Thermalization and collectivity in small and large systems

Aleksas Mazeliauskas

Institute for Theoretical Physics, Heidelberg University

June 6, 2023

Emmy
Noether-
Programm

DFG Deutsche
Forschungsgemeinschaft

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

IsoQuant
SFB1225

1. Motivation

2. QCD thermalisation in large systems

3. Collectivity in small systems

4. Connections to ultracold atom experiments

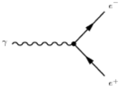
5. Summary

Motivation

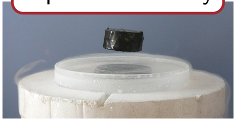
Condensed matter phenomena at extreme conditions

"More is different" Anderson (1972) ⇒ *new unexpected phenomena emerge from interactions of large number of constituents.*

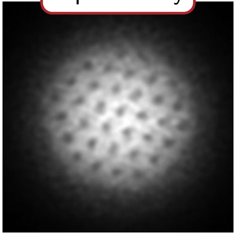
Quantum Electrodynamics (QED) - abelian gauge theory – photons have no charge



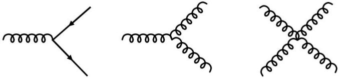
superconductivity



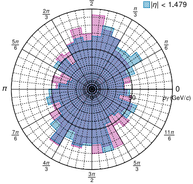
superfluidity



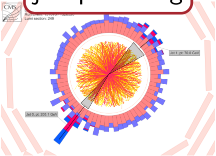
Quantum Chromodynamics (QCD) - non-abelian gauge theory – gluons have color charge



collective flow



jet quenching

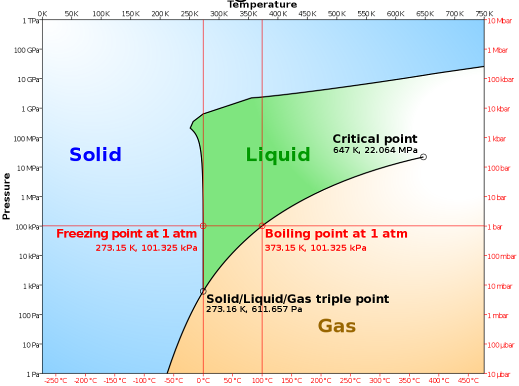


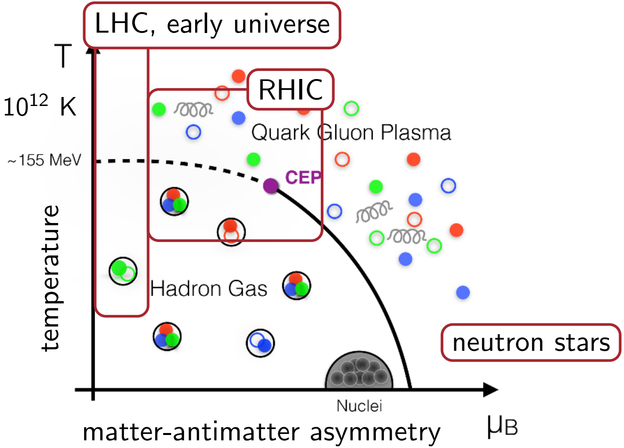
Experimental and theoretical challenge: discover and study new phenomena and new phases of matter consisting of strongly interacting quarks and gluons.

Phase diagram of QCD matter

- Transition from hadrons to quark-gluon plasma: $T = 155\,\mathrm{MeV} \sim 10^{12}\,\mathrm{K}$ at $\mu_B = 0$.

Phase diagram of water

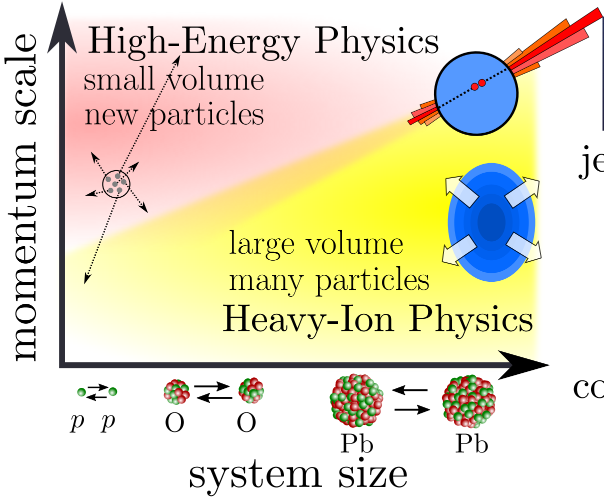


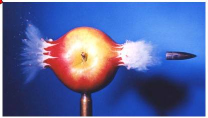


New phase of QCD matter at extreme conditions – quark-gluon plasma (QGP)

Two approaches to particle collisions

1. **HEP**: concentrate higher energy in smaller and smaller volume ⇒ *use electrons or protons*
2. **HIP**: distribute high energy/nucleon density over a large volume ⇒ *use large nuclei*



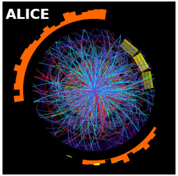


jet quenching

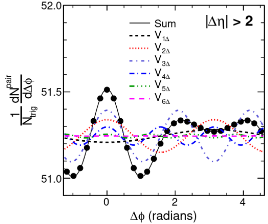
collective flow

Experimental evidence I: Elliptic, triangular and other flows

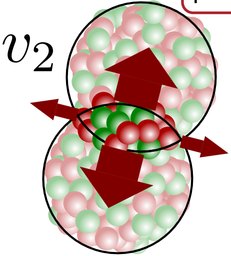
Produced particles show significant angular modulations v_n in the azimuthal angle ϕ

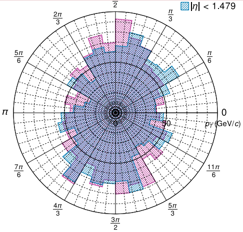


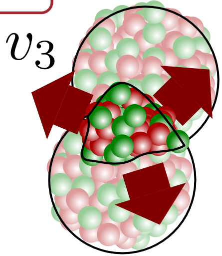
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2v_2\cos(2\phi) + 2v_3\cos(3\phi)\dots\right).$$

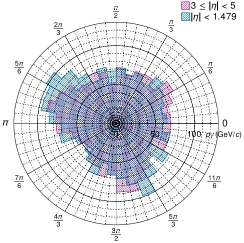


particle momentum deposition



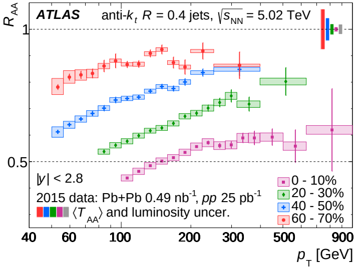






Collective particle flow is explained by pressure gradient driven transverse QGP expansion.

Experimental evidence II: High-p_T parton energy loss — jet quenching

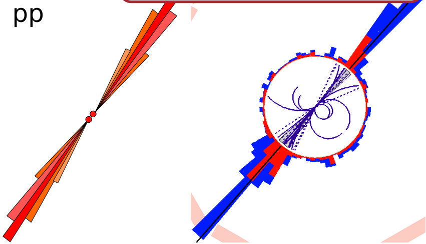


Jet spectrum is suppressed in nuclear collisions compared to proton-proton collisions

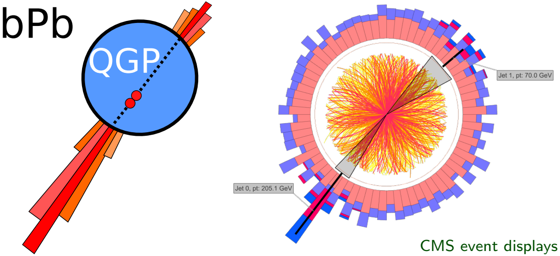
$$R_{AA} = \frac{dN^j_{AA}/dp_T}{N_{\rm coll} \times dN^j_{pp}/dp_T} < 1$$

back-to-back jets in *pp*

pp



PbPb



CMS event displays

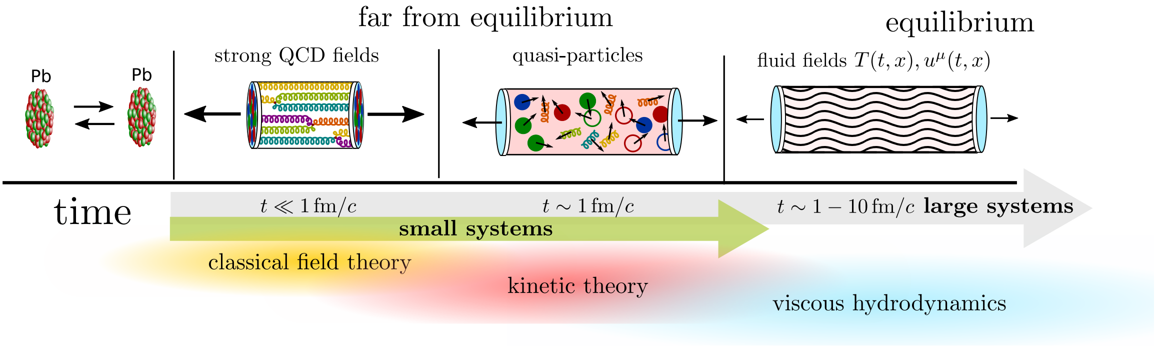
Jet quenching is explained by energy loss in strongly interacting plasma.

QCD thermalisation in large systems

QCD thermalisation: *ab initio* approaches

Berges, Heller, AM, Venugopalan *RMP* (2021)

At high-energy/density limit use descriptions from first principles of QCD.



far from equilibrium

equilibrium

strong QCD fields

quasi-particles

fluid fields $T(t,x), u^\mu(t,x)$

time

$t \ll 1\,\mathrm{fm}/c$

$t \sim 1\,\mathrm{fm}/c$

$t \sim 1-10\,\mathrm{fm}/c$ **large systems**

small systems

classical field theory

kinetic theory

viscous hydrodynamics

- Very early times: gluon occupancies $f_g \sim \frac{1}{\alpha_s} \Rightarrow$ *classical statistical description*.
- Intermediate times: $1 < f_g \ll \frac{1}{\alpha_s} \Rightarrow$ *quasi-particle picture*, see Boguslavski, Lappi, Mace, Schlichting (2022)

QCD effective kinetic theory — bridge between initial state and equilibrium.

High-temperature QCD kinetic theory

Arnold, Moore, Yaffe *JHEP* (2003)

underlying quantum field theory

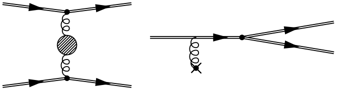
2-point correlations

$$\mathcal{L}_{\text{QCD}} = \bar{q}\left(i\gamma^\mu D_\mu - m\right) q - \tfrac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$$

effective kinetic description

phase-space distribution

$$\partial_t f(t, \mathbf{x}, \mathbf{p}) + \frac{\mathbf{p}}{|p|} \cdot \nabla_{\mathbf{x}} f(t, \mathbf{x}, \mathbf{p}) = -\mathcal{C}_{2\leftrightarrow 2}[f] - \mathcal{C}_{1\leftrightarrow 2}[f]$$



- Leading order scattering processes:
  - elastic scattering
  - medium-induced collinear radiation
- Out-of-equilibrium description of quark gluon plasma.
- *Low momentum thermalisation $\Longleftrightarrow$ high momentum energy loss.*

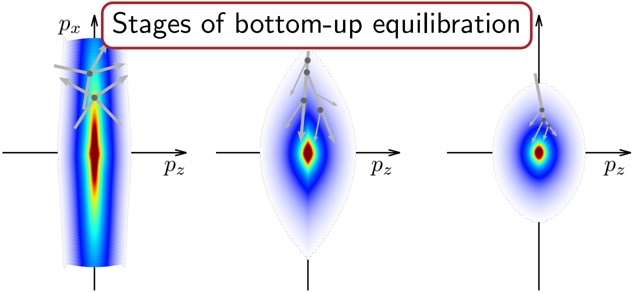
Challenge: Solving **multidimensional Boltzmann equation** for $f(t, \mathbf{x}, \mathbf{p})$

For homogeneous systems: $f(t, \mathbf{x}, \mathbf{p}) \Longrightarrow f(\tau, \mathbf{p})$

Bottom-up thermalisation scenario

Baier, Mueller, Schiff and Son (2001)

$$\text{Boltzmann eq.:}\quad \partial_\tau f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{longitudinal expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow2}[f] - \mathcal{C}_{1\leftrightarrow2}[f]}_{\text{in-medium QCD collisions}}$$

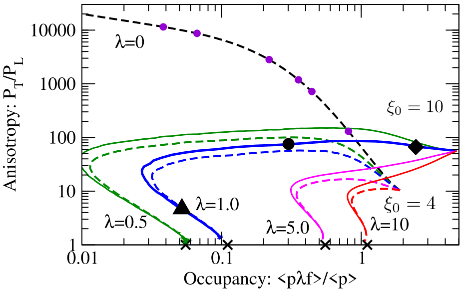


Stages of bottom-up equilibration

2 ↔ 2 broadening

collinear cascade

mini-jet quench



Kurkela and Zhu (2015), Keegan, Kurkela, AM and Teaney (2016), Kurkela, AM, Paquet, Schlichting and Teaney (2018)

Non-thermal fixed point as the first stage of bottom-up

- Self-similar evolution of highly occupied gluons in classical-statistical Yang-Mills

Berges, Schenke, Schlichting, Venugopalan (2014)

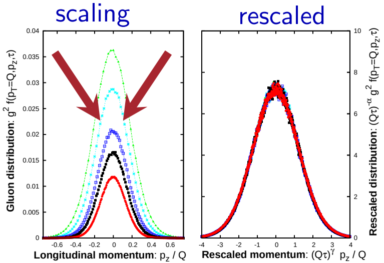
$$f_g(p_\perp, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_\perp, \tau^\gamma p_z), \quad \tau = \sqrt{t^2 - z^2}$$

Universal exponents: $\alpha \approx -\frac{2}{3}, \quad \beta \approx 0, \quad \gamma \approx \frac{1}{3}$

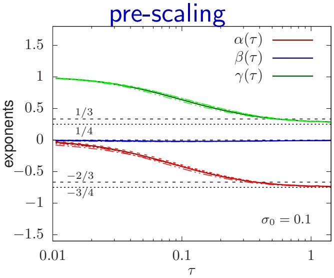
- Time dependent scaling exponents → pre-scaling

AM, Berges (2019) Schmied, Mikheev, Gasenzer (2019)

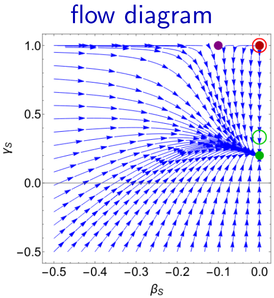
- *Stability analysis and corrections to scaling exponents*



Berges, Schenke, Schlichting, Venugopalan (2014)



AM, Berges (2019)

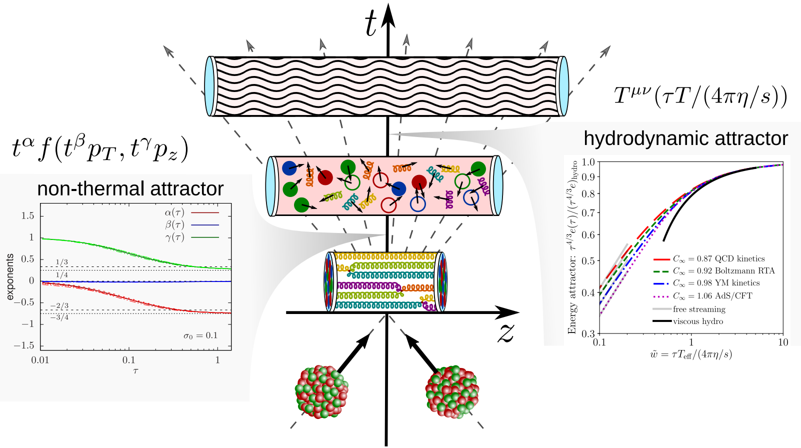


Brewer, Scheihing-Hitschfeld, Yin, 2203.02427
see also Mikheev, AM, Berges, 2203.02299

See also Preis, Heller and Berges 2209.14883, Heller, AM, Preis, *in progress*

Attractors in QCD thermalisation

- Non-thermal fixed point – simplification of dynamics through self-similar evolution
- Hydrodynamic attractor – *collapse to macroscopic fluid dynamics behaviour.*



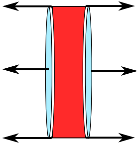
See reviews by Berges, Heller, AM, Venugopalan (2020), Florkowski, Heller and Spalinksi (2017), Romatschke and Romatschke (2017)

Beyond conventional gradient expansion: hydrodynamic attractors

Pressure anisotropy collapse to a hydrodynamic attractor

Heller, Janik, Witaszczyk (2011)

$$\frac{\tau\partial_\tau e}{e} = -1 - \frac{P_L}{e} = -\frac{4}{3} + \underbrace{\frac{16}{9}\,\frac{\eta/s}{\tau T}}_{\text{1st gradient}} \; \cdots$$



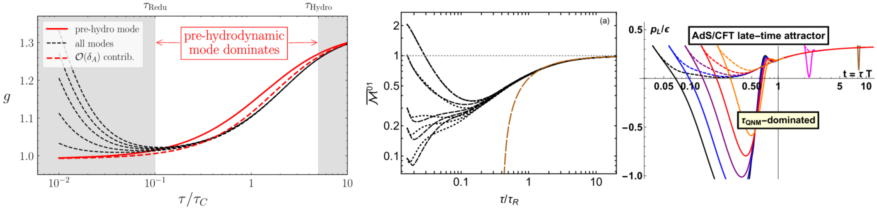
$\tau < R$

RTA

EKT

AdS/CFT

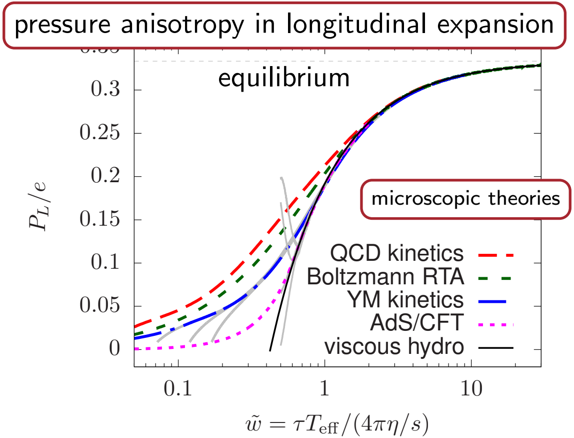
anisotropy



time

Brewer, Yan, Yi (2019), Almaalol, Kurkela, Strickland (2020), Kurkela, van der Schee, Widemann, Wu (2019)

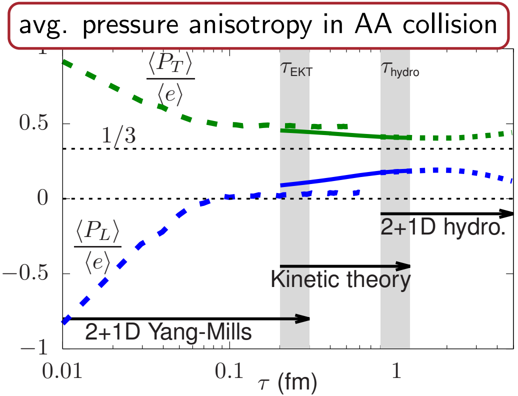
Applications of hydrodynamic attractors in heavy-ion collisions



pressure anisotropy in longitudinal expansion

microscopic theories

Giacalone, AM, Schlichting, (2019)



avg. pressure anisotropy in AA collision

Kurkela, AM, Paquet, Schlichting and Teaney (2018)

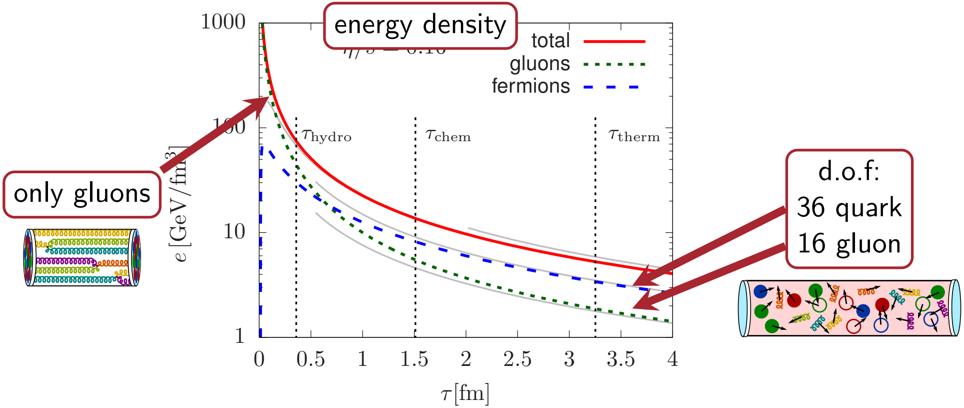
- Calculated the universal entropy production during equilibration.

$$(dS/dy)_\infty = 4/3 C_\infty^{3/4} (4\pi\eta/s)^{1/3} \kappa^{1/3} (dE/dy)_0^{2/3}$$

- Practical event-by-event kinetic pre-equilibrium for heavy ion collisions: KøMPøST

Hydrodynamic, chemical and thermal equilibration

- τ_{hydro}: total energy density follows viscous hydro
- τ_{chem}: quark and gluon energy ratios follow chemical equilibrium
- τ_{therm}: total energy density follows ideal hydro



energy density

only gluons

d.o.f:
36 quark
16 gluon

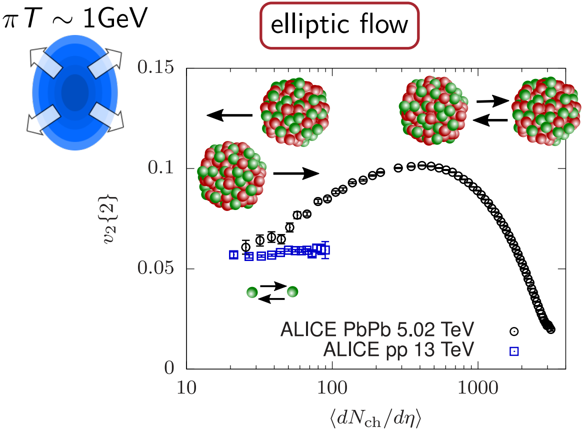
Kurkela, AM (2018) for finite μ_B, see Du and Schlichting, (2020)

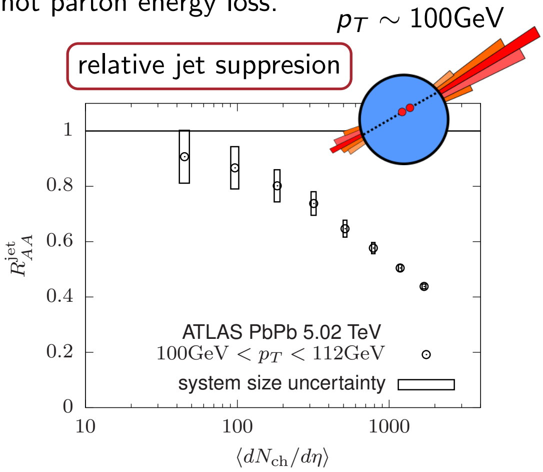
Chemical equilibration is a unique feature of QCD kinetic theory

Collectivity in small systems

Outstanding problem: origin of collective behaviour in small systems

Collective flow seen in all hadron collisions, but not parton energy loss.





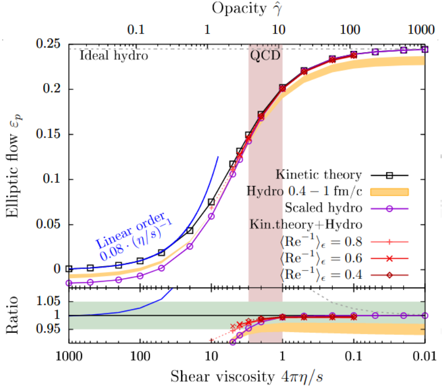
Big question: Is Quark Gluon Plasma created in small systems?

Transverse flow in RTA kinetic theory

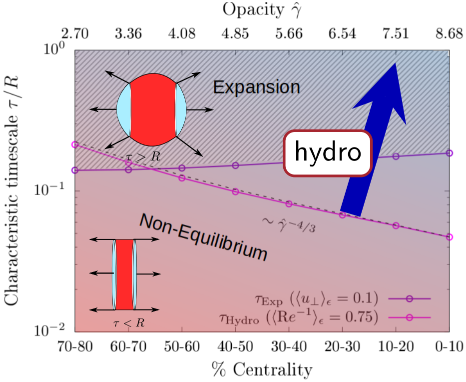
Relaxation scale $\tau_R = 5\eta/sT$, opacity $\hat{\gamma} \propto R/\tau_R \propto$ system size in units of mean-free-path

$$\text{RTA boltzmann eq.:} \quad \partial_t f + \frac{\mathbf{p}}{|p|} \cdot \nabla f = -\frac{p_\mu u^\mu}{\tau_R}\left(f - f_{\text{eq}}\left(\frac{p_\mu u^\mu}{T}\right)\right)$$

Flow response for 30-40% PbPb



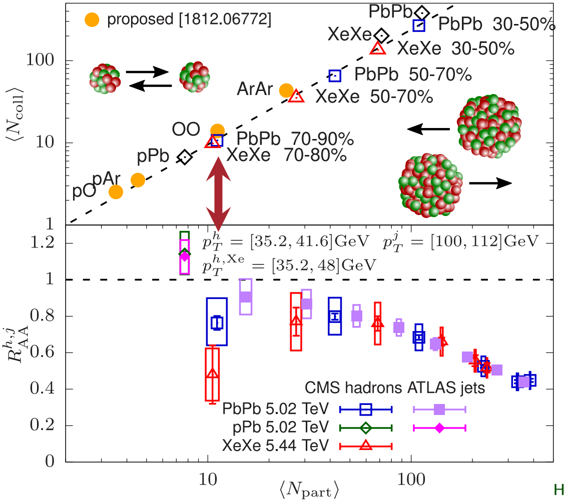
Applicability of hydrodynamics



Ambrus, Schlichting, Werthmann (2021,2023), see also Kurkela, Wiedemann, Wu (2019)

For QCD kinetic theory result see: single-hit approx. Kurkela, AM, Törnkvist (2021), full expansion work in progress with Fabian Zhou.

Energy loss in small systems



Huss, Kurkela, AM, Paatelainen, van der Schee, Wiedemann *PRL,PRC* (2020)

$\sqrt{s_{NN}} \sim 7$ TeV OO at LHC in 2024
STAR collected $\mathcal{L}_{\mathrm{OO}} = 32\,\mathrm{nb}^{-1}$

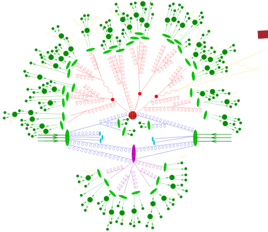
Brewer, AM, van der Schee (2021), 2103.01939

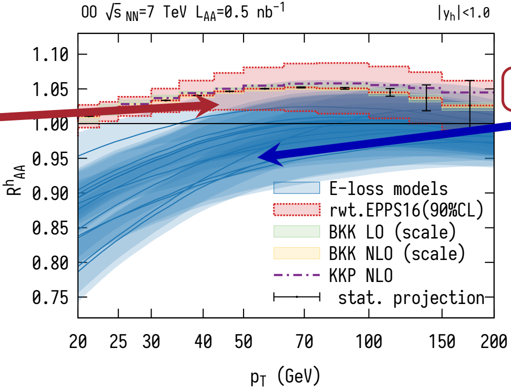
- No jet quenching signals in peripheral PbPb and *p*Pb collisions seen
- *Minimum bias oxygen-oxygen collisions probe the relevant size regime!*

Oxygen-oxygen collisions — unique opportunity to discover energy loss

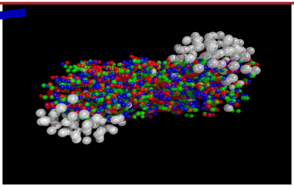
- Discovery of small energy loss ⇒ *important to quantify uncertainties in the baseline*
- We performed next-to-leading order computations of perturbative baseline.
- Extrapolated energy loss models down to oxygen-oxygen collisions.

HEP: no-rescattering





HIP: prediction for energy loss



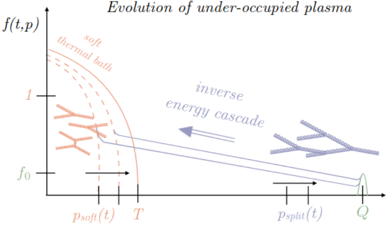
Measurable difference between the baseline and modelled medium effect!

Huss, Kurkela, AM, Paatelainen, van der Schee, Wiedemann, *PRC,PRL* (2021)

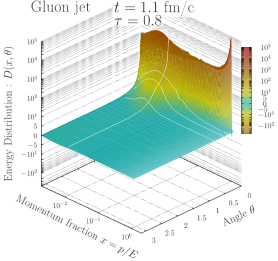
Energy loss studies with QCD kinetic theory

- BDMPS-Z in collinear-limit $\Longrightarrow$ QCD kinetic theory
- QCD kinetic theory describes both energy loss and thermalisation
  $\Rightarrow$ *jet wake contains remnants of QCD thermalisation*

1D jet-quenching

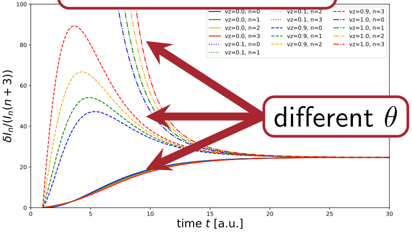


2D jet-quenching



Mehtar-Tani, Schlichting, Soudi, 2209.10569

2D jet thermalisation



different θ

work in progress with Fabian Zhou

For recent work on change in jet chemistry, see Sirimanna et al. 2211.15553

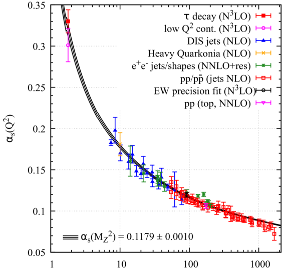
Future goal: non-equilibrium jet-medium interactions.

Connections to ultracold atom experiments

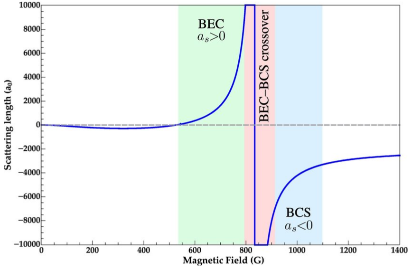
Strongly interacting quantum systems

Running of QCD coupling α_s

Interaction strength in cold gas $g = \frac{4\pi\hbar^2 a}{m}$



Energy scale in GeV



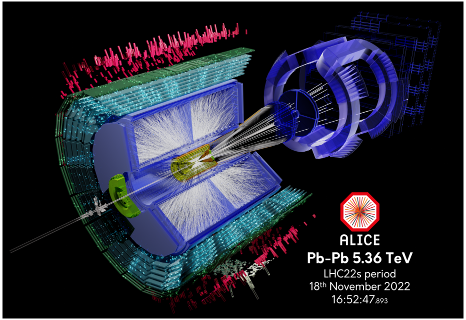
External magnetic field 2008.05046

- *Strongly interacting QCD matter:*
  - At low temperature/density α_s grows ⇒ non-perturbative QCD.
  - Large gluon occupancy $f_g \sim \frac{1}{\alpha_s}$ ⇒ weakly coupled, but strongly interacting system.
- *Strongly interacting ultra-cold quantum gas*
  - Divergent scattering length at Feshbach resonance (degenerate bound and un-bound state).

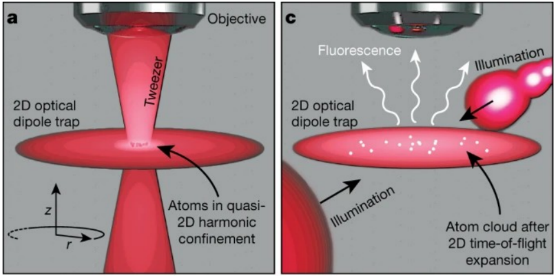
Comparison of experimental setups

- Definite quantum state of cold bose of fermi gas prepared in a trap.
- Set desired interaction strength (or switch it off at will)
- Positions of atoms imaged by destructive illumination ⇒ repeat to map evolution.
- Free expansion in a harmonic trap ⇒ convert momentum into coordinate.

momentum only imaging



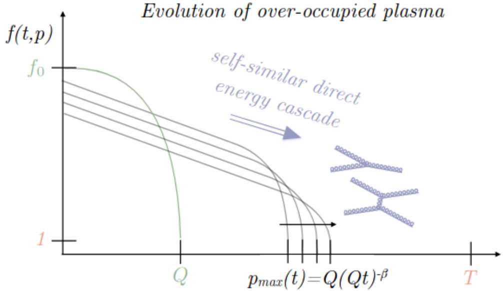
momentum and space imaging



Holten et al. Nature (2022)

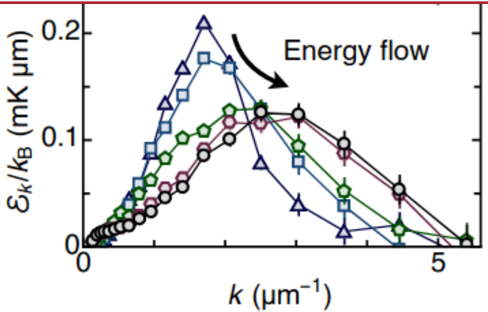
Cold atom experiments offer unprecedented abilities to observe the system evolution.

Thermalisation of over-occupied plasma (non-expanding case)



from review by Schlichting and Teaney (2019)

Thermalization of quench-cooled atomic gas



Glidden at al. Nature Physics (2021)

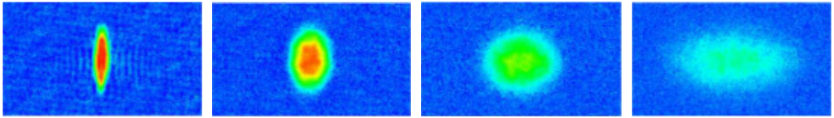
- Over-occupied scaling $f(t,p) = t^{-\frac{4}{7}} f(pt^{-\frac{1}{7}})$ Schlichting (2012) Kurkela and Moore (2012)
- Scaling of spinor Bose gas far from equilibrium Prüfer et al. (2018)

Universality of far-from-equilibrium evolution!

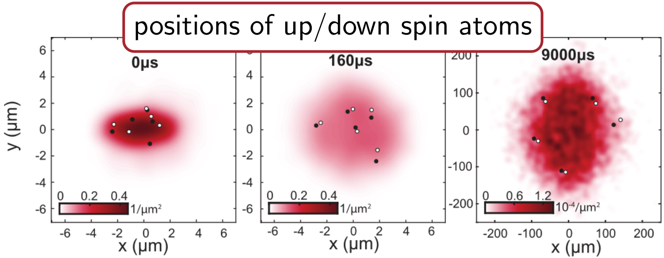
Geometry inversion as a sign of hydrodynamic flow

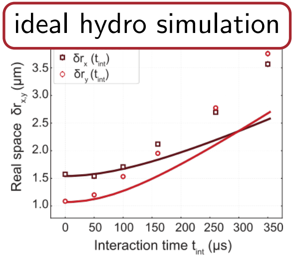
- Elliptic expansion of $\sim 10^5$ strongly interacting lithium atoms

O'Hara et al. Science (2002)



- Elliptic expansion of 10 ^{6}Li atoms — *mesoscopic fluid*!


positions of up/down spin atoms


ideal hydro simulation

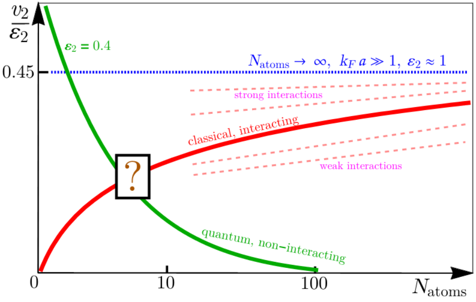
Brandstetter, Lunt, Heintze, Giacalone, Heyen, Gałka, Holten, Subramanian, Preiss, Flörchinger, Jochim (to appear soon)

Expansion qualitatively described by ideal hydro with many-body equation of state.

System size dependence

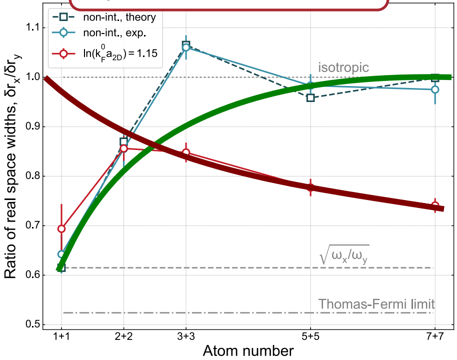
- Experimental techniques allow precise control of initial number of atoms
- For non-interacting systems momentum anisotropy determined by uncertainty principle.
- Interactions generate additional anisotropy.

theory expecation of elliptic flow



Floerchinger, Giacalone, Heyen, Tharwat, 2111.13591

aspect ratio of the cloud



Branstetter et al. (to appear soon)

Emergence of gradient driven expansion for $N \sim 10$ atoms!

Summary

Summary

High-energy proton and nuclear collisions:

- Unique access to rich real-time dynamics of **many-body QCD physics**.
- Multi-faceted problems with **interdisciplinary connections**.
- Detailed understanding of QCD thermalisation in **large systems**.

Outstanding challenges:

- Uncovering the physical *origins of collective behaviour* in all hadronic collisions and explaining their *dependence on the size of the collision system*.
- Emergence of macroscopic behaviour in few-body cold atom systems

⇒ rich area for inderciplinary collaboration