Parton Interpretation of Parton Distributions

J.P. Ma

Institute of Theoretical Physics, CAS, Beijing

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Outline:

1. QCD factorization and parton distributions

2. Twist-4 parton distributions

3. Some general statements

4. Summary

1. QCD factorization and parton distributions

QCD predictions for hadron-scattering without knowing the inner structure of hadrons ??

QCD factorization theorems:

DIS at leading power: $\gamma^*(q) + H(P) \rightarrow X$

$$d\sigma \sim \sum_{a=q,\bar{q},g} \int_0^1 dx C_a(x, Q, \mu^2) f_a(x, \mu^2) \left\{ 1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2} \right) \right\}$$

Q^2 : - square of momentum of the virtual photon, large…

$C_a(x, Q^2, \mu^2)$: Perturbative coefficient function, no soft divergence

$f_a(x, \mu^2)$: Parton distribution function at twist-2

Physical picture of DIS:

the virtual photon interacts with one parton "a" inside the hadron, the parton carries a momentum fraction x of the hadron, i.e., xP

Partons are quarks, antiquarks or gluons.

Taking the leading order of C_a , it is the naïve parton model(Feynman) in early time.

In a quantum field theory like QCD, it is proven to all orders and every thing in the formula is well-defined….

The power correction involves twist-4 parton distributions……

In parton model, parton distributions are interpreted as probabilities. Therefore, it is expected they have no support for $x < 0$ and $x > 1$.

But this has to be proven in QCD….. !!

Definitions of parton distributions in QCD as a quantum field theory

Light-cone coordinate system:

A vector in the system is given by: $$A^\mu = (A^+, A^-, A^1, A^2)$$

with the relation:

$$A^+ = \frac{A^0 + A^3}{\sqrt{2}}, A^- = \frac{A^0 - A^3}{\sqrt{2}}, A^{1,2} = A^{1,2}, A_\perp^\mu = (A^1, A^2),$$

If a hadron moves in the z-direction with large energy, then its momentum is given as $P^\mu \approx (P^+, 0, 0, 0)$, only one component!

For simplifying the discussion, we take the light-cone gauge:

$$n \cdot G(x) = G^+(x) = 0, \quad n^\mu = (0, 1, 0, 0).$$

The defintion of twist-2 quark distribution:

$$f_q(x) = \int \frac{d\lambda}{4\pi} e^{-i\lambda x P^+} \langle P|\bar{\psi}(\lambda n)\gamma^+ \psi(0)|P\rangle$$

and twist-2 antiquark distribution:

$$f_{\bar{q}}(x) = \int \frac{d\lambda}{4\pi} e^{-i\lambda x P^+} \langle P|\mathrm{tr}\left[\gamma^+ \psi(\lambda \mathrm{n})\bar{\psi}(0)\right]|\mathrm{P}\rangle.$$

Taking only connected diagrams of Green's function into account as usual, one has:

$$f_{\bar{q}}(x) = -f_q(-x)$$

The order of operators is irrelevant… No need to do time-ordering !!!

Sandwiching the sum between two operators: $1 = \sum_X |X\rangle\langle X|$

One can show $f_q(x) = 0, \quad \text{for} \quad |x| > 1.$

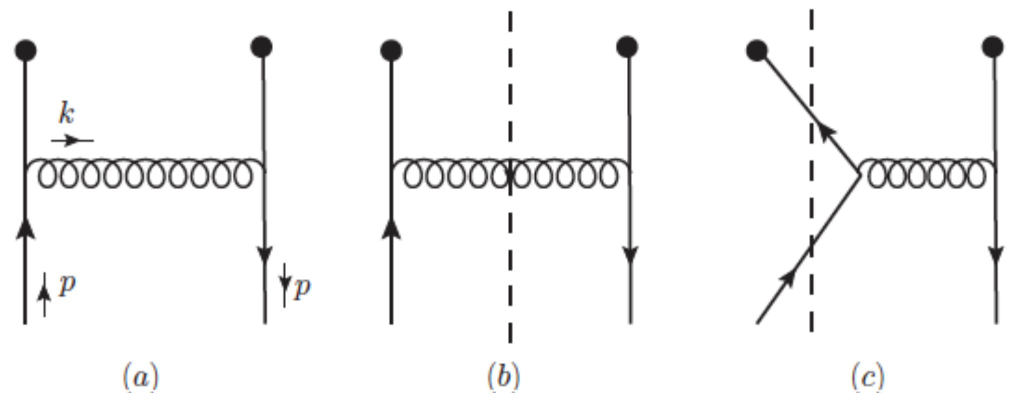
Parton interpretation: A parton inside a hadron can not move faster than the hadron!! Also can not move in the opposite direction.

It is interesting to examine the interpretation with perturbative theory to calculate the quark distribution of a quark with the momentum P.

There are two ways to do it with or without time-ordering.
At tree-level:

$$f_q(x) = \delta(1-x)$$

At one-loop:



(a): the contribution for T-ordering

(b)+(c+c.c): the contribution without time-ordering

For the case of time-ordering:

$$f_q(x)\Big|_a = 2\alpha_s C_F \theta(x)\theta(1-x) \int \frac{d^2k_\perp}{(2\pi)^2} \left[\frac{1+x^2}{1-x} \frac{1}{k_\perp^2+(1-x)^2m^2} + \frac{m^2x}{(k_\perp^2+(1-x)^2m^2)^2}\right].$$

It is zero for x<0 or x>1 as expected.

In the case without time-ordering:

$$f_q(x)\Big|_b = 2\alpha_s C_F \theta(1-x) \int \frac{d^2k_\perp}{(2\pi)^2} \left[\frac{1+x^2}{1-x} \frac{1}{k_\perp^2+(1-x)^2m^2} + \frac{m^2x}{(k_\perp^2+(1-x)^2m^2)^2}\right].$$

$$f_q(x)\Big|_{c+c.c.} = -2\alpha_s C_F \theta(-x) \int \frac{d^2k_\perp}{(2\pi)^2} \left[\frac{1+x^2}{1-x} \frac{1}{k_\perp^2+(1-x)^2m^2} + \frac{m^2x}{(k_\perp^2+(1-x)^2m^2)^2}\right].$$

The sum is zero for x<0 and x>1. m is the quark mass

It is tricky to obtain the contribution from (c)… divergent

The quark distribution at twist-2 has the parton interpretation at one-loop level, and the order of the operators is irrelevant.

Similarly, one can show it in the case of twist-2 gluon distributions.

How about parton distributions beyond twist-2 ?

A "well-known" result: Parton distributions at twist-3, twist-4 have the parton interpretation, and the order of operators used to define these distributions are irrelevant……

R.L. Jaffe, Nucl Phys. B229 (1983) 205

If one reads the paper carefully……

2. Twist-4 parton distributions

The following twist-4 parton distribution is involved in the power correction in DIS:

$$T_1(x) = \int \frac{d\lambda}{8\pi} e^{i\lambda x P^+} \langle P|\bar{\psi}(0)\gamma_\alpha \gamma^+ \gamma_\beta D_\perp^\alpha(0) D_\perp^\beta(\lambda n)\psi(\lambda n)|P\rangle,$$

there are four field operators.

We introduce:

$$M^2 E_q(x) = (P^+)^2 \int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \langle P|\bar{\psi}(\lambda n)\gamma^- \psi(0)|P\rangle,$$

One can show:

$$M^2 E_q(x) = -\frac{2}{x^2} T_1(x) + 2\frac{m}{x} M e(x) - \frac{m^2}{x^2} f_q(x) \approx -\frac{2}{x^2} T_1(x),$$

m: quark mass
e(x): a twist-3 parton disribution.

We examine the parton interpretation of Eq(x) by calculating it with a quark state.

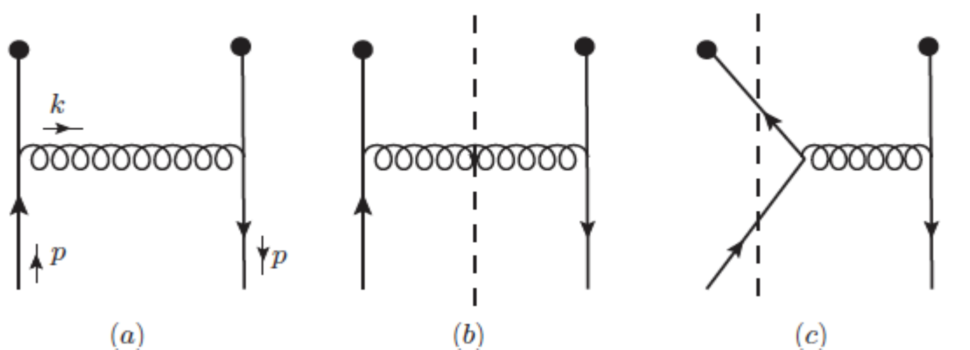
To check the relevance of ordering, we introduce the similar distribution with time-ordered product of operators: :

$$M^2 \tilde{E}_q(x) = (P^+)^2 \int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \langle P|T(\bar{\psi}(\lambda n)\gamma^- \psi(0))|P\rangle .$$

The results for a quark state:

At tree-level: $$M^2 \tilde{E}_q(x) = M^2 E_q(x) = m^2 \delta(1-x).$$

At one-loop level, for $x \neq 1$, the same diagrams contribute



From (a) we have:

$$M^2\tilde{E}_q(x) = \alpha_s C_F \int \frac{d^2k_\perp}{(2\pi)^2}\Bigg[2\theta(x)\theta(1-x)\frac{k_\perp^2(k_\perp^2+m^2(x(x-4)+5))}{(1-x)(k_\perp^2+m^2(1-x)^2)^2} - \frac{1}{x^2(1-x)}\Big(\theta(x)-\theta(-x)\Big)\Bigg].$$

It is not zero for x<0 and x>1 !! And it is power divergent.

From (b) ,+(c + c.c.):

$$M^2 E_q(x) = \alpha_s C_F \int \frac{d^2k_\perp}{(2\pi)^2}\Bigg[2\theta(x)\theta(1-x)\frac{k_\perp^2(k_\perp^2+m^2(x(x-4)+5))}{(1-x)(k_\perp^2+m^2(1-x)^2)^2} + \frac{2}{x^2(1-x)}\theta(-x)\Bigg].$$

It is not zero for x< 0, power divergent.

There is no parton interpretation of the twist-4 distribution!!

The difference is not zero:

$$M^2E_q(x) - M^2\tilde{E}_q(x) = \alpha_s C_F \frac{1}{x^2(1-x)}\int\frac{d^2k_\perp}{(2\pi)^2}, \quad -\infty < x < \infty$$

Distributions are different with different ordering!!!

The difference is related to a commutator of two quark fields.

We introduce

$$\psi(x) = \psi^{(+)}(x) + \psi^{(-)}(x), \quad \psi^{(+)}(x) = \frac{1}{2}\gamma^-\gamma^+\psi(x), \quad \psi^{(-)}(x) = \frac{1}{2}\gamma^+\gamma^-\psi(x)$$

Only the –fields are involved in the difference:

$$M^2 E_q(x) - M^2 \tilde{E}_q(x) = (P^+)^2 \int \frac{d\lambda}{2\pi} e^{-i\lambda x P^+} \theta(-\lambda) \langle P | \left\{ \bar{\psi}^{(-)}(\lambda n), \gamma^- \psi^{(-)}(0) \right\} | P \rangle,$$

$\psi^{(+)}(x)$: The true dynamical freedom

$\psi^{(-)}(x)$: Determined by equation of motion

For gluon field in the light-cone gauge: $G^+(x) = 0$

$G_\perp^\mu(x)$: The true dynamical freedom

$G^-(x)$: Determined by equation of motion

To calculate the commutator, the best way is to use light-cone quantization.

In the quantization: X^+ is taking as the time

The quantization rule:

$$\left\{\psi^{(+)}(x), \bar{\psi}^{(+)}(0)\right\}\Big|_{x^+=0} = \frac{1}{2}\gamma^-\delta(x^-)\delta^2(x_\perp),$$

$$\left[G_\perp^{a,\mu}(y), G_\perp^{b,\nu}(0)\right]\Big|_{y^+=0} = -\frac{1}{4}i\delta^{\mu\nu}\delta_{ab}\epsilon(y^-)\delta^2(\vec{y}_\perp).$$

We obtain the difference from the commutator

$$M^2E_q(x) - M^2\tilde{E}_q(x) = \alpha_s C_F \frac{1}{x^2(1-x)}\delta^2(0), \quad \delta^2(0) = \int \frac{d^2k_\perp}{(2\pi)^2}.$$

It is the same as that obtained from calculation of Feymann diagrams !

For the twist-4 quark distribution:

a: No parton interpretation. The contribution is power divergent in the region of $x<0$ and $x>1$.

b: The ordering is relevant, different ordering of operators gives different twist-4 distributions.

We have also studied the following twist-4 gluon distributions:

$$M^2 E(x) = P^+ \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle P | G^{a,+-}(\lambda n) G^{a,+-}(0) | P \rangle,$$

$$M^2 \tilde{E}(x) = P^+ \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle P | T\Big(G^{a,+-}(\lambda n) G^{a,+-}(0) \Big) | P \rangle.$$

The conclusion is the same as that for the quark distributions

The corresponding "twist-4" TMD parton distributions:

$$E_q(x,k_\perp) = (P^+)^2 \int \frac{d\lambda d^2x_\perp}{(2\pi)^3} e^{-i\lambda x P^+ - i x_\perp\cdot k_\perp} \langle P|\bar{\psi}(\lambda n + x_\perp)\gamma^-\psi(0)|P\rangle,$$

$$\tilde{E}_q(x,k_\perp) = (P^+)^2 \int \frac{d\lambda d^2x_\perp}{(2\pi)^3} e^{-i\lambda x P^+ - i x_\perp\cdot k_\perp} \langle P|T(\bar{\psi}(\lambda n + x_\perp)\gamma^-\psi(0))|P\rangle,$$

One-loop results:

$$\tilde{E}_q(x,k_\perp) = \alpha_s C_F\left[2\theta(x)\theta(1-x)\frac{k_\perp^2(k_\perp^2+m^2(x(x-4)+5))}{(1-x)(k_\perp^2+m^2(1-x)^2)^2} - \frac{1}{x^2(1-x)}\Big(\theta(x)-\theta(-x)\Big)\right],$$

$$E_q(x,k_\perp) = \alpha_s C_F\left[2\theta(x)\theta(1-x)\frac{k_\perp^2(k_\perp^2+m^2(x(x-4)+5))}{(1-x)(k_\perp^2+m^2(1-x)^2)^2} + \frac{2}{x^2(1-x)}\theta(-x)\right],$$

$$E_q(x,k_\perp) = \tilde{E}_q(x,k_\perp) = \alpha_s C_F\frac{1}{x^2(1-x)}.$$

They are not zero for x<0 and x>1, but finite in these regions…

Details: JPM, Z.Y. Pang and G.P. Zhang, PLB820 (2021) 136472

3. Some general statements

The analysis can be generalized to generalized parton distributions,

light-cone distribution amplitudes, TMD light-cone wave functions…

About ordering of operators:
A generic twist-4 distribution is defined with

$$\langle P|\phi_1(\lambda_1 n)\phi_2(\lambda_2 n)\phi_3(\lambda_3 n)\phi_4(0)|P\rangle,$$

$\phi_i (i = 1, 2, 3, 4)$ can be any of $\psi^{(+)}, \quad \bar{\psi}^{(+)} \quad \text{or} \quad G_\perp^\mu$

If we change the order , e,g.,

$$\langle P|\phi_1(\lambda_1 n)\phi_2(\lambda_2 n)\phi_3(\lambda_3 n)\phi_4(0)|P\rangle \pm \langle P|\phi_2(\lambda_2 n)\phi_1(\lambda_1 n)\phi_3(\lambda_3 n)\phi_4(0)|P\rangle$$

$$\langle P|\phi_1(\lambda_1 n)\phi_2(\lambda_2 n)\phi_3(\lambda_3 n)\phi_4(0)|P\rangle \pm \langle P|\phi_2(\lambda_2 n)\phi_1(\lambda_1 n)\phi_3(\lambda_3 n)\phi_4(0)|P\rangle$$
$$\propto \langle P|\phi_3(\lambda_3 n)\phi_4(0)|P\rangle \neq 0$$

Two consequences:

1. Different orders are different at twist-4 or higher..

2. The dependence on λ_1, λ_2 is trivial

➔ The associated momenta are not constrained, no parton interpretation!

An interesting observation:

For light-ray operators:

$$\left\{\psi^{(+)}(\lambda_1 n), \bar{\psi}^{(+)}(\lambda_2 n)\right\} = \frac{1}{2}\gamma^{-}\delta(\lambda_1 - \lambda_2)\delta^2(0),$$

The commutator is power-divergent…

If we can discard power-divergences like if one uses dimensional regulation, then the commutator is zero….

Parton interpretation can be restored…. But not for TMD distributions.

An interesting question is:

If we discard power-divergences, what is quantum effect in the product of light-ray field operators ???

4. Summary

Parton distributions beyond twist-3 have no parton interpretation.