强相互作用系统相变与对称性破缺

刘玉鑫

北京大学物理学院 理论物理研究所

Outline

Ⅰ. 引言 —— 一些基本概念

Ⅱ. 原子核的相与相变

Ⅲ. 强相互作用物质的相与相变

Ⅳ. 小结

第十六届粒子物理、核物理和宇宙学交叉学科前沿问题研讨会，

2023-07-01

I. 引言 — 一些基本概念

1. 强相互作用与强相互作用物质

♠ 强相互作用

微观粒子之间的力程短、强度大的相互作用。

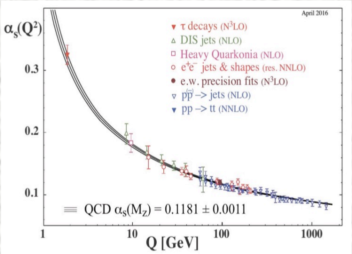
- 核力：r ~ fm 量级，

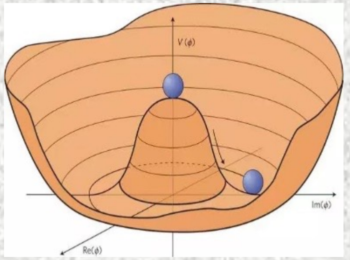
  基本特征：电荷无关、有心力+非有心力、LS耦合、短程排斥芯、etc.

- 夸克间相互作用：

  基本特征：渐近自由、手征对称性破缺、色禁闭。

  $\Lambda_{QCD} = 200 \sim 300$ MeV .





♠ 强相互作用物质

原子核、核物质、夸克胶子物质、强子与夸克胶子的混杂物质

♠ 原子核物理即强相互作用物质物理，

不仅是研究至少两个结构层次上的物质的性质、结构、反应的科学，

还是揭示可见物质及其质量起源机制和宇宙物质演化的科学！

2. 相与相变的概念及物质演化过程的物理描述方案

♣ 如何具体描述物质的演化是物理学人致力探索的瑰宝！

♠ 相：不均匀的状态中，可以由力学手段分离的组分相同、物理和化学性质相同的均匀部分的状态。

♠ 相变：确定的外部条件下，由一相到另一相的演化。

♠ 相即确定对称性的态，相变即对称性破缺or恢复的过程！

♠ 强相互作用物质的相

强相互作用物质即处于不同相的夸克胶子物质状态；

- 夸克和胶子有手征对称 (CS) 相、CSB相，不禁闭相和禁闭相，
- 原子核有多种集体运动模式相（形状相）。

➔ 可见物质演化过程可表述为强相互作用物质的相变！

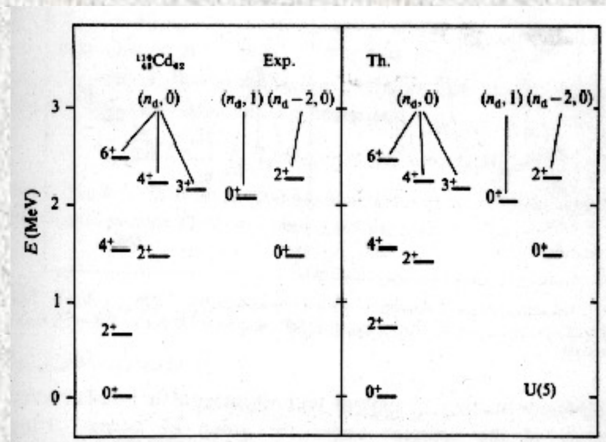
➔ 可见物质及其质量的产生即强相互作用系统的不同对称性破缺的表现，亦即不同的"涌现"现象。

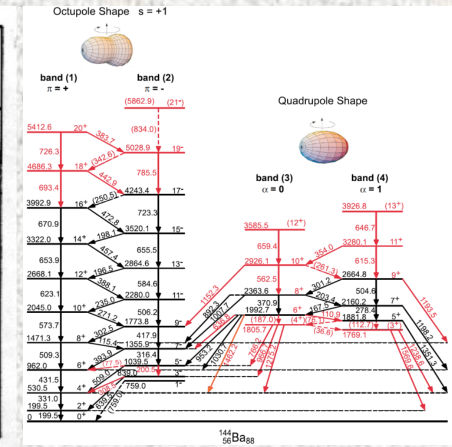
II. 原子核的相与相变

1. 原子核具有多种模式的集体运动，即有丰富的相

♠ 实验发现：

• 原子核具有近似规则的振动谱、转动谱



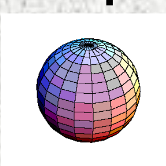


• 原子核具有远大于单粒子的跃迁几率、电四极矩、磁矩、等

♠ 原子核具有多种集体运动模式

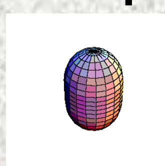
• 振动

Monopole

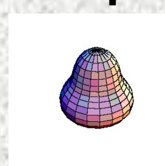


Dipole

Qudropole

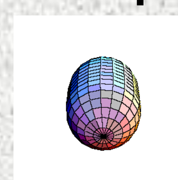


Octupole

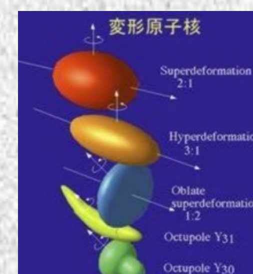


• 转动

Qudropole



Other modes



➔ 原子核具有丰富的集体运动相（形状相）

♠ 理论研究方法

- **扩展的壳模型**

  BCS、无规位相近似(RPA)、投影壳模型(Projected SM)

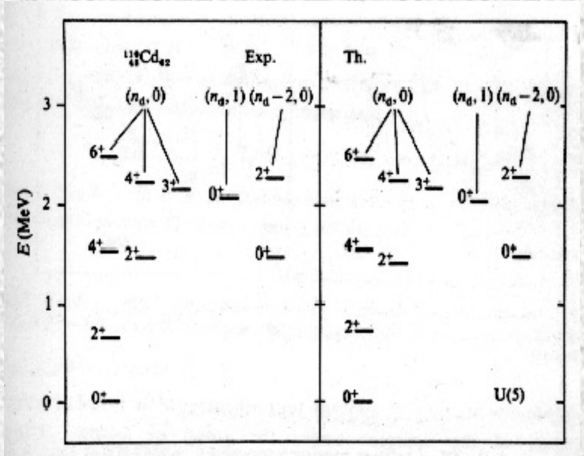
- **集体模型**

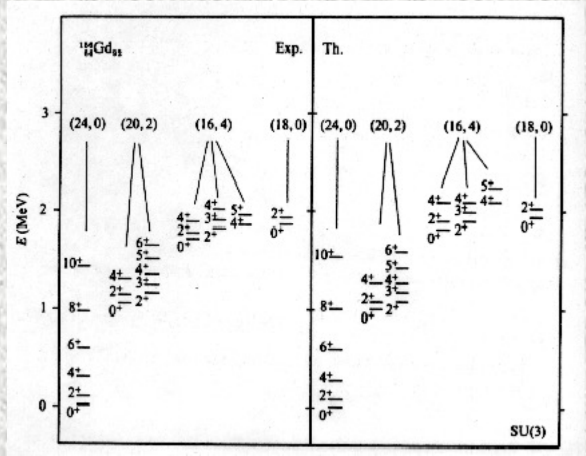
$$H=-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin3\gamma}\frac{\partial}{\partial\gamma}\left(\sin3\gamma\frac{\partial}{\partial\gamma}\right)\right]+\sum_{\lambda}\frac{\hbar^2\omega^2L^2}{2\Im_\lambda}+V(\beta,\gamma)$$

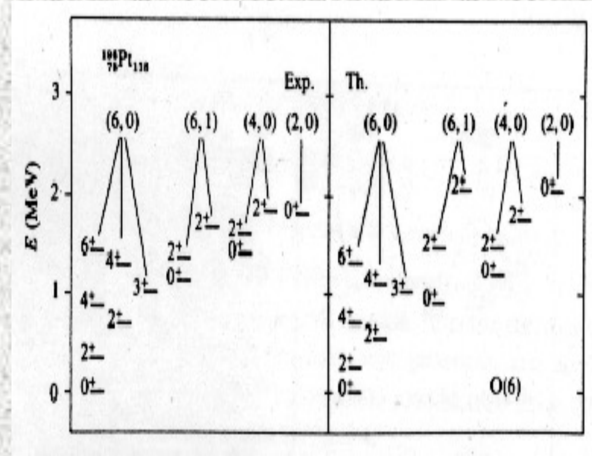
- **代数模型**

  SU(3)、Pseudo-SU(3)、IBM、IBFM、FDSM,

最简单的IBM (sdIBM1) 下的能谱及与实验测量结果的比较







2. 原子核的集体运动模式相变

♠ 原子核具有丰富的集体运动相（形状相）。

♠ 在一些因素影响下，原子核的集体运动相会发生变化，即有集体运动模式相变（形状相变）。

例如：

Isospin
excitation energy (ε, T)
angular momentum
Interaction Strengths ……

Deformation parameter

Control Parameter

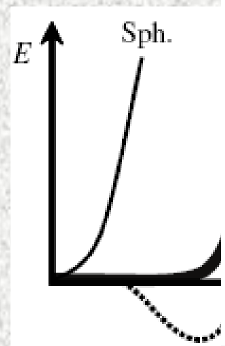
Order Parameter

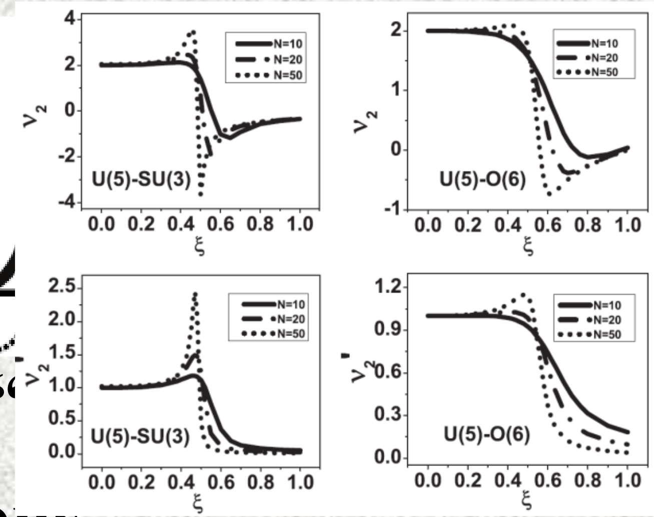
Nuclear Shape Phase Transition
and
Shape Coexistence

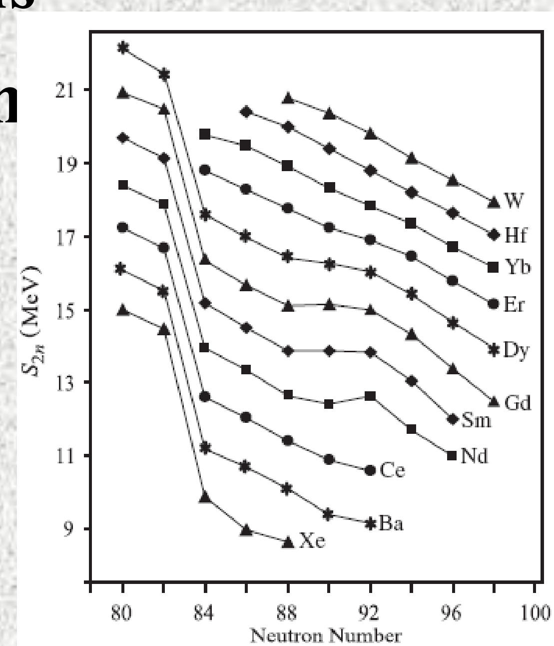
♠ 表征原子核形状相变的标志量

- Potential energy surface
- Energy spectrum
- B(E2) ratios
- Ratio of E2 gamma-ray energy over “
- Overlap of ground state wave functions
- Fraction of the “deformed” boson num
- Enhanced E0 transition
- Two neutron separation energy

••••••••







3. IBM下对原子核集体运动相及相变的研究

♠ 基本方法 (以具有U(6)对称性的IBM1为例)

♣ 哈密顿量

$$\hat{H} = E_0 + \varepsilon_d C_{1U(5)} + A_1 C_{2U(5)} + A_2 C_{2O(6)} + A_3 C_{2SU(3)} + B C_{2O(5)} + C C_{3O(3)},$$

♣ 投影相干态（近似表述）

N个 s、d 玻色子系统的投影(内禀)相干态近似表述为:

$$|N,\beta,\gamma\rangle = \left[s^{+} + \beta(d_0^{+}\cos\gamma + \tfrac{1}{\sqrt{2}}\sin\gamma(d_2^{+} + d_{-2}^{+}))\right]^N |0\rangle ,$$

♣ 玻色子表述与集体模型中参数的对应关系

原理：不同表象中E2跃迁矩阵元相等。

$$\langle \beta_2,\gamma_2 | \hat{T}_{CM}(E2) | \beta_2,\gamma_2 \rangle = \langle N,\beta,\gamma | \hat{T}_b(E2) | N,\beta,\gamma \rangle ,$$

$$eQ_0(CM) \approx \tfrac{4}{5} eZ \{\sum_{k=1}^{Z} r_k^2\} \beta_2, \quad e_B Q_0(b) \approx e_B N \frac{2\beta - \sqrt{2/7}\chi\beta^2}{1+\beta^2}, \quad \rightarrow \beta_2 \approx \frac{5 e_B N}{4eZR_0^{\,2}} \frac{2\beta - \sqrt{2/7}\chi\beta^2}{1+\beta^2}.$$

于是，(近似) 有 $\beta \propto \beta_2$ ， $\gamma = \gamma_2$.

Ginocchio， P. Van Isacker，Chen，Dieperink, Feng, …

理论基础介绍见刘玉鑫《物理学家用李群李代数》。

♠ 位能曲面与集体运动相

♣ 一般情况

- **不考虑角动量投影**

位能面泛函： $E(N,\beta,\gamma)=\frac{<N;\beta,\gamma|\hat{H}|N;\beta,\gamma>}{<N;\beta,\gamma|N;\beta,\gamma>}$,

稳定态条件： $\frac{\partial E}{\partial \beta}=0$ ， $\frac{\partial E}{\partial \gamma}=0$.

- **考虑角动量投影**

角动量投影算符： $P_{MK}^{L}=\frac{2L+1}{8\pi^2}\int D_{MK}^{L}{}^{*}(\Omega)R(\Omega)d\Omega$,

位能面泛函： $E_{gsb}(N,L,\beta,\gamma)=\frac{\langle N;\beta,\gamma|\,\hat{H}P_{00}^{L}\,|N;\beta,\gamma\rangle}{\langle N;\beta,\gamma|\,P_{00}^{L}\,|N;\beta,\gamma\rangle}$.

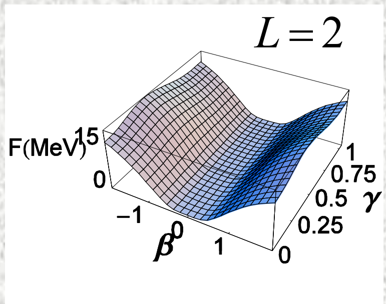
稳定条件： $\frac{\partial E}{\partial \beta}=0$, $\frac{\partial E}{\partial \gamma}=0$.

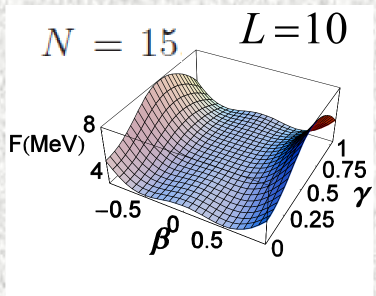
Ring-Schuck's Book, Sun & Hara, Kuyucack & Morrison, … , Leviatan & Ginocchio, PRL 90, 212501 ('03)

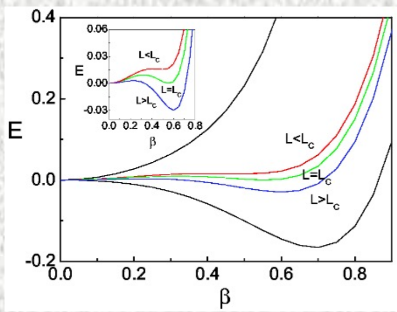
♣ Numerical Results

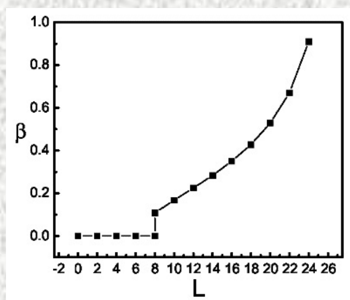
• U(5) Symmetry

For $A+B<0,\ B<0$





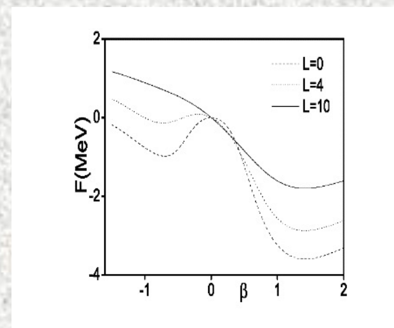




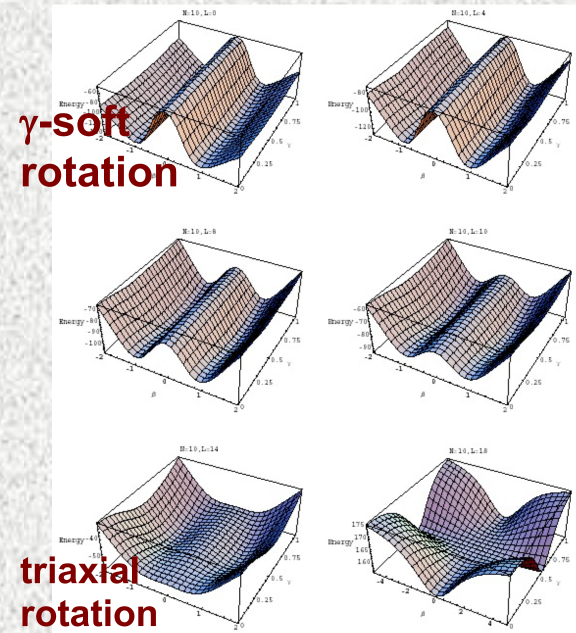
➔ L induces vibrational state becomes axial deformation (rotational) state.

Liu & Mu, Phys. Lett. B 633, 49 (2006), Zhao, Liu, Mu, & Liu, IJMPE 15 (2006), 1711

• SU(3) Symmetry



• O(6) Symmetry



➔ With the increasing of angular momentum,
the γ-soft rotational state becomes triaxial rotational state,
further shape-coexistence.

Zhao, Liu, Mu, & Liu, IJMPE 15 (2006), 1711

♣ 同位旋驱动的集体运动模式相变

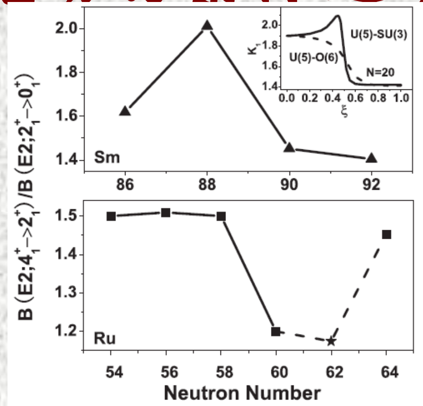
- **哈密顿量**

$$
\begin{aligned}
\hat{H} &= \xi \hat{n}_d + \frac{1-\xi}{N}\hat{Q}(\chi)\cdot\hat{Q}(\chi) \\
&= \Big[\xi - \frac{2(1-\xi)\chi}{7N}(\chi+\frac{\sqrt{7}}{2})\Big]C_{1,U(5)} + \frac{2(\xi-1)\chi}{7N}(\chi+\frac{\sqrt{7}}{2})C_{2,U(5)} \\
&\quad - \frac{(\xi-1)\chi}{\sqrt{7}N}C_{2,SU(3)} + \frac{2(\xi-1)(\chi+\frac{\sqrt{7}}{2})}{\sqrt{7}N}C_{2,O(6)} \\
&\quad - \frac{2(\xi-1)(\chi+\frac{\sqrt{7}}{2})(\chi+\sqrt{7})}{7N}C_{2,O(5)} - \frac{(1-\xi)\chi(\chi+2\sqrt{7})}{14N}C_{2,O(3)}
\end{aligned}
$$

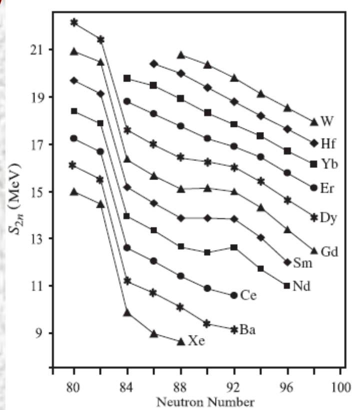
其中的$\xi \propto N_{对}$，因此可由该哈密顿量出发通过数值计算进行研究。

已发现各质量区都有同位旋驱动的形状相变。

- **同位旋驱动的集体运动模式相变实例**

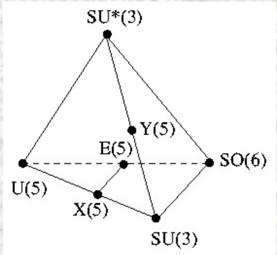


两个同位素链核的E2跃迁分支比

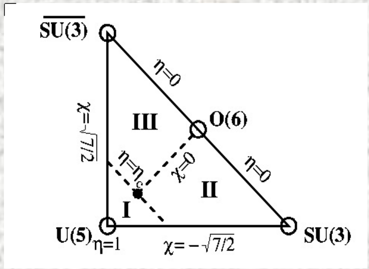


稀土区原子核的双中子分离能

- **并存在临界点对称性**



Iachello, PRL 91, 132502 ('03)

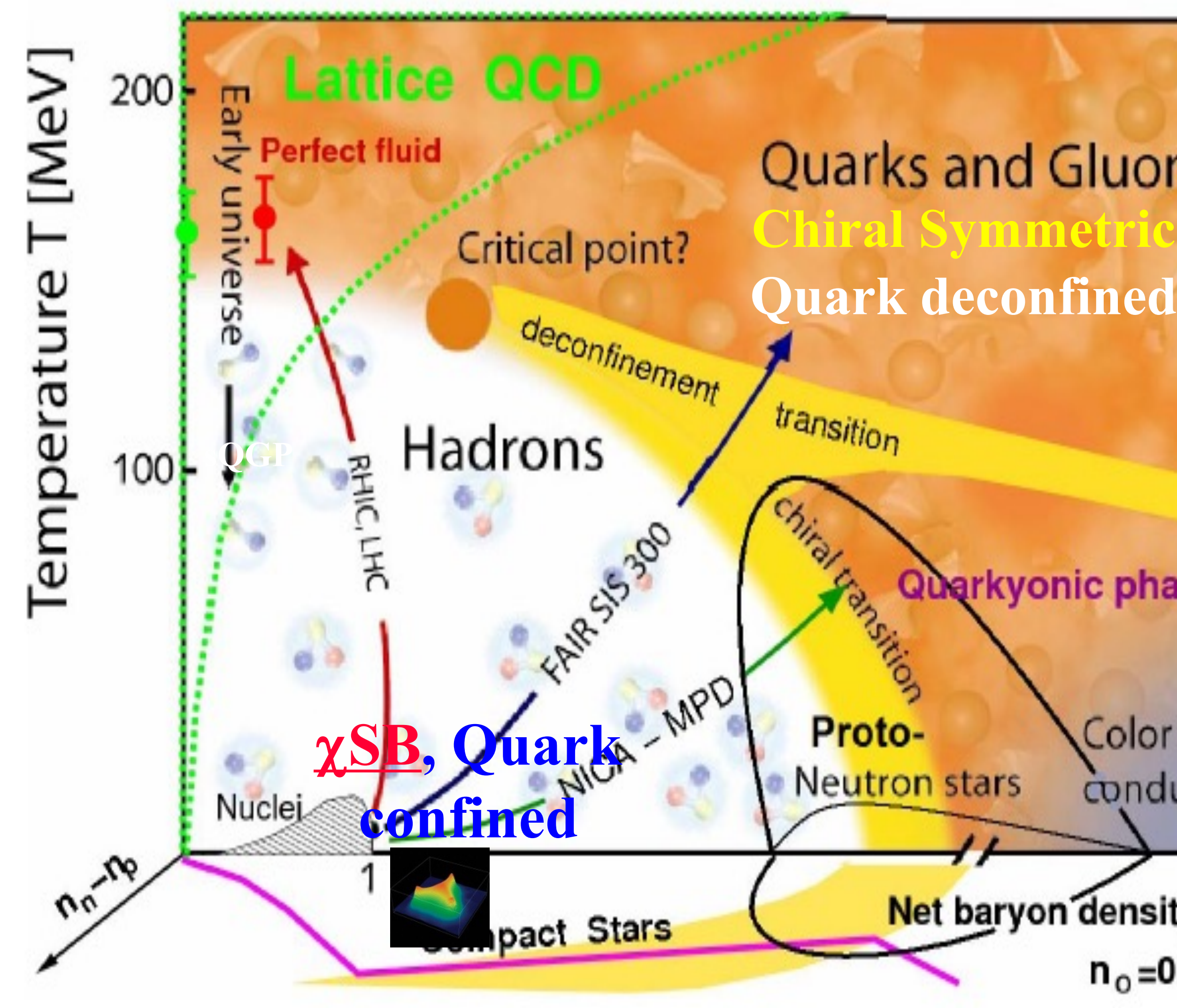


Jolie, et. al., PRL 89, 182503 ('02);
Y. Zhang, YXL, et al., PLB 732, 55 (2014).

➔ **不同模式的对称性破缺使得不同模式的集体运动得以涌现!**

III. 强相互作用物质的相与相变 (QCD相变)

1. 概述



Schematic QCD phase diagram for nuclear matter. The solid lines show the
phase boundaries for the indicated phases. The solid circle depicts the critical

December 2007

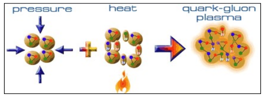
涉及相变：

禁闭（强子化）– 退禁闭

手征对称性破缺 – 恢复

影响QCD相变的因素：

介质效应： 温度，



密度 (化学势)

有限尺度

内禀因素： 流质量，

跑动耦合强度，

色味结构，•••

研究方法：

实验： RHIC、Ast-Obs.

理论： 离散场论、连续场论

计算： 实现理论、模拟

2. QCD相变的基本特征及对相关理论研究的基本要求

♣ 基本特征

- 涉及手征和禁闭两类相变;
- 非微扰效应

  两类相变都发生在非微扰能区(10^2 MeV);

♣ 对理论方法的基本要求:

(1) 统一包含两类相变

- DCSB & its Restoration
- Confinement & Deconfinement

的特征和性质,

(2) 强相互作用的非微扰方法。

3. QCD的手征对称性及其破缺

♠ 两味系统 有对称性 $SU_L(2)\otimes SU_R(2) \cong SU_V(2)\otimes SU_{AV}(2)$.

♣ 手征对称性破缺

前述对称性表明，存在宇称相反的手征伙伴强子态，

但事实上不存在，

例如： $m_\pi \approx 138$ MeV, $m_\sigma \approx (400\sim550)$ MeV;

$m_\rho \approx 775$ MeV, $m_{a1} \approx 1230$ MeV .

→前述的手征对称性一定被破缺。

由于$SU_V(2)$一直近似保持，则有破缺

$$SU_L(2)\otimes SU_R(2) \supset SU_V(2)\otimes U_A(1) \ .$$

手征极限下，→ 强子和Goldstone玻色子；

超越手征极限下，

Goldstone玻色子实际为赝Goldstone玻色子，如π,

并有GOR关系： $m_\pi^2 f_\pi^2 = -(m_u^0 + m_d^0)\langle \bar{q}q \rangle$.

♠ 三味系统：性质类似，但空间更大、关系表述较复杂，

♠ 产生可见质量的手征对称性破缺是动力学破缺

♣ The conventional Higgs mechanism can not generate the observable mass !

♣ Dynamical chiral symmetry breaking

$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle \bar{q}q \rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}$$

$$\langle \bar{q}q \rangle \neq 0$$ ➡ **DCSB**

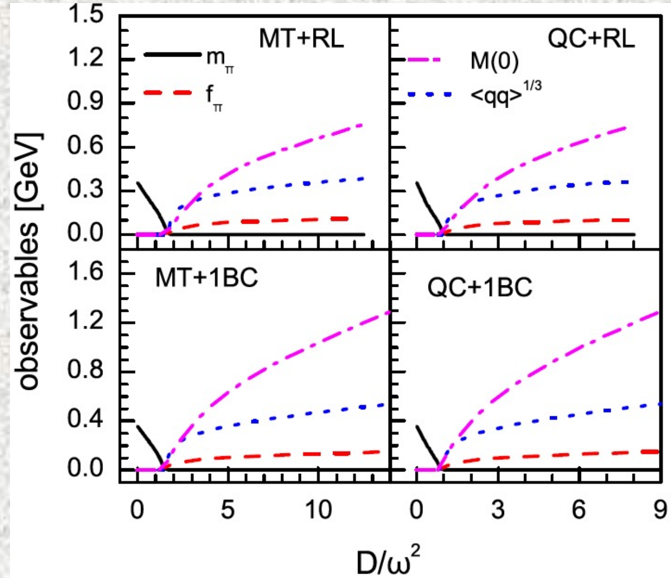
In Dyson-Schwinger Eq. approach

$$M(p^2) = \frac{B(p^2)}{A(p^2)}$$

Numerical results in chiral limit

➔ Increasing the interaction strength induces the dynamical mass generation

i.e., the CSB is DCSB !

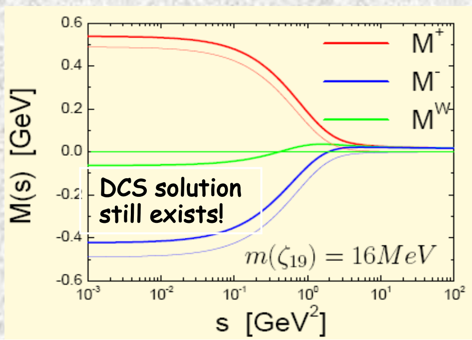


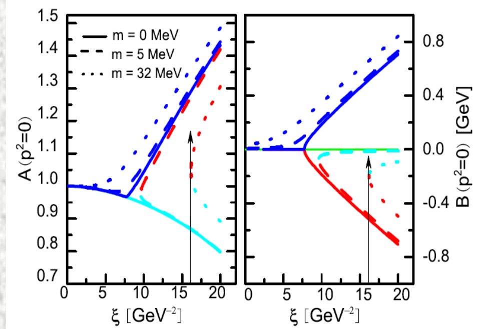
K.L. Wang, YXL, et al., Phys. Rev. D 86,114001(2012); …

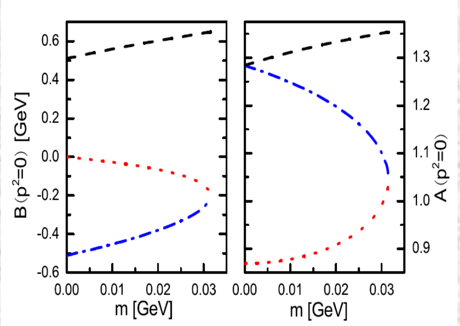
♣ Dynamical Chiral Symmetry Breaking (DCSB) still exists when there has been ECSB

L. Chang, Y. X. Liu, C. D. Roberts, et al, arXiv: nucl-th/0605058;
R. Williams, C.S. Fischer, M.R. Pennington, arXiv: hep-ph/0612061;
K. L. Wang, Y. X. Liu, & C. D. Roberts, Phys. Rev. D 86, 114001 (2012).

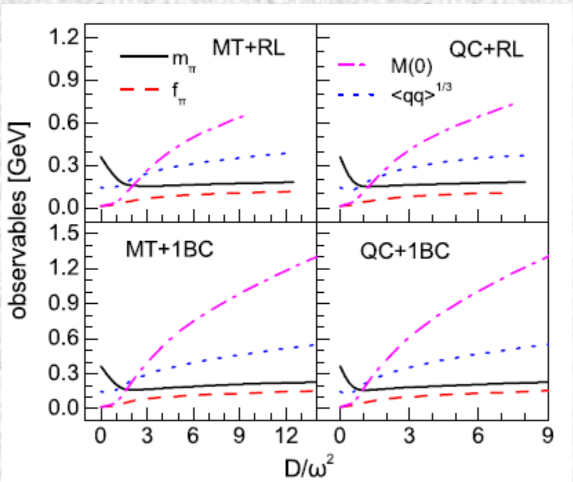
Solutions of the DSE with MT model and QC model for the gluon propagator, bare & 1BC model for the quark-gluon vertex :

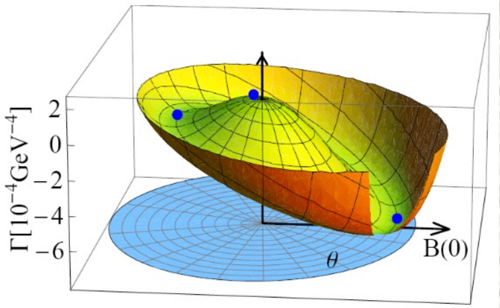






Conditions: Interaction strength large enough, m not very large.





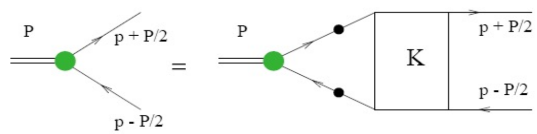
2nd order Phase Transition shifts to crossover.

♣ The masses of observable particles can have been described in DSE approach of QCD

• Meson — Poincare covariant BSE with DSE

Quantum field theory bound states: BSE

$$\Gamma_M(p;P) = \int_k^{\Lambda} K(p,k;P)\, S(k_+)\, \Gamma_M(k;P)\, S(k_-)$$



L. Chang, C.D. Roberts, PRL 103, 081601 (2009);

……

Interaction	Eq. (6)	Eq. (8)	Eq. (8)	Eq. (8)	Eq. (8)
$(D\omega)^{1/3}$	0.72	0.8	0.8	0.8	0.8
ω	0.4	0.4	0.5	0.6	0.7
$m_{u,d}^{\zeta}$	0.0037	0.0034	0.0034	0.0034	0.0034
m_s^{ζ}	0.084	0.082	0.082	0.082	0.082
$A(0)$	1.58	2.07	1.70	1.38	1.16
$M(0)$	0.50	0.62	0.52	0.42	0.29
m_π	0.138	0.139	0.134	0.136	0.139
f_π	0.093	0.094	0.093	0.090	0.081
$\rho_\pi^{1/2}$	0.48	0.49	0.49	0.49	0.48
m_K	0.496	0.496	0.495	0.497	0.503
f_K	0.11	0.11	0.11	0.11	0.10
$\rho_K^{1/2}$	0.54	0.55	0.55	0.55	0.55
m_ρ	0.74	0.76	0.74	0.72	0.67
f_ρ	0.15	0.14	0.15	0.14	0.12
m_ϕ	1.07	1.09	1.08	1.07	1.05
f_ϕ	0.18	0.19	0.19	0.19	0.18
m_σ	0.67	0.67	0.65	0.59	0.46
$\rho_\sigma^{1/2}$	0.52	0.53	0.53	0.51	0.48

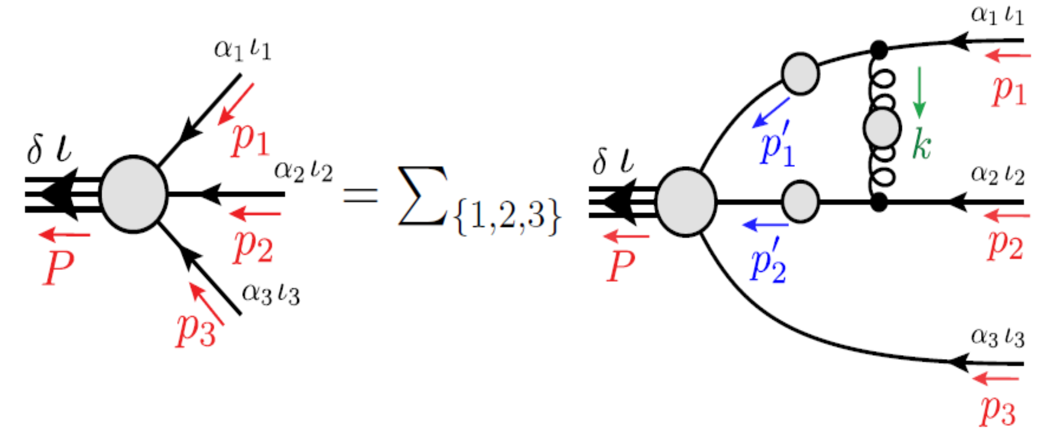
(S.X. Qin, et al., Phys. Rev. C 84, 042202(R) (2011))

	CLRQ vertex	Expt.	RL-Padé	RL-direct
m_π	0.138	0.138	0.138	0.137
m_ρ	0.84 ± 0.03	0.777	0.754	0.758
m_σ	1.13 ± 0.01	0.4 – 1.2	0.645	0.645
m_{a_1}	1.28 ± 0.01	1.24 ± 0.04	0.938	0.927
m_{b_1}	1.24 ± 0.10	1.21 ± 0.02	0.904	0.912
$m_{a_1} - m_\rho$	0.44 ± 0.04	0.46 ± 0.04	0.18	0.17
$m_{b_1} - m_\rho$	0.40 ± 0.14	0.43 ± 0.02	0.15	0.15

(L. Chang, et al., Phys. Rev. C 85, 052201(R) (2012))

• Nucleon — Relativistic three-body problem,

Poincare covariant Faddeev Equation



$$\Psi^{\alpha_1\alpha_2\alpha_3,\delta}_{\iota_1\iota_2\iota_3,\iota}(p_1, p_2, p_3) = \sum_{j=1,2,3} [\mathcal{K}SS\Psi]_j,$$

$$[\mathcal{K}SS\Psi]_3 = \int_{dk} \mathcal{K}^{\alpha_1\alpha_1',\alpha_2\alpha_2'}_{\iota_1\iota_1'\iota_2\iota_2'}(p_1, p_2; p_1', p_2') S^{\alpha_1'\alpha_1''}_{\iota_1'\iota_1''}(p_1') S^{\alpha_2'\alpha_2''}_{\iota_2'\iota_2''}(p_2') \Psi^{\alpha_1''\alpha_2''\alpha_3;\delta}_{\iota_1''\iota_2''\iota_3;\iota}(p_1', p_2', p_3),$$

$$\mathcal{K}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'},$$

$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(k^2)T_{\mu\nu}(k), \quad k^2 T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu.$$

$$\frac{1}{Z_2^2}\tilde{\mathcal{G}}(s) = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2\gamma_m \mathcal{F}(s)}{\ln[\tau + (1 + s/\Lambda_{QCD}^2)^2]},$$

• Numerical results in RL approximation

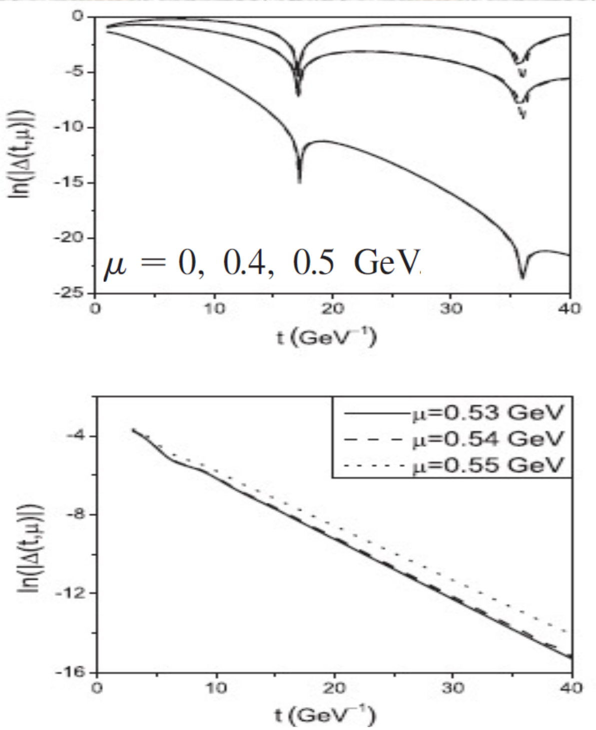
Baryon	quarks	Empirical [90]			Herein				
		$(0,\frac{1}{2}^+)_3$	$(1,\frac{1}{2}^+)_4$	$(0,\frac{1}{2}^-)_5$	$(0,\frac{1}{2}^+)_6$	$(1,\frac{1}{2}^+)_7$	$(0,\frac{1}{2}^-)_8$	$(1,\frac{1}{2}^+)_9^*$	$(0,\frac{1}{2}^-)_{10}^*$
N	*uud*	0.938	1.440	1.535	0.948	1.279	1.144	1.440	1.542
Λ	*uds*	1.116	1.600	1.670	1.114	1.474	1.316	1.582	1.581
Σ	*uus*	1.189	1.660	1.620	1.114	1.474	1.316	1.582	1.581
Ξ	*uss*	1.315			1.279	1.670	1.487	1.723	1.620
Λ_c	*udc*	2.286		2.595	2.184	2.543	2.401	2.650	2.666
Σ_c	*uuc*	2.455			2.184	2.543	2.401	2.650	2.666
Λ_b	*udb*	5.619		5.912	5.394	5.809	5.650	5.916	5.916
Σ_b	*uub*	5.811			5.394	5.809	5.650	5.916	5.916
Ξ_c	*usc*	2.468		2.790	2.350	2.738	2.572	2.792	2.705
Ξ_c'	*usc*	2.577			2.350	2.738	2.572	2.792	2.705
Ξ_{cc}	*ucc*	3.621			3.421	3.807	3.657	3.861	3.790
Ξ_b	*usb*	5.792			5.560	6.004	5.822	6.058	5.955
Ξ_b'	*usb*	5.945			5.560	6.004	5.822	6.058	5.955
Ξ_{cb}	*ucb*				6.631	7.073	6.907	7.127	7.040
Ξ_{cb}'	*ucb*				6.631	7.073	6.907	7.127	7.040
Ξ_{bb}	*ubb*				9.841	10.339	10.157	10.393	10.289
Ω_c	*ssc*	2.695			2.516	2.934	2.744	2.934	2.744
Ω_{cc}	*scc*				3.586	4.002	3.829	4.002	3.829

S.X. Qin, C.D. Roberts, S.M. Schmidt, Few-body Syst. 60, 26 (2019); S.X. Qin, et al., Phys. Rev. D 97,114017('18); etc.

♠ Confinement

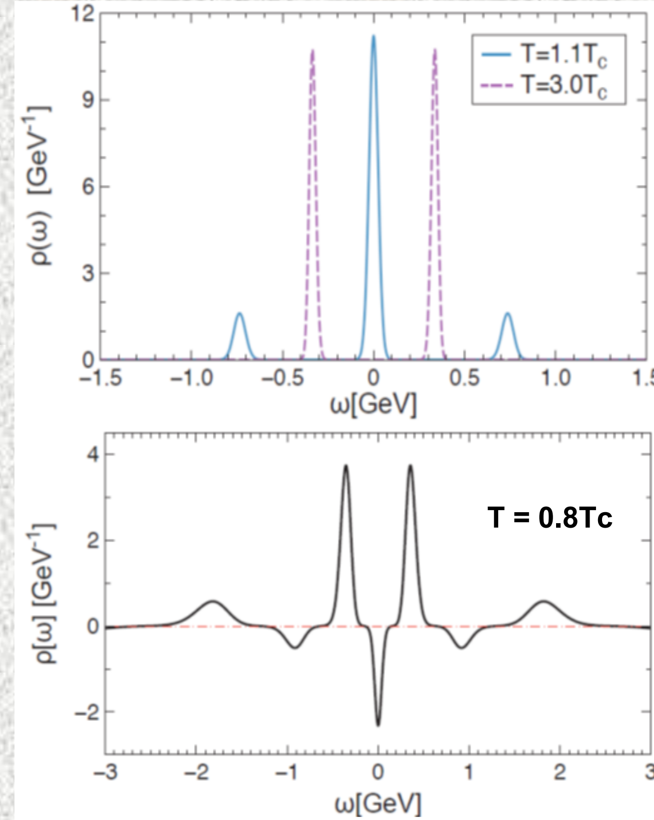
Practical particle: spectral density is positive !

positivity violation → quarks are confined.



$$\Delta(\tau,\mu) = \int \frac{d^4p}{(2\pi)^4} e^{i\vec{p}\cdot\vec{x}+ip_4\tau}\delta(\vec{p})\sigma_B(p;\mu).$$

H. Chen, YXL, et al., Phys. Rev. D 78, 116015 (2008)



S.X. Qin, D. Rischke, Phys. Rev. D 88, 056007 (2013)

$$S^R(\omega,\vec{p}) = S(i\omega_n,\vec{p})|_{i\omega_n\to\omega+i\epsilon}$$

$$S(i\omega_n,\vec{p}) = \int_{-\infty}^{+\infty}\frac{d\omega'}{2\pi}\frac{\rho(\omega',\vec{p})}{i\omega_n-\omega'}$$

$$\rho(\omega,\vec{p}) = -i\vec{\gamma}\cdot\vec{p}\,\rho_v(\omega,\vec{p}^2) + \gamma_4\omega\,\rho_e(\omega,\vec{p}^2) + \rho_s(\omega,\vec{p}^2)$$

In MEM

$$P[\rho|M(\alpha)] = \frac{1}{Z_S}e^{\alpha S[\rho,m]},$$

$$S[\rho,m] = \int_{-\infty}^{+\infty} d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$$

$$m(\omega) = m_0\theta(\Lambda^2-\omega^2).$$

→ 不同模式的对称性破缺使得不同模式的粒子得以涌现!

4. 关于QCD相变的研究需要新的相变判据

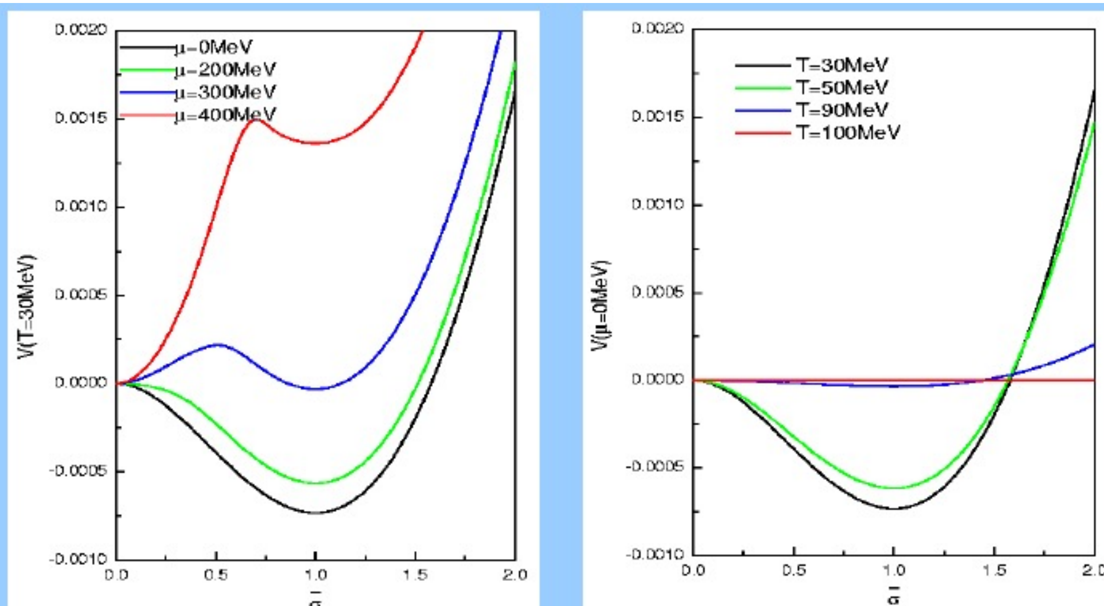
♠ Criterion determining the phase boundary line & the position of the CEP

♣ Conventional Criterion

Order Parameter: *chiral cond.* $\langle \bar{q} q \rangle$!

$$\mathcal{M}(p) \simeq m_0 \left[\ln p/\Lambda_{QCD}\right]^d + C \frac{-\langle \bar{q} q \rangle}{p^2 \left[\ln p/\Lambda_{QCD}\right]^d}$$

Procedure: Analyzing the ThDyn Potential



Signature of PT: $\frac{\partial^2 \Omega}{\partial T^2}$, $\frac{\partial^2 \Omega}{\partial \mu^2}$, etc., change sign .

Question:

In case of completely nonperturbative,
one can not have the thermodynamic potential.

The conventional criterion fails.

One needs then new criterion!

♣ New Criterion: Chiral Susceptibility

- **Def.:** Response of the order parameter to control variables

$$\frac{\partial M}{\partial T},\ \frac{\partial M}{\partial \mu};\quad \frac{\partial\langle\bar{q}q\rangle}{\partial T},\ \frac{\partial\langle\bar{q}q\rangle}{\partial \mu};\quad \frac{\partial B}{\partial T},\ \frac{\partial B}{\partial \mu};\quad \frac{\partial B}{\partial m_0};$$

- **Simple Demonstration Equiv. of New-C to Conv-C**

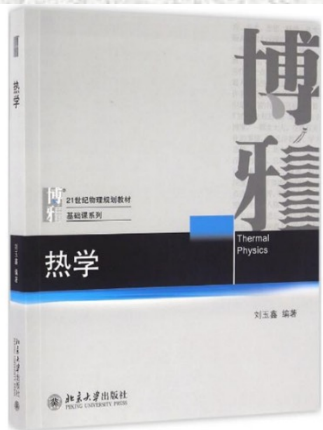
TD Potential:

$$\Omega(T,\eta) = \Omega_0(T) + \frac{1}{2}\alpha\eta^2 + \frac{1}{4}\beta(\eta^2)^2 + \frac{1}{6}\gamma(\eta^2)^3 + \cdots$$

Stability Condition:

$$\frac{\partial\Omega}{\partial\eta} = \alpha\eta + \beta\eta^3 + \gamma\eta^5 = 0$$

$$\frac{\partial^2\Omega}{\partial\eta^2} = \alpha + 3\beta\eta^2 + 5\gamma\eta^4 > 0,\ St.;\ < 0,\ Unst..$$



Derivative of stab. cond. against control var.:

$$[\alpha + 3\beta\eta^2 + 5\gamma\eta^4]\left(\frac{\partial\eta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta\left(\frac{\partial\alpha}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial\beta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial\gamma}{\partial\varsigma}\right)_{\varsigma=\zeta_c} = 0$$

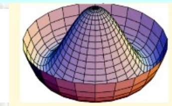
we have:

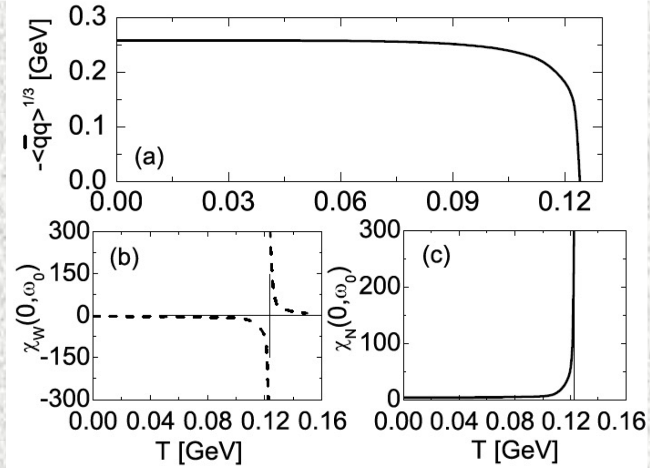
$$\chi = \left(\frac{\partial\eta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} = -\frac{\eta\left(\frac{\partial\alpha}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^3\left(\frac{\partial\beta}{\partial\varsigma}\right)_{\varsigma=\zeta_c} + \eta^5\left(\frac{\partial\gamma}{\partial\varsigma}\right)_{\varsigma=\zeta_c}}{\left(\frac{\partial^2\Omega}{\partial\eta^2}\right)_{\frac{\partial\Omega}{\partial\eta}=0}}$$

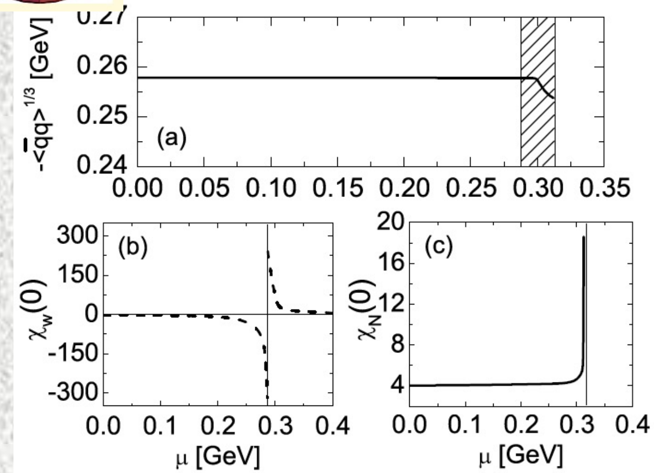
At field theory level, see: Fei Gao, & Y.X. Liu, Phys. Rev. D 94, 076009 (2016).

♣Demonstration of the New Criterion

In chiral limit（$m_0 = 0$）

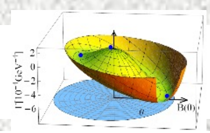




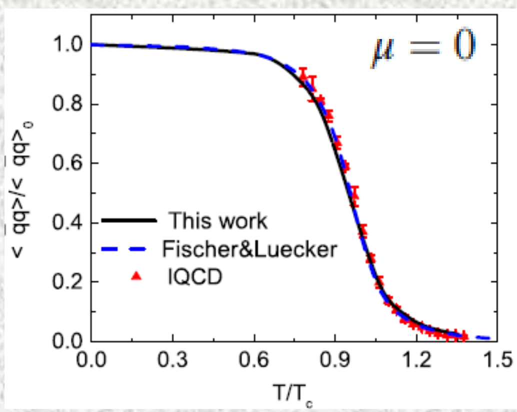


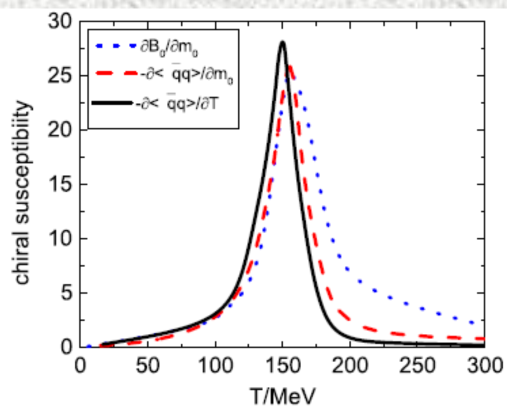
S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

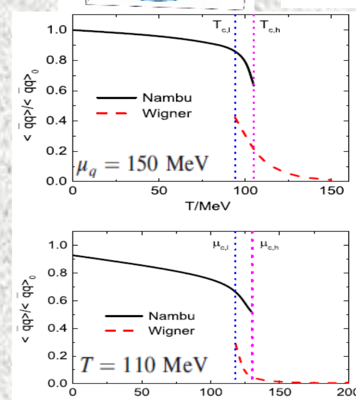
Beyond chiral limit（$m_0 \neq 0$）

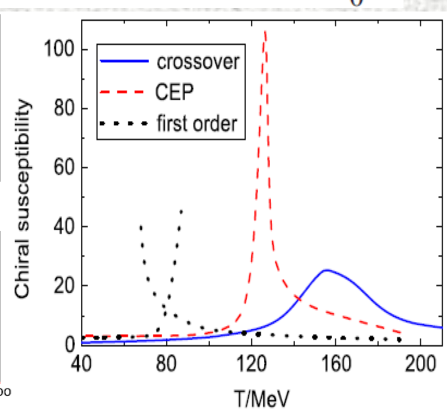


$$\langle\bar{q}q\rangle = \langle\bar{q}q\rangle_{m_0} - m_0 \frac{\partial\langle\bar{q}q\rangle_{m_0}}{\partial m_0}$$









Fei Gao, Y.X. Liu, Phys. Rev. D 94, 076009 (2016) .

♣ Characteristic of the New Criterion

As 2nd order PT (Crossover) occurs,
the χs of the two (DCS, DCSB) phases
diverge (take maximum) at same states.

As 1st order PT takes place,
χs of the two phases diverge at dif. states.

➔ the χ criterion can not only give the phase boundary, but also determine the position the CEP.

For multi-flavor system,
one should analyze the maximal eigenvalue of the susceptibility matrix (L.J. Jiang, YXL, et al., PRD 88, 016008),
or the mixed susceptibility (F. Gao, YXL, PRD 94, 076009).

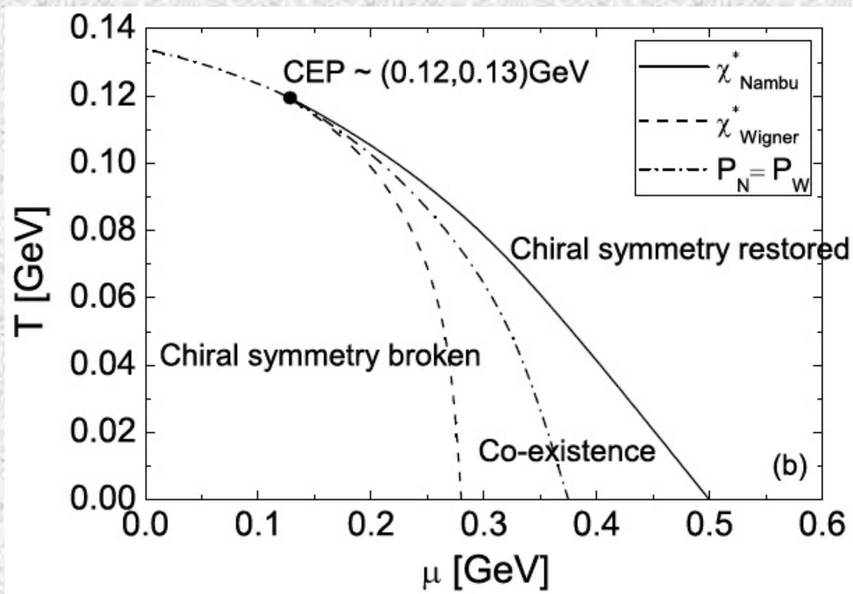
♠ QCD相图的数值结果

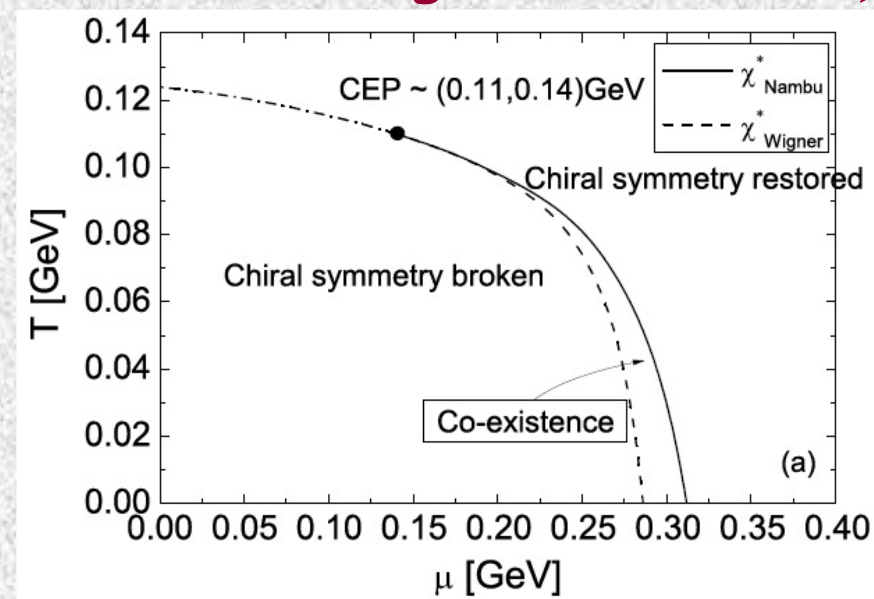
♣ QCD Phase Diagrams & the position of the CEP have been given in the DSE

- In chiral limit

With bare vertex
(ETP is available, the PB is shown as the dot-dashed line)

With Ball-Chiu vertex
(ETP is not available, but the coexistence region is obtained)

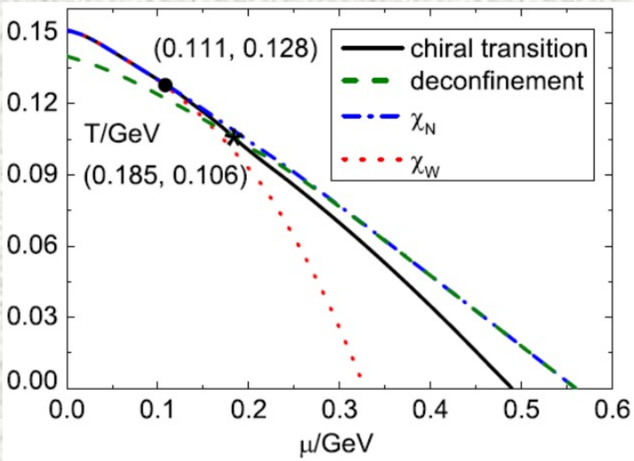




S.X. Qin, L. Chang, H. Chen, YXL, et al., Phys. Rev. Lett. 106, 172301 (2011).

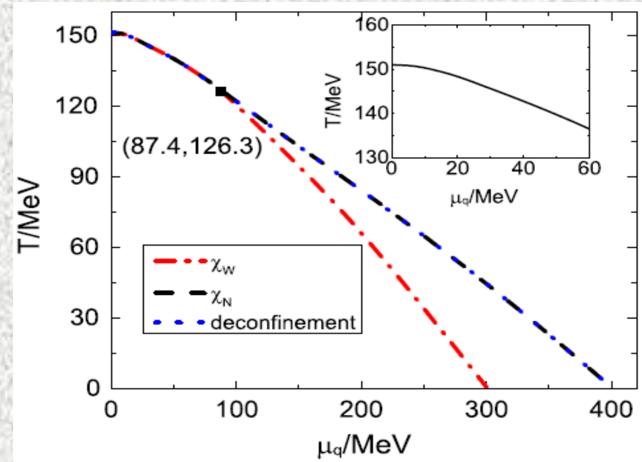
• Beyond Chiral limit

With bare vertex
(ETP is available, the PB is shown as the dot-dashed line)



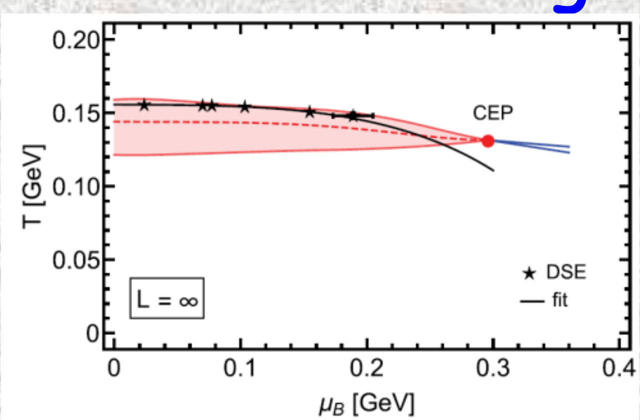
F. Gao, et al., PRD 93, 094019 ('16).

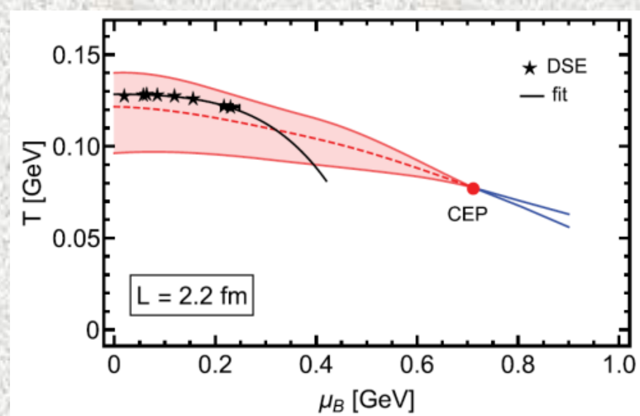
With CLR vertex
(ETP is not available, but the coexistence region is obtained)



F. Gao, Y.X. Liu, PRD 94, 076009 ('16).

• When considering the finite size effect

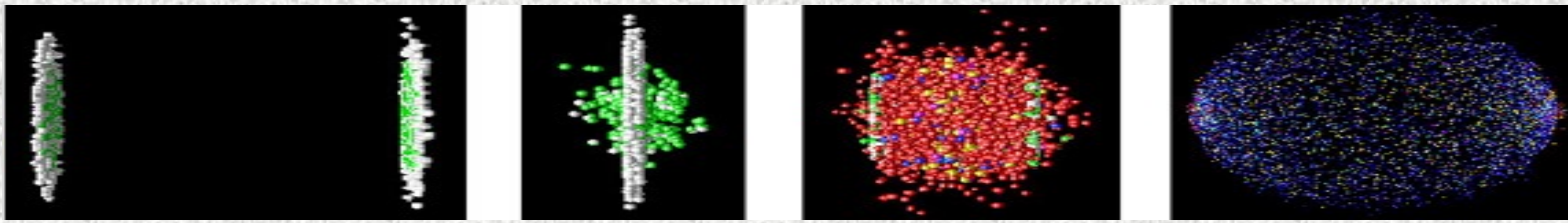




Y. Lu, M.Y. Chen, et al., PRD 105, 034012 ('22).

5. QCD相变的实验室观测

♠ 方法：相对论性重离子碰撞（RHIC）等 (u23u)

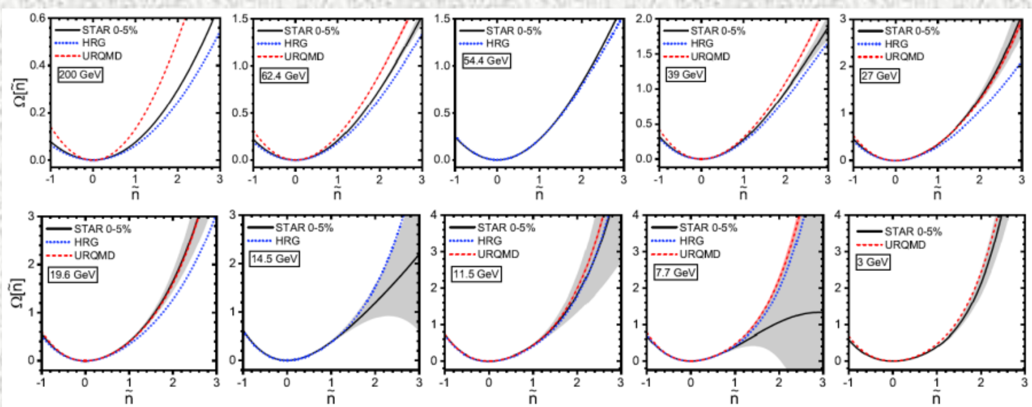


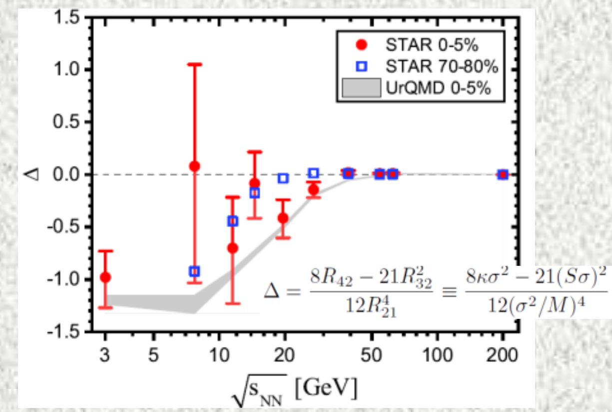
Thin Pancakes Lorentz γ=100	Nuclei pass thru each other < 1 fm/c	Huge Stretch Transverse Expansion High Temperature (?!)	The Last Epoch: Final Freezeout-- Large Volume

We measure the "final" state,
we are most interested in the "intermediate" state,
we need to understand the "initial" state…

♠ 困难：实验测得的是再强子化后的信息，而非相变时的信息！

♠ 一个可能的解决方案：由测得的重子数涨落确定热力学势，提取相变信息。





Y. Lu, F. Gao, X.F. Luo, L. Chang, and Y.X. Liu, arViv: 2211.03401.

IV. 小结

♠ 简述了物质的相、相变及对称性等概念，
讨论了构建物质演化过程的物理描述方案的分析方法。

♠ 简述了原子核的集体运动相和相变的概念，以及研究现状和目前的一些热点。

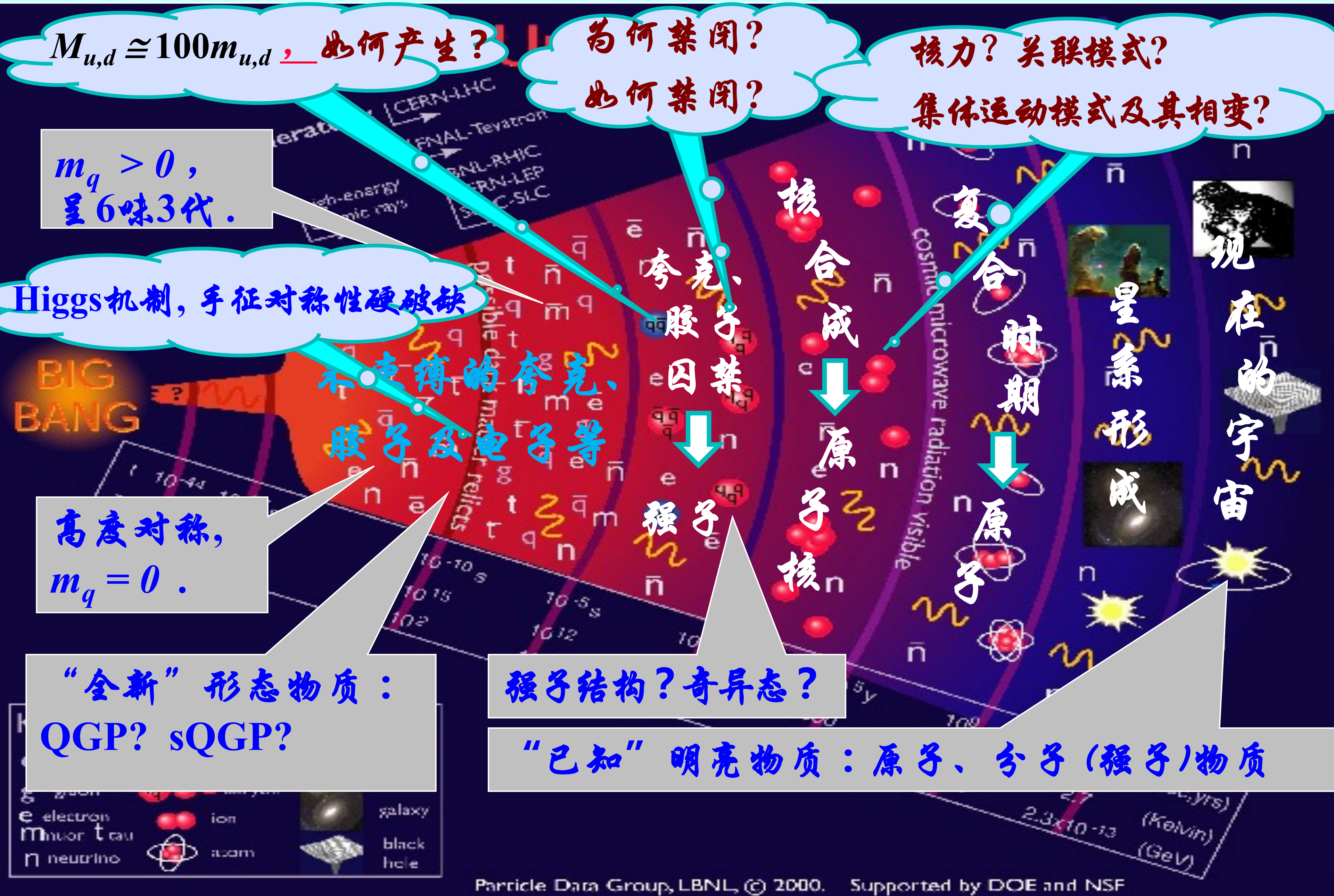
♠ 简述了强相互作用物质的相及相变的概念、范畴和基本研究方法，以及一些研究进展情况和热点问题。

♠ 强相互作用系统的相与相变是不同能标下不同“涌现”的表现，是协调“还原论”与“整体论”的极佳平台。

♠ 强相互作用系统的相与相变的研究
是揭示可见物质及其质量起源的机制和规律的重大课题，
是建立研究有限多体系统相变的一般方法的具体平台，
充满挑战和重大创新机遇，已取得很大进展，但任重道远！

谢 谢 ！

♣ 我们所处宇宙强相互作用物质演化过程概貌



♣ 美国2007年核科学长期规划目录

The Frontiers of Nuclear Science
A LONG RANGE PLAN
December 2007
arXiv:0809.3137

Preface . . . .

1. Overview and Recommendations . . . .

2. The Science 13

Quantum Chromodynamics: From the Structure of Hadrons to the Phases of Nuclear Matter 14

QCD and the Structure of Hadrons 16

The Phases of Nuclear Matter 35

The Emerging QCD Frontier: The Electron-Ion Collider 50

Nuclei: From Structure to Exploding Stars 57

In Search of the New Standard Model 75

3. The Tools of Nuclear Science 93

Facilities for Nuclear Science 94

International Collaborations and Facilities 112

The 2015

LONG RANGE PLAN for NUCLEAR SCIENCE

最新的可参见arXiv:2211.02224

Preface . . . .

1. Summary and Recommendations 3

2. Quantum Chromodynamics: The Fundamental Description of the Heart of Visible Matter 9

   2.1 QCD and the Structure of Hadrons and Nuclei 11

   2.2 QCD and the Phases of Strongly Interacting Matter 21

   2.3 Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics 31

3. Nuclear Structure and Reactions 43

4. Nuclear Astrophysics 53

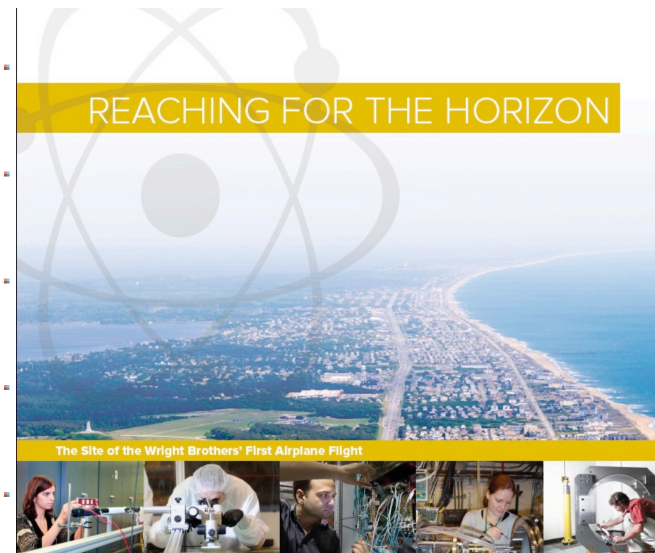
5. Fundamental Symmetries and Neutrinos 63

6. Theoretical Nuclear Physics 81

7. Facilities and Tools 87

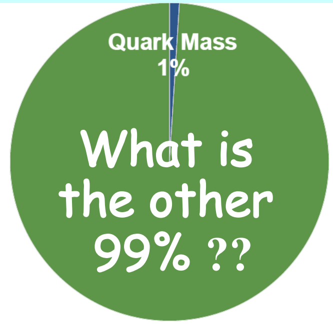
8. Workforce, Education, and Outreach in Nuclear Science 107

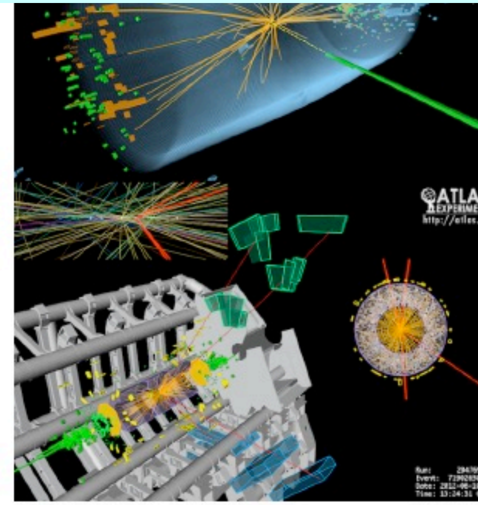
9. Broader Impacts 119

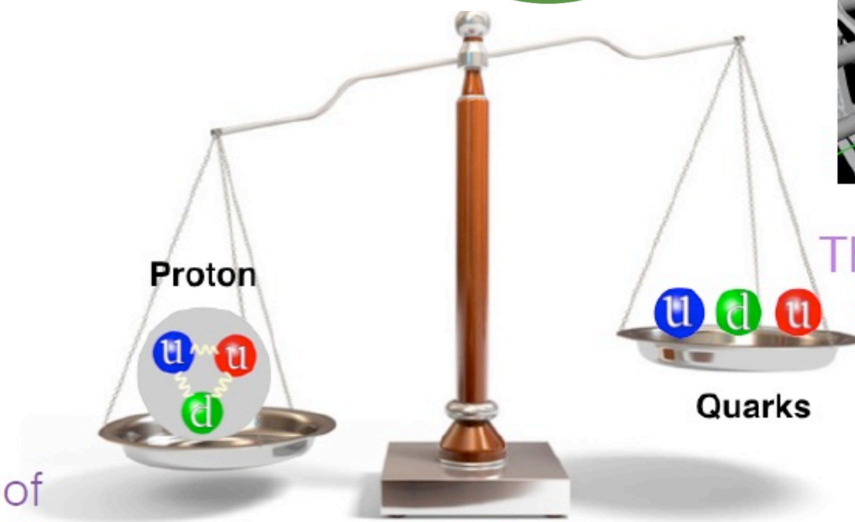


♠ The Nucleon Mass Crisis

How does the mass of nucleon arise ??





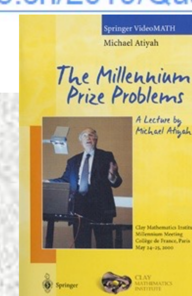


The Higgs boson make the u/d quark having masses (2GeV MS-bar):

But the mass of the proton is
938.272046(21) MeV.
~**100 times of the sum of the quark masses!**

m_u = 2.08(09) MeV
m_d = 4.73(12) MeV

http://flag.unibe.ch/2019/Quark%20masses



"核子质量起源"与"夸克囚禁"
共同构成一个世纪大奖问题!

♠微观粒子具有对称性和对称性破缺

♣对称性：变换下的不变性。

♣群与代数：满足一定条件的对称性变换的集合称为群，群的无穷小生成元的集合称为其相应的代数。

♣常见对称性变换和典型李群及李代数

- U(N) 群 （A_{N-1}李代数）：保模变换的元素的集合；

$$[E^m_{m'},\ E^n_{n'}] = E^m_{n'}\delta_{m'n} - E^n_{m'}\delta_{mn'},$$

其中$E^m_{m'}$为仅m行m'列矩阵元为1、其它矩阵元都为0的矩阵。

- SP(2N)群（C_N李代数）：辛变换不变的元素的集合；

$$[\mathcal{S}_{ij}\ \ \mathcal{S}_{kl}] = \mathcal{H}_{jk}\mathcal{S}_{il} + \mathcal{H}_{jl}\mathcal{S}_{ik} + \mathcal{H}_{ik}\mathcal{S}_{jl} + \mathcal{H}_{il}\mathcal{S}_{jk}\ ,$$

其中 $\mathcal{S}_{ij} = e_i\bar{e}_j + e_j\bar{e}_i,\quad \mathcal{H}_{ij} = \bar{e}_i e_j = -\bar{e}_j e_i,\quad \bar{e}_i = Ge_i,$

- SO(N) 群（ $B_{[N/2]}$ 或 $D_{N/2}$李代数）：正交变换的集合；

$$[\Xi_{ij}\ \ \Xi_{kl}] = G_{ij}\Xi_{kl} - G_{ki}\Xi_{jl} + G_{lj}\Xi_{ki} - G_{li}\Xi_{kj}\ ,$$

其中 $\Xi_{ij} = G_{ik}E^k_j - G_{jk}E^k_i$ ， $G_{ij} = G_{ji}$.

♣ 费密子系统的动力学对称性及其破缺

- **对称性及其破缺**

计算知，费米子产生算符和湮灭算符的二次型 $a_{jm}^{\dagger}a_{jm'}$ 及其叠加的对易关系与U(2*j*+1)、SP(2*j*+1)李代数的对易关系完全相同，这表明，费米子系统具有对称性和对称性破缺

$$\mathrm{U}(2j+1)\supset \mathrm{SP}(2j+1)\supset \mathrm{SO}(3)\,.$$

- **生成元与表示**

(子) 群	生成元	Casimir算子	不可约表示
U(N)	$E_{m\,m'}=a_m^{\dagger}a_{m'}$ $(a^{\dagger}\tilde{a})_q^k$	$\hat{n}_f$ $\frac{\hat{n}_f(N+1-\hat{n}_f)}{2N}$	$[1^n]$
SP(N)	$\Xi_{m\,m'}=a_m^{\dagger}\tilde{a}_{m'}+a_{m'}^{\dagger}\tilde{a}_m$ $(a^{\dagger}\tilde{a})_q^k\quad k=$ 奇数	$\hat{n}_f(N+2-\hat{n}_f)/2+P^{\dagger}\tilde{P}=$ $\sum_{k=\text{奇数},q}(-)^q(a^{\dagger}\tilde{a})_q^k(a^{\dagger}\tilde{a})_{-q}^k$	(1^{ν})
SO(3)	$J_q=\sqrt{j(j+1)(2j+1)/3}(a^{\dagger}\tilde{a})_q^1$	$J^2=-J_{+1}J_{-1}-J_{-1}J_{+1}+J_0^2$	J
SU(2)$_q$	$Q_0=N/4-\hat{n}_f/2,$ $Q_{-1}=P^{\dagger}/2,\ Q_{+1}=\tilde{P}/2$	$Q_0(Q_0+1)-\frac{1}{2}P^{\dagger}\tilde{P}$	Q

其中 $P^{+}=\sqrt{\frac{2j+1}{2}}(a_j^{+}a_j^{+})_0^0.$

♣ 玻色子系统的动力学对称性及其破缺

- **对称性及其破缺**

计算知，玻色子产生算符和湮灭算符的二次型 $b^{\dagger}_{lm}b_{m'}$ 及其叠加的对易关系与U(2*l*+1)、SO(2*l*+1)李代数的对易关系完全相同，这表明，费米子系统具有对称性和对称性破缺

$$\mathrm{U}(2l+1) \supset \mathrm{SO}(2l+1) \supset \mathrm{SO}(3)\,.$$

- **生成元与表示**

(子) 群	生成元	Casimir算子	不可约表示
U(N)	$E_{m\,m'} = b^{\dagger}_{m}b_{m'}$ $(b^{\dagger}\tilde{b})^{k}_{q}$	$\hat{n}_b$ $\dfrac{\hat{n}_b(N-1+\hat{n}_b)}{2N}$	$[n_b]$
SO(N)	$\Xi_{m\,m'} = b^{\dagger}_{m}\tilde{b}_{m'} - b^{\dagger}_{m'}\tilde{b}_{m}$ $(b^{\dagger}\tilde{b})^{k}_{q}\quad k=$ 奇数	$\hat{n}_b(N-2+\hat{n}_b)/2 - P^{\dagger}\tilde{P} =$ $\sum_{k=\text{奇数},\,q}(-)^{q+1}(b^{\dagger}\tilde{b})^{k}_{q}(b^{\dagger}\tilde{b})^{k}_{-q}$	(ν)
SO(3)	$J_q = \sqrt{\dfrac{l(l+1)(2l+1)}{3}}\,(b^{\dagger}\tilde{b})^{1}_{q}$	$L^2 = -L_{+1}L_{-1} - L_{-1}L_{+1} + L_0^2$	L
$\mathrm{SU(2)}_q$	$Q_0 = \frac{N}{4} + \frac{\hat{n}_b}{2}\,,$ $Q_{+1} = P^{\dagger}/2\,,$ $Q_{-1} = \tilde{P}/2$	$Q_0(Q_0-1) - \dfrac{P^{\dagger}\tilde{P}}{2} =$ $\dfrac{\hat{n}_b(N-2+\hat{n}_b)}{4} - \dfrac{P^{\dagger}\tilde{P}}{2} + \dfrac{N(N-4)}{16}$	Q

其中 $P^{\dagger} = \sqrt{\dfrac{2l+1}{2}}\,(b^{\dagger}_{l}b^{\dagger}_{l})^{0}_{0}\,.$

4. 原子核集体运动模式相变临界点的对称性

♠ 求解玻尔哈密顿量

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin3\gamma}\frac{\partial}{\partial\gamma}\sin3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_\kappa\frac{Q_\kappa^2}{\sin^2(\gamma - \frac{2}{3}\pi\kappa)}\right] + V(\beta,\gamma).$$

➔

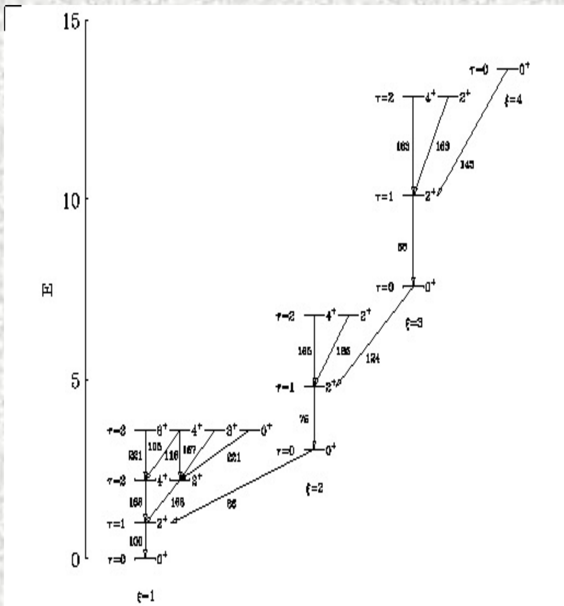
$V(\beta,\gamma) = U(\beta)$ = **infinite well** → **E(5)**

$V(\beta,\gamma) = U(\beta) + U(\gamma) =$ **Inf-W + HO** → **X(5)**

$V(\beta,\gamma) = U(\beta) + U(\gamma) =$ **HO + Inf-W** → **Y(5)**

Spectrum of E(5) Symmetry

$$E_{E(5)} = \frac{\hbar^2}{2B}\left(\frac{\chi_{\xi,\tau}}{\beta_w}\right)^2$$



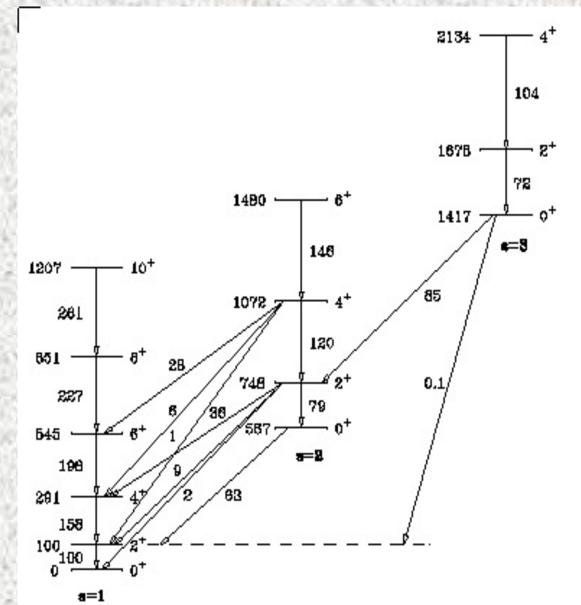
(Iachello, PRL 85, 3580 (2000))

^{134}Ba, ^{108}Pd, ^{130}Xe, ···

(Casten, PRL 85, 3584 (2000);
Ginocchio, PRL 90, 212501 (2003) ;
Zhang & Liu, PRC 65, 057301 (2002);
Clark, et al., PRC 69, 064322 (2004);
Garrel, et al., PRC 73, 054315 (2006);
··········)

Spectrum of X(5) Symmetry

$$E_{X(5)} = \frac{\hbar^2}{2B}\left(\frac{\chi_{s,L}}{\beta_w}\right)^2 + \frac{3a}{\sqrt{<\beta^2>}}(n_\gamma + 1)$$



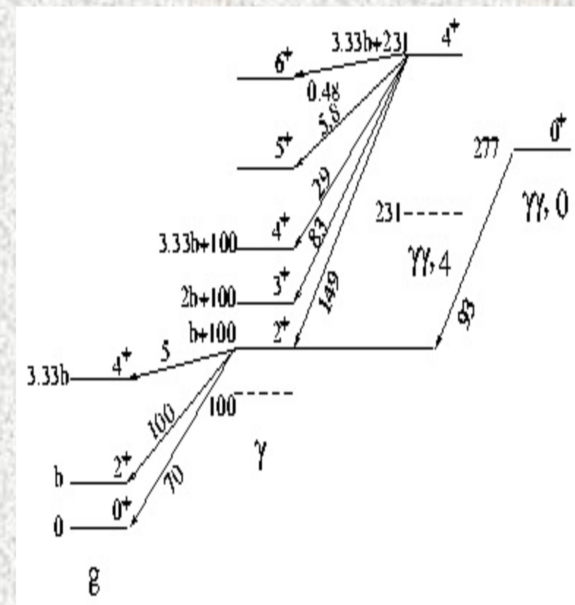
(Iachello, PRL 87, **052502**(2001))

^{152}Sm, ^{154}Gd, ^{156}Dy, ^{150}Nd, ···

(Casten, PRL 87, 052503 (2001);
Capirio, PRC 66, 054310 (2002);
Tonev, PRC 69, 034334 (2004);
Zhang & Liu, CPL 20, 1028 (03);
Dawald, et al., EPJA 20, 173 (04);
Moller, et al., PRC 74, 024313 (06); Mertz, et al., 77, 014307 (08); ·········)

Spectrum of Y(5) Symmetry

$$E(n_\beta, L, s', K, M) = E_0 + B' n_\beta + BL(L+1) + A(x_{s',M'})^2,$$

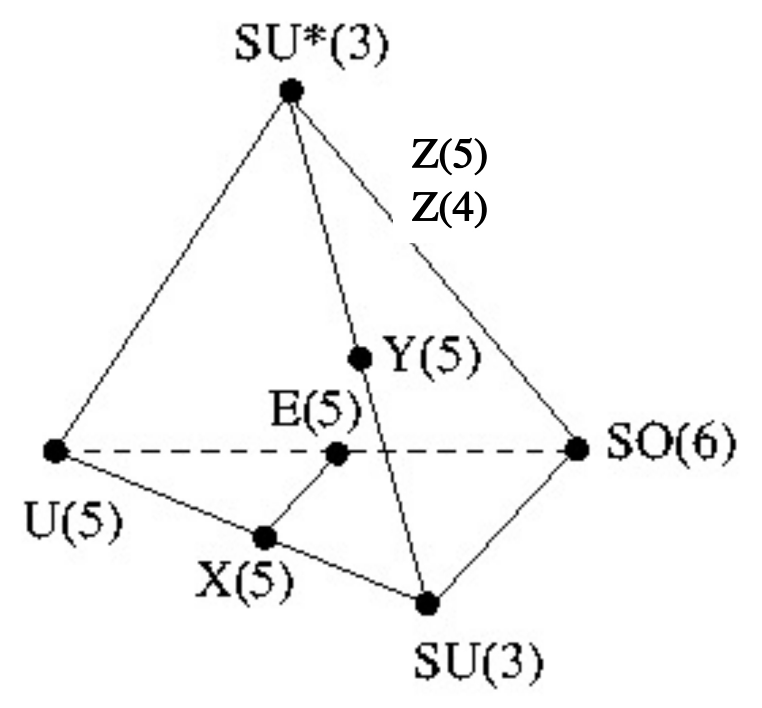


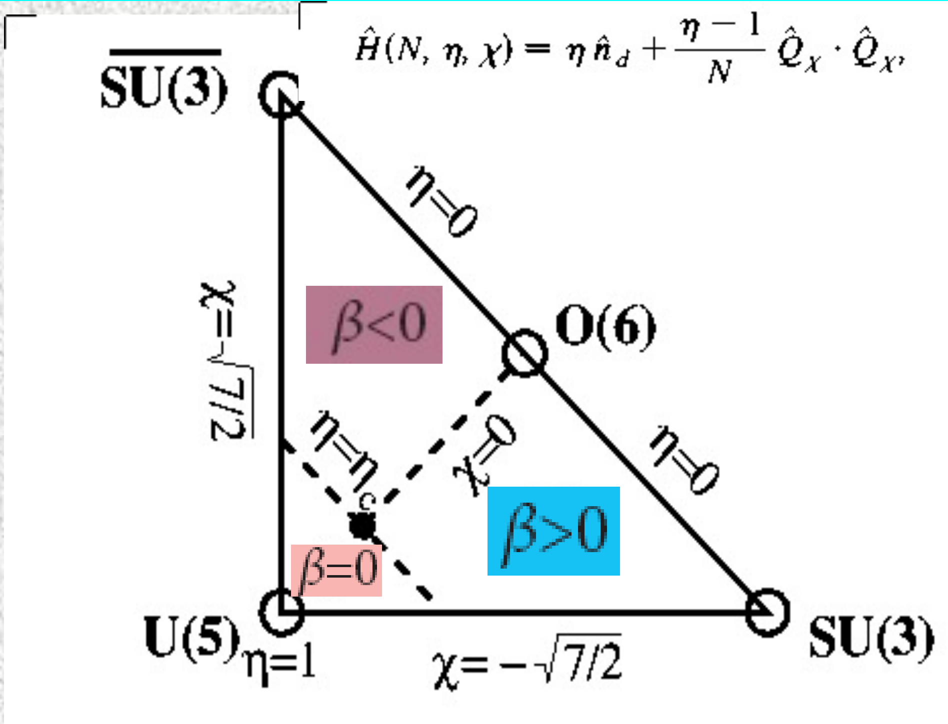
(Iachello, PRL 91, 132502(2003))

^{166}Er, ^{168}Er, ···

(PRC 68 , 024307 (2003); …

♣ 能谱和跃迁几率的计算结果表明这些解对应核形状相变的临界点对称状态





Iachello, PRL 91, 132502 (2003)

Observables distinguishing 1st from 2nd :

PRC76, 011305(R)(2007); PRL 100, 142501(2008);

Jolie, et al., PRL 89, 182503 (2002);

Warner, Nature 420, 614 (2002) ;

Casten, Nature Phys. 2, 811 (2006);

Iachello & Arima's Book (1987);

Casten, Nature Phys. 2, 811 (2006); Casten, et al., J. Phys. G 34, R285 (2007);

Cejnar, Jolie, & Casten, Rev. Mod. Phys. 82, 2155 (2010);

••••• ••••••

♠ 临界点态的对称性的代数结构

♣ General Discussion & the Algebrac Structure

Five-dimensional infinite square well:

translational and rotational inv. → $H = \alpha C_{2,\mathrm{Eu}(5)} = \alpha p^2$

Defining: $d_m = \frac{1}{\sqrt{2}}[(-1)^m q_{-m} + ip_m]$, $d_m^{\dagger} = \frac{1}{\sqrt{2}}[q_m - (-)^m ip_{-m}]$,

→ $$\hat{H}_{\mathrm{F}(5)} = \alpha\left[\hat{n}_d + \frac{5}{2} - \frac{1}{2}(\hat{P}_d^{\dagger} + \hat{P}_d)\right],$$

where $\hat{n}_d = \sum_m d_m^{\dagger} d_m$, $P_d^{\dagger} = \sum_m (-)^m d_m^{\dagger} d_{-m}^{\dagger}$, $P_d = \sum_m (-)^m d_m d_{-m}$.

For the U(5) — O(6) transition in the IBM,

$$\hat{H}_{\mathrm{cri}} = \varepsilon\left[\frac{1}{2}\hat{n}_d - \frac{1}{8N}(d^{\dagger}s + s^{\dagger}\tilde{d}) \cdot (d^{\dagger}s + s^{\dagger}\tilde{d})\right]$$

For low-lying states in large N limit,

$$\hat{H}_{cri,Large-N} = \tfrac{1}{4}\varepsilon\left[\hat{n}_d - \tfrac{5}{2} - \tfrac{1}{2}(\hat{P}_d^{+} + \hat{P}_d)\right]$$

Then, $H_{E(5)} = H_{cri,Large\text{-}N} = H_{F(5)}$, except for a constant.

♣ Algebraic Description of the X(5) symmetry and the evolution from E(5) to X(5)

Recalling: the wavefunctions in E(5) and X(5) symmetries are all Bessel functions, but in different rank:

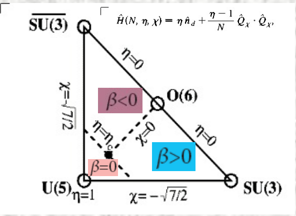
$\nu_{E(5)} = \tau + \frac{3}{2}$, with τ: the IRREP of the SO(5) group,

$\nu_{X(5)} = \sqrt{\frac{L(L+1)}{3} + \frac{9}{4}}$. with L : the IRREP of the SO(3) group,

→ **$H_{X(5)} = H_{cri,Large\text{-}N} = H_{F(5)}$, but with different QNs.**

Branching rule of SO(5) ⊃ SO(3) $L = 2\tau,\ 2\tau-2,\ 2\tau-3,\ \cdots, \tau$

→ the E(5) - X(5) evolution can be realized with



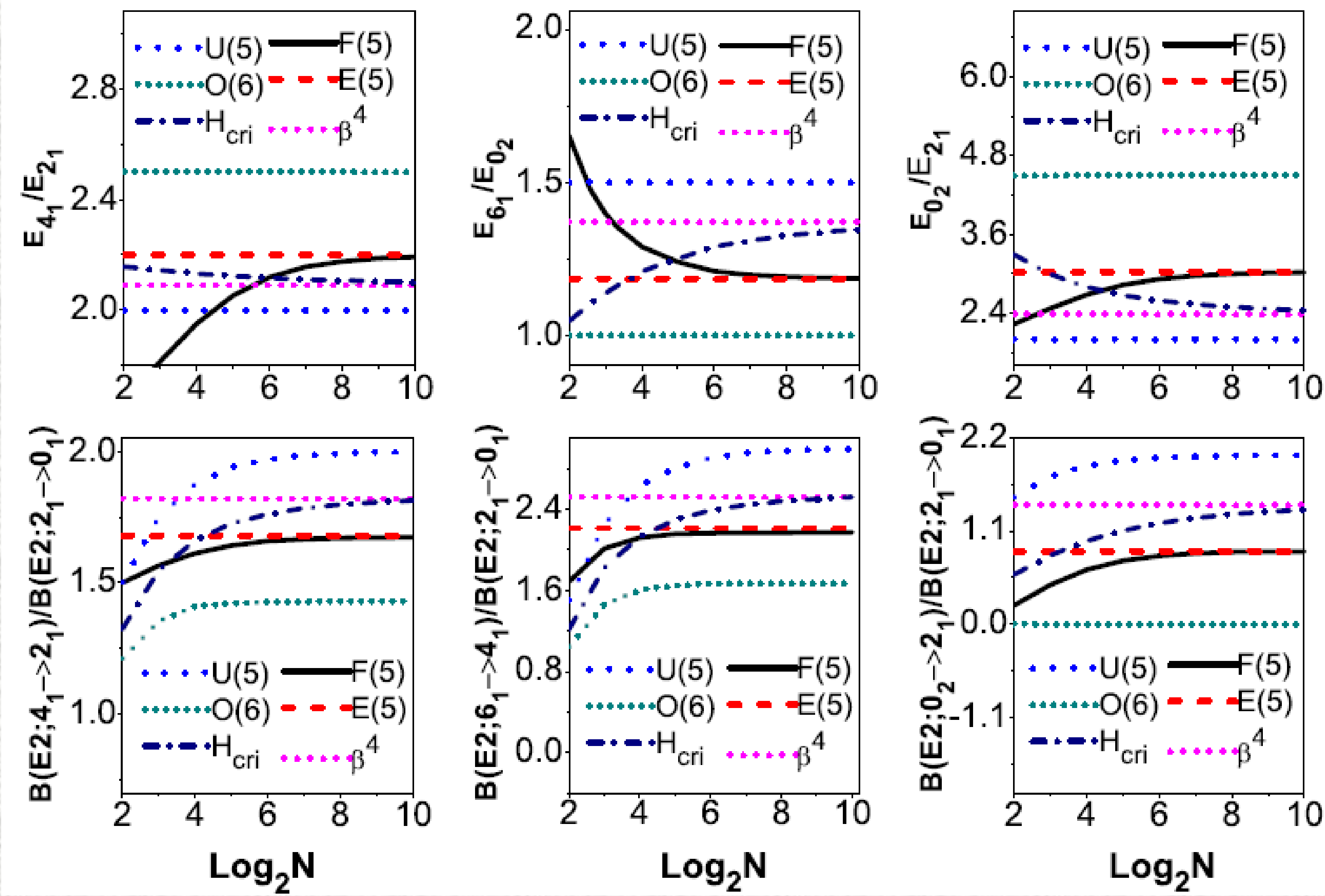
$$\nu = \left(1+\frac{2}{\sqrt{7}}\chi\right)\frac{L}{2} - \frac{2\chi}{\sqrt{7}}\left[\frac{-3+\sqrt{9+4L(L+1)/3}}{2}\right] + \frac{3}{2}$$

and $\chi \in [-\frac{\sqrt{7}}{2}, 0]$

Y. Zhang, Y.X. Liu, F. Pan, et al., Phys. Lett. B 732, 55 (2014).

♣ Numerical Comparison

Diagonalizing the Hamiltonian $H_{F(5)}$ with basis $|N\ \tau\ \Delta\ L>$

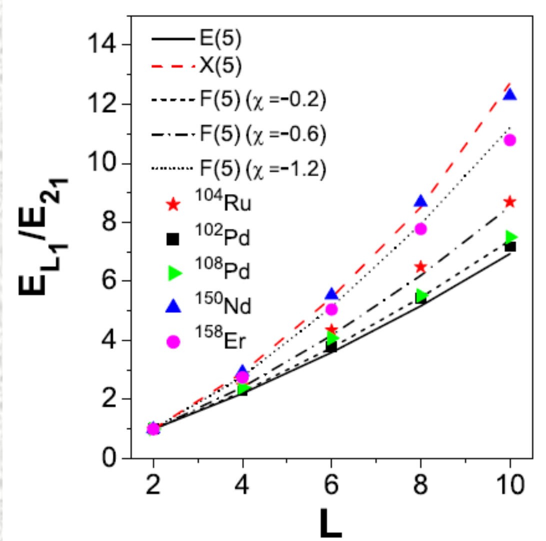


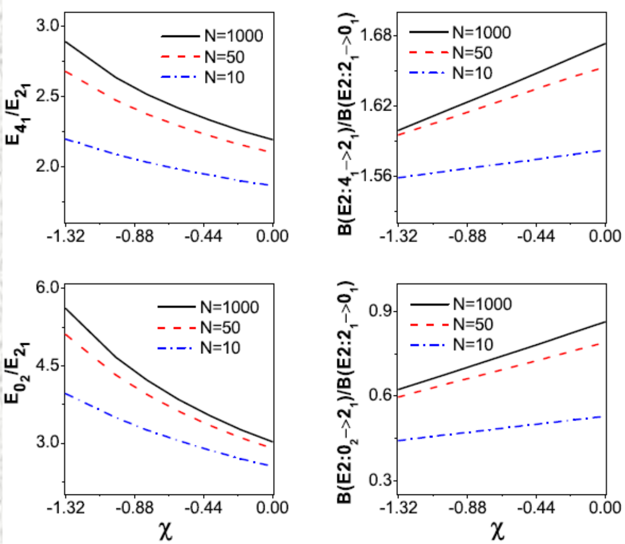
Y. Zhang, Y. X. Liu, et al.,

♠ Numerical Demonstration

	E(5)	F(5) at $N=1000$, $\chi=-0.8$	F(5) at $N=1000$, $\chi=-1.0$	F(5) at $N=1000$, $\chi=-1.1$	F(5) at $N=1000$, $\chi=-1.32$	X(5)	^{158}Er	^{152}Sm
E_{4_1}/E_{2_1}	2.20	2.51	2.63	2.71	2.89	2.91	2.74	3.01
E_{6_1}/E_{2_1}	3.59	4.43	4.74	4.93	5.41	5.45	5.05	5.80
E_{0_2}/E_{2_1}	3.03	4.22	4.67	4.93	5.61	5.67	4.2	5.62
$\frac{B(E2;\,4_1\to 2_1)}{B(E2;\,2_1\to 0_1)}$	1.68	1.63	1.62	1.61	1.60	1.58	1.49	1.45
$\frac{B(E2;\,0_2\to 2_1)}{B(E1;\,2_1\to 0_1)}$	0.86	0.72	0.68	0.66	0.62	0.63	∼	0.22

Y. Zhang, Y.X. Liu, F. Pan, et al., Phys. Lett. B 732, 55 (2014).

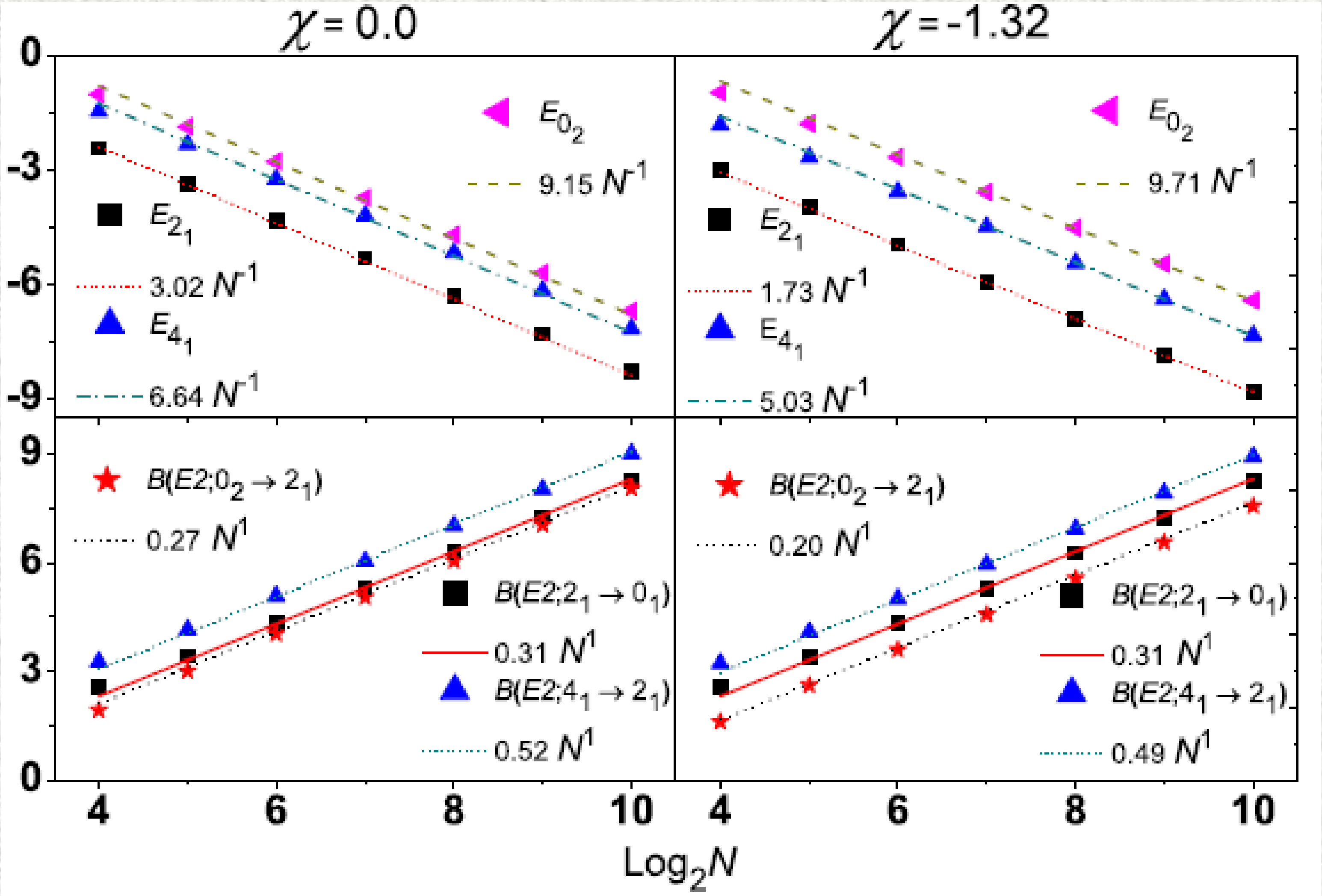




➔ **“more” induces not only quantitative difference, but also qualitative one as the same as “SB”.**

♠ Numerical Demonstration

Scaling Behavior



Y. Zhang, Y.X. Liu, F. Pan, et al., Phys. Lett. B 732, 55 (2014)

♠ Describing Odd Nuclei in F(5) model

Recalling:

wavefunction in E(5/(2*j*+1)) sym. is the Bessel function in rank:

$$\nu_{E(5/4)} = \sqrt{\tau_1(\tau_1+3)+3}\ ,$$

with τ_1: the QN in the IRREP $(\tau_1, 1/2)$ of the Spin(5) group,

→ $H_{X(5/(2j+1))} = H_{F(5)}$, but with different QNs.

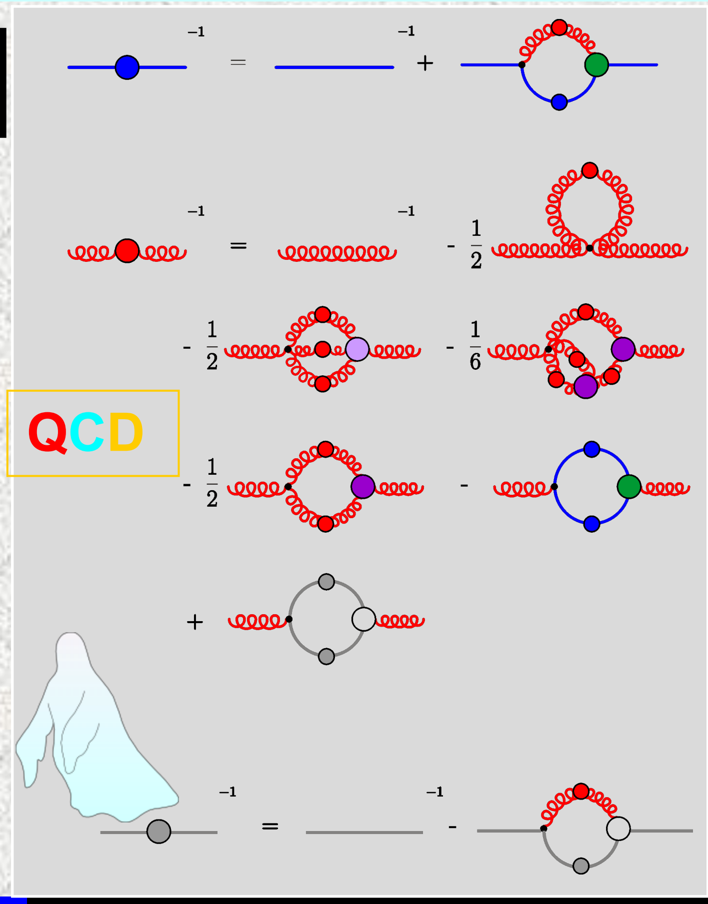
Numerical result:

	F(5) at SUSY		E(5/4)
	$N=10$	$N=1000$	
$E_{\frac{11}{2}_1}/E_{\frac{7}{2}_1}$	2.87	2.20	2.20
$E_{\frac{3}{2}_2}/E_{\frac{7}{2}_1}$	4.23	3.34	3.33
$E_{\frac{7}{2}_2}/E_{\frac{7}{2}_1}$	5.84	5.02	5.02
$\frac{B(E2;\ \frac{11}{2}_1 \to \frac{7}{2}_1)}{B(E2;\ \frac{7}{2}_1 \to \frac{3}{2}_1)}$	1.65	1.60	1.60
$\frac{B(E2;\ \frac{3}{2}_2 \to \frac{7}{2}_1)}{B(E2;\ \frac{7}{2}_1 \to \frac{3}{2}_1)}$	0.37	0.25	0.24

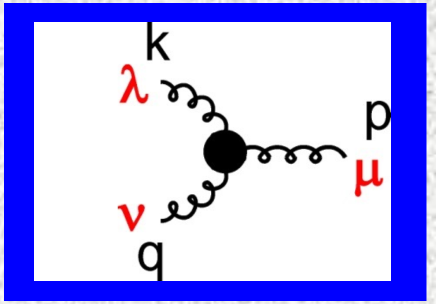
Y. Zhang, F. Pan, Y. X. Liu, Z. F. Hou, et al., Phys. Rev. *C* 82, 034327 (2010); etc.

♠ Dyson-Schwinger Equations
— A Nonperturbative QCD Approach

♣ Outline of the DS Equations



Slavnov-Taylor Identity



axial gauges

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) \;=\; \Pi^{\mu\nu}(p) \;-\; \Pi^{\mu\nu}(q)$$

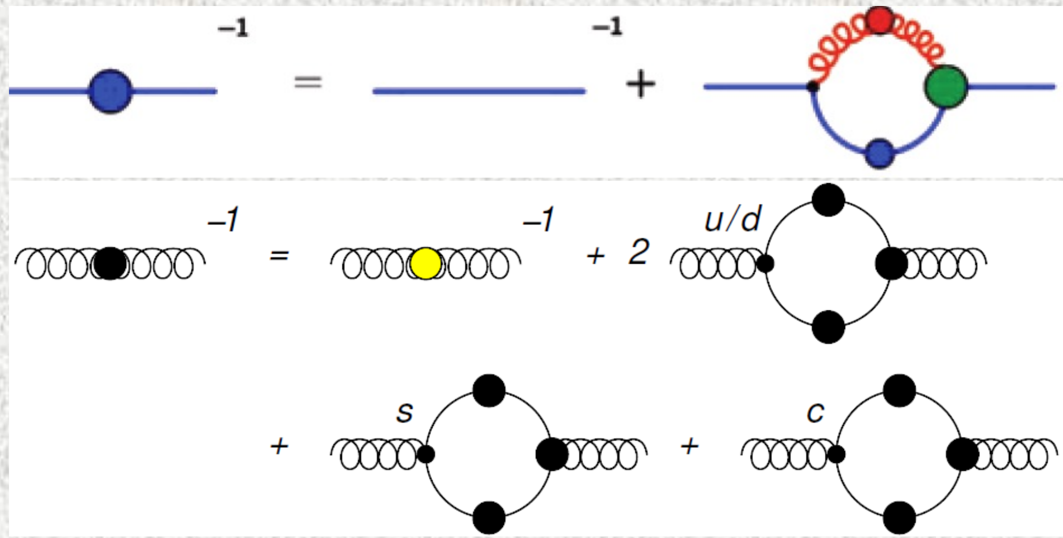
covariant gauges

$$k_\lambda \, \Gamma^{\lambda\mu\nu}(k,p,q) \;=\; H(k^2) \left[G_{\mu,\sigma}(q,-k)\, \Pi^{T}_{\sigma,\nu}(p) \;-\; G_{\nu\sigma}(p,-k)\, \Pi^{T}_{\sigma\mu}(q) \right]$$

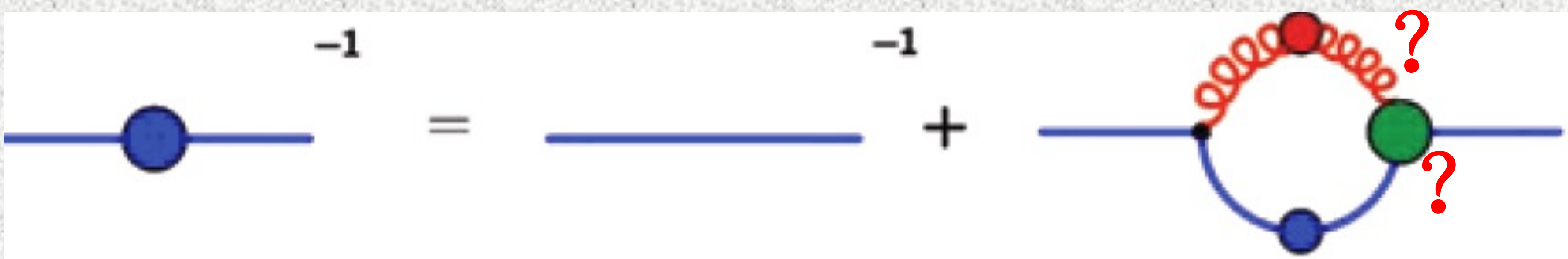
C. D. Roberts, et al, PPNP 33(1994), 477; R. Alkofer, et. al, Phys. Rep. 353(2001), 281;
A. Bashir, ···, LYX, Roberts, et al., CTP 58 (2012), 79; C.S. Fischer, PPNP 105(2019), 1; ··

♣ Algorithm of Solving the DSEs of QCD

- Solving the coupled quark, ghost and gluon equations (parts of the diagrams) :



- Solving the truncated quark equation with the symmetries being preserved.



♣ Expression of the quark gap equation

- Truncation: Preserving Symmetry. ➔ Quark Eq.

$$S^{-1}(p) = Z_2(-i\not{p} + Z_m m) + Z_1 g^2 \int \frac{d^4q}{(2\pi)^4} [t^a \gamma_\mu S(q) \Gamma^b_\nu(p,q) D^{ab}_{\mu\nu}(p-q)]$$

- Lorenz Structure ➔ Quark Eq. in Vacuum :

$$S^{-1}(p) = i\not{p} A(p^2, \Lambda^2) + B(p^2, \Lambda^2)$$

with

$$\begin{cases} A(x) = 1 + \frac{1}{6\pi^3} \int dy \frac{yA(y)}{yA^2(y) + B^2(y)} \Theta_A(x,y) \\ B(x) = \frac{1}{2\pi^3} \int dy \frac{yB(y)}{yA^2(y) + B^2(y)} \Theta_B(x,y) \end{cases}$$

$\Theta_A(x,y)$ & $\Theta_B(x,y)$ are functions of the vertex & the gluon.

- Quark Eq. in Medium

Tempt. T : ➔ Matsubara Frequency $\omega_n = (2n+1)\pi T$

Density ρ : ➔ Chemical Potential μ

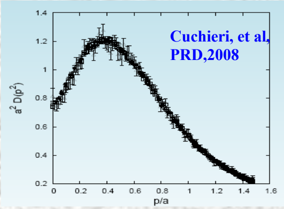
$$S^{-1}(p) \Longrightarrow S^{-1}(p, \omega_n, \mu) = iA(p, \omega_n, \mu)\vec{\gamma} \cdot \vec{p} + iC(p, \mu)\gamma_4(\omega_n + i\mu) + B(\tilde{p}) + \cdots$$

♣ Models of the effective gluon propagator

$$g^2 D_{\rho\sigma}(k) = 4\pi \frac{\mathcal{G}(k^2)}{k^2} \left(\delta_{\rho\sigma} - \frac{k_\rho k_\sigma}{k^2}\right)$$

- Commonly Used: Maris-Tandy Model **(PRC 56, 3369)**

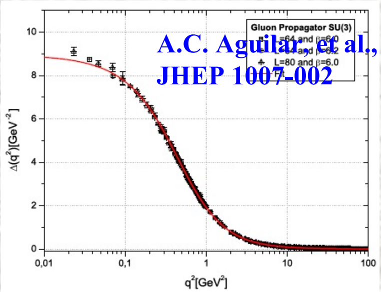
$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D\, t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln\left[\tau + \left(1 + t/\Lambda_{\rm QCD}^2\right)^2\right]} \frac{1 - \exp(-t/[4m_{\mathcal{F}}^2])}{t} \quad (3)$$



- Recently Proposed: Infrared Constant Model

(Qin, Chang, Liu, Roberts, Wilson, Phys. Rev. C 84, 042202(R), (2011).)

Taking $t/\omega^2 = k^2/\omega^2 = 1$ the coefficient of the above expression



- General derivation and analysis (Zwanziger, PRD 87, 085039 (2013)) show that the one in 4-D should really be infrared constant.

♣ Models of quark-gluon interaction vertex

$$\Gamma^a_\mu(q,p) = t^a \Gamma_\mu(q,p)$$

- Bare Ansatz

$$\Gamma_\mu(q,p) = \gamma_\mu$$ (Rainbow Approximation)

- Ball-Chiu (BC) Ansatz

$$\Gamma^{BC}_\mu(p,q) = \frac{A(p^2)+A(q^2)}{2}\gamma_\mu + \frac{(p+q)_\mu}{p^2-q^2}\{[A(p^2)-A(q^2)]\frac{(\gamma\cdot p+\gamma\cdot q)}{2} -i[B(p^2)-B(q^2)]\},$$

Satisfying W-T Identity, L-C. restricted

- Curtis-Pennington (CP) Ansatz

$$\Gamma^{CP}_\mu(p,q) = \Gamma^{BC}_\mu(p,q) + \frac{1}{2}(A(p^2)-A(q^2))\frac{\gamma_\mu(p^2-q^2)-(k+p)_\mu\gamma\cdot(p+q)}{d(p,q)},$$

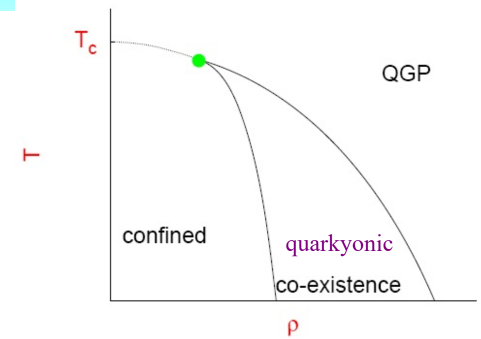
$$d(p,q) = \frac{(p^2-q^2)^2+[M^2(p^2)+M^2(q^2)]^2}{p^2+q^2}.$$

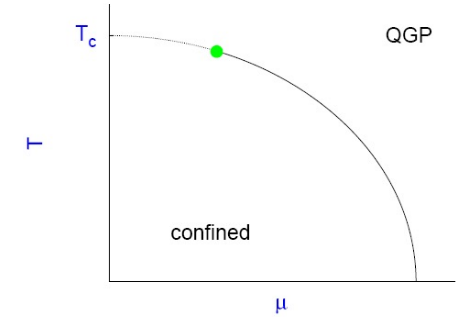
Satisfying Prod. Ren.

- **CLRQ** (BC+ACM, Chang, etc, PRL 106,072001('11); Qin, etc, PLB 722,384('13); C. Tang, F. Gao, & YXL, Phys. Rev. D 100, 056001 (2019))

$$\Gamma^{\rm acm}_\mu(p_f,p_i) = \Gamma^{\rm acm_4}_\mu(p_f,p_i) + \Gamma^{\rm acm_5}_\mu(p_f,p_i),$$

♠ Relation between the Chiral PT & the Confinement-Deconfinement PT





♣ **Lattice QCD Calculation**

de Forcrand, et al.,
Nucl. Phys. B Proc. Suppl. 153, 62 (2006); …

and **General (large-N_c) Analysis**

McLerran, et al., NPA 796, 83 ('07);
NPA 808, 117 ('08);
NPA 824, 86 ('09), …

claim that there exists a quarkyonic phase.

♣ **Coleman-Witten Theorem (PRL 45, 100 ('80)):**
Confinement coincides with DCSB !!

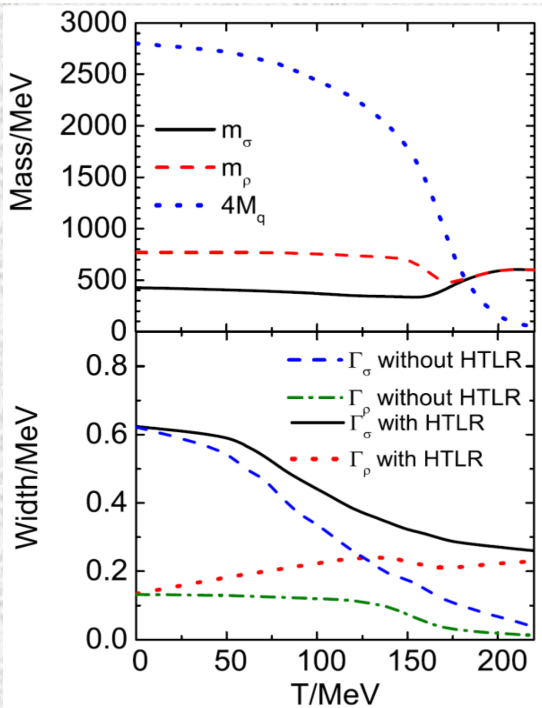
♣ Inconsistence really exists?!
Nature of the Quarkyonic Phase ?!

♣ Identifying the chiral phase transition with the masses of some hadrons

GT Relation

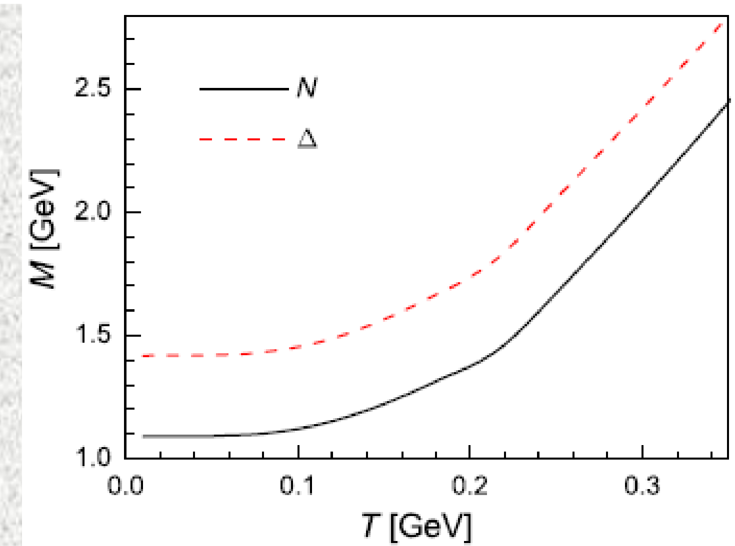
$$M_\sigma^2 = M_\pi^2 + 4M_q^2$$

➔ $M_\sigma \cong M_\pi$ can be a signal of the DCS.



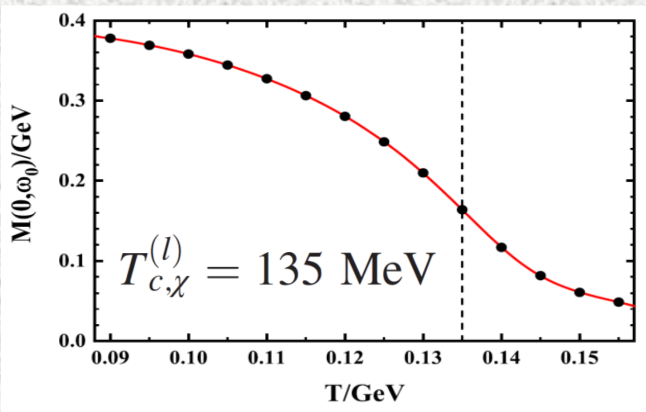
$r_S \propto 1/M_S$, when $r_S < r_{md}$, the color gets deconfined.

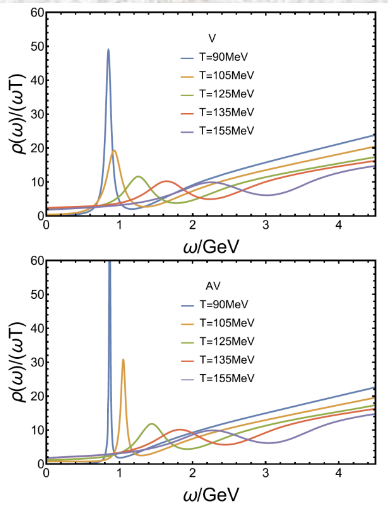
Hadron properties provide signals for not only the chiral phase transt. but also the confinement-deconfnmt. phase transition.

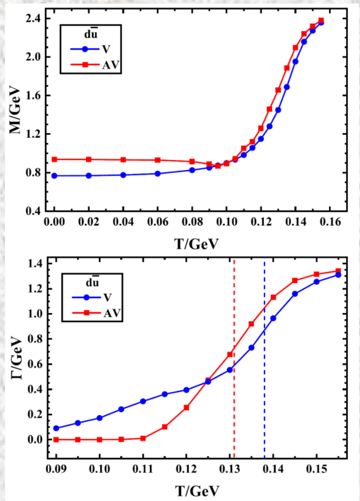


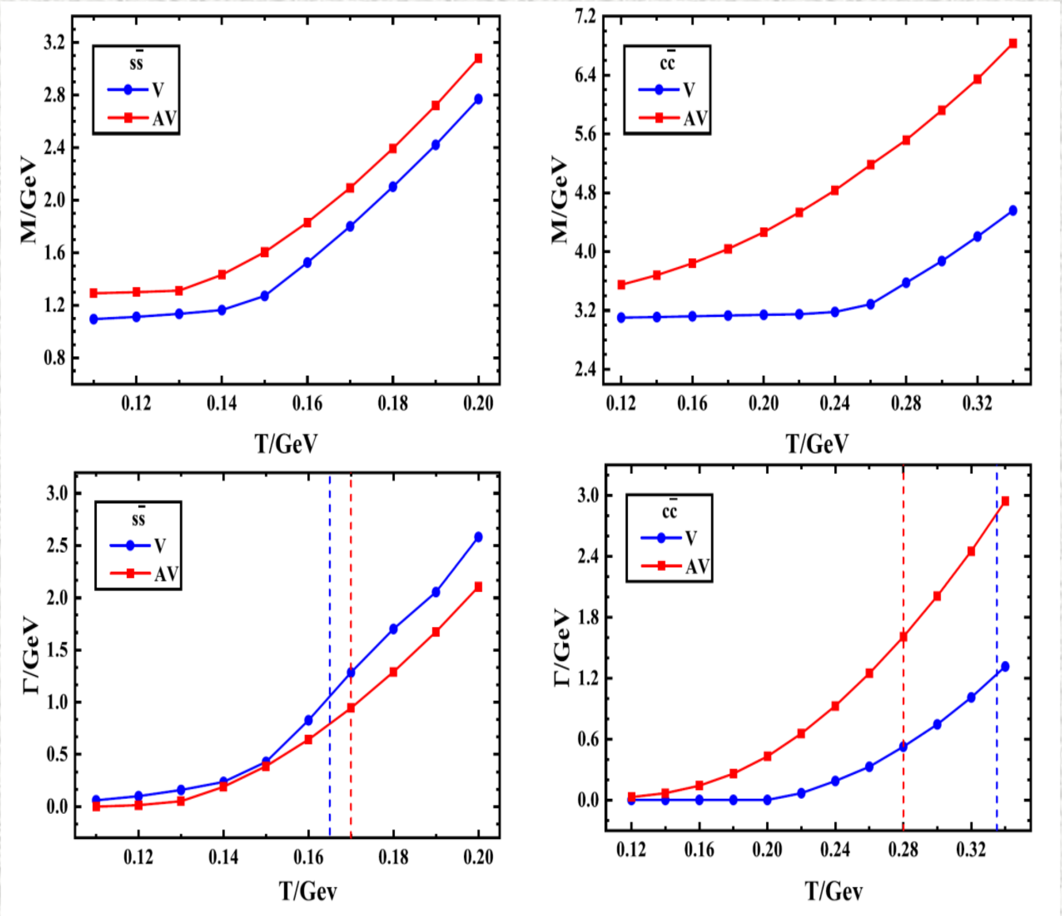
Wei-jie Fu, and Yu-xin Liu, Phys. Rev. D 79, 074011 (2009) ;
K.L. Wang, Y.X. Liu, C.D. Roberts, Phys. Rev. D 87, 074038 (2013);
Fei Gao, and Yu-xin Liu, Phys. Rev. D 97, 056011 (2018) ;

♣ Flavor-dependence of the dissociation Temperature & the relation with the $T_{c,\chi}$







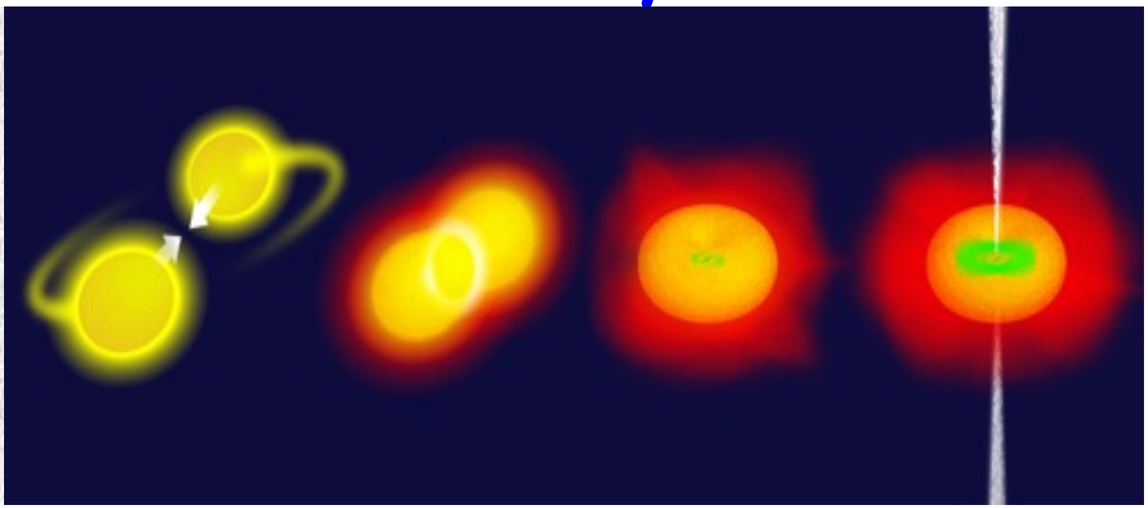


Light flavor: coincident; Heavy flavor: $T_{c,d} > T_{c,\chi}$!

Ling-feng Chen, Si-xue Qin, and YXL, Phys. Rev. D 102, 054012 (2020)

♠ An Excellent Astronomic Observation Signal: Gravitational Mode Oscillation Frequency

- **G-Wave in Binary Neutron Star Merger**



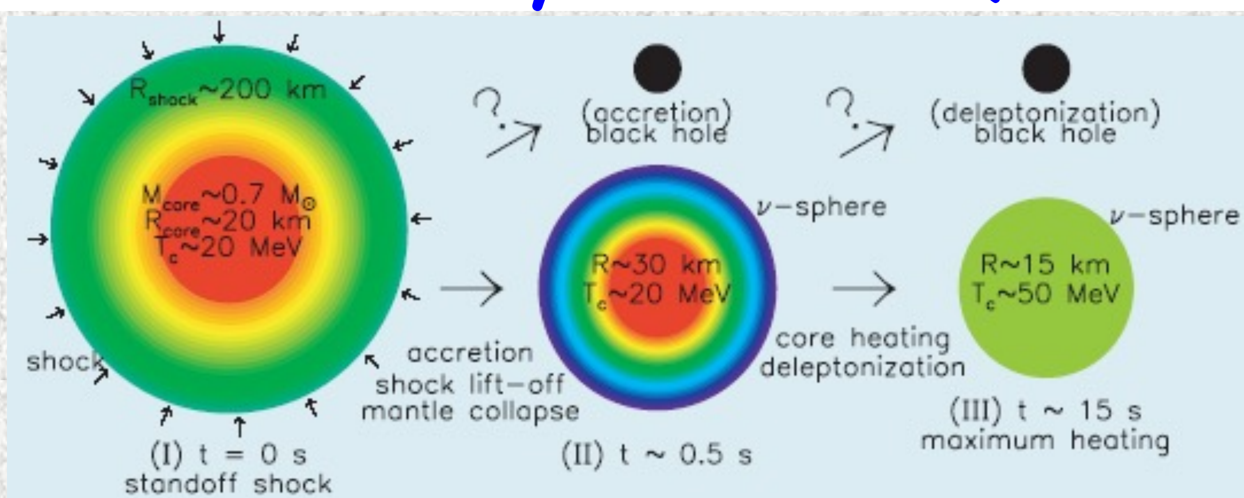
$F_{postmerger} \in (1.84,\ 3.73)$kHz,

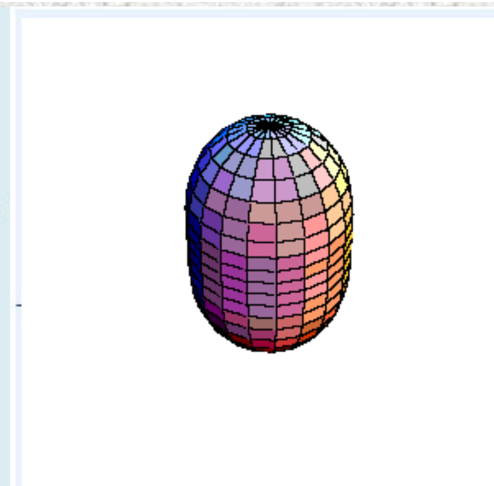
with width<200Hz,

(PRD 86, 063001(2012))

$F_{spiral} < F_{postmerger}$

- **G-Wave in Newly Born NS/QS after the SNE**





- Comparison of G-mode Oscillation Frequencies of the two kind nb Stars

Neutron Star: RMF, Quark Star: Bag Model

→ Frequency of the G-mode oscillation

Radial order of g-mode	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
$n=1$	717.6	774.6	780.3	82.3	78.0	63.1
$n=2$	443.5	467.3	464.2	52.6	45.5	40.0
$n=3$	323.8	339.0	337.5	35.3	30.8	27.8

W.J. Fu, H.Q. Wei, and Y.X. Liu, arXiv: 0810.1084, Phys. Rev. Lett. 101, 181102 (2008)

• Comparison with other modes

Neutron Star: RMF, Quark Star: Bag Model

→ Frequencies of the f- & p-mode oscillations

Modes	Neutron Star			Strange Quark Star		
	$t=100$	$t=200$	$t=300$	$t=100$	$t=200$	$t=300$
${}_2f$	1103	1133	1176	2980	2997	3016
${}_2p_1$	2265	2426	2494	18282	17330	16792
${}_2p_2$	3780	4054	4179	28792	27288	26438
${}_2p_3$	5319	5702	5869	38988	36950	35798

♣ G-mode oscillation in quark star has very low freq. !

W.J. Fu, Z. Bai, Y.X. Liu, arXve:1701.00418