Some of recent progress in standard model effective field theory

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Disclaimer: quoted refs are not always the first ones.

Status of standard model in a few words

- No new particles found up to mass ~ 1 TeV > $\Lambda_{\mathrm{EW}} \approx 100\ \mathrm{GeV}$
  although some apparent tension exists between SM and expts.
  → SM phenomenologically very healthy
- Still, two practical issues remain to be addressed:
  $m_\nu < 1\ \mathrm{eV}$, believed to originate from phys well above Λ_{EW}
  If DM is of particle nature, SM cannot offer a candidate.
- There are more advanced theoretical challenges:
  flavor puzzle
  origin of electroweak symmetry breaking
  ......

Status of standard model in a few words

- Thus new phys is called for, which must involve particles of either mass $\gg \Lambda_{\mathrm{EW}}$

  $\rightarrow$ not directly reachable at colliders

  or mass $\leq \Lambda_{\mathrm{EW}}$, interacting feebly with SM particles

  $\rightarrow$ not yet detected in precision measurements
- Question:

  How to investigate new phys in such a circumstance?

Modern view of standard model

- All quantum field theories are effective field theories appropriate to a certain range of energy scales.
- SM is based on QFT.

  It should be considered the leading part of an EFT appropriate to

  $\mathrm{E} \leq \Lambda_{\mathrm{EW}}$.
- SM is successful because it parameterizes all possible interactions permitted by gauge symmetry.

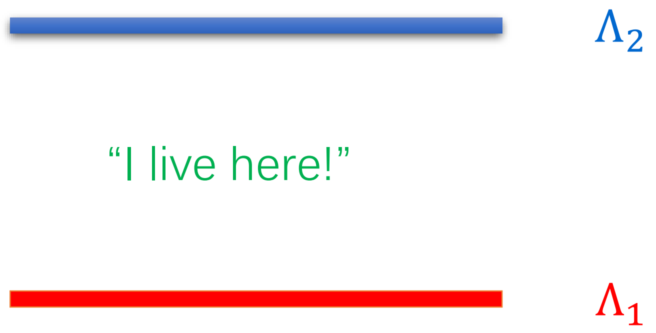
  It is self-contained in that it is “closed” under renormalization.

  — a very important property for

  self-consistency and predictability.

EFT: general discussion

- An EFT is an infinite tower of effective interactions organized by their relative importance.
- Given an accuracy expected for a measurement, only a finite number of effective interactions are important, which are also self-contained in a similar sense as in a renormalizable theory.
- An EFT defined in an energy range $\Lambda_1 < E < \Lambda_2$ is always a low-energy EFT relative to Λ_2.



EFT: general discussion

Three essential elements to specify an EFT:

- Dynamical degrees of freedom.

  — *what are prepared and produced?*

- Symmetries as a guiding principle for constructing interactions.

  — *most sacred are gauge symmetries and dynamically broken symmetries*

- A power counting rule telling what's more important.

  — *low-energy EFT: importance decreases with increasing power of*

  p/Λ_2 *in amplitude* $\leftrightarrow$ ∂/Λ_2 *in Lagrangian*

→ ✓ to establish a basis of effective interactions/operators at each order in low-energy expansion;
✓ to renormalize them to improve perturbation calc, i.e., RGE

EFT： general discussion

- Usually, the characteristic scale of a physical process lies well below the scale at which the mechanism for the process occurs.

  — *a sequence of EFTs is required to connect data with physical origin*

→ matching is required at the boundary of two neighboring EFTs to connect them

- Two types of matching:

✓Strong dynamics involved

— completely new dynamical DoFs appear,

e.g., chiral symmetry breaking in QCD at Λ_χ

✓Perturbative interactions only

— from $\mu > \Lambda_2$ to $\mu < \Lambda_2$, integrate out heavy fields of mass $O(\Lambda_2)$.

How EFT works: $K^- \to \pi^+ l^- l^-$

The process occurs at $\mu \sim 10^2$ MeV.

It violates lepton number

— its mechanism is new phys at $\mu \gg \Lambda_{EW}$

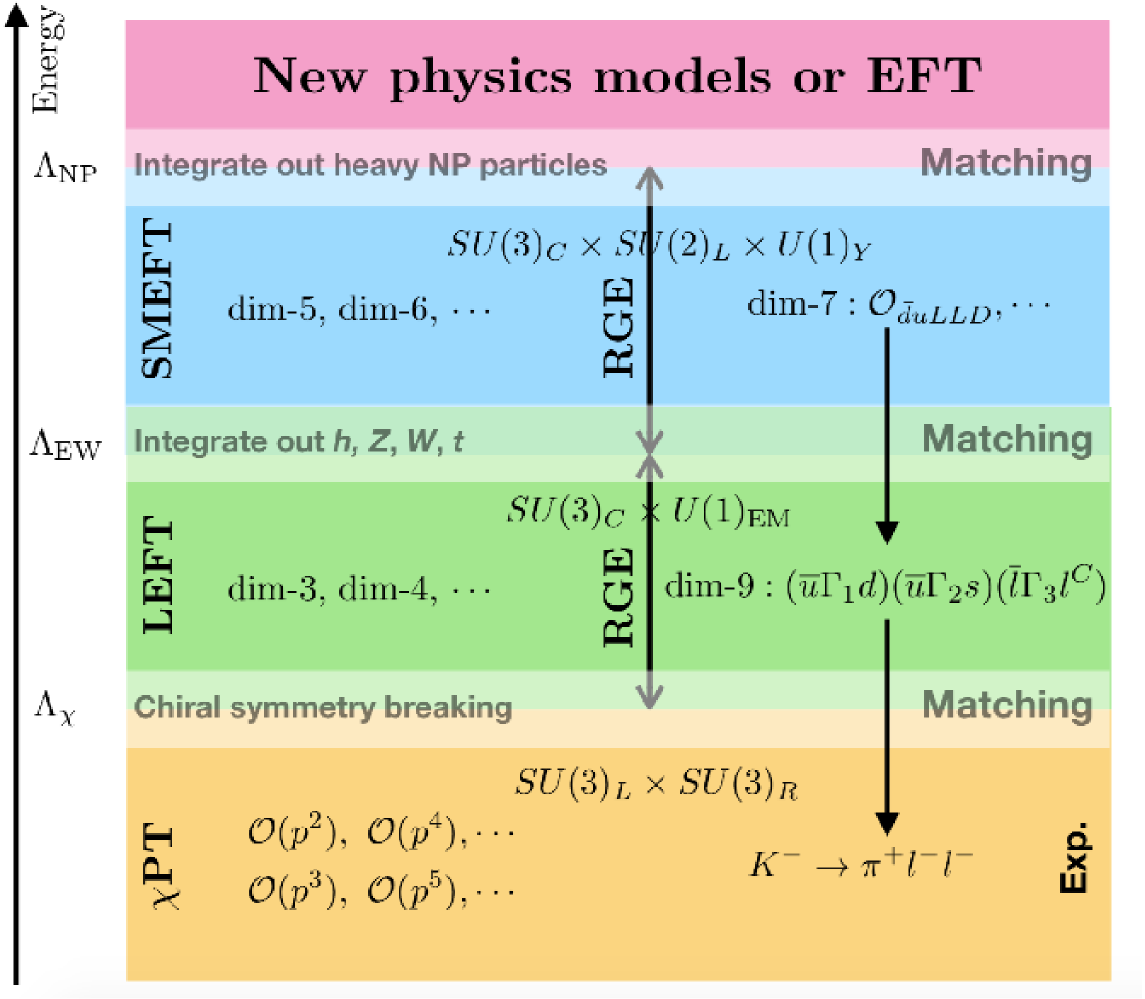
→ a sequence of EFTs required to connect them

- A sequence of EFTs: SMEFT, LEFT, χPT

✓Bases of operators and RGE in each EFT;

✓Matching between SMEFT and LEFT, and between LEFT and χPT.

- Matching between SMEFT and your desired new phys model.



How EFT works: $K^- \to \pi^+ l^- l^-$

Symbolically,

LNV Wilson coefficient at Λ_{NP}

$\mathscr{A}(K^- \to \pi^+ l^- l^-)$ is a sum of products:

$$\Downarrow$$

$$f_\chi R_\chi \quad \otimes \quad R_{\text{LEFT}} \quad \otimes \quad R_{\text{SMEFT}} \quad \otimes \quad C_{\text{LNV}}(\Lambda_{\text{NP}})$$

$$\Uparrow \qquad \Uparrow \qquad \Uparrow$$

matching between	matching between	matching between
χPT and LEFT	LEFT and SMEFT	SMEFT and NP

f_χ: χPT amplitude with low-energy strong constants (expt's, lattice, etc)

R_χ: RGE in χPT, $\Lambda_\chi \to m_K$

R_{LEFT}: RGE in LEFT, $\Lambda_{\text{EW}} \to \Lambda_\chi$

R_{SMEFT}: RGE in SMEFT, $\Lambda_{\text{NP}} \to \Lambda_{\text{EW}}$

Standard model EFT (SMEFT)

Defined between Λ_{NP} and Λ_{EW}:

- Dynamical degrees of freedom (DoFs) restricted to SM fields;
- Symmetries – $SU(3)_C \times SU(2)_L \times U(1)_Y$, no L or B conservation requirement etc;
- Power counting – expansion in p/Λ_{NP}.

SMEFT is an infinite tower of effective interactions involving higher and higher dimensional operators:

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{\mathrm{SM}} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \mathscr{L}_8 + \mathscr{L}_9 + \cdots$$

$\mathscr{L}_{\mathrm{SM}}$: GSW 1960s

$\mathscr{L}_5$: Weinberg 1979

$\mathscr{L}_6$: Buchmuller-Wyler 1986; ……
Grzadkowski et al 2010
– Warsaw basis

$\mathscr{L}_7$: Lehman 2014
Liao-Ma 2016

$\mathscr{L}_8$: Li et al, arXiv: 2005.00008
Murphy, arXiv: 2005.00059

$\mathscr{L}_9$: Li et al, arXiv: 2007.07899
Liao-Ma, arXiv: 2007.08125

Latest: up to dim-12, Harlander et al, 2305.06832

SMEFT: dim-5

- Unique Weinberg operator for Majorana m_ν, $\Delta L = 2$ Weinberg 1979

$$\varepsilon_{ij}\varepsilon_{mn}(L_p^i C L_r^m) H^j H^n$$

L : LH lepton doublet
H : Higgs doublet
i, j, m, n : SU(2) indices
p, r, s, t : flavor indices

- 1-loop RGE Babu et al 1993, Antusch et al 2001
- Responsible for "standard mass mechanism" for nuclear neutrinoless double beta ($0\nu\beta\beta$). … Cirigliano et al 2017, 2018
- No other interesting phys.

SMEFT: dim-6

- Long history on basis of operators.

  Started with Buchmüller-Wyler 1986,

  Corrected and improved by efforts by many groups,

  Culminated with Warsaw basis Grzadkowski et al 2010 –

- 63 operators $\begin{cases} 59:\ \Delta B = \Delta L = 0 \\ 4:\ \Delta B = \Delta L = 1 \end{cases}$

  without counting flavors (easy with trivial flavor relations) and Hermitian conjugate.

- 1-loop RGE by UC San Diego group in 2013, 2014 Barcelona group in 2013

- Rich phenomenology, especially for LHC phys, vast literature skipped

  Commonly quoted proton decay: $p \to e^{+}\pi^{0}$

SMEFT: dim-7

- Early partial analysis by Weinberg 1980 Weldon-Zee 1980
- 1st systematic analysis by Lehman 2014
- Final answer by Liao-Ma 2016:

  18 operators = $12\ (\Delta B = 0,\ \Delta L = 2) + 6\ (-\Delta B = \Delta L = 1)$

  Flavors not counted above; but must be done for applications –

  Nontrivial flavor relations first appear at dim 7 – involving Yukawas Liao-Ma 2019
- Consistent with independent counting by Hilbert series approach Henning et al 2015.
- 1-loop RGE Liao-Ma 2016 Liao-Ma 2019
- Phenomenology limited to L- (and B-) violating phys:

  unusual proton decay $p \to \nu\pi^+$ Liao-Ma 2016

  Liao et al, 2019,2020,2021

  various long- and short-range contri. to $0\nu\beta\beta$, $M_1^- \to M_2^+ l^- l^-$, $\tau^- \to l^+ M_1^- M_2^-$, etc

  ⋯ Cirigliano et al 2017, 2018, ⋯, Feng et al 2019

SMEFT: dim-8

- Many independent operators: Li et al, 2020; Murphy, 2020

  mostly conserve L and B, others break $\Delta B = \Delta L = 1$
- RGE done for purely bosonic operators: Chala et al, 2021; Bakshi et al, 2022
- Phenomenology partly explored, mainly with bosonic operators:

  electroweak precision data, triple gauge couplings, diboson production:

  Degrande and Li, 2023; Corbett et al, 2023

SMEFT: dim-9

- Basis of complete and independent operators established; 2 studies consistent

  Li et al, 2020; Liao-Ma, 2020

- Number of terms in $\mathscr{L}_9$: Number of operators with 3 generations:

	Number of terms in $\mathscr{L}_9$	Number of operators with 3 generations
$L = \pm 2,\ B = 0$:	384	44874
$L = 0,\ \ B = \pm 2$:	10	2862
$L = \pm 3,\ B = \pm 1$:	4	486
$L = \mp 1\ \ B = \pm 1$:	236	42234

  most violate both $L \pm B$ except for the last group which conserves $L + B$.

- Renormalization to be done
- Phenomenology partly done:

  nuclear $0\nu\beta\beta$ decays, neutron-antineutron oscillation, rare nucleon decays

SMEFT: higher-dim operators less important?

- Generally yes, barring one caveat.
- L- or B-violating effects are much smaller than conserving effects

  $\rightarrow$ L or B violation should originate at a higher scale

  $\rightarrow$ Wilson coeffs. for operators of different L or B pattern cannot be compared in a model-independent manner.
- General results on L or B pattern in SMEFT:

Kobach, 2016

✓ $(\Delta B - \Delta L)/2$ and dimension d of an operator share the same odd or even nature.

✓ Imposing flavor symmetry postpones occurrence of L or B violation at a higher d:

L or B violation impossible for $d < 9$ except for $|\Delta L| = 2$;

Helset and Kobach, 2019

e.g., proton decay severely suppressed:

$d = 9$: 2 operators involve $3l3q$ but necessarily with c or t $\rightarrow$ tree level impossible

$d = 10$: 4-body decay with $\Delta B = -\frac{\Delta L}{3} = 1$; $d = 11$: 3-body decay with $\Delta B = \frac{\Delta L}{3} = 1$

Low-energy EFT

When $E < \Lambda_{\rm EW}$, electroweak SSB manifests itself.

Heavy particles $(h, W^{\pm}, Z^0, t)$ of mass $\sim\Lambda_{\rm EW}$ are integrated out $\to$ LEFT

Defined between $\Lambda_{\rm EW}$ and $\Lambda_\chi \sim 1\ \mathrm{GeV}$:

- Dynamical DoFs = SM fields other than above heavy ones;
- Symmetries – $SU(3)_C \times U(1)_Q$;
- Power counting – expansion in $p/\Lambda_{\rm EW}$.

Actually well applied, e.g., in b phys, although not studied systematically.

Jenkins et al, 2017 ($\mathscr{L}_5$, $\mathscr{L}_6$); Murphy, 2020 ($\mathscr{L}_8$); $|\Delta L| = 2$ sector: Liao et al, 2019 ($\mathscr{L}_9$)

$$\mathscr{L}_{\rm LEFT} = \mathscr{L}_\nu + \mathscr{L}_{\rm QED} + \mathscr{L}_{\rm QCD} + \mathscr{L}_5 + \mathscr{L}_6 + \mathscr{L}_7 + \mathscr{L}_8 + \mathscr{L}_9 + \cdots$$

Liao et al, 2020 ($\mathscr{L}_7$); Li et al, 2020 ($\mathscr{L}_8$, $\mathscr{L}_9$)

Important: combined power counting in $1/\Lambda_{\rm EW}$ and $1/\Lambda_{\rm NP}$

LEFT: RGE and matching to SMEFT

To get prepared for analysis of precision measurements at low energy,
both RGE in LEFT and matching between LEFT and SMEFT are demanded.

- tree-level up to dim-6 operators in both EFTs Jenkins et al, 1709.04486
- tree-level up to dim-7 operators in both EFTs Liao et al, 2005.08013
- tree-level up to dim-8 operators in both EFTs: partly done, Hamoudou et al, 2207.08856
  by either setting $H \to \mathrm{vev}$ or integrating out $h, W^{\pm}, Z$ and keeping p-indept terms
- one-loop up to dim-6 operators in both EFTs Dekens and Stoffer, 1908.05295
  delicacy appears with evanescent operators in DR

- one-loop RGE for dim-6 operators Jenkins et al, 1711.05270
- one-loop QCD RGE for dim-9 $|\Delta L| = 2$ operators involving $2l$ Liao et al, 1909.06272
  for dim-9 $|\Delta L| = 2$ operators specific to $0\nu\beta\beta$ Cirigliano et al, 1806.02780
  QCD RGE for dim-9 operators in $n\bar{n}$ oscillation: one-loop Caswell et al, PLB122
  two-loop Buchoff and Wagman, 1506.00647

Matching NP to SMEFT

- EFT is useful not only for bottom-up but also for top-down approach.
- Assuming NP lives at $\Lambda_{\mathrm{NP}} \gg \Lambda_{\mathrm{EW}}$ and all new particles have mass $\gg \Lambda_{\mathrm{EW}}$, its low-energy effects on SM particles can be incorporated by integrating out new particles
  — matching NP and SMEFT at $\mu = \Lambda_{\mathrm{NP}}$
- Matching in perturbation theory is a double-expansion:
  in inverse powers of heavy mass $\rightarrow$ higher-dim operators in SMEFT
  in loop expansion $\rightarrow$ Wilson coeffi. a series in couplings
- Matching at tree level:
  substituting in L_{NP} EoMs for heavy particles and expanding in inverse masses
  $\rightarrow$ tree level Wilson coeffi.

Matching NP to SMEFT at one loop

- Past years have witnessed significant progress in 1-loop matching based on:

✓ Functional approach augmented by covariant derivative expansion

✓ Loop integration by method of regions ……; Cohen-Lu-Zhang, 2011.02484

- Features:

✓ The result is directly the 1-loop contribution to $\mathcal{L}_{\rm SMEFT}$ whose operators and Wilson coeffs. are obtained simultaneously.

✓ One only has to work with NP theory without computing in SMEFT!

Examples of 1-loop functional matching

Obtain 1-loop contribution tc $\mathcal{L}_{\rm SMEFT}$ by integrating out heavy

- ✓ superpartners in MSSM Henning et al, 2014; ⋯
- ✓ singlet or triplet scalar ⋯; Jiang et al, 2018; ⋯ ⋯;.Zhang, 1610.00710
- ✓ vectorlike fermions Huo, 1506.00840
- ✓ triplet vector boson Brivio et al, 2108.01094
- ✓ fermions or scalars in type-I, -II, and –III neutrino seesaw models Zhang-Zhou, 2107.12133 Du et al, 2201.04646 Li et al, 2201.05082
- ✓ dark-sector particles in scotogenic neutrino mass models Liao-Ma, 2210.04270
- ✓ ……

Ongoing activities not covered here

- In the existence of new particles of mass $< \Lambda_{\mathrm{EW}}$, SMEFT/LEFT has to be enlarged to include them as dynamical DoFs:
  - ✓ νSMEFT, with sterile neutrinos;
  - ✓ DM EFT, including axion-like particles or particles of various spin, with or without DM discrete symmetry.
- Higgs EFT vs SMEFT:
  Is *the Higgs boson* completely responsible for electroweak SSB?
  Do new particles gain mass from electroweak SSB?
- Various extensions of Hilbert series to count operators in theory with nonlinearly realized symmetry, with supersymmetry, with definite CP, etc.
- Evanescent operators in operator reduction and matching at one loop, and in RGE at two loops.
- Phenomenology especially at colliders and global fitting.
- On-shell methods, positivity bounds at tree level → one loop