Probing the Supersymmetric Grand Unified Theories at the Future Proton-Proton Colliders and Hyper-Kamiokande Experiment

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Waqas Ahmed, TL, Shabbar Raza and Fang-Zhou Xu, Phys. Lett. B **819**, 136378 (2021) [arXiv:2007.15059 [hep-ph]]; Waqas Ahmed, TL, and Shabbar Raza, arXiv:2208.05257 [hep-ph]; in preparation.

Supersymmetry and the SSMs with R-Parity

- A natural solution to the gauge hierarchy problem in the SM.
- Gauge coupling unification can be achieved.
- The Lightest Supersymmetric Particle (LSP) such as the LSP neutralino etc can be a dark matter candidate.
- The electroweak gauge symmetry can be broken radiatively due to the large top quark Yukawa coupling.
- . . . .

The Grand Unified Theories: $SU(5)$ and $SO(10)$

- Gauge interaction unification.
- Fermion unification.

  In $SO(10)$ model, one family of the SM fermion forms a spinor **16** representation.
- Yukawa coupling unification.

  $b - \tau$ and $b - \tau - t$ Yukawa coupling unifications in $SU(5)$ and $SO(10)$ models, respectively.
- Charge quantization.
- Weak mixing angle at weak scale M_Z.
- Neutrino masses and mixings by seesaw mechanism.
- Prediction: dimension-six proton decay via heavy gauge boson exchange.

Problems

- Gauge symmetry breaking.
- Doublet-triplet splitting problem.
- Dimension-five Proton decay problem.
- Fermion mass problem.

  The wrong prediction on the fermion mass ratios: $m_e/m_\mu = m_d/m_s$.

String Models

- Calabi-Yau compactification of heterotic $E_8 \times E_8$ string theory
- Orbifold compactification of heterotic $E_8 \times E_8$ string theory
- D-brane models on Type II orientifolds
- Free fermionic string model builing
- $\mathcal{F}$-Theory Model Building

Supersymmetry is a bridge between the promising low energy phenomenology and high-energy fundamental physics.

Grand Particle Physics Paradigm

$$\text{String Theory} \rightarrow \text{String Models} \rightarrow \text{SUSY GUTs} \rightarrow \text{SSMs} \rightarrow \text{SM}$$

Are supersymmetric Standard Models natural?

Can we probe this grand particle physics paradiagm at the future pp colliders and other experiments? If yes, what is the center-of-mass energy needed?

We can probe the SUSY GUTs at the future experiments!

The LHC Supersymmetry Search Contraints

- The first two-generation squark mass low bounds are around 1.6 (1.75) TeV.
- The gluino mass low bound is around 2.25 (2.46) TeV.
- The stop and sbottom mass low bounds are around 1.16 (1.3) and 1.35 (1.45) TeV, respectively.

The SSMs are fine-tuned!!!

Supersymmetry at the Current and Future Colliders

- The wrong impression is that supersymmetry was excluded at the LHC?
- Can we rule out supersymmetry at the LHC, VLHC, $\mathrm{FCC_{hh}}$ and SppC?
  No! No!! No!!!
- Points: supersymmetry breaking soft mass scale can be pushed to be much higher than 1 TeV, while gauge coupling unification can still be realized due to the logarithmic RGE running and threshold corrections around the GUT scale.
- Conclusion: supersymmetry will defintely not die in the near future!!!

Natural Supersymmetry

The interesting question: can we rule out the natural supersymmetry at the $\mathrm{FCC_{hh}}$ and SppC? Or can we solve the supersymmetry electroweak fine-tuning problem naturally?

Fine-Tuning Definition

- Fine-tuning Definition [1]: the quantitative measure $\Delta_{\rm FT}^{\rm EENZ-BG}$ for fine-tuning is the maximum of the logarithmic derivative of M_Z with respect to all the fundamental parameters a_i at the GUT scale

$$\Delta_{\rm FT}^{\rm EENZ-BG} = {\rm Max}\{\Delta_i^{\rm GUT}\} ~, \quad \Delta_i^{\rm GUT} = \left| \frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})} \right| ~.$$

[1] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986); R. Barbieri and G. F. Giudice, Nucl. Phys. B **306**, 63 (1988).

Question: Super-Natural Supersymmetry

Can we propose a supersymmetry scenario whose the EENZ-BG fine-tuning measure is automatically 1 or order 1 ($\mathcal{O}(1)$)?

Fundamental physics principles: simplicity and naturalness.

Super-Natural Supersymmetry [2]

- Fine-Tuning Definition:

$$\Delta_{\rm FT} ~=~ {\rm Max}\{\Delta_i^{\rm GUT}\}~, \quad \Delta_i^{\rm GUT} ~=~ \left| \frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})} \right| ~.$$

- Natural Solution:

$$M_Z^n ~=~ f_n \left(\frac{M_Z}{M_*} \right) ~ M_*^n ~.$$

$$\frac{\partial {\rm ln}(M_Z^n)}{\partial {\rm ln}(M_*^n)} ~\simeq~ \frac{M_*^n}{M_Z^n} \frac{\partial M_Z^n}{\partial M_*^n} ~\simeq~ \frac{1}{f_n} f_n \simeq \mathcal{O}(1) ~.$$

- For no-scale supergravity and M-theory on S^1/Z_2, we have $M_* = M_{1/2}$ and $M_* = M_{3/2}$, respectively.

[2] T. Leggett, T. Li, J. A. Maxin, D. V. Nanopoulos and J. W. Walker, arXiv:1403.3099 [hep-ph]; Phys. Lett. B **740**, 66 (2015) [arXiv:1408.4459 [hep-ph]]; G. Du, T. Li, D. V. Nanopoulos and S. Raza, Phys. Rev. D **92**, no. 2, 025038 (2015) [arXiv:1502.06893 [hep-ph]]; T. Li, S. Raza and X. C. Wang, Phys. Rev. D **93**, no. 11, 115014 (2016) [arXiv:1510.06851 [hep-ph]].

Predictions

- The LSP neutralino is Bino-like.
- The right-handed charged sleptons are light.
- The dark matter relic density can be explained via the LSP neutralino-light stau coannihilation.
- If the Bino mass is less than 120 GeV, the dark matter relic density can be explained by the LSP neutralino-light stau annihilation as in the bulk region. This scenario can be probed at the future CEPC.

Grand Particle Physics Paradigm

String Theory $\rightarrow$ String Models $\rightarrow$ SUSY GUTs $\rightarrow$ SSMs $\rightarrow$ SM

Are supersymmetric Standard Models natural?

Can we probe this grand particle physics paradiagm at the future pp colliders and other experiments? If yes, what is the center-of-mass energy needed?

We can probe the SUSY GUTs at the future experiments!

Future Colliders

- Lepton colliders: CEPC, CLIC, $\mathrm{FCC_{ee}}$, and ILC.
- Hadron colliders: HL-LHC, HE-LHC, $\mathrm{FCC_{hh}}$, SppC, and VLHC.

To probe the new physics beyond the SM, we do need future proton-proton colliders.

Future Proton-Proton Colliders

- Question: what is the concrete scientific goal for the future pp colliders?
- Question: what is the center-of-mass energy needed for this scientific goal?

The Scientific Goal for the Future PP Colliders

- Supersymmetry cannot be the scientific goal since the sparticles can be very heavy and then we cannot probe supersymmetry.
- The supersymmetric GUTs with grand desert hypothesis can be the scientific goal [3]!!!

[3] Waqas Ahmed, TL, Shabbar Raza and Fang-Zhou Xu, Phys. Lett. B **819**, 136378 (2021) [arXiv:2007.15059 [hep-ph]]; ; Waqas Ahmed, TL, and Shabbar Raza, arXiv:2208.05257 [hep-ph]; in preparation.

The Supersymmetry Breaking Mediation Mechanisms

- Gravity mediated supersymmetry breaking.
- Anomaly mediated supersymmetry breaking.
- Gauge mediated supersymmetry breaking.

The Proof

- Grand desert hypothesis: no new physics between the sparticle mass scale and the GUT scale.
- For the GUTs with the GUT scale $M_{GUT} \leq 1.2 \times 10^{16}$ GeV we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment.
- For the GUTs with $M_{GUT} \geq 1.2 \times 10^{16}$ GeV, we can probe the gluino and/or squarks at the $\mathrm{FCC_{hh}}$ and SppC for gravity mediation, gauge mediation, and anomaly mediation.
- Providing the "Concrete Scienctific Goal" for the future pp colliders $\mathrm{FCC_{hh}}$/SppC and Hyper-Kamiokande experiment.

The First Part of the Proof

For the GUTs with the GUT scale $M_{GUT} \leq 1.2 \times 10^{16}$ GeV, we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment.

The Predictions of the SUSY GUTs: Proton Decays

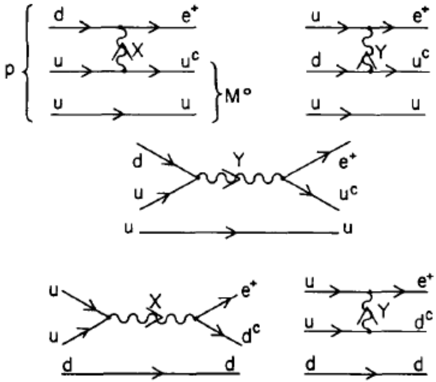
- The dimension-six proton decay via superheavy $(X_\mu, \; Y_\mu)$ gauge boson exchanges

$$SU(5) \; = \; \left(\begin{array}{cc} SU(3)_C & (\overline{X}_\mu, \overline{Y}_\mu) \\ (X_\mu, Y_\mu) & SU(2)_L \end{array} \right) \; .$$

- The dimension-five proton decays via colored Higgsino exchanges in the supersymmetric GUTs.
- We do not consider the dimension-five proton decays.

We can have the dimension-five proton decays at renormalizable level by introducing vector-like particles, and can nelect the dimension-five proton decays in some models such as the flipped $SU(5)$ models.

The Dimension-Six Proton Decay via (X_μ, Y_μ) Exchanges



The GUTs with $M_{GUT} \leq 1.2 \times 10^{16}$ GeV

- The proton lifetime from the dimension-six proton decay $p \to e^+\pi^0$ via heavy gauge boson exchange is

$$\tau_p \simeq 1.0 \times 10^{35} \times \left(\frac{2.5}{A_R}\right)^2 \times \left(\frac{0.04}{\alpha_{\rm GUT}}\right)^2 \times \left(\frac{M_{\rm GUT}}{1.0 \times 10^{16}~{\rm GeV}}\right)^4 ~{\rm years}~.$$

- At the future Hyper-Kamiokande experiment with 186 kt water, we can probe the GUTs with proton lifetime via dimension-6 proton decay at least above 1.0×10^{35} years [4]. The original Hyper-Kamiokande experimental proposal has 1,000 kt water, therefore, we can probe the GUTs with proton lifetime via dimension-6 proton decay at least above 5.37634×10^{35} years.

[4] K. Abe *et al.* [Hyper-Kamiokande], [arXiv:1805.04163 [physics.ins-det]].

The GUTs with $M_{GUT} \leq 1.2 \times 10^{16}$ GeV

- For the original Hyper-Kamiokande experimental proposal, we can probe the GUTs with GUT scale up to 1.46726×10^{16} GeV.
- Therefore, we can probe the GUTs with GUT scale up to 1.2×10^{16} GeV.

Comment

- In fact, if we did not observe the dimension-six proton decay from the Super-Kamiokande experiment and future Hyper-Kamiokande experiment, we obtain $M_{X_\mu/Y_\mu} \geq 1.2 \times 10^{16}$ GeV, not $M_{\rm GUT} \geq 1.2 \times 10^{16}$ GeV.
- There indeed exists some subtleties to define the GUT scale due to threshold corrections.
- Because $M_{X_\mu/Y_\mu} \leq M_{GUT}$, our study is not affected by these subtleties.

The Second Part of the Proof

For the GUTs with $M_{GUT} \geq 1.2 \times 10^{16}$ GeV for gravity mediation, and for the GUTs with $M_{GUT} \geq 1.0 \times 10^{16}$ GeV for gauge mediation and anomaly mediation, we can probe the gluino and/or squarks at the future pp colliders.

The Supersymmetry Searches at the Future pp Colliders

- At the future 100 TeV pp Colliders such as $\mathrm{FCC_{hh}}$ and SppC, Wino via Bino decay, gluino $\tilde{g}$ via heavy flavor decay, gluino via light flavor decay, first-two generation squarks $\tilde{q}$, and stop can be discovered for their masses up to about 6.5 TeV, 11 TeV, 17 TeV, 14 TeV, and 11 TeV, respectively. If the gluino and first-two generation squark masses are similar, they can be probed up to 20 TeV.
- To discover the gluino $\tilde{g}$ with mass around 15 TeV via heavy flavor decay, we need the 160 TeV pp collider such as the VLHC.

The Phenomenological Constraints

- The SM-like Higgs boson mass $m_h \subset [123,\ 127]\ \mathrm{GeV}$, and gluino mass $m_{\tilde{g}} \geq 2.2\ \mathrm{TeV}$.
- The constraints from rare decay processes $B_s \to \mu^+\mu^-$, $b \to s\gamma$, and $B_u \to \tau\nu_\tau$.
- To be general, we do not require the relic abundance of the LSP neutralino to satisfy the Planck bound within 5σ $0.114 \leq \Omega_{\mathrm{CDM}} h^2 \leq 0.126$.

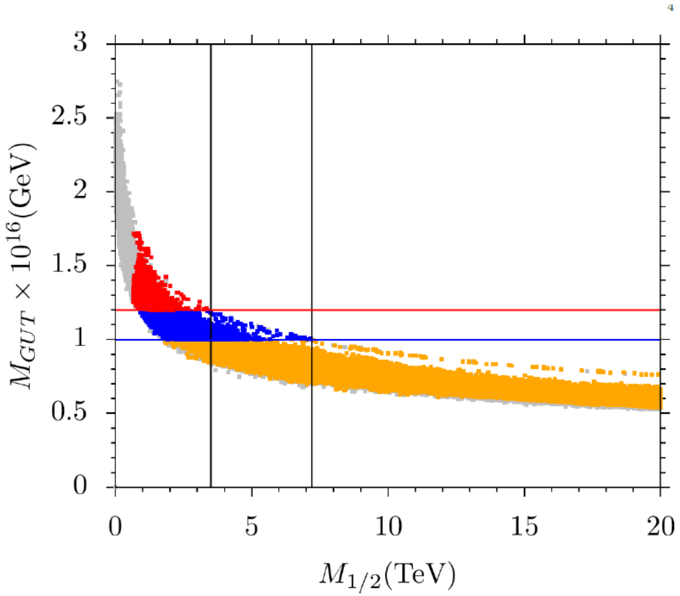
Gravity Mediated Supersymmetry Breaking

We perform the random scans for the following mSUGRA/CMSSM parameter space

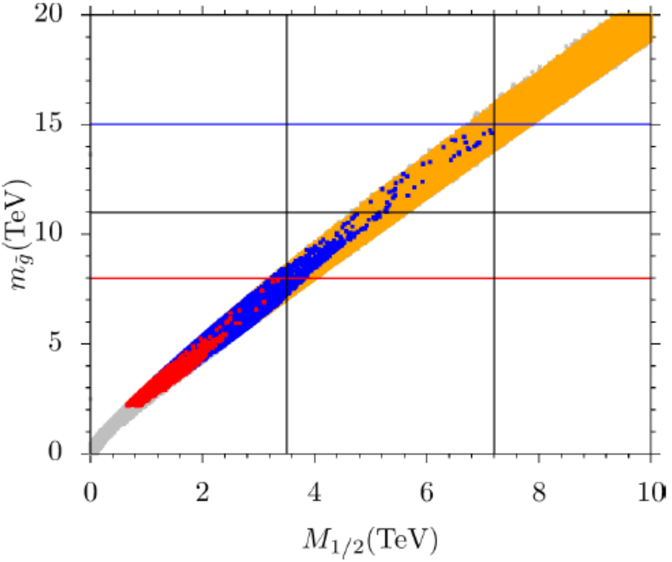
$$\begin{aligned} 0 \leq M_0 &\leq 90\,\mathrm{TeV}, \\ 0 \leq M_{1/2} &\leq 30\,\mathrm{TeV}, \\ -3 \leq A_0/M_0 &\leq 3, \\ 2 \leq \tan\beta &\leq 60 \end{aligned}$$

with $\mu > 0$ and $m_t = 173.2\,\mathrm{GeV}$.

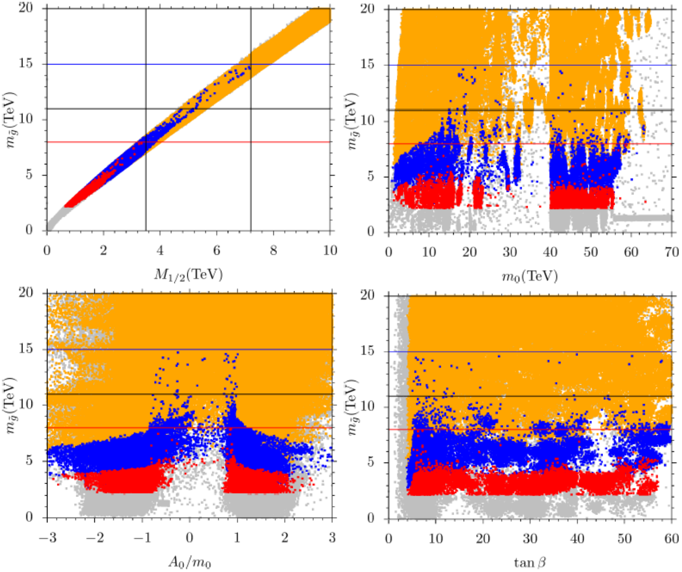
Gravity Mediated Supersymmetry Breaking



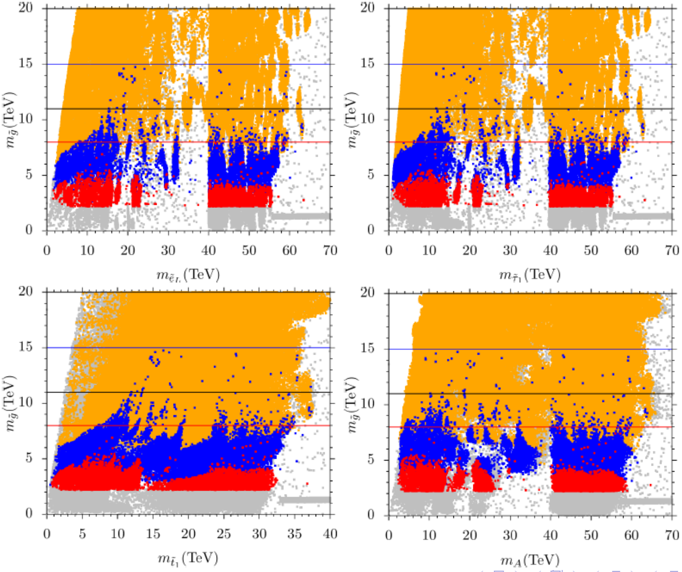
Gravity Mediated Supersymmetry Breaking



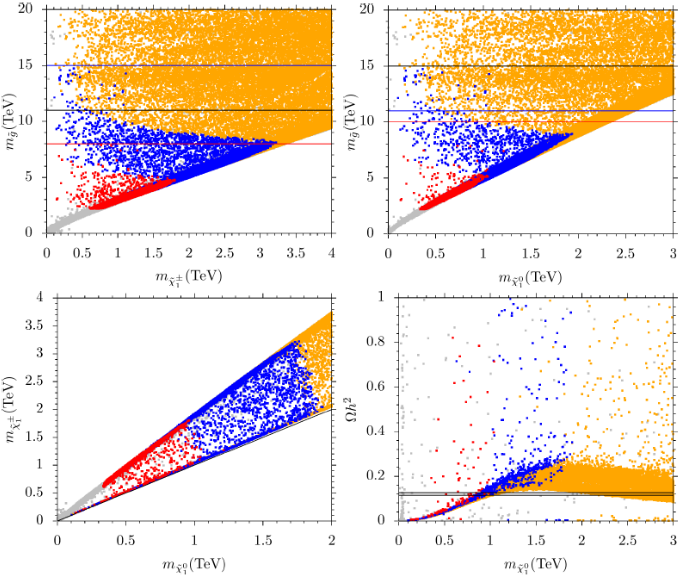
Gravity Mediated Supersymmetry Breaking



Gravity Mediated Supersymmetry Breaking



Gravity Mediated Supersymmetry Breaking



Gravity Mediated Supersymmetry Breaking

	Point 1	Point 2	Point 3	Point 4
m_0	53650	56830	13170	59140
$M_{1/2}$	2659	3056	4223	6457
A_0/m_0	0.8865	0.84	-0.038	0.9733
$\tan\beta$	7.15	6.2	20.6	7.32
M_{GUT}	1.272×10^{16}	1.226×10^{16}	1.035×10^{16}	1.01×10^{16}
m_h	126	125	125	126
m_H	54011	57565	12686	59618
m_A	53658	57189	12603	59229
$m_{H^\pm}$	54011	57565	12686	59619
$m_{\tilde{\chi}^0_{1,2}}$	936, 943	989, 994	870, 873	1061, 1063
$m_{\tilde{\chi}^0_{3,4}}$	1231, 2346	1424, 2705	1962, 3633	3070, 5703
$m_{\tilde{\chi}^\pm_{1,2}}$	984, 2270	1038, 2619	901, 3570	1113, 5557
$m_{\tilde{g}}$	6520	7399	9009	14286
$m_{\tilde{u}_{L,R}}$	53667, 53825	56887, 57050	15042, 14898	59898, 59990
$m_{\tilde{t}_{1,2}}$	31163, 44180	33309, 46976	10067, 12811	35365, 49562
$m_{\tilde{d}_{L,R}}$	53667, 53851	56887, 57077	15042,14898	59898, 60007
$m_{\tilde{b}_{1,2}}$	44077, 53590	46856, 56860	12763, 14506	49431, 59701
$m_{\tilde{\nu}_1}$	53656	56842	13429	59259
$m_{\tilde{\nu}_3}$	53507	56842	13192	59085
$m_{\tilde{e}_{L,R}}$	53630, 53617	56815, 56796	13426,13247	59231, 59141
$m_{\tilde{\tau}_{1,2}}$	53321, 53472	56564,56688	12764, 13187	58796, 59048
$\sigma_{SI}(\mathrm{cm}^2)$	1.127×10^{-45}	6.64×10^{-46}	1.3×10^{-46}	4.36×10^{-47}
$\Omega_{CDM}h^2$	0.119	0.123	0.115	0.125

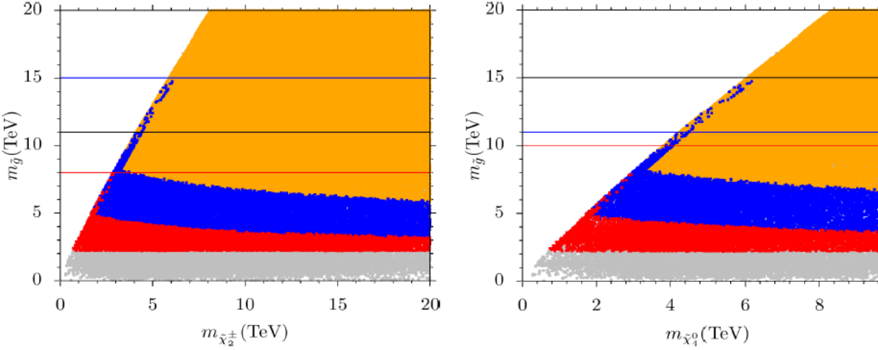
Gravity Mediated Supersymmetry Breaking: Gluino Searches

- For GUTs with $M_{GUT} \geq 1 \times 10^{16}\,\mathrm{GeV}$, the upper bound on the universal gaugino mass $M_{1/2}$ is about 7.2 TeV, and then the upper bound on gluino mass is 15 TeV.
- In our viable parameter space, the lightest squark is generically to be light stop, and thus we do have gluino via heavy flavor decay. To probe such gluino with mass up to 15 TeV, we find that the center-of-mass energy of the future pp collider needs to be about 160 TeV.
- For GUTs with $M_{GUT} \geq 1.2 \times 10^{16}\,\mathrm{GeV}$, the upper bound on the universal gaugino mass $M_{1/2}$ is about 3.5 TeV, and then the upper bound on gluino mass is 8 TeV.
- We can probe the GUTs with $M_{GUT} \geq 1.2 \times 10^{16}\,\mathrm{GeV}$ at the $\mathrm{FCC_{hh}}$ and SppC.

Gravity Mediated Supersymmetry Breaking: Wino Searches

- For GUTs with $M_{GUT} \geq 1 \times 10^{16}\,\mathrm{GeV}$, the upper bound on the universal gaugino mass $M_{1/2}$ is about 7.2 TeV, and then the upper bound on Wino mass is about 6 TeV.
- The Wino via Bino decay can be probed at the $\mathrm{FCC_{hh}}$ and SppC.
- Question: Wino via Higgsino decay?
- We might probe the GUTs with $M_{GUT} \geq 1.0 \times 10^{16}\,\mathrm{GeV}$ at the $\mathrm{FCC_{hh}}$ and SppC via Wino searches, and further study is under investigation.

Gravity Mediated Supersymmetry Breaking



For gluino mass lighter than or equal to 9 TeV, we can probe gluino at the $\mathrm{FCC_{hh}}$ and SppC. While for gluino mass heavier than 9 TeV, we obtain that both χ_4^0 and $\chi_2^\pm$ are lighter than about 6 TeV. Thus, Wino is lighter than about 6 TeV.

The GUTs with Generic Gravity Mediation

- Universal gaugino mass.
- The fermions and sfermions form complete GUT chiral multiplets
- The μ term is determined by electroweak symmetry breaking.
- The key difference between the GUTs with generic gravity mediation and mSUGRA is the scalar Higgs soft masses.
- Because Higgs fields are scalars and then their contributions to the RGEs are small, our conclusion is still valid. Of course, the sparticle spectra can be different.
- We shall study the $SU(5)$ models with the following supersymmetry breaking soft masses in details

$$M_{1/2},~M_{\bar{5}},~M_{10},~M_{H_u},~M_{H_d},~A_t,~A_b=A_\tau,~\tan\beta,~sign(\mu)~.$$

Anomaly Mediated Supersymmetry Breaking

- We consider the GUTs with $M_{GUT} \geq 1.0 \times 10^{16}\,\mathrm{GeV}$.
- We perform the random scans over the following parameter space of the minimal AMSB

$$
\begin{aligned}
1\,\mathrm{TeV} &\leq M_0 \leq 7.5\,\mathrm{TeV}, \\
100\,\mathrm{TeV} &\leq M_{3/2} \leq 3000\,\mathrm{TeV}, \\
2 &\leq \tan\beta \leq 60
\end{aligned}
$$

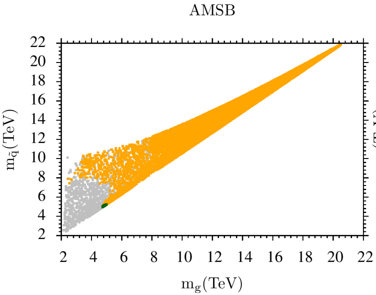
with $\mu > 0$ and $m_t = 173.2\,\mathrm{GeV}$.

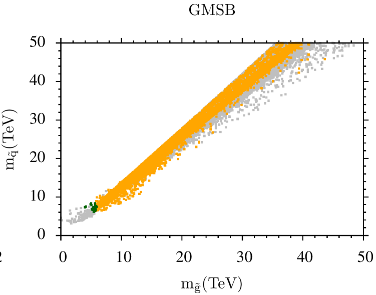
Gauge Mediated Supersymmetry Breaking

- We consider the GUTs with $M_{GUT} \geq 1.0 \times 10^{16}\,\mathrm{GeV}$.
- With one pair of the messenger fields in the **5** and $\overline{\mathbf{5}}$ representations of $SU(5)$, we perform random scans over the following parameter space

$$5 \times 10^5\,\mathrm{GeV} \leq \Lambda \equiv F/M_{\mathrm{mess}} \leq 10^{10}\,\mathrm{GeV},$$
$$2 \times \Lambda \leq M_{\mathrm{mess}} \leq 10^{15}\,\mathrm{GeV},$$
$$2 \leq \tan\beta \leq 60$$

with $\mu > 0$ and $m_t = 173.2\,\mathrm{GeV}$.


AMSB
22
20
18
16
14
12
10
8
6
4
2
2 4 6 8 10 12 14 16 18 20 22
$m_{\tilde{q}}$(TeV)
m_{g}(TeV)


GMSB
50
40
30
20
10
0
0 10 20 30 40 50
$m_{\tilde{q}}$(TeV)
$m_{\tilde{g}}$(TeV)

Anomaly and Gauge Mediated Supersymmetry Breakings

- For anomaly mediation, the squark and gluino masses are less than about 5 TeV.
- For anomaly mediation, the squark and gluino masses are less than about 8 TeV and 6 TeV, respectively.
- The GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reaches of the future 100 TeV pp colliders such as the $\mathrm{FCC_{hh}}$ and SppC.

Summary

- Supersymmetry is a bridge between the low energy phenomenology and high-energy fundamental physics, and thus is the promising new physics beyond the SM.
- Gauge coupling unification in the supersymmetric SM strongly implies the GUTs.
- With the grand desert hypothesis, we show that the supersymmetric GUTs and some string models can be probed at the future pp colliders $\mathrm{FCC_{hh}}$/SppC and Hyper-Kamiokande experiment.

The Proof

- Grand desert hypothesis: no new physics between the sparticle mass scale and the GUT scale.
- For the GUTs with the GUT scale $M_{GUT} \leq 1.2 \times 10^{16}$ GeV we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment.
- For the GUTs with $M_{GUT} \geq 1.2 \times 10^{16}$ GeV, we can probe the gluino and/or squarks at the $\mathrm{FCC_{hh}}$ and SppC for gravity mediation, gauge mediation, and anomaly mediation.
- Providing the "Concrete Scienctific Goal" for the future pp colliders $\mathrm{FCC_{hh}}$/SppC and Hyper-Kamiokande experiment.

Thank You Very Much
for Your Attention!