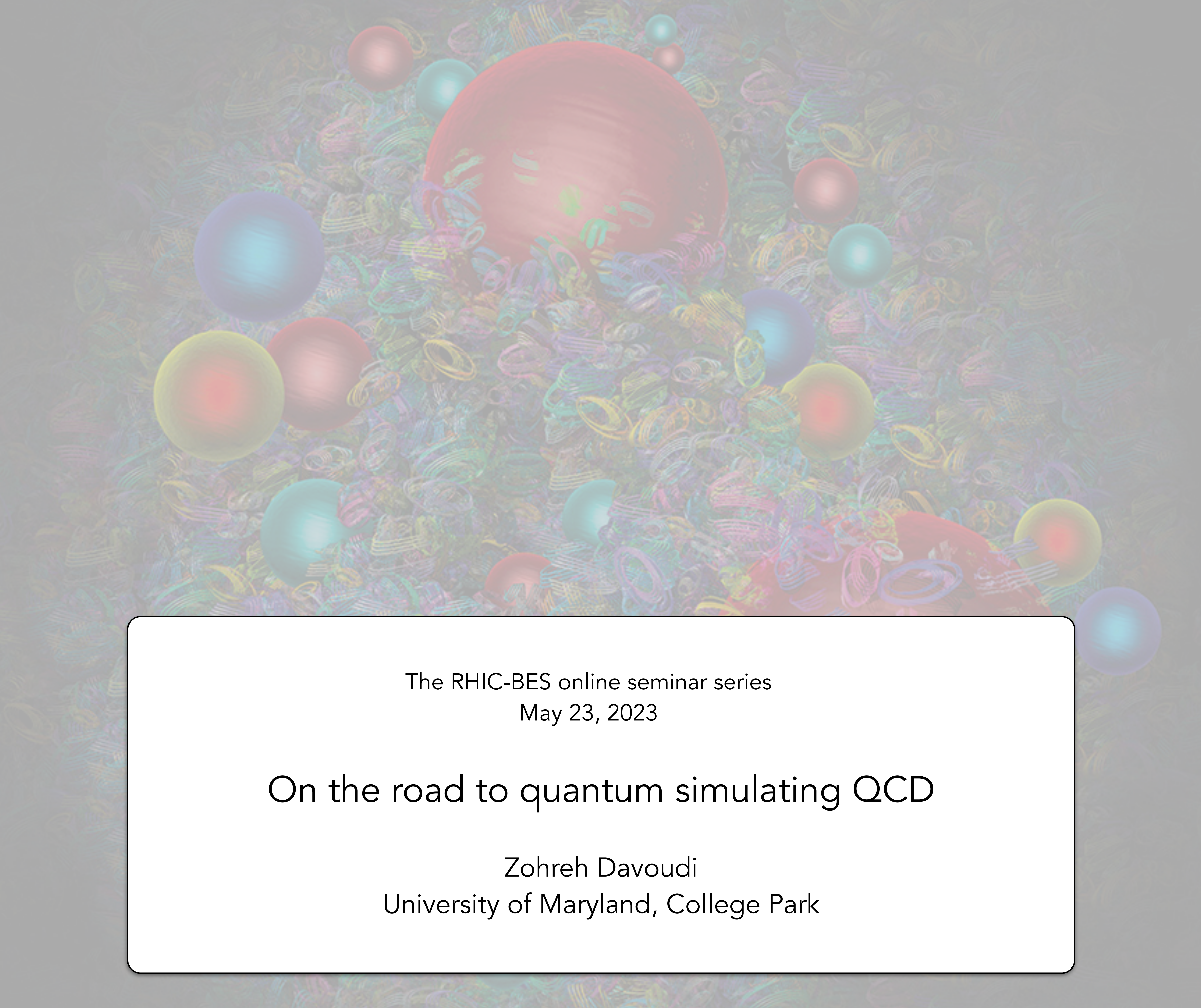
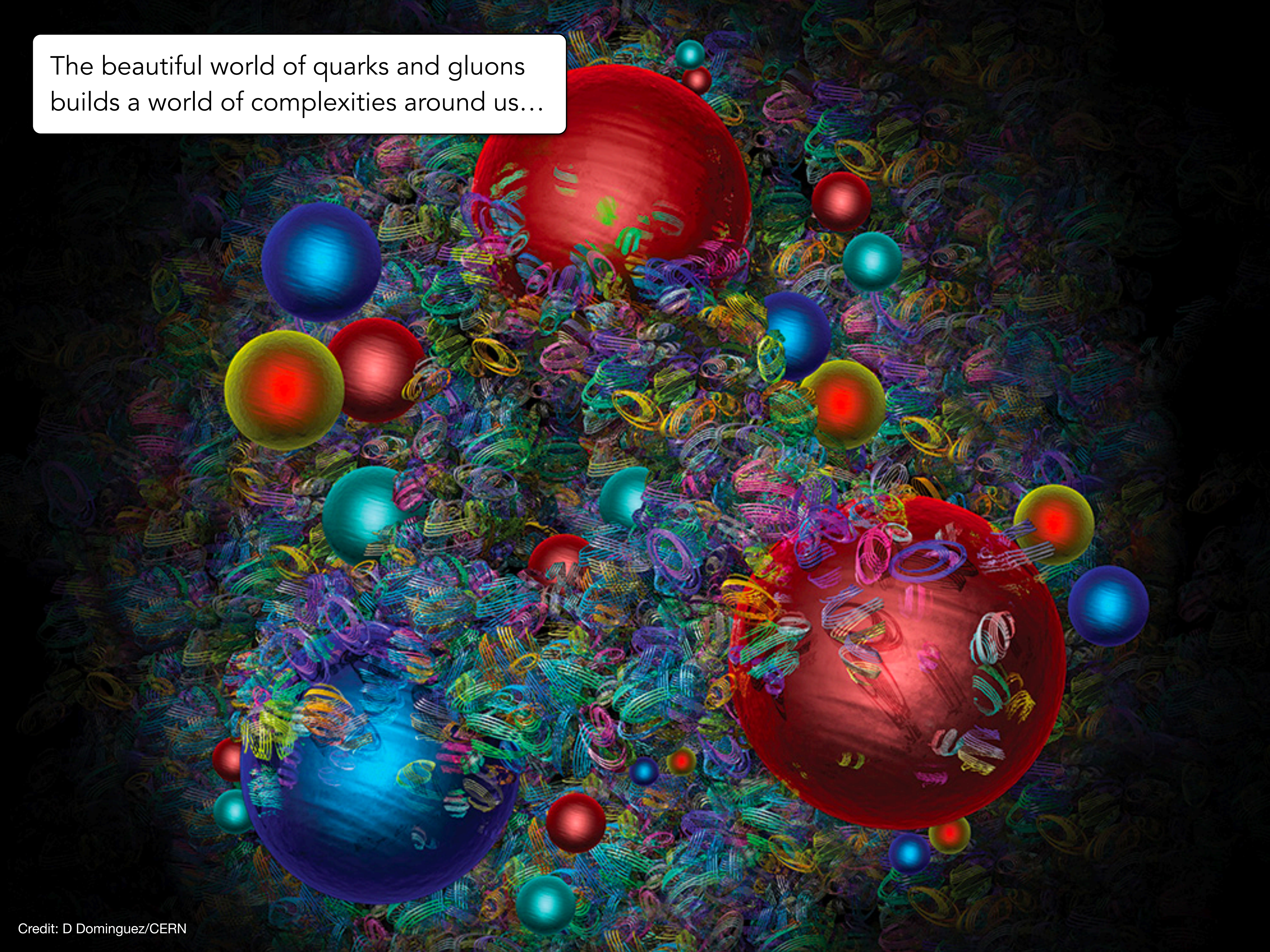


The RHIC-BES online seminar series
May 23, 2023

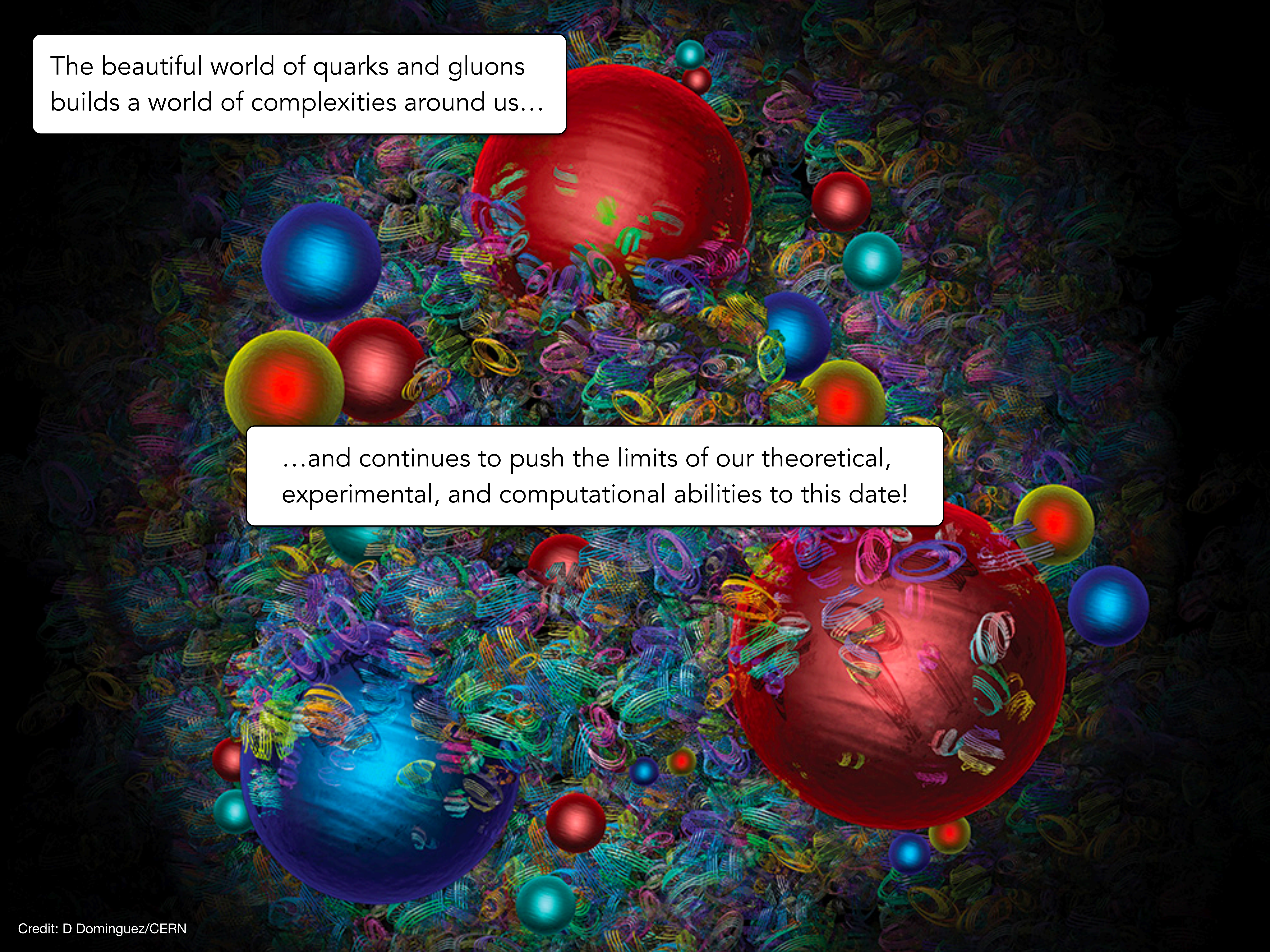
On the road to quantum simulating QCD

Zohreh Davoudi
University of Maryland, College Park

The beautiful world of quarks and gluons builds a world of complexities around us…



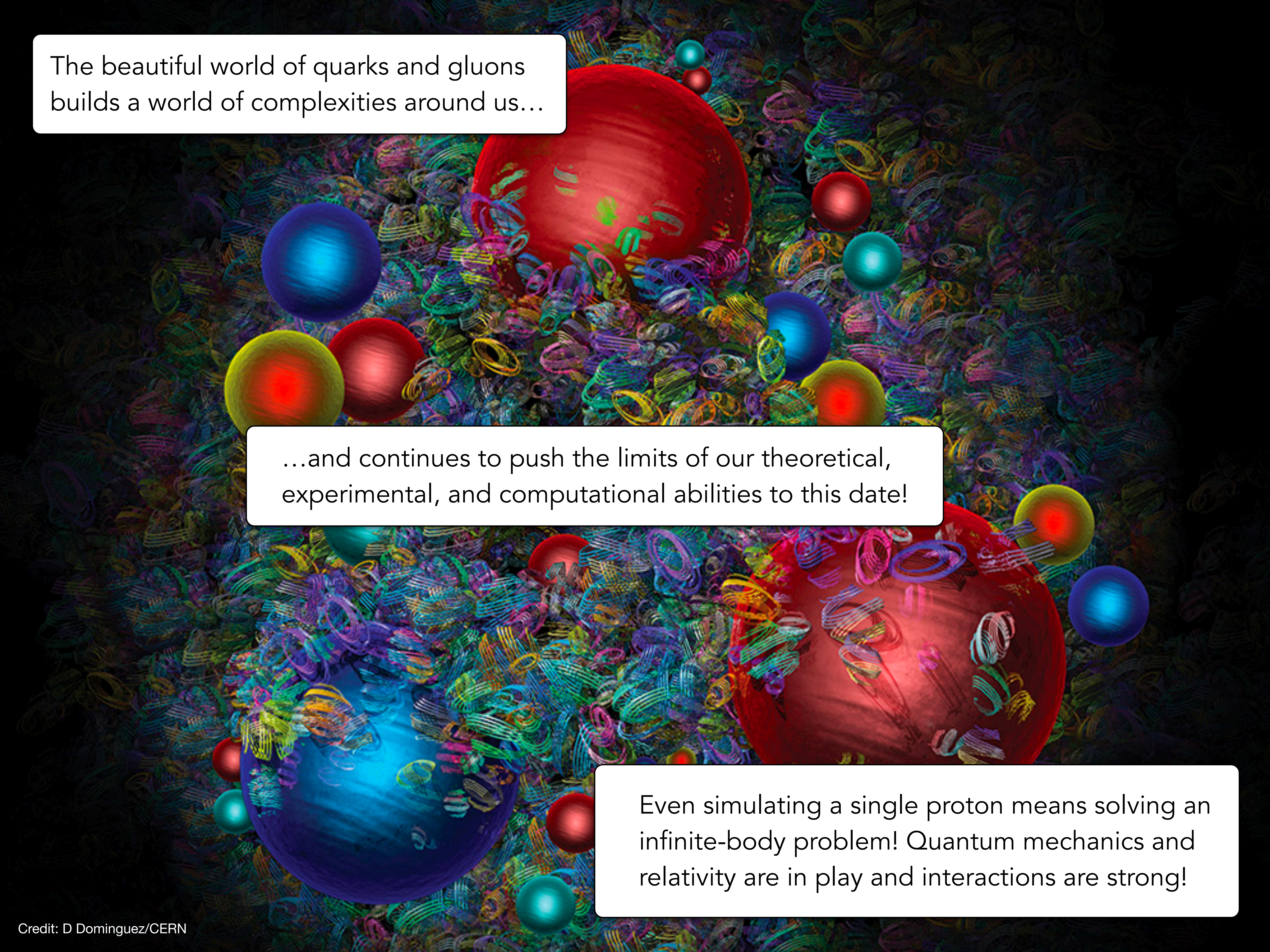
Credit: D Dominguez/CERN



The beautiful world of quarks and gluons builds a world of complexities around us…

…and continues to push the limits of our theoretical, experimental, and computational abilities to this date!

Credit: D Dominguez/CERN



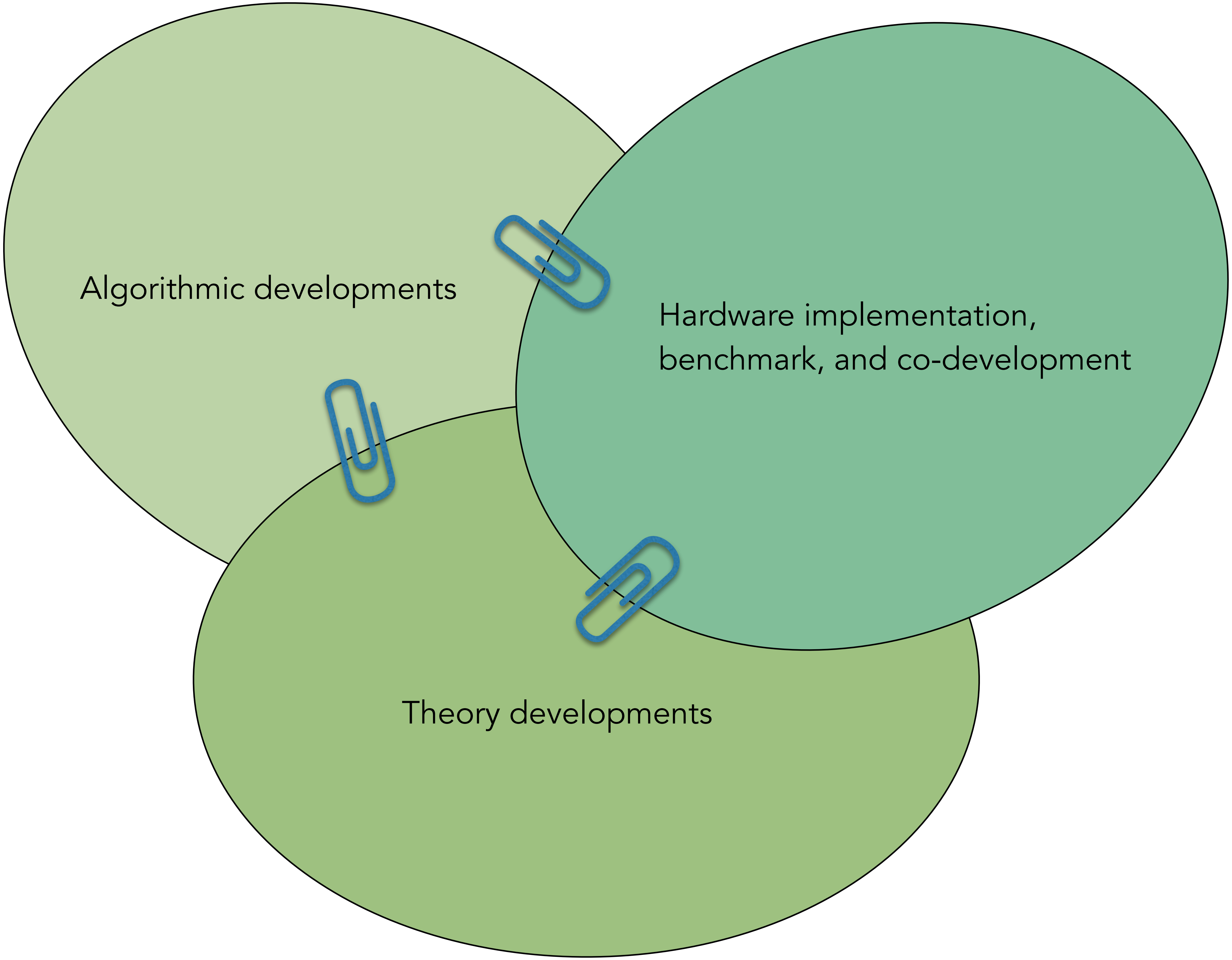
The beautiful world of quarks and gluons builds a world of complexities around us…

…and continues to push the limits of our theoretical, experimental, and computational abilities to this date!

Even simulating a single proton means solving an infinite-body problem! Quantum mechanics and relativity are in play and interactions are strong!

Credit: D Dominguez/CERN

LATTICE QCD: A MULTI-PRONG PROGRAM THAT SIMULATES QCD NON-PERTURBATIVELY



Algorithmic developments

Hardware implementation,
benchmark, and co-development

Theory developments

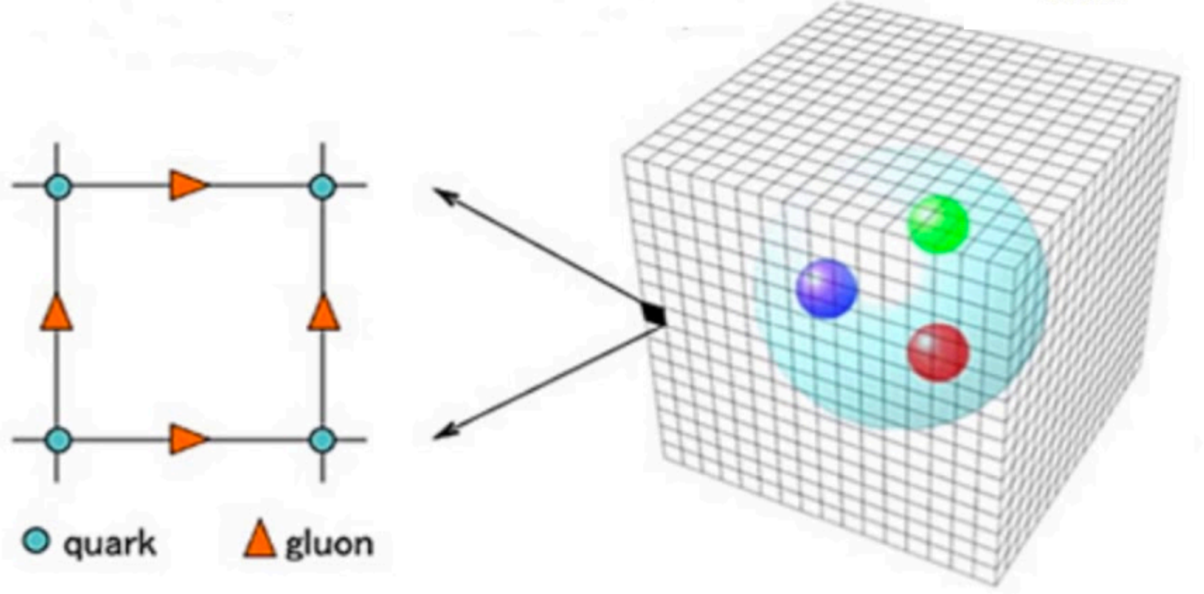
Theory developments

How to define QCD/Standard Model on a finite grid?

How to preserve/recover symmetries, e.g., gauge symmetry, chiral symmetry, rotational symmetry?

How to take infinite-volume and continuum limits? How to quantify systematics?

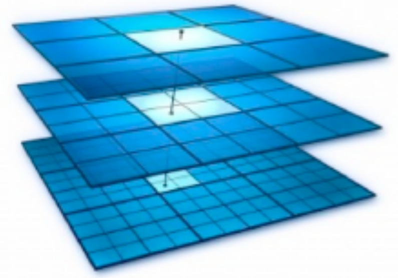
How to obtain scattering amplitudes?



Theory developments

Algorithmic developments

Algorithmic developments



e.g. How to solve **optimally** ?
A.x=b

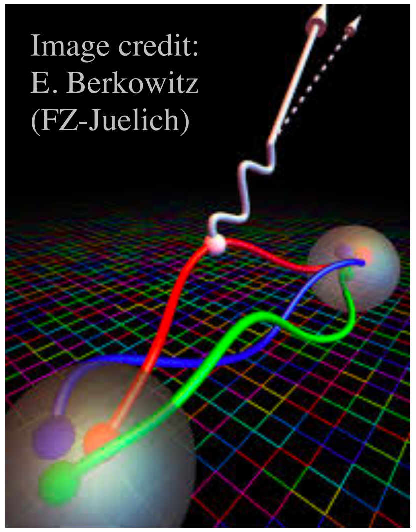


Image credit:
E. Berkowitz
(FZ-Juelich)

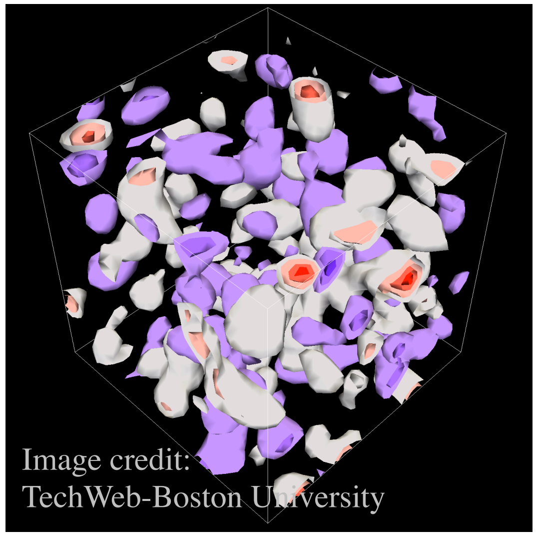


Image credit:
TechWeb-Boston University

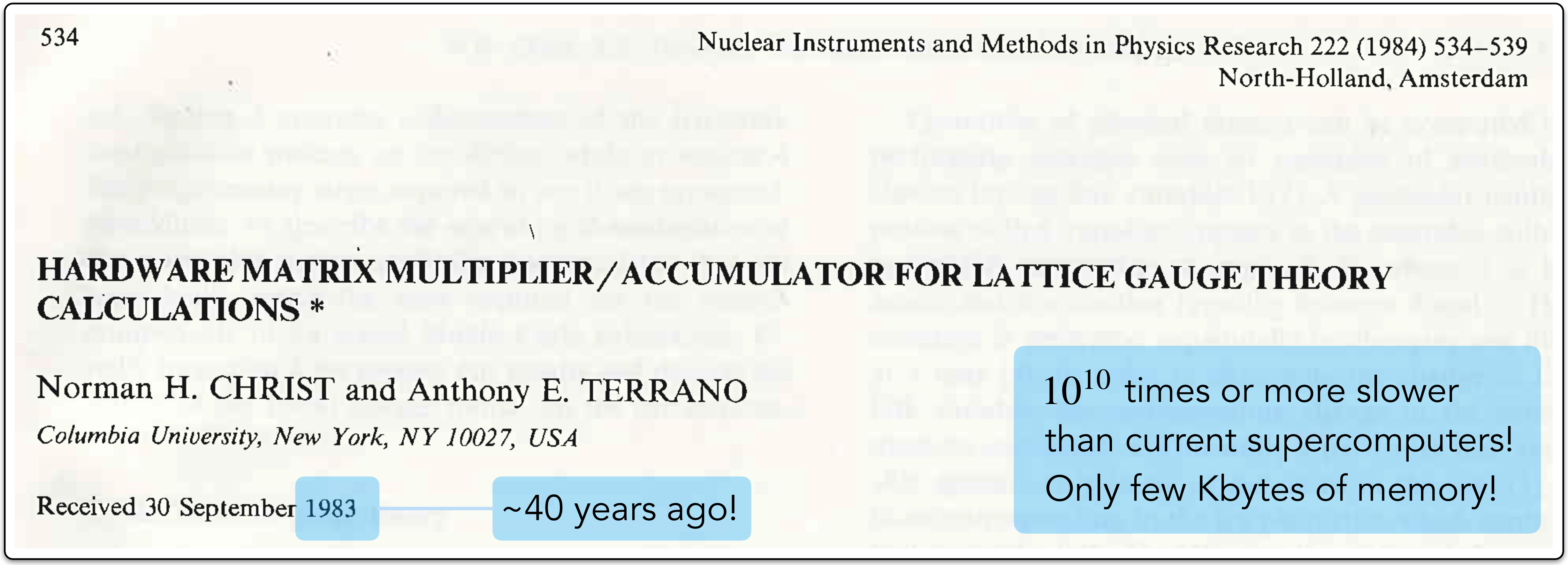
How to importance sample vacuum gauge-filed configurations?

How to evaluate quark propagators (invert large matrices)?

How to contract quarks and form correlation functions efficiently?

Hardware implementation, benchmark, and co-development

QCD ON A CHIP!



534

Nuclear Instruments and Methods in Physics Research 222 (1984) 534–539
North-Holland, Amsterdam

HARDWARE MATRIX MULTIPLIER / ACCUMULATOR FOR LATTICE GAUGE THEORY CALCULATIONS *

Norman H. CHRIST and Anthony E. TERRANO

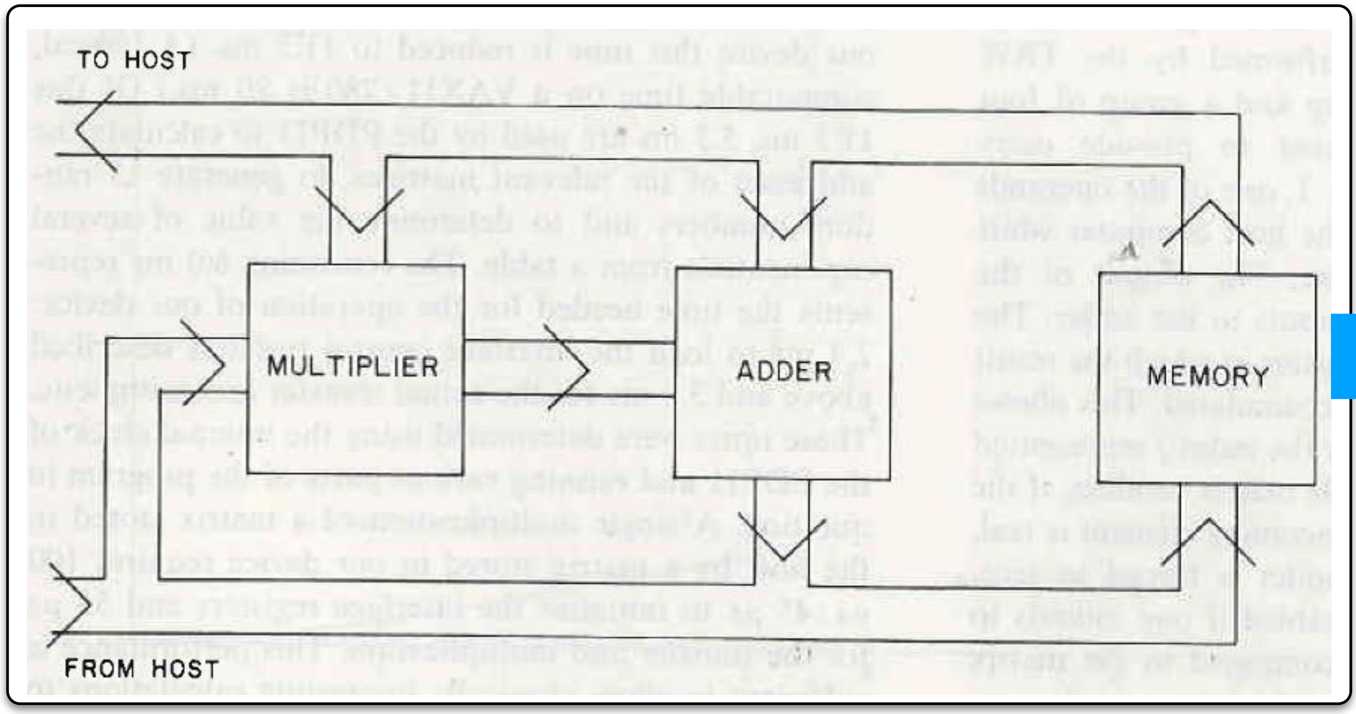
Columbia University, New York, NY 10027, USA

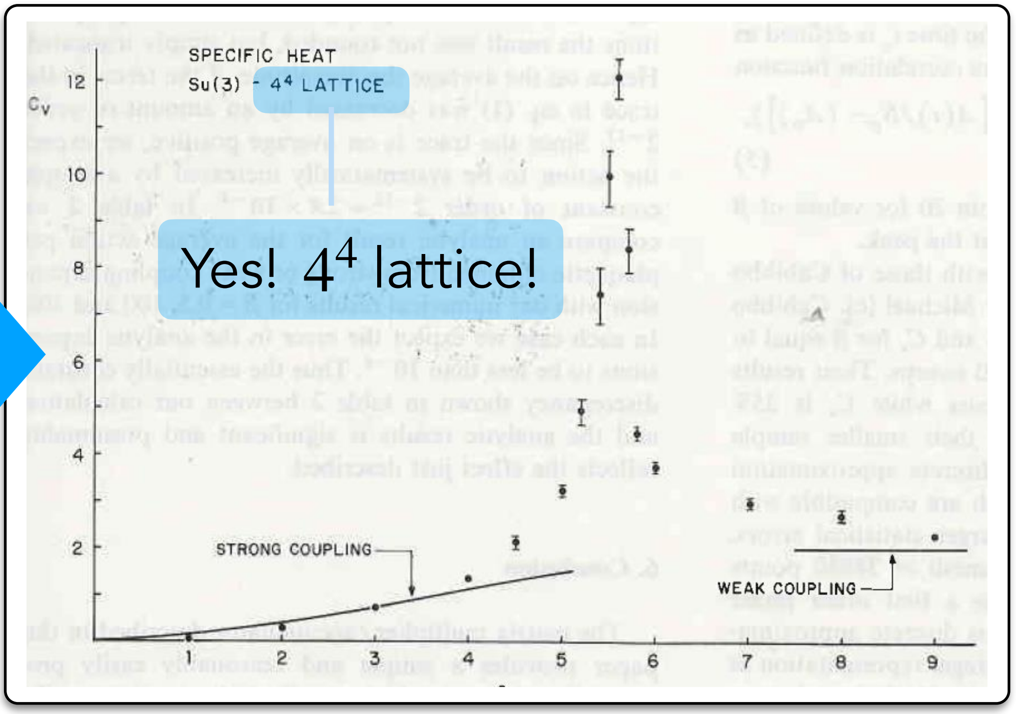
Received 30 September 1983

~40 years ago!

10^{10} times or more slower than current supercomputers!

Only few Kbytes of memory!





Yes! 4^4 lattice!

Hardware implementation, benchmark, and co-development

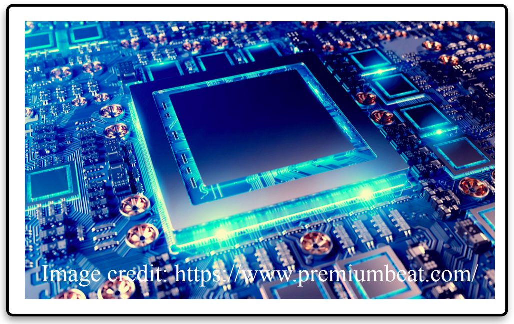


Image credit: https://www.premiumbeat.com/

Image credit: equinox.com

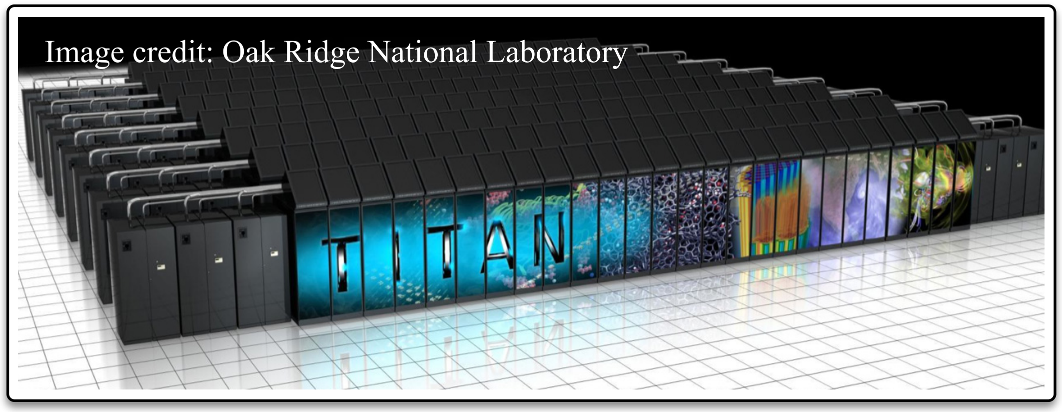


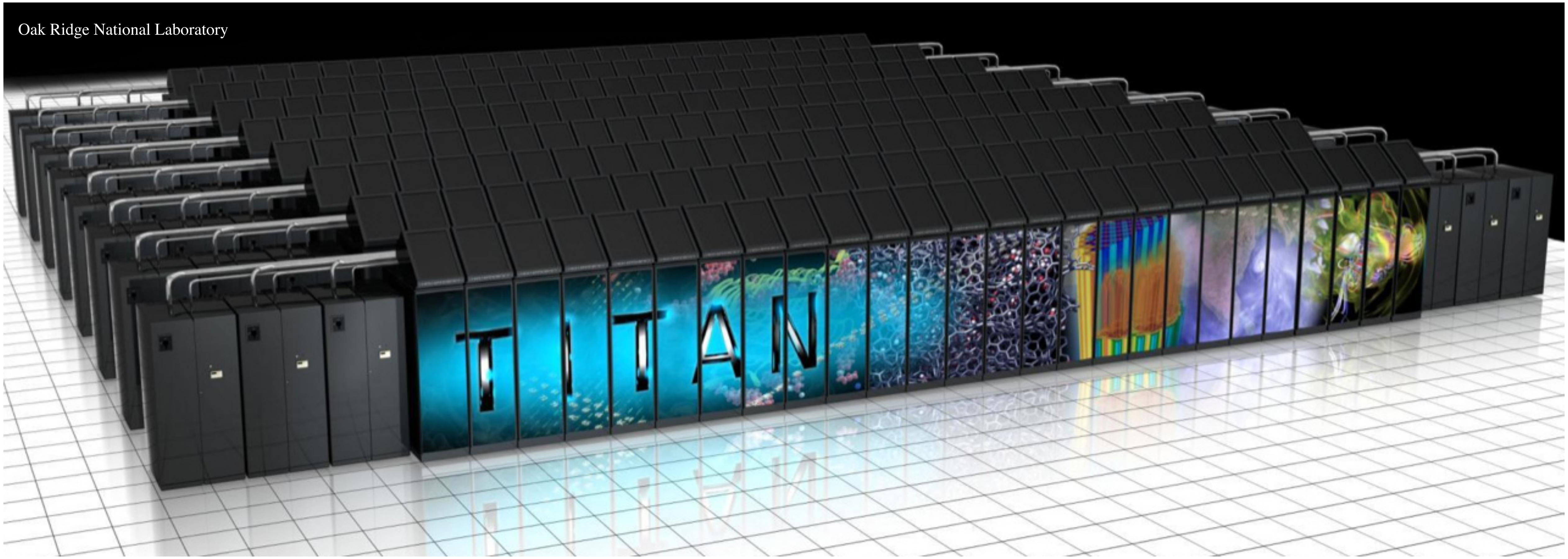
Image credit: Oak Ridge National Laboratory

Which tasks can be parallelized and which tasks are done in series?

What are the memory requirements and what kind of node connectivity is required?

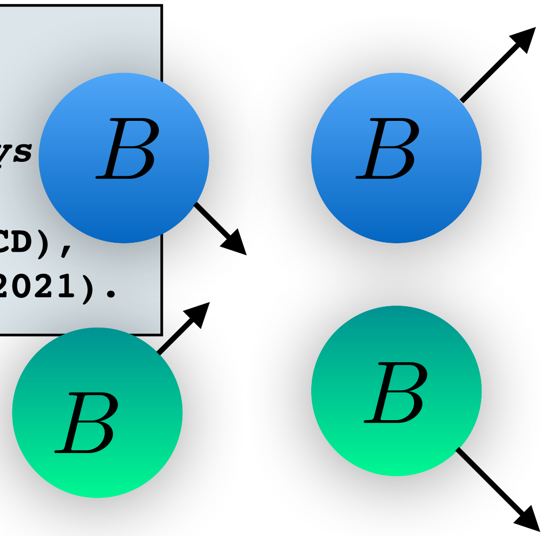
Can we take advantage of GPUs? Which parts of the computations are more suitable for given architecture?

PUTTING ALL THESE HEROIC THEORY, ALGORITHM, AND CO-DESIGN EFFORTS TO WORK AND HAVING ACCESS TO HUNDREDS OF MILLION CPU HOURS ON THE LARGEST SUPERCOMPUTERS IN THE U.S. HAS LED TO MANY IMPRESSIVE RESULTS.



Titan supercomputer, Oak Ridge National Laboratory, USA

Wagman et al (ZD) (NPLQCD), Phys.Rev.D96,114510 (2017).
Illa et al (ZD) (NPLQCD), *Phys Rev. D*103, 5, 054508 (2021).
Amarasinghe et al (ZD) (NPLQCD), arXiv:2108.10835 [hep-lat] (2021).

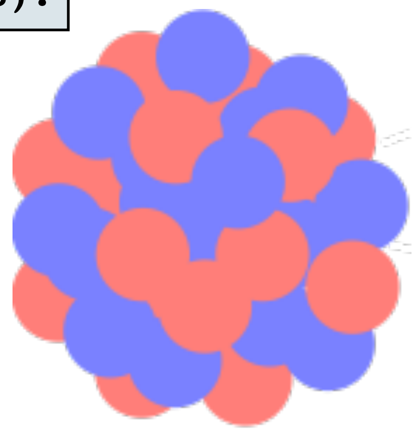


$$c_1, \dots, c_6 \longrightarrow a, b \longrightarrow a$$

$$SU(N_f = 3) \longrightarrow SU(2N_f = 6) \longrightarrow SU(16)$$

Chang et al (ZD) (NPLQCD), Phys. Rev. Lett. 120, 5, 152002 (2018).

$$g_X^{(f)}(A) = \langle A|\bar{q}_f \Gamma_X q_f|A\rangle$$



$= \sum$ ●, ● ?

For a recent review see: ZD, Detmold, Orginos, Parreño, Savage, Shanahan, Wagman, *Phys. Rept.* 900, 1-74 (2021).

Savage et al (ZD) [NPLQCD], Phys. Rev. Lett. 119,062002 (2017).
[Highlighted by Department of Energy]

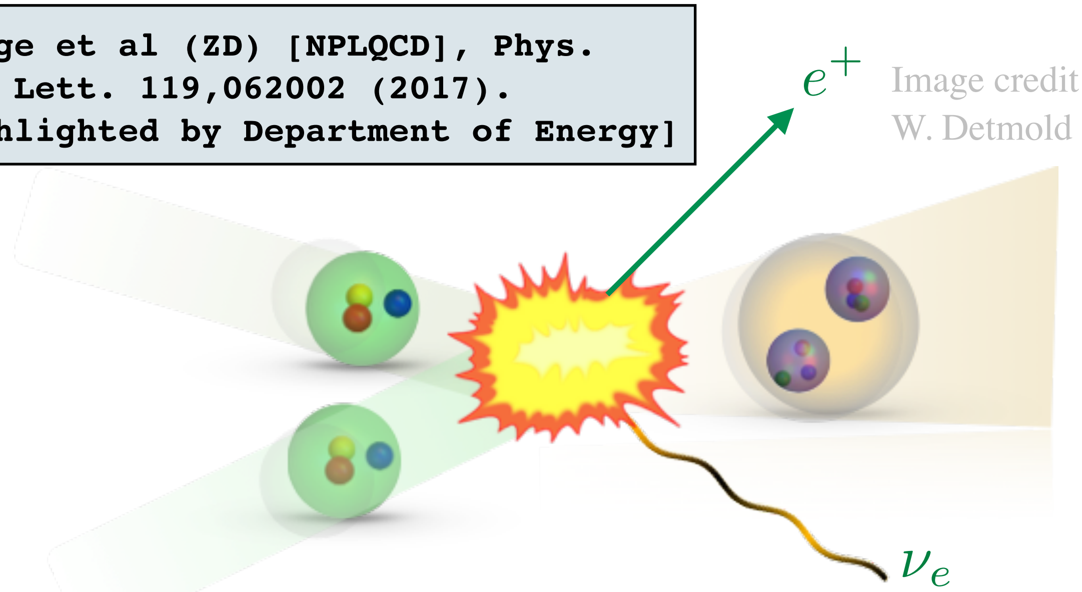
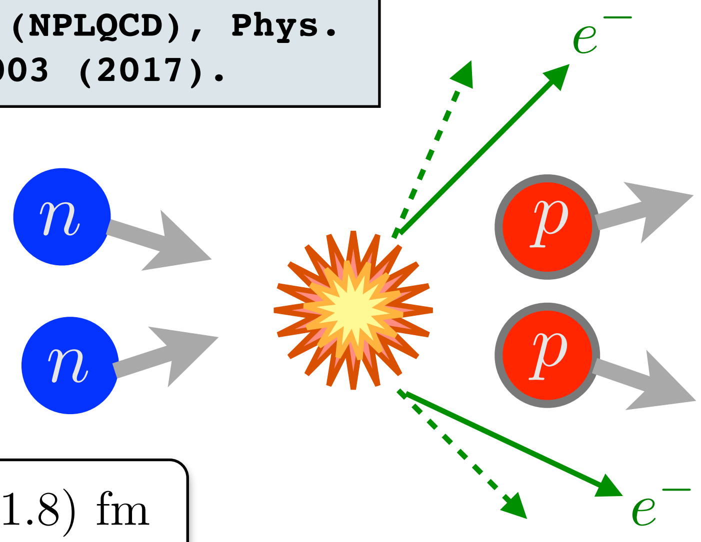


Image credit:
W. Detmold (MIT)

$$L_{1,A} = 3.9(0.1)(1.0)(0.3)(0.9) \text{ fm}^3 \ \ @ \ \mu = m_\pi^{\text{phys.}} = 140 \text{ MeV}$$

Tiburzi et al (ZD) (NPLQCD), Phys. Rev. D96, 054505 (2017).
Shanahan et al (ZD) (NPLQCD), Phys. Rev. Lett. 119, 062003 (2017).



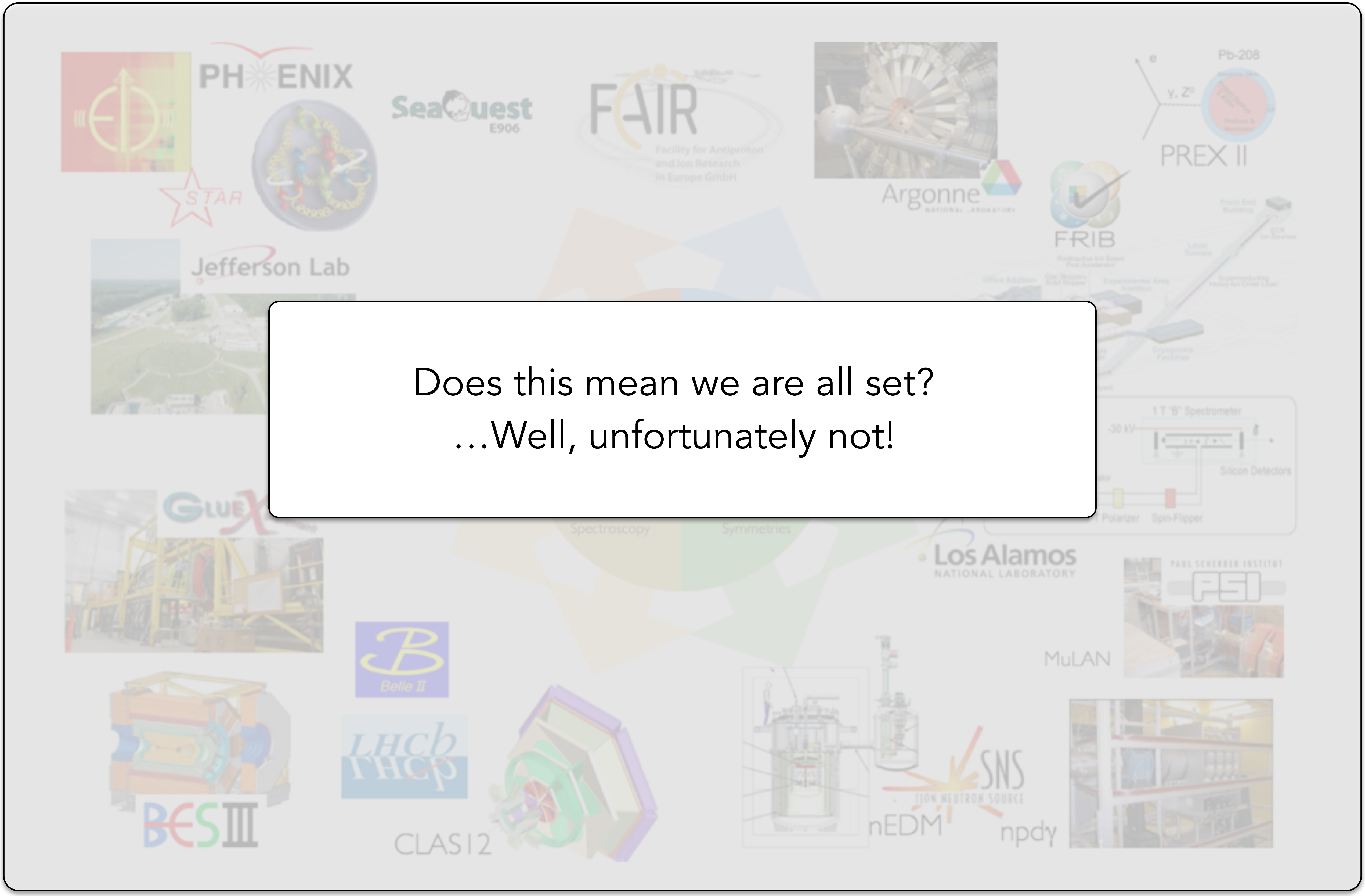
NEW

$$\mathbb{H}_{2,S} = 4.7(1.3)(1.8) \text{ fm}$$
$$@ \ m_\pi = 806 \text{ MeV}$$

LATTICE QCD IS SUPPORTING A MULTI-BILLION DOLLAR EXPERIMENTAL PROGRAM!

Slide content courtesy of Martin Savage.

LATTICE QCD IS SUPPORTING A MULTI-BILLION DOLLAR EXPERIMENTAL PROGRAM!



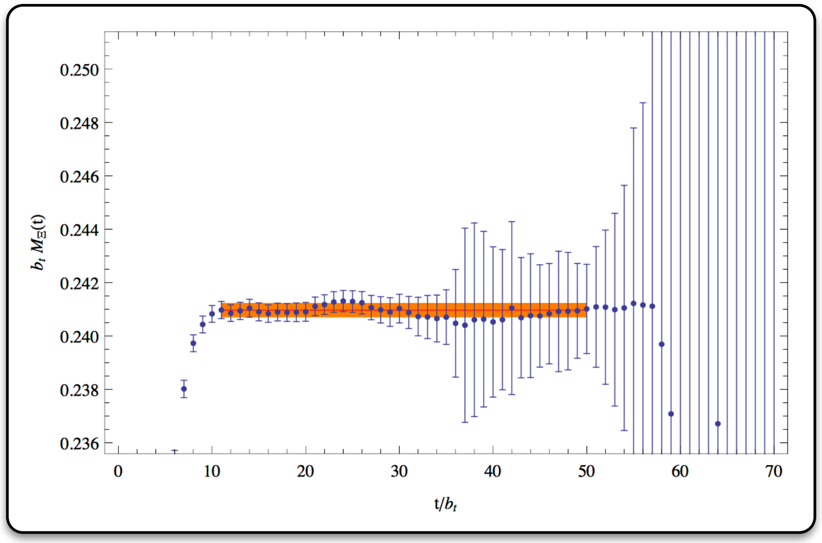
Does this mean we are all set?

…Well, unfortunately not!

THREE FEATURES MAKE LATTICE QCD CALCULATIONS OF NUCLEI HARD:

i) The complexity of systems grows factorially with the number of quarks.

Detmold and Orginos (2013)
Detmold and Savage (2010)
Doi and Endres (2013)

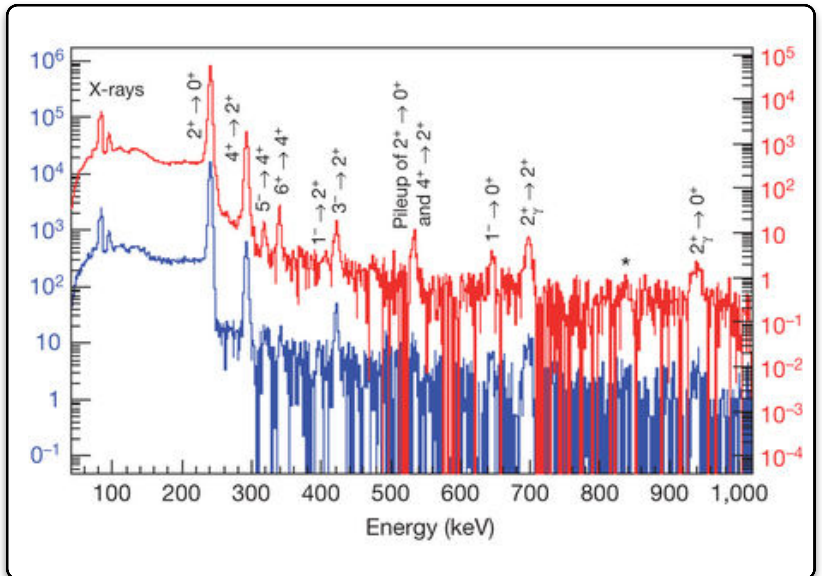


ii) There is a severe signal-to-noise degradation.

Paris (1984) and Lepage (1989)
Wagman and Savage (2017, 2018)

iii) Excitation energies of nuclei are much smaller than the QCD scale.

Beane at al (NPLQCD) (2009)
Beane, Detmold, Orginos, Savage (2011)
ZD (2018)
Briceno, Dudek and Young (2018)



ADDITIONALLY THE SIGN PROBLEM FORBIDS:

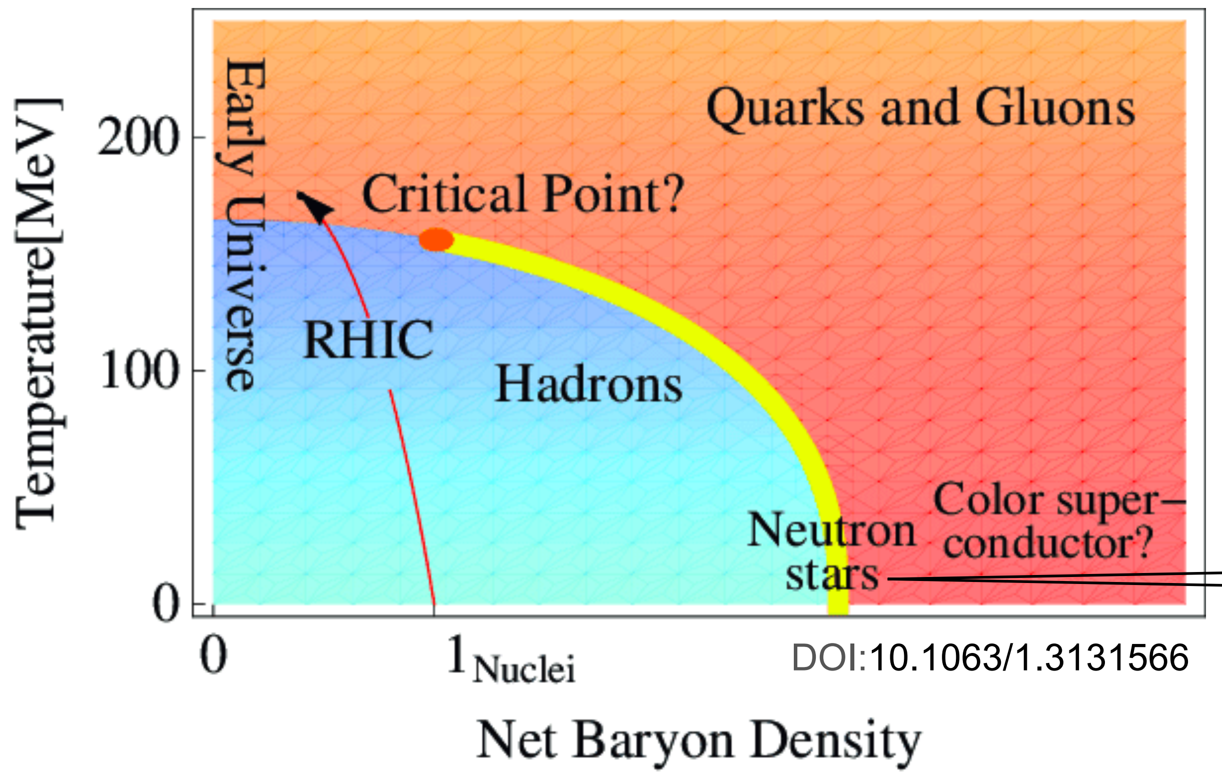
i) Studies of nuclear isotopes, dense matter, and phase diagram of QCD…both with lattice QCD and with *ab initio* nuclear many-body methods.

Path integral formulation:

$$e^{-S[U,q,\bar{q}]}$$

with a complex action:

$$\mathcal{L}_{\rm QCD} \to \mathcal{L}_{\rm QCD} - i\mu \sum_f \bar{q}_f \gamma^0 q_f$$



DOI:10.1063/1.3131566

Image credit: NSF/LIGO/Sonoma State University/A. Simonnet

ADDITIONALLY THE SIGN PROBLEM FORBIDS:

ii) Real-time dynamics of matter in heavy-ion collisions or after Big Bang…

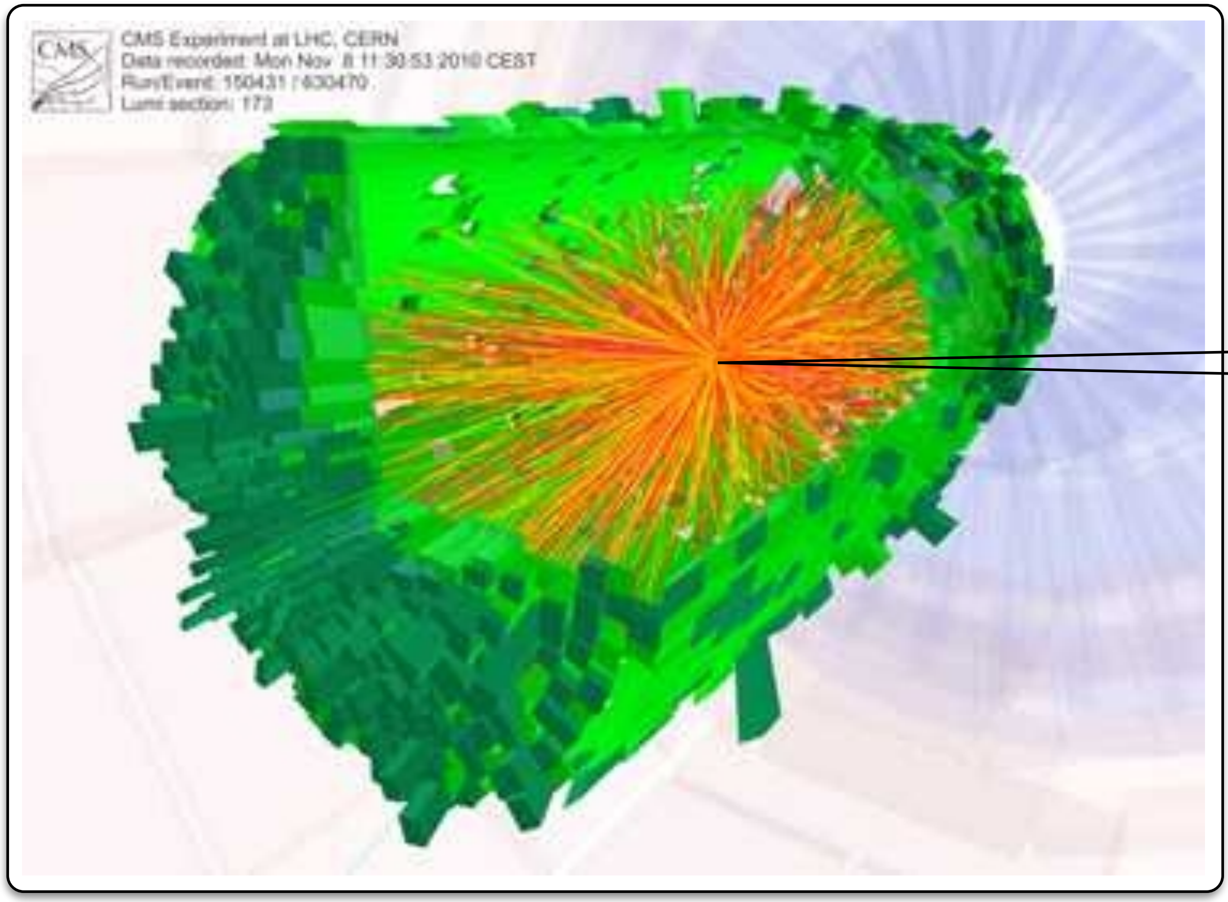
…and a wealth of dynamical response functions, transport properties, hadron distribution functions, and non-equilibrium physics of QCD.

Path integral formulation:

$$e^{iS[U,q,\bar{q}]}$$

Hamiltonian evolution:

$$U(t) = e^{-iHt}$$



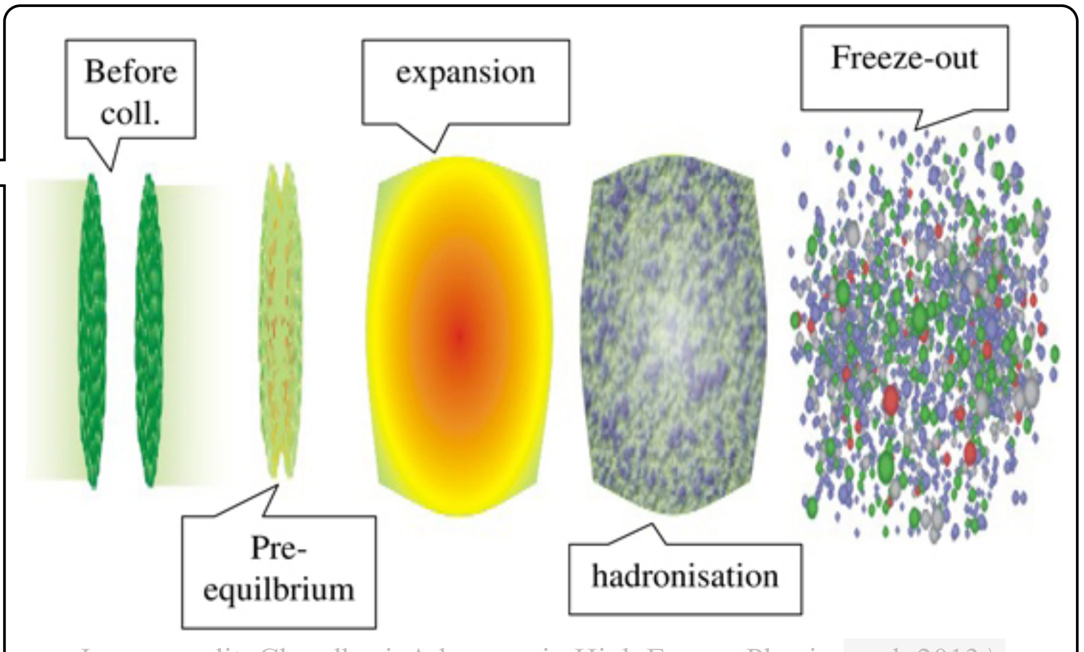
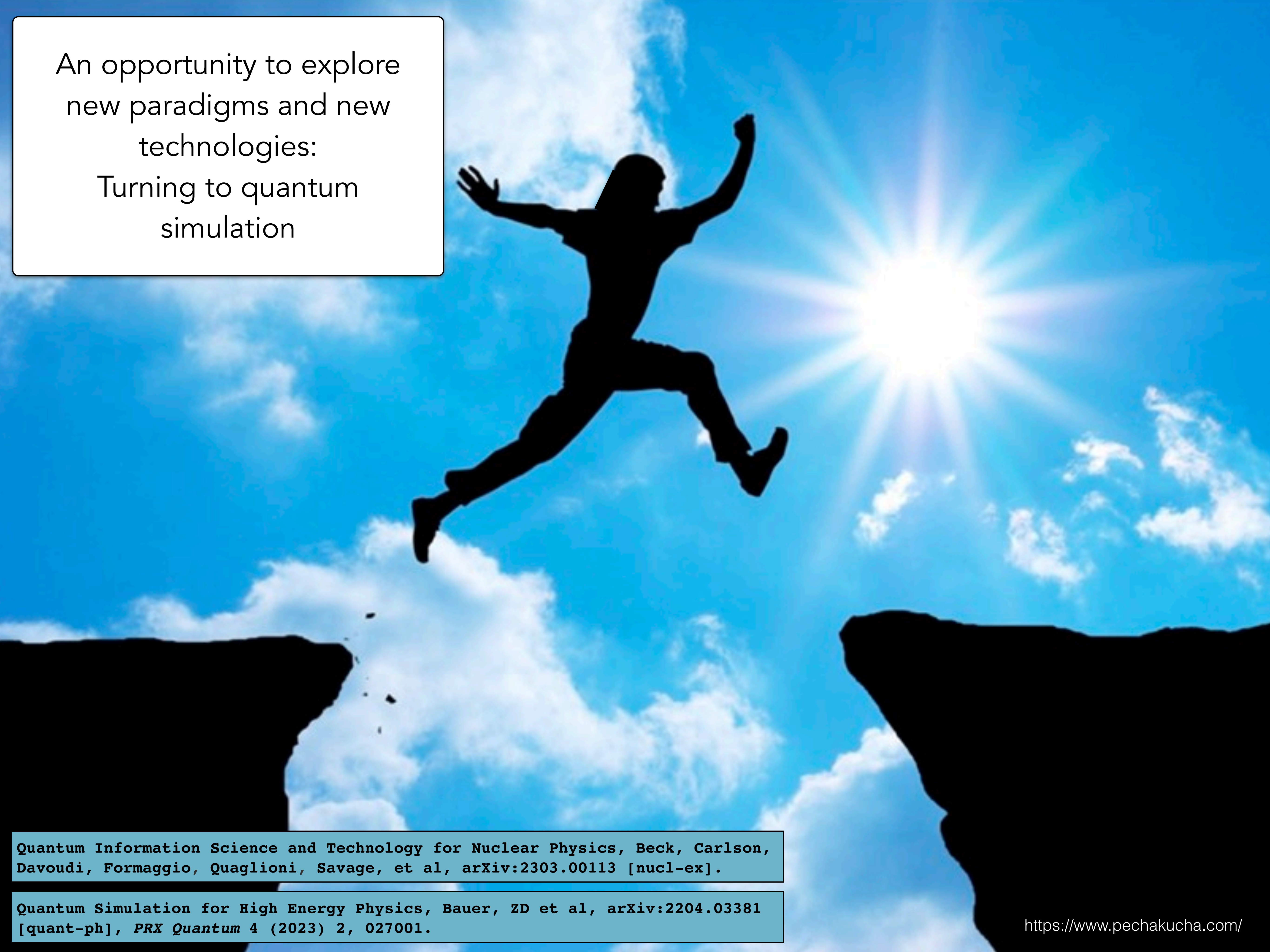


Image credit: Chaudhuri, Advances in High Energy Physics, vol. 2013,\.

An opportunity to explore new paradigms and new technologies: Turning to quantum simulation



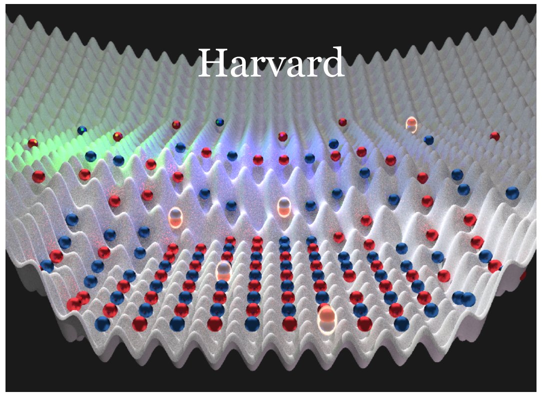
Quantum Information Science and Technology for Nuclear Physics, Beck, Carlson, Davoudi, Formaggio, Quaglioni, Savage, et al, arXiv:2303.00113 [nucl-ex].

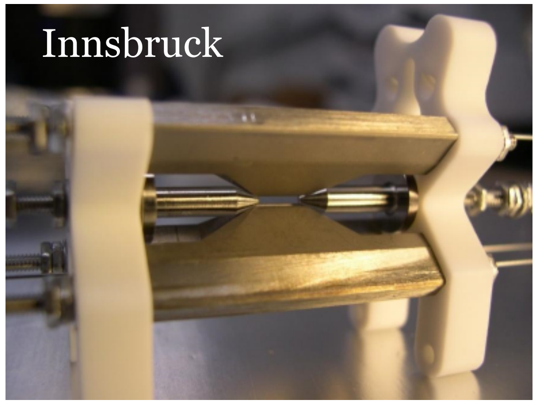
Quantum Simulation for High Energy Physics, Bauer, ZD et al, arXiv:2204.03381 [quant-ph], *PRX Quantum* 4 (2023) 2, 027001.

https://www.pechakucha.com/

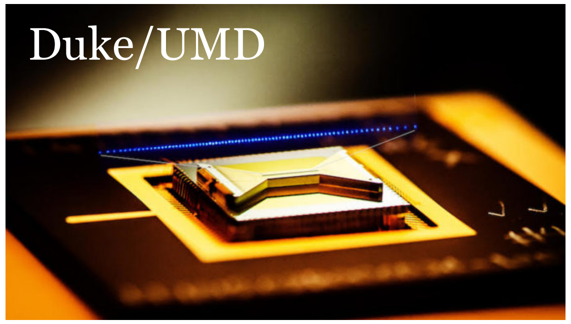
A RANGE OF QUANTUM SIMULATORS WITH VARING CAPACITY AND CAPABILITY

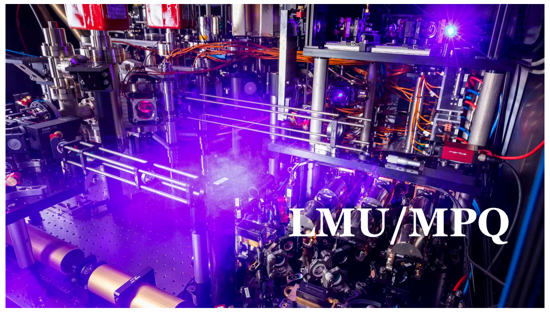
- Atomic systems (trapped ions, cold atoms, Rydbergs)
- Condensed matter systems (superconducting circuits, dopants in semiconductors such as in Silicon, NV centers in diamond)
- Laser-cooled polar molecules
- Optical quantum computing

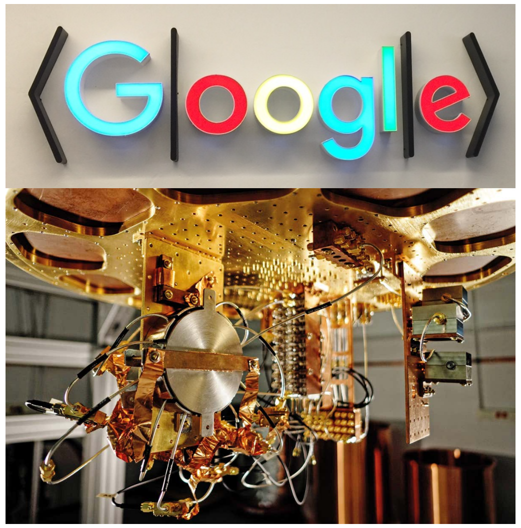




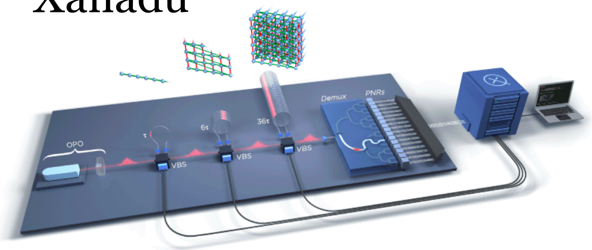
IONQ







Xanadu



HOW SIMILAR TO QUANTUM CHEMISTRY SIMULATIONS?

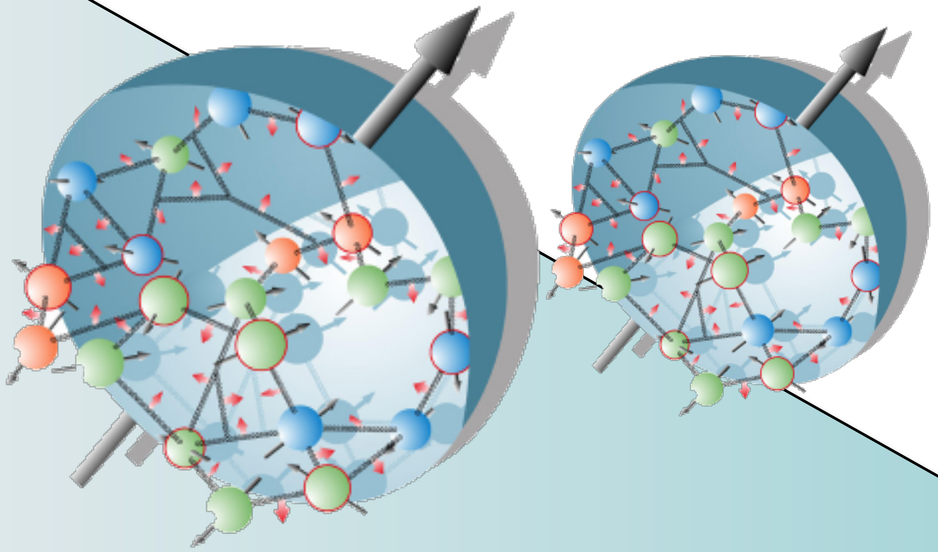


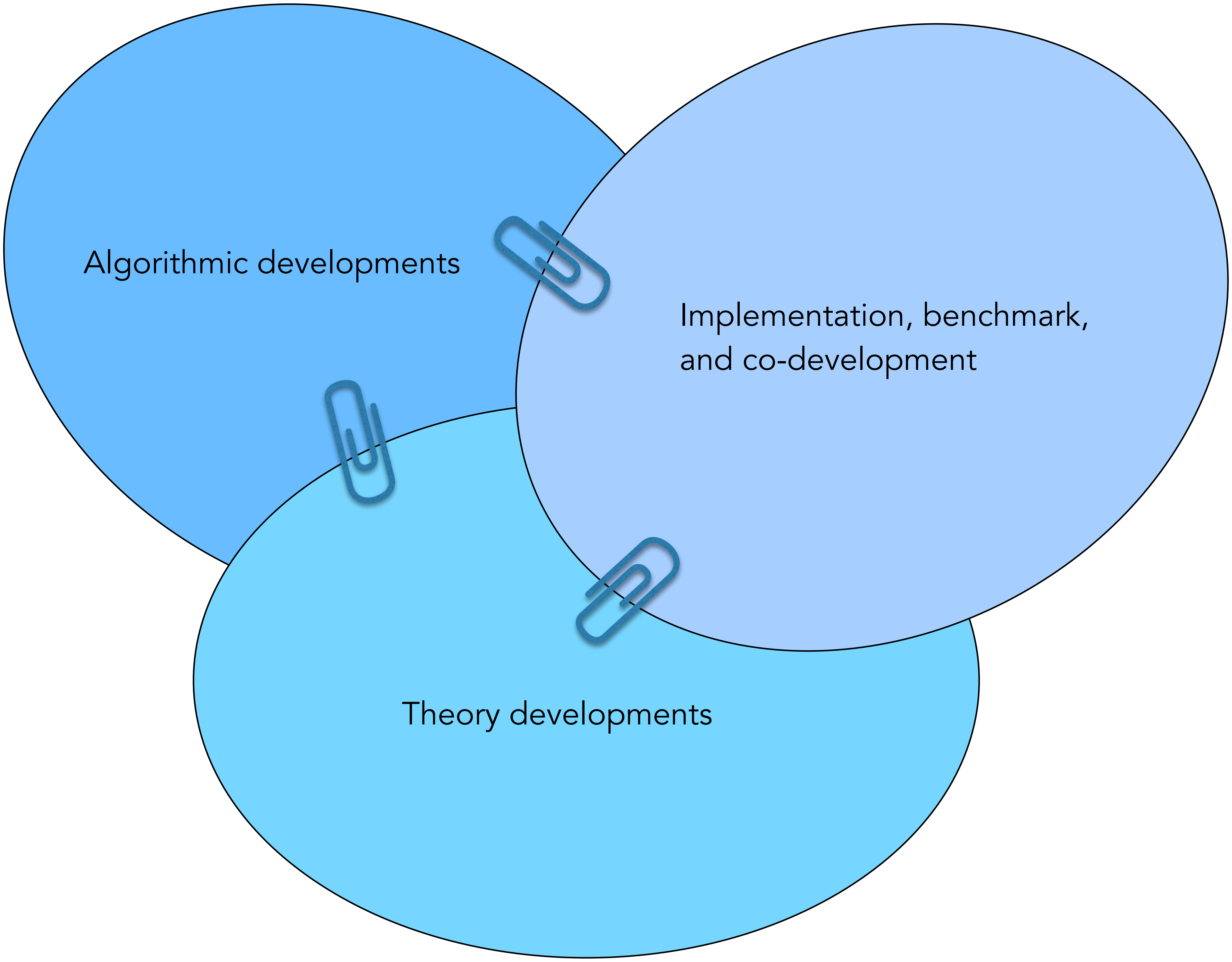
Image credit: CERN courier

Starting from the Standard Model

Both bosonic and fermionic DOF are dynamical and coupled, exhibit both global and local (gauge) symmetries, relativistic hence particle number not conserved, vacuum state nontrivial in strongly interacting theories.

Attempts to cast QFT problems in a language closer to quantum chemistry and NR simulations: Kreshchuk, Kirby, Goldstein, Beauchemin, Love, arXiv:2002.04016 [quant-ph], Kreshchuk, Jia, Kirby, Goldstein, Vary, Love, Entropy 2021, 23, 597, Liu, Xin, arXiv:2004.13234 [hep-th], Barata , Mueller, Tarasov, Venugopalan (2020)

QUANTUM SIMULATION OF QUANTUM FIELD THEORIES: A MULTI-PRONG EFFORT

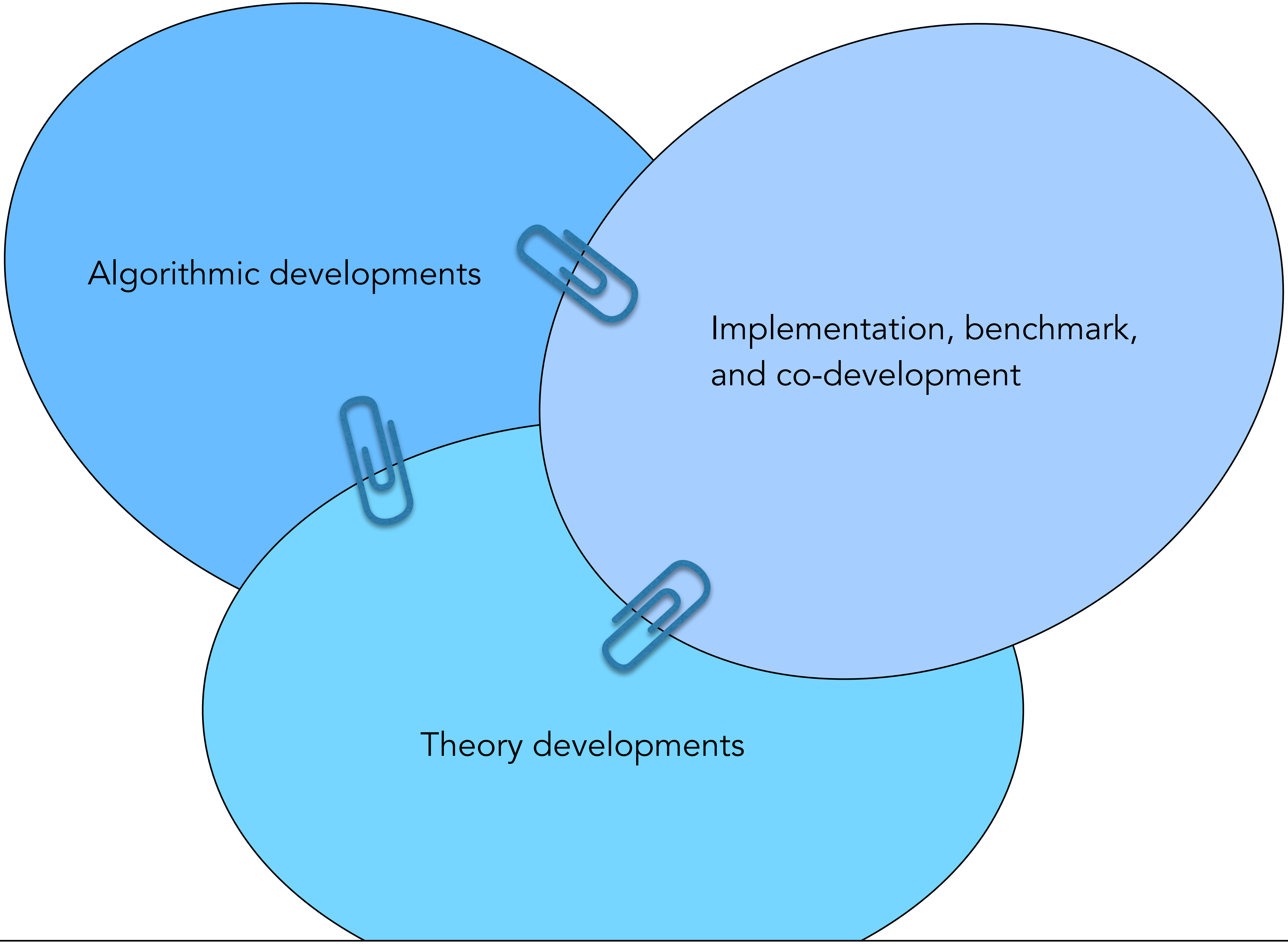


Algorithmic developments

Implementation, benchmark,
and co-development

Theory developments

QUANTUM SIMULATION OF QUANTUM FIELD THEORIES: A MULTI-PRONG EFFORT



Algorithmic developments

Implementation, benchmark,
and co-development

Theory developments

Major focus on my work with my group and collaborators! Apologies for the missing references.

How to formulate QCD in the Hamiltonian language?

What are the efficient formulations? Which bases will be most optimal toward the continuum limit?

How to preserve the symmetries? How much should we care to retain gauge invariance?

How to quantify systematics such as finite volume, discretization, boson truncation, time digitization, etc?

Theory developments

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

Hamiltonian formalism is a more natural than the path integral formalism for quantum simulation/computation:

Kogut and Susskind formulation:

$$H_{\mathrm{QCD}} = \underbrace{-t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right)}_{\text{Fermion hopping term}} + \underbrace{m\sum_x s_x \psi_x^\dagger \psi_x}_{\text{Fermion mass}} + \underbrace{\frac{g^2}{2}\sum_{\langle xy\rangle}\left(L_{xy}^2 + R_{xy}^2\right)}_{\text{Energy of color electric field}} - \underbrace{\frac{1}{4g^2}\sum_{\Box}\mathrm{Tr}\left(U_\Box + U_\Box^\dagger\right)}_{\text{Energy of color magnetic field}}.$$

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

Hamiltonian formalism is a more natural than the path integral formalism for quantum simulation/computation:

An infinite-dimensional Hilbert space!

Kogut and Susskind formulation:

$$H_{\rm QCD} = -t\sum_{\langle xy\rangle} s_{xy}\left(\psi_x^\dagger U_{xy}\psi_y + \psi_y^\dagger U_{xy}^\dagger \psi_x\right) + m\sum_x s_x \psi_x^\dagger \psi_x + \frac{g^2}{2}\sum_{\langle xy\rangle}\left(L_{xy}^2 + R_{xy}^2\right) - \frac{1}{4g^2}\sum_{\square}{\rm Tr}\left(U_\square + U_\square^\dagger\right).$$

Fermion hopping term

Fermion mass

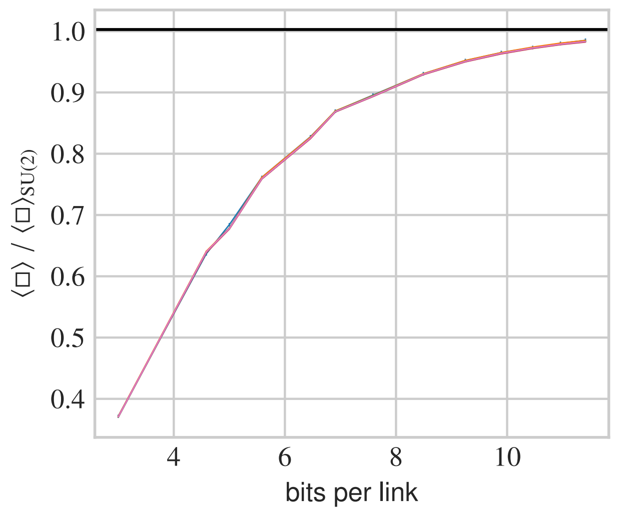
Energy of color electric field

Energy of color magnetic field

Tong, Albert, McClean, Preskill, and Su (2021).

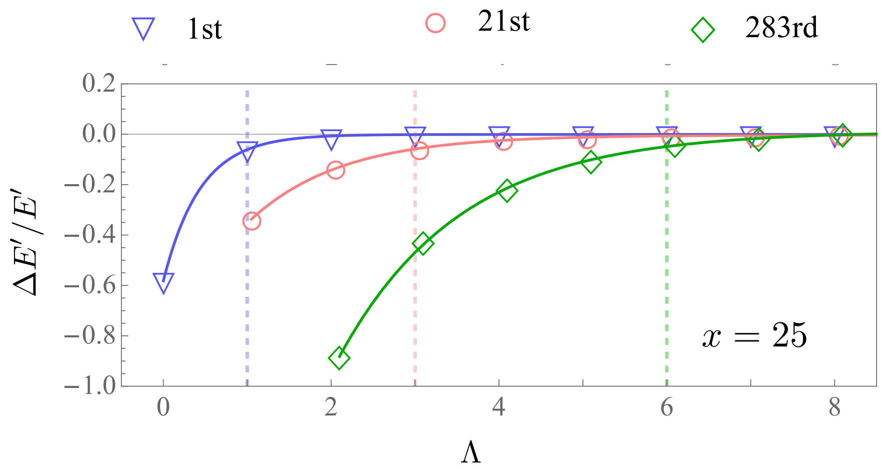
Gauge-field truncation

SU(2) pure gauge in 3+1 D in group element basis



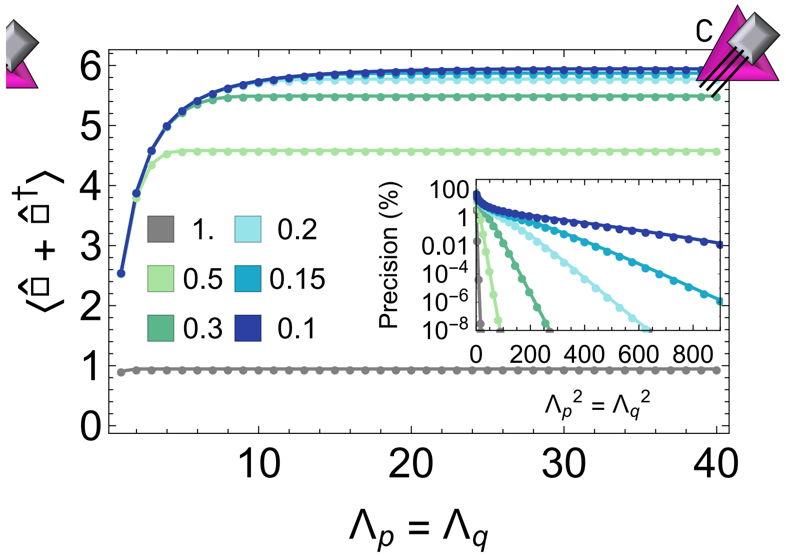
Hackett et al, Phys. Rev. A 99, 062341 (2019).

SU(2) with matter in 1+1 D in electric-field basis



ZD, Raychowdhury, and Shaw, Phys. Rev. D 104, 074505 (2021).

SU(3) pure gauge in 2+1 D in local-irreps basis



iavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).

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Fermion hopping term

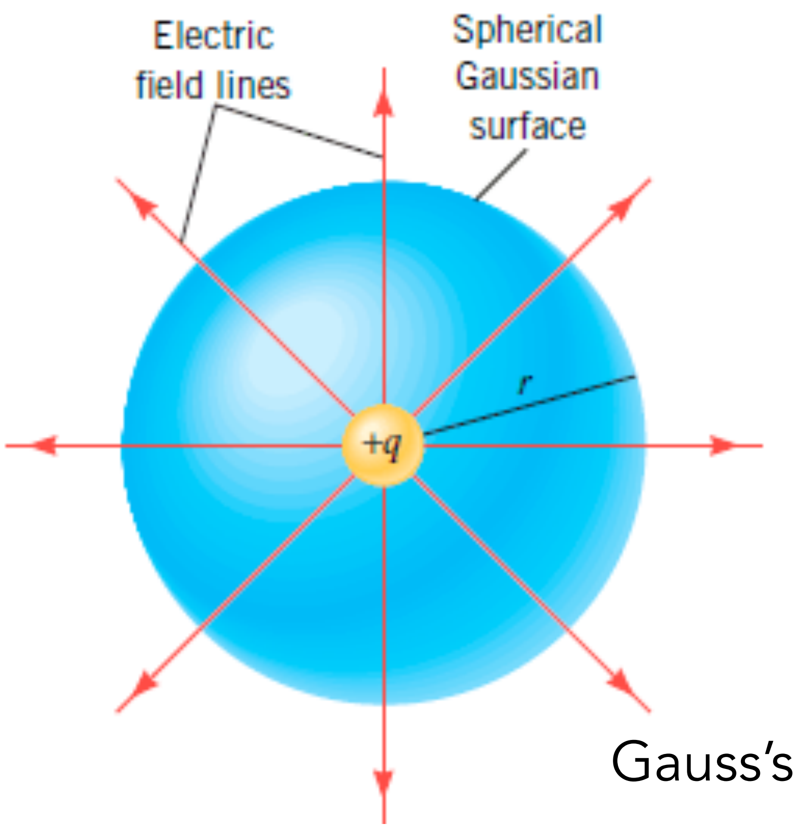
Fermion mass

Energy of color electric field

Energy of color magnetic field

Generator of infinitesimal gauge transformation

$$G_x^a = \psi_x^{i\dagger}\lambda_{ij}^a\psi_x^j + \sum_k\left(L^a_{x,x+\hat{k}} + R^a_{x-\hat{k},x}\right) \Longrightarrow G_x^i|\psi(\{q_x^{(i)}\})\rangle = q_x^{(i)}|\psi(\{q_x^{(i)}\})\rangle$$



Gauss's law

Image credit: https://physicsteacher.in/

QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

Hamiltonian formalism is a more natural than the path integral formalism for quantum simulation/computation:

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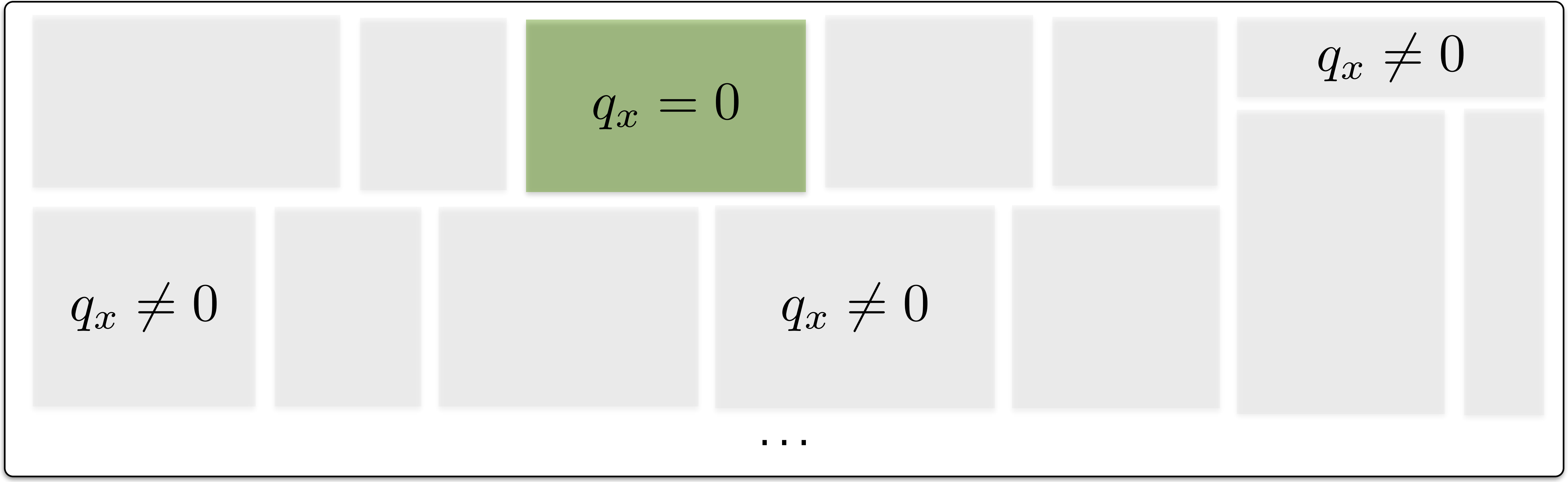
Fermion hopping term

Fermion mass

Energy of color electric field

Energy of color magnetic field

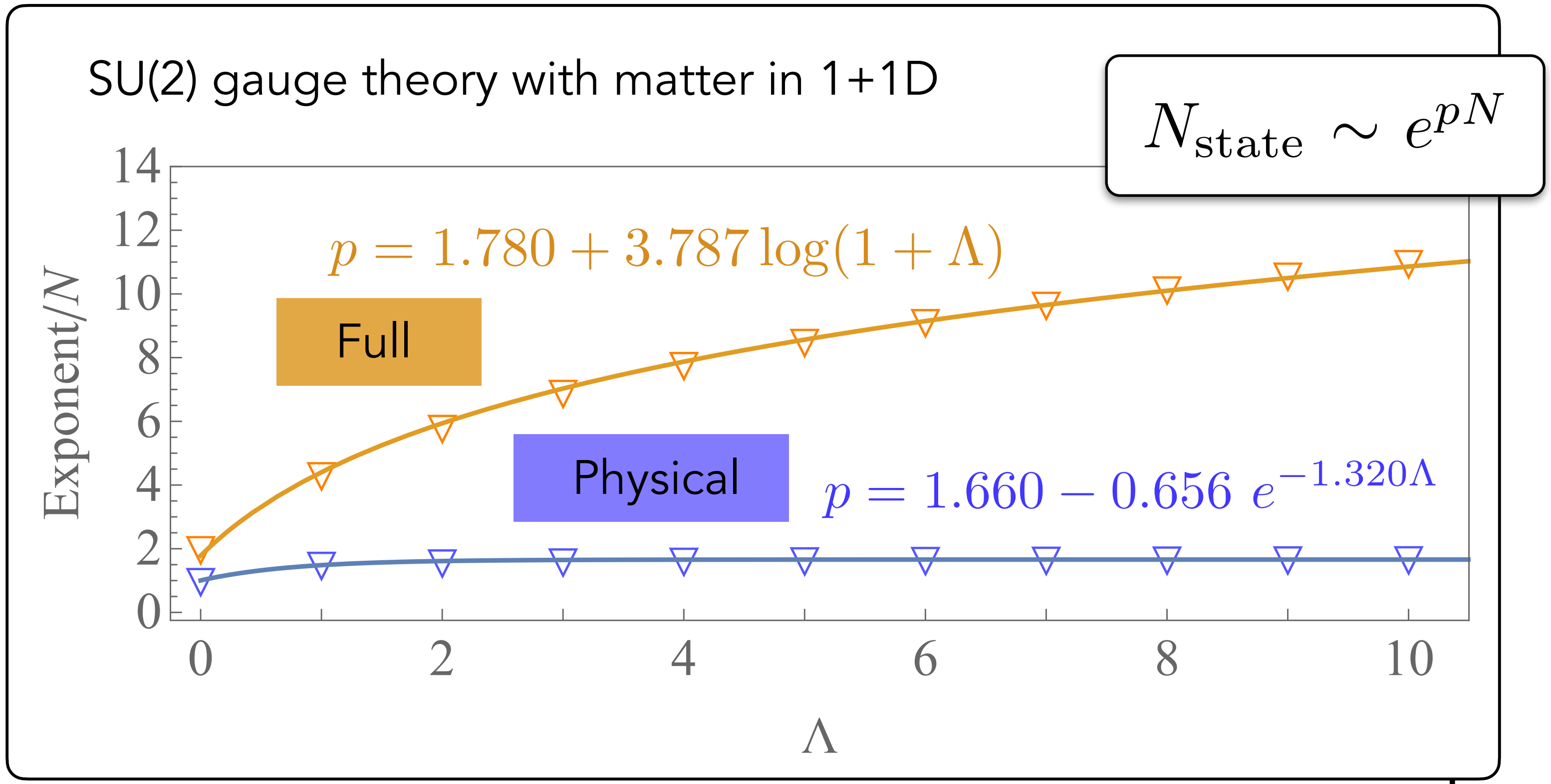
Generator of infinitesimal gauge transformation $G_x^a = \psi_x^{i\dagger}\lambda_{ij}^a\psi_x^j + \sum_k\left(L^a_{x,x+\hat{k}} + R^a_{x-\hat{k},x}\right) \Rightarrow G_x^i|\psi(\{q_x^{(i)}\})\rangle = q_x^{(i)}|\psi(\{q_x^{(i)}\})\rangle$



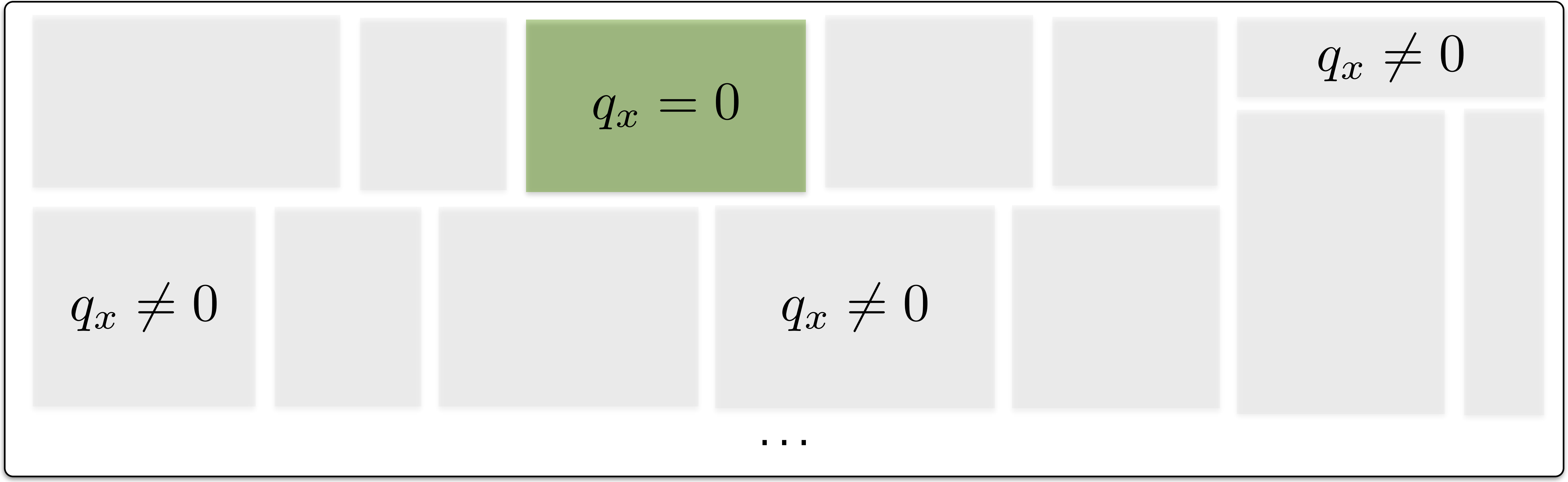
QUANTUM SIMULATION OF GAUGE FIELD THEORIES: THEORY DEVELOPMENTS

SU(2) gauge theory with matter in 1+1D

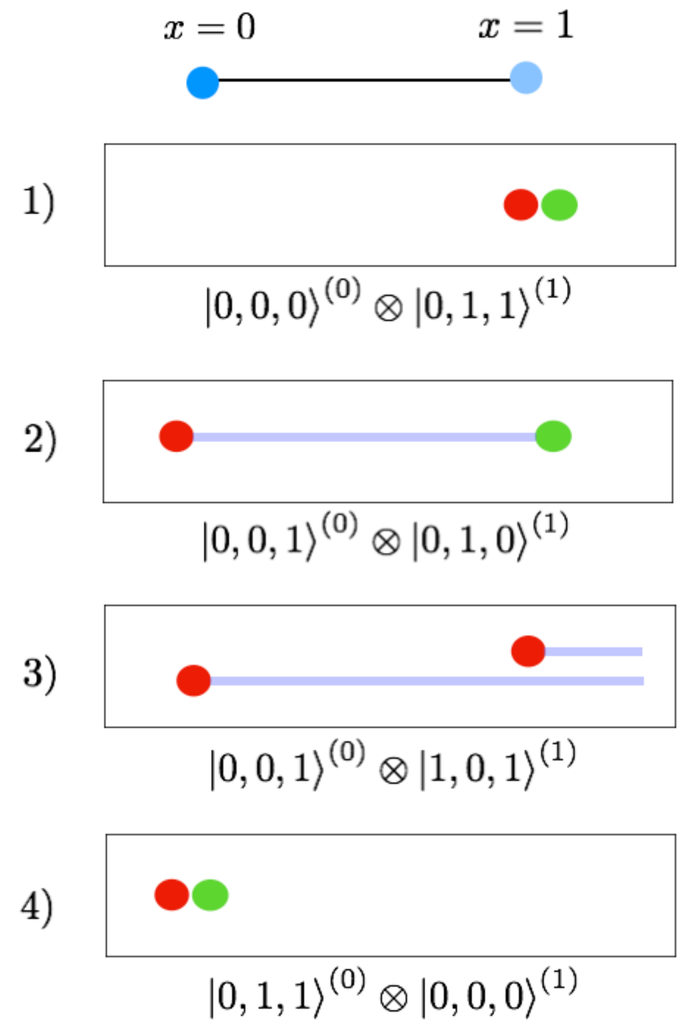
$$N_{\text{state}} \sim e^{pN}$$



ZD, Raychowdhury, and Shaw, Phys. Rev. D 104, 074505, (2020).



IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION



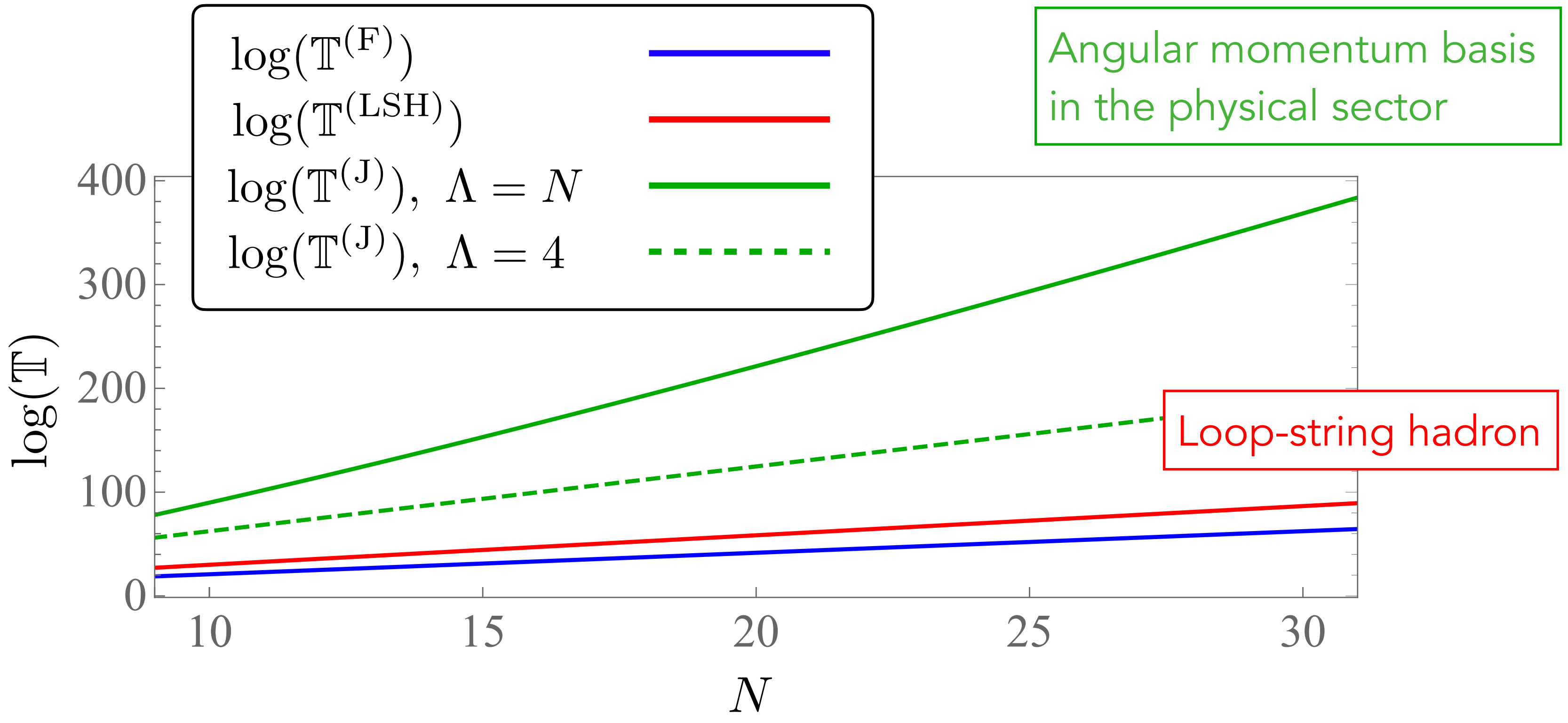
Loop-string-hadron formulation: Building blocks (quantum numbers) are already local and gauge-invariant.

Raychowdhury, Stryker, Phys. Rev. D 101, 114502 (2020).

SU(3) extension: Kadam,, Raychowdhury, Stryker, arXiv:2212.04490 [hep-lat] (2022).

IDEAS TO SUPPRESS LEAKAGE TO UNPHYSICAL SECTOR IN THE SIMULATION

The time complexity of classical Hamiltonian-simulation algorithms for each formulation.

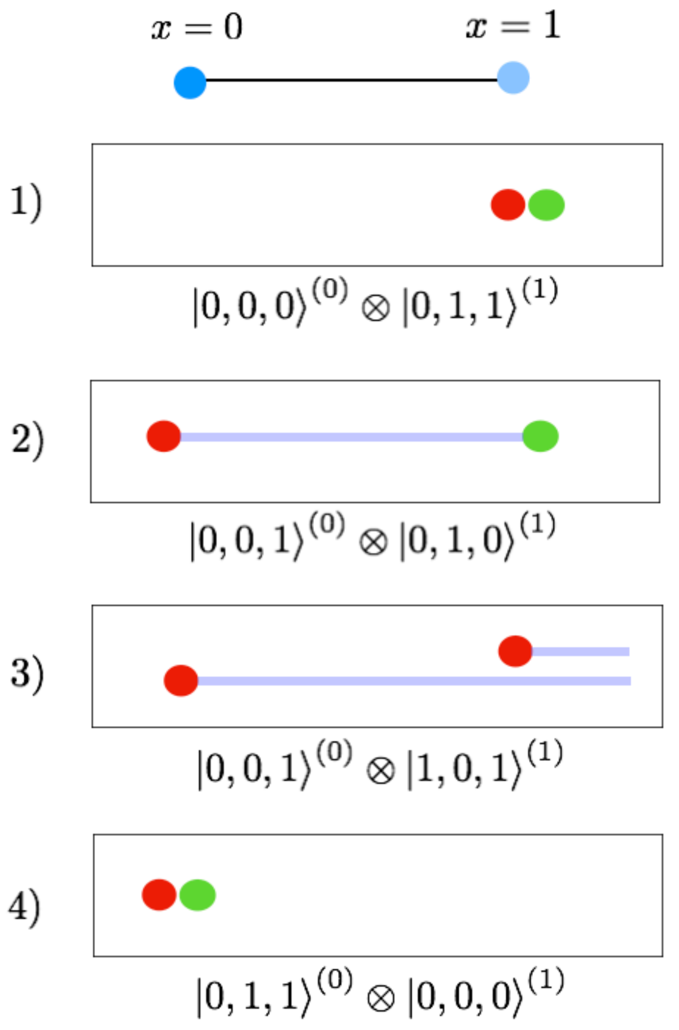


Angular momentum basis in the physical sector

Loop-string hadron

ZD, Raychowdhury, and Shaw, Phys. Rev. D 104, 074505 (2020).

Quantum algorithm scalings in: ZD, Shaw (2), and Stryker, arXiv:2212.14030 [hep-lat] (2022).



Loop-string-hadron formulation: Building blocks (quantum numbers) are already local and gauge-invariant.

Raychowdhury, Stryker, Phys. Rev. D 101, 114502 (2020).

MANY HAMILTONIAN FORMULATIONS OF GAUGE THEORIES EXIST, BUT WHICH ONE TO PICK?

Gauge-field theories (Abelian and non-Abelian):

Group-element representation
Zohar et al; Lamm et al

Prepotential formulation
Mathur, Raychowdhury et al

Loop-String-Hadron basis
Raychowdhury, Stryker, Kadam

Link models, qubitization
Chandrasekharan, Wiese et al,
Alexandru, Bedaque, et al, Hersch et al.

Fermionic basis
Hamer et al; Martinez et al;
Banuls et al

Bosonic basis
Cirac and Zohar

Light-front quantization
Kreshchuk, Love, Goldstien,
Vary et al.; Ortega Rico at al

Local irreducible representations
Byrnes and Yamamoto;
Ciavarella, Klco, and Savage

Manifold lattices
Buser et al

Dual plaquette (magnetic) basis
Bender, Zohar et al; Kaplan and Styker; Unmuth-Yockey; Hasse et al; Bauer and Grabowska

Spin-dual representation
Mathur et al

Scalar field theory

Field basis
Jordan, Lee, and Preskill

Continuous-variable basis
Pooser, Siopsis et al

Harmonic-oscillator basis
Klco and Savage

Single-particle basis
Barata , Mueller, Tarasov, and Venugopalan.

Algorithmic developments [Digital]

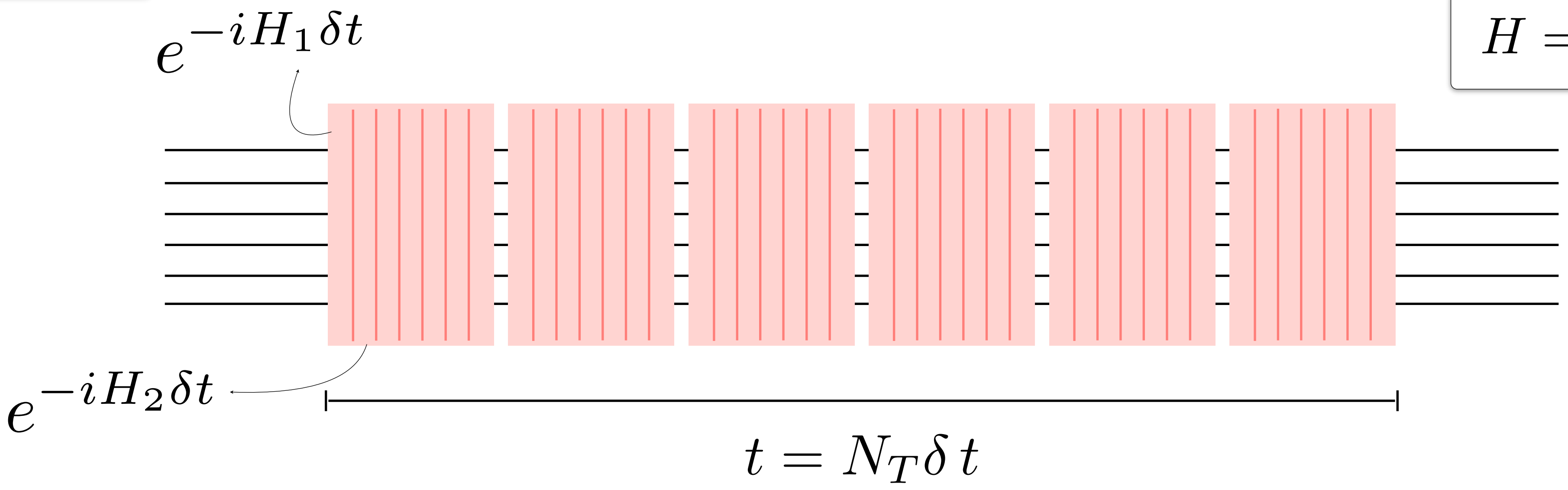
- Near- and far-term algorithms with bounded errors and resource requirement for gauge theories?
- Can given formulation/encoding reduce qubit and gate resources?
- Should we develop gauge-invariant simulation algorithms?
- How do we do state preparation and compute observables like scattering amplitudes?

Algorithmic developments
[Digital]

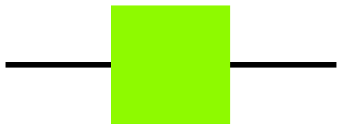
- Near- and far-term algorithms with bounded errors and resource requirement for gauge theories?
- Can given formulation/encoding reduce qubit and gate resources?
- Can we develop gauge-invariant simulation algorithms?
- How do we do state preparation and compute observables like scattering amplitudes?

Digital

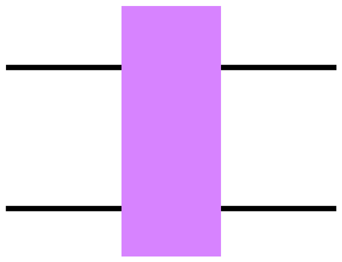
$$H = H_1 + H_2 + \cdots$$



Single-qubit gates



Two-qubit entangling gate



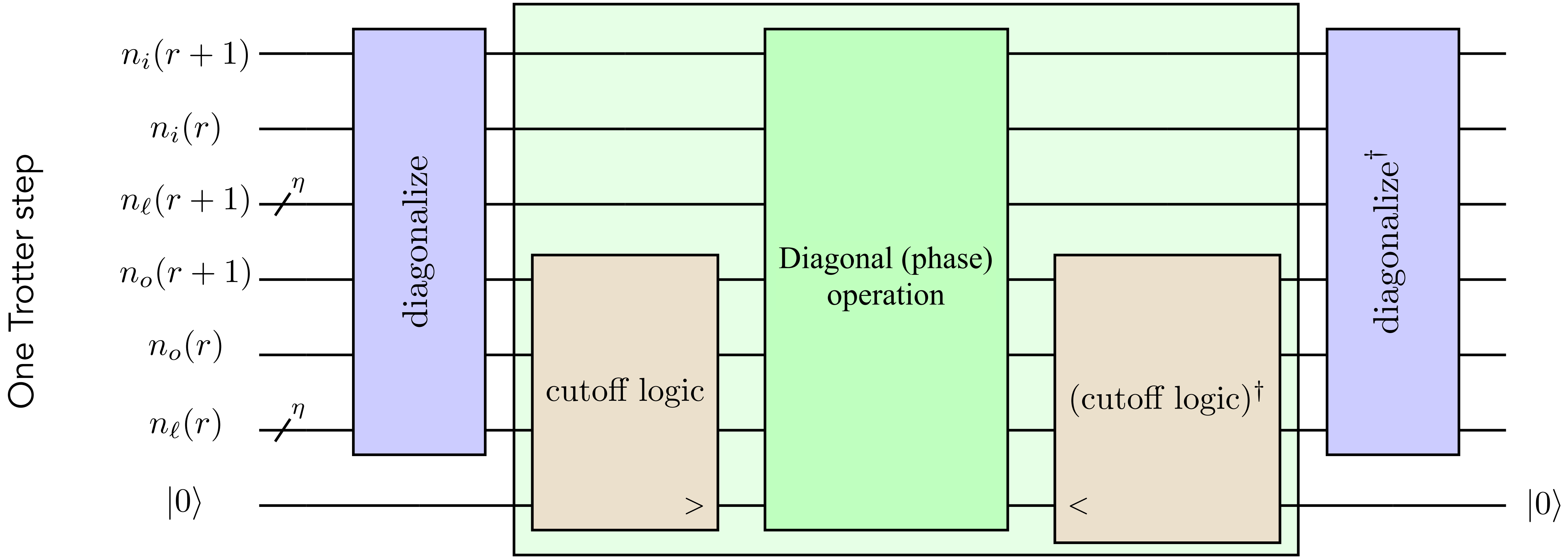
How many qubits and gates are required to achieve accuracy ϵ in a given observables? Are there algorithms that scale optimally?

Important algorithmic progress for U(1), SU(2), and SU(3) theories can be found in:
Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).
Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).
Kan and Nam, arXiv:2107.12769 [quant-ph].
ZD, Shaw (2), and Stryker, arXiv:2212.14030 [hep-lat] (2022).
Gustafson, Lamm, Lovelace, and Musk, arXiv:2208.12309 [quant-ph] (2022).

Example: SU(2) gauge theory coupled to matter in 1+1 D with loops, strings, hadrons

ZD, Shaw, and Stryker, General Quantum Algorithms for Hamiltonian Simulation with Applications to a Non-Abelian Lattice Gauge Theory, arXiv:2212.14030 [hep-lat] (2022).

How do we exponentiate each term of the Hamiltonian in a digitized approach?



Make sure the boson cutoff is not exceeded. Decompose the terms and diagonalize them wisely to not break the Abelian constraint.

Each Trotter step amounts to:

1) First diagonalizing each term.
2) Second evaluating the diagonal operators .

Example: SU(2) gauge theory coupled to matter in 1+1 D with loops, strings, hadrons

ZD, Shaw, and Stryker, General Quantum Algorithms for Hamiltonian Simulation with Applications to a Non-Abelian Lattice Gauge Theory, arXiv:2212.14030 [hep-lat] (2022).

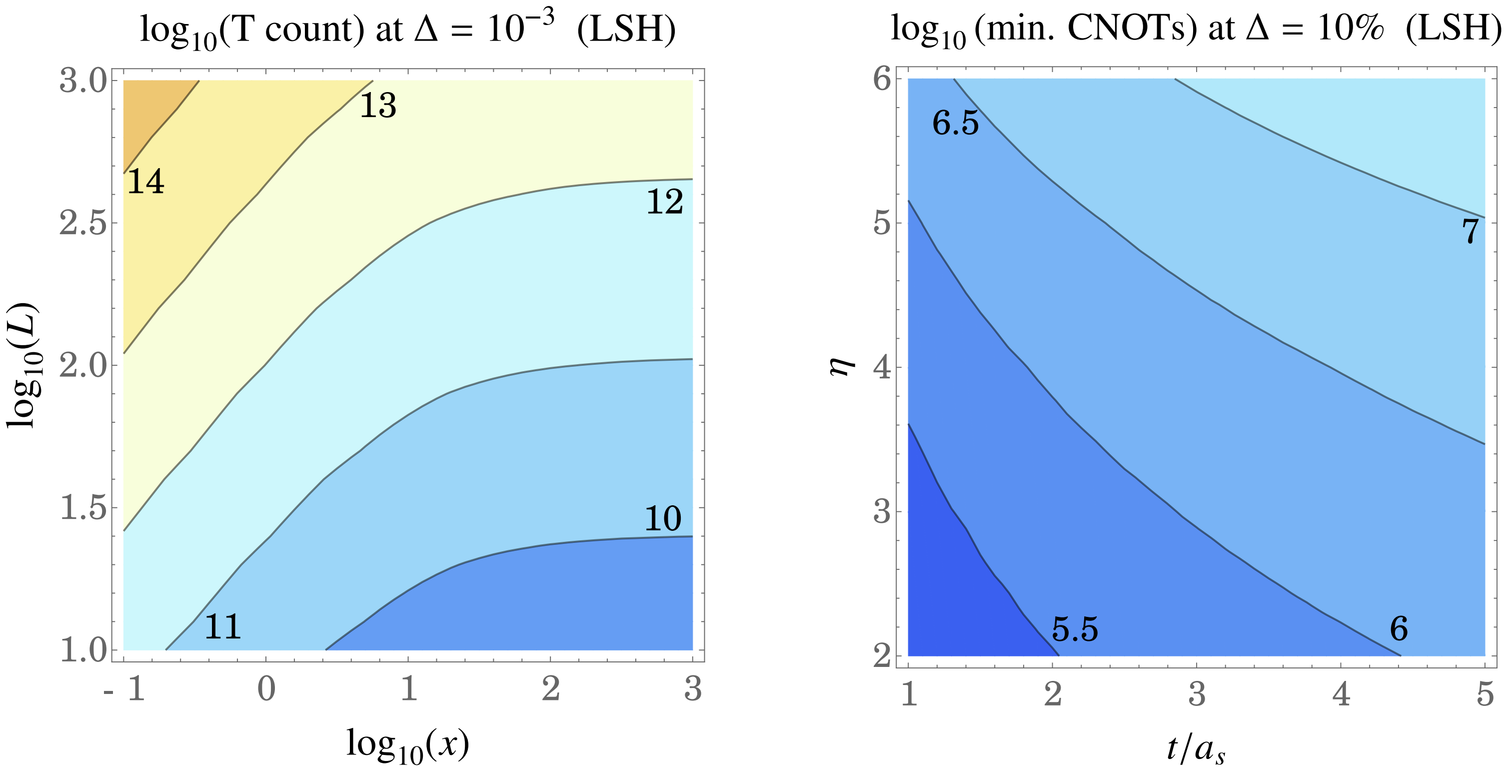
Conclusions:

- Taking the continuum limit takes tens to hundreds of thousands of qubits and $> 10^{15}$ gates. Improvements possible.

- In the near term, SU(2) in 1D (LSH) is about 10^8 times more expensive than U(1).

- In the far term, SU(2) in 1D (LSH) is about 10^5 times more expensive than U(1).

- More intelligent decomposition of QCD Hamiltonian according to our procedure will save many orders of magnitude in resources compared to current estimates (Kan and Nam), but this is still costly!



Algorithmic developments
[Analog]

Can practical proposals for current hardware be developed?

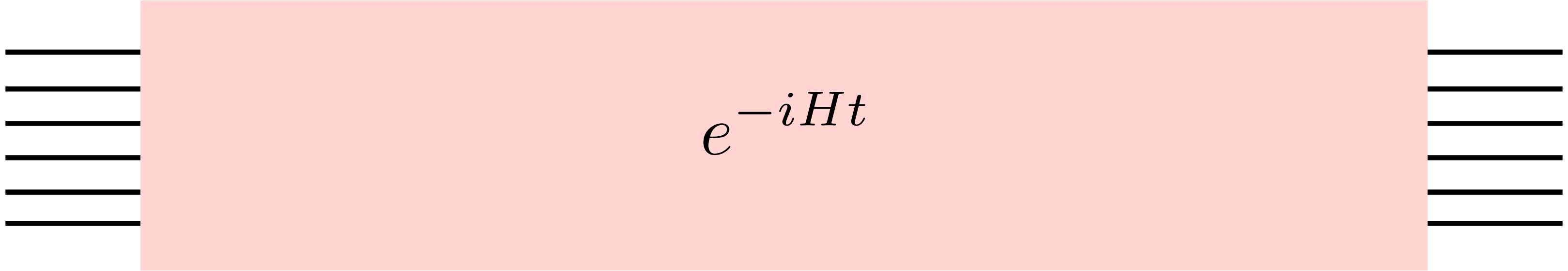
Can we simulate higher-dimensional gauge theories?

Can non-Abelian gauge theories be realized in an analog simulator?

Can we robustly bound the errors in the analog simulation? What quantities are more robust to errors?

Analog

No gates



$$e^{-iHt}$$

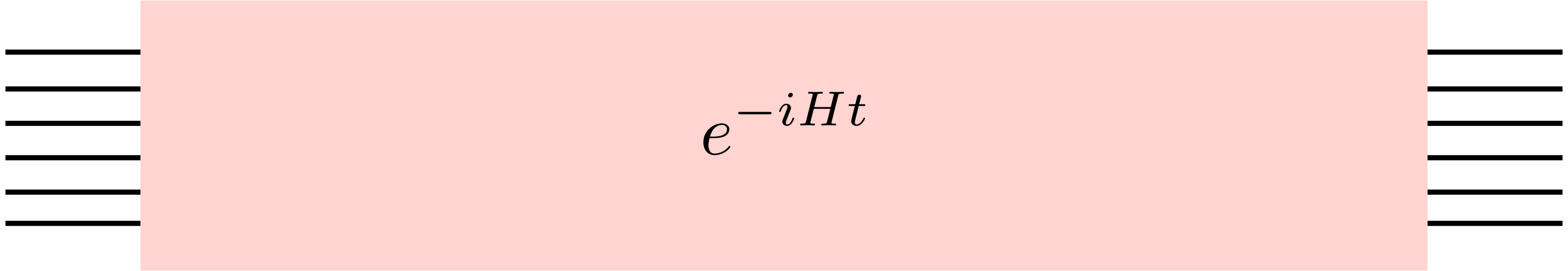
Wineland et al, J.Res.Natl.Inst.Stand.Tech. 103 (1998) 259, Schneider et al, Rep. Prog. Phys. 75 024401 (2012).

$$H_{\text{eff}} = \sum_{i<j} J_{ij} \sigma_i^x \sigma_j^x + B_z \sum_i \sigma_i^z .$$

$$J_{ij} \approx J_0 / |i-j|^\alpha$$

Analog

No gates



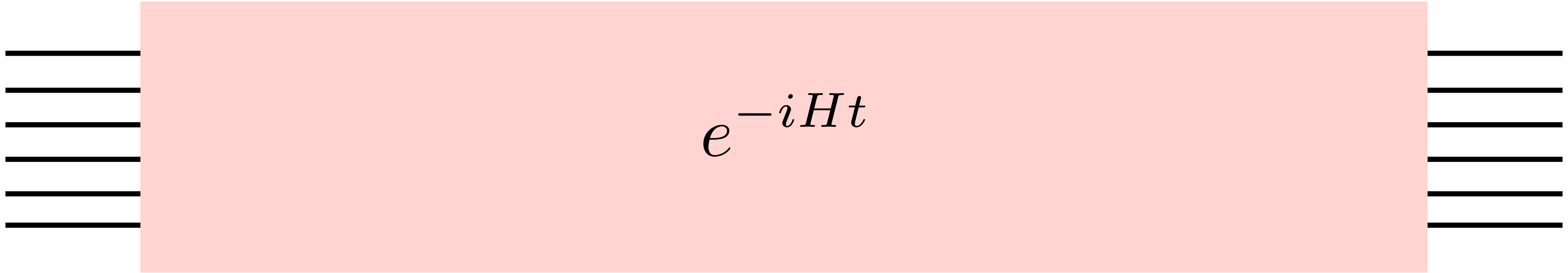
ZD, Hafezi, Monroe, Pagano, Seif, Shaw, Phys. Rev. Research, 2, 023015 (2020), arXiv: 1908.03210 [quant-ph].

See for other ideas: Ciavarella, Caspar, Singh, Savage, Lougovski, arXiv:2207.09438 [quant-ph].

Analog

$$H_{\text{eff}} = \sum_{\substack{i,j \\ j<i}} \left[J^{(xx)}_{i,j} \sigma_x^{(i)} \otimes \sigma_x^{(j)} + J^{(yy)}_{i,j} \sigma_y^{(i)} \otimes \sigma_y^{(j)} + J^{(zz)}_{i,j} \sigma_z^{(i)} \otimes \sigma_z^{(j)} \right] - \frac{1}{2} \sum_{i=1}^{N} B_z^{(i)} \sigma_z^{(i)}$$

No gates



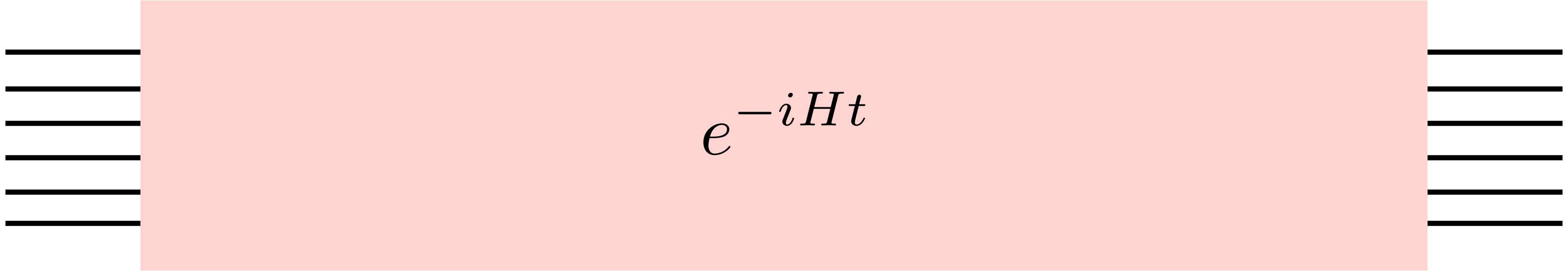
Recent development: N-body interactions: Katz, Centina, Monroe, Phys. Rev. Lett. 129, 063603 (2022).

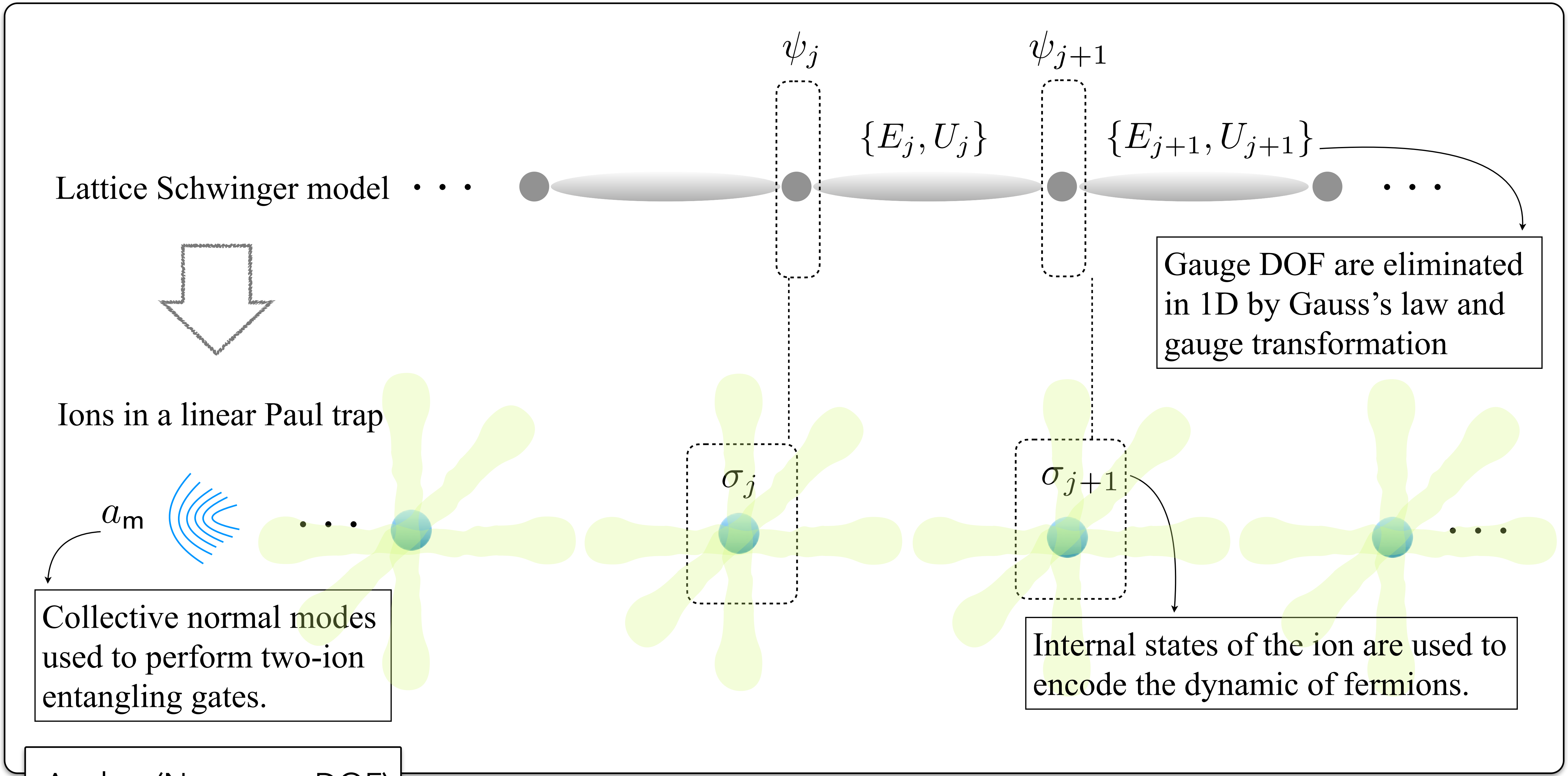
Andrade, ZD, Grass, Hafezi, Pagano, Seif, arXiv:2108.01022 [quant-ph], See also: Bermudez et al, Pays.Rev.A79, 060303 R (2009).

Analog

$$H_{\text{eff}} = \sum_i J_i^{(\sigma)} \sigma_z^{(i)} + \sum_{i,j} J_{i,j}^{(\sigma\sigma)} \sigma_+^{(i)} \otimes \sigma_+^{(j)} + \sum_{i,j,k} J_{i,j,k}^{(\sigma\sigma\sigma)} \sigma_+^{(i)} \otimes \sigma_+^{(j)} \otimes \sigma_+^{(k)} + \text{h.c.}$$

No gates





Lattice Schwinger model · · ·

Ions in a linear Paul trap

Gauge DOF are eliminated in 1D by Gauss's law and gauge transformation

Collective normal modes used to perform two-ion entangling gates.

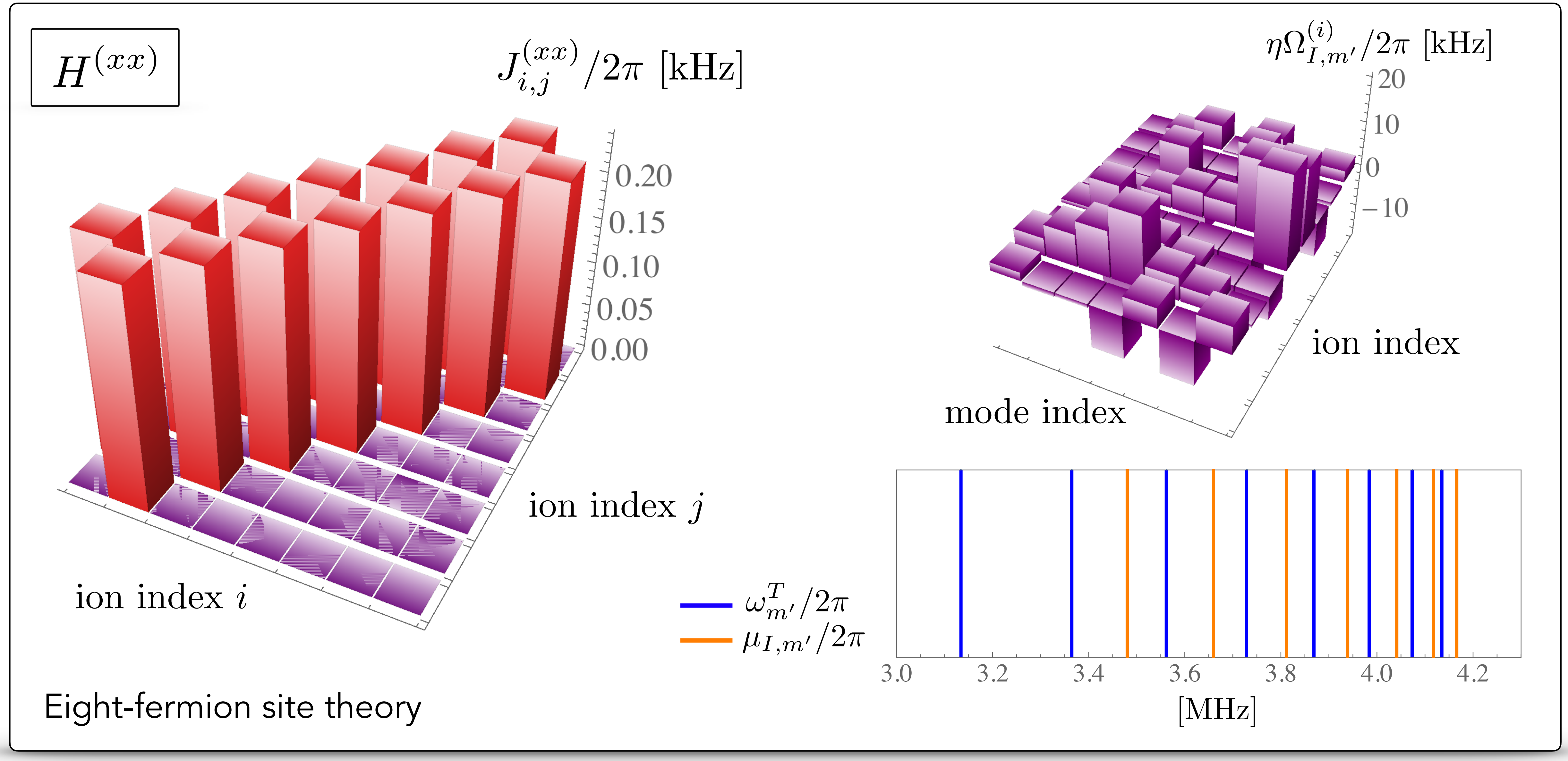
Internal states of the ion are used to encode the dynamic of fermions.

Analog (No gauge DOF)

`ZD, Hafezi, Monroe, Pagano, Seif and Shaw, Phys. Rev. R 2, 023015 (2020).`

`See also Yang et al, Physical Review A 94, 052321 (2016) for a phonon-ion based analog proposal of lattice Schwinger Model.`

$$H = x\sum_{n=1}^{N-1}\left[\sigma_+^{(n)}\sigma_-^{(n+1)} + \sigma_+^{(n+1)}\sigma_-^{(n)}\right] + \sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left(\sigma_z^{(m)} + (-1)^m\right)\right]^2 + \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_z^{(n)}$$



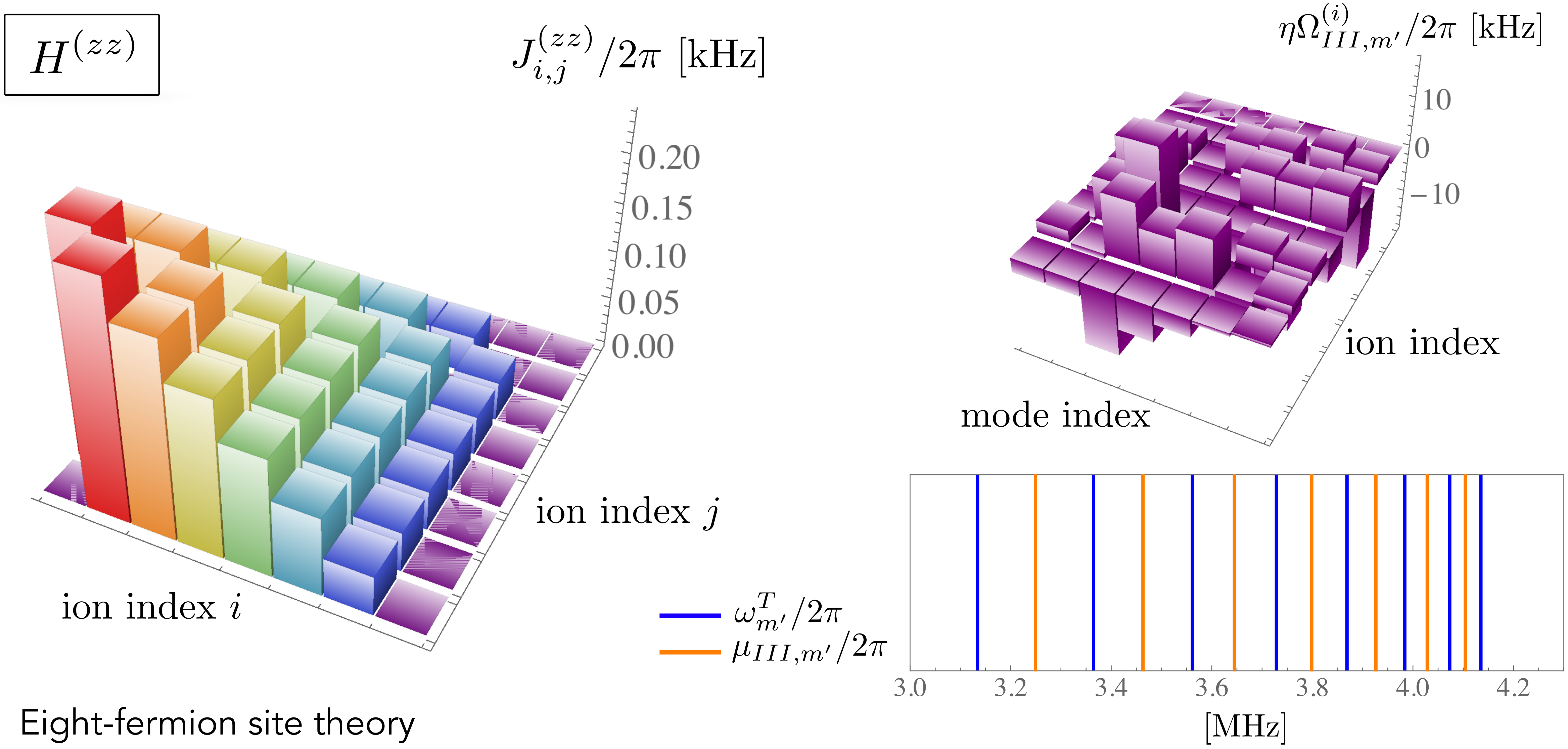
$H^{(xx)}$

Eight-fermion site theory

ZD, Hafezi, Monroe, Pagano, Seif and Shaw, Phys. Rev. R 2, 023015 (2020).

$$H = x\sum_{n=1}^{N-1}\left[\sigma_+^{(n)}\sigma_-^{(n+1)} + \sigma_+^{(n+1)}\sigma_-^{(n)}\right] + \sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left(\sigma_z^{(m)} + (-1)^m\right)\right]^2 + \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_z^{(n)}$$

$H^{(zz)}$



Eight-fermion site theory

`ZD, Hafezi, Monroe, Pagano, Seif and Shaw, Phys. Rev. R 2, 023015 (2020).`

$$H = x\sum_{n=1}^{N-1}\left[\sigma_+^{(n)}\sigma_-^{(n+1)} + \sigma_+^{(n+1)}\sigma_-^{(n)}\right] + \sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left(\sigma_z^{(m)} + (-1)^m\right)\right]^2 + \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_z^{(n)}$$

Implementation, benchmark, and co-development

- What is the capability limit of the hardware for gauge-theory simulations so far?
- What is the nature of noise in hardware and how can it best be mitigated?
- Can we co-develop dedicated systems for gauge-theory simulations?
- Can digital and analog ideas be combined to facilitate simulations of field theories?

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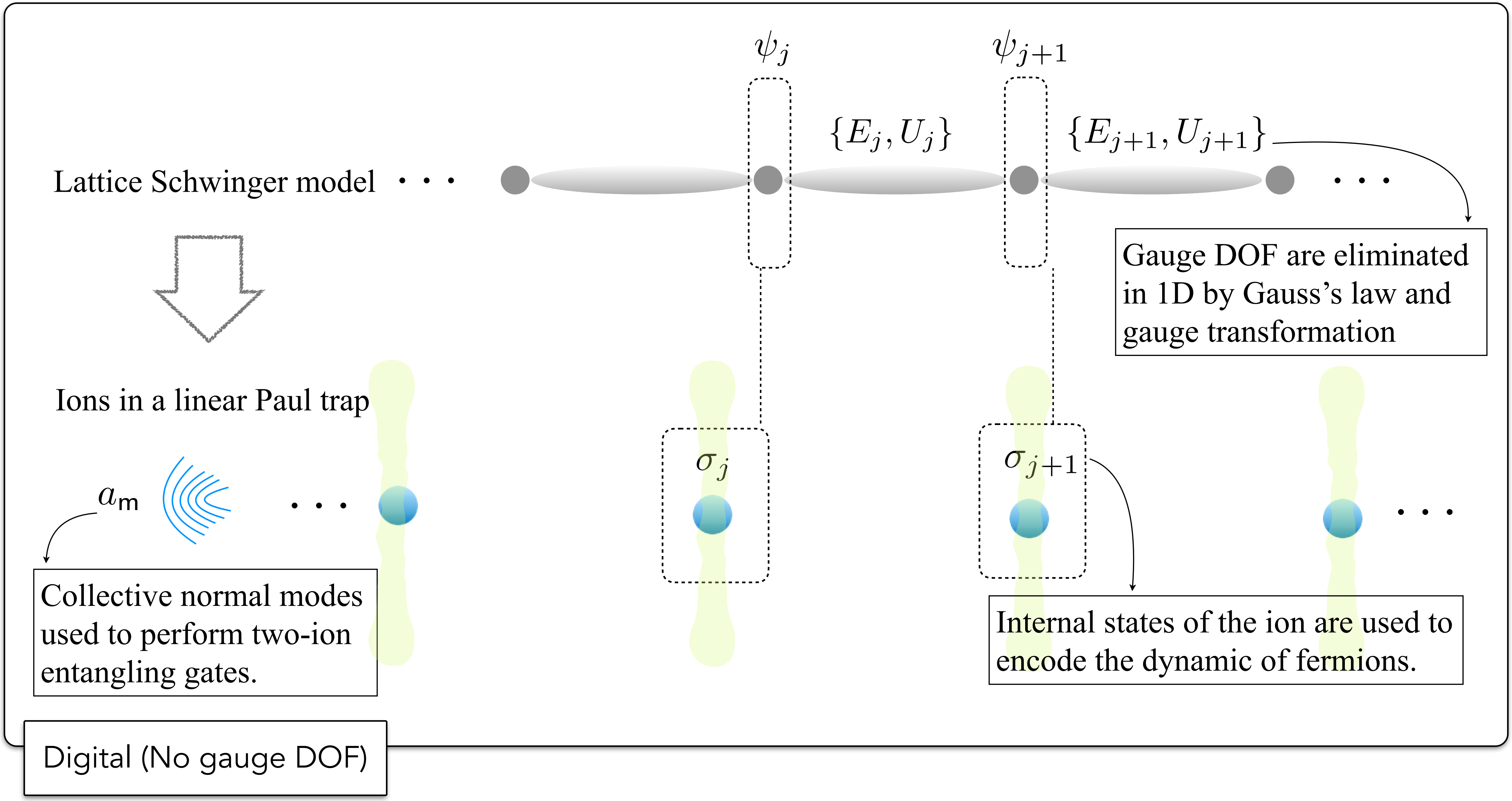


Lattice Schwinger model $\cdots$

ψ_j

ψ_{j+1}

$\{E_j, U_j\}$

$\{E_{j+1}, U_{j+1}\}$ $\cdots$

Gauge DOF are eliminated in 1D by Gauss's law and gauge transformation

Ions in a linear Paul trap

a_m

$\cdots$

σ_j

σ_{j+1}

$\cdots$

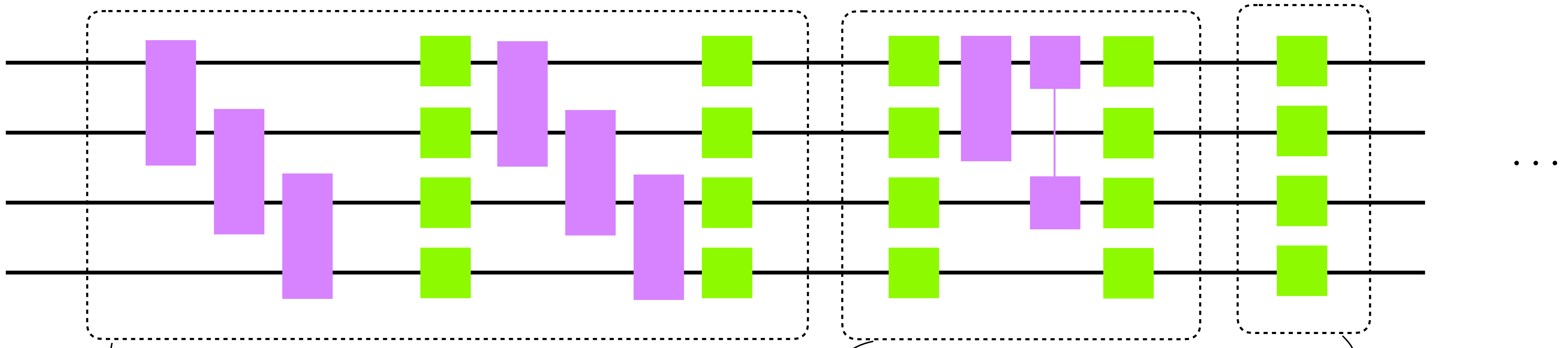
Collective normal modes used to perform two-ion entangling gates.

Internal states of the ion are used to encode the dynamic of fermions.

Digital (No gauge DOF)

$$H = x\sum_{n=1}^{N-1}\left[\sigma_+^{(n)}\sigma_-^{(n+1)} + \sigma_+^{(n+1)}\sigma_-^{(n)}\right] + \sum_{n=1}^{N-1}\left[\epsilon_0 + \frac{1}{2}\sum_{m=1}^{n}\left(\sigma_z^{(m)} + (-1)^m\right)\right]^2 + \frac{\mu}{2}\sum_{n=1}^{N}(-1)^n\sigma_z^{(n)}$$

Associated quantum circuit for Trotterized evolution:



Fermion-gauge interactions

Gauge-field interactions

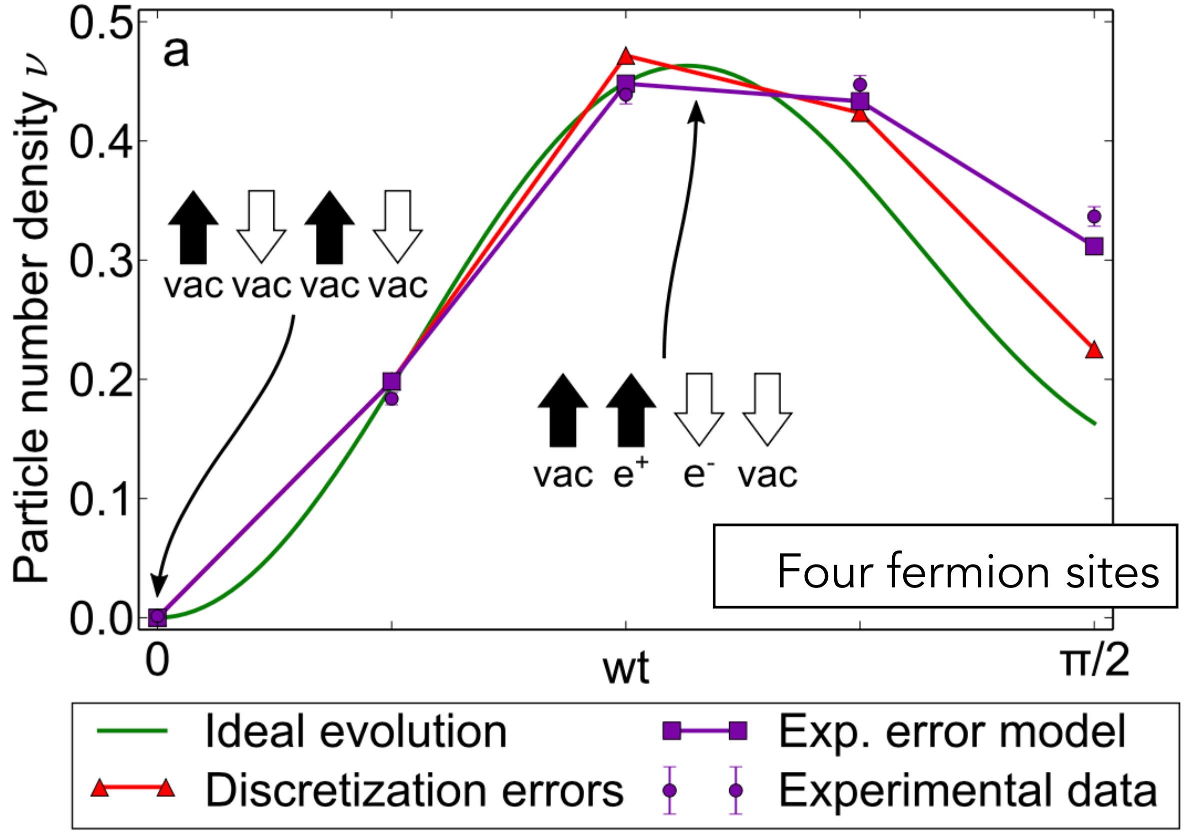
Fermion mass term

Four-fermion site theory, one Trotter step

$$H = x \sum_{n=1}^{N-1} \left[\sigma_{+}^{(n)} \sigma_{-}^{(n+1)} + \sigma_{+}^{(n+1)} \sigma_{-}^{(n)} \right] + \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left(\sigma_z^{(m)} + (-1)^m \right) \right]^2 + \frac{\mu}{2} \sum_{n=1}^{N} (-1)^n \sigma_z^{(n)}$$

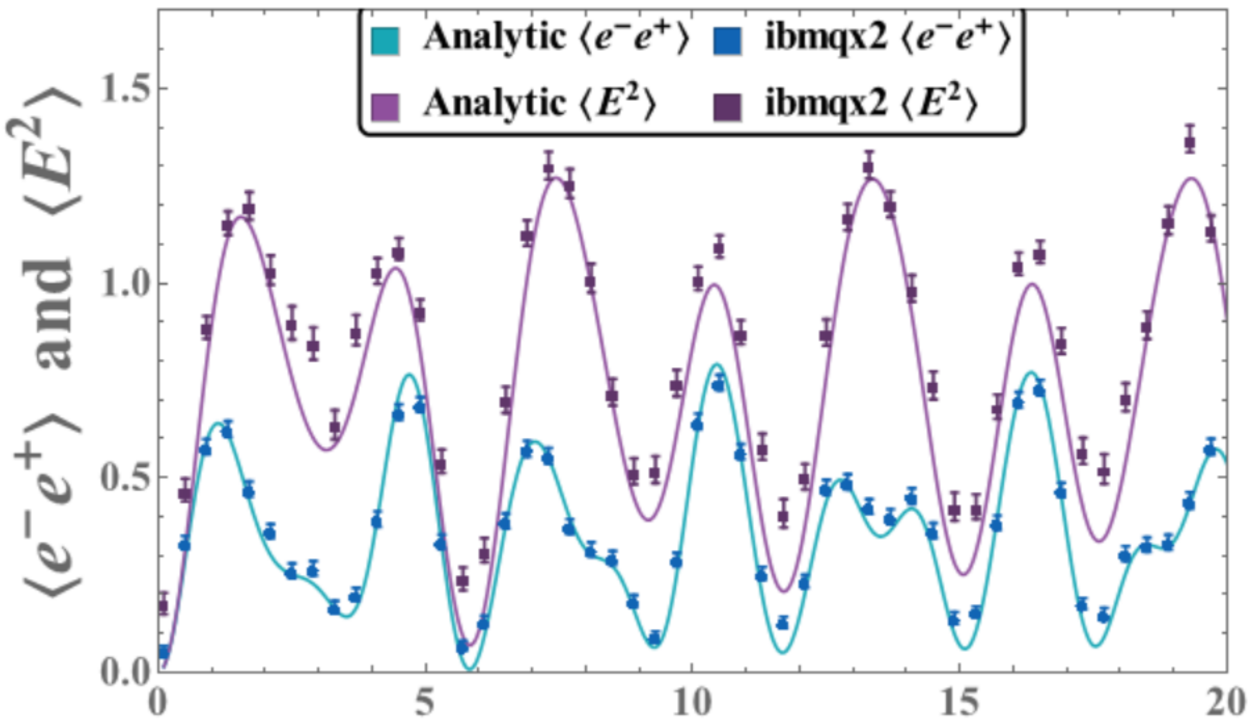
IMPLEMENTATIONS ON THE ACTUAL QUANTUM HARDWARE

`Martinez et al, Nature 534, 516 EP (2016).`



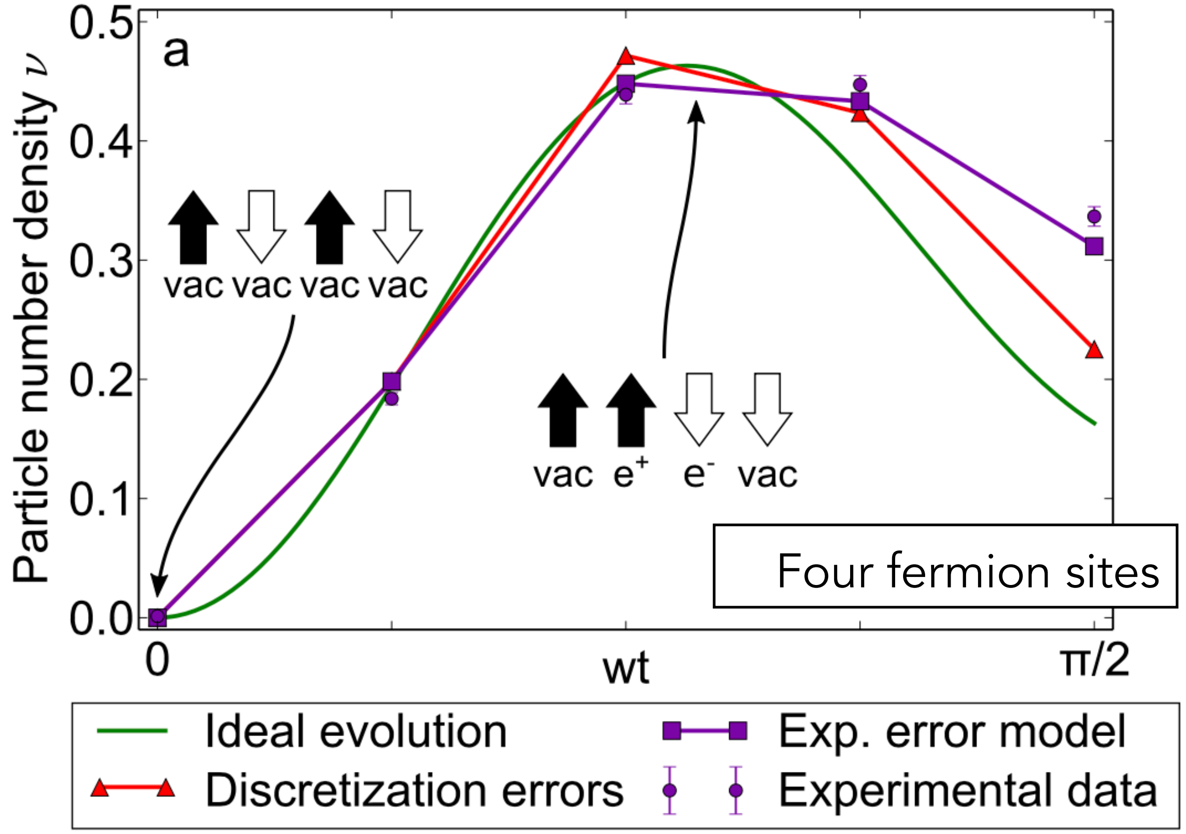
IMPLEMENTATIONS ON THE ACTUAL QUANTUM HARDWARE

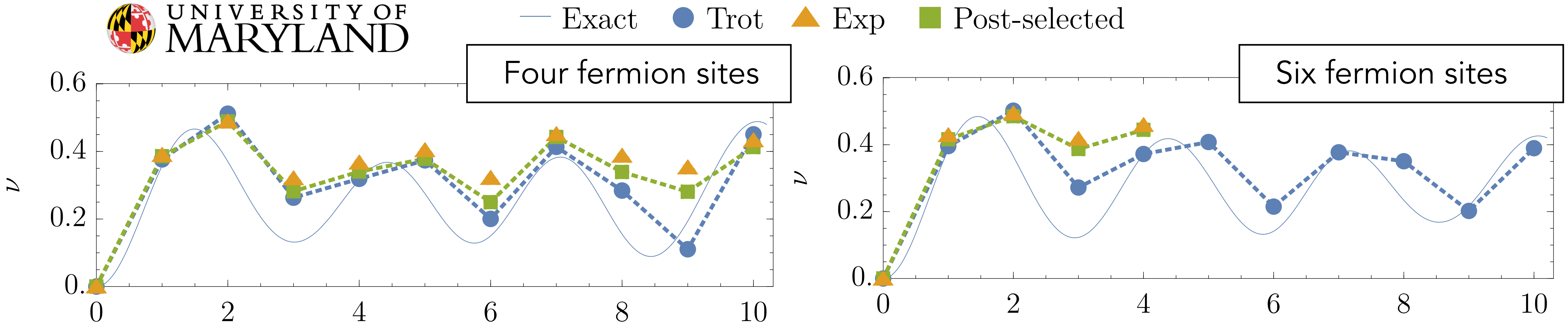
Klco, Savage, et al, Phys. Rev. A 98, 032331 (2018).



A hybrid classical-quantum approach allows a 2-qubit reduction of 4-qubit simulation.

Martinez et al, Nature 534, 516 EP (2016).



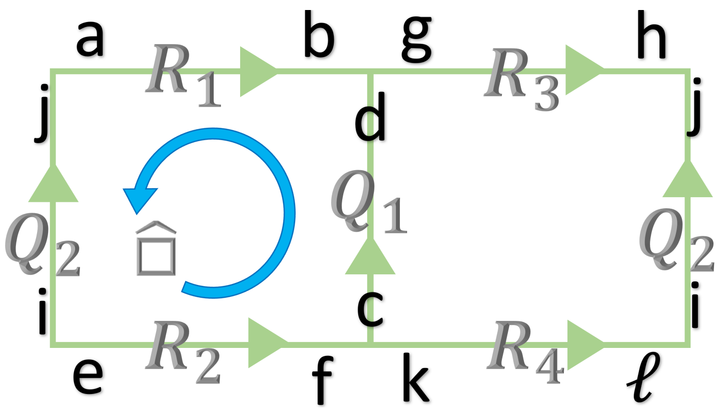


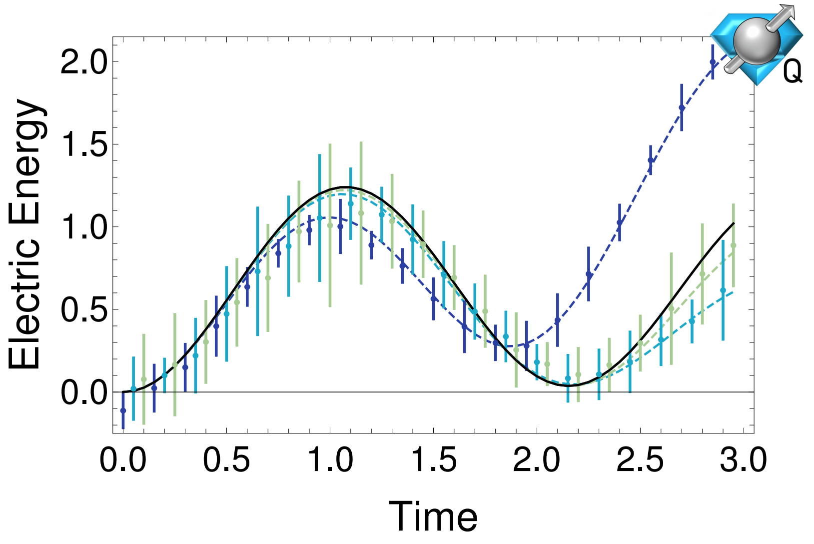
80 entangling gates!

90 entangling gates!

Nguyen, Tran, Zhu, Green, Huerta Alderete, ZD, Linke, *PRX Quantum* 3 (2022) 2, 020324.

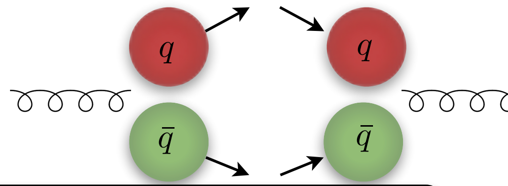
NON-ABELIAN GAUGE THEORIES: HARDWARE IMPLEMENATTION





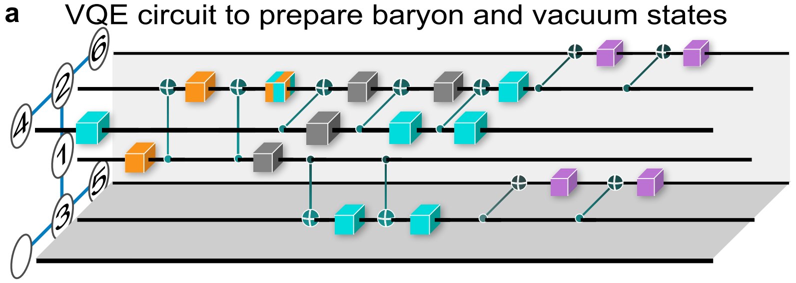
Real-time dynamic of pure SU(3) with global irrupts on IBM

Ciavarella, Klco, and Savage, Phys. Rev. D 103, 094501 (2021).

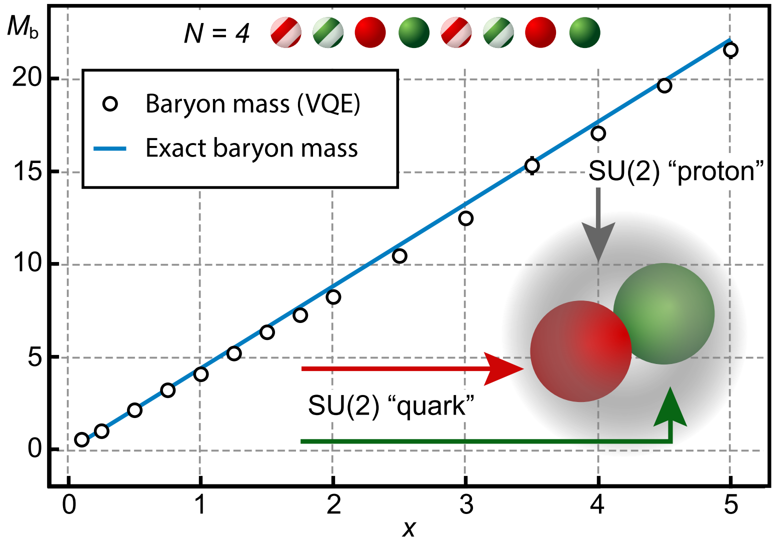


Low-lying spectrum of SU(2) with matter in 1+1 D on IBM

a VQE circuit to prepare baryon and vacuum states

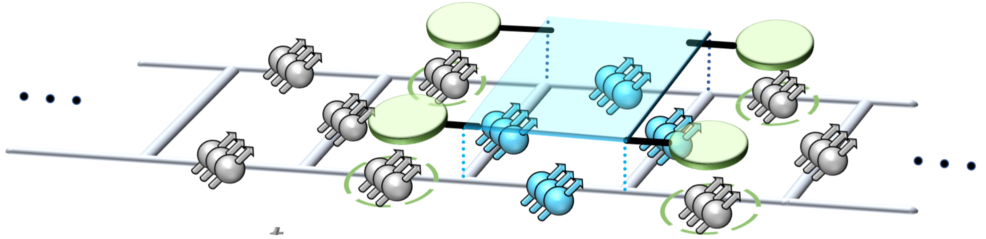


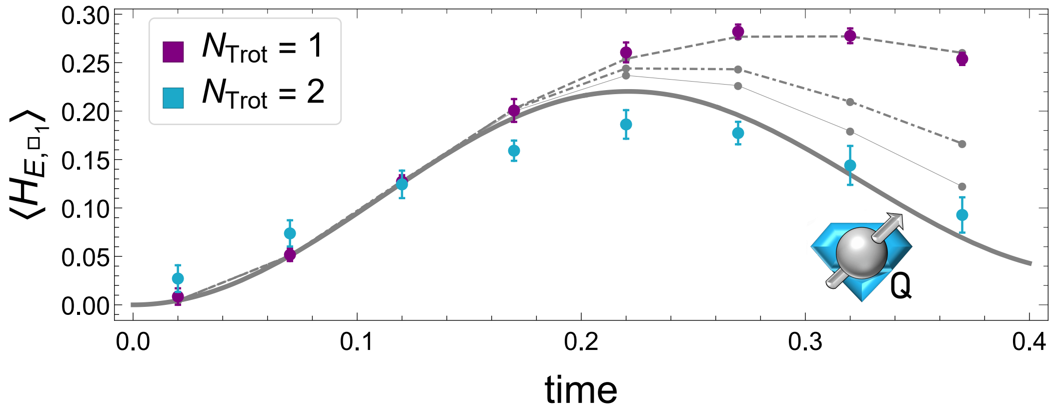
b VQE preparation of the baryon mass



Atas et al, Nature Communications 12, 6499 (2021). SU(3) example: Atas et al: arXiv:2207.03473 [quant-ph].

Real-time dynamic of pure SU(2) with global irreps on IBM





Klco, Savage, and Stryker, Phys. Rev. D 101, 074512 (2020).

See also studies on D-wave annealers: Rahman et al, Phys. Rev. D 104, 034501 (2021), Illa and Savage, arXiv:2202.12340 [quant-ph], Farrel et al, arXiv:2207.01731 [quant-ph].

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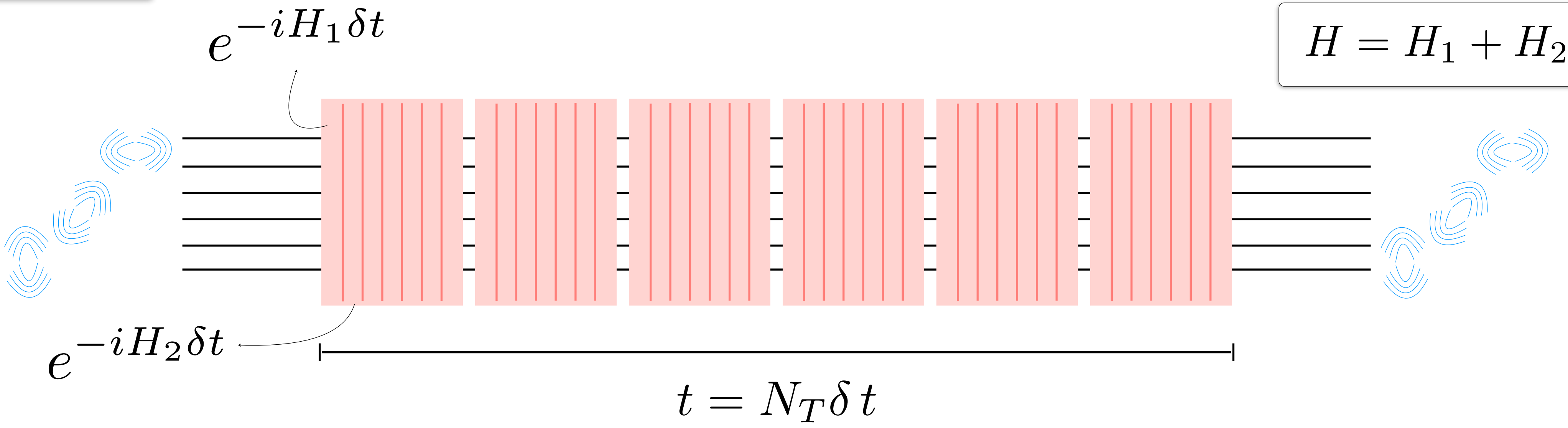
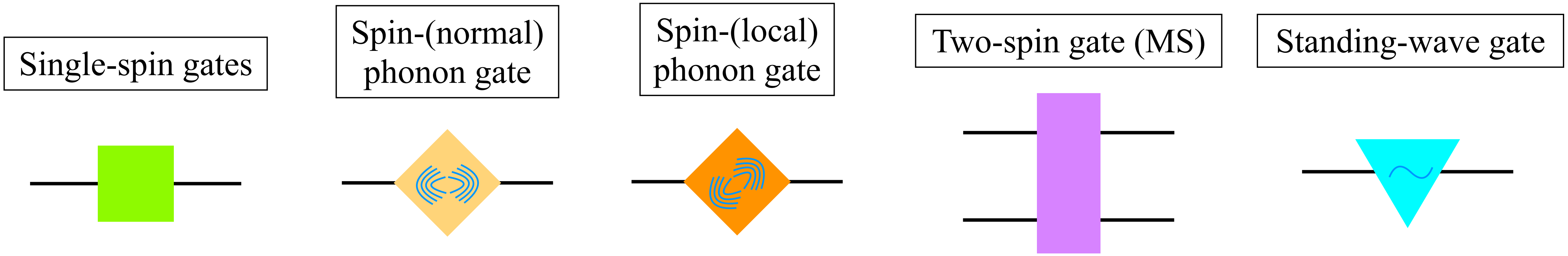
Analog-Digital



$$H = H_1 + H_2 + \cdots$$



Single-spin gates

Spin-(normal) phonon gate

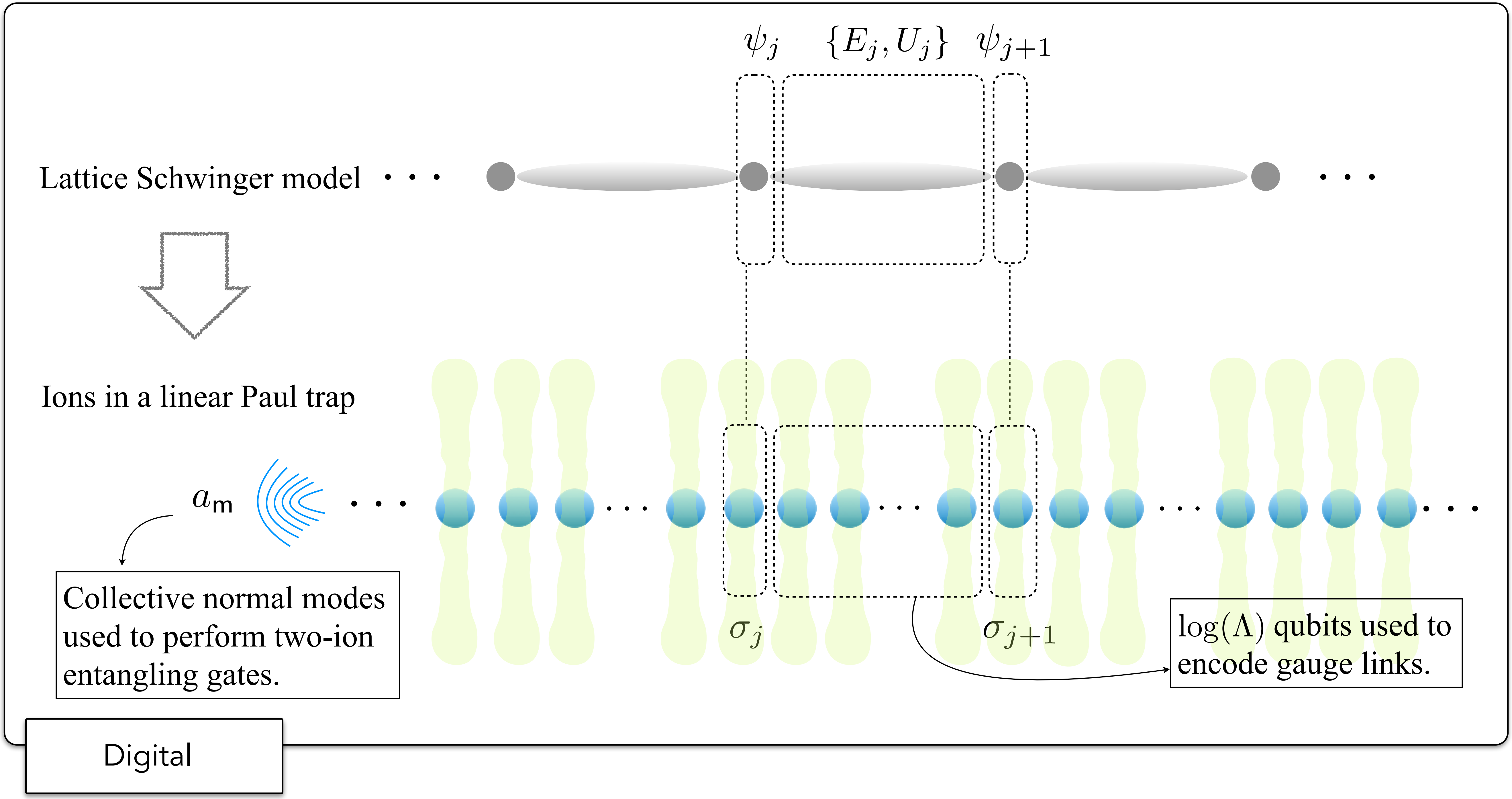
Spin-(local) phonon gate

Two-spin gate (MS)

Standing-wave gate

ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

How many qubits and gates are required to achieve accuracy ϵ in a given observables? Are there algorithms that scale optimally?



Lattice Schwinger model

Ions in a linear Paul trap

Collective normal modes used to perform two-ion entangling gates.

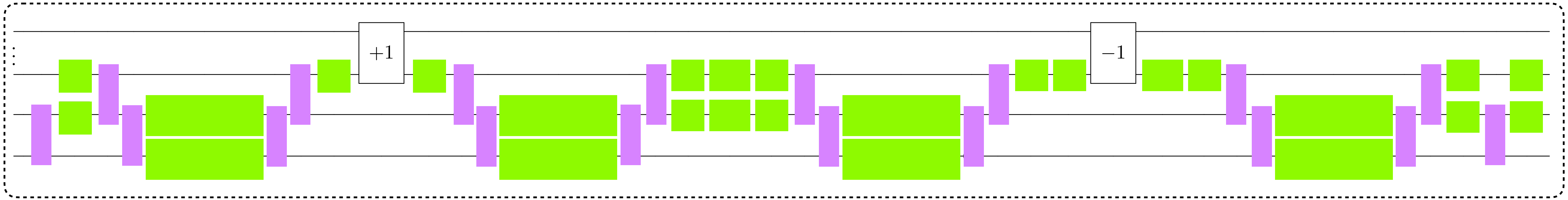
$\log(\Lambda)$ qubits used to encode gauge links.

Digital

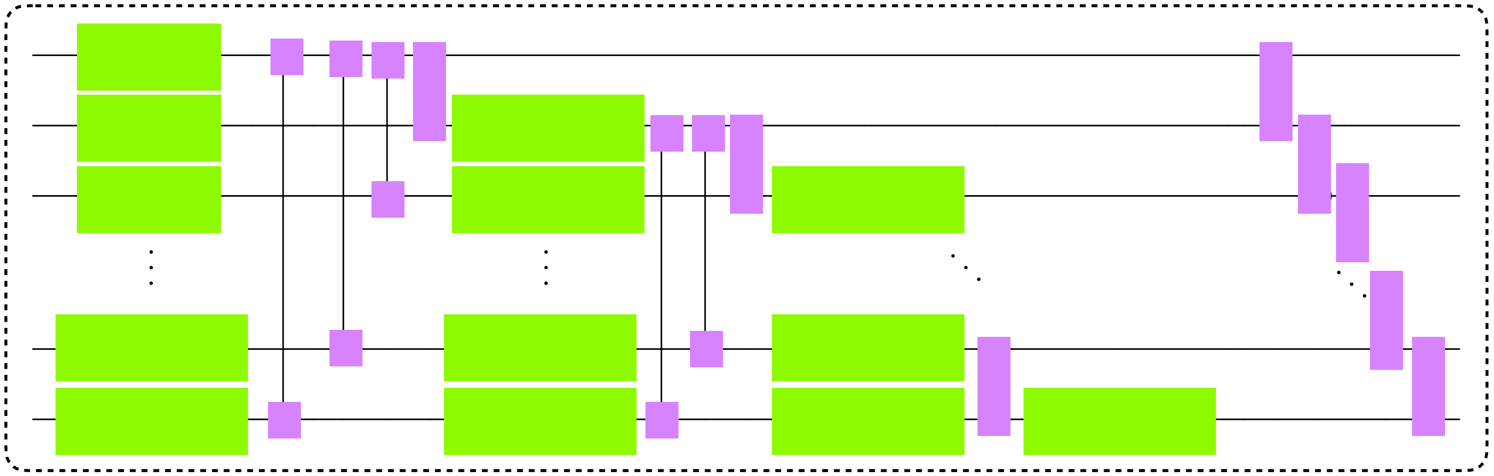
$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$

Circuit and recourse analysis

Shaw, Lougovski, Stryker, Wiebe, Quantum 4, 306 (2020).



Sample gauge-fermion interaction block



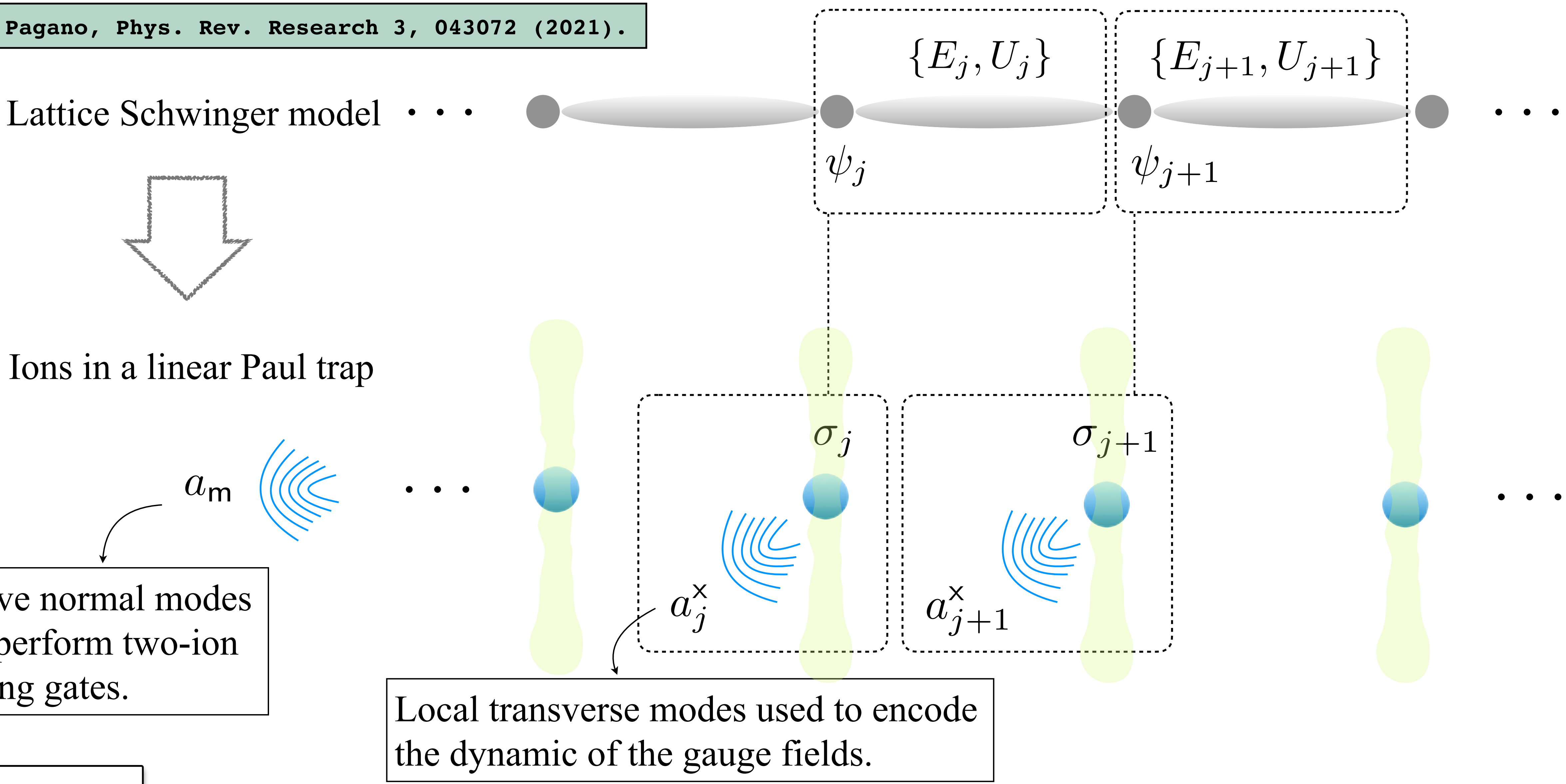
Part of electric field interactions acting on gauge DOF registers

Near term cost

	$\delta_g = 10^{-3}$		$\delta_g = 10^{-4}$		$\delta_g = 10^{-5}$		$\delta_g = 10^{-6}$		$\delta_g = 10^{-7}$	
	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT	$\tilde{\epsilon}^2$	CNOT
$x = 10^{-2}$	—	7.3e4	—	1.6e5	—	3.4e5	—	7.3e5	5.6e-2	1.6e6
$x = 10^{-1}$	—	1.6e4	—	3.5e4	—	7.5e4	5.9e-2	1.6e5	2.7e-3	3.5e5
$x = 1$	—	4.6e3	—	9.9e3	1.0e-1	2.1e4	4.7e-3	4.6e4	2.2e-4	9.9e4
$x = 10^2$	—	2.8e3	8.3e-1	6.1e3	3.8e-2	1.3e4	1.8e-3	2.8e4	8.2e-5	6.0e4

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$

ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).



Lattice Schwinger model

Ions in a linear Paul trap

Collective normal modes used to perform two-ion entangling gates.

Local transverse modes used to encode the dynamic of the gauge fields.

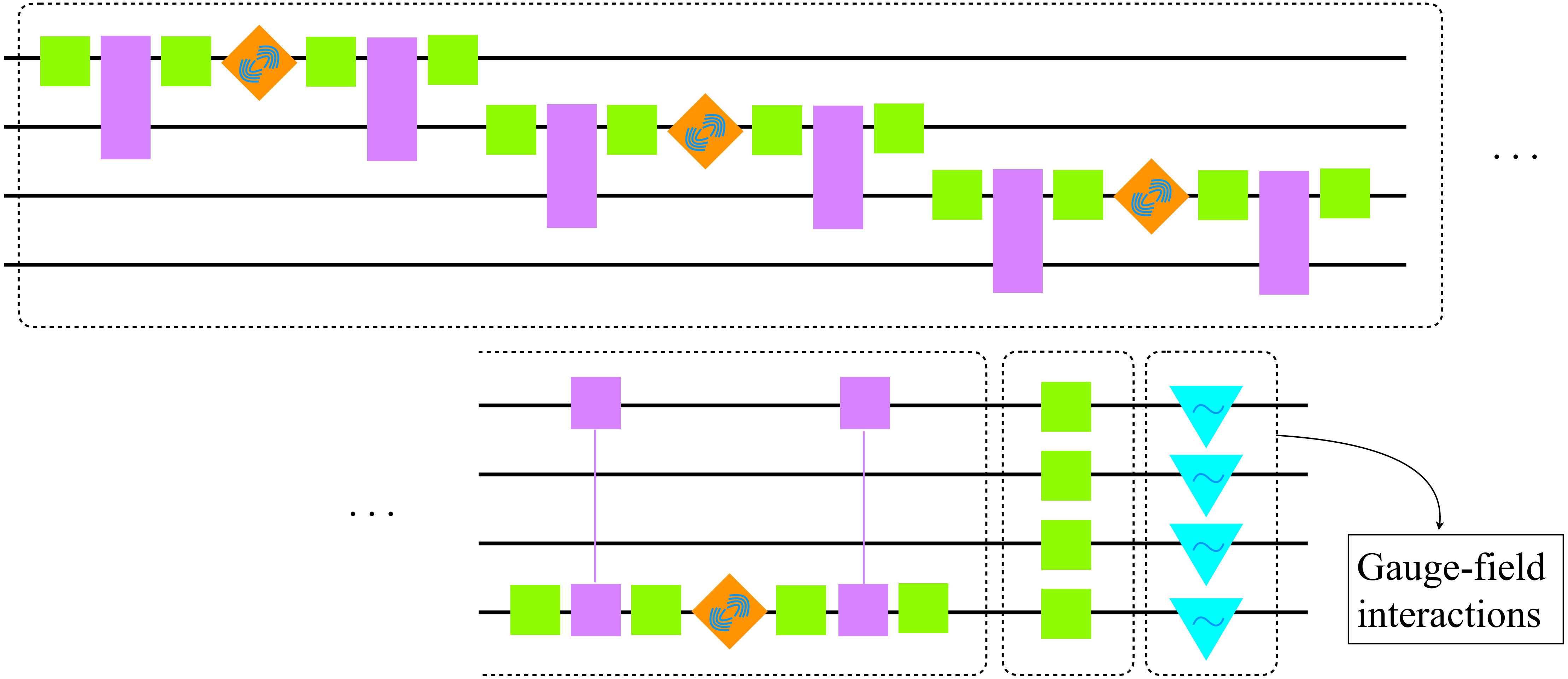
Analog-Digital

See Yang et al, Physical Review A 94, 052321 (2016) for the highly-occupied bosonic model of the Schwinger model.

See also Casanova et al, Phys. Rev. Lett. 108, 190502 (2012), Lamata et al, EPJ Quant. Technol. 1, 9 (2014), and Mezzacapo et al, Phys. Rev. lett. 109, 200501 (2012) for analog-digital approaches to other interacting fermion-boson theories.

$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$

Analog-Digital



Fermion-gauge interactions

Gauge-field interactions

Fermion mass term

ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

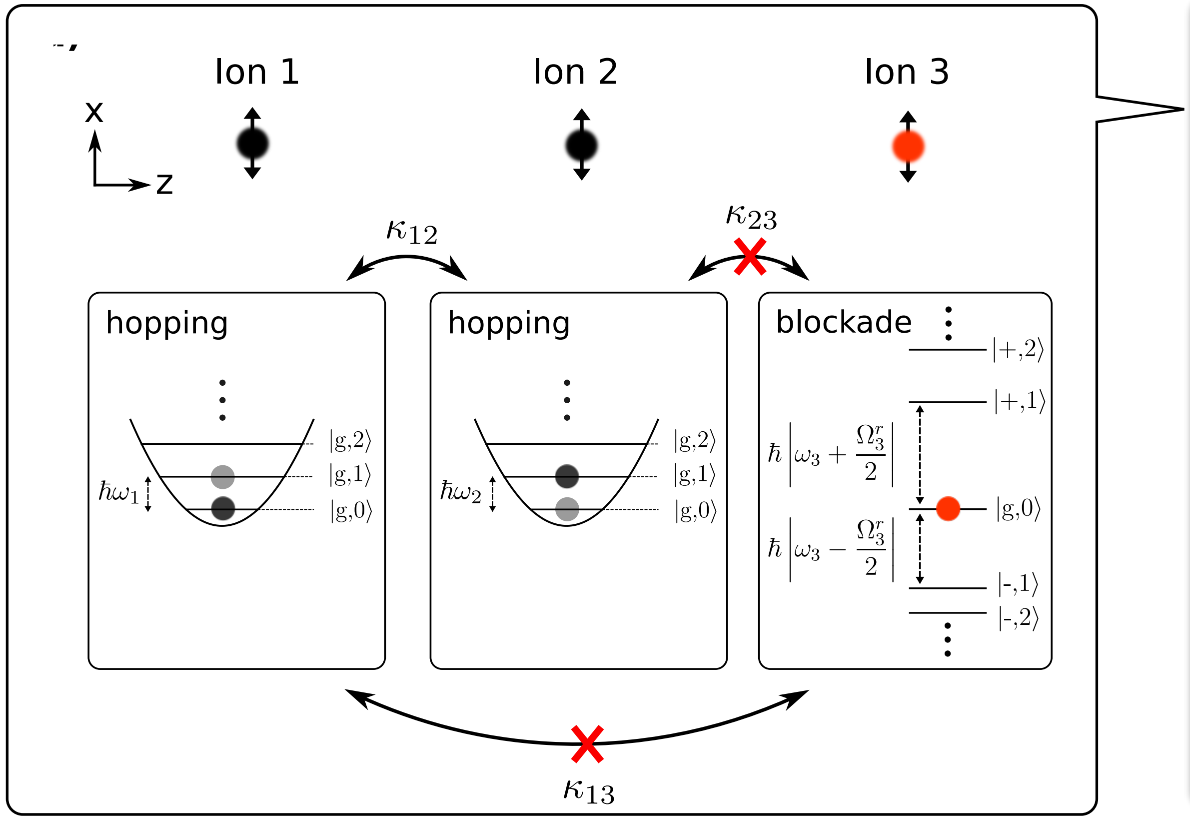
$$H = -ix \sum_{n=1}^{N-1} \left[\psi_n^\dagger U_n \psi_{n+1} - \text{h.c.}\right] + \sum_{n=1}^{N-1} E_n^2 + \mu \sum_{n=1}^{N} (-1)^n \psi_n^\dagger \psi_n$$

Let us compare the circuit structure of digital and analog-digital cases when gauge DOF are present:

Schwinger model			
	Fermion-gauge interaction	Fermion mass	Electric-field term
Analog-digital	$\mathcal{O}\left(N\right)$	$\mathcal{O}\left(1\right)$	$\mathcal{O}(N)$
Digital	$\mathcal{O}\left(N^2\left(\log\Lambda\right)^2\right)$	$\mathcal{O}\left(1\right)$	$\mathcal{O}\left(N\left(\log\Lambda\right)^2\right)$

ZD, Linke, Pagano, Phys. Rev. Research 3, 043072 (2021).

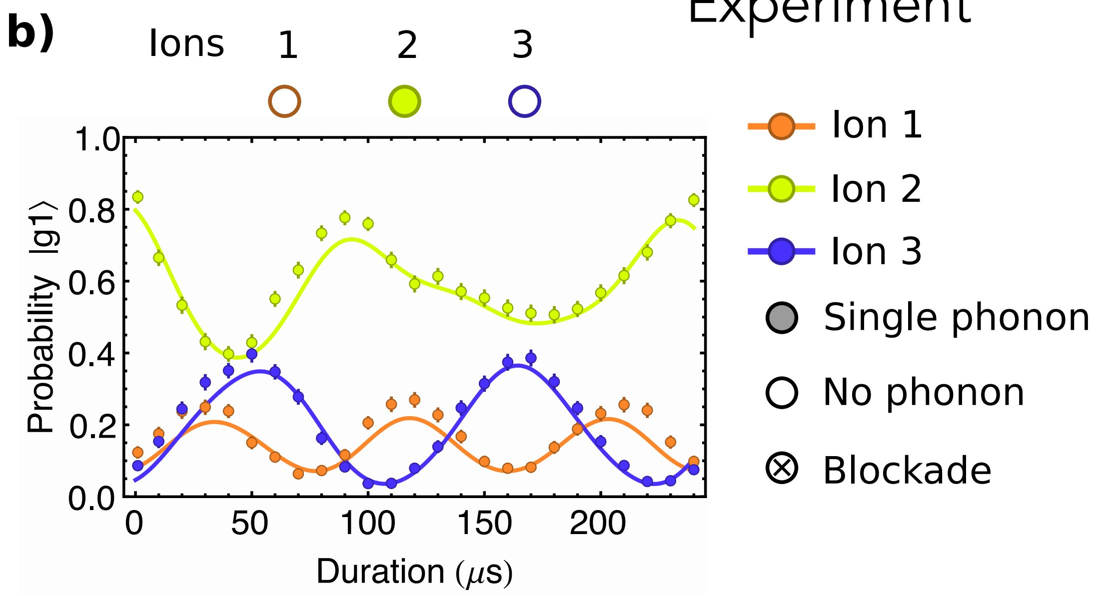
Is phonon control experimentally feasible? Yes…at least for small systems so far!



Debnath et al, Phys. Rev. Lett. 120, 073001 (2018).

UNIVERSITY OF MARYLAND

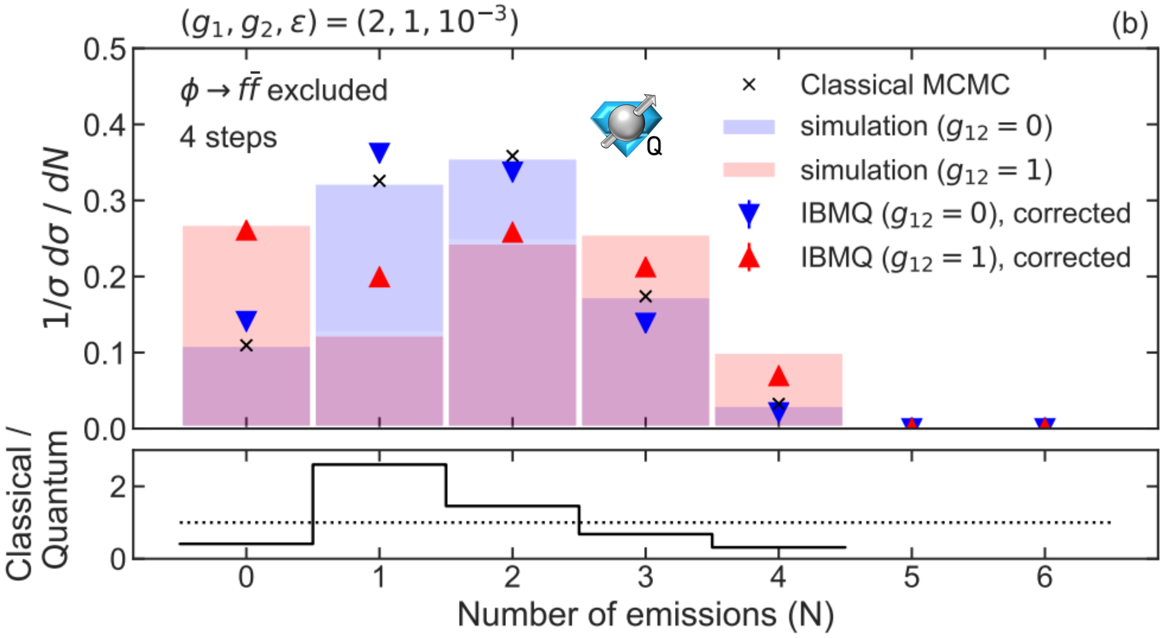
Monroe-Linke Experiment



Finally a few more examples showcasing progress in hardware implementation of a range of QCD-inspired problems…

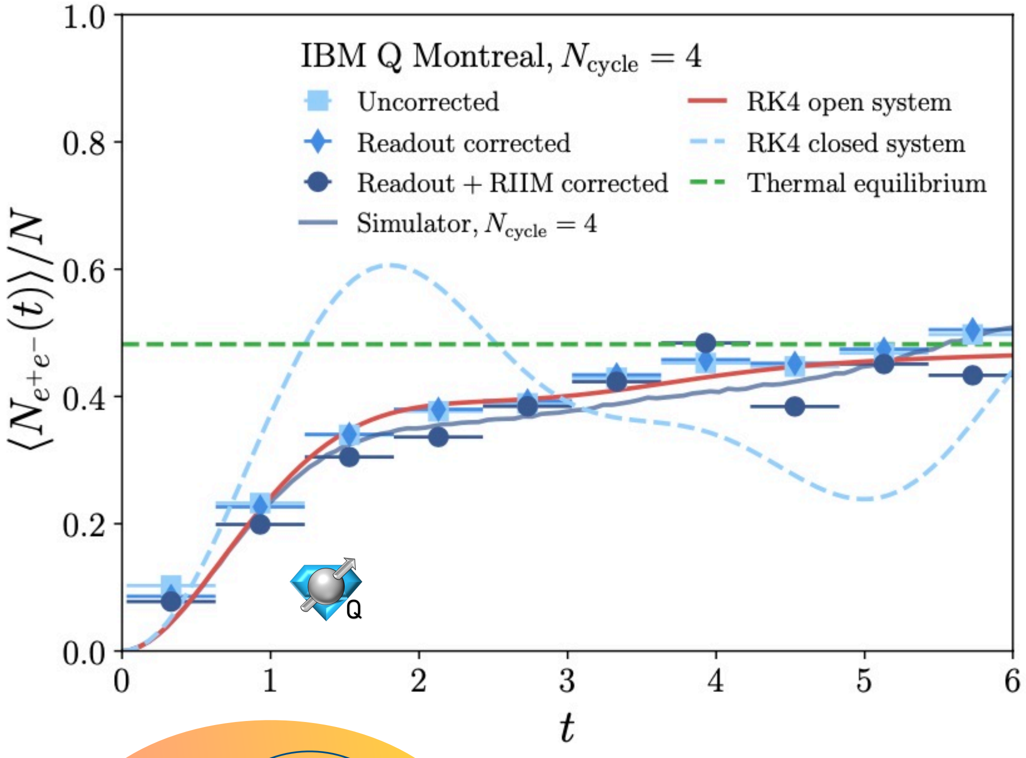
PARTON SHOWER ALGORITHMS AND HEAVY QUARKONIA MOTION IN QGP

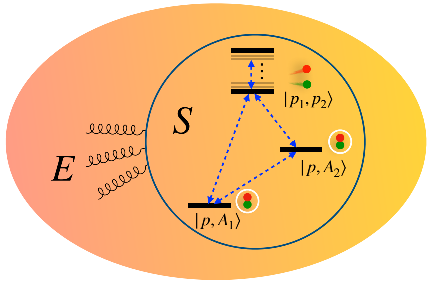
Nachman, Provasoli, and Bauer‡, *Phys. Rev. Lett.* 126 (2021) 6, 062001.



A polynomial time quantum final state shower algorithm that accurately models the effects of intermediate spin states similar to those present in electroweak showers.

See also Bepari, Malik, Spannowsky, Williams, Phys. Rev. D 103, 076020 (2021), Williams, Malik, Spannowsky, Bepari, Phys. Rev. D 106 (2022) 056002, Gustafson, Prestel, Spannowsky, Williams, J. High Energ. Phys. 2022, 35 (2022).

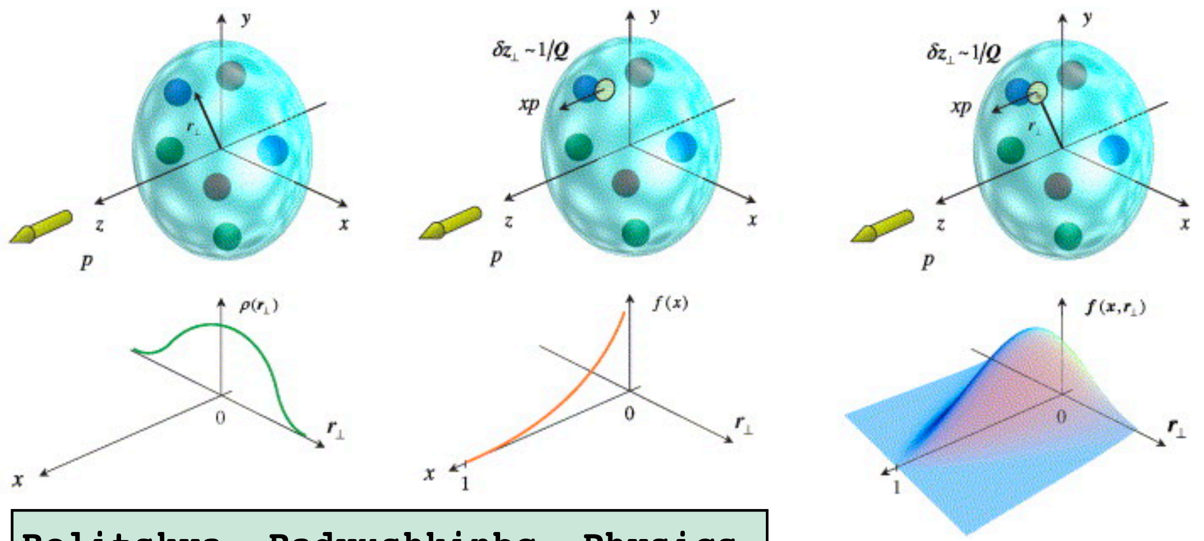




$q\bar{q}$ moving in medium

de Jong, Metcal, Mulligan, Ploskon, Ringer, and, Yao, *Phys.Rev.D* 104 (2021) 5, 051501.

PARTON DISTRIBUTION FUNCTIONS, DECAY AMPLITUDES



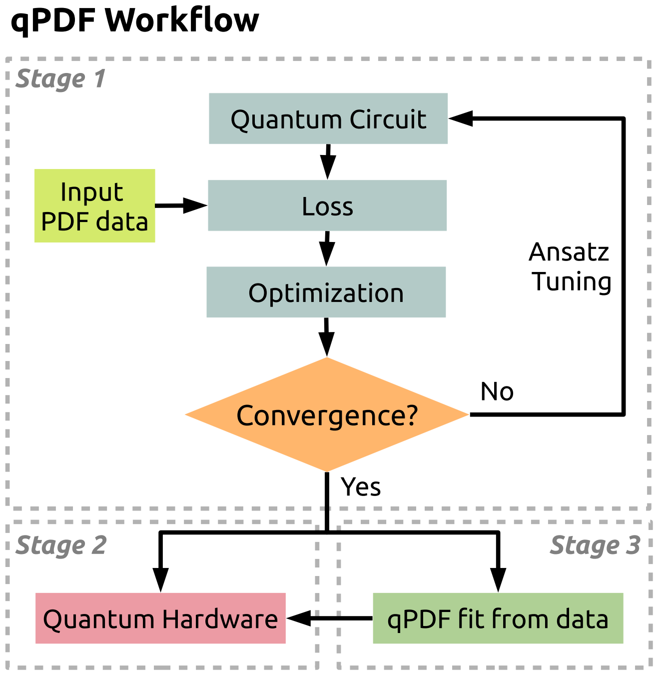
Belitskya, Radyushkinbc, Physics Reports 418 (2005), 1-387.

Either calculate PDFs directly since non-equal time amplitudes are possible on quantum computers…

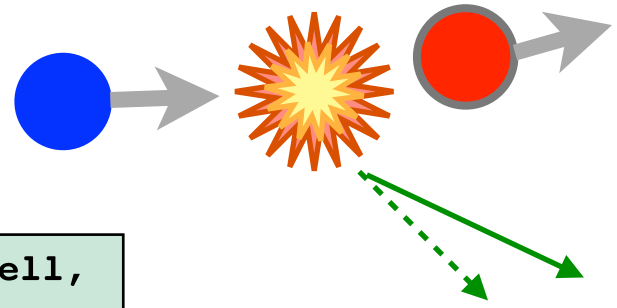
Mueller, Tarasov, and Raju Venugopalan, PRD 102, 016007 (2020), Lamm, Lawrence, and Yamauchi, Phys. Rev. Res. 2, 013272 (2020), Echevarria, Egusquiza, Rico, and G Schnell, PRD 104, 014512 (2021).

…or expedite global fitting of PDFs with variational quantum eigensolvers…

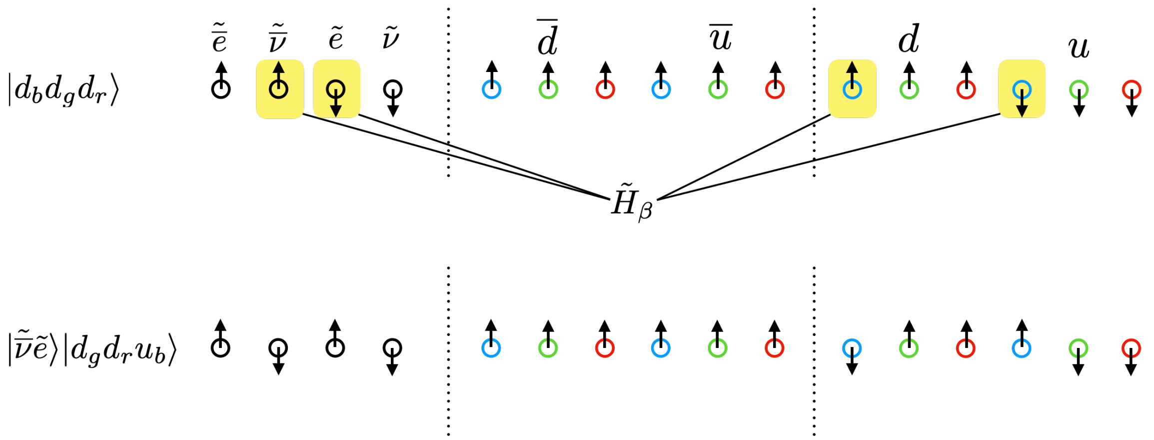
Perez-Salinas, Cruz-Martinez, Alhajri, and Carrazza , PRD 103, 034027 (2021), Qian, Basili, Pal, Luecke, and Vary, arXiv:2112.01927 (2021).

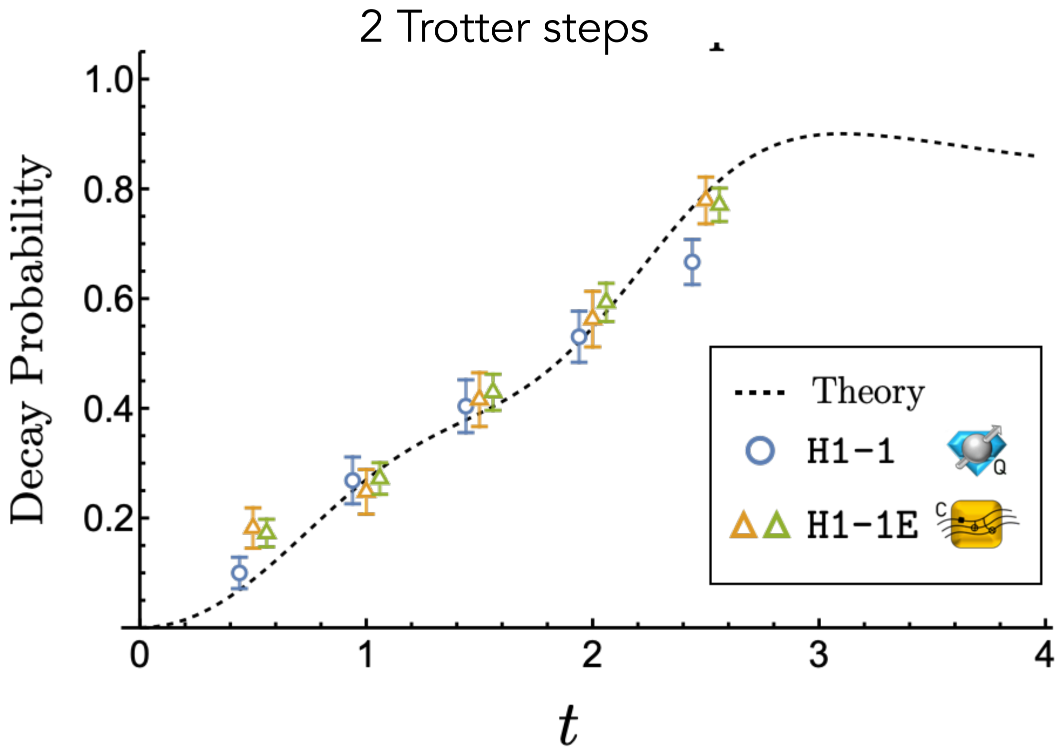


Quantum computing β decay in 1+1 QCD

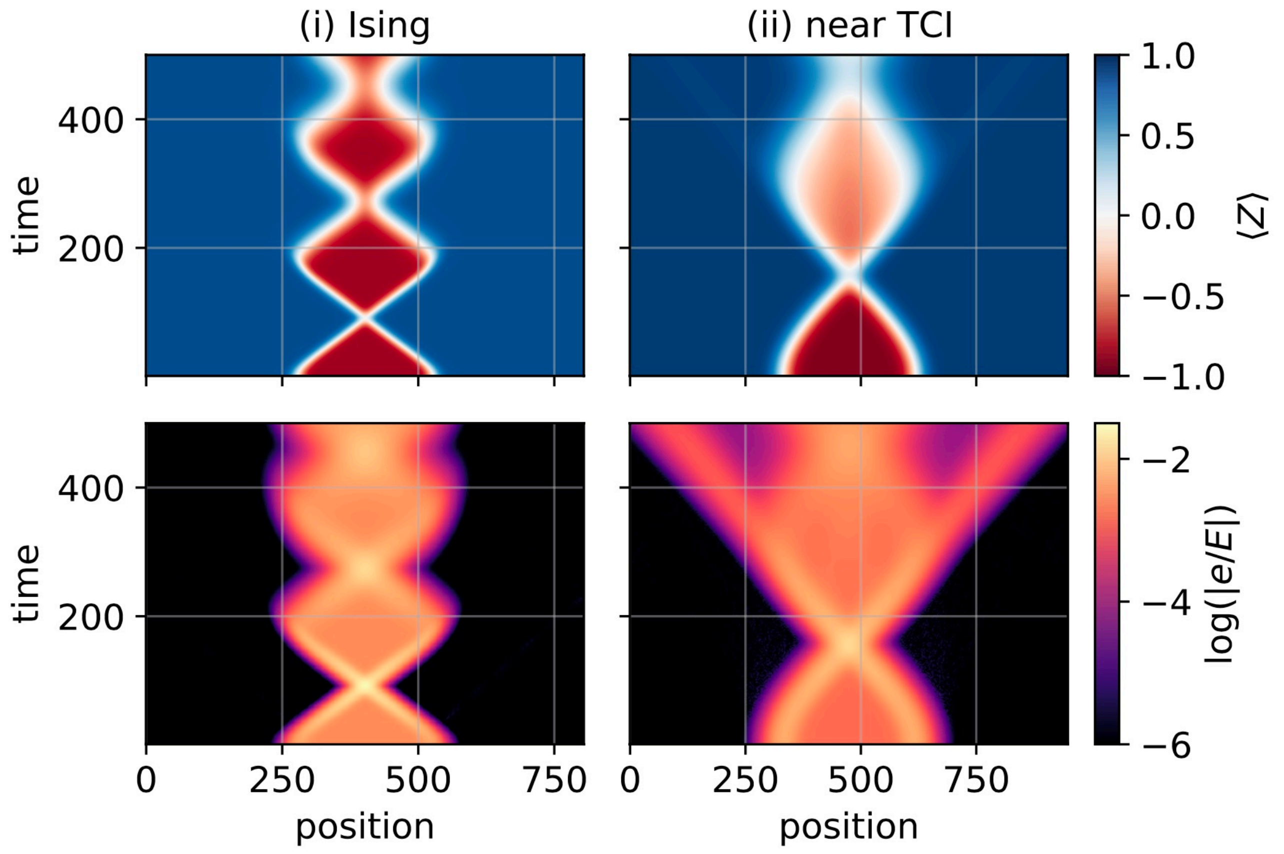


Farrell, Chernyshev, Powell, Zemlevskiy, Illa, and Savage, arXiv:2209.10781 [quant-ph].

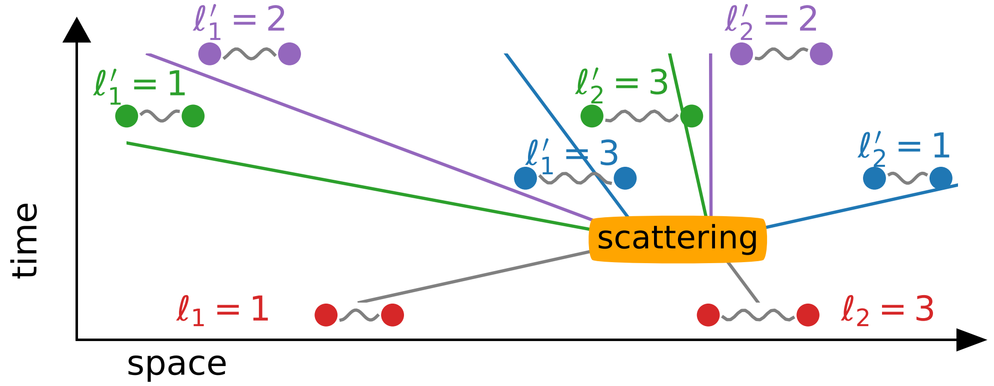


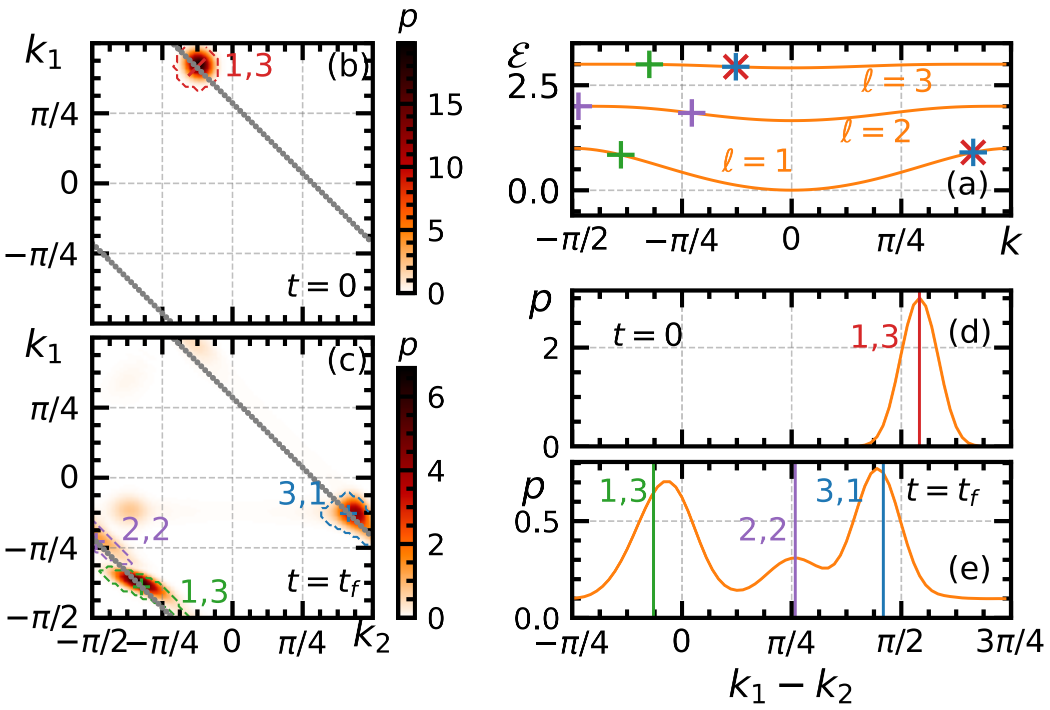


FIRST STEPS TOWARD SCATTERING IN SPIN SYSTEMS
— NUMERICAL SIMULATIONS —



Ashley Milsted, Liu, John Preskill, and Vidal, PRX Quantum 3 (2022) 2, 020316.

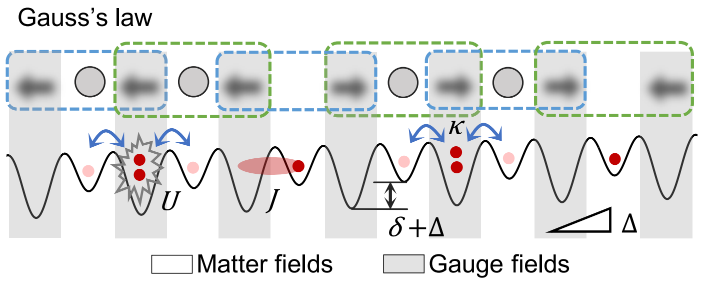




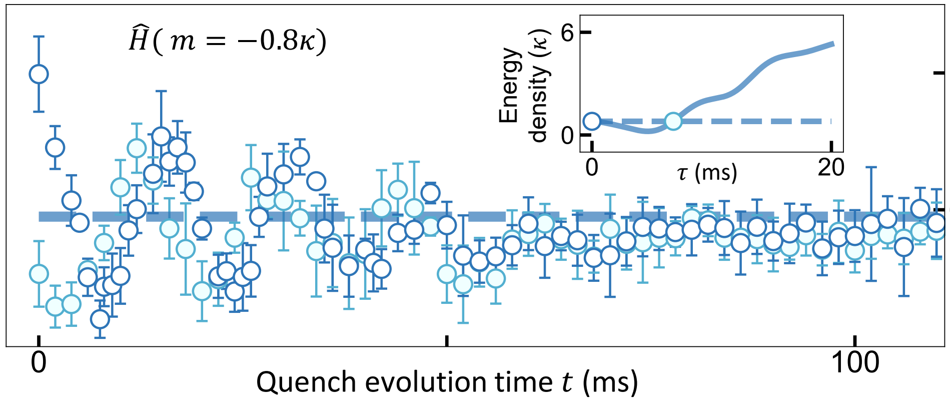
Surace, Lerose, New J. Phys. 23 (2021) 062001.

THERMALIZATION AND NON-EQUILIBRIUM PROPERTIES

Thermalization dynamics of U(1) Quantum Link Model in a 71-site analog simulator

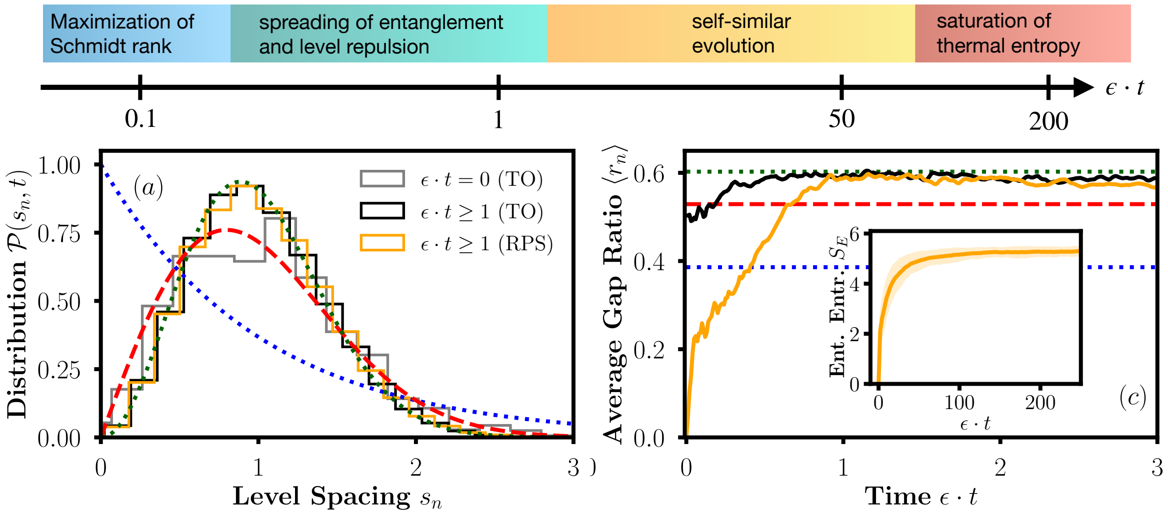


Zhou et al, Science 377 (2022) 6603.

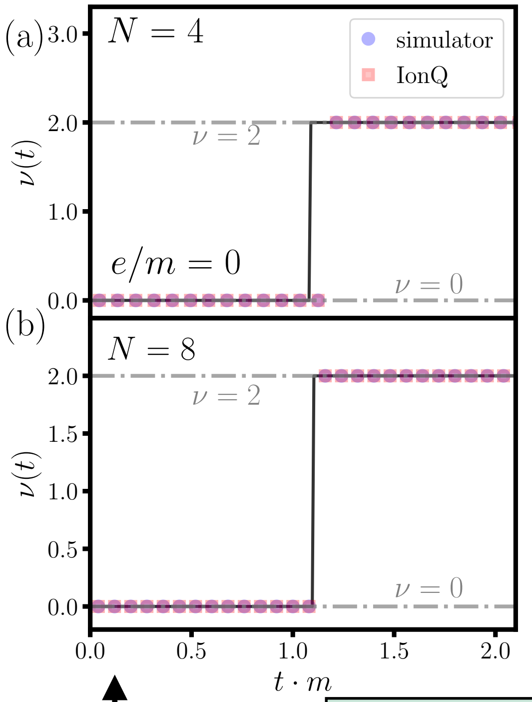


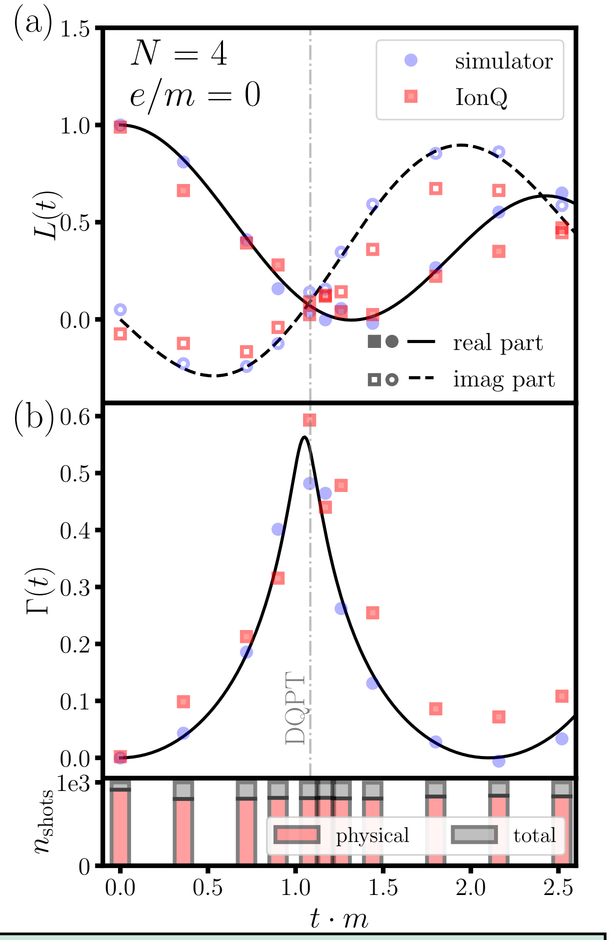
Stages of thermalization dynamics of Z2 LGT in 2+1 D from entanglement spectrum

Mueller, Zache, Ott, Phys. Rev. Lett. 129, 011601 (2022).

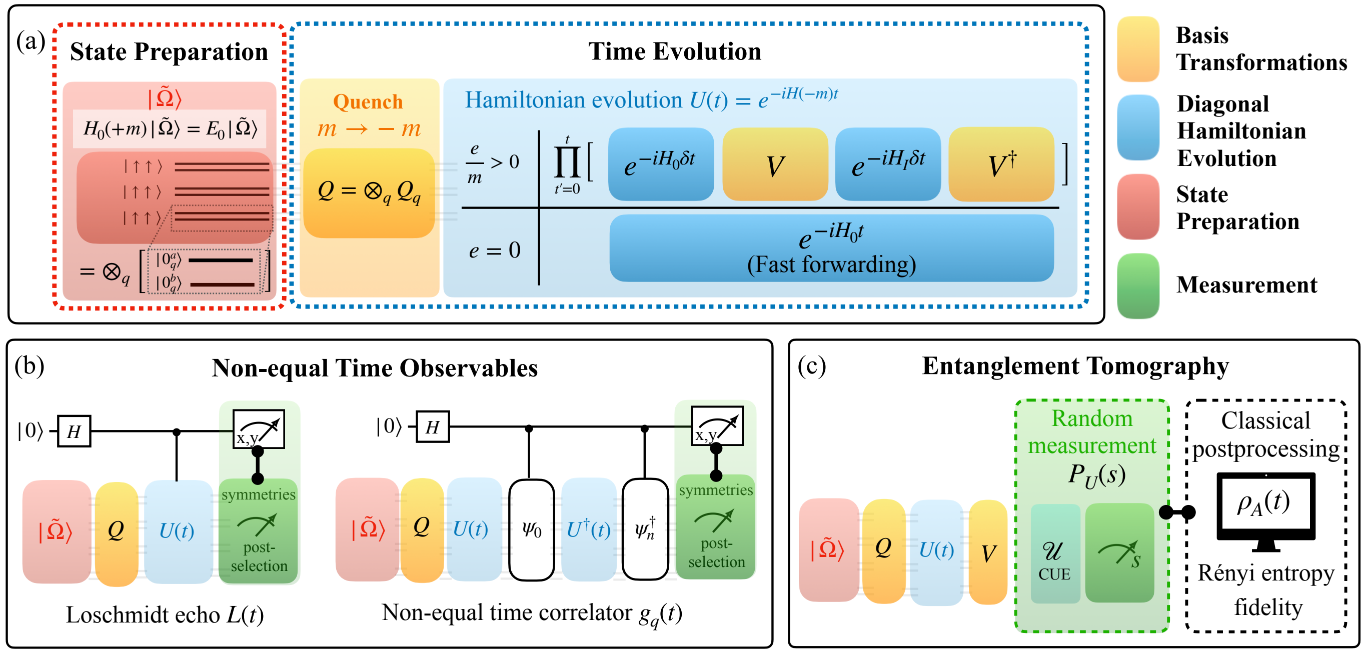


A dynamical phase transition and topological order in lattice Schwinger model with an IonQ quantum computer:



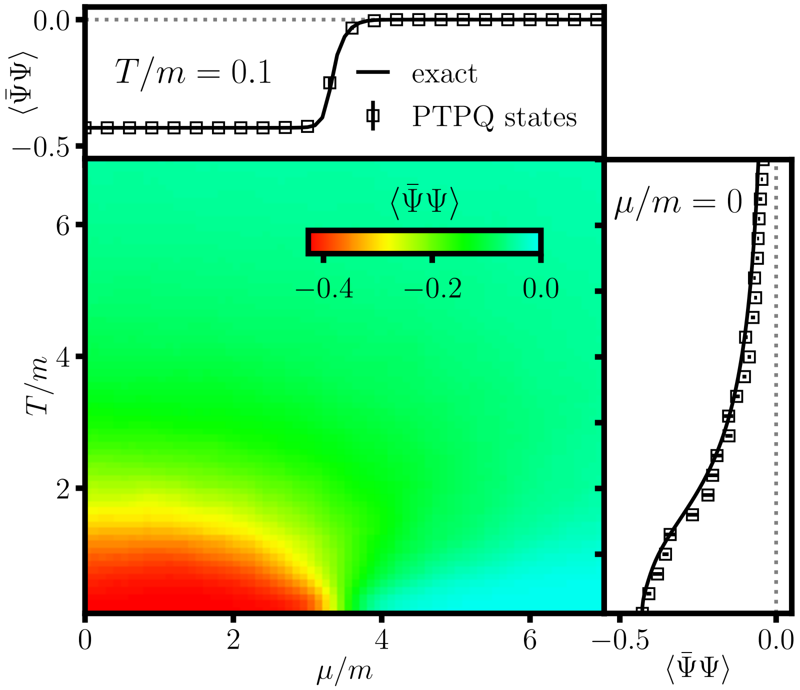


Mueller, Carolan, Connelly, Dumitrescu, ZD, Mueller, Yeter-Aydeniz, to be released (2022).



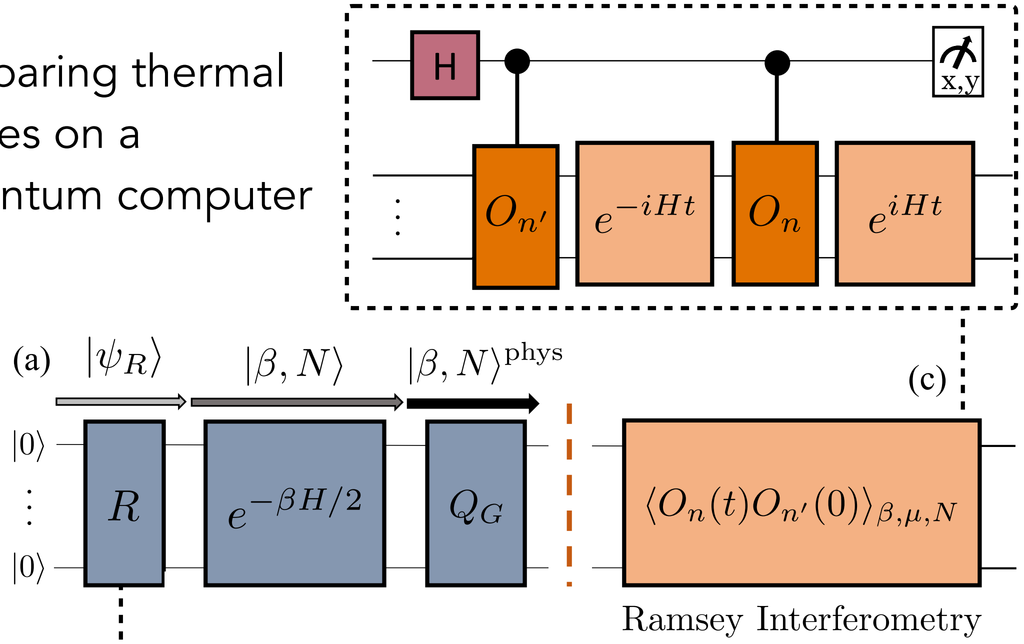
FINITE TEMPERATURE AND FINTIE DENSITY PHASE DIAGRAM

Toward Quantum Computing Phase Diagrams of Gauge Theories with Thermal Pure Quantum States, ZD, Mueller, Powers, arXiv:2208.13112 [hep-lat] (2022).

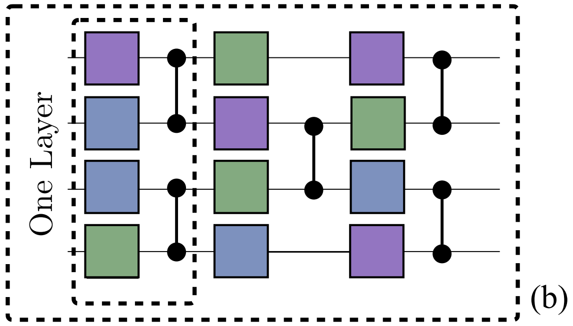


Phase diagram of Z_2^{1+1} with fermions

Preparing thermal states on a quantum computer



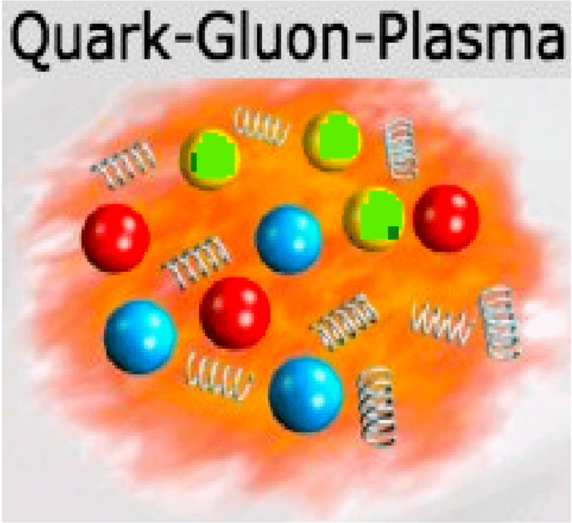
Ramsey Interferometry



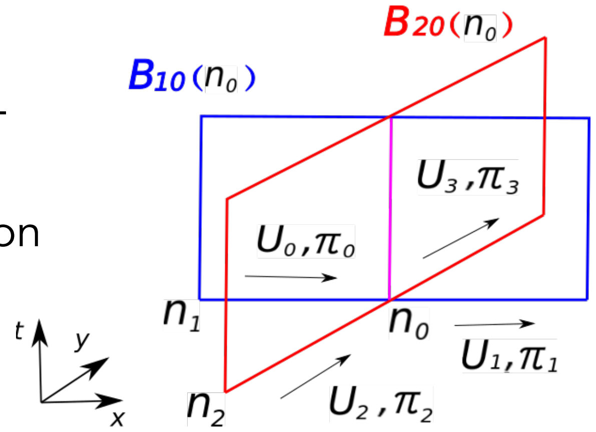
See also: Quantum Simulation of Chiral Phase Transitions, Czajkaa, Kang, Ma , Zhaoa, *JHEP* 08 (2022) 209.

Transport coefficients from real-time correlators of energy momentum tensor

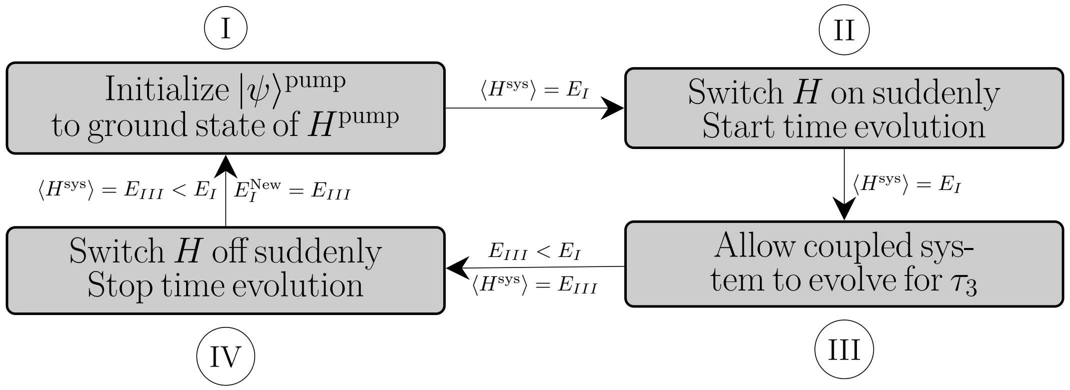
Cohen, Lamm, Lawrence, and Yamauchi, Phys. Rev. D 104, 094514 (2021).



How to define energy-momentum tensor in Hamiltonian formulation



How to prepare a proton state? [Generally not developed sufficiently.]



Ⅰ Initialize $|\psi\rangle^{\text{pump}}$ to ground state of H^{pump}

$\langle H^{\text{sys}}\rangle = E_I$

Ⅱ Switch H on suddenly Start time evolution

$\langle H^{\text{sys}}\rangle = E_I$

Ⅲ Allow coupled system to evolve for τ_3

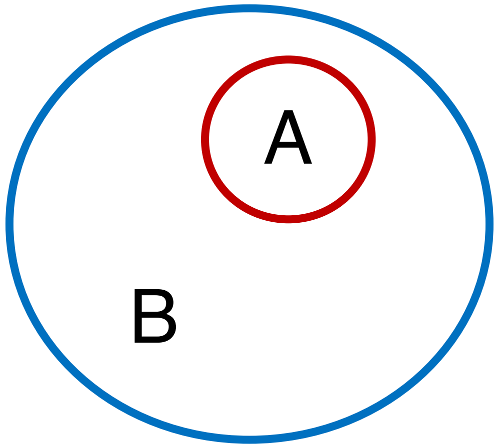
$E_{III} < E_I$ $\langle H^{\text{sys}}\rangle = E_{III}$

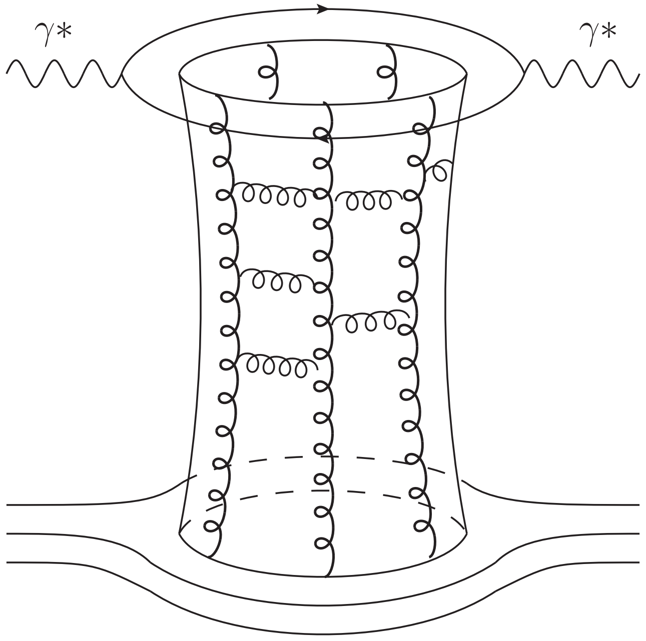
Ⅳ Switch H off suddenly Stop time evolution

$\langle H^{\text{sys}}\rangle = E_{III} < E_I$ $E_I^{\text{New}} = E_{III}$

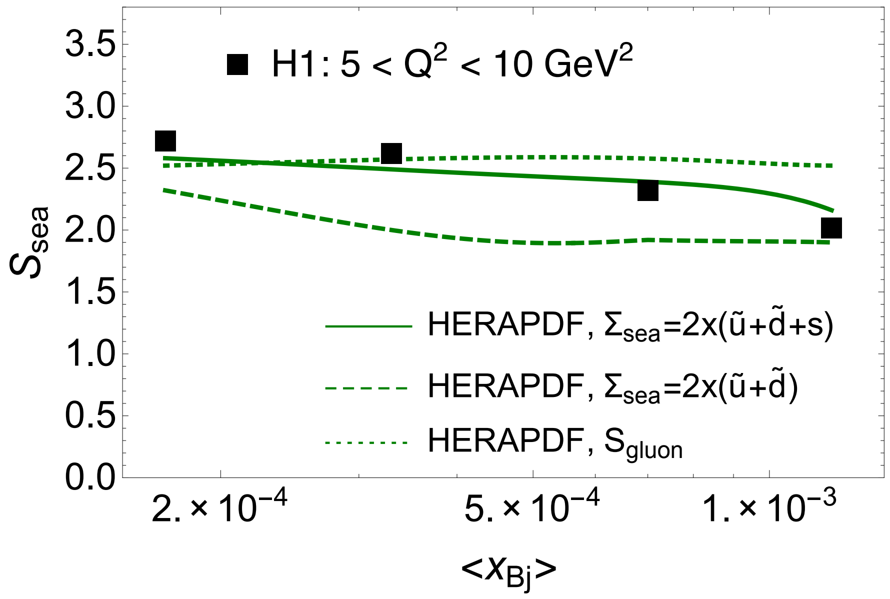
QUANTUM ENTANGLEMENT IN HIGH- AND LOW-ENERGY NUCLEAR PHYSICS

Deep inelastic scattering as a probe of entanglement?



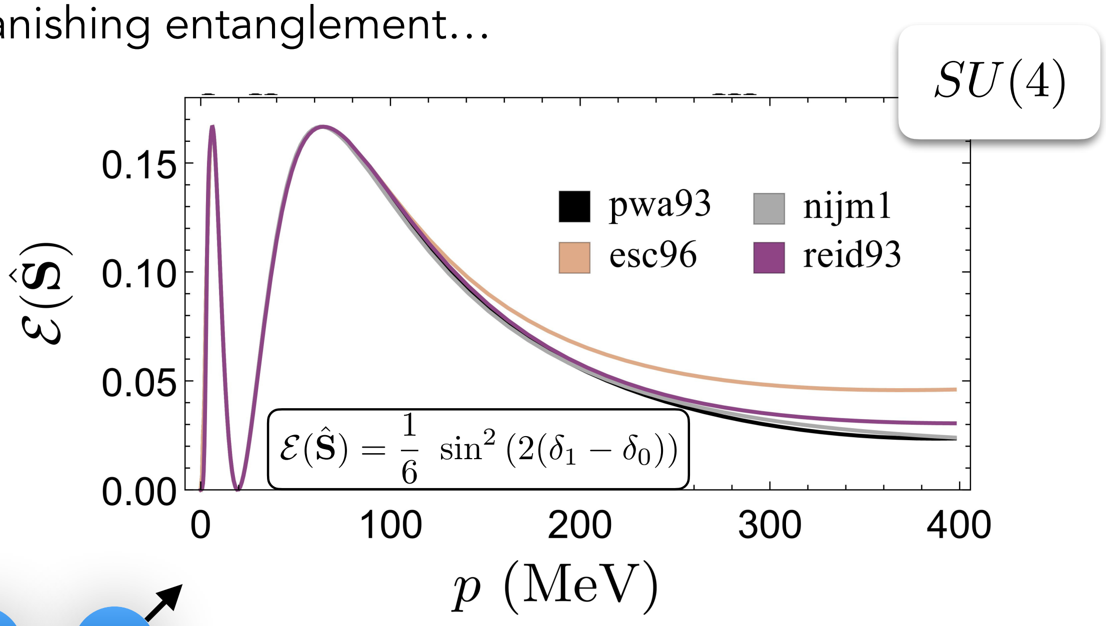


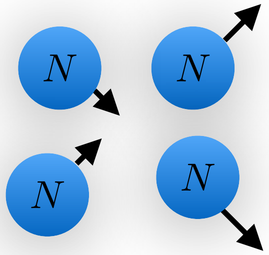
Entropy of hadrons derived from PDFs can be related to entanglement entropy.



Kharzeev and Levin,, Phys. Rev. D 95, 114008 (2017), Zhang, Hao, Kharzeev, and Korepin, Phys. Rev. D 105, 014002 (2022).

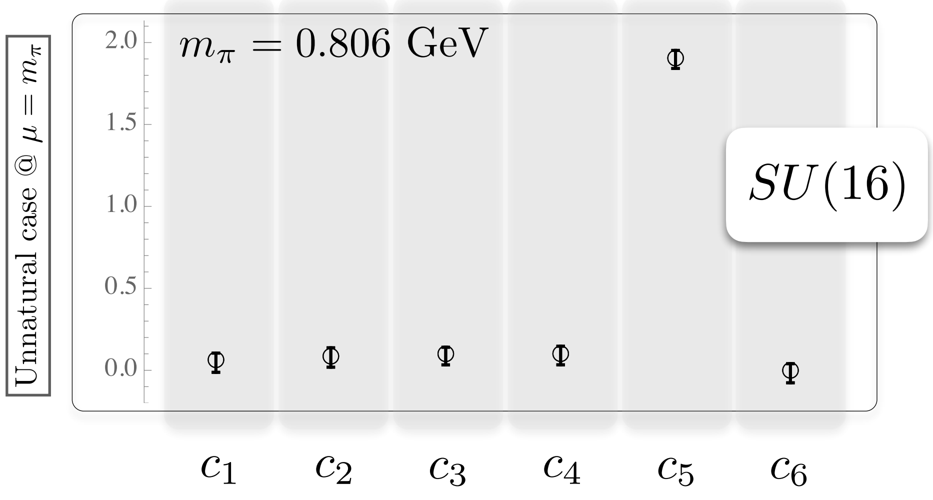
NN interactions at low energies are consistent with vanishing entanglement…

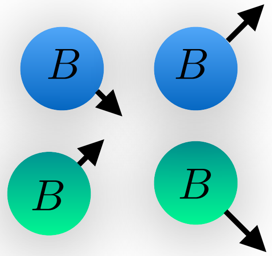




Beane, Kaplan, Klco and Savage, Phys. Rev. Lett. 122, 102001 (2019)

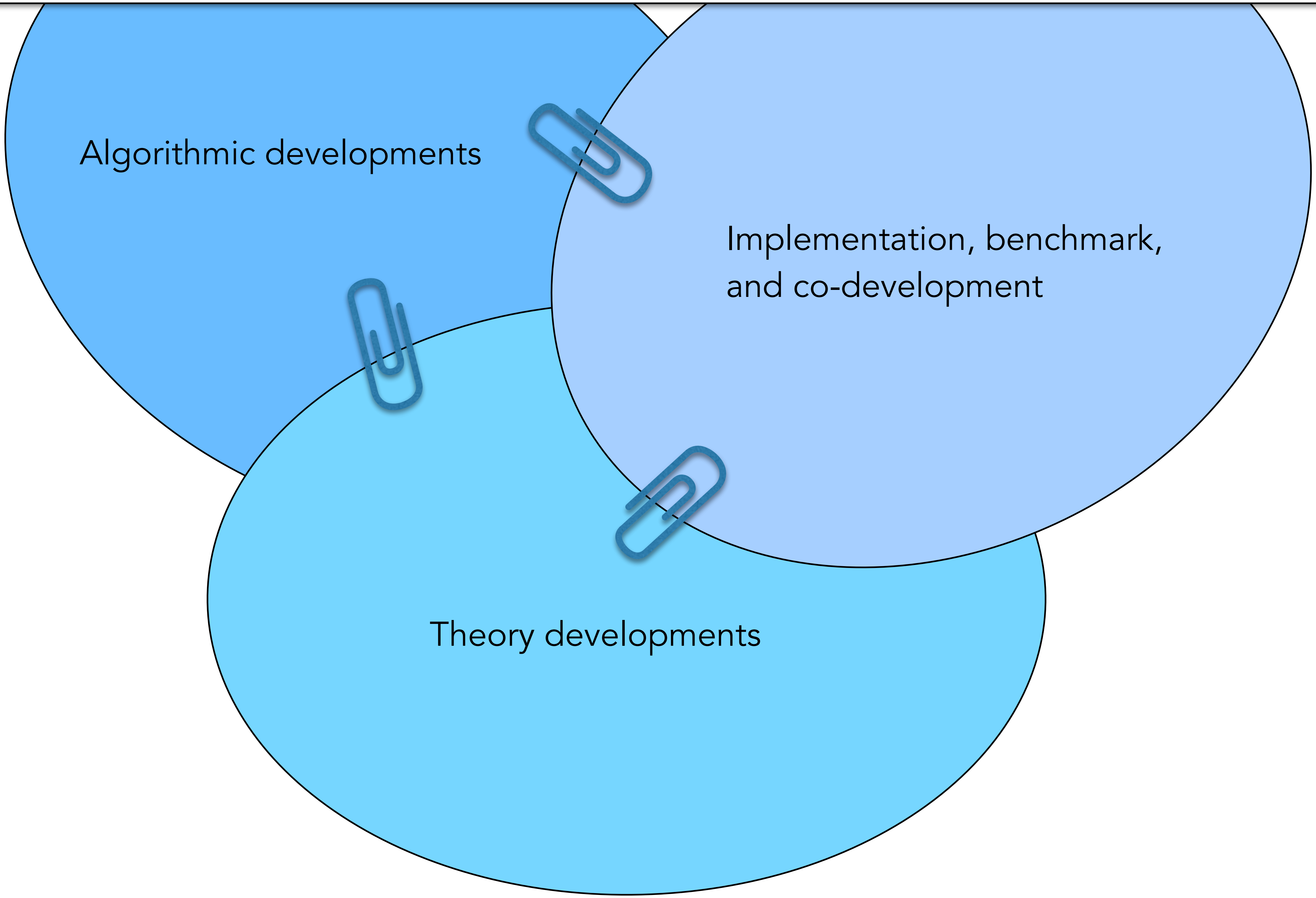
…as are low-energy BB interactions as obtained with lattice QCD.





Wagman, Winter, Chang, ZD, Detmold, Orginos, Savage, Shanahan (NPLQCD),Phys. Rev. D 96, 114510 (2017)

We’ve got a long way to go to get to **QCD** but we know what to do! If one thing we learned from the successful conventional lattice-QCD program is that **theory/algorithm/experiment** collaborations will be the key. It is even more important in the quantum-computing era since our computers are themselves physical systems!



Algorithmic developments

Implementation, benchmark, and co-development

Theory developments

MY COLLABORATORS IN QUANTUM SIMULATION…

* (Current or former) group member

NP

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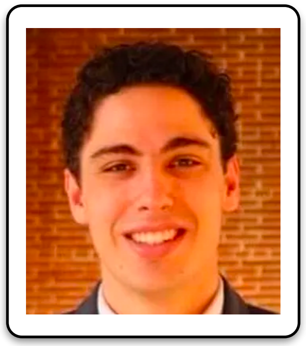
* J. STRYKER (P)

* A. SHAW I (S)

* C. POWERS (S)



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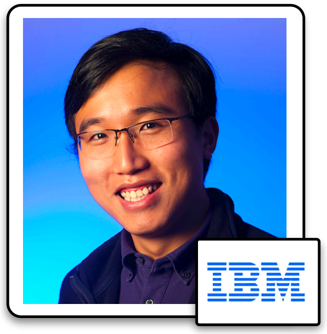

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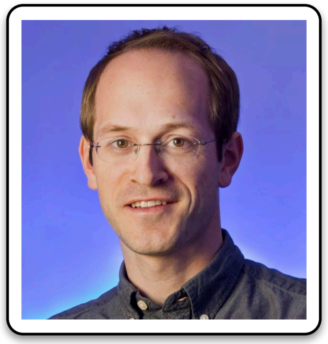
A. CHILDS



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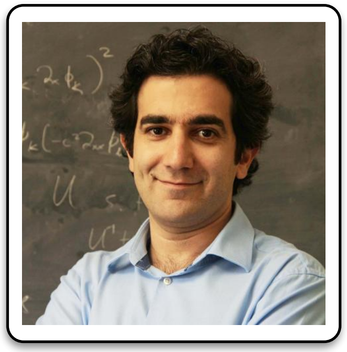

N. LINKE

C. MONROE

OTHER COLLABORATORS I AM ENJOYING WORKING WITH IN ONGOING PROJECTS:

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Lei Feng (P) @Duke

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* Chung-Chun Hsieh (S) @UMD

Or Katz (P) @Duke

Alessio Lerose (P) @U of Geneva

Will Morong (P) @UMD

Alexander Schuckert (P) @UMD

Federica Surace (P) @Caltech

Brayden Ware (P) @UMD

* Christopher White (P) @UMD

Seth Whitsitt (P) @UMD

* Group member

AND FINALLY THANKS TO FUNDING AGENCIES FOR SUPPORTING THIS RESEARCH.

U.S. DEPARTMENT OF ENERGY | Office of Science

Alfred P. Sloan FOUNDATION

NSF | Institute for Robust Quantum Simulation

MCFP MARYLAND CENTER for Fundamental Physics

RIKEN

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THANK YOU