A Solution of $B \to \pi\pi, K\pi$ Puzzles Based on Perturbative QCD

报告人：杨茂志

南开大学

合作者：吕晟，王汝轩

Based on works:

[1] S. Lü and M. Z. Yang, NPB972, 115550 (2021).

[2] S. Lü and M. Z. Yang, PRD107, 013004 (2023).

[2] R.X. Wang, M.Z. Yang, arxiv:2212.09054， to appear in PRD.

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Content

I Introduction

- High Precision data are collected by B factories after years’ runing.
- Serious discrepancies have been revealed between experimental data and theoretical predictions.
- $B \to \pi\pi,\ \pi K$ puzzles are remarkable problems existing in B decays

(1) $B \to \pi\pi$ puzzle:

Perturbative QCD (PQCD) and QCD factorization (QCDF) are two main theoretical methods for calculating B decays in these several decades.

Decay Mode	Exp. Data	PQCD and QCDF
$B^+ \to \pi^+\pi^0$	$(5.5 \pm 0.4) \times 10^{-6}$	$\sim 5.0 \times 10^{-6}$
$B^0 \to \pi^+\pi^-$	$(5.12 \pm 0.19) \times 10^{-6}$	$\sim 5.0 \times 10^{-6}$
$B^0 \to \pi^0\pi^0$	$(1.59 \pm 0.26) \times 10^{-6}$	$\sim 1.0 \times 10^{-7}$

Theoretical prediction gives a much small branching ratio for $B^0 \to \pi^0\pi^0$ decay mode.

Calculation of NLO can't solve this problem either.

(2) $B \to \pi K$ puzzle:

Experimental data for $\mathrm{B} \to \pi\mathrm{K}$ decays

Decay Mode	Exp. Br	Exp. CP Violation
$B^+ \to K^0\pi^+$	$(2.37 \pm 0.08) \times 10^{-5}$	-0.017 ± 0.016
$B^+ \to K^+\pi^0$	$(1.29 \pm 0.05) \times 10^{-5}$	0.037 ± 0.021
$B^0 \to K^+\pi^-$	$(1.96 \pm 0.05) \times 10^{-5}$	-0.083 ± 0.004
$B^0 \to K^0\pi^0$	$(9.9 \pm 0.5) \times 10^{-6}$	0.00 ± 0.13

From theoretical side, the amplitude for $B \to \pi K$ decays are generally like

$$A(B^+ \to \pi^+ K^0) = P',$$

$$\sqrt{2}A(B^+ \to \pi^0 K^+) = -P'\left[1 + \frac{P'_{\rm ew}}{P'} + \left(\frac{T'}{P'} + \frac{C'}{P'}\right)e^{i\phi_3}\right]$$

$$A(B^0 \to \pi^- K^+) = -P'\left(1 + \frac{T'}{P'}e^{i\phi_3}\right),$$

$$\sqrt{2}A(B^0 \to \pi^0 K^0) = P'\left(1 - \frac{P'_{\rm ew}}{P'} - \frac{C'}{P'}e^{i\phi_3}\right),$$

T' ~tree, C'~ color suppressed tree, P'~penguin, P'_{ew}~electroweak penguin

The amplitudes obey the counting rule In the SM

$$\frac{T'}{P'} \sim \lambda, \qquad \frac{P'_{\rm ew}}{P'} \sim \lambda, \qquad \frac{C'}{P'} \sim \lambda^2. \qquad \lambda \sim 0.22$$

H.N. Li, S. Mishima, A.I. Sanda, PRD72,114005(2005)

The relation about CP violation is expected

$$A_{CP}(B^+ \to \pi^0 \mathrm{K}^+) \simeq A_{CP}(B^0 \to \pi^- \mathrm{K}^+)$$

Experimental data give:

$$A_{CP}(B^+ \to \pi^0 \mathrm{K}^+) - A_{CP}(B^0 \to \pi^- \mathrm{K}^+) = 0.120 \pm 0.021$$

This is contradictory to the theoretical expectation in the SM

The amplitude is also puzzling when confronting the SM expectation to the experimental data

based on factorization method

A.J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, EPJC 32, 45 (2003)

(3) The brief status for solving $B \to \pi\pi,\ \pi K$ puzzles

a) Calculation up to next-to-leading order in QCD (PQCD)

[1] H. N. Li, S. Mishima, and A. I. Sanda, PRD 72, 114005 (2005).
[2] Y. L. Zhang, X. Y. Liu, Y. Y. Fan, S. Cheng, and Z. J. Xiao, PRD 90, 014029 (2014).
[3] W. Bai, M. Liu, Y.Y. Fan, W.F. Wang, S. Cheng, and Z.J. Xiao, CPC 38, 033101 (2014).
[4] J. Chai, S. Cheng, Y. H. Ju, D. C. Yan, C. D. Lü, and Z. J. Xiao, CPC 46, 123103 (2022).
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b) Penguin annihilation and power correction (QCDF)

- Endpoint divergence in annihilation is modelled as

$$X_A \equiv \int_0^1 dx/\bar{x} \quad \rightarrow \quad X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A})$$

- Power corrections to the color-suppressed topology are parametrized as

$$a_2 \rightarrow a_2\ (1 + \rho_C e^{i\phi_C})$$

[1] H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 074031 (2009).
[2] H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).
[3] Q. Chang, J. Sun, Y. Yang, and X. Li, Phys. Rev. D 90, 054019 (2014)

c) A soft factor associated with pseudoscalar is introduced due to the residuel divergence in kT factorization in loop process (PQCD)

[1] H.N. Li and S. Mishima, Phys. Rev. D 83, 034023 (2011).
[2] H.N. Li and S. Mishima, Phys. Rev. D 90, 074018 (2014).
[3] X. Liu, H.N. Li, Z.J. Xiao, Phys. Rev. D 93, 014024 (2016).

d) New physics is invoked

[1] V. Barger, C.W. Chiang, P. Langacker, H.S. Lee, Phys.Lett. B 598, 218 (2004).
[2] S. Baek, P. Hamel, D. London, A. Datta, D.A. Suprun, Phys. Rev. D 71, 057502 (2005).
[3] R. Arnowitt, B. Dutta, B. Hu, S. Oh, Phys. Lett. B 633, 748 (2006).
[4] C. Kim, S. Oh, and Y.W. Yoon, Phys. Lett. B 665, 231 (2008).
[5] N.B. Beaudry, A. Datta, D. London, A. Rashed, J.S. Roux, JHEP 01, 074 (2018).
[6] A. Datta, J. Waite, and D. Sachdeva, Phys. Rev. D 100, 055015 (2019).
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etc.

II B decays in perturbative QCD based on k_T factorization (PQCD)

The decay amplitude of $B \to M_1 M_2$ is

$$M = \int d^3k_1 \int d^3k_2 \int d^3k_3 \; \Phi^B(k_1, \mu)$$

$$\cdot\, C(\mu) H(k_1, k_2, k_3, \mu) \Phi^{M_1}(k_2, \mu)\, \Phi^{M_2}(k_3, \mu)$$

for $\mu > \mu_c = 1$ GeV

The spinor wave function of B meson is taken from solving relativistic potential model

$$
\begin{aligned}
\Phi^B_{\alpha\beta}(\vec{k}) = & \frac{-i f_B m_B}{4} K(\vec{k}) \\
& \times \left\{ (E_Q + m_Q) \frac{1+ \not{v}}{2} \left[\left(\frac{k_+}{\sqrt{2}} + \frac{m_q}{2} \right) \not{n}_+ \right.\right. \\
& \left. + \left(\frac{k_-}{\sqrt{2}} + \frac{m_q}{2} \right) \not{n}_- - k_\perp^\mu \gamma_\mu \right] \gamma_5 \\
& - (E_q + m_q) \frac{1- \not{v}}{2} \left[\left(\frac{k_+}{\sqrt{2}} - \frac{m_q}{2} \right) \not{n}_+ \right. \\
& \left.\left. + \left(\frac{k_-}{\sqrt{2}} - \frac{m_q}{2} \right) \not{n}_- - k_\perp^\mu \gamma_\mu \right] \gamma_5 \right\}_{\alpha\beta},
\end{aligned}
$$

$$
K(\vec{k}) = \frac{2 N_B \Psi_0(\vec{k})}{\sqrt{E_q E_Q (E_q + m_q)(E_Q + m_Q)}},
$$

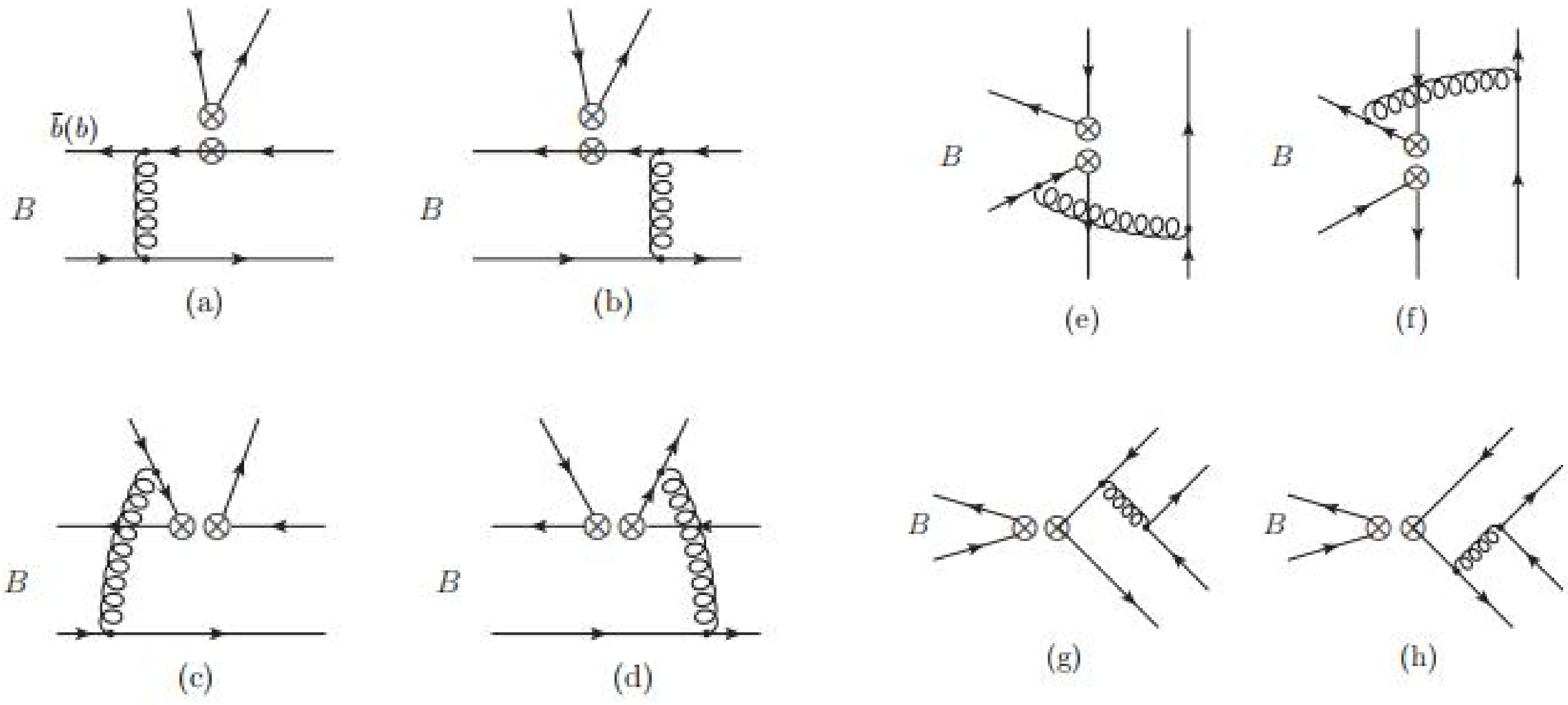
[1] M.Z. Yang, EPJC 72, 1880 (2012).
[2] J.B. Liu and M.Z. Yang, JHEP 2014, 106 (2014).
[3] J.B. Liu and M.Z. Yang, PRD 91, 094004 (2015).
[4] H.K. Sun and M.Z. Yang, PRD 95, 113001 (2017).
[5] H.K. Sun and M.Z. Yang, PRD 99, 093002 (2019).

The spinor wave function for light meson

$$\Phi^{\pi}_{\gamma\delta} = \frac{if_\pi}{4}\left[\not{p}\gamma_5\phi_\pi(u,k_{q\perp}) - \mu_\pi\gamma_5\phi^\pi_P(u,k_{q\perp})\right.$$
$$\left. + \mu_\pi\gamma_5\sigma^{\mu\nu}p_\mu z_\nu\frac{\phi^\pi_\sigma(u,k_{q\perp})}{6}\right]_{\gamma\delta},$$

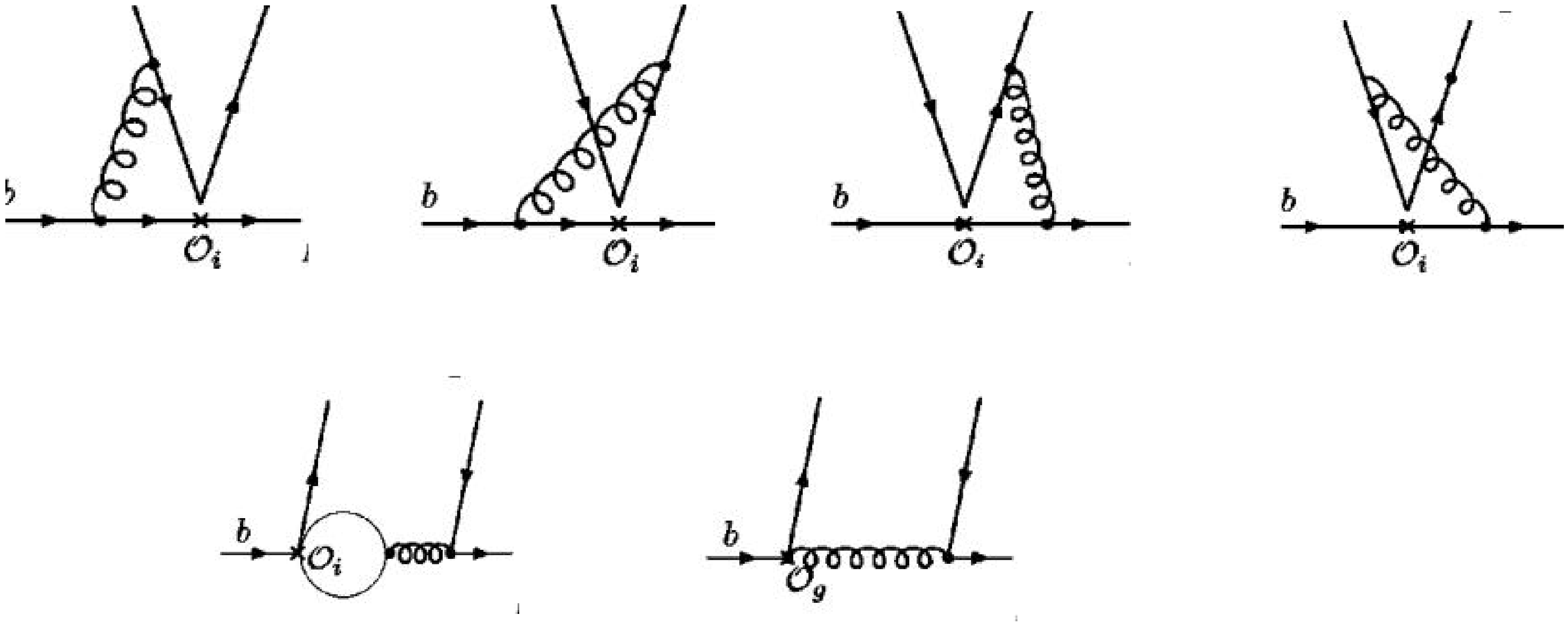
[1] V.M. Braun and I. Filyanov, Z Phys. C 48, 239 (1990).
[2] P. Ball, JHEP 01, 010 (1999).
[3] P. Ball, V.M. Braun, and A. Lenz, JHEP 05, 004 (2006).

Leading order diagrams in QCD



- Transverse momenta are kept to remove endpoint singlarity
- Sudakov factor is introduced to suppress long-distance contribution

Most important next-to-leading order diagrams in QCD



Naive calculation of the hard diagrams in PQCD

●With the B meson wave function taken from Relativistic potential model, the suppression of Sudakov factor to soft contribution becomes weak.

●Soft contributions are still large

(1)For dirgrams (a) (b) (g) (h):

more than 40% in the range $\alpha_s/\pi > 0.2$

(2) For dirgrams (c) (d) :

more than 93% in the range $\alpha_s/\pi < 0.2$

(3) For dirgrams (e) (f) :

contribution only a few percent, very small

[1] S. Lü and M. Z. Yang, NPB972, 115550 (2021).

[2] S. Lü and M. Z. Yang, PRD107, 013004 (2023).

[3] R.X. Wang, M.Z. Yang, Eprint, arxiv:2212.09054, to appear in PRD.

III. Introduction of soft form factors

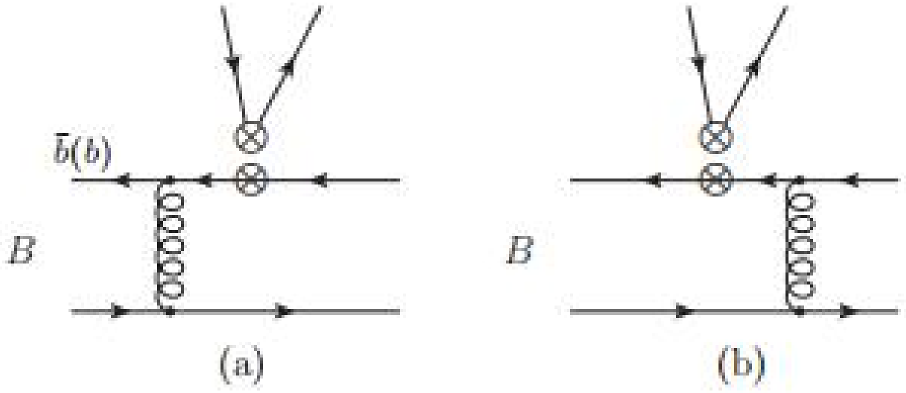
A momentum cutoff is introduced for perturbative calculation:

Taking μ_c as a critical scale that separating soft and hard interaction

(1) $\mu > \mu_c$ for hard scale

(2) $\mu < \mu_c$ for soft scale

$\mu_c = 1$ GeV



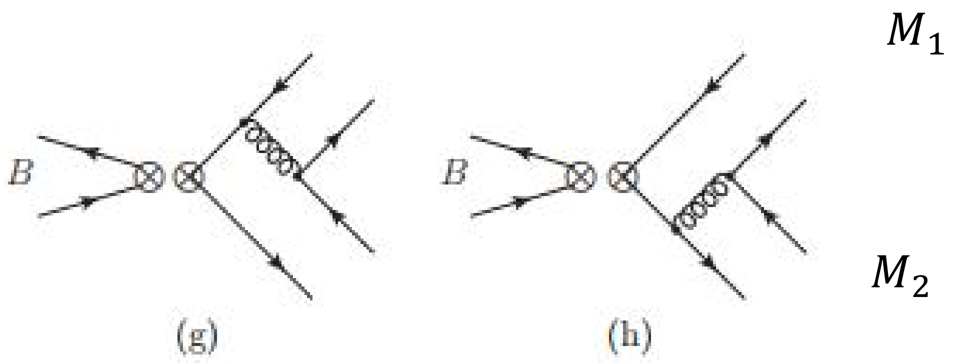
Transition form factor

$$F_0^{BM} = h_0^{BM} + \xi_0^{BM}$$

soft form factor

$$h_0^{BM} = \Phi^B \otimes H \otimes \Phi^\pi$$

hard form factor



Production form factor

$$\langle M_1 M_2 | S | 0 \rangle \equiv -\frac{1}{2}\sqrt{\mu_{M1}\mu_{M2}} F_+^{M_1 M_2}$$

$$\mu_M = m_M^2/(m_{q1} + m_{q2})$$

Separation of hard and soft form factor:

$$F_+^{M_1 M_2} = h_+^{M_1 M_2} + \xi_+^{M_1 M_2}$$

hard

soft

The $B\pi$ and BK transition form factors calculated perturbatively with $\mu > \mu_c$ are：

$$h_0^{B\pi} = 0.23 \pm 0.01 \qquad h_0^{BK} = 0.29 \pm 0.02$$

The total $B\pi$ and BK transition form factors from LQCD and experimental constraint are

$$F_0^{B\pi} = 0.27 \pm 0.02 \qquad F_0^{BK} = 0.33 \pm 0.04$$

which result in

$$\xi_0^{B\pi} = 0.04 \pm 0.01 \qquad \xi_0^{BK} = 0.04 \pm 0.02$$

IV. Cotribution of color-octet quark-antiquark compoents

There is a relation for the generators of the color SU(3) group

$$T^a_{ik}T^a_{jl} = -\frac{1}{2N_c}\delta_{ik}\delta_{jl} + \frac{1}{2}\delta_{il}\delta_{jk}$$

It can change the color non-singlet quark current into color singlet and octet operators

$$(\bar{q}_{1i}q_{2j})(\bar{q}_{3j}q_{4i}) = \frac{1}{N}(\bar{q}_{1i}q_{2i})(\bar{q}_{3j}q_{4j}) + 2(\bar{q}_1 T^a q_2)(\bar{q}_3 T^a q_4).$$

The first term is the color suppressed trem, and the second color-octet one

Hadronic matrix element of color-octet operators

For example, the contribution of the operator O_1 to the decay of $\overline{B}^0 \to K^-\pi^+$

$$O_1 = \overline{s}_i\gamma^\mu(1-\gamma_5)u_j\overline{u}_j\gamma_\mu(1-\gamma_5)b_i$$

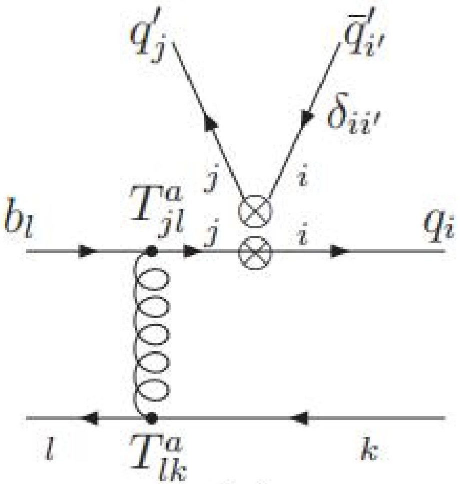
$$A = \langle K^-\pi^+|C_1O_1|\overline{B}^0\rangle$$

$$= \frac{C_1}{N_c}\left\langle K^-\pi^+\middle|\overline{s}_i\gamma^\mu(1-\gamma_5)u_i\overline{u}_j\gamma_\mu(1-\gamma_5)b_j\middle|\overline{B}^0\right\rangle$$
$$+2C_1\left\langle K^-\pi^+\middle|\overline{s}\gamma^\mu(1-\gamma_5)T^a u\ \overline{u}\gamma_\mu(1-\gamma_5)T^a b\middle|\overline{B}^0\right\rangle$$

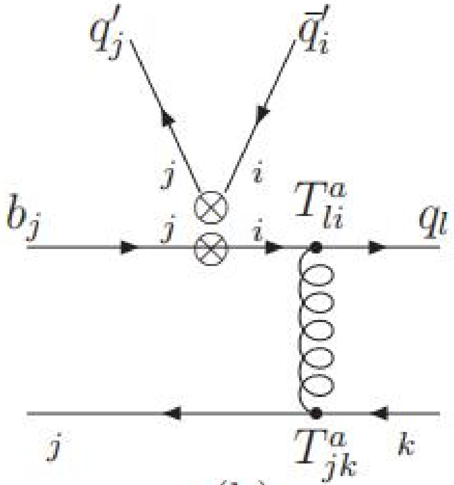
color-octet matrix element

Such matrix element may have nonzero value!

The contributions of the final quark pair in color octet state are considered by treating the final quark-antiquark pairs in the hard decay diagrams in non-singlet states



(a)



(b)

$$\sum_{ijkl} T^a_{jl}T^a_{lk} = \sum_{ijk} C_F\delta_{jk} = \sum_{ijki'} C_F\delta_{jk}\delta_{ii'}$$
$$= \sum_{ijki'} C_F\left(\frac{1}{N_c}\delta_{ji'}\delta_{ik} + 2T^a_{ji'}T^a_{ik}\right),$$

$$\sum_{ijkl} T^a_{li}T^a_{jk} = \sum_{ijkl}\left[-\frac{1}{2N_c}\delta_{li}\delta_{jk} + \frac{1}{2}\delta_{lk}\delta_{ji}\right]$$
$$= \sum_{ijkl}\left[-\frac{1}{2N_c}\left(\frac{1}{N_c}\delta_{lk}\delta_{ji} + 2T^b_{lk}T^b_{ji}\right) + \frac{1}{2}\delta_{lk}\delta_{ji}\right]$$
$$= \sum_{ijkl}\left(\frac{C_F}{N_c}\delta_{lk}\delta_{ji} - \frac{1}{N_c}T^b_{lk}T^b_{ji}\right),$$

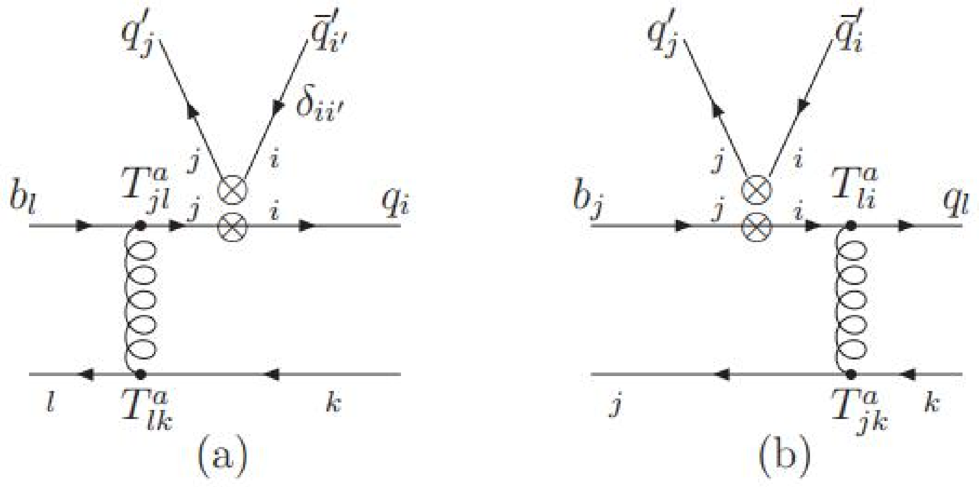
●The contributions of the other diagrams can be analyzed similarly.

●Two parameters Y_F^8 and Y_M^8 need to be introduced, which descibe the effect of color-octet quark pair changing to color singlet states by exchanging soft gluons.

Y_F^8 is for the factorizable diagrams

Y_M^8 is for the non-factorizable diagrams

For (V-A)(V-A) and (S+P)(S-P) operators, these two diagrams' controbutions are:



$$Y_F^8 F_e^{(P)8}$$

$$F_e^{(P)8} \equiv 2N_c^2 F_e^{(P)a} - \frac{N_c}{C_F} F_e^{(P)b}$$

The color-octet contributions of the other diagrams and operator insertions can be also obtained

V. Confronting the theoretical framework to experimental data

The residual free parameters are

$$\xi^{M_1M_2} = d_1 e^{i\phi_1} \qquad Y_F^8(M_1M_2) = d_2 e^{i\phi_2} \qquad Y_M^8(M_1M_2) = d_3 e^{i\phi_3}$$

which can be determined by fitting data

For $\pi\pi$ final state:

$$\xi^{\pi\pi} = (0.17 \pm 0.04)e^{i(-0.84\pm0.07)\pi}$$

$$Y_F^8(\pi\pi) = (0.11 \pm 0.02)e^{i(-0.49\pm0.04)\pi}$$

$$Y_M^8(\pi\pi) = (0.05 \pm 0.02)e^{i(0.75\pm0.10)\pi}$$

For $K\pi$ final state:

$$\xi^{K\pi} = (0.10 \pm 0.01)e^{i(-0.48\pm0.04)\pi}$$

$$Y_F^8(K\pi) = (0.12^{+0.01}_{-0.02})e^{i(1.00^{+0.03}_{-0.04})\pi}$$

$$Y_M^8(K\pi) = (0.05 \pm 0.03)e^{i(0.93^{+0.17}_{-0.14})\pi}$$

$$B(B^0 \to \pi^+\pi^-) = (5.12 \pm 0.95) \times 10^{-6}, \qquad A_{CP}(B^0 \to \pi^+\pi^-) = 0.31 \pm 0.08,$$
$$B(B^+ \to \pi^+\pi^0) = (5.37 \pm 0.62) \times 10^{-6}, \qquad A_{CP}(B^+ \to \pi^+\pi^0) = 0.0045 \pm 0.0010,$$
$$B(B^0 \to \pi^0\pi^0) = (1.69 \pm 0.54) \times 10^{-6}, \qquad A_{CP}(B^0 \to \pi^0\pi^0) = 0.35 \pm 0.11.$$

TABLE I: Branching ratio and direct CP violation with different NLO contributions .

	LO_{NLOWC}	+VC	+QL	+MP	NLO	NLO+soft	Data
$Br(B^0 \to \pi^+\pi^-)\times 10^{-6}$	3.90	4.04	4.84	3.81	4.82	5.12	5.12 ± 0.19
$Br(B^+ \to \pi^+\pi^0)\times 10^{-6}$	3.59	3.24	3.59	3.59	3.24	5.37	5.5 ± 0.4
$Br(B^0 \to \pi^0\pi^0)\times 10^{-6}$	0.36	0.14	0.41	0.32	0.12	1.69	1.59 ± 0.26
$A_{CP}(B^0 \to \pi^+\pi^-)$	0.27	0.27	0.15	0.28	0.16	0.31	0.32 ± 0.04
$A_{CP}(B^+ \to \pi^+\pi^0)$	0.00	0.00	0.00	0.00	0.00	0.00	0.03 ± 0.04
$A_{CP}(B^0 \to \pi^0\pi^0)$	-0.60	0.31	-0.61	-0.70	0.30	0.35	0.33 ± 0.22

* $Y_f^8 = 0.11e^{-0.49\pi i}, Y_m^8 = 0.05e^{0.75\pi i}, \xi^{\pi\pi} = 0.17e^{-0.84\pi i}$

$$B(B^+ \to K^0\pi^+) = 24.3^{+4.5+2.4}_{-4.7-2.3} \times 10^{-6}, \qquad A_{CP}(B^+ \to K^0\pi^+) = 0.012^{+0.001+0.001}_{-0.001-0.001},$$
$$B(B^+ \to K^+\pi^0) = 12.6^{+2.3+1.1}_{-2.5-1.0} \times 10^{-6}, \qquad A_{CP}(B^+ \to K^+\pi^0) = 0.041^{+0.034+0.012}_{-0.028-0.015},$$
$$B(B^0 \to K^+\pi^-) = 20.0^{+3.4+1.3}_{-3.7-1.2} \times 10^{-6}, \qquad A_{CP}(B^0 \to K^+\pi^-) = -0.084^{+0.035+0.044}_{-0.038-0.049},$$
$$B(B^0 \to K^0\pi^0) = 9.4^{+1.7+0.8}_{-1.8-0.8} \times 10^{-6}, \qquad A_{CP}(B^0 \to K^0\pi^0) = -0.112^{+0.050+0.036}_{-0.055-0.040},$$

TABLE II. $B \to K\pi$ branching ratios and CP violations.

Mode	LO	$\mathrm{LO_{NLOWC}}$	NLO	$+\xi^{BK(\pi)}$	$+\xi^{K\pi}$	$+Y^8_F$	$+Y^8_M$	$+\xi^{BK(\pi)}+\xi^{K\pi}+Y^8_{F,M}$	Data[3]
$B(B^+ \to K^0\pi^+) \times 10^{-6}$	8.5	13.4	13.8	20.8	14.0	22.9	11.2	$24.3^{+4.5+2.4}_{-4.7-2.3}$	23.7 ± 0.8
$B(B^+ \to K^+\pi^0) \times 10^{-6}$	6.0	9.0	8.4	12.0	7.6	12.5	6.8	$12.6^{+2.3+1.1}_{-2.5-1.0}$	12.9 ± 0.5
$B(B^0 \to K^+\pi^-) \times 10^{-6}$	8.8	13.7	13.2	18.8	11.5	21.4	11.5	$20.0^{+3.4+1.3}_{-3.7-1.2}$	19.6 ± 0.5
$B(B^0 \to K^0\pi^0) \times 10^{-6}$	2.9	4.9	5.2	7.9	5.2	9.6	4.7	$9.4^{+1.7+0.8}_{-1.8-0.8}$	9.9 ± 0.5
$A_{CP}(B^+ \to K^0\pi^+)$	-0.006	-0.004	0.010	0.013	0.013	0.006	0.010	$0.012^{+0.001+0.001}_{-0.001-0.001}$	-0.017 ± 0.016
$A_{CP}(B^+ \to K^+\pi^0)$	-0.185	-0.153	-0.039	-0.001	0.073	-0.032	-0.003	$0.041^{+0.034+0.012}_{-0.028-0.015}$	0.037 ± 0.021
$A_{CP}(B^0 \to K^+\pi^-)$	-0.239	-0.175	-0.107	-0.063	0.025	-0.195	-0.126	$-0.084^{+0.035+0.044}_{-0.038-0.049}$	-0.083 ± 0.004
$A_{CP}(B^0 \to K^0\pi^0)$	0.004	0.018	-0.036	-0.040	-0.048	-0.147	-0.094	$-0.112^{+0.050+0.036}_{-0.055-0.040}$	0.00 ± 0.13

Discussion of μ_c-dependence:

TABLE III. $B \to K\pi$ branching ratios and CP violations varying with the critical cutoff scale μ_c, where the total from factors are fixed with $F_0^{BK}(0) = 0.33$, $F_0^{B\pi}(0) = 0.27$ and$F_+^{K\pi} = 0.20\,\exp(-0.47i\pi)$ [a]

Mode	$\mu_c = 0.9$GeV	1.0GeV	1.1GeV	1.3GeV	1.5GeV	2.0GeV	Data [3]
$B(B^+ \to K^0\pi^+) \times 10^{-6}$	24.5	24.3	24.3	23.5	23.0	20.9	23.7 ± 0.8
$B(B^+ \to K^+\pi^0) \times 10^{-6}$	12.4	12.6	12.7	12.5	12.4	11.5	12.9 ± 0.5
$B(B^0 \to K^+\pi^-) \times 10^{-6}$	19.5	20.0	20.4	20.5	20.4	19.0	19.6 ± 0.5
$B(B^0 \to K^0\pi^0) \times 10^{-6}$	9.3	9.4	9.5	9.3	9.1	8.4	9.9 ± 0.5
$A_{CP}(B^+ \to K^0\pi^+)$	0.012	0.011	0.011	0.010	0.009	0.008	-0.017 ± 0.016
$A_{CP}(B^+ \to K^+\pi^0)$	0.055	0.041	0.031	0.012	0.001	-0.011	0.037 ± 0.021
$A_{CP}(B^0 \to K^+\pi^-)$	-0.055	-0.084	-0.102	-0.133	-0.150	-0.178	-0.083 ± 0.004
$A_{CP}(B^0 \to K^0\pi^0)$	-0.100	-0.112	-0.118	-0.128	-0.133	-0.148	0.00 ± 0.13

[a] The total form factors should not vary with the critical cutoff scale. So the value of them can be obtained by adding the hard and soft part at any value of μ_c. Here $F_+^{K\pi}$ is taken by adding the values of $h^{K\pi}$ and $\xi^{K\pi}$ at $\mu_c = 1$ GeV. The color-octet contributions are taken as μ_c-independent quantities.

- Br and CPV are not changed much around $\mu_c \sim 1$GeV
- The change becomes large when $\mu_c > 2$ GeV, where the scale of soft interaction is pushed too high

VI. Summary

1) We used the B meson wave function obtained from relativistic potential model. Then the suppression of Sudakov factor to LD contribution is no longer sufficient.

2) A critical cutoff scale μ_c is introduced to insure perturbation calculation applicable.

3) Soft form factors, transition and production form factors, have to be introduced.

4) Contribution of color-octet final quark-antiquark pair is considered, which is crucial to explain the experimental data.