Hybrids from lattice QCD

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Outline

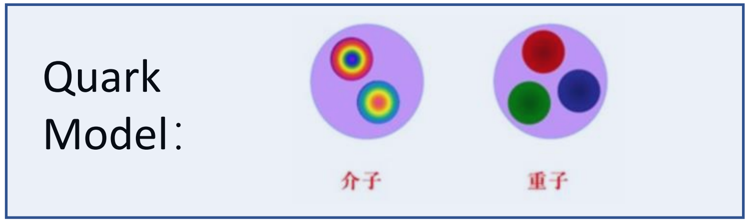
I. Motivation
II. Light $0^{+}1^{-+}$ hybrid η_1 in the J/ψ radiative decay
III. Charmoniumlike 1^{-+} hybrid η_{c1} decay
V. Summary and perspectives

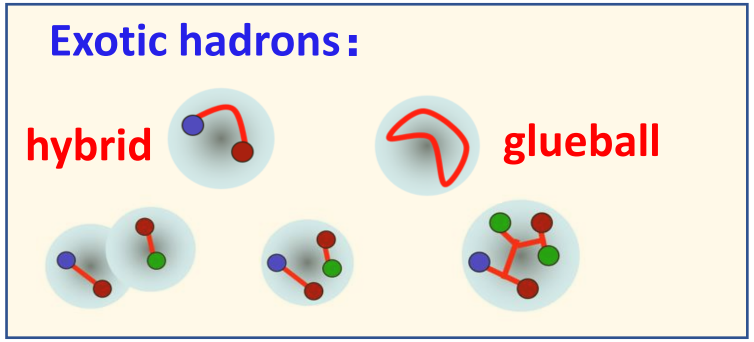
I. Introduction

1. Conventional and exotic light hadrons

- Quark model sort light hadrons into
  $\bar{q}q$ multiplets (mesons)
  qqq multiplets (baryons)

- Gluons are also fundamental degrees of freedom of QCD, therefore there may exist glueballs $(gg\cdots)$ and hybrids $(\bar{q}qg)$.





2. Experimental candidates for hybrids

- Isovector 1^{-+} states: $\pi_1(1400)$, $\pi_1(1600)$, $\pi_1(2105)$
- Isoscalar 1^{-+} state: $\eta_1(1855)$
- Charmoniumlike 1^{-+} hybrid η_{c1}: counterpart of π_1 and η_1, no evidence yet!

3. Masses of hybrids predicted by Lattice QCD

$$m_{\pi_1} \sim 1.8-2.0\ GeV, \quad m_{\eta_1} \sim 2.0-2.3\ GeV, \quad m_{\eta_{c1}} \sim 4.2-4.4\ GeV$$

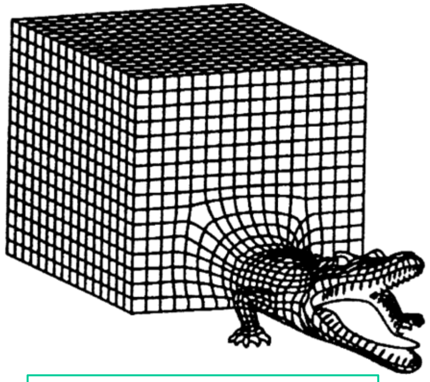
4. Formalism of Lattice QCD

- **Path integral quantization on finite Euclidean spacetime lattices**

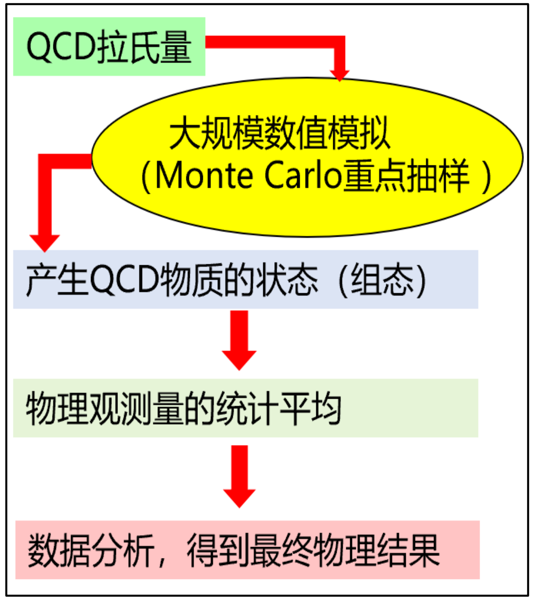
$$Z = \int DAD\psi D\bar{\psi}\, e^{iS[A,\psi,\bar{\psi}]} \rightarrow \int DU \det M[U]\ e^{-S_g[U]}$$

$$\langle \hat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{Z}\int DU\ \det M[U]\ e^{-S_g[U]}\ \mathcal{O}[U]$$

Green's functions **Field product**



Spacetime discretization



- **Very similar to a statistical physics system**
- **Monte Carlo simulation——importance sampling according to** $\mathcal{P}[U] \propto \det M[U]\ e^{-S_g[U]}$

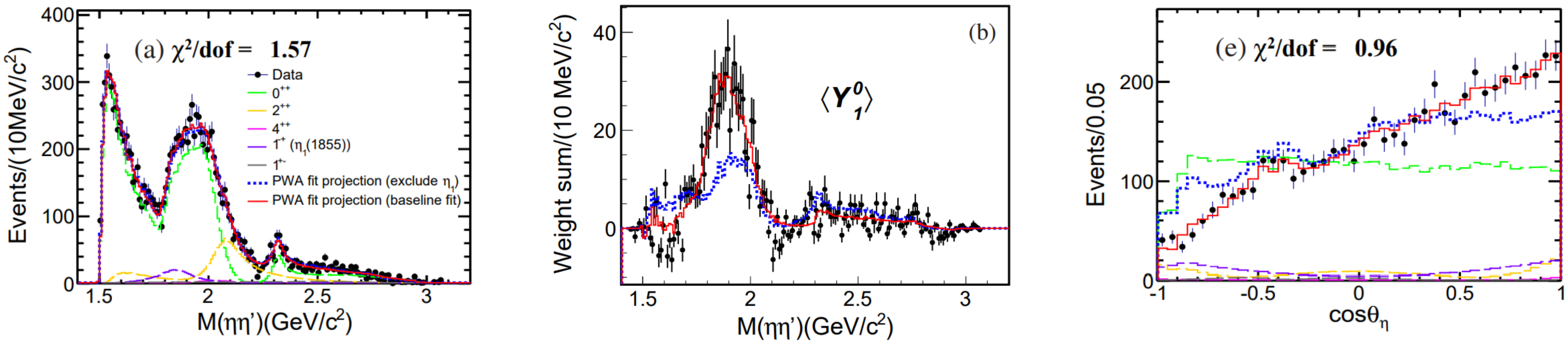
Gauge ensemble: $\{U_i(\text{spacetime}), i = 1, \dots, N\}$ ➡ $\langle \hat{\mathcal{O}}[U,\psi,\bar{\psi}]\rangle = \frac{1}{N}\sum_i \mathcal{O}[U_i] + O\left(\frac{1}{\sqrt{N}}\right)$

III. Light $0^{+}1^{-+}$ hybrid η_1 in J/ψ radiative decays

1. $\eta_1(1855)$ ($I^G J^{PC} = 0^{+}1^{-+}$) observed by BESIII

(BESIII, Phys. Rev. Lett. 129, 192002 (2022), arXiv:2202.00621(hep-ex))

- Partial wave analysis of the process $J/\psi \to \gamma\eta\eta'$



- Resonance parameters of $\eta_1(1855)$: $m_{\eta_1} = 1855 \pm 9^{6}_{-1}$ MeV, $\Gamma_{\eta_1} = 188 \pm 18^{+3}_{-8}$ MeV
  Combined branching fraction: $\mathrm{Br}(J/\psi \to \gamma\eta_1 \to \gamma\eta\eta') = (2.70 \pm 0.41^{+0.16}_{-0.35}) \times 10^{-6}$

- The first candidate for isoscalar 1^{-+} hybrid.

- Theoretical prediction of η_1 production rate in J/ψ radiative decays?

2. Lattice QCD calculation of partial width of $J/\psi \to \gamma X$

- **Radiative decay width:** $$\Gamma(i \to \gamma f) = \frac{1}{2J_i + 1} \frac{1}{32\pi^2} \int d\Omega_q \frac{|\vec{q}|}{M_i^2} \sum_{r_i, r_j, r_\gamma} \left| \mathcal{M}_{r_i, r_j, r_\gamma} \right|^2$$
- **Transition amplitudes:** $$\mathcal{M}_{r_i, r_f, r_\gamma} = \epsilon_\mu^*(q, r_\gamma) \langle f(p_f, r_f) | j_{em}^\mu(0) | i(p_i, r_i) \rangle$$ $$Q^2 = -(p_f - p_i)^2$$
- **Multipole decomposition:** $$\langle f(p_f, r_f) | j_{em}^\mu(0) | i(p_i, r_i) \rangle = \sum_k \alpha_k^\mu(p_f, p_i, \epsilon_f, \epsilon_i) F_k(Q^2)$$
- **Decay width in terms of the form factors:** $$\Gamma(i \to \gamma f) \propto \sum_k F_k^2(0)$$
- **So the major task is to calculate the matrix elements, which can be derived from the three-point functions on the lattice**

$$\Gamma_3^{i\mu j}(\vec{p}_f, \vec{q}; t_f, t) = \sum_{\vec{y}} e^{i\vec{q}\cdot\vec{y}} \langle \Omega | \mathcal{O}_f^i(\vec{p}_f, t_f) j_{em}^\mu(\vec{y}, t) \mathcal{O}_i^+(\vec{p}_i, 0) | \Omega \rangle$$

3. Lattice setup

- **Anisotropic lattice** ($\xi = \frac{a_s}{a_t} \gg 1$): **heavy particles ($J/\psi$) involved, compromise of the resolution in the time direction and the computational expenses.**

- **Large statistics: decay process takes place through disconnected diagrams (OZI suppressed)**
  A large statistics is mandatory for good S/N.

- **Lattice actions: Tadpole improved Symanzik's gauge action (C. Morningstar, PRD60(1999)034509)**
  Tadpole impoved Clover action for $N_f = 2$ degenerate u, d sea quarks
  and also the valence charm quark.

$L^3 \times T$	β	a_t^{-1}(GeV)	ξ	m_π(MeV)	N_{cfg}
$16^3 \times 128$	2.0	6.894(51)	~ 5.3	348.5(1.0)	6991

Jiang et al,
Phys.Rev.D 107 (2023) 094510

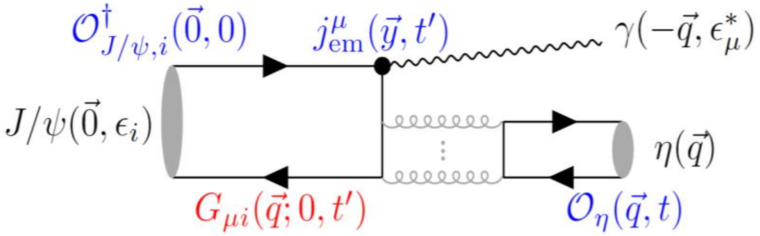
- **Calculation of disconnected diagrams: distillation method (M. Peardon et al. (HSC), PRD80(2009)054506).**

4. Partial width of $J/\psi \to \gamma\eta$ as a calibration (X. Jiang et al., Phys. Rev. Lett. 130, 061901 (2023))

1) Extraction of the form factor $M(Q^2)$

$$\Gamma(J/\psi \to \gamma\eta) = \frac{4\alpha}{27}|p_\gamma|^3 M^2(0)$$

$M(Q^2)$ is defined through the multipole decomposition



$$\langle \eta(p_\eta)|j^\mu_{em}(0)|\psi(p_\psi,\lambda)\rangle = M(Q^2)\epsilon^{\mu\nu\rho\sigma}p_{\psi,\nu}p_{\eta,\rho}\epsilon_\sigma(p_\psi,\lambda)$$

$$Q^2 = -(p_\psi - p_\eta)^2 = \vec{q}^2 - \left(m_\psi - E_\eta(\vec{q})\right)^2$$

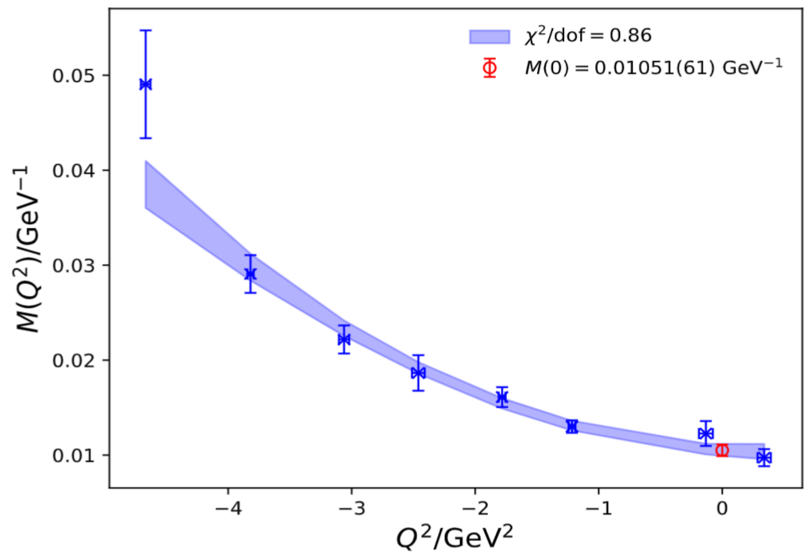
The matrix element is encoded in the three-point function as (for $t \gg t' \gg 0$)

$$\Gamma^{(3)}_{\mu i}(\vec{q};t,t') \approx \frac{Z_\eta(\vec{q})Z^*_\psi}{4VE_\eta(\vec{q})m_\psi}e^{-E_\eta(\vec{q})(t-t')}e^{-m_\psi t'}\sum_\lambda \langle \eta(\vec{q})|j^\mu_{em}|\psi\left(\vec{0},\lambda\right)\rangle \epsilon^*\left(\vec{0},\lambda\right)$$

Thus $M(Q^2)$ is obtained along with the following two-point functions

$$\Gamma^{(2)}_\eta(\vec{q},t) \approx \frac{1}{2E_\eta(\vec{q})V}|Z_\eta(\vec{q})|^2 e^{-E_\eta(\vec{q})t} \qquad \Gamma^{(2)}_\psi(t) \approx \frac{1}{2m_\psi V}|Z_\psi|^2 e^{-m_\psi t}$$

2) Prediction of $\Gamma(J/\psi \to \gamma\eta)$



- Interpolation of $M(Q^2)$ to the on-shell value at $Q^2 = 0$

$$M(Q^2) = M(0) + aQ^2 + bQ^4 + \mathcal{O}(Q^6)$$
$$M(0) = 0.01051(61)\ \text{GeV}^{-1}$$

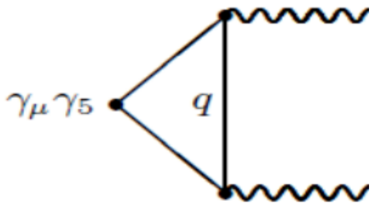
- Partial decay width

$$\Gamma(J/\psi \to \gamma\eta) = \frac{4\alpha}{27}|p_\gamma|^3 M^2(0) = 0.385(45)\ \text{keV}$$
$$\text{Br}(J/\psi \to \gamma\eta) = 4.16(49) \times 10^{-3} \quad (\Gamma_{tot} = 92.6(1.7)\text{keV})$$

This can be compared with the branching fraction already : $\text{Br}(J/\psi \to \gamma\eta') = 5.25(7) \times 10^{-3}$

3) Mechanism behind the large production ratio——$U_A(1)$ anomaly?

$$\partial_\mu j_5^\mu(x) = 2im\, j_5(x) + \sqrt{N_f}\frac{g^2}{32\pi^2}G^a_{\mu\nu}(x)\tilde{G}^{a,\mu\nu}(x) \qquad j_5^\mu = \frac{1}{\sqrt{N_f}}\sum_i \bar{\psi}_i\gamma^\mu\gamma_5\psi_i,$$



$U_A(1)$ anomaly introduces an anomalous coupling between singlet pseudoscalar and gluons.

- Assuming the $U_A(1)$ anomaly dominance for the production of η states (OZI evading)

$SU_I(2)$ case: $M_{N_f=2}(0)(N_f=2) = 0.01051(61)\ \text{GeV}^{-1}$

$SU_F(3)$ case: $M_{N_f=3}(0) = \sqrt{3/2}\, M_{N_f=2}(0) = 0.0129(8)\ \text{GeV}^{-1}$

$\eta - \eta'$ mixing:

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}, \quad m_\eta = 547\ \text{MeV},\ m_{\eta'} = 958\ \text{MeV}$$

$\theta_{qrad} = -11.3°$: $Br(J/\psi \to \gamma\eta) = 0.256(30) \times 10^{-3}$, $Br(J/\psi \to \gamma\eta') = 5.21(62) \times 10^{-3}$

$\theta_{lin} = -24.5°$: $Br(J/\psi \to \gamma\eta) = 1.15(14) \times 10^{-3}$, $Br(J/\psi \to \gamma\eta') = 4.49(53) \times 10^{-3}$

Expt. (PDG2020)

$Br(J/\psi \to \gamma\eta) = 1.11(3) \times 10^{-3}$, $Br(J/\psi \to \gamma\eta') = 5.25(7) \times 10^{-3}$

Agree better!

- A recent lattice study of $\eta(\eta')$ mass and decay constant gives (G. Bali et al., JHEP08(2021)137)

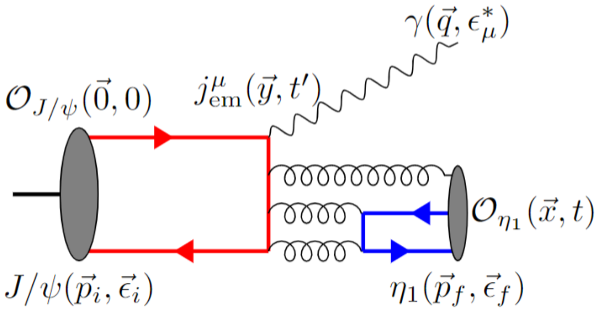
$$a_\eta \propto \langle \Omega | \alpha_s G\tilde{G} | \eta \rangle, \qquad a_{\eta'} \approx \langle \Omega | \alpha_s G\tilde{G} | \eta' \rangle$$

From which the gluonic mixing angle of $\eta - \eta'$ is derived as $\theta_g = -\arctan \frac{a_\eta}{a_{\eta'}} \approx -24(4)°$

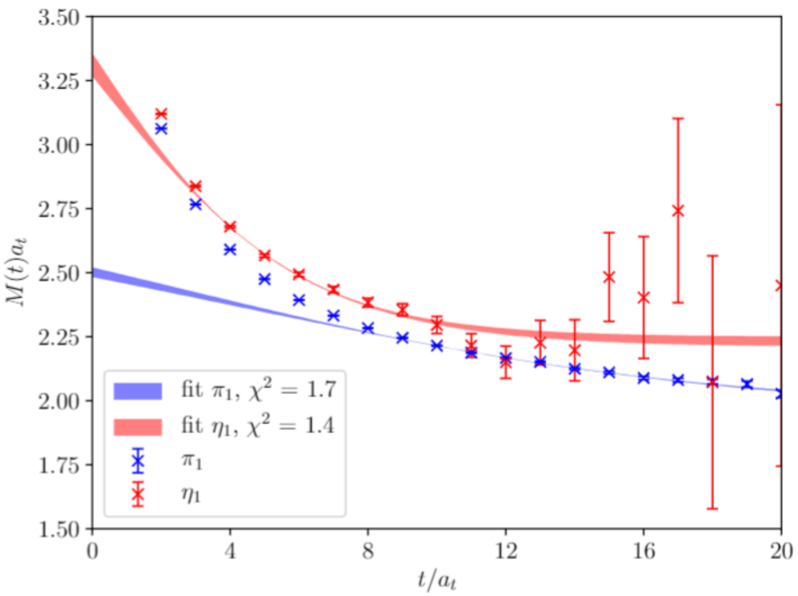
4. Partial width of $J/\psi \to \gamma\eta_1(1^{-+})$ (F. Chen et al., Phys. Rev. D 107, 054511 (2023), arXiv: 2207.04694 (hep-lat))

$$\Gamma(J/\psi \to \gamma\eta_1) = \frac{4\alpha}{27}\frac{|\vec{p}_\gamma|}{2m_\psi^2}\left(M_1^2(0) + E_2^2(0)\right)$$

1) Extraction of the form factors $M_1(Q^2)$ and $E_2(Q^2)$



$$\Gamma^{(3)}_{i\mu j} = \frac{1}{T}\sum_{\tau}^{T}\langle \mathcal{O}^i_{\eta_1}(0, t+\tau) G_{\mu j}(\vec{p}, \vec{p}; t'+\tau, \tau)\rangle$$



Blue: effective mass of isovector π_1
Red: effective mass of isoscalar η_1

$$m_{\pi_1} = 1.950(28)\ \text{GeV}$$

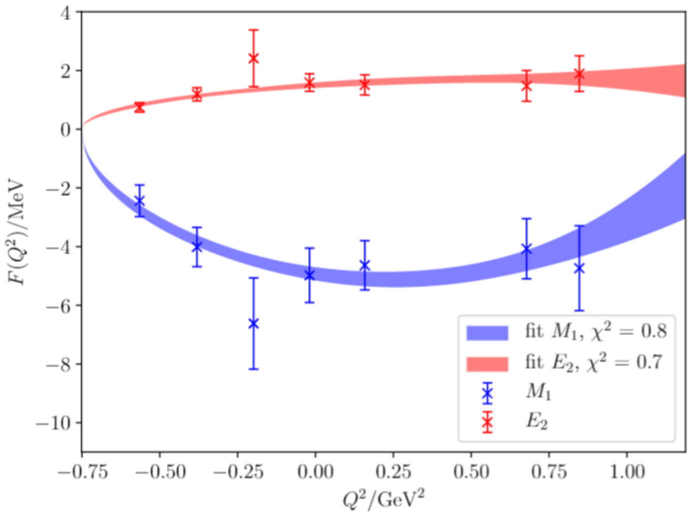
$$m_{\eta_1} = 2.230(39)\ \text{GeV}$$

2) Prediction of the decay width and branching fraction

$$M_1(0) = -4.73(74)\ \text{MeV}, \qquad E_2(0) = 1.18(22)\ \text{MeV}$$

$$\Gamma(J/\psi \to \gamma\eta_1) = \frac{4\alpha}{27}\frac{|\vec{p}_\gamma|}{2m_\psi^2}\left(M_1^2(0) + E_2^2(0)\right)$$

$$\boxed{\Gamma(J/\psi \to \gamma\eta_1) = 2.04(61)\ \text{eV}}$$



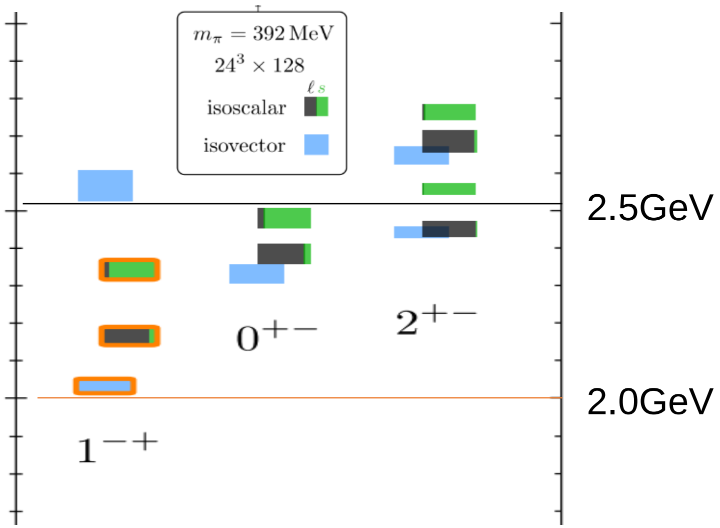
3) $\eta_1^{(l)}$ and $\eta_1^{(h)}$ in the SU(3) case

There are two mass eigen states $\eta_1^{(l)}, \eta_1^{(h)}$

$$\begin{pmatrix} |\eta_1^{(l)}\rangle \\ |\eta_1^{(h)}\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} |l\bar{l}\rangle \\ |s\bar{s}\rangle \end{pmatrix} \qquad \begin{pmatrix} |\eta_1^{(l)}\rangle \\ |\eta_1^{(h)}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\eta_1^{8}\rangle \\ |\eta_1^{1}\rangle \end{pmatrix}$$

$$\boxed{\theta = 54.7^\circ - \alpha}$$

Lattice calculation: $\alpha \approx 22.7(2.1)^\circ$ or $\theta = 32.0(2.1)^\circ$



J. Dudek et al. (HSC),
PRD 88(2013) 094505 (2013)

Intuitively, gluons in J/ψ radiative decay couple to flavor singlets. Therefore

$$\Gamma\left(J/\psi \to \gamma\eta_1^{(l)}\right) = \sin^2\theta\ \Gamma\left(J/\psi \to \gamma\eta_1^1\right)\chi^{(l)}$$
$$\Gamma\left(J/\psi \to \gamma\eta_1^{(h)}\right) = \cos^2\theta\ \Gamma\left(J/\psi \to \gamma\eta_1^1\right)\chi^{(h)}$$

$$\chi^{(x)} = \frac{m_{\eta_1}^2 \left|\vec{p}_\gamma\left(\eta_1^{(x)}\right)\right|^3}{m_{\eta_1^{(x)}}^2 \left|\vec{p}_\gamma(\eta_1)\right|^3}$$

On the other hand, $\eta\eta'$ only appears as a flavor octet, so $\eta_1^{(h,l)} \to \eta\eta'$ must take place through its octet component:

$$\left\langle \eta\eta'|H_I|\eta_1^{(l)}\right\rangle = \cos\theta \left\langle \eta\eta'|H_I|\eta_1^{(8)}\right\rangle \equiv g\cos\theta \left|\vec{k}^{(l)}\right|$$
$$\left\langle \eta\eta'|H_I|\eta_1^{(h)}\right\rangle = \sin\theta \left\langle \eta\eta'|H_I|\eta_1^{(8)}\right\rangle \equiv g\sin\theta \left|\vec{k}^{(h)}\right|$$

$$\Gamma\left(\eta^{(l)} \to \eta\eta'\right) \propto g^2 \frac{\left|\vec{k}^{(l)}\right|^3}{m_{\eta_1^{(l)}}^2}\cos^2\theta$$

$$\Gamma\left(\eta^{(l)} \to \eta\eta'\right) \propto g^2 \frac{\left|\vec{k}^{(h)}\right|^3}{m_{\eta_1^{(h)}}^2}\sin^2\theta$$

$$r = \frac{\mathrm{Br}\left(J/\psi \to \gamma\eta_1^{(l)} \to \gamma\eta\eta'\right)}{\mathrm{Br}\left(J/\psi \to \gamma\eta_1^{(h)} \to \gamma\eta\eta'\right)} = \frac{\chi^{(l)}\left|\vec{k}^{(l)}\right|^3 m_{\eta_1^{(h)}}^2}{\chi^{(h)}\left|\vec{k}^{(h)}\right|^3 m_{\eta_1^{(l)}}^2}\frac{\Gamma_{\eta_1^{(h)}}}{\Gamma_{\eta_1^{(l)}}} \sim \frac{\Gamma_{\eta_1^{(h)}}}{\Gamma_{\eta_1^{(l)}}}\mathcal{O}(1)$$

BESIII observation: $\mathbf{Br}(J/\psi \to \gamma\eta_1(1855) \to \gamma\eta\eta') = (2.70 \pm 0.41^{+0.16}_{-0.35}) \times 10^{-6}$

If $\eta_1(1855)$ is the $\eta_1^{(l)}$, then

$$\Gamma(J/\psi \to \gamma\eta_1(1855)) = (2.0 \pm 0.7)\ \mathrm{eV}$$
$$\mathrm{Br}(J/\psi \to \gamma\eta_1(1855)) = (2.1 \pm 0.7) \times 10^{-5}$$
$$\mathrm{Br}(\eta_1(1855) \to \eta\eta') = (13 \pm 5)\%$$

If $\eta_1(1855)$ is the $\eta_1^{(h)}$, then

$$\Gamma(J/\psi \to \gamma\eta_1(1855)) = (5.0 \pm 1.6)\ \mathrm{eV}$$
$$\mathrm{Br}(J/\psi \to \gamma\eta_1(1855)) = (5.4 \pm 1.8) \times 10^{-5}$$
$$\mathrm{Br}(\eta_1(1855) \to \eta\eta') = (5.0 \pm 1.9)\%$$

- **Hint of the existence of the second state ----**

conventional quantum numbers are found. The most significant additional contribution (4.4σ) comes from an exotic 1^{-+} component around 2.2 GeV. Changing the J^{PC} assignment

M. Ablikim [BESIII], Phys. Rev. D 072012 (2022)

- **The production rate and the width of this state are desired!**

III. Charmoniumlike 1^{-+} hybrid η_{c1} decay

1. Hybrids in the flux-tube model (Isgur & Paton, Phys. Rev. D 31 (1985) 2910)

- A vibrational string along the $Q\bar{Q}$ axis.
- The picture is originated from the lattice QCD formulation.

$$H = -\frac{1}{2\mu}\frac{\partial^2}{\partial r^2} + \frac{L(L+1)-\Lambda^2}{2\mu r^2} + E^1(r)$$

$$E^1(r) = -\frac{4\alpha_s}{3r} + c + br + \frac{\pi}{r}\left(1 - e^{-fb^{1/2}r}\right)$$

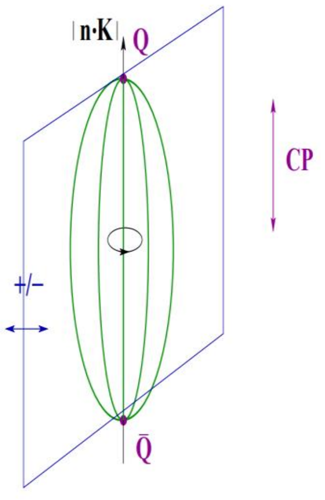


Figure 1: Quarkonium hybrid symmetries.

In analogy with di-atomic molecule:

Λ_η^ϵ ——cylinder symmetry along the $Q\bar{Q}$ axis

$|\vec{n}\cdot K| = 0, 1, 2, \cdots \Rightarrow \Lambda = \Sigma, \Pi, \Delta, \cdots$

η: CP of the flux tube, $CP = +, \ -\ (g, u)$

ϵ: reflection symmetry with respect to the plane including the $Q\bar{Q}$ axis

Spin of the meson : $J = L + S$

$L = L_{Q\bar{Q}} + K$

$L_{Q\bar{Q}}^2 = L^2 - 2\Lambda^2 + K^2 + \cdots$

PC of the meson :

$P = \epsilon(-1)^{L+\Lambda+1}, C = \eta\epsilon(-1)^{L+\Lambda+S}$

TABLE I. Some low-lying meson hybrids.

Flavor	J^{PC} or J^P	Mass (GeV) for $f=1$	$\frac{dm}{df}$ (GeV)	Δm[a] (GeV)	m[b] (GeV)
$I=1$	$2^{\pm\mp},1^{\pm\mp},0^{\pm\mp},1^{\pm\pm}$	1.67	0.08	0.19	~1.9
$I=\frac{1}{2}$	$2^{\pm},1^{\pm},0^{\pm},1^{\pm}$	1.80	0.10	0.17	~2.0
$I=0$ $\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)$	$2^{\pm\mp},1^{\pm\mp},0^{\pm\mp},1^{\pm\pm}$	1.67	0.08	0.19	~1.9
$I=0$ $(s\bar{s})$	$2^{\pm\mp},1^{\pm\mp},0^{\pm\mp},1^{\pm\pm}$	1.91	0.12	0.14	~2.1
$c\bar{c}$	$2^{\pm\mp},1^{\pm\mp},0^{\pm\mp},1^{\pm\pm}$	4.19	0.18	0.06	~4.3
$b\bar{b}$	$2^{\pm\mp},1^{\pm\mp},0^{\pm\mp},1^{\pm\pm}$	10.79	0.28	0.02	~10.8

[a]Contribution to the mass from nonadiabatic effects, taken from Ref. 14.
[b]A "best guess" based on the previous columns.

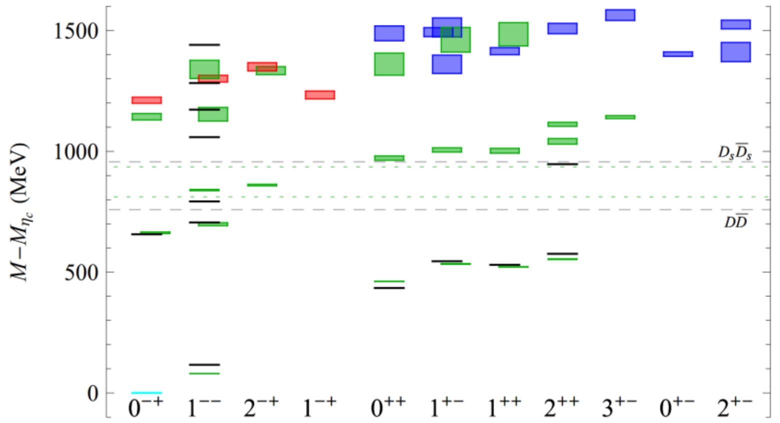
K. Juge et al., Phys. Rev. Lett. 82, 4400 (1999)

Isgur & Paton, Phys. Rev. D 31 (1985) 2910

2. Lattice prediction of the spectrum of charmoniumlike states
(L. Liu [HSC Collab.], JHEP 07 (2012) 126)

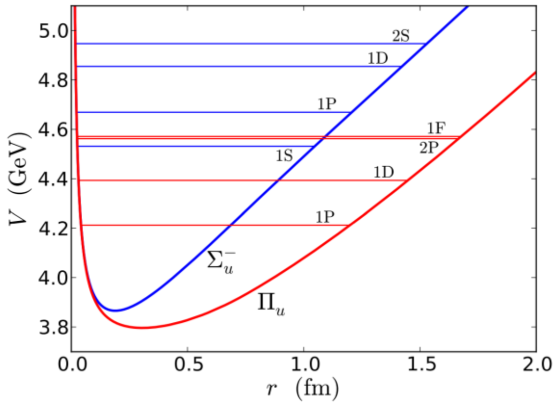
- Super-multiplet $((0,1,2)^{-+}, 1^{--})$ observed around 4.2 GeV
- Consistent with the phenomenological expectation.
- Non-consistence appears in the spectrum of excited states
  Flux-tube: $\Delta m(2P-1P) \sim 0.38$ GeV
  QLQCD: $\Delta m(2P-1P) \sim 1.2-1.3$ GeV

- The internal stucture of these hybrids reflected by the BS wave functions seems different from the flux-tube picture.

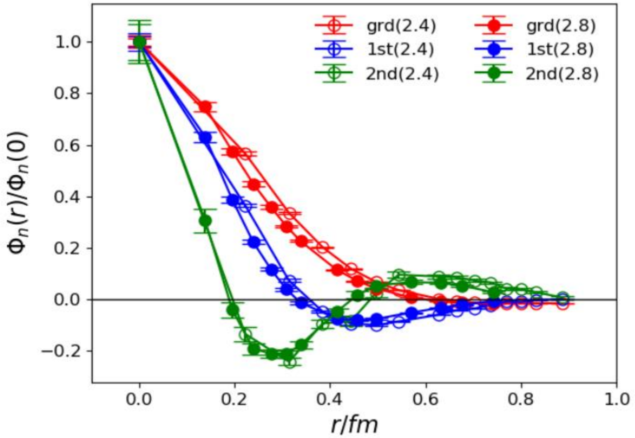


$m_{\eta_{c1}} \approx 4.23$ GeV $(N_f = 2+1)$

L. Liu [HSC Collab.], JHEP 07 (2012) 126



E. Braaten et al.,
Phys. Rev. D 90 (2014) 014044



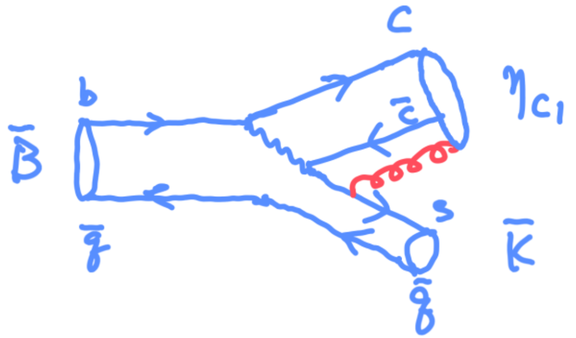
#node	$m(1^{--})$ (GeV)	$m(0^{-+})$ (GeV)	$m(1^{-+})$ (GeV)	$m(2^{-+})$ (GeV)
0	3.109(5)	3.010(4)	-	-
0	3.703(82)	3.672(76)	-	-
0	4.591(69)	4.551(63)	4.309(2)	4.419(3)
1	5.460(31)	5.393(28)	5.693(12)	5.779(12)
2	8.226(99)	8.286(109)	7.661(31)	7.708(29)

Y. Ma et al., Chin. Phys. C 45 (2021) 093111

3. Possible production processes in experiments

- η_{c1} production on e^+e^- collider $e^+e^- \to \psi(nS) \to \gamma\eta_{c1}$ $(\psi(4415)\ \ etc.)$
- η_{c1} production in B meson decays (LHCb and Belle II)

$B \to \bar{K}X, \quad X = X(3872), Z_c(4430), Z_c(3900), \quad etc.$



4. η_{c1} decay modes

Flux-tube model selection rule:

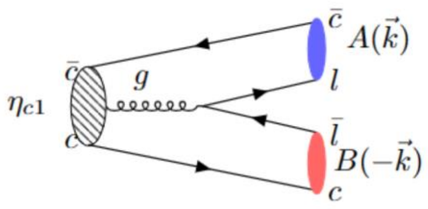
1) Modes of two S-wave mesons are suppressed, SP-modes are favored.
2) Modes of two identical mesons are prohibited.

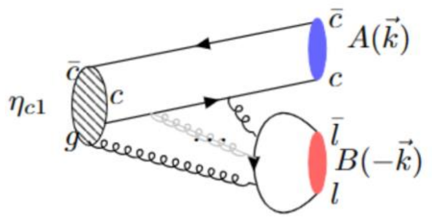
$$\langle AB|H_I|H\rangle \propto \int d^3\vec{r}\ (\phi_H(\vec{r})\cdots)\int_0^1 d\xi \cos(\xi\pi)\ \phi_A(\xi\vec{r})\phi_B((1-\xi)\vec{r})$$

(P. Page et al., Phys. Rev. D 59 (1999) 034016)

Open-charm and closed-charm decay modes

1) SP modes: $D_1(2420)\bar{D},\ \chi_{c1}\eta(\eta')$
2) SS modes: $D^*\bar{D},\ D^*\bar{D}^*,\ \eta_c\eta(\eta'), J/\psi\omega(\phi)$





5. Lattice methodology (C. McNeile & C. Michael, Phys. Lett. B 556 (2003) 177)

For the two-body decay $\eta_{c1} \to AB$, in the space spanned by $|\eta_{c1}\rangle$ and $|AB\rangle$ $(m_{\eta_{c1}} > E_{AB})$

$$|\eta_{c1}\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad |AB\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

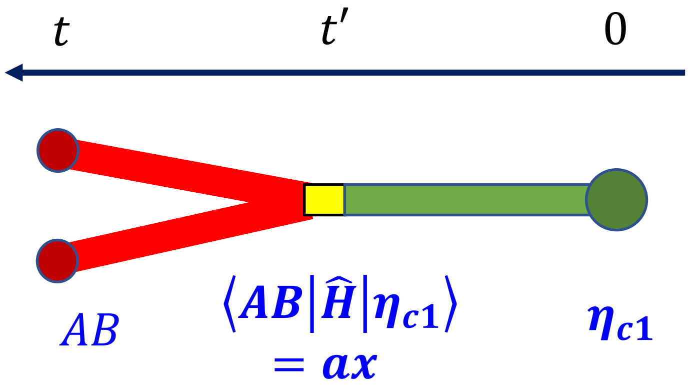
$$\hat{H} = \begin{pmatrix} m_{\eta_{c1}} & x \\ x & E_{AB}\end{pmatrix}$$

$$\hat{T}(a) = e^{-a\hat{H}} = e^{-a\bar{E}} \begin{pmatrix} e^{-a\Delta/2} & ax \\ ax & e^{a\Delta/2}\end{pmatrix}$$

$$\bar{E} = \frac{m_{\eta_{c1}} + E_{AB}}{2}, \qquad \Delta = m_{\eta_{c1}} - E_{AB}$$

The transition takes place at any t' between 0 and t:

$$\langle \Omega | \mathcal{O}_{AB} | \eta_{c1}\rangle \approx 0 \qquad \langle \Omega | \mathcal{O}_{\eta_{c1}} | AB \rangle \approx 0$$



$$\mathcal{C}_{\eta_{c1},AB}(t) = \langle \Omega | \mathcal{O}_{AB}(t)\mathcal{O}^+_{\eta_{c1}}(0)|\Omega\rangle$$
$$= \langle \Omega | \mathcal{O}_{AB}(0) e^{-t\,a\hat{H}} \mathcal{O}^+_{\eta_{c1}}(0)|\Omega\rangle$$
$$\to -axt\, e^{-t\,a\bar{E}} \langle \Omega|\mathcal{O}_{AB}|AB\rangle \langle \eta_{c1}|\mathcal{O}^+_{\eta_{c1}}|\Omega\rangle$$

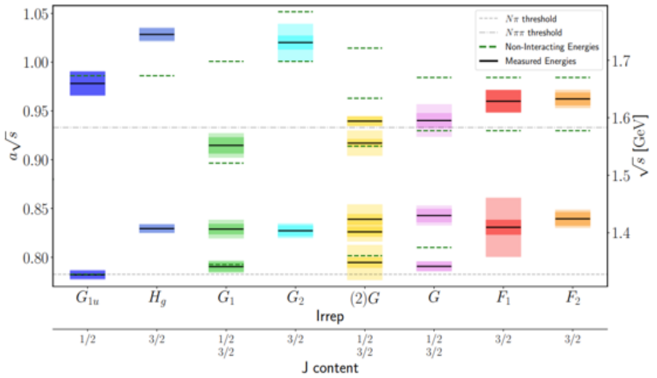
$$\langle \Omega|\mathcal{O}_{AB}|AB\rangle \approx \langle \Omega|\mathcal{O}_{A}|A\rangle \langle \mathcal{O}|\mathcal{O}_{B}|B\rangle$$

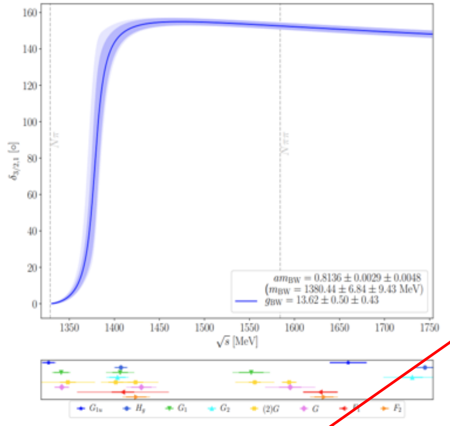
$$\frac{\mathcal{C}_{\eta_{c1},AB}(t)}{\sqrt{\mathcal{C}_{\eta_{c1}}(t)\mathcal{C}_A(t)\mathcal{C}_B(t)}} \to -ax\,t\left(1 + \frac{1}{24}(a\Delta\,t)^2\right)$$

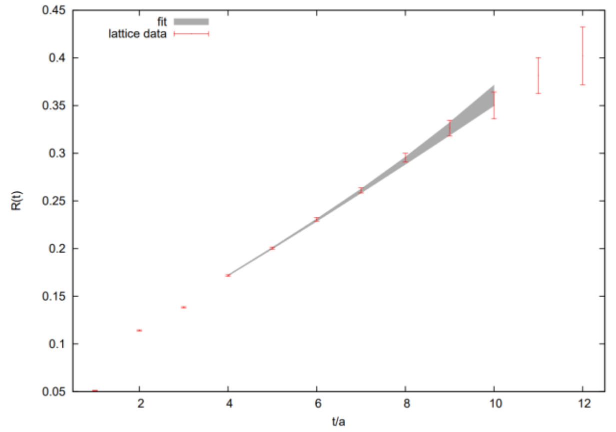
$N\pi$ scattering and the Δ resonance

G. Silvi et al., PRD103 (2021) 094508 (arXiv:2101.00689) and references therein

$N_s^3 \times N_t$	$24^3 \times 48$
β	3.31
$am_{u,d}$	-0.09530
am_s	-0.040
a [fm]	0.1163(4)
L [fm]	2.791(9)
m_π [MeV]	255.4(1.6)
$m_\pi L$	3.61(2)
N_{config}	600
N_{meas}	9600







C. Alexandrou et al.,
Phys. Rev. D 88 (2013) 031501

K-matrix rescaled: $K = \rho^{1/2}\widehat{K}\rho^{1/2}$

K relates to the phase shift: $K^{Jl} = \tan\delta_{Jl}$

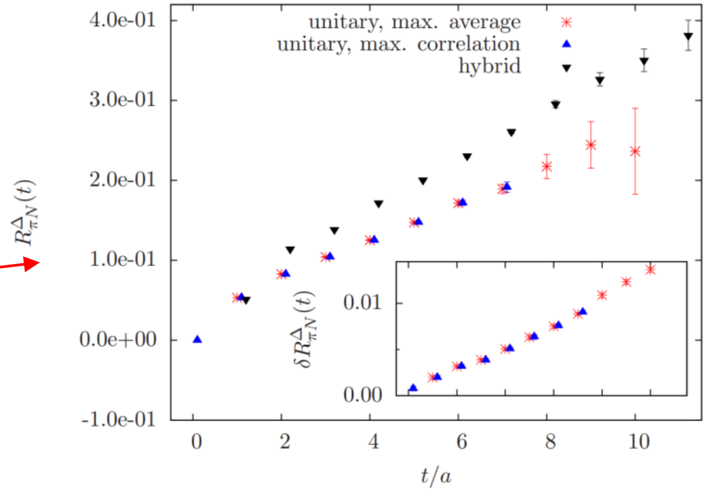
$m_\Delta = (1378.3 \pm 6.6 \pm 9.0)$ MeV

$\Gamma_\Delta = (\ \ 16.4 \pm 1.0 \pm 1.4)$ MeV

Breit Wigner: $\widehat{K}^{(3/2,1)} = \frac{\sqrt{s}\,\Gamma(s)}{(m_{BW}^2 - s)\rho}$

$$\Gamma(s) = \frac{g_{BW}^2}{6\pi}\frac{k^3}{s}$$

$$\Gamma_{\rm EFT}^{\rm LO} = \frac{g_{\Delta\text{-}\pi N}^2}{48\pi}\frac{E_N + m_N}{E_N + E_\pi}\frac{k^3}{m_N^2}$$



Collaboration	m_π [MeV]	Methodology	m_Δ [MeV]	$g_{\Delta\text{-}\pi N}$
Verduci 2014 [38]	266(3)	Distillation, Lüscher	$1396(19)_{\rm BW}$	19.90(83)
Alexandrou et al. 2013 [37]	360	Michael, McNeile	1535(25)	27.0(0.6)(1.5)
Alexandrou et al. 2016 [39]	180	Michael, McNeile	1350(50)	23.7(0.7)(1.1)
Andersen et al. 2018 [41]	280	Stoch. distillation, Lüscher	$1344(20)_{\rm BW}$	37.1(9.2)
Our result	255.4(1.6)	Smeared sources, Lüscher	$1380(7)(9)_{\rm BW}$, $1378(7)(9)_{\rm pole}$	23.8(2.7)(0.9)
Physical value [5]	139.5704(2)	phenomenology, K-matrix	$1232(1)_{\rm BW}$, $1210(1)_{\rm pole}$	29.4(3) [79], 28.6(3) [80]

C. Alexandrou et al.,
Phys. Rev. D 93 (2016) 114515

5. Decay amplitudes (C. Shi et al., arXiv: 2306.12884 (hep-lat))

- **The effective Lagrangian for η_{c1} two-body decays**

$$\mathcal{L}_I^{cc} \sim -g_{\chi\eta} m_{\eta_{c1}} H_\mu A^\mu \eta - i g_{\eta_c \eta} H_\mu \eta_c \overleftrightarrow{\partial}^\mu \eta + i H_\mu \left(g \psi_\nu \partial^\nu \omega^\mu + g' \omega_\nu \partial^\nu \psi^\mu + g_0 \psi_\nu \overleftrightarrow{\partial}^\mu \omega^\nu \right)$$

$$\mathcal{L}_I^{oc} \sim g_{D_1 D} m_{\eta_{c1}} H_\mu \frac{1}{2}\left(D_1^{\mu,+} D + D^+ D_1^\mu\right) + g_{D^*\bar{D}^*} H^\mu \frac{i}{\sqrt{2}}\left(D^{\nu,+} \partial_\nu D_\mu + \partial_\nu D_\mu^+ D^\nu\right)$$
$$+ g_{D^*\bar{D}} \epsilon^{\mu\nu\rho\sigma} (\partial_\mu H_\nu) \frac{1}{2} \left[(\partial_\rho D_\sigma^+) - D^+ (\partial_\rho D_\sigma) \right]$$

- **The flavor wave functions of the open-charm modes**

$$\left|D\bar{D}'\right\rangle^{(I=0)}_{(C=+)} = \frac{1}{2}\left(\left|D^+ D'^-\right\rangle + \left|D^0 \bar{D}'^0\right\rangle\right) \pm \frac{1}{2}\left(\left|D^- D'^+\right\rangle + \left|\bar{D}^0 D'^0\right\rangle\right)$$

$D' = D^*, D_1$,
"+" for $D\bar{D}^*$
"-" for $D\bar{D}_1$

$$\left|D^*\bar{D}^*\right\rangle^{(I=0)}_{(C=+)} = \frac{1}{\sqrt{2}}\left(\left|D^{*+} D^{*-}\right\rangle + \left|D^{0*} \bar{D}^{0*}\right\rangle\right)^{(S=1)}_{(L=1)}$$

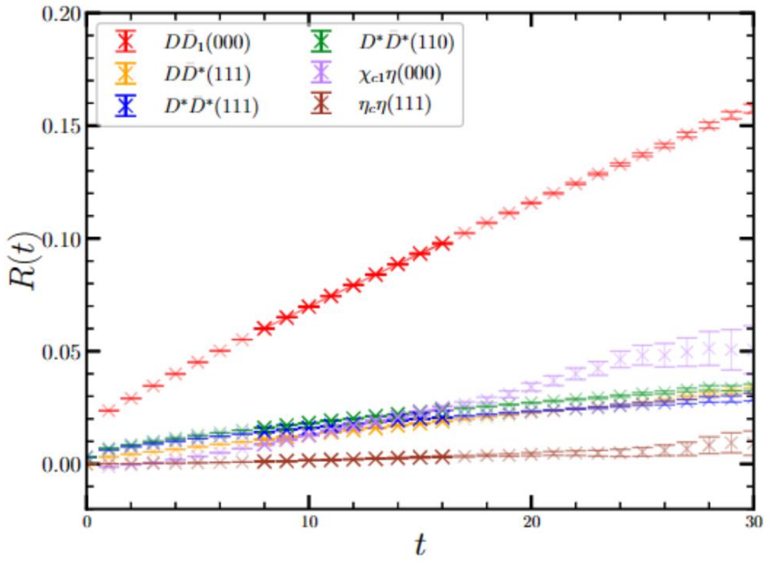
$L + S =$ even

- **Amplitudes for $\eta_{c1} \to AB$ from the Lagrangian**

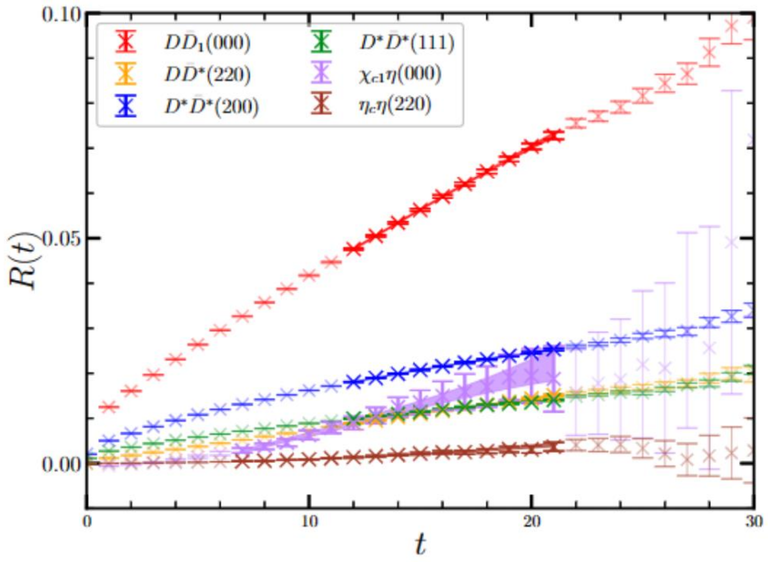
IE	$N_s^3 \times N_t$	β	a_t^{-1}(GeV)	ξ	m_π(MeV)	N_V	$N_{\rm cfg}$
L16	$16^3 \times 128$	2.0	6.894(51)	~ 5.3	~ 350	70	708
L24	$24^3 \times 192$	2.0	6.894(51)	~ 5.3	~ 350	160	171

$$
\begin{aligned}
x_{AP}^{\lambda'\lambda} &= g_{AP} m_{\eta_{c1}} \vec{\epsilon}_\lambda(\vec{0}) \cdot \vec{\epsilon}^{\,*}_{\lambda'}(\vec{k}), \\
x_{PP}^{\lambda} &= 2 g_{PP} \vec{\epsilon}_\lambda(\vec{0}) \cdot \vec{k}, \\
x_{D^*\bar{D}}^{\lambda'\lambda} &= g_{D^*\bar{D}} \vec{\epsilon}_\lambda(\vec{0}) \cdot (\vec{\epsilon}^{\,*}_{\lambda'}(\vec{k}) \times \vec{k}), \\
x_{D^*\bar{D}^*}^{\lambda'\lambda''\lambda} &= 2 g_{D^*\bar{D}^*} \vec{\epsilon}_\lambda(\vec{0}) \cdot \left(\vec{k} \times \left[\vec{\epsilon}^{\,*}_{\lambda'}(\vec{k}) \times \vec{\epsilon}^{\,*}_{\lambda''}(-\vec{k})\right]\right)
\end{aligned}
$$

$$
\frac{\mathcal{C}_{\eta_{c1},AB}(t)}{\sqrt{\mathcal{C}_{\eta_{c1}}(t)\mathcal{C}_A(t)\mathcal{C}_B(t)}} \to -(ax)\,t\left(1 + \frac{1}{24}(a\Delta\, t)^2\right)
$$



L16



L24

- **The final results**

Mode (AB)	$\hat{k}$(IE)	r_1 $(\times 10^{-3})$	g_{AB}	g_{AB} (ave.)	Γ_{AB} (MeV)
$D_1\bar{D}$	$(0,0,0)$(L16)	4.95(5)	4.27(5)	4.6(6)	258(133)
	$(0,0,0)$(L24)	3.10(26)	4.92(41)		
$D^*\bar{D}$	$(1,1,1)$(L16)	1.11(3)	8.35(21)	8.3(7)	88(18)
	$(2,2,0)$(L24)	0.78(7)	8.34(74)		
$D^*\bar{D}^*$	$(1,1,1)$(L16)	1.00(3)	3.44(12)	4.6(1.8)	150(118)
	$(1,1,0)$(L16)	1.15(4)	3.79(12)		
	$(2,0,0)$(L24)	1.05(9)	5.06(42)		
	$(1,1,1)$(L24)	0.67(7)	6.31(58)		
$\chi_{c1}\eta_{(2)}$	$(0,0,0)$(L16)	2.04(26)	1.31(2)	1.35(45)	–
	$(0,0,0)$(L24)	1.18(38)	1.39(45)		
$\eta_c\eta_{(2)}$	$(1,1,1)$(L16)	0.20(6)	0.62(18)	0.55(22)	–
	$(2,2,0)$(L24)	0.10(3)	0.47(12)		

$$\delta g_{AB} = \frac{\max g_{AB} - \min g_{AB}}{2}$$

- $D_1\bar{D}$ **dominates.**
- $D^*\bar{D}$ **and** $D^*\bar{D}^*$ **are important.**

This observation is in striking contrast to the expectation of the flux-tube model.

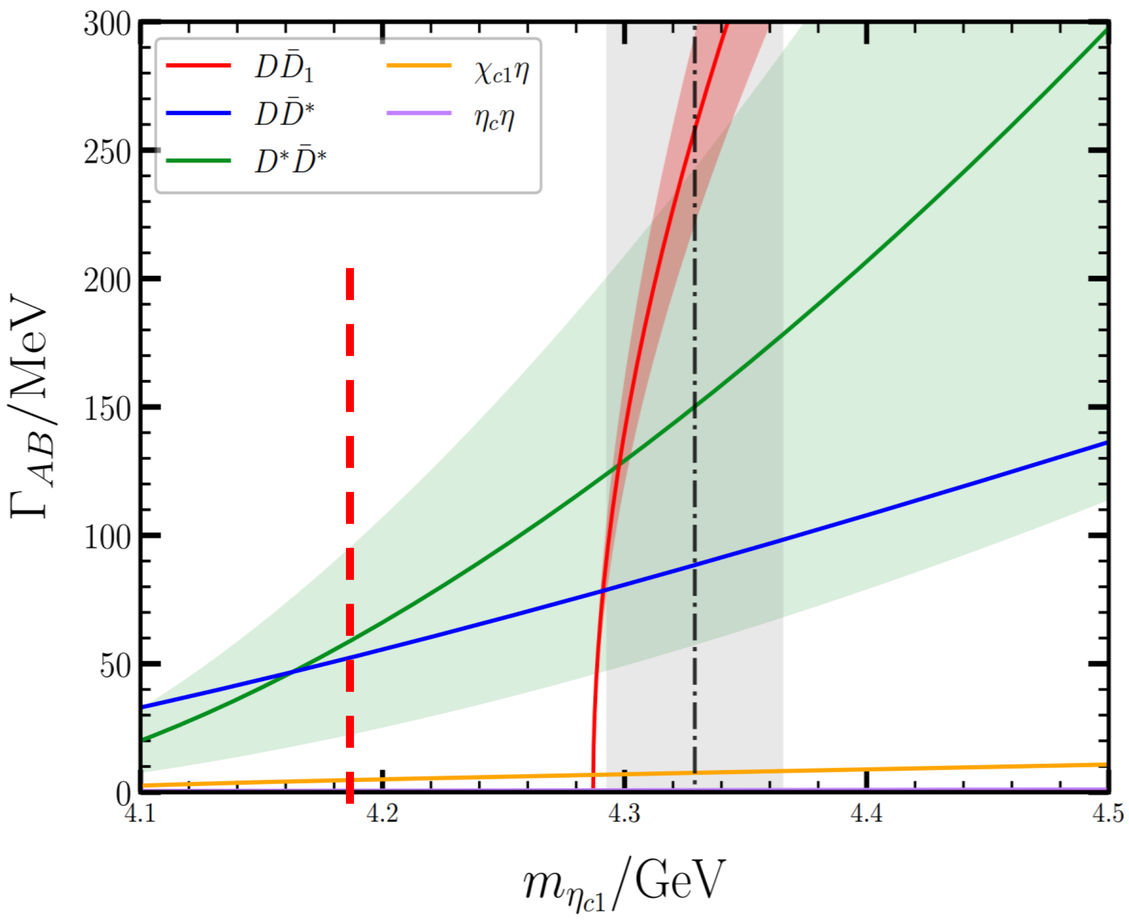
$$\overline{|\mathcal{M}(\eta_{c1}\to AP)|^2} = \frac{1}{3} g_{AP}^2 m_{\eta_{c1}} \left(3 + \frac{k_{\rm ex}^2}{m_A^2}\right),$$

$$\overline{|\mathcal{M}(\eta_{c1}\to PP)|^2} = \frac{4}{3} g_{PP}^2 k_{\rm ex}^2,$$

$$\overline{|\mathcal{M}(\eta_{c1}\to VP)|^2} = \frac{2}{3} g_{VP}^2 k_{\rm ex}^2,$$

$$\overline{|\mathcal{M}(\eta_{c1}\to D^*\bar{D}^*)|^2} = \frac{4}{3} g^2 k_{\rm ex}^2 \frac{m_{\eta_{c1}}^2}{m_{D^*}^2}.$$

$$\Gamma_{AB} = \frac{1}{8\pi}\frac{k_{\rm ex}}{m_{\eta_{c1}}^2}\overline{|\mathcal{M}(\eta_{c1}\to AB)|^2}$$



The $m_{\eta_{c1}}$-dependence of partial decay widths

- For $m_{\eta_{c1}} = 4329(36)$ MeV, we have

$$\Gamma_{D_1\bar{D}} = 258(133)\ \mathrm{MeV}$$
$$\Gamma_{D^*\bar{D}^*} = 150(118)\ \mathrm{MeV}$$
$$\Gamma_{D^*\bar{D}^*} = 88(18)\ \mathrm{MeV}$$

$$\Gamma_{\chi_{c1}\eta} = \sin^2\theta \cdot 44(29)\ \mathrm{MeV}$$
$$\Gamma_{\eta_c\eta'} = \cos^2\theta \cdot 0.93(77)\ \mathrm{MeV}$$

- Given the mass above, η_{c1} seems too wide to be identified easily in experiments.

- However, $\Gamma_{\eta_{c1}}$ is very sensitive to $m_{\eta_{c1}}$.
- If $m_{\eta_{c1}} \sim 4.2$ GeV, then $\Gamma_{\eta_{c1}} \sim 100$ MeV.
  The dominant decay channels are $D^*\bar{D}$ and $D^*\bar{D}^*$.
- Especially for $D^*\bar{D}^*$, the measurement of the polarization of D^* and $\bar{D}^*$ will help distinguish a 1^{-+} states from 1^{--} states.

We suggest LHCb, BelleII and BESIII to search for η_{c1} in $D^*\bar{D}$ and $D^*\bar{D}^*$ systems !

In comparison with the decay of the $1^{+}1^{-+}$ hybrid

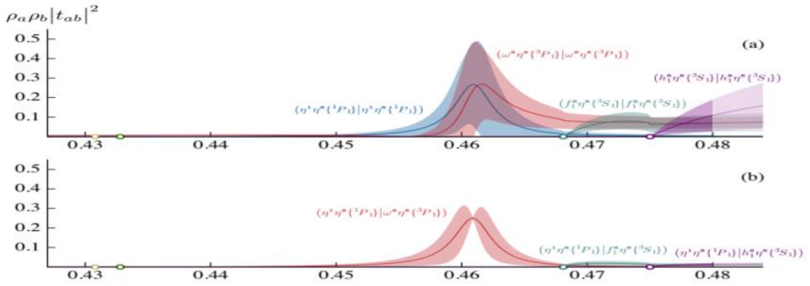
A.J. Woss (HSC Collaboration), Phys. Rev. D 103, 054502 (2021)

- **Luescher quantization condition** for hadron-hadron scatterings on finite lattices

$$\det[1 + i\rho t(1 + iM)] = 0$$

- Possible composition of the final states of $\pi_1 \to M_1 M_2$ (in SU(3) symmetric limit)

$$\eta^1\eta^8, \omega^8\eta^8, \omega^8\omega^8, \omega^1\omega^8,$$
$$f_1^8\omega^8, h_1^8\eta^8, f_1^1\eta^8$$



$$t_{\ell SJa,\ell' S' Jb} \sim \frac{c_{\ell SJa}\, c_{\ell' S' Jb}}{s_0 - s}$$

	thr./MeV	$\lvert c_i^{\rm phys}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\overline{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
			$\Gamma = \sum_i \Gamma_i = 139 \to 590$

Discorvery modes

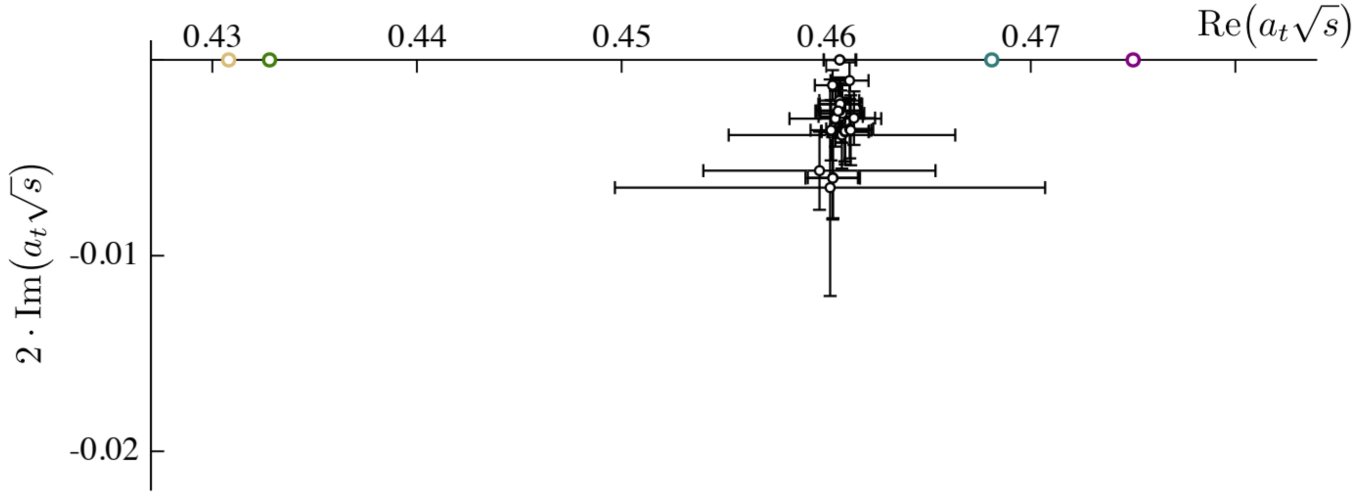
The partial widths of of $\pi_1(1564)$

$b_1\pi$ dominates the decay of $\pi_1(1564)$!

V. Summary

- The spectrum of hybrids are extensively explored in lattice QCD
- The production rate of η_1 in J/ψ radiative decay is predicted for the first time.
- We give the first lattice QCD prediction of the partial decay widths of the charmoniumlike η_{c1}
- We provide useful theoretical information of hybrids for experiments.

Thanks!


Re$(a_t\sqrt{s})$
0.43
0.44
0.45
0.46
0.47
$2 \cdot$ Im$(a_t\sqrt{s})$
-0.01
-0.02