On the Majorana nature of massive neutrinos

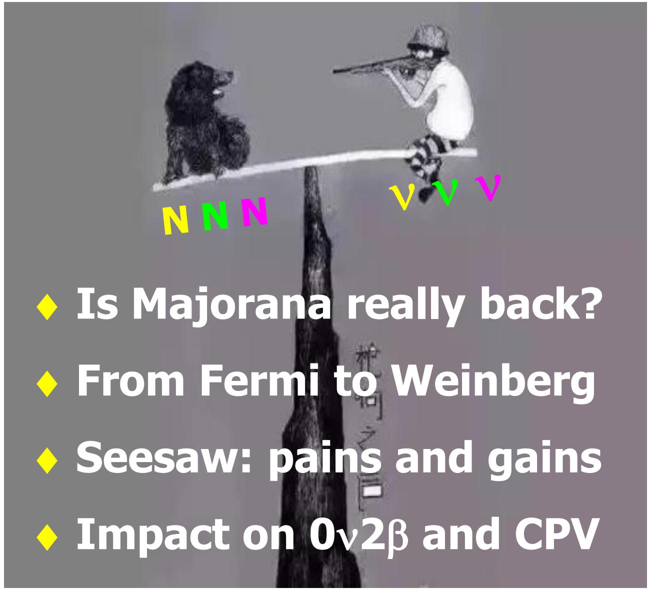
Zhi-zhong Xing
【IHEP Beijing】

- **Neutrino counting with one (or more) right-handed Majorana neutrino**

  李学潜、陶志坚，Mod. Phys. Lett. A 5 (1990) 1933

- **Neutrino mixing effects, tau leptonic decays, and the solar neutrino problem**

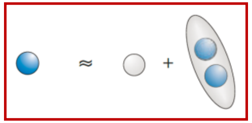
  李学潜、陶志坚，Phys. Rev. D 43 (1991) 3691



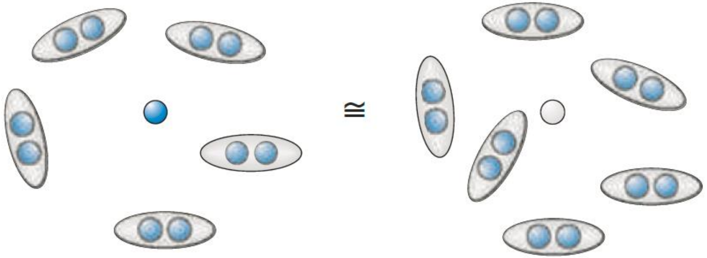
It’s said that Majorana was returning

perspective

Nature Physics 5, 614–618 (2009)



Majorana zero mode



Majorana returns

Frank Wilczek

In his short career, Ettore Majorana made several profound contributions. One of them, his concept of ‘Majorana fermions’ — particles that are their own antiparticle — is finding ever wider relevance in modern physics.

用甜言蜜语哄骗　　衍生物、结果

Enrico Fermi had to cajole his friend Ettore Majorana into publishing his big idea: a modification of the Dirac equation that would have profound ramifications for particle physics. Shortly afterwards, in 1938, Majorana mysteriously disappeared, and for 70 years his modified equation remained a rather obscure footnote in theoretical physics. Now suddenly, it seems, Majorana’s concept is ubiquitous, and his equation is central to recent work not only in neutrino physics, supersymmetry and dark matter, but also on some exotic states of ordinary matter.

晦涩的

无所不在的、普遍存在的

A fake signal of 0ν2β

Modern Physics Letters A, Vol. 16, No. 37 (2001) 2409–2420

Citations > 680

EVIDENCE FOR NEUTRINOLESS DOUBLE BETA DECAY

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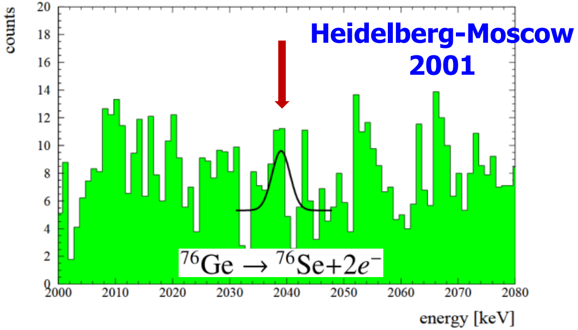
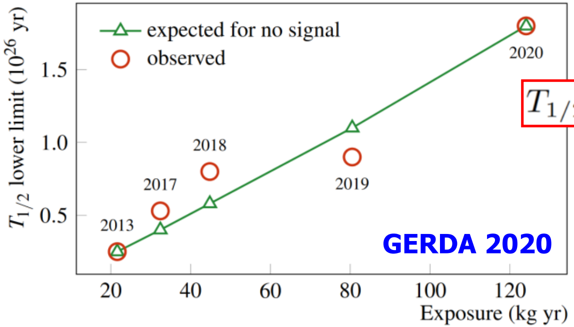
H.V. Klapdor





$T_{1/2} > 1.8 \times 10^{26}$ yr at 90% C.L.

It is not me!

Ettore Majorana

The abstract: First evidence for neutrinoless double beta decay is observed giving first evidence for lepton number violation. The evidence for this decay mode is 97% (2.2σ) with the Bayesian method, and 99.8% c.l. (3.1σ) with the method recommended by the Particle Data Group. The half-life of the process is found with the Bayesian method to be $T^{0\nu}_{1/2} = (0.8 - 18.3) \times 10^{25}$ y (95% c.l.) with a best value of 1.5×10^{25} y. The deduced value of the effective neutrino mass is, with the nuclear matrix elements from [1], $\langle m \rangle$ = (0.11 - 0.56) eV (95% c.l.), with a best value of 0.39 eV. Uncertainties in the nuclear matrix elements

Another fake evidence?

RESEARCH

TOPOLOGICAL MATTER SCIENCE · 21 Jul 2017 · Vol 357, Issue 6348 · pp. 294-299 · DOI: 10.1126/science.aag2792

Chiral Majorana fermion modes in a quantum anomalous Hall insulator–superconductor structure

Qing Lin He,[1*†] Lei Pan,[1†] Alexander L. Stern,[3] Edward C. Burks,[4] Xiaoyu Che,[1] Gen Yin,[1] Jing Wang,[5,6] Biao Lian,[6] Quan Zhou,[6] Eun Sang Choi,[7] Koichi Murata,[1] Xufeng Kou,[1,8*] Zhijie Chen,[4] Tianxiao Nie,[1] Qiming Shao,[1] Yabin Fan,[1] Shou-Cheng Zhang,[6*] Kai Liu,[4] Jing Xia,[3] Kang L. Wang[1,2*]

Majorana fermion is a hypothetical particle that is its own antiparticle. We report transport measurements that suggest the existence of one-dimensional chiral Majorana fermion modes in the hybrid system of a quantum anomalous Hall insulator thin film coupled with a superconductor. As the external magnetic field is swept, half-integer quantized conductance plateaus are observed at the locations of magnetization reversals, giving a distinct signature of the Majorana fermion modes. This transport signature is reproducible over many magnetic field sweeps and appears at different temperatures. This finding may open up an avenue to control Majorana fermions for implementing robust topological quantum computing.

过去我们认为有粒子必有其反粒子，正如有天使必有魔鬼。但今天，我们找到了一个没有反粒子的粒子，一个只有天使，没有魔鬼的完美世界——张首晟

Editorial retraction 2022-11-18

Good news to Majorana hunters

Phase A (1930 —— 1939):

- ♦ **1930 Wolfgang Pauli**, a postcard / letter. — $1\nu 1\beta$
- ♦ **1933 Enrico Fermi**, Ric. Sci. 4, 491. — $1\nu 1\beta$
- ♦ **1935 Maria Goeppert-Mayer**, Phys. Rev. D 48, 512. — $2\nu 2\beta$
- ♦ **1937 Ettore Majorana**, Nuovo Cim. 14, 171. — $\nu^c = \nu$
- ♦ **1939 Wendell Furry**, Phys. Rev. D 56, 1184. — $0\nu 2\beta$



Phase B (1957 —— 1998):

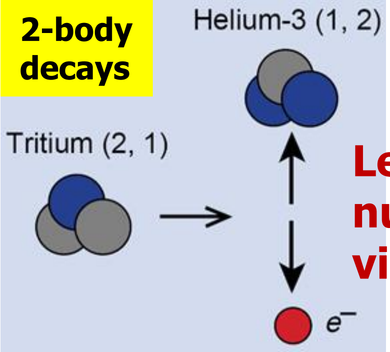
- ♦ **1956 T.D. Lee**, **C.N. Yang**, Phys. Rev. D 105, 167. — $m_\nu = 0$
- ♦ **1957 Bruno Pontecorvo**, Sov. Phys. JETP 6, 429. — $m_\nu \neq 0$
- ♦ **1967 Steven Weinberg**, Phys. Rev. Lett. 19, 1264. — $m_\nu = 0$
- ♦ **1987 S. Elliot**, **A. Hahn**, **M. Moe**, Phys. Rev. Lett. 59, 2020. — $2\nu 2\beta$
- ♦ **1998 SK collaboration**, Phys. Rev. Lett. 81, 1562. — $m_\nu \neq 0$


The observations of $2\nu 2\beta$ decays + ν-oscillations are so encouraging that most hunters believe the existence of $0\nu 2\beta$, a unique signal for the Majorana nature of ν's.

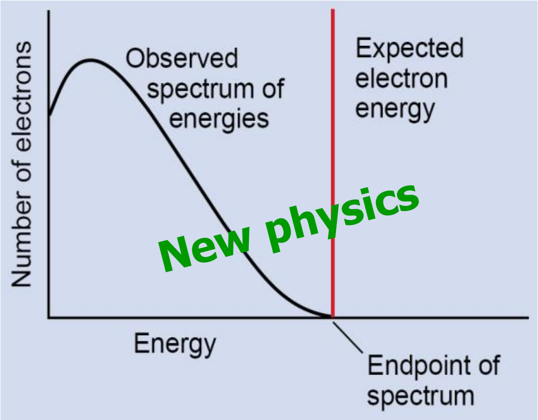
From Pauli to Fermi

The Energy crisis in β decays

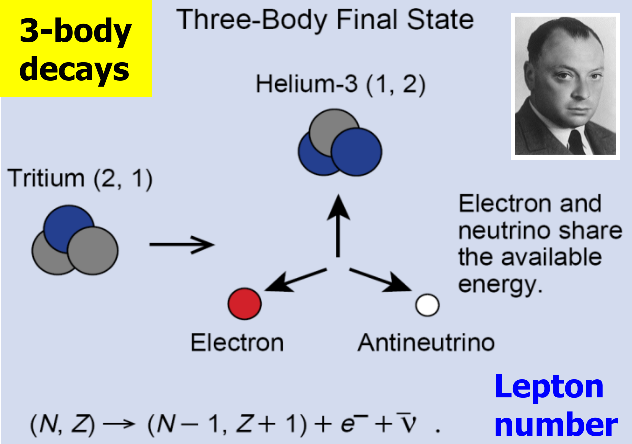
J. Chadwick 1914 , C. Ellis 1920~1927



Lepton number violation

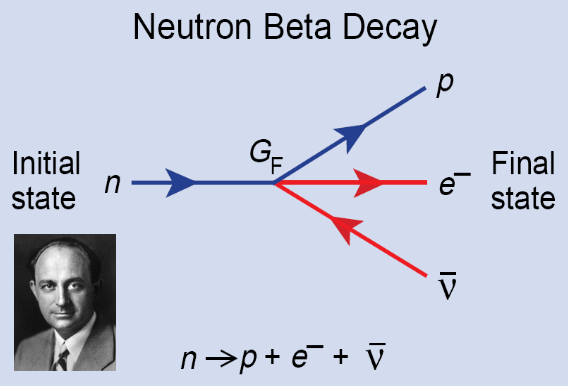


Wolfgang Pauli's new idea (1930)



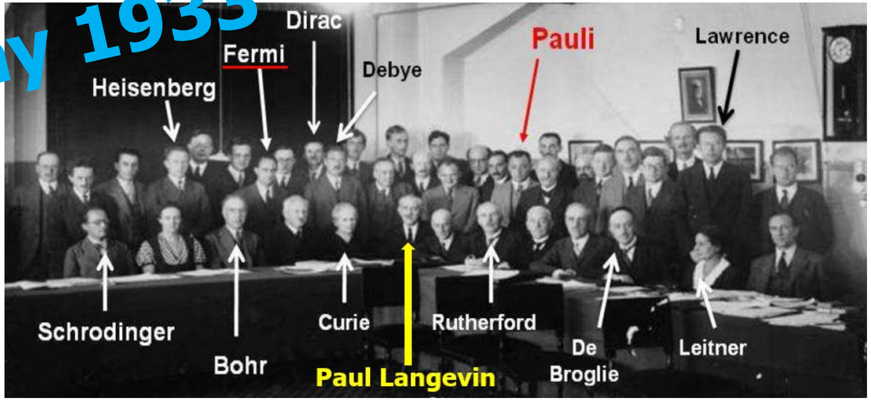
Lepton number conservation

Enrico Fermi's β-theory (1933)



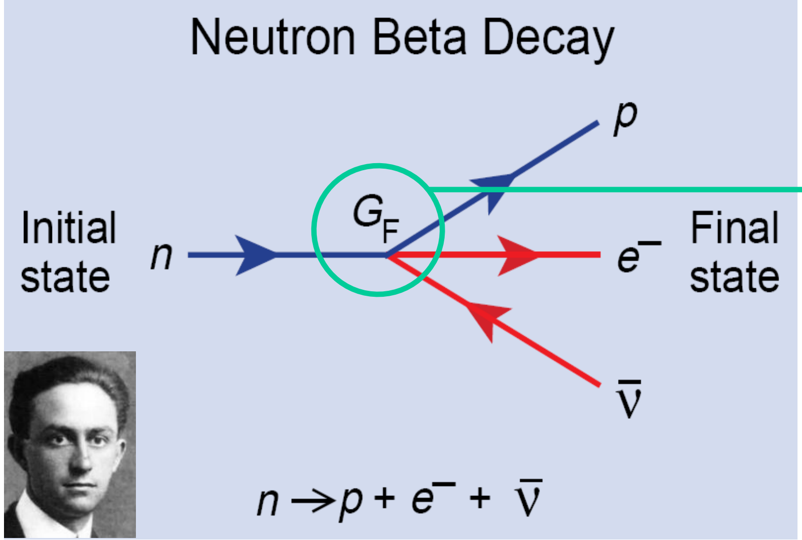
"I will be remembered for this paper!"

Solvay 1933



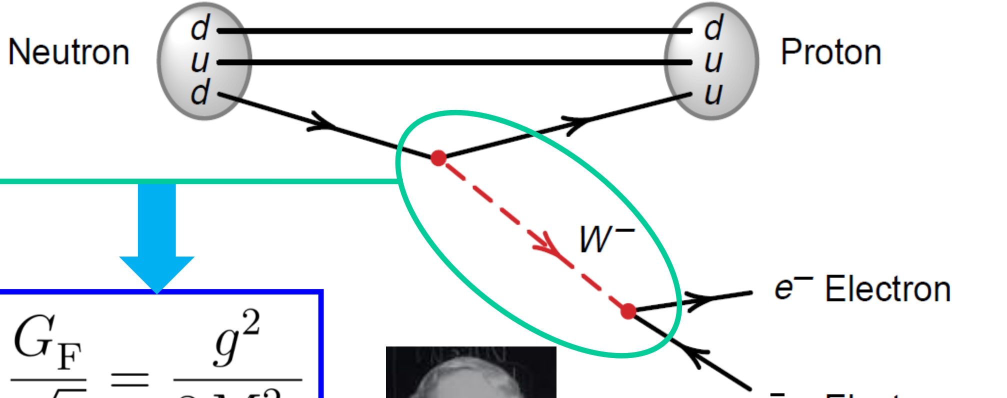
Untie Fermi's knot

It's Steven Weinberg who untied Fermi's knot by introducing new heavy degrees of freedom (1967)



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

gauge seesaw



Fermi coupling constant | **Weak interaction coupling constant**

$$G_F \simeq 1.166 \times 10^{-5}\ \text{GeV}^{-2}$$

$$g \simeq 0.65 \quad \textbf{vs} \quad M_W \simeq 80.4\ \text{GeV}$$

A good lesson some effective quantities at low energies are very likely to originate from new heavy degrees of freedom in a more fundamental theory at much higher energy scales.

Weinberg's model

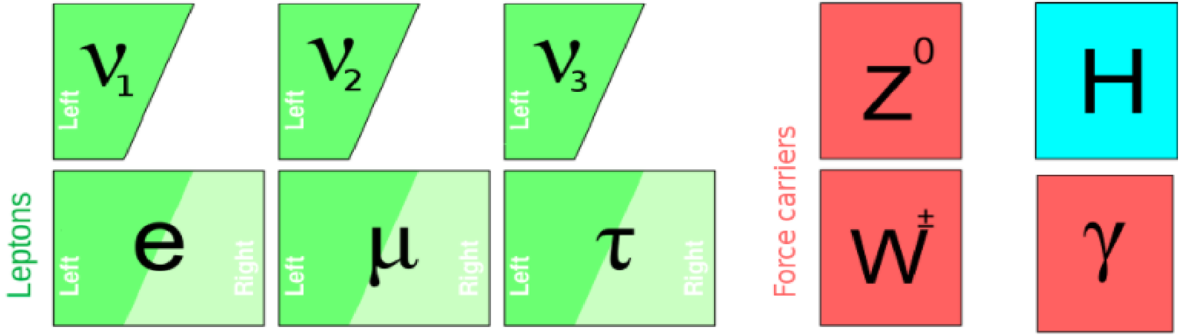
VOLUME 19, NUMBER 21 PHYSICAL REVIEW LETTERS 20 NOVEMBER 1967

Citations ~ 14000

A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)



- ♦ Its **theoretical ingredients** are really perfect!
- ♦ Its **particle content** looks very strange: **no** left-handed ν-fields.

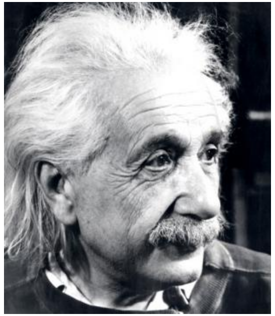
My style is usually **not to propose specific models** that will lead to specific experimental predictions, but rather to interpret in a broad way what is going on, and **make very general remarks**, like with the development of the point of view associated with **effective field theory** (2021@CERN Courier)

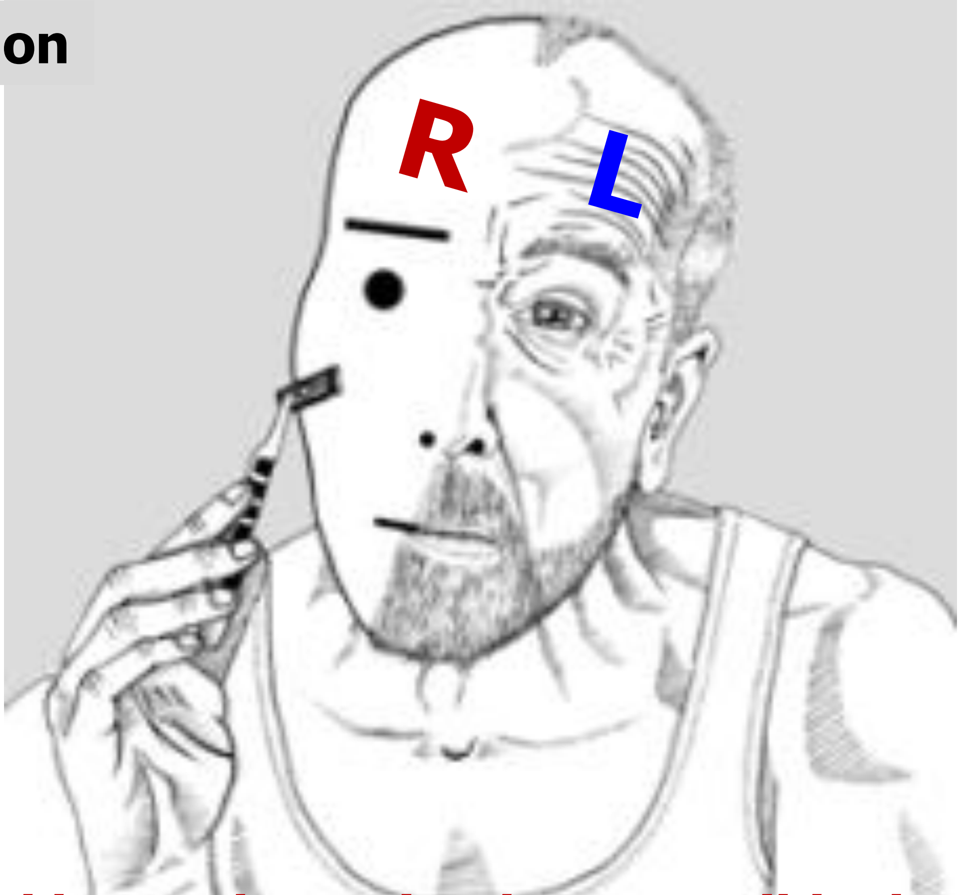
A wrong use of Occam's razor!

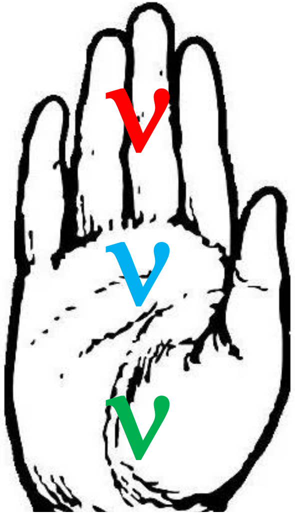
maximal P violation

$$\begin{pmatrix} u_\mathrm{L} \\ d_\mathrm{L} \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} u_\mathrm{R} \\ d_\mathrm{R} \end{matrix}$$

$$\begin{pmatrix} \nu_{e\mathrm{L}} \\ e_\mathrm{L} \end{pmatrix} \begin{matrix} \longleftrightarrow \\ \longleftrightarrow \end{matrix} \begin{matrix} ? \\ e_\mathrm{R} \end{matrix}$$







Right-handed neutrinos

Albert Einstein:

Everything should be made as simple as possible, but not simpler!

The solution at lowest costs

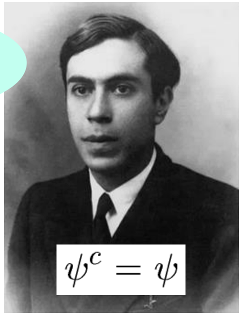
Dirac or Majorana?

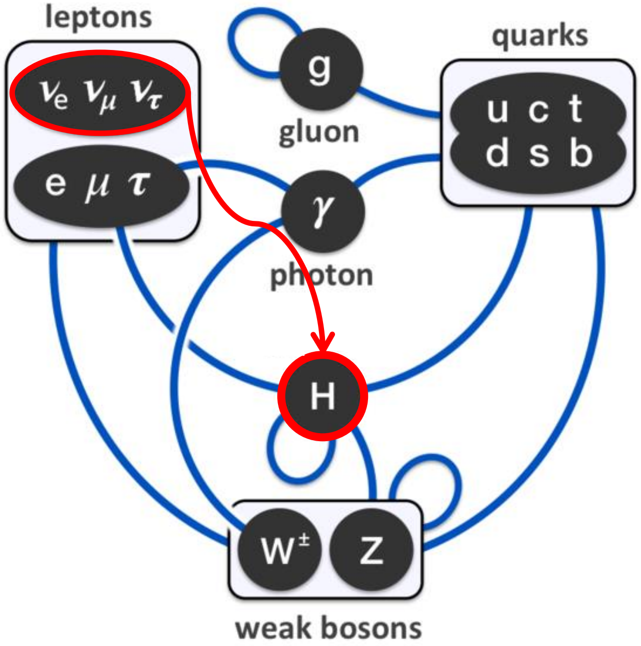
That is a question!

$$-\mathcal{L}_{\rm SS} = \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

Dirac (IQ>90) **Majorana (IQ>130)**

1937





What is the Majorana nature?

The antiparticle of a free fermion is itself.

Costs: heavy sterile neutrinos which are inaccessible in today's experiments (and probably the day after tomorrow's).

Rewards: the fundamental structures of the SM are unaffected, seesaw + leptogenesis works (1 stone kills 2 birds)….

Is there a convincing guiding principle to do so?

Gell-Mann's totalitarian principle (1956): Everything not forbidden is compulsory !

Weinberg's 3rd law of progress in TH physics (1983): You may use any degree of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry.

How seesaw works?

The canonical seesaw mechanism formally works far above the Fermi scale (ZZX, 2301.10461, NPB)

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$$-\mathcal{L}_{\rm lepton} = \overline{\ell_{\rm L}} Y_l H l_{\rm R} + \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$= \overline{l_{\rm L}} Y_l l_{\rm R} \phi^0 + \frac{1}{2}\overline{\begin{bmatrix}\nu_{\rm L} & (N_{\rm R})^c\end{bmatrix}} \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \begin{bmatrix}(\nu_{\rm L})^c \\ N_{\rm R}\end{bmatrix} + \overline{\nu_{\rm L}} Y_l l_{\rm R} \phi^+ - \overline{l_{\rm L}} Y_\nu N_{\rm R} \phi^- + {\rm h.c.}$$

The basis transformation related to the origin of active Majorana neutrino masses before SSB:

$$\mathbb{U}^\dagger \begin{pmatrix} \mathbf{0} & Y_\nu \phi^{0*} \\ Y_\nu^T \phi^{0*} & M_{\rm R}\end{pmatrix} \mathbb{U}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix}$$

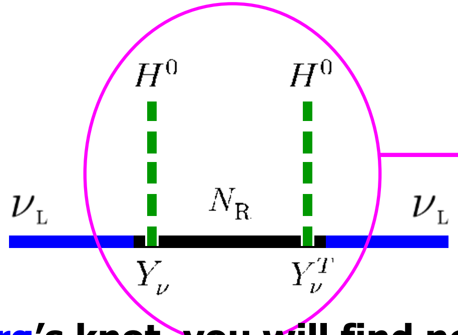
working masses:

$$\begin{cases} D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\} \\ D_N \equiv {\rm Diag}\{M_1, M_2, M_3\} \end{cases}$$

SSB

$$\begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R}\end{pmatrix}$$

6×6 mass matrix



Integrating out the heavy degrees of freedom:

$$-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^c + {\rm h.c.} \qquad M_\nu \simeq -Y_\nu \frac{\langle H\rangle^2}{M_{\rm R}} Y_\nu^T$$

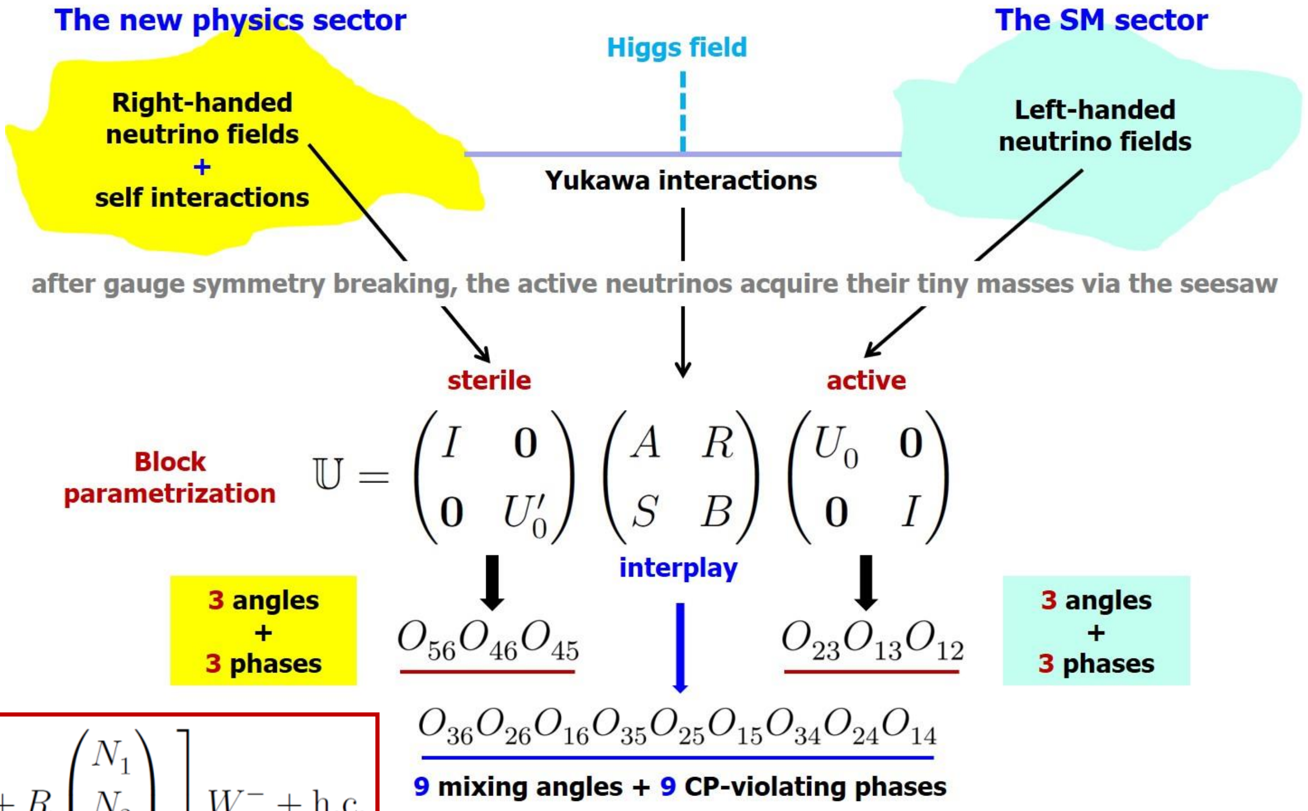
consistent with the Weinberg operator (1979)

If you can untie Weinberg's knot, you will find new heavy Majorana neutrinos at a superhigh scale.

A block parametrization

A **block parametrization** of active-sterile flavor mixing in the seesaw framework:

- ♦ reflects salient features of the seesaw dynamics;
- ♦ offers generic + explicit expressions of observables using the Euler-like angles and phases (ZZX, 1110.0083)



The weak **charged-current** interactions of leptons:

$U = AU_0$: the PMNS matrix;
R : an analogue for heavy.

seesaw + unitarity:

$$\begin{cases} U D_\nu U^T + R D_N R^T = \mathbf{0} \\ UU^\dagger + RR^\dagger = I \end{cases}$$

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\, \overline{(e \ \ \mu \ \ \tau)_{\rm L}}\, \gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W_\mu^- + {\rm h.c.}$$

oscillations ⟵ light heavy ⟶ leptogenesis

Euler-like parameters

$$U_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

derivational parameters

$$c_{ij} \equiv \cos\theta_{ij}$$
$$\hat{s}_{ij} \equiv e^{i\delta_{ij}}\sin\theta_{ij} \ (\text{for } 1 \le i < j \le 6)$$

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36} \end{pmatrix}$$

$$R = \begin{pmatrix} \hat{s}^*_{14}c_{15}c_{16} & \hat{s}^*_{15}c_{16} & \hat{s}^*_{16} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ +c_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & -\hat{s}^*_{15}\hat{s}_{16}\hat{s}^*_{26} + c_{15}\hat{s}^*_{25}c_{26} & c_{16}\hat{s}^*_{26} \\ \begin{array}{l} -\hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + \hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} - c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} + c_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +c_{15}c_{25}\hat{s}^*_{35}c_{36} \end{array} & c_{16}c_{26}\hat{s}^*_{36} \end{pmatrix}$$

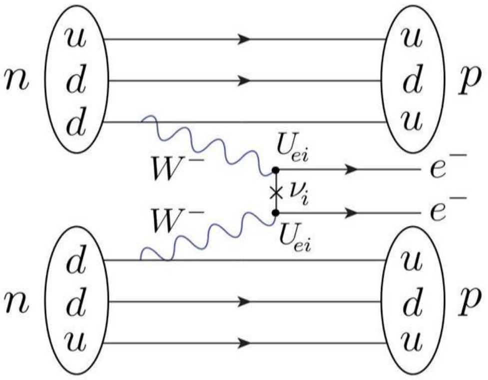
The original seesaw parameters in A+R:

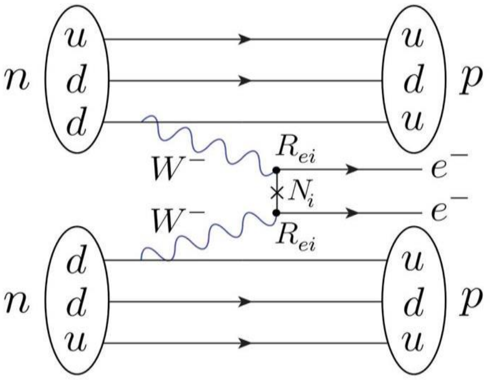
9 angles + 6 phases

Now you can calculate everything that can in principle be measured (see, e.g., xzz, 2305.xxxxx).

Impacts on 0ν2β decays

The seesaw-induced Majorana nature of massive neutrinos allows for lepton-number-violating 0ν2β decays to occur, a unique way to hunt for Mr. Majorana.



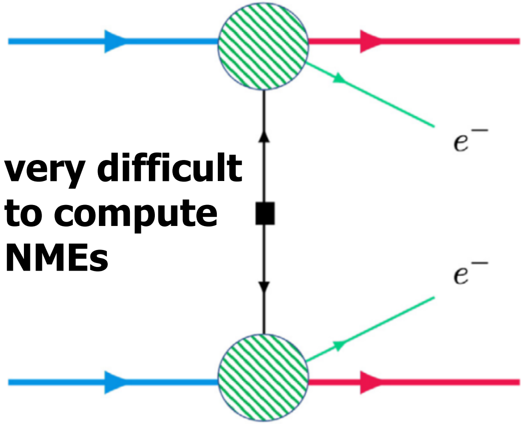


Seesaw + Unitarity:

$$\sum_{i=1}^{3} m_i U_{ei}^2 + \sum_{j=1}^{3} M_j R_{ej}^2 = 0$$

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{j=1}^{3} |R_{ej}|^2 = 1$$

Interplay between propagators + NMEs



J.M. Yao, J. Meng, Y.F. Niu, P. Ring, PPNP 2022

A stupid question: which channel is more fundamental?

Smart answer: they are equally fundamental thanks to Yukawa interactions.

In most cases, the contributions from heavy Majorana neutrinos to 0ν2β are negligibly small in the canonical seesaw mechanism. (ZZX, 0907.3014; W. Rodejohann, 0912.3388)

But…(e.g., D.L. Fang, Y.F. Li, Y.Y. Zhang, 2112.12779)

Impacts on CP violation

- The exact seesaw formula —— a bridge between the original and derivational flavor parameters:

$$U D_\nu U^T + R D_N R^T = \mathbf{0} \longrightarrow M_\nu \equiv U_0 D_\nu U_0^T = \left(\mathrm{i} A^{-1} R\right) D_N \left(\mathrm{i} A^{-1} R\right)^T$$

Degrees of freedom (mass + mixing angle + CPV phase): 3 + 3 + 3 (derivational) ⟵ 3 + 9 + 6 (original)

- To calculate the Jarlskog invariant of CP violation in the active neutrino oscillations, which is defined by

$$\mathcal{J}_\nu \sum_\gamma \varepsilon_{\alpha\beta\gamma} \sum_k \varepsilon_{ijk} = \mathrm{Im}\left[(U_0)_{\alpha i} (U_0)_{\beta j} (U_0)^*_{\alpha j} (U_0)^*_{\beta i}\right]$$

$$\begin{cases} D_\nu \equiv \mathrm{Diag}\{m_1, m_2, m_3\} \\ D_N \equiv \mathrm{Diag}\{M_1, M_2, M_3\} \\ \Delta m^2_{ij} \equiv m_i^2 - m_j^2 \end{cases}$$

- On the one hand, we use the light degrees of freedom to obtain the relation:

$$\mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu} \left(M_\nu M_\nu^\dagger\right)_{\mu\tau} \left(M_\nu M_\nu^\dagger\right)_{\tau e}\right] = \mathcal{J}_\nu \Delta m^2_{21} \Delta m^2_{31} \Delta m^2_{32}$$

- On the other hand, we use the original seesaw-related parameters to calculate the same quantity *in the leading order approximation* of $A^{-1}R$.

$$A^{-1} R \simeq \begin{pmatrix} \hat{s}^*_{14} & \hat{s}^*_{15} & \hat{s}^*_{16} \\ \hat{s}^*_{24} & \hat{s}^*_{25} & \hat{s}^*_{26} \\ \hat{s}^*_{34} & \hat{s}^*_{35} & \hat{s}^*_{36} \end{pmatrix}$$

But here, we switch off one heavy neutrino for simplicity.

Simplified result in the minimal seesaw

- The minimal seesaw framework with only 2 heavy Majorana neutrinos —— a benchmark scenario
- Consequences: (1) only 2 massive light (active) neutrinos; (2) only 2 CPV phases in the unitary PMNS matrix.
- Parameter counting: Original —— 2 masses + 6 mixing angles + 3 CPV phases;
  Derivational —— 2 masses + 3 mixing angles + 2 CPV phases.

$$\alpha \equiv \delta_{14} - \delta_{15}\ , \ \beta \equiv \delta_{24} - \delta_{25}\ , \ \gamma \equiv \delta_{34} - \delta_{35}$$

$$\begin{aligned} \mathrm{Im}\left[\left(M_\nu M_\nu^\dagger\right)_{e\mu}\left(M_\nu M_\nu^\dagger\right)_{\mu\tau}\left(M_\nu M_\nu^\dagger\right)_{\tau e}\right] = & \ C_0\left[C_\alpha \sin 2\alpha + C_\beta \sin 2\beta + C_\gamma \sin 2\gamma \right. \\ & + C_{\alpha+\beta}\sin(\alpha+\beta) + C_{\beta+\gamma}\sin(\beta+\gamma) + C_{\gamma+\alpha}\sin(\gamma+\alpha) \\ & \left. + C_{\alpha-\beta}\sin(\alpha-\beta) + C_{\beta-\gamma}\sin(\beta-\gamma) + C_{\gamma-\alpha}\sin(\gamma-\alpha)\right] = \mathcal{J}_\nu \Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32} \end{aligned}$$

- Coefficients

$$\begin{aligned} C_0 = & \ M_1^2 M_2^2\left[s_{14}^2\left(s_{25}^2+s_{35}^2\right)+s_{24}^2\left(s_{15}^2+s_{35}^2\right)+s_{34}^2\left(s_{15}^2+s_{25}^2\right)\right. \\ & \left. -2s_{14}s_{15}s_{24}s_{25}\cos(\alpha-\beta) - 2s_{24}s_{25}s_{34}s_{35}\cos(\beta-\gamma) - 2s_{14}s_{15}s_{34}s_{35}\cos(\gamma-\alpha)\right] \end{aligned}$$

ZZX, 2306.02362
to appear in PLB

$$C_\alpha = M_1 M_2 s_{14}^2 s_{15}^2\left(s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2\right)$$

$$C_\beta = M_1 M_2 s_{24}^2 s_{25}^2\left(s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2\right)$$

$$C_\gamma = M_1 M_2 s_{34}^2 s_{35}^2\left(s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2\right)$$

$$C_{\alpha+\beta} = M_1 M_2 s_{14}s_{15}s_{24}s_{25}\left(s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2\right)$$

$$C_{\beta+\gamma} = M_1 M_2 s_{24}s_{25}s_{34}s_{35}\left(s_{14}^2 s_{35}^2 - s_{34}^2 s_{15}^2 + s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2\right)$$

$$C_{\gamma+\alpha} = M_1 M_2 s_{14}s_{15}s_{34}s_{35}\left(s_{24}^2 s_{15}^2 - s_{14}^2 s_{25}^2 + s_{34}^2 s_{25}^2 - s_{24}^2 s_{35}^2\right)$$

$$C_{\alpha-\beta} = \left[M_1^2\left(s_{14}^2+s_{24}^2+s_{34}^2\right)s_{34}^2 - M_2^2\left(s_{15}^2+s_{25}^2+s_{35}^2\right)s_{35}^2\right]s_{14}s_{15}s_{24}s_{25}$$

$$C_{\beta-\gamma} = \left[M_1^2\left(s_{14}^2+s_{24}^2+s_{34}^2\right)s_{14}^2 - M_2^2\left(s_{15}^2+s_{25}^2+s_{35}^2\right)s_{15}^2\right]s_{24}s_{25}s_{34}s_{35}$$

$$C_{\gamma-\alpha} = \left[M_1^2\left(s_{14}^2+s_{24}^2+s_{34}^2\right)s_{24}^2 - M_2^2\left(s_{15}^2+s_{25}^2+s_{35}^2\right)s_{25}^2\right]s_{14}s_{15}s_{34}s_{35}$$

- Conclusion:

low — $\mathcal{J}_\nu$ ←(9 ways)— seesaw scale — $\alpha\ ,\ \beta\ ,\ \gamma$

CPV in heavy Majorana neutrino decays

- **The flavor-dependent CP-violating asymmetries in LNV decays of heavy Majorana neutrinos:**

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma(N_i \to \ell_\alpha + H) - \Gamma(N_i \to \overline{\ell_\alpha} + \overline{H})}{\sum_\alpha \left[\Gamma(N_i \to \ell_\alpha + H) + \Gamma(N_i \to \overline{\ell_\alpha} + \overline{H})\right]}$$

$$\mathcal{Y}_\nu \equiv Y_\nu U_0^{\prime *}$$

$$x_{ji} \equiv M_j^2 / M_i^2$$

$$= \frac{1}{8\pi(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ii}} \sum_{j\neq i} \left\{ \mathrm{Im}\left[(\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij} \xi(x_{ji}) + (\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)^*_{ij} \zeta(x_{ji}) \right] \right\}$$

where $\xi(x_{ji}) = \sqrt{x_{ji}} \left\{ 1 + 1/\left(1 - x_{ji}\right) + \left(1 + x_{ji}\right) \ln\left[x_{ji} / \left(1 + x_{ji}\right) \right] \right\}$ **,** $\zeta(x_{ji}) = 1/\left(1 - x_{ji}\right)$

- **Baryogenesis via leptogenesis in the early Universe**

- **A net lepton number asymmetry:**

$$Y_\mathrm{L} \equiv \frac{n_\mathrm{L} - n_{\overline{\mathrm{L}}}}{s} = \frac{1}{g_*} \sum_{i,\alpha} \kappa_{i\alpha} \varepsilon_{i\alpha}$$

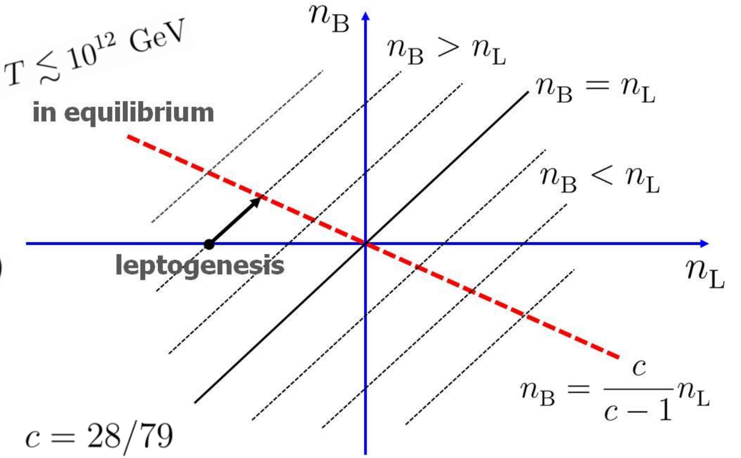
$$\Gamma(N_i \to \ell_\alpha + H) < H(T = M_i)$$

$$10^2~\mathrm{GeV} \lesssim T \lesssim 10^{12}~\mathrm{GeV}$$

sphaleron-induced (B-L)-conserving process in thermal equilibrium

$$Y_\mathrm{B} \equiv \frac{n_\mathrm{B} - n_{\overline{\mathrm{B}}}}{s} = -c Y_\mathrm{L}$$

A net baryon number asymmetry



CPV in 1st heavy Majorana neutrino decays

♦ **The CP-violating asymmetries in LNV decays of the first heavy Majorana neutrino:**

$$\varepsilon_{1e} = \frac{M_1^2 s_{14} s_{15}}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[x_{21}\xi(x_{21}) \left[s_{14}s_{15}\sin 2\alpha + s_{24}s_{25}\sin(\alpha+\beta) + s_{34}s_{35}\sin(\alpha+\gamma) \right] + x_{21}\zeta(x_{21}) \left[s_{24}s_{25}\sin(\alpha-\beta) + s_{34}s_{35}\sin(\alpha-\gamma) \right] \Big]$$

$$\varepsilon_{1\mu} = \frac{M_1^2 s_{24} s_{25}}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[x_{21}\xi(x_{21}) \left[s_{14}s_{15}\sin(\alpha+\beta) + s_{24}s_{25}\sin 2\beta + s_{34}s_{35}\sin(\beta+\gamma) \right] + x_{21}\zeta(x_{21}) \left[s_{14}s_{15}\sin(\beta-\alpha) + s_{34}s_{35}\sin(\beta-\gamma) \right] \Big]$$

$$\varepsilon_{1\tau} = \frac{M_1^2 s_{34} s_{35}}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \Big[x_{21}\xi(x_{21}) \left[s_{14}s_{15}\sin(\alpha+\gamma) + s_{24}s_{25}\sin(\beta+\gamma) + s_{34}s_{35}\sin 2\gamma \right] + x_{21}\zeta(x_{21}) \left[s_{14}s_{15}\sin(\gamma-\alpha) + s_{24}s_{25}\sin(\gamma-\beta) \right] \Big]$$

$\alpha \equiv \delta_{14} - \delta_{15}\ ,\ \beta \equiv \delta_{24} - \delta_{25}\ ,\ \gamma \equiv \delta_{34} - \delta_{35}$

$$\varepsilon_1 = +\frac{M_1^2 x_{21}\xi(x_{21})}{8\pi\langle\phi^0\rangle^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)} \left[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma + 2 s_{14}s_{15}s_{24}s_{25}\sin(\alpha+\beta) + 2 s_{14}s_{15}s_{34}s_{35}\sin(\alpha+\gamma) + 2 s_{24}s_{25}s_{34}s_{35}\sin(\beta+\gamma) \right]$$

CPV in 2nd heavy Majorana neutrino decays

♦ **The CP-violating asymmetries in LNV decays of the second heavy Majorana neutrino:**

$$\varepsilon_{2e} = -\frac{M_2^2 s_{14} s_{15}}{8\pi \langle \phi^0 \rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[x_{12} \xi(x_{12}) \left[s_{14} s_{15} \sin 2\alpha + s_{24} s_{25} \sin(\alpha+\beta) + s_{34} s_{35} \sin(\alpha+\gamma) \right] + x_{12} \zeta(x_{12}) \left[s_{24} s_{25} \sin(\alpha-\beta) + s_{34} s_{35} \sin(\alpha-\gamma) \right] \Big]$$

$$\varepsilon_{2\mu} = -\frac{M_2^2 s_{24} s_{25}}{8\pi \langle \phi^0 \rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[x_{12} \xi(x_{12}) \left[s_{14} s_{15} \sin(\alpha+\beta) + s_{24} s_{25} \sin 2\beta + s_{34} s_{35} \sin(\beta+\gamma) \right] + x_{12} \zeta(x_{12}) \left[s_{14} s_{15} \sin(\beta-\alpha) + s_{34} s_{35} \sin(\beta-\gamma) \right] \Big]$$

$$\varepsilon_{2\tau} = -\frac{M_2^2 s_{34} s_{35}}{8\pi \langle \phi^0 \rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[x_{12} \xi(x_{12}) \left[s_{14} s_{15} \sin(\alpha+\gamma) + s_{24} s_{25} \sin(\beta+\gamma) + s_{34} s_{35} \sin 2\gamma \right] + x_{12} \zeta(x_{12}) \left[s_{14} s_{15} \sin(\gamma-\alpha) + s_{24} s_{25} \sin(\gamma-\beta) \right] \Big]$$

$\alpha \equiv \delta_{14} - \delta_{15}\,, \ \beta \equiv \delta_{24} - \delta_{25}\,, \ \gamma \equiv \delta_{34} - \delta_{35}$

$$\varepsilon_2 = -\frac{M_2^2 x_{12} \xi(x_{12})}{8\pi \langle \phi^0 \rangle^2 \left(s_{15}^2 + s_{25}^2 + s_{35}^2\right)} \Big[s_{14}^2 s_{15}^2 \sin 2\alpha + s_{24}^2 s_{25}^2 \sin 2\beta + s_{34}^2 s_{35}^2 \sin 2\gamma + 2 s_{14} s_{15} s_{24} s_{25} \sin(\alpha+\beta) + 2 s_{14} s_{15} s_{34} s_{35} \sin(\alpha+\gamma) + 2 s_{24} s_{25} s_{34} s_{35} \sin(\beta+\gamma) \Big]$$

A summary of the phase dependence

- The Jarlskog invariant and CPV asymmetries depend on 3 independent phases (zzx, 2306.02362).

	2α	2β	2γ	$\alpha+\beta$	$\beta+\gamma$	$\alpha+\gamma$	$\alpha-\beta$	$\beta-\gamma$	$\gamma-\alpha$
$\mathcal{J}_\nu$	√	√	√	√	√	√	√	√	√
ε_{1e}	√			√		√	√		√
$\varepsilon_{1\mu}$		√		√	√		√	√	
$\varepsilon_{1\tau}$			√		√	√		√	√
ε_{1}	√	√	√	√	√	√			
ε_{2e}	√			√		√	√		√
$\varepsilon_{2\mu}$		√		√	√		√	√	
$\varepsilon_{2\tau}$			√		√	√		√	√
ε_{2}	√	√	√	√	√	√			

CP不守恒量之间一般不存在正比关系

- A more straightforward correlation between the two CPV observables needs some assumptions.

Concluding remarks

- The canonical seesaw mechanism is the most natural and economical mechanism to produce tiny Majorana neutrino masses and interpret the cosmological matter-antimatter asymmetry by thermal leptogenesis mechanism, but all these only work qualitatively.

- *For the first time*, we have derived the generic + explicit expressions of the Jarlskog invariant for light neutrino oscillations in terms of the original seesaw flavor parameters, and of CP asymmetries for heavy neutrino decays based on a block parametrization of the active-sterile flavor texture.

- A full numerical exploration of the seesaw parameter space can be done, using a good computer. Testing the seesaw mechanism and identifying the Majorana nature of massive neutrinos is largely possible in the precision measurement era. The $0\nu2\beta$ decay will be a smoking gun.