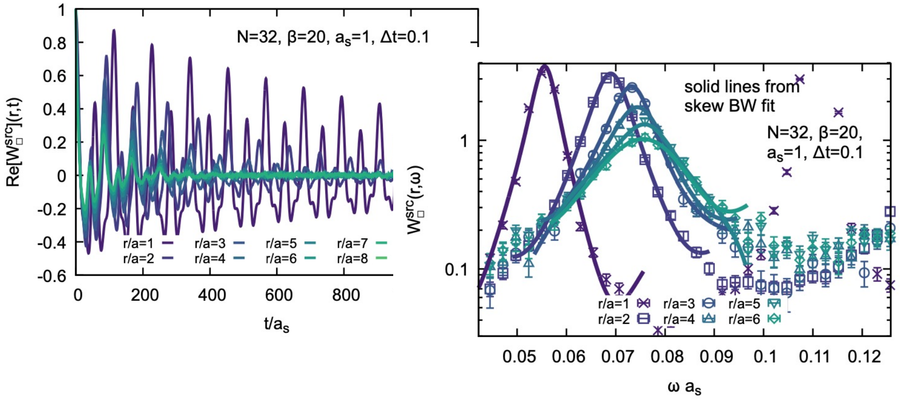

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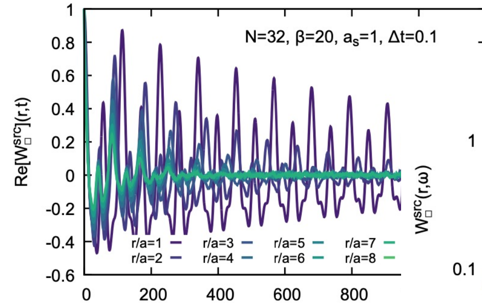

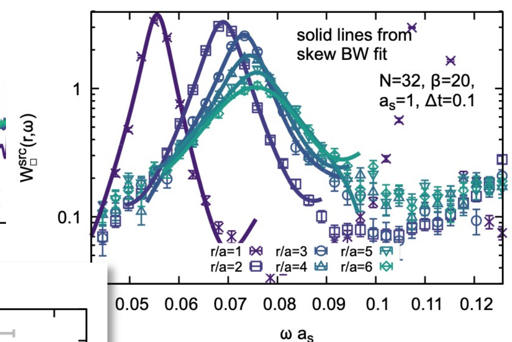

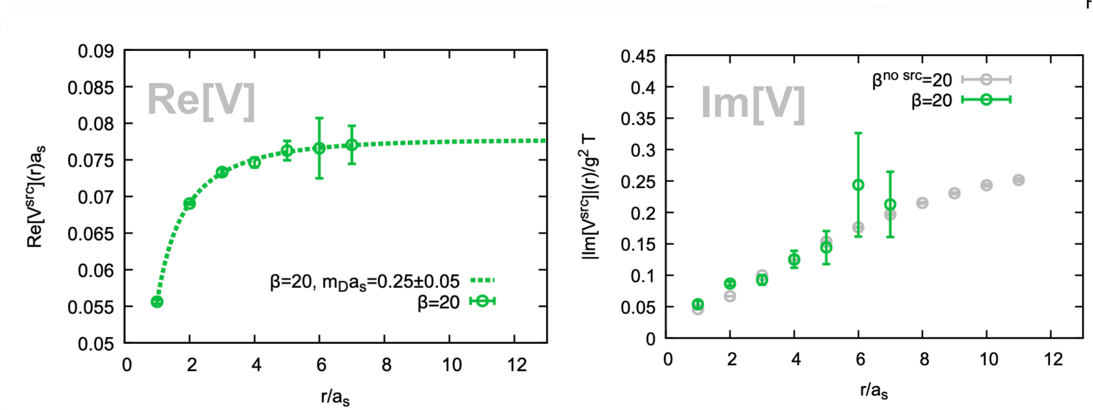

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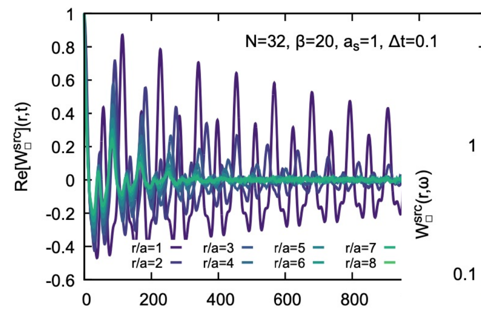

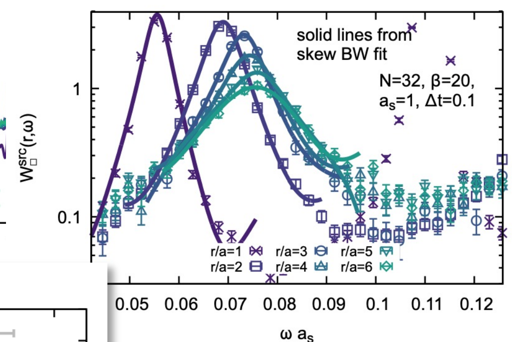

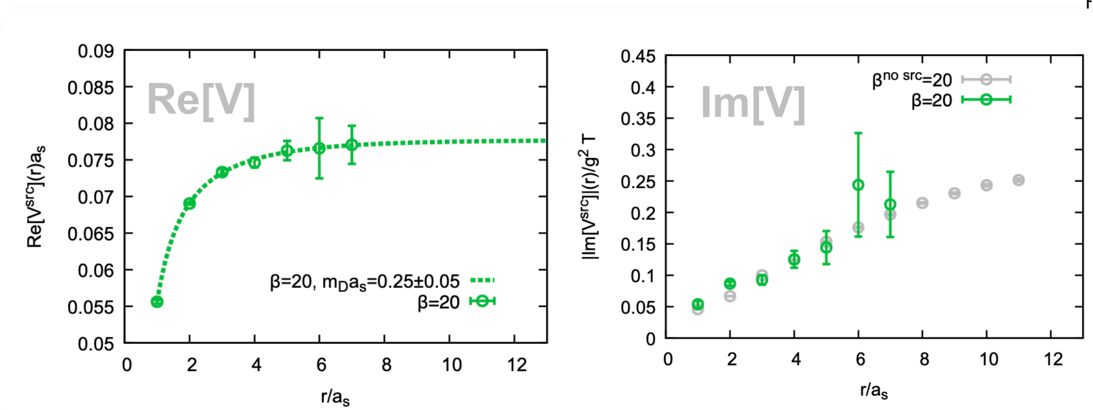

- Re[V] shows screening, Im[V]>0 present (as known from prior studies)

Part II

University of Stavanger

select recent Lattice insights on T>0 quarkonium

T>0 static potential

Euclidean quantum

Y. Burnier, O. Kaczmarek, A.R.
JHEP 12 (2015) 101
Y. Burnier, A.R. PRD 95 (2017) 5, 054511
HotQCD et.al. PRD 105 (2022) 5, 054513

classical statistical

A. Lehmann, A.R. JHEP 07 (2021) 067
K. Boguslavski, B. Kasmai, M. Strickland
JHEP 10 (2021) 083

T>0 quarkonium spectra

relativistic formulation

Y. Burnier et. al. JHEP 11 (2017) 206

using lattice EFT (NRQCD)

S. Kim, P. Petreczky, A.R. JHEP 11 (2018) 088
FASTSUM PoS LATTICE2019 (2019) 076
R. Larsen et. al. Phys.Lett.B 800 (2020) 135119

Transport coefficients

heavy quark diffusion

N. Brambilla et.al. PRD 102 (2020) 7, 074503
L. Altenkort et.al. PRD 103 (2021) 1, 014511
TUMQCD PRD 107 (2023) 5, 054508

Sommerfeld enhancement

Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

In-medium bound states (relativistic)

University of Stavanger

- **Modelling** of (η_c,η_b) spectral function via fit to **continuum extrapolated** Euclidean correlators on **high resolution** lattices of a **gluonic medium**

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UV part of spectral function
high order perturbative computation
careful choice of renorm. scheme

Threshold part of spectral function
weak-coupling potential
(vacuum at small r, thermal at interm. r

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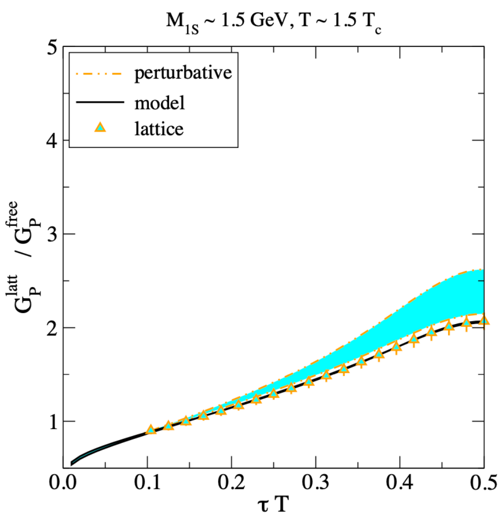

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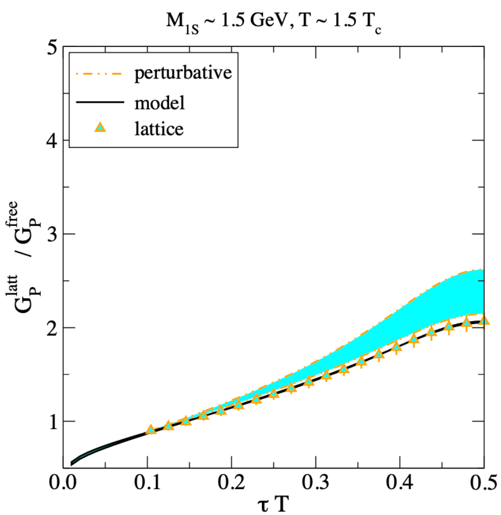

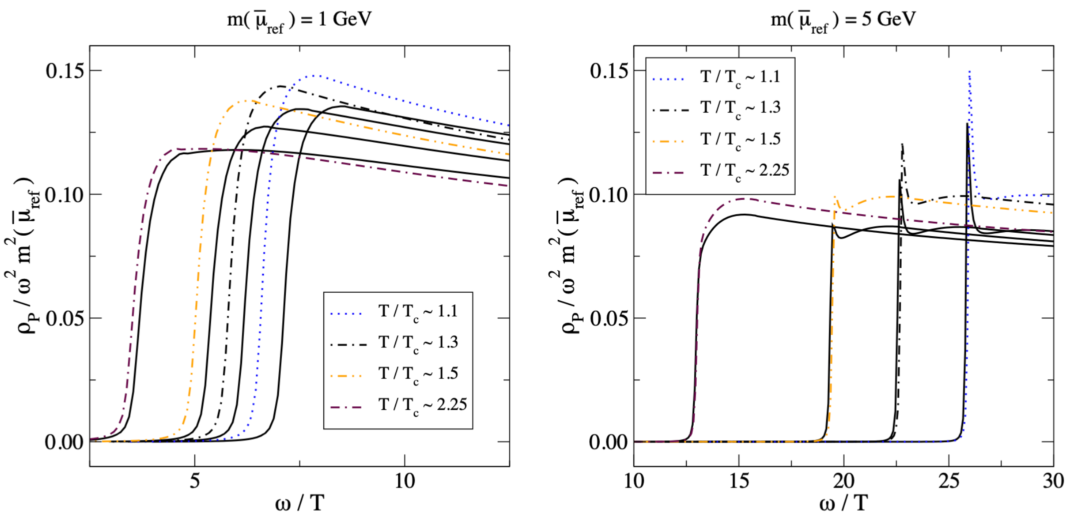

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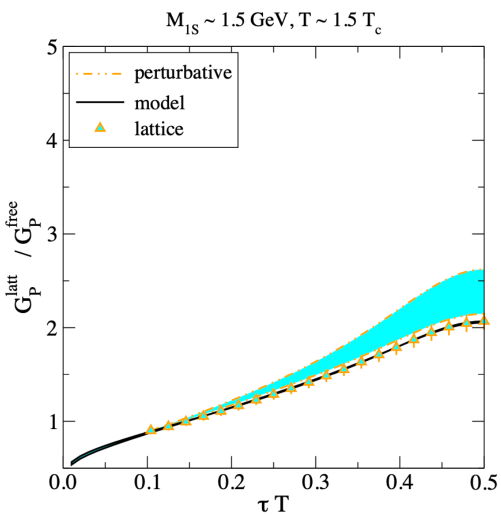

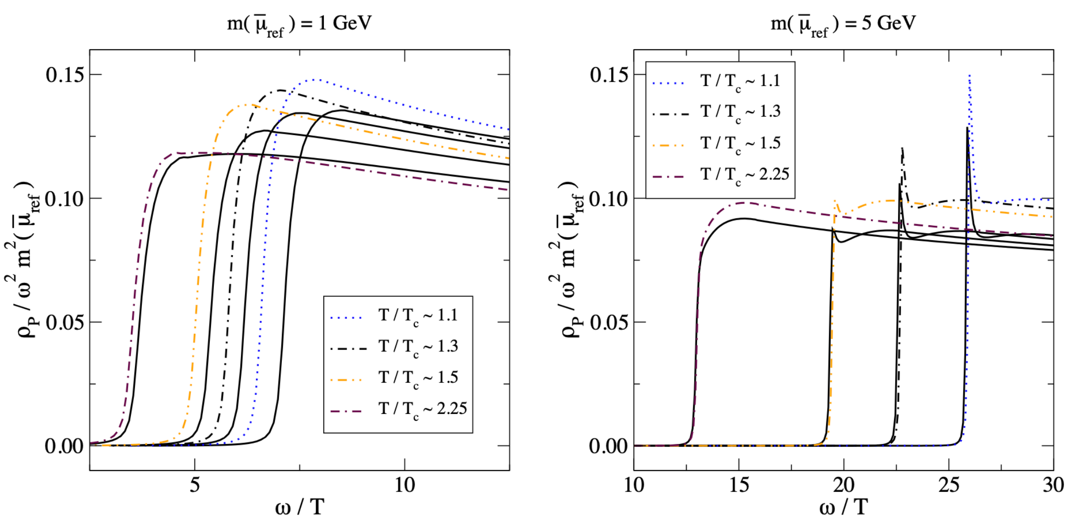

- Excellent agreement with lattice data & relatively rapid melting

In-medium bound states (non-rel.)

University of Stavanger

- **Bayesian reconstruction** of spectral functions from Euclidean **NRQCD correlators** on lattices with **realistic medium** d.o.f. (N_f=2+1) S.Kim, P. Petreczky, A.R. JHEP 11 (2018) 088

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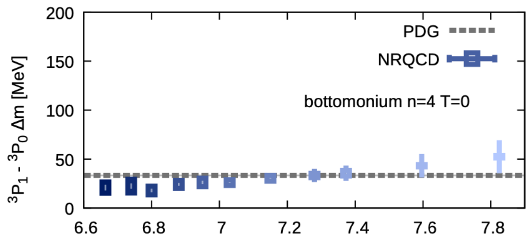

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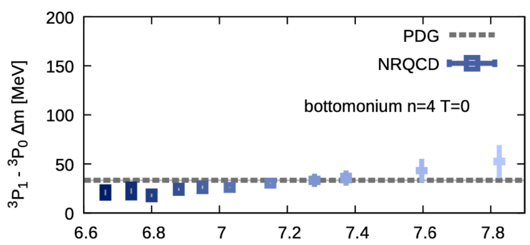
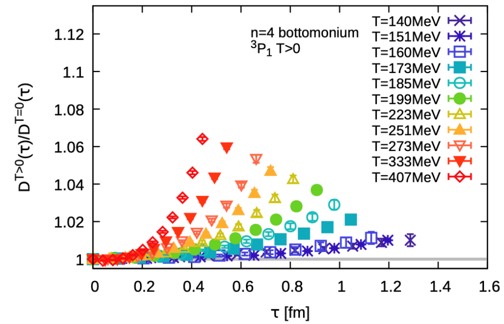


Test via spectral function models
ratio of T=0 and T>0 correlator: ground state peak moves to smaller ω

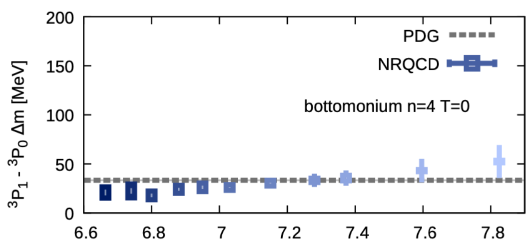


In-medium bound states (non-rel.)

University of Stavanger

- **Bayesian reconstruction** of spectral functions from Euclidean **NRQCD correlators** on lattices with **realistic medium** d.o.f. (N_f=2+1) S.Kim, P. Petreczky, A.R. JHEP 11 (2018) 088

T=0 mass splitting:
establish that NRQCD approximation works well at these lattice spacings



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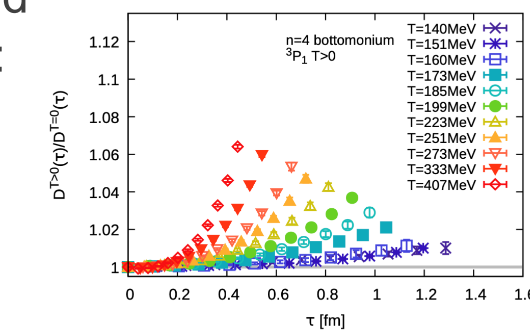
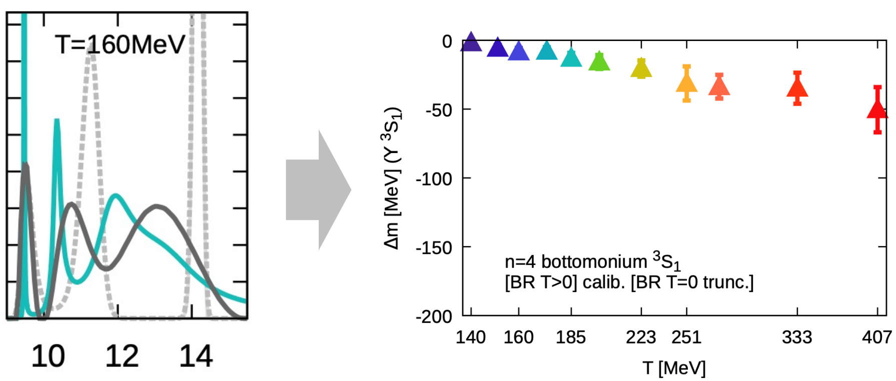


Careful study of reconstruction artifacts: Bayesian T>0 and T=0 results can only be compared at same Euclidean time extent.



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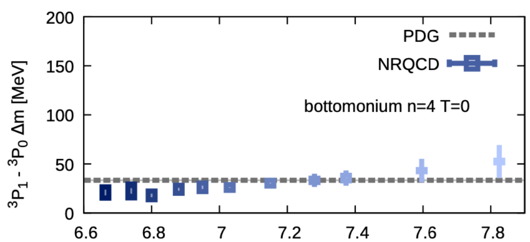

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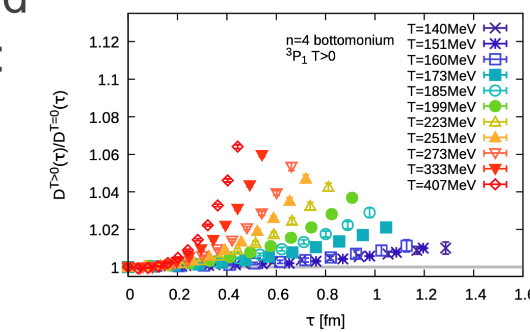

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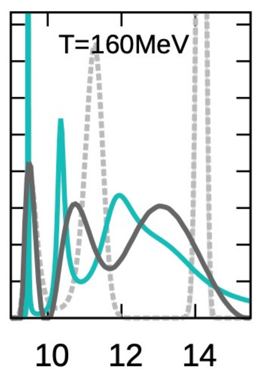

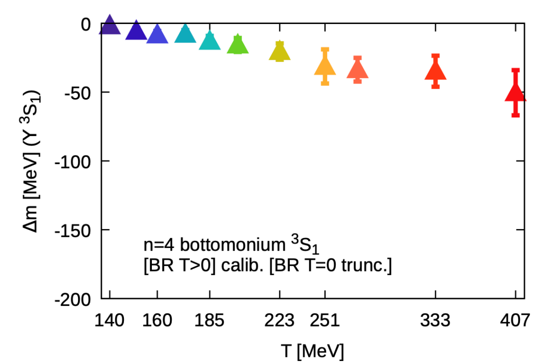

- Disappearance of peaks consistent with FASTSUM & negative mass shift, no access to T>0 excited states from first principles c.f. FASTSUM JHEP 07 (2014) 097

In-medium bound states (non-rel.) II

University of Stavanger

- **Modelling** of spectral function via fit to **NRQCD** Euclidean correlators w/ **pNRQCD model input** on lattices with **realistic medium** d.o.f. (N_f=2+1)

R. Larsen et.al. PLB 800 (2020) 135119

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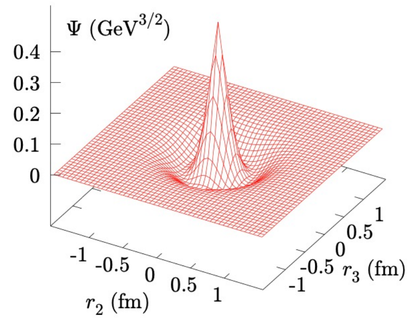
Potential model wavefunctions

attempt to project out ground or excited state contribution in T>0 Euclidean correlators



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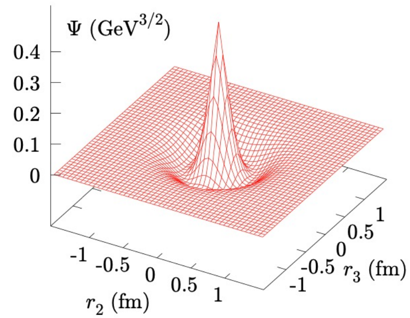

Subtract unknown UV contrib
Assuming high ω spectral function is T independent, subtract on level of correlators

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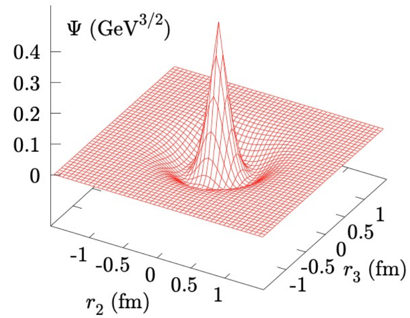

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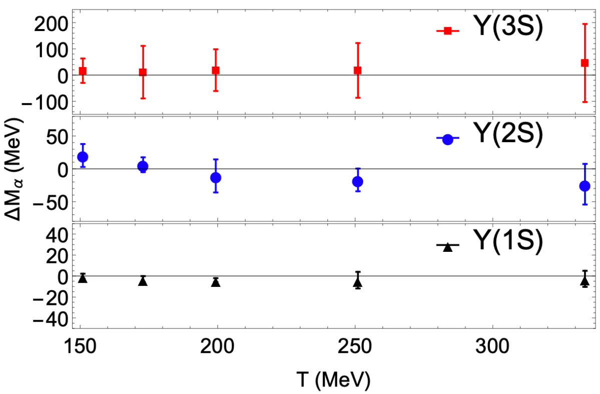

Gaussian model spectral function

with extra delta peak to capture Lorentzian tail.

$$\rho_\alpha^{\rm med}(\omega,T) = A_\alpha^{\rm cut}(T)\,\delta\left(\omega-\omega_\alpha^{\rm cut}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega-M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$$

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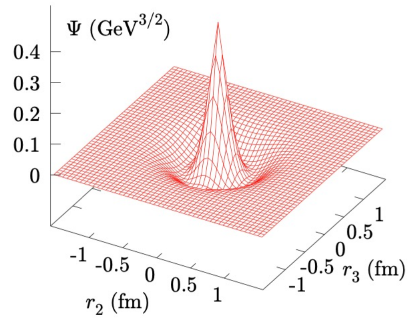

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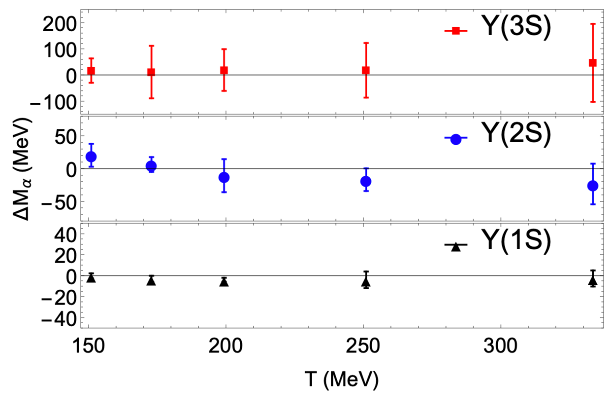

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- No change in the in-medium masses within estimated error budget.

Part III

University of Stavanger

select recent Lattice insights on T>0 quarkonium

T>0 static potential

Euclidean quantum

Y. Burnier, O. Kaczmarek, A.R.
JHEP 12 (2015) 101
Y. Burnier, A.R. PRD 95 (2017) 5, 054511
HotQCD et.al. PRD 105 (2022) 5, 054513

classical statistical

A. Lehmann, A.R. JHEP 07 (2021) 067
K. Boguslavski, B. Kasmai, M. Strickland
JHEP 10 (2021) 083

T>0 quarkonium spectra

relativistic formulation

Y. Burnier et. al. JHEP 11 (2017) 206

using lattice EFT (NRQCD)

S. Kim, P. Petreczky, A.R. JHEP 11 (2018) 088
FASTSUM PoS LATTICE2019 (2019) 076
R. Larsen et. al. Phys.Lett.B 800 (2020) 135119

Transport coefficients

heavy quark diffusion

N. Brambilla et.al. PRD 102 (2020) 7, 074503
L. Altenkort et.al. PRD 103 (2021) 1, 014511
TUMQCD PRD 107 (2023) 5, 054508

Sommerfeld enhancement

Kim Laine JHEP 07 (2016) 143,
Biondini, Kim, Laine JCAP 10 (2019) 078

Heavy quark diffusion coefficient

- **Modelling** of spectral function via fit to **continuum extrapolated effective** Euclidean correlators on **gluonic medium** lattices

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Transport peak in standard $Q\bar{Q}$ spectra **too sharp and small:** use **effective operator** with same transport physics

S. Caron-Huot, M. Laine, G. Moore JHEP 04 (2009) 053

$$G_E(\tau) = -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\beta,\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau,0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}\left[U(\beta,0)\right]\right\rangle}$$

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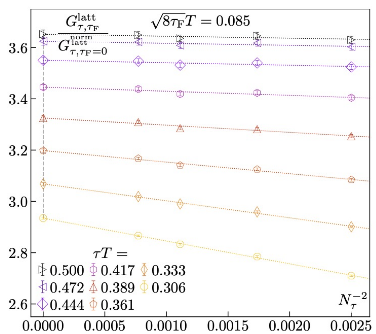
Continuum extrapolation using high precision data from **Gradient Flow** or **Multilevel algorithm**



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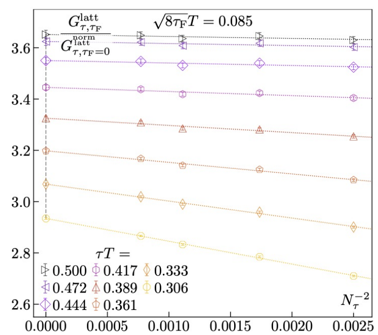
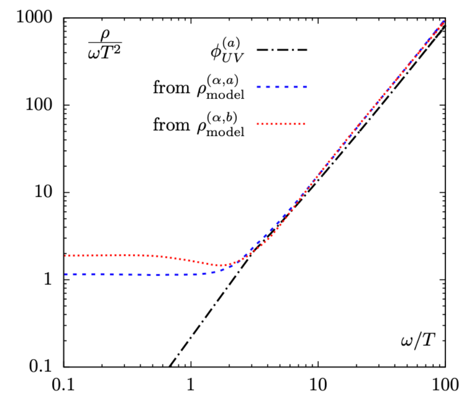


UV spectra from perturbation theory
IR part from diffusion process,
weighted for fit to Euclidean correlator

$$\rho^{(UV)}(\omega) = \frac{g^2 C_F \omega^3}{6\pi} \qquad \rho^{(IR)}(\omega) = \frac{\kappa\omega}{2T}$$



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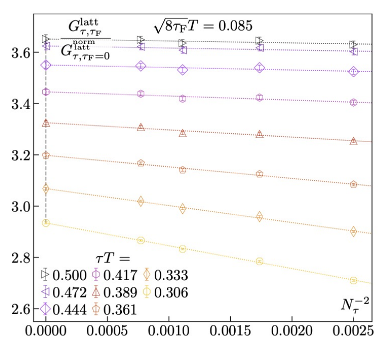

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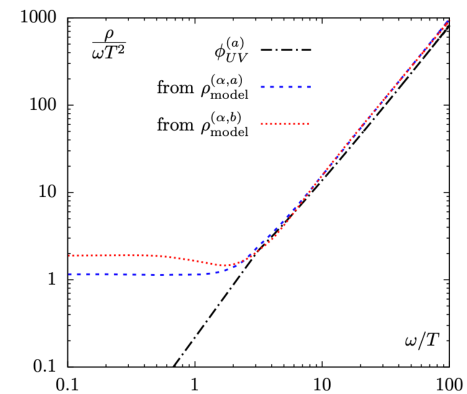


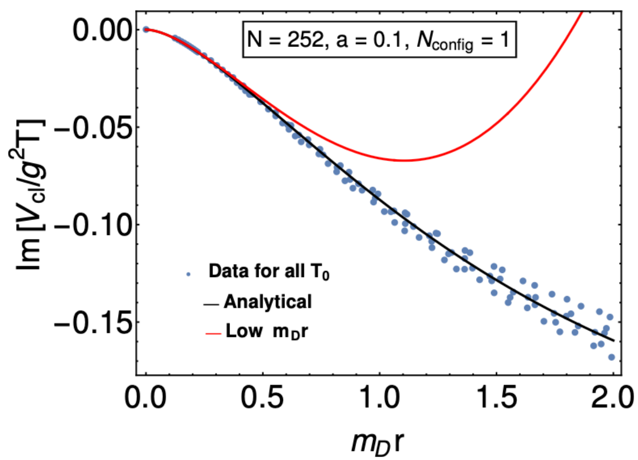
- Using different model ansätze and fitting ranges: $\kappa/T^3 = 2.31\ldots3.70$

Heavy quark transport II

- Exploit relation of transport coefficients to other quantities that are accessible on the lattice

Small distance behavior of Im[V] can be related to κ in HTL perturbation theory. (here classical statistical sims.)

K. Boguslavski, B. Kasmei, M. Strickland JHEP 10 (2021) 083



$$\kappa_V^{\rm latt} \approx C_F\, g^2 T m_{D,\rm HCL}^2 \left(0.173 \log\left(\frac{T}{m_{D,\rm HCL}}\right) - 0.023\right)$$

Masss shifts and decay widths can be related to κ and γ in EFT for Coulombic quarkonium states

N. Brambilla et.al. PRD 100 (2019) 5, 054025

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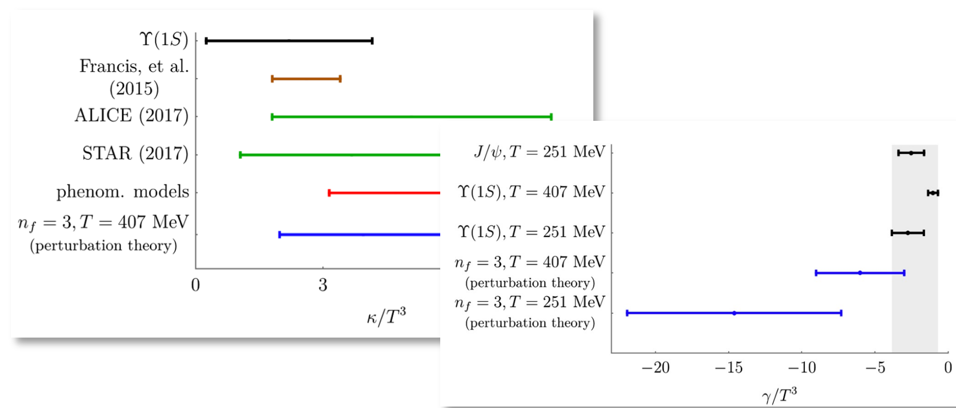

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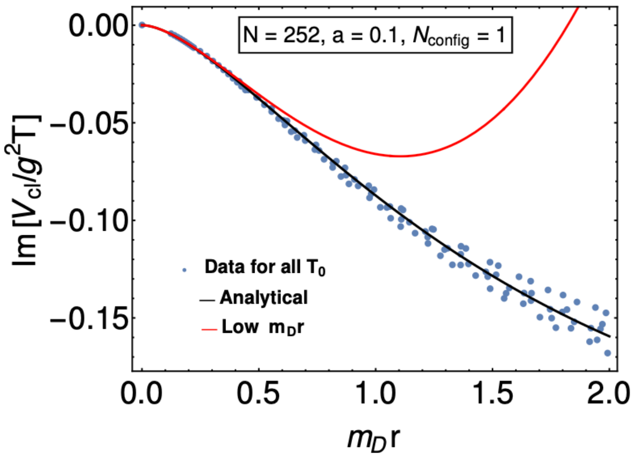

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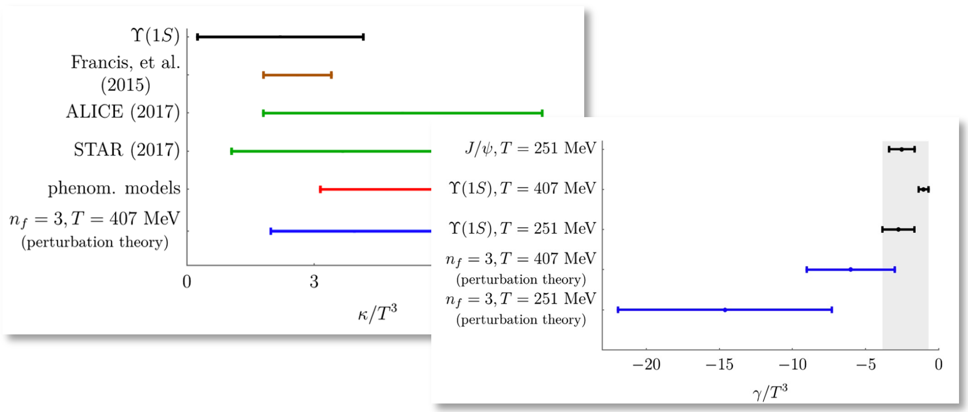

- Constraints to well known & recently identified transport coefficients

Conclusion

University of Stavanger

- Lattice QCD can provide non-perturbative and first principles insight into heavy quark **in-medium bound states & transport**
- Phenomenologically relevant quarkonium physics requires access to real-time dynamics: **spectral functions**
- Community entered an era of **high precision Euclidean data:** spectral function modelling and direct spectral reconstruction
- Some unexpected findings requiring community effort: apparent absence of screening in Re[V] (despite screening of F^1) and absence of mass shifts in recent studies on HISQ lattices.

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Thank you for your attention

Bayesian spectral reconstruction

University of Stavanger

- Inversion of Laplace transform required to obtain spectra from correlators

$$D(\tau) = \int_{-2M_Q}^{\infty} d\omega e^{-\tau\omega} \rho(\omega)$$

Bayesian spectral reconstruction

University of Stavanger

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$$D_i = \sum_{l=1}^{N_\omega} \exp[-\omega_l \tau_i]\, \rho_l\, \Delta\omega_l$$

1. N_ω parameters ρ_l >> N_τ datapoints
2. simulation input D_i has finite precision

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- Bayes: regulate χ^2 fit (P[D|ρ]=exp[-L]) with prior information (P[ρ|I]=exp[S])

$$P[\rho|D,I] \propto P[D|\rho]\; P[\rho|I] \qquad \left.\frac{\delta}{\delta\rho} P[\rho|D,I]\right|_{\rho=\rho^{BR}} = 0$$

for standard MEM see e.g.
Asakawa, Hatsuda, Nakahara
Prog.Part.Nucl.Phys. 46 (2001) 459

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Standard BR method (BR)

$$S_{BR} = \alpha \int d\omega \left(1 - \frac{\rho}{m} + \log\left[\frac{\rho}{m}\right]\right)$$

- Resolves narrow peaked structures with high accuracy
- Ringing in broad structures if reconstructed from small # of datapoints

Y. Burnier, A.R. PRL 111 (2013) 182003

„high gain – high noise“

Low ringing BR method (BR_l)

$$S_{BR_\ell} = \alpha \int d\omega \left(\kappa\left(\frac{\partial\rho}{\partial\omega}\right)^2 + 1 - \frac{\rho}{m} + \log\left[\frac{\rho}{m}\right]\right)$$

- Introduces penalty on arc length of reconstruction $(dL/dw)^2=1+(d\rho/dw)^2$
- Efficiently removes ringing but may lead to overestimated peak widths

c.f. C. Fsicher et.al. PRD98 (2018) 014009

„low gain – low noise“