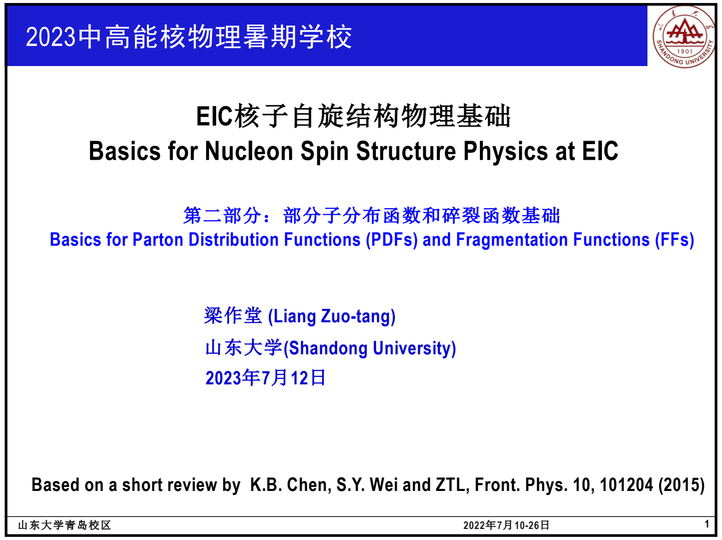

2023中高能核物理暑期学校

EIC核子自旋结构物理基础
Basics for Nucleon Spin Structure Physics at EIC

第二部分：部分子分布函数和碎裂函数基础
Basics for Parton Distribution Functions (PDFs) and Fragmentation Functions (FFs)

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山东大学(Shandong University)
2023年7月12日

Based on a short review by K.B. Chen, S.Y. Wei and ZTL, Front. Phys. 10, 101204 (2015)

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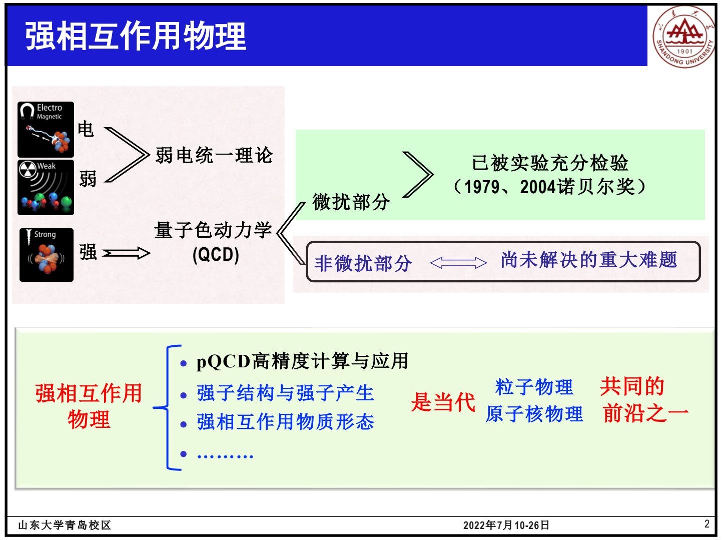


强相互作用物理

电
弱
弱电统一理论
强
量子色动力学
(QCD)
微扰部分
已被实验充分检验
（1979、2004诺贝尔奖）
非微扰部分 ⟺ 尚未解决的重大难题

强相互作用物理
- pQCD高精度计算与应用
- 强子结构与强子产生
- 强相互作用物质形态
- ………

是当代 粒子物理 原子核物理 共同的前沿之一

2

强相互作用物理：强相互作用物质形态

电磁
- 气体
- 液体
- 固体
- 晶体
- 超导体
- 超流体
- 等离子体
- 电磁波
- ………

强
- 核子/强子 (nucleon/hadron)
- 原子核 (nuclei)

束缚态

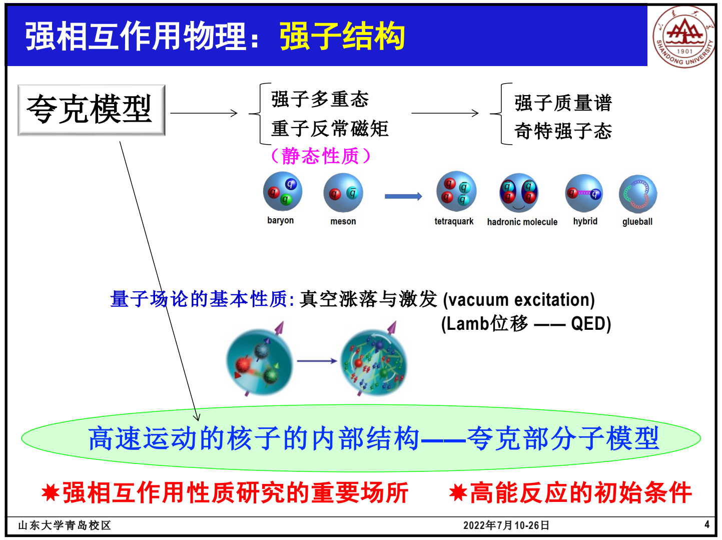
- 色超导体？(color super conductor)
- 色玻璃体？(color glass condensate)
- 夸克胶子等离子体？(quark gluon plasmas)
- ………

特殊条件下

原子分子物理
凝聚态物理
光学
等离子体物理
声学
无线电物理

材料科学
电子科学
化学

3



强相互作用物理：强子结构

夸克模型 → 强子多重态 / 重子反常磁矩 （静态性质） → 强子质量谱 / 奇特强子态

量子场论的基本性质: 真空涨落与激发 (vacuum excitation)
(Lamb位移 —— QED)

高速运动的核子的内部结构——夸克部分子模型

✷强相互作用性质研究的重要场所 ✷高能反应的初始条件

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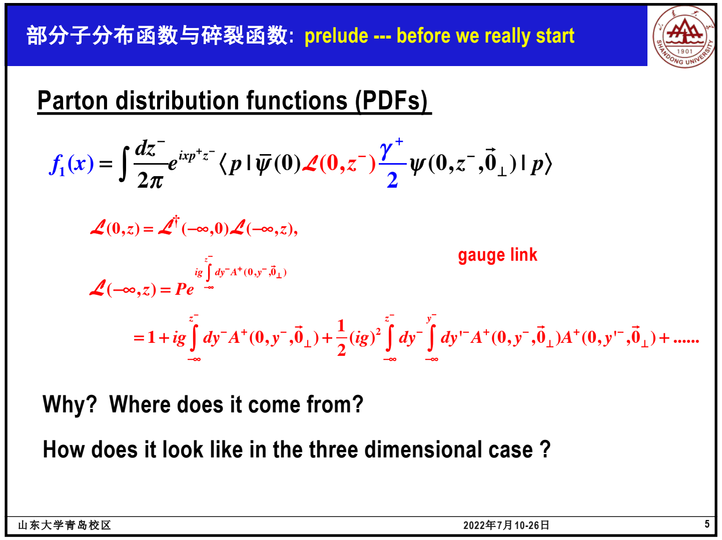

部分子分布函数与碎裂函数：prelude --- before we really start

Parton distribution functions (PDFs)

$$f_1(x)=\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle p\,|\,\overline{\psi}(0)\mathcal{L}(0,z^-)\frac{\gamma^+}{2}\psi(0,z^-,\vec{0}_\perp)\,|\,p\rangle$$

$$\mathcal{L}(0,z)=\mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z),$$

gauge link

$$\mathcal{L}(-\infty,z)=Pe^{ig\int_{-\infty}^{z^-}dy^-A^+(0,y^-,\vec{0}_\perp)}$$

$$=1+ig\int_{-\infty}^{z^-}dy^-A^+(0,y^-,\vec{0}_\perp)+\frac{1}{2}(ig)^2\int_{-\infty}^{z^-}dy^-\int_{-\infty}^{y^-}dy'^-A^+(0,y^-,\vec{0}_\perp)A^+(0,y'^-,\vec{0}_\perp)+\ldots\ldots$$

Why? Where does it come from?

How does it look like in the three dimensional case ?

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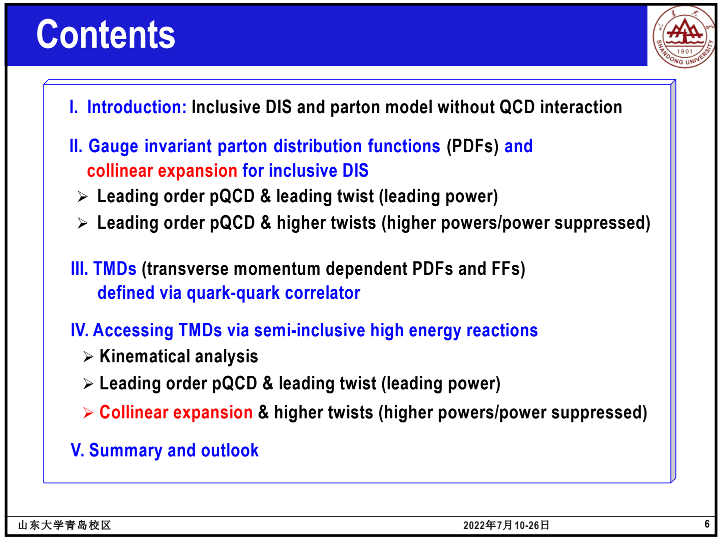

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  - ➢ Leading order pQCD & higher twists (higher powers/power suppressed)
- III. TMDs (transverse momentum dependent PDFs and FFs) defined via quark-quark correlator
- IV. Accessing TMDs via semi-inclusive high energy reactions
  - ➢ Kinematical analysis
  - ➢ Leading order pQCD & leading twist (leading power)
  - ➢ Collinear expansion & higher twists (higher powers/power suppressed)
- V. Summary and outlook

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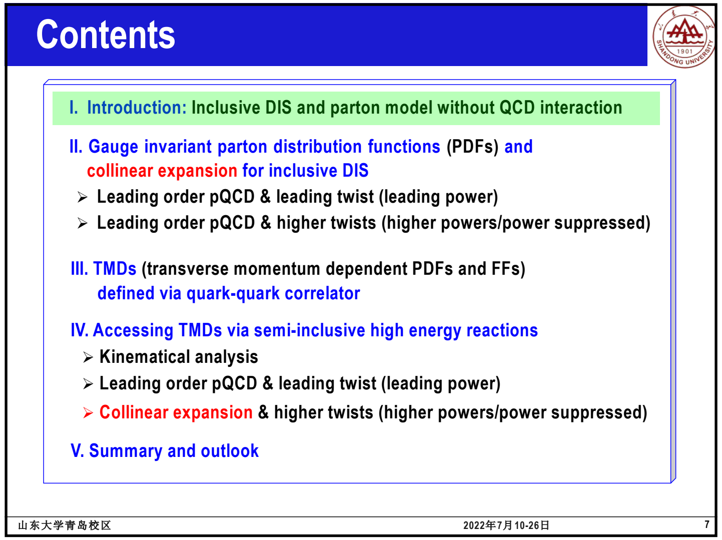

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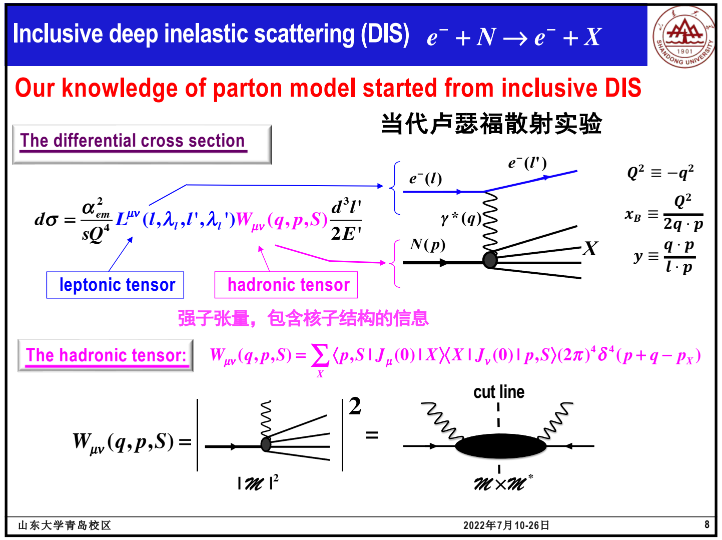

Inclusive deep inelastic scattering (DIS) $e^-+N\rightarrow e^-+X$

Our knowledge of parton model started from inclusive DIS

当代卢瑟福散射实验

The differential cross section

$$d\sigma=\frac{\alpha_{em}^2}{sQ^4}L^{\mu\nu}(l,\lambda_l,l',\lambda_l')W_{\mu\nu}(q,p,S)\frac{d^3l'}{2E'}$$

leptonic tensor

hadronic tensor

$Q^2\equiv-q^2$

$x_B\equiv\frac{Q^2}{2q\cdot p}$

$y\equiv\frac{q\cdot p}{l\cdot p}$

强子张量，包含核子结构的信息

The hadronic tensor: $W_{\mu\nu}(q,p,S)=\sum_X\langle p,S\,|\,J_\mu(0)\,|\,X\rangle\langle X\,|\,J_\nu(0)\,|\,p,S\rangle(2\pi)^4\delta^4(p+q-p_X)$

$W_{\mu\nu}(q,p,S)=\left|\quad\right|^2=$

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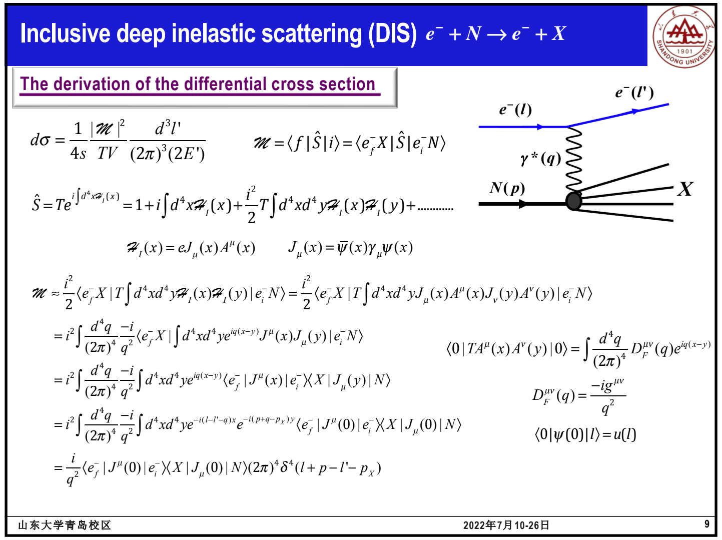


Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

The derivation of the differential cross section

$$d\sigma = \frac{1}{4s}\frac{|\mathscr{M}|^2}{TV}\frac{d^3l'}{(2\pi)^3(2E')} \qquad \mathscr{M} = \langle f|\hat{S}|i\rangle = \langle e_f^- X|\hat{S}|e_i^- N\rangle$$

$$\hat{S} = Te^{i\int d^4x\mathscr{H}_I(x)} = 1 + i\int d^4x\mathscr{H}_I(x) + \frac{i^2}{2}T\int d^4xd^4y\mathscr{H}_I(x)\mathscr{H}_I(y) + \text{...........}$$

$$\mathscr{H}_I(x) = eJ_\mu(x)A^\mu(x) \qquad J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$$

$$\mathscr{M} \approx \frac{i^2}{2}\langle e_f^- X|T\int d^4xd^4y\mathscr{H}_I(x)\mathscr{H}_I(y)|e_i^- N\rangle = \frac{i^2}{2}\langle e_f^- X|T\int d^4xd^4yJ_\mu(x)A^\mu(x)J_\nu(y)A^\nu(y)|e_i^- N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\langle e_f^- X|\int d^4xd^4ye^{iq(x-y)}J^\mu(x)J_\mu(y)|e_i^- N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\int d^4xd^4ye^{iq(x-y)}\langle e_f^-|J^\mu(x)|e_i^-\rangle\langle X|J_\mu(y)|N\rangle$$

$$= i^2\int\frac{d^4q}{(2\pi)^4}\frac{-i}{q^2}\int d^4xd^4ye^{-i(l-l'-q)x}e^{-i(p+q-p_X)y}\langle e_f^-|J^\mu(0)|e_i^-\rangle\langle X|J_\mu(0)|N\rangle$$

$$= \frac{i}{q^2}\langle e_f^-|J^\mu(0)|e_i^-\rangle\langle X|J_\mu(0)|N\rangle(2\pi)^4\delta^4(l+p-l'-p_X)$$

$$\langle 0|TA^\mu(x)A^\nu(y)|0\rangle = \int\frac{d^4q}{(2\pi)^4}D_F^{\mu\nu}(q)e^{iq(x-y)}$$

$$D_F^{\mu\nu}(q) = \frac{-ig^{\mu\nu}}{q^2}$$

$$\langle 0|\psi(0)|l\rangle = u(l)$$

Inclusive deep inelastic scattering (DIS) $e^- + N \to e^- + X$

Kinematic analysis: find the complete set of the "basic Lorentz tensors" and the general form of the hadronic tensor

The constraints: Gauge invariance $q^\mu W_{\mu\nu}(q,p,S) = 0$ Hermiticity $W_{\mu\nu}^*(q,p,S) = W_{\nu\mu}(q,p,S)$

Parity invariance $W_{\mu\nu}(\tilde{q},\tilde{p},-\tilde{S}) = W^{\mu\nu}(q,p,S)$

The unpolarized set: $\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$, $(q+2xp)_\mu(q+2xp)_\nu$

The polarized (spin dependent) set: $\varepsilon_{\mu\nu\rho\sigma}q^\sigma S^\sigma$, $\varepsilon_{\mu\nu\rho\sigma}q^\sigma\left(S^\sigma - \frac{S\cdot q}{p\cdot q}p^\sigma\right)$

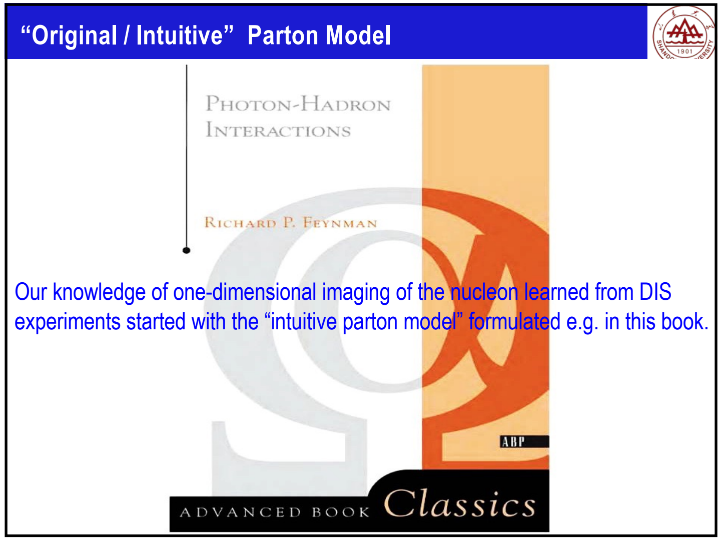
$$\Longrightarrow W_{\mu\nu}(q,p,S) = W_{\mu\nu}^{(S)}(q,p) + iW_{\mu\nu}^{(A)}(q,p,S)$$

$$W_{\mu\nu}^{(S)}(q,p) = 2\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \frac{1}{xQ^2}(q+2xp)_\mu(q+2xp)_\nu F_2(x,Q^2)$$

$$W_{\mu\nu}^{(A)}(q,p,S) = \frac{2M}{p\cdot q}\varepsilon_{\mu\nu\rho\sigma}q^\sigma S^\sigma g_1(x,Q^2) + \frac{2M}{p\cdot q}\varepsilon_{\mu\nu\rho\sigma}q^\sigma\left(S^\sigma - \frac{S\cdot q}{p\cdot q}p^\sigma\right)g_2(x,Q^2)$$

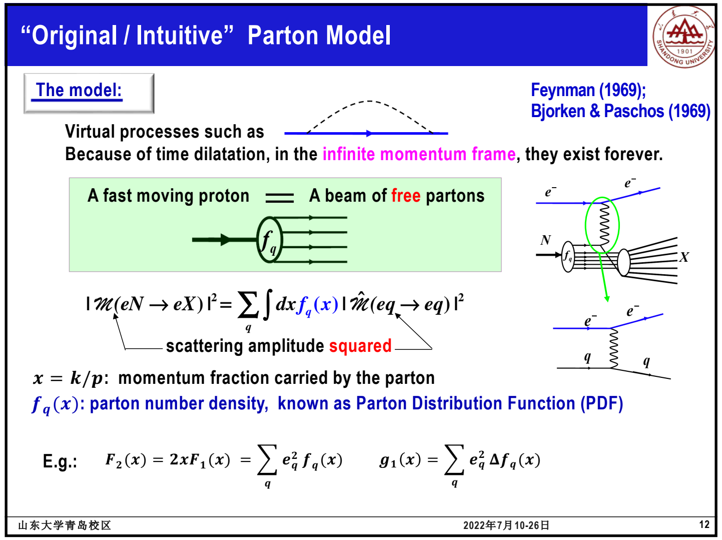
4 independent "basic Lorentz tensors" and correspondingly 4 structure functions

包含核子结构的信息



"Original / Intuitive" Parton Model

Our knowledge of one-dimensional imaging of the nucleon learned from DIS experiments started with the "intuitive parton model" formulated e.g. in this book.



"Original / Intuitive" Parton Model

The model:

Feynman (1969);
Bjorken & Paschos (1969)

Virtual processes such as

Because of time dilatation, in the infinite momentum frame, they exist forever.

A fast moving proton = A beam of free partons

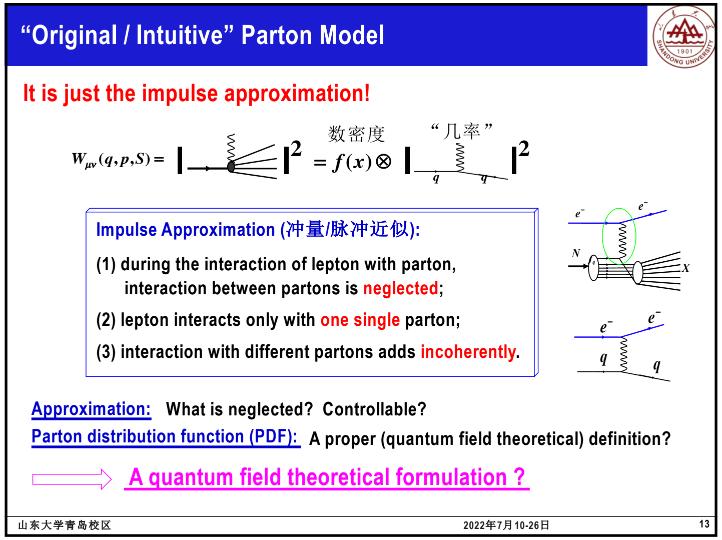
$$|\mathscr{M}(eN \to eX)|^2 = \sum_q\int dxf_q(x)|\hat{\mathscr{M}}(eq \to eq)|^2$$

scattering amplitude squared

$x = k/p$: momentum fraction carried by the parton

$f_q(x)$: parton number density, known as Parton Distribution Function (PDF)

E.g.: $F_2(x) = 2xF_1(x) = \sum_q e_q^2 f_q(x)$ $\qquad g_1(x) = \sum_q e_q^2 \Delta f_q(x)$



“Original / Intuitive” Parton Model

It is just the impulse approximation!

$W_{\mu\nu}(q,p,S) = |\quad|^2 = f(x)\otimes|\quad|^2$

Impulse Approximation (冲量/脉冲近似):

(1) during the interaction of lepton with parton, interaction between partons is neglected;

(2) lepton interacts only with one single parton;

(3) interaction with different partons adds incoherently.

Approximation: What is neglected? Controllable?

Parton distribution function (PDF): A proper (quantum field theoretical) definition?

⇨ A quantum field theoretical formulation ?

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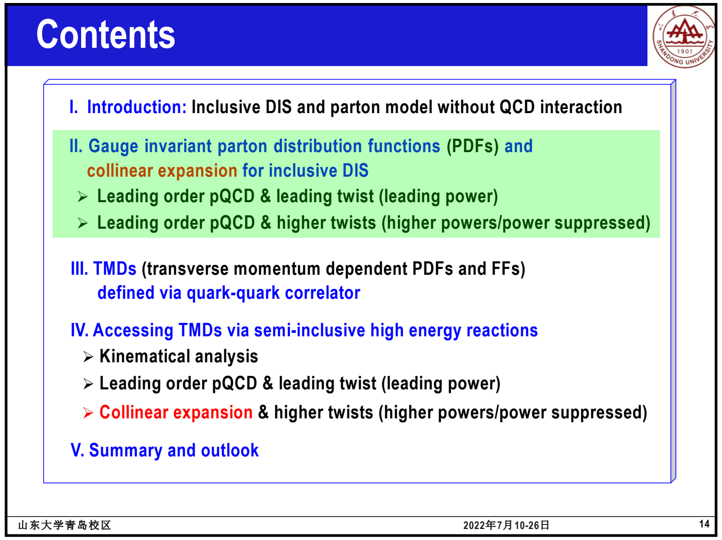

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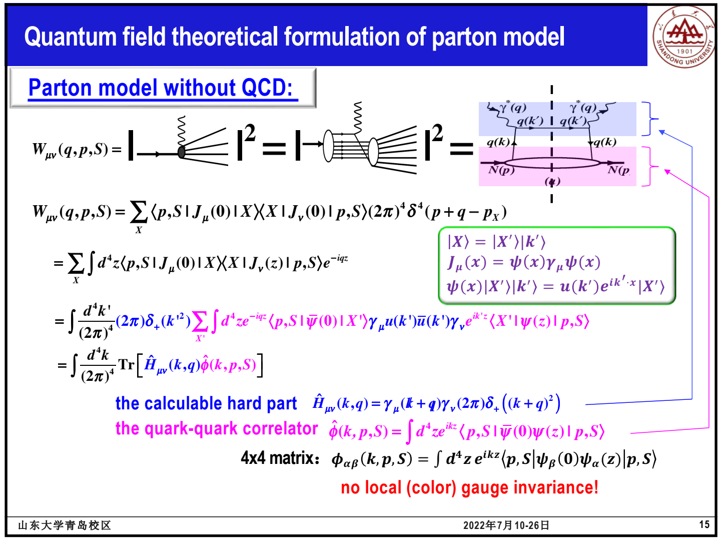

Quantum field theoretical formulation of parton model

Parton model without QCD:

$W_{\mu\nu}(q,p,S) = |\quad|^2 = |\quad|^2 =$

$$W_{\mu\nu}(q,p,S) = \sum_X \langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p,S\rangle(2\pi)^4\delta^4(p+q-p_X)$$

$$= \sum_X \int d^4z \langle p,S|J_\mu(0)|X\rangle\langle X|J_\nu(z)|p,S\rangle e^{-iqz}$$

$$|X\rangle = |X'\rangle|k'\rangle$$
$$J_\mu(x) = \psi(x)\gamma_\mu\psi(x)$$
$$\psi(x)|X'\rangle|k'\rangle = u(k')e^{ik'\cdot x}|X'\rangle$$

$$= \int\frac{d^4k'}{(2\pi)^4}(2\pi)\delta_+(k'^2)\sum_{X'}\int d^4z e^{-iqz}\langle p,S|\bar\psi(0)|X'\rangle\gamma_\mu u(k')\bar u(k')\gamma_\nu e^{ik'z}\langle X'|\psi(z)|p,S\rangle$$

$$= \int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(k,q)\hat\phi(k,p,S)\right]$$

the calculable hard part $\hat H_{\mu\nu}(k,q) = \gamma_\mu(\not k+\not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

the quark-quark correlator $\hat\phi(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar\psi(0)\psi(z)|p,S\rangle$

4x4 matrix: $\phi_{\alpha\beta}(k,p,S) = \int d^4z\, e^{ikz}\langle p,S|\psi_\beta(0)\psi_\alpha(z)|p,S\rangle$

no local (color) gauge invariance!

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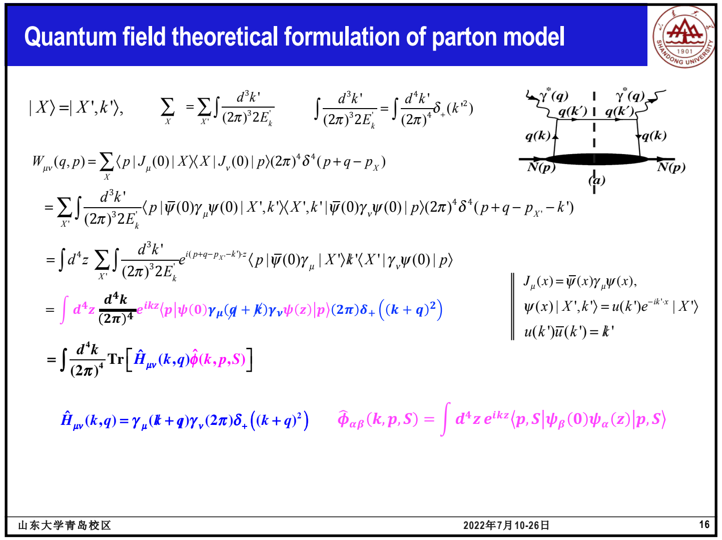

Quantum field theoretical formulation of parton model

$$|X\rangle = |X',k'\rangle, \qquad \sum_X = \sum_{X'}\int\frac{d^3k'}{(2\pi)^3 2E'_k} \qquad \int\frac{d^3k'}{(2\pi)^3 2E'_k} = \int\frac{d^4k'}{(2\pi)^4}\delta_+(k'^2)$$

$$W_{\mu\nu}(q,p) = \sum_X \langle p|J_\mu(0)|X\rangle\langle X|J_\nu(0)|p\rangle(2\pi)^4\delta^4(p+q-p_X)$$

$$= \sum_{X'}\int\frac{d^3k'}{(2\pi)^3 2E'_k}\langle p|\bar\psi(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\bar\psi(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= \int d^4z \sum_{X'}\int\frac{d^3k'}{(2\pi)^3 2E'_k}e^{i(p+q-p_{X'}-k')\cdot z}\langle p|\bar\psi(0)\gamma_\mu|X'\rangle\not k'\langle X'|\gamma_\nu\psi(0)|p\rangle$$

$$J_\mu(x) = \bar\psi(x)\gamma_\mu\psi(x),$$
$$\psi(x)|X',k'\rangle = u(k')e^{-ik'\cdot x}|X'\rangle$$
$$u(k')\bar u(k') = \not k'$$

$$= \int d^4z\frac{d^4k}{(2\pi)^4}e^{ikz}\langle p|\psi(0)\gamma_\mu(\not q+\not k)\gamma_\nu\psi(z)|p\rangle(2\pi)\delta_+\left((k+q)^2\right)$$

$$= \int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat H_{\mu\nu}(k,q)\hat\phi(k,p,S)\right]$$

$\hat H_{\mu\nu}(k,q) = \gamma_\mu(\not k+\not q)\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$ $\quad\hat\phi_{\alpha\beta}(k,p,S) = \int d^4z\, e^{ikz}\langle p,S|\psi_\beta(0)\psi_\alpha(z)|p,S\rangle$

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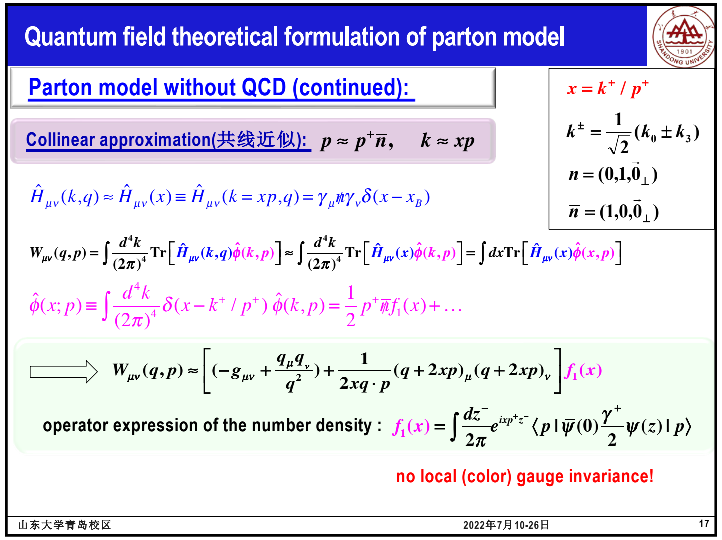


Quantum field theoretical formulation of parton model

Parton model without QCD (continued):

Collinear approximation(共线近似): $p \approx p^+\bar{n}, \quad k \approx xp$

$x = k^+ / p^+$

$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$

$n = (0,1,\vec{0}_\perp)$

$\bar{n} = (1,0,\vec{0}_\perp)$

$$\hat{H}_{\mu\nu}(k,q) \approx \hat{H}_{\mu\nu}(x) \equiv \hat{H}_{\mu\nu}(k = xp, q) = \gamma_\mu \not{n} \gamma_\nu \delta(x - x_B)$$

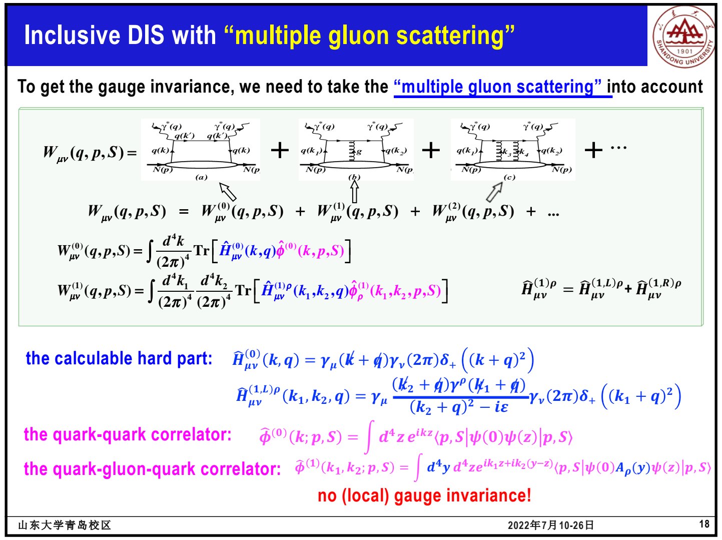
$$W_{\mu\nu}(q,p) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}(k,q)\hat{\phi}(k,p)\right] \approx \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}(x)\hat{\phi}(k,p)\right] = \int dx \mathrm{Tr}\left[\hat{H}_{\mu\nu}(x)\hat{\phi}(x,p)\right]$$

$$\hat{\phi}(x;p) \equiv \int \frac{d^4k}{(2\pi)^4} \delta(x - k^+ / p^+)\, \hat{\phi}(k,p) = \frac{1}{2} p^+ \not{\bar{n}} f_1(x) + \dots$$

$$\Longrightarrow \quad W_{\mu\nu}(q,p) \approx \left[(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{1}{2xq \cdot p}(q+2xp)_\mu (q+2xp)_\nu\right] f_1(x)$$

operator expression of the number density : $f_1(x) = \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p | \bar{\psi}(0) \frac{\gamma^+}{2} \psi(z) | p \rangle$

no local (color) gauge invariance!



Inclusive DIS with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$$W_{\mu\nu}(q,p,S) = \text{(a)} + \text{(b)} + \text{(c)} + \cdots$$

$$W_{\mu\nu}(q,p,S) = W^{(0)}_{\mu\nu}(q,p,S) + W^{(1)}_{\mu\nu}(q,p,S) + W^{(2)}_{\mu\nu}(q,p,S) + \dots$$

$$W^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]$$

$$W^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q)\hat{\phi}^{(1)}_\rho(k_1,k_2,p,S)\right]$$

$$\hat{H}^{(1)\rho}_{\mu\nu} = \hat{H}^{(1,L)\rho}_{\mu\nu} + \hat{H}^{(1,R)\rho}_{\mu\nu}$$

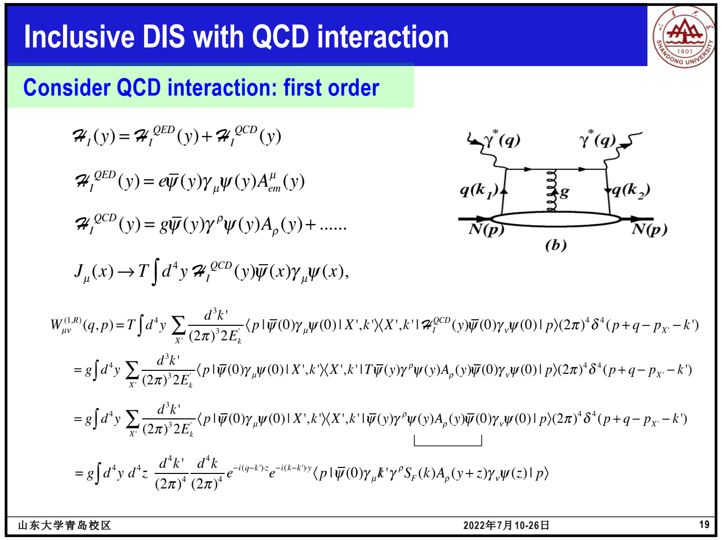
the calculable hard part: $\hat{H}^{(0)}_{\mu\nu}(k,q) = \gamma_\mu(\not{k} + \not{q})\gamma_\nu(2\pi)\delta_+\left((k+q)^2\right)$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q) = \gamma_\mu \frac{(\not{k}_2 + \not{q})\gamma^\rho(\not{k}_1 + \not{q})}{(k_2+q)^2 - i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$

the quark-quark correlator: $\hat{\phi}^{(0)}(k;p,S) = \int d^4z\, e^{ikz}\langle p,S|\psi(0)\psi(z)|p,S\rangle$

the quark-gluon-quark correlator: $\hat{\phi}^{(1)}(k_1,k_2;p,S) = \int d^4y\, d^4z e^{ik_1z+ik_2(y-z)}\langle p,S|\psi(0)A_\rho(y)\psi(z)|p,S\rangle$

no (local) gauge invariance!



Inclusive DIS with QCD interaction

Consider QCD interaction: first order

$$\mathcal{H}_I(y) = \mathcal{H}_I^{QED}(y) + \mathcal{H}_I^{QCD}(y)$$

$$\mathcal{H}_I^{QED}(y) = e\bar{\psi}(y)\gamma_\mu\psi(y)A^\mu_{em}(y)$$

$$\mathcal{H}_I^{QCD}(y) = g\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y) + \dots\dots$$

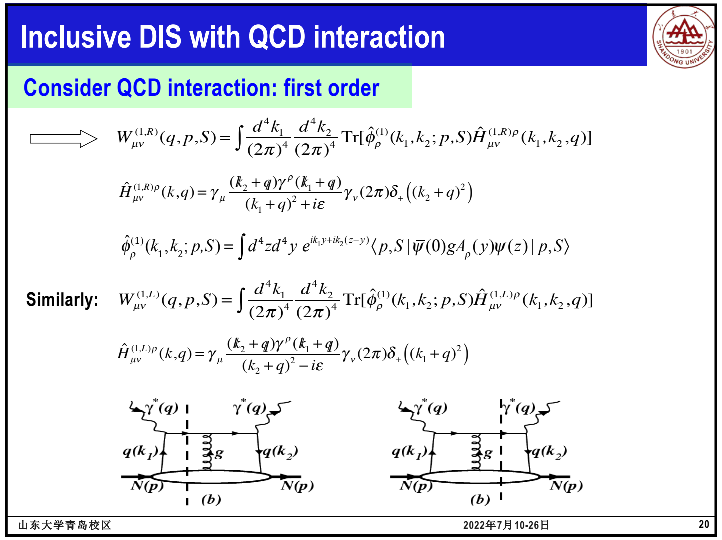
$$J_\mu(x) \to T\int d^4y\, \mathcal{H}_I^{QCD}(y)\bar{\psi}(x)\gamma_\mu\psi(x),$$

$$W^{(1,R)}_{\mu\nu}(q,p) = T\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E'_k} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\mathcal{H}_I^{QCD}(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E'_k} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|T\bar{\psi}(y)\gamma^\rho\psi(y)A_\rho(y)\bar{\psi}(0)\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y \sum_{X'} \frac{d^3k'}{(2\pi)^3 2E'_k} \langle p|\bar{\psi}(0)\gamma_\mu\psi(0)|X',k'\rangle\langle X',k'|\bar{\psi}(y)\gamma^\rho\underbrace{\psi(y)A_\rho(y)\bar{\psi}(0)}\gamma_\nu\psi(0)|p\rangle(2\pi)^4\delta^4(p+q-p_{X'}-k')$$

$$= g\int d^4y\, d^4z\; \frac{d^4k'}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} e^{-i(q-k')\cdot z}e^{-i(k-k')\cdot y}\langle p|\bar{\psi}(0)\gamma_\mu\not{k}'\gamma^\rho S_F(k)A_\rho(y+z)\gamma_\nu\psi(z)|p\rangle$$



Inclusive DIS with QCD interaction

Consider QCD interaction: first order

$$\Longrightarrow \quad W^{(1,R)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}[\hat{\phi}^{(1)}_\rho(k_1,k_2;p,S)\hat{H}^{(1,R)\rho}_{\mu\nu}(k_1,k_2,q)]$$

$$\hat{H}^{(1,R)\rho}_{\mu\nu}(k,q) = \gamma_\mu \frac{(\not{k}_2 + \not{q})\gamma^\rho(\not{k}_1 + \not{q})}{(k_1+q)^2 + i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_2+q)^2\right)$$

$$\hat{\phi}^{(1)}_\rho(k_1,k_2;p,S) = \int d^4zd^4y\, e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)gA_\rho(y)\psi(z)|p,S\rangle$$

Similarly: $W^{(1,L)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}[\hat{\phi}^{(1)}_\rho(k_1,k_2;p,S)\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q)]$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(k,q) = \gamma_\mu \frac{(\not{k}_2 + \not{q})\gamma^\rho(\not{k}_1 + \not{q})}{(k_2+q)^2 - i\varepsilon}\gamma_\nu(2\pi)\delta_+\left((k_1+q)^2\right)$$

Inclusive DIS: LO pQCD, leading twist

Collinear approximation:

$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$

$\hat{H}^{(1)}_{\mu\nu}(x_1, x_2) \equiv \hat{H}^{(1)}_{\mu\nu}(k_1 = x_1 p, k_2 = x_2 p, q$

✪ Approximating the **hard part** as that at *k =xp:*

$$\hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x)$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) \approx \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)$$

$x = k^+ / p^+$

$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$

$n = (0,1,\vec{0}_\perp)$

$\bar{n} = (1,0,\vec{0}_\perp)$

✪ Keep only the longitudinal component of the gluon field:

$$A_\rho(y) \approx n \cdot A(y) \frac{p_\rho}{n \cdot p} = A^+(y) \frac{p_\rho}{p^+}$$

✪ Using the Ward identities such as,

$$p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace hard parts for diagrams with multiple gluon scatterings by $\hat{H}^{(0)}_{\mu\nu}(x)$.

✪ Adding all terms together ⇨

Inclusive DIS: LO pQCD, leading twist

$$\Rightarrow \quad W_{\mu\nu}(q,p,S) \approx \tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k;p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

LO & leading twist

$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z e^{ikz} \langle p,S | \bar{\psi}(0) \mathcal{L}(0,z) \psi(z) | p,S \rangle$$

The gauge invariant un-integrated quark-quark correlator: contain QCD interaction!

gauge link

$$\mathcal{L}(0,z) = \mathcal{L}^\dagger(-\infty,0)\mathcal{L}(-\infty,z),$$

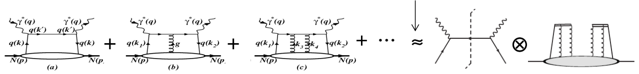
$$\mathcal{L}(-\infty,z) = Pe^{ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp)}$$

$$= 1 + ig\int_{-\infty}^{z^-} dy^- A^+(0,y^-,\vec{0}_\perp) + \frac{1}{2}(ig)^2 \int_{-\infty}^{z^-} dy^- \int_{-\infty}^{y^-} dy'^- A^+(0,y^-,\vec{0}_\perp) A^+(0,y'^-,\vec{0}_\perp) + \ldots\ldots$$

Gauge link comes from the multiple gluon scattering.

Graphically:

collinear approximation



Inclusive DIS: LO pQCD, leading & higher twists

Collinear expansion: Ellis, Furmanski, Petronzio (1982,1983); Qiu, Sterman (1990,1991)

✪ Expanding the **hard part** at *k =xp* :

$\hat{H}^{(0)}_{\mu\nu}(x) \equiv \hat{H}^{(0)}_{\mu\nu}(k = xp, q)$

$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \equiv \left.\frac{\partial \hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=xp}$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} \omega_\rho{}^{\rho'} k_{\rho'} + \ldots\ldots$$

$$\hat{H}^{(1)\rho}_{\mu\nu}(k_1,k_2,q) = \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2) + \frac{\partial \hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)}{\partial k_1^\sigma} \omega_\sigma{}^{\sigma'} k_{1\sigma'} + \ldots\ldots$$

✪ Decomposition of the gluon field:

$$A_\rho(y) = n \cdot A(y) \frac{p_\rho}{n \cdot p} + \omega_\rho{}^{\rho'} A_{\rho'}(y)$$

$x = k^+ / p^+$

$\omega_\rho{}^{\rho'} \equiv g_\rho{}^{\rho'} - \bar{n}_\rho n^{\rho'}$

$\omega_\rho{}^{\rho'} k_{\rho'} = (k - xp)_\rho$

$k^\pm = \frac{1}{\sqrt{2}}(k_0 \pm k_3)$

$n = (0,1,\vec{0}_\perp)$

$\bar{n} = (1,0,\vec{0}_\perp)$

✪ Using the Ward identities such as,

$$\frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k^\rho} = -\hat{H}^{(1)\rho}_{\mu\nu}(x,x), \quad p_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\varepsilon}$$

to replace the derivatives etc.

✪ Adding all terms with the same hard part together ⇨

Inclusive DIS: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S) = \tilde{W}^{(0)}_{\mu\nu}(q,p,S) + \tilde{W}^{(1)}_{\mu\nu}(q,p,S) + \tilde{W}^{(2)}_{\mu\nu}(q,p,S) + \ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(x)\right]$$

$$\hat{\Phi}^{(0)}(k;p,S) = \int d^4z\, e^{ikz} \langle p,S | \psi(0) \mathcal{L}(0,z) \psi(z) | p,S \rangle$$

gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{\Phi}^{(1)}_{\rho'}(k_1,k_2,p,S)\hat{H}^{(1)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho{}^{\rho'}\right]$$

$$\hat{\Phi}^{(1)}(k_1,k_2;p,S) = \int d^4y\, d^4z e^{ik_1z+ik_2(y-z)} \langle p,S | \psi(0) \mathcal{L}(0,y) D_\rho(y) \mathcal{L}(y,z) \psi(z) | p,S \rangle$$

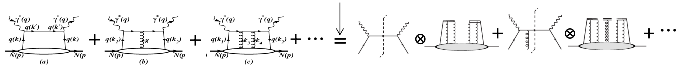
$$D_\rho(y) = -i\partial_\rho + gA_\rho(y)$$

gauge invariant quark-gluon-quark correlator

⇨ A consistent framework for inclusive DIS $e^- N \to e^- X$ including leading & higher twists

Graphically

collinear expansion



Inclusive DIS: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The “collinearly expanded hard parts” take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(x)=\hat{h}^{(0)}_{\mu\nu}\delta(x-x_B),\qquad \hat{h}^{(0)}_{\mu\nu}=\gamma_\mu\not{n}\gamma_\nu$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1,x_2)\omega_\rho^{\ \rho'}=\frac{\pi}{2q\cdot p}\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\delta(x_1-x_B),\quad \hat{h}^{(1)\rho}_{\mu\nu}=\gamma_\mu\bar{\not{n}}\gamma^\rho\not{n}\gamma_\nu$$

depends only on ONE variable!

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int dx\mathrm{Tr}\left[\hat{\Phi}^{(0)}(x;p,S)\ h^{(0)}_{\mu\nu}\right]\delta(x-x_B)$$

$$\hat{\Phi}^{(0)}(x;p,S)\equiv\int\frac{d^4k}{(2\pi)^4}\delta(x-\frac{k^+}{p^+})\hat{\Phi}^{(0)}(k;p,S)=\int\frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

one-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

$$\tilde{W}^{(1)}_{\mu\nu}(q,p,S)=\frac{\pi}{2q\cdot p}\mathrm{Re}\int dx\mathrm{Tr}\left[\hat{\varphi}^{(1)}_{\rho'}(x;p,S)\ h^{(1)\rho}_{\mu\nu}\omega_\rho^{\ \rho'}\right]\delta(x-x_B)$$

$$\hat{\varphi}^{(1)}_\rho(x;p,S)\equiv\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x-\frac{k_1^+}{p^+})\hat{\Phi}^{(1)}_\rho(k_1,k_2;p,S)=\int\frac{p^+dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z^-)\psi(z^-)|p,S\rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

⇨ Only ONE dimensional imaging of the nucleon is involved in inclusive DIS.

PDFs defined via quark-quark correlator

- Expand the quark-quark correlator in terms of the Γ-matrices:

$$\hat{\Phi}^{(0)}(x;p,S)=\frac{1}{2}\left[\Phi^{(0)}(x;p,S)+i\gamma_5\tilde{\Phi}^{(0)}(x;p,S)+\gamma^\alpha\Phi^{(0)}_\alpha(x;p,S)+\gamma_5\gamma^\alpha\tilde{\Phi}^{(0)}_\alpha(x;p,S)+i\gamma_5{}^\alpha\sigma^{\alpha\beta}\Phi^{(0)}_{\alpha\beta}(x;p,S)\right]$$

(scalar) (pseudo-scalar) (vector) (axial vector) (tensor)

- Make Lorentz decompositions

$$p=p^+\bar{n}+\frac{M^2}{2p^+}n,\quad S=\lambda\frac{p^+}{M}\bar{n}+S_T-\lambda\frac{M^2}{2p^+}n$$

$$\Phi^{(0)}(x;p,S)=Me(x)$$

$$\tilde{\Phi}^{(0)}(x;p,S)=\lambda Me_L(x)$$

$$\Phi^{(0)}_\alpha(x;p,S)=p^+\bar{n}_\alpha f_1(x)+M\varepsilon_{\perp\alpha\rho}S_T^\rho f_T(x)+\frac{M^2}{p^+}n_\alpha f_3(x)$$

$$\tilde{\Phi}^{(0)}_\alpha(x;p,S)=\lambda p^+\bar{n}_\alpha g_{1L}(x)+MS_{T\alpha}g_T(x)+\lambda\frac{M^2}{p^+}n_\alpha g_{3L}(x)$$

$$\Phi^{(0)}_{\rho\alpha}(x;p,S)=p^+\bar{n}_{[\rho}S_{T\alpha]}h_{1T}(x)-M\varepsilon_{T\rho\alpha}h_T(x)+\lambda M\bar{n}_{[\rho}n_{\alpha]}h_L(x)+\frac{M^2}{p^+}n_{[\rho}S_{T\alpha]}h_{3T}(x)$$

3+6+3

blue: twist-2

black: twist-3, M/Q suppressed

brown: twist-4, (M/Q)^2 suppressed

$$A_{[\alpha}B_{\beta]}\equiv A_\alpha B_\beta-A_\beta B_\alpha$$

$$\varepsilon_{\perp\alpha\beta}\equiv\varepsilon_{\rho\sigma\alpha\beta}\bar{n}^\rho n^\sigma$$

the scalar functions are the one-dimensional PDFs, e.g.,

$$f_1(x)=\frac{1}{p^+}n^\alpha\Phi^{(0)}_\alpha(x;p,S)=\int\frac{dz^-}{2\pi}e^{ixp^+z^-}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z^-)\frac{\gamma^+}{2}\psi(z^-)|p,S\rangle$$

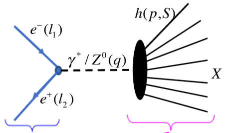
Inclusive hadron production in e⁺e⁻-annihilation $e^-+e^+\to h+X$

The differential cross section ⇨ definition of the fragmentation function

$$d\sigma=\chi\frac{\alpha_{em}^2}{sQ^4}L^{\mu\nu}(l_1,\lambda_1,l_2,\lambda_2)W_{\mu\nu}(q,p,S)\frac{d^3p}{2E}$$

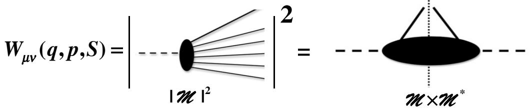
leptonic tensor

hadronic tensor



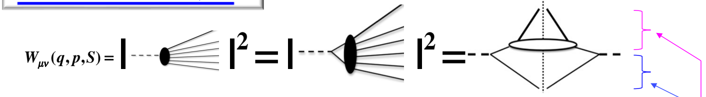
$$-q^2=Q^2\qquad z_B=\frac{2q\cdot p}{Q^2}$$

The hadronic tensor: $W_{\mu\nu}(q,p,S)=\sum_X\langle p,S;X|J_\mu(0)|0\rangle\langle 0|J_\nu(0)|p,S;X\rangle(2\pi)^4\delta^4(q-p-p_X)$



Quantum field theoretical formulation $e^-+e^+\to h+X$

Parton model without QCD:



$$W_{\mu\nu}(q,p,S)=\sum_X\langle p,S;X|J_\mu(0)|0\rangle\langle 0|J_\nu(0)|p,S;X\rangle(2\pi)^4\delta^4(q-p-p_X)$$

$$=\sum_X\int d^4z\langle p,S;X|J_\mu(0)|X\rangle\langle 0|J_\nu(z)|p,S;X\rangle e^{-iqz}$$

$$J_\mu(x)=\bar{\psi}(x)\Gamma_\mu\psi(x),$$
$$|X\rangle=|X'\rangle|k'\rangle,$$
$$\bar{\psi}(x)|X'\rangle|k'\rangle=\bar{v}(k')e^{ik'\cdot x}|X'\rangle$$

$$=\int\frac{d^4k'}{(2\pi)^4}(2\pi)\delta_+(k'^2)\sum_{X'}\int d^4ze^{-iqz}\langle p,S;X'|\bar{\psi}(0)|0\rangle\Gamma_\mu v(k')\bar{v}(k')\Gamma_\nu\langle 0|\psi(z)|p,S;X'\rangle e^{-ik'z}$$

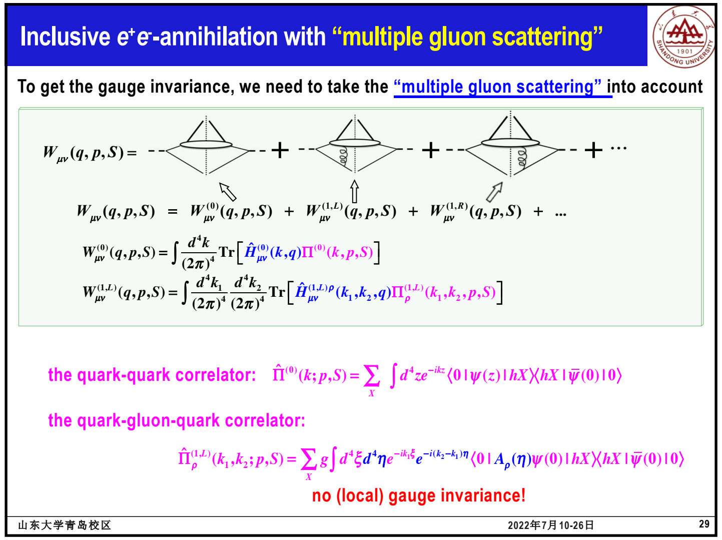
$$=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{H}_{\mu\nu}(k,q)\hat{\Pi}(k,p,S)\right]$$

$$\Gamma_\mu=\begin{cases}\gamma_\mu\\ \gamma_\mu(c_V-c_A\gamma_5)\end{cases}$$

the calculable hard part $\hat{H}_{\mu\nu}(k,q)=\Gamma_\mu(\not{q}-\not{k})\Gamma_\nu(2\pi)\delta_+\left((q-k)^2\right)$

the quark-quark correlator $\hat{\Pi}(k;p,S)=\sum_X\int d^4ze^{-ikz}\langle 0|\psi(z)|p,S;X\rangle\langle p,S;X|\bar{\psi}(0)|0\rangle$

no local (color) gauge invariance!



Inclusive e⁺e⁻-annihilation with "multiple gluon scattering"

To get the gauge invariance, we need to take the "multiple gluon scattering" into account

$$W_{\mu\nu}(q,p,S)= \cdots + \cdots + \cdots + \cdots$$

$$W_{\mu\nu}(q,p,S) = W^{(0)}_{\mu\nu}(q,p,S) + W^{(1,L)}_{\mu\nu}(q,p,S) + W^{(1,R)}_{\mu\nu}(q,p,S) + \ldots$$

$$W^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\Pi^{(0)}(k,p,S)\right]$$

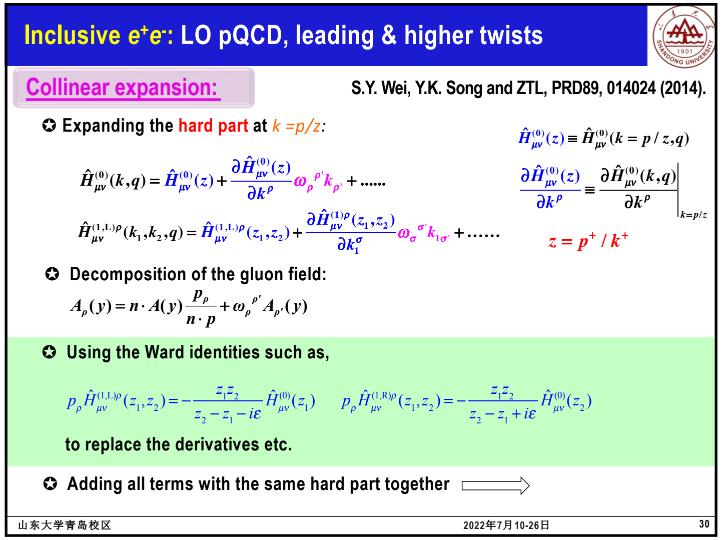
$$W^{(1,L)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{H}^{(1,L)\rho}_{\mu\nu}(k_1,k_2,q)\Pi^{(1,L)}_{\rho}(k_1,k_2,p,S)\right]$$

the quark-quark correlator: $\hat{\Pi}^{(0)}(k;p,S)=\sum_X\int d^4z e^{-ikz}\langle 0|\psi(z)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$

the quark-gluon-quark correlator:

$$\hat{\Pi}^{(1,L)}_{\rho}(k_1,k_2;p,S)=\sum_X g\int d^4\xi d^4\eta e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|A_\rho(\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(0)|0\rangle$$

no (local) gauge invariance!



Inclusive e⁺e⁻: LO pQCD, leading & higher twists

Collinear expansion:

S.Y. Wei, Y.K. Song and ZTL, PRD89, 014024 (2014).

- Expanding the hard part at $k = p/z$:

$$\hat{H}^{(0)}_{\mu\nu}(k,q)=\hat{H}^{(0)}_{\mu\nu}(z)+\frac{\partial\hat{H}^{(0)}_{\mu\nu}(z)}{\partial k^\rho}\omega_\rho{}^{\rho'}k_{\rho'}+\ldots\ldots$$

$$\hat{H}^{(1,\mathrm{L})\rho}_{\mu\nu}(k_1,k_2,q)=\hat{H}^{(1,\mathrm{L})\rho}_{\mu\nu}(z_1,z_2)+\frac{\partial\hat{H}^{(1)\rho}_{\mu\nu}(z_1,z_2)}{\partial k_1^\sigma}\omega_\sigma{}^{\sigma'}k_{1\sigma'}+\ldots\ldots$$

$$\hat{H}^{(0)}_{\mu\nu}(z)\equiv\hat{H}^{(0)}_{\mu\nu}(k=p/z,q)$$

$$\frac{\partial\hat{H}^{(0)}_{\mu\nu}(z)}{\partial k^\rho}\equiv\left.\frac{\partial\hat{H}^{(0)}_{\mu\nu}(k,q)}{\partial k^\rho}\right|_{k=p/z}$$

$$z=p^+/k^+$$

- Decomposition of the gluon field:

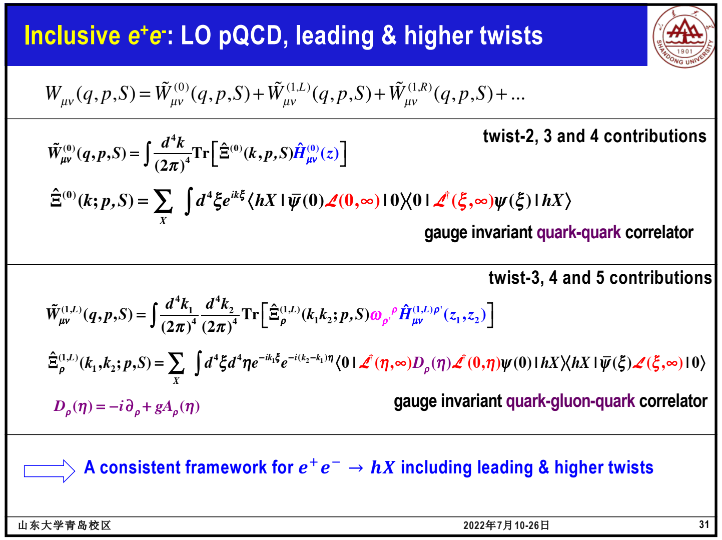
$$A_\rho(y)=n\cdot A(y)\frac{p_\rho}{n\cdot p}+\omega_\rho{}^{\rho'}A_{\rho'}(y)$$

- Using the Ward identities such as,

$$p_\rho\hat{H}^{(1,\mathrm{L})\rho}_{\mu\nu}(z_1,z_2)=-\frac{z_1z_2}{z_2-z_1-i\varepsilon}\hat{H}^{(0)}_{\mu\nu}(z_1)\qquad p_\rho\hat{H}^{(1,\mathrm{R})\rho}_{\mu\nu}(z_1,z_2)=-\frac{z_1z_2}{z_2-z_1+i\varepsilon}\hat{H}^{(0)}_{\mu\nu}(z_2)$$

to replace the derivatives etc.

- Adding all terms with the same hard part together ⇨



Inclusive e⁺e⁻: LO pQCD, leading & higher twists

$$W_{\mu\nu}(q,p,S)=\tilde{W}^{(0)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S)+\tilde{W}^{(1,R)}_{\mu\nu}(q,p,S)+\ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(0)}(k,p,S)\hat{H}^{(0)}_{\mu\nu}(z)\right]$$

$$\hat{\Xi}^{(0)}(k;p,S)=\sum_X\int d^4\xi e^{ik\xi}\langle hX|\bar{\psi}(0)\mathcal{L}(0,\infty)|0\rangle\langle 0|\mathcal{L}^\dagger(\xi,\infty)\psi(\xi)|hX\rangle$$

gauge invariant quark-quark correlator

twist-3, 4 and 5 contributions

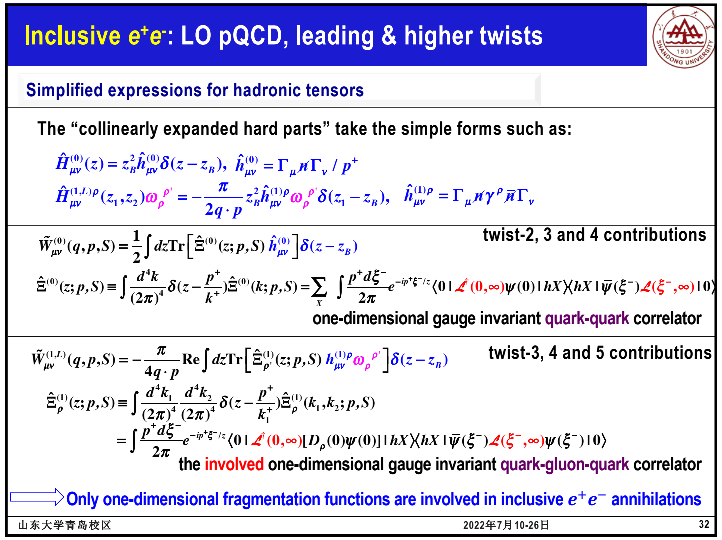
$$\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S)=\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(1,L)}_{\rho}(k_1k_2;p,S)\omega_{\rho'}{}^{\rho}\hat{H}^{(1,L)\rho'}_{\mu\nu}(z_1,z_2)\right]$$

$$\hat{\Xi}^{(1,L)}_{\rho}(k_1,k_2;p,S)=\sum_X\int d^4\xi d^4\eta e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|\mathcal{L}^\dagger(\eta,\infty)D_\rho(\eta)\mathcal{L}^\dagger(0,\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$D_\rho(\eta)=-i\partial_\rho+gA_\rho(\eta)$

gauge invariant quark-gluon-quark correlator

⇨ A consistent framework for $e^+e^-\to hX$ including leading & higher twists



Inclusive e⁺e⁻: LO pQCD, leading & higher twists

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}^{(0)}_{\mu\nu}(z)=z_B^2\hat{h}^{(0)}_{\mu\nu}\delta(z-z_B),\quad \hat{h}^{(0)}_{\mu\nu}=\Gamma_\mu\not{n}\Gamma_\nu/p^+$$

$$\hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2)\omega_\rho{}^{\rho'}=-\frac{\pi}{2q\cdot p}z_B^2\hat{h}^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\delta(z_1-z_B),\quad \hat{h}^{(1)\rho}_{\mu\nu}=\Gamma_\mu\not{n}\gamma^\rho\bar{\not{n}}\Gamma_\nu$$

twist-2, 3 and 4 contributions

$$\tilde{W}^{(0)}_{\mu\nu}(q,p,S)=\frac{1}{2}\int dz\mathrm{Tr}\left[\hat{\Xi}^{(0)}(z;p,S)\,\hat{h}^{(0)}_{\mu\nu}\right]\delta(z-z_B)$$

$$\hat{\Xi}^{(0)}(z;p,S)\equiv\int\frac{d^4k}{(2\pi)^4}\delta(z-\frac{p^+}{k^+})\hat{\Xi}^{(0)}(k;p,S)=\sum_X\int\frac{p^+d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

one-dimensional gauge invariant quark-quark correlator

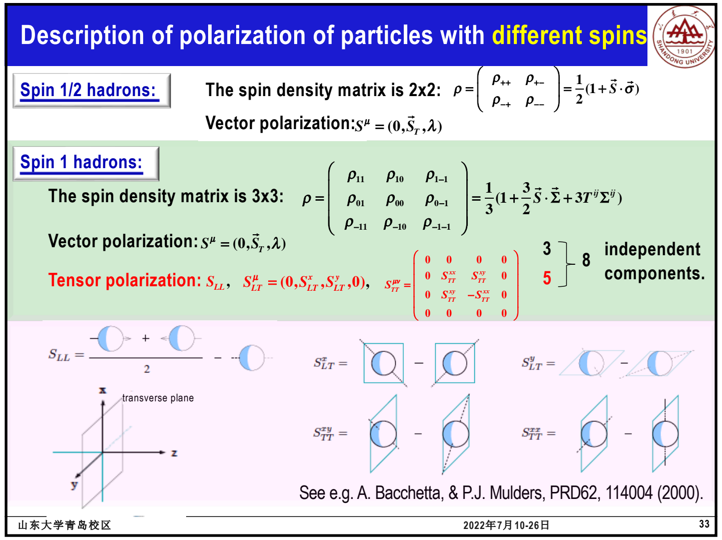
twist-3, 4 and 5 contributions

$$\tilde{W}^{(1,L)}_{\mu\nu}(q,p,S)=-\frac{\pi}{4q\cdot p}\mathrm{Re}\int dz\mathrm{Tr}\left[\hat{\Xi}^{(1)}_{\rho'}(z;p,S)\,h^{(1)\rho}_{\mu\nu}\omega_\rho{}^{\rho'}\right]\delta(z-z_B)$$

$$\hat{\Xi}^{(1)}_{\rho}(z;p,S)\equiv\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(z-\frac{p^+}{k_1^+})\hat{\Xi}^{(1)}_{\rho}(k_1,k_2;p,S)$$
$$=\int\frac{p^+d\xi^-}{2\pi}e^{-ip^+\xi^-/z}\langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)\psi(\xi^-)|0\rangle$$

the involved one-dimensional gauge invariant quark-gluon-quark correlator

⇨ Only one-dimensional fragmentation functions are involved in inclusive e^+e^- annihilations



Description of polarization of particles with different spins

Spin 1/2 hadrons:

The spin density matrix is 2x2: $\rho=\begin{pmatrix}\rho_{++} & \rho_{+-}\\ \rho_{-+} & \rho_{--}\end{pmatrix}=\frac{1}{2}(1+\vec{S}\cdot\vec{\sigma})$

Vector polarization: $S^{\mu}=(0,\vec{S}_T,\lambda)$

Spin 1 hadrons:

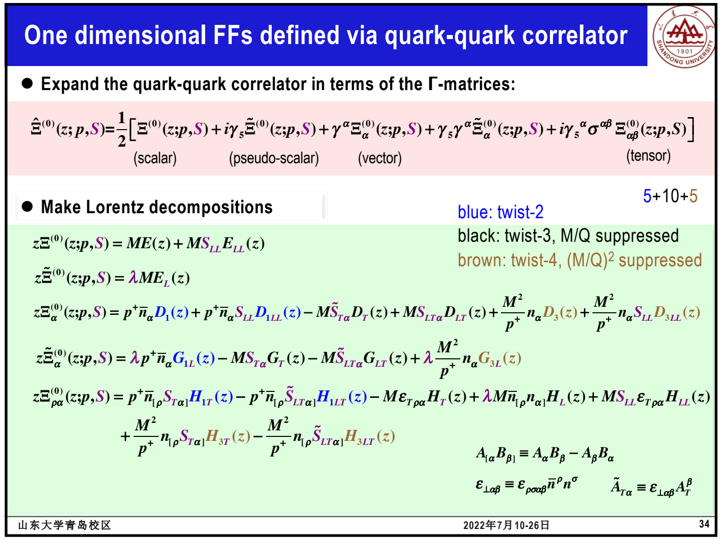
The spin density matrix is 3x3: $\rho=\begin{pmatrix}\rho_{11} & \rho_{10} & \rho_{1-1}\\ \rho_{01} & \rho_{00} & \rho_{0-1}\\ \rho_{-11} & \rho_{-10} & \rho_{-1-1}\end{pmatrix}=\frac{1}{3}(1+\frac{3}{2}\vec{S}\cdot\vec{\Sigma}+3T^{ij}\Sigma^{ij})$

Vector polarization: $S^{\mu}=(0,\vec{S}_T,\lambda)$ 3

Tensor polarization: S_{LL}, $S_{LT}^{\mu}=(0,S_{LT}^{x},S_{LT}^{y},0)$, $S_{TT}^{\mu\nu}=\begin{pmatrix}0&0&0&0\\0&S_{TT}^{xx}&S_{TT}^{xy}&0\\0&S_{TT}^{xy}&-S_{TT}^{xx}&0\\0&0&0&0\end{pmatrix}$ 5

8 independent components.

See e.g. A. Bacchetta, & P.J. Mulders, PRD62, 114004 (2000).



One dimensional FFs defined via quark-quark correlator

- Expand the quark-quark correlator in terms of the Γ-matrices:

$$\hat{\Xi}^{(0)}(z;p,S)=\frac{1}{2}\Big[\Xi^{(0)}(z;p,S)+i\gamma_5\tilde{\Xi}^{(0)}(z;p,S)+\gamma^{\alpha}\Xi_{\alpha}^{(0)}(z;p,S)+\gamma_5\gamma^{\alpha}\tilde{\Xi}_{\alpha}^{(0)}(z;p,S)+i\gamma_5{}^{\alpha}\sigma^{\alpha\beta}\,\Xi_{\alpha\beta}^{(0)}(z;p,S)\Big]$$

(scalar) (pseudo-scalar) (vector) (tensor)

- Make Lorentz decompositions

5+10+5

blue: twist-2

black: twist-3, M/Q suppressed

brown: twist-4, (M/Q)^2 suppressed

$$z\Xi^{(0)}(z;p,S)=ME(z)+MS_{LL}E_{LL}(z)$$

$$z\tilde{\Xi}^{(0)}(z;p,S)=\lambda ME_{L}(z)$$

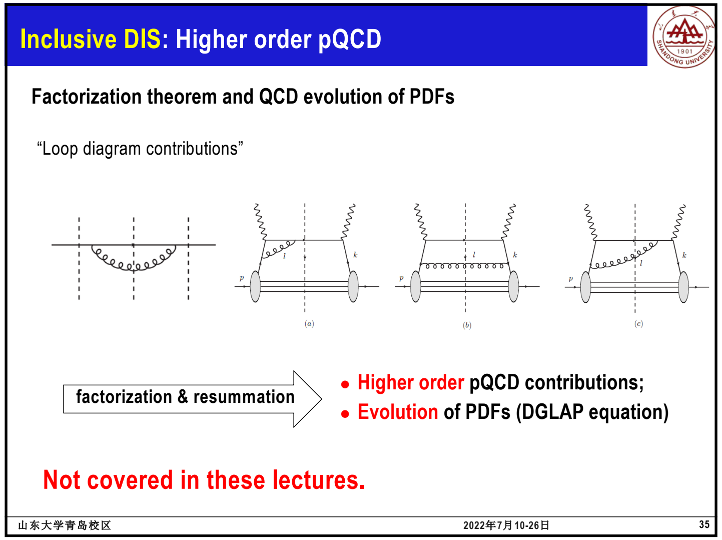
$$z\Xi_{\alpha}^{(0)}(z;p,S)=p^{+}\bar{n}_{\alpha}D_{1}(z)+p^{+}\bar{n}_{\alpha}S_{LL}D_{1LL}(z)-M\tilde{S}_{T\alpha}D_{T}(z)+MS_{LT\alpha}D_{LT}(z)+\frac{M^{2}}{p^{+}}n_{\alpha}D_{3}(z)+\frac{M^{2}}{p^{+}}n_{\alpha}S_{LL}D_{3LL}(z)$$

$$z\tilde{\Xi}_{\alpha}^{(0)}(z;p,S)=\lambda p^{+}\bar{n}_{\alpha}G_{1L}(z)-MS_{T\alpha}G_{T}(z)-M\tilde{S}_{LT\alpha}G_{LT}(z)+\lambda\frac{M^{2}}{p^{+}}n_{\alpha}G_{3L}(z)$$

$$z\Xi_{\rho\alpha}^{(0)}(z;p,S)=p^{+}\bar{n}_{[\rho}S_{T\alpha]}H_{1T}(z)-p^{+}\bar{n}_{[\rho}\tilde{S}_{LT\alpha]}H_{1LT}(z)-M\varepsilon_{T\rho\alpha}H_{T}(z)+\lambda M\bar{n}_{[\rho}n_{\alpha]}H_{L}(z)+MS_{LL}\varepsilon_{T\rho\alpha}H_{LL}(z)$$
$$+\frac{M^{2}}{p^{+}}n_{[\rho}S_{T\alpha]}H_{3T}(z)-\frac{M^{2}}{p^{+}}n_{[\rho}\tilde{S}_{LT\alpha]}H_{3LT}(z)$$

$$A_{[\alpha}B_{\beta]}\equiv A_{\alpha}B_{\beta}-A_{\beta}B_{\alpha}$$

$$\varepsilon_{\perp\alpha\beta}\equiv\varepsilon_{\rho\sigma\alpha\beta}\bar{n}^{\rho}n^{\sigma}\qquad\tilde{A}_{T\alpha}\equiv\varepsilon_{\perp\alpha\beta}A_{T}^{\beta}$$



Inclusive DIS: Higher order pQCD

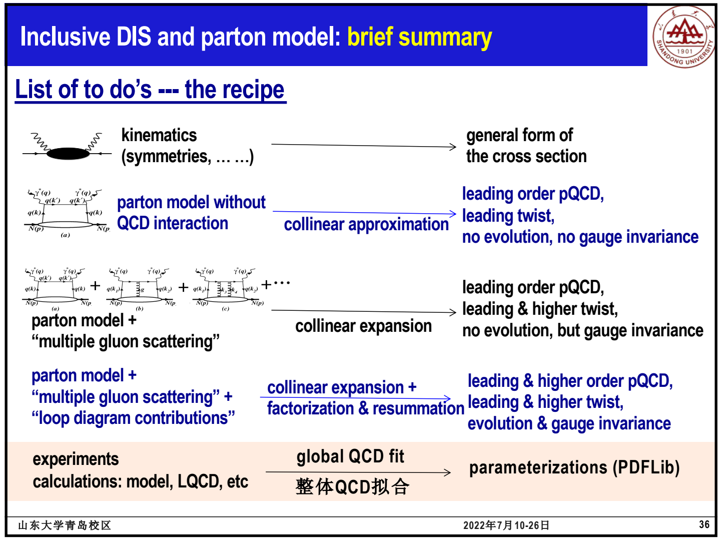
Factorization theorem and QCD evolution of PDFs

"Loop diagram contributions"

factorization & resummation

- Higher order pQCD contributions;
- Evolution of PDFs (DGLAP equation)

Not covered in these lectures.



Inclusive DIS and parton model: brief summary

List of to do's --- the recipe

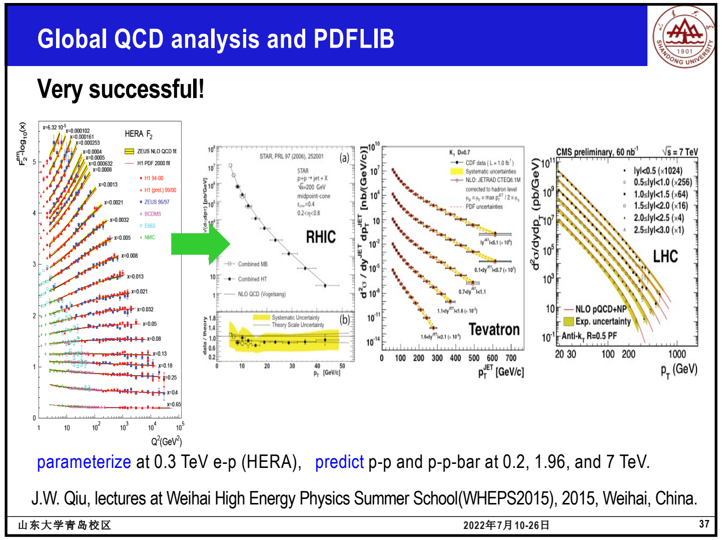
kinematics (symmetries, … …) → general form of the cross section

parton model without QCD interaction → collinear approximation → leading order pQCD, leading twist, no evolution, no gauge invariance

parton model + "multiple gluon scattering" → collinear expansion → leading order pQCD, leading & higher twist, no evolution, but gauge invariance

parton model + "multiple gluon scattering" + "loop diagram contributions" → collinear expansion + factorization & resummation → leading & higher order pQCD, leading & higher twist, evolution & gauge invariance

experiments
calculations: model, LQCD, etc → global QCD fit 整体QCD拟合 → parameterizations (PDFLib)



Global QCD analysis and PDFLIB

Very successful!

parameterize at 0.3 TeV e-p (HERA), predict p-p and p-p-bar at 0.2, 1.96, and 7 TeV.

J.W. Qiu, lectures at Weihai High Energy Physics Summer School(WHEPS2015), 2015, Weihai, China.

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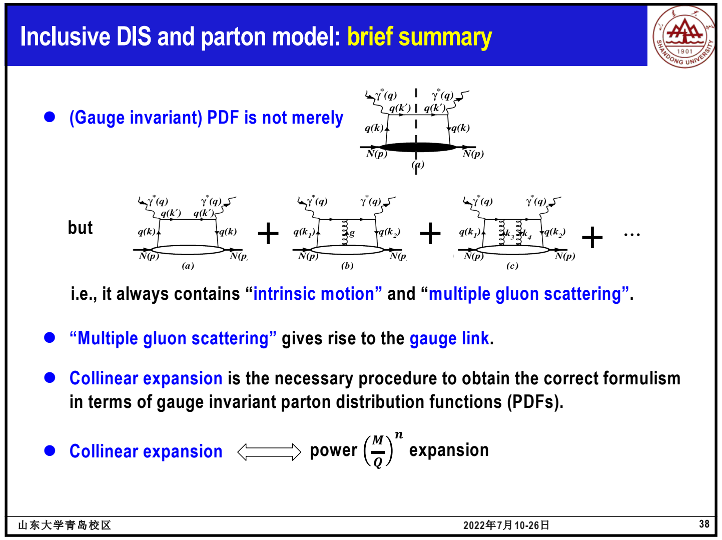

Inclusive DIS and parton model: brief summary

- (Gauge invariant) PDF is not merely

but

i.e., it always contains “intrinsic motion” and “multiple gluon scattering”.

- “Multiple gluon scattering” gives rise to the gauge link.
- Collinear expansion is the necessary procedure to obtain the correct formulism in terms of gauge invariant parton distribution functions (PDFs).
- Collinear expansion $\Longleftrightarrow$ power $\left(\frac{M}{Q}\right)^n$ expansion

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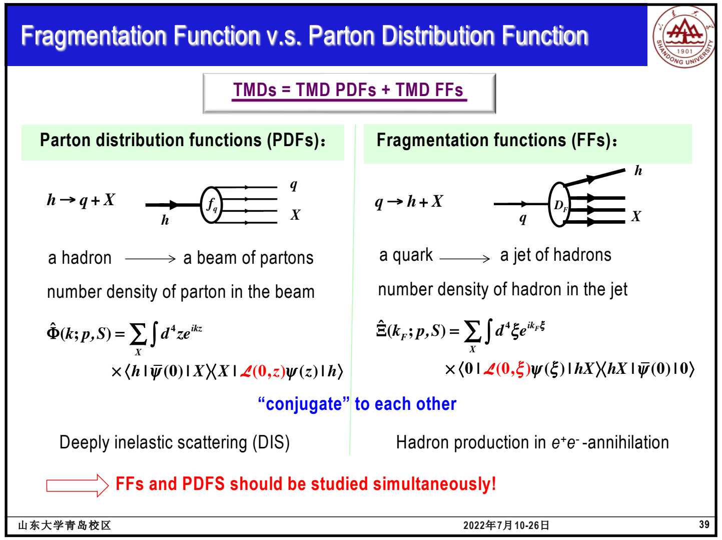

Fragmentation Function v.s. Parton Distribution Function

TMDs = TMD PDFs + TMD FFs

Parton distribution functions (PDFs)：

$h \to q + X$

a hadron ⟶ a beam of partons

number density of parton in the beam

$$\hat{\Phi}(k;p,S) = \sum_X \int d^4 z e^{ikz}$$
$$\times \langle h | \bar{\psi}(0) | X \rangle\langle X | \mathcal{L}(0,z)\psi(z) | h \rangle$$

Deeply inelastic scattering (DIS)

Fragmentation functions (FFs)：

$q \to h + X$

a quark ⟶ a jet of hadrons

number density of hadron in the jet

$$\hat{\Xi}(k_F;p,S) = \sum_X \int d^4 \xi e^{ik_F\xi}$$
$$\times \langle 0 | \mathcal{L}(0,\xi)\psi(\xi) | hX \rangle\langle hX | \bar{\psi}(0) | 0 \rangle$$

Hadron production in e^+e^--annihilation

“conjugate” to each other

⇨ FFs and PDFS should be studied simultaneously!

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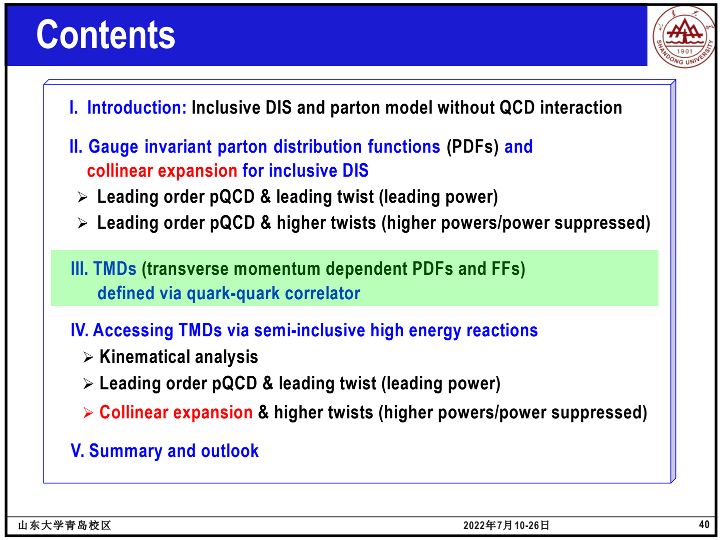

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I. Introduction: Inclusive DIS and parton model without QCD interaction

II. Gauge invariant parton distribution functions (PDFs) and collinear expansion for inclusive DIS
- ➢ Leading order pQCD & leading twist (leading power)
- ➢ Leading order pQCD & higher twists (higher powers/power suppressed)

III. TMDs (transverse momentum dependent PDFs and FFs) defined via quark-quark correlator

IV. Accessing TMDs via semi-inclusive high energy reactions
- ➢ Kinematical analysis
- ➢ Leading order pQCD & leading twist (leading power)
- ➢ Collinear expansion & higher twists (higher powers/power suppressed)

V. Summary and outlook

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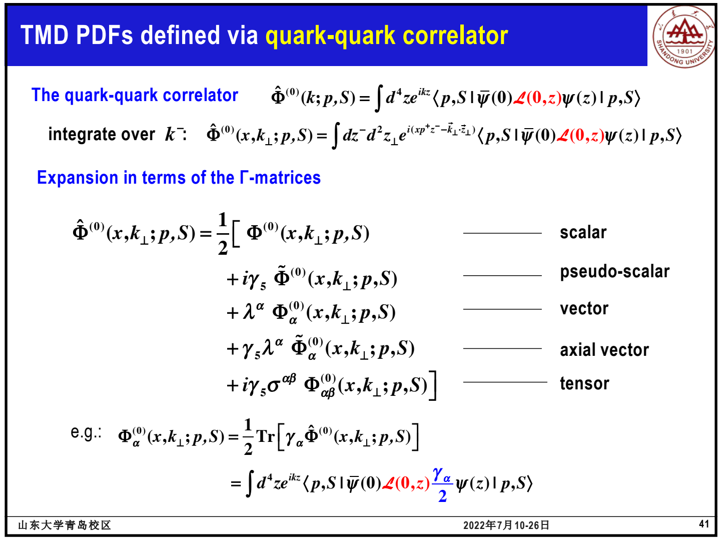

TMD PDFs defined via quark-quark correlator

The quark-quark correlator $\hat{\Phi}^{(0)}(k;p,S) = \int d^4 z e^{ikz} \langle p,S | \bar{\psi}(0) \mathcal{L}(0,z) \psi(z) | p,S \rangle$

integrate over k^-: $\hat{\Phi}^{(0)}(x,k_\perp;p,S) = \int dz^- d^2 z_\perp e^{i(xp^+z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \langle p,S | \bar{\psi}(0) \mathcal{L}(0,z) \psi(z) | p,S \rangle$

Expansion in terms of the Γ-matrices

$$\hat{\Phi}^{(0)}(x,k_\perp;p,S) = \frac{1}{2}\Big[\Phi^{(0)}(x,k_\perp;p,S) \quad \text{------ scalar}$$
$$+ i\gamma_5 \tilde{\Phi}^{(0)}(x,k_\perp;p,S) \quad \text{------ pseudo-scalar}$$
$$+ \lambda^\alpha \Phi^{(0)}_\alpha(x,k_\perp;p,S) \quad \text{------ vector}$$
$$+ \gamma_5 \lambda^\alpha \tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S) \quad \text{------ axial vector}$$
$$+ i\gamma_5 \sigma^{\alpha\beta} \Phi^{(0)}_{\alpha\beta}(x,k_\perp;p,S) \Big] \quad \text{------ tensor}$$

e.g.: $\Phi^{(0)}_\alpha(x,k_\perp;p,S) = \frac{1}{2} \mathrm{Tr}\left[\gamma_\alpha \hat{\Phi}^{(0)}(x,k_\perp;p,S)\right]$

$$= \int d^4 z e^{ikz} \langle p,S | \bar{\psi}(0) \mathcal{L}(0,z) \frac{\gamma_\alpha}{2} \psi(z) | p,S \rangle$$

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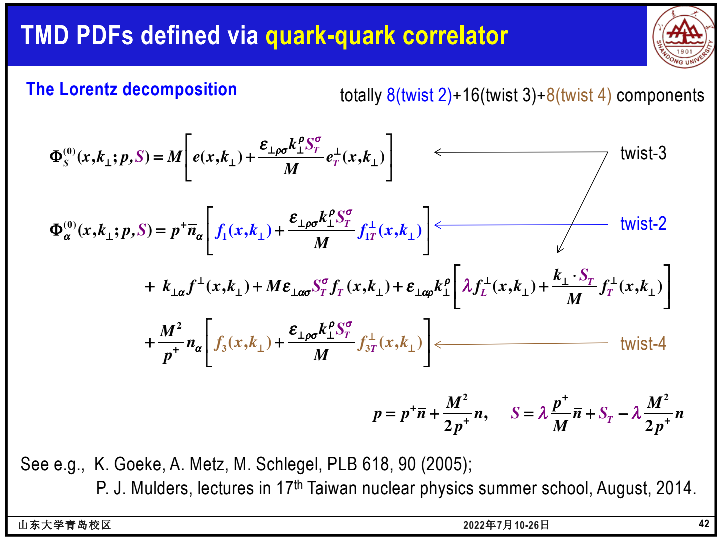

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\Phi^{(0)}_S(x,k_\perp;p,S) = M\left[e(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma} k_\perp^\rho S_T^\sigma}{M} e_T^\perp(x,k_\perp)\right] \leftarrow \text{twist-3}$$

$$\Phi^{(0)}_\alpha(x,k_\perp;p,S) = p^+ \bar{n}_\alpha \left[f_1(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma} k_\perp^\rho S_T^\sigma}{M} f_{1T}^\perp(x,k_\perp)\right] \leftarrow \text{twist-2}$$
$$+ k_{\perp\alpha} f^\perp(x,k_\perp) + M\varepsilon_{\perp\alpha\sigma} S_T^\sigma f_T(x,k_\perp) + \varepsilon_{\perp\alpha\rho} k_\perp^\rho \left[\lambda f_L^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} f_T^\perp(x,k_\perp)\right]$$
$$+ \frac{M^2}{p^+} n_\alpha \left[f_3(x,k_\perp) + \frac{\varepsilon_{\perp\rho\sigma} k_\perp^\rho S_T^\sigma}{M} f_{3T}^\perp(x,k_\perp)\right] \leftarrow \text{twist-4}$$

$$p = p^+ \bar{n} + \frac{M^2}{2p^+} n, \qquad S = \lambda \frac{p^+}{M} \bar{n} + S_T - \lambda \frac{M^2}{2p^+} n$$

See e.g., K. Goeke, A. Metz, M. Schlegel, PLB 618, 90 (2005);
P. J. Mulders, lectures in 17th Taiwan nuclear physics summer school, August, 2014.

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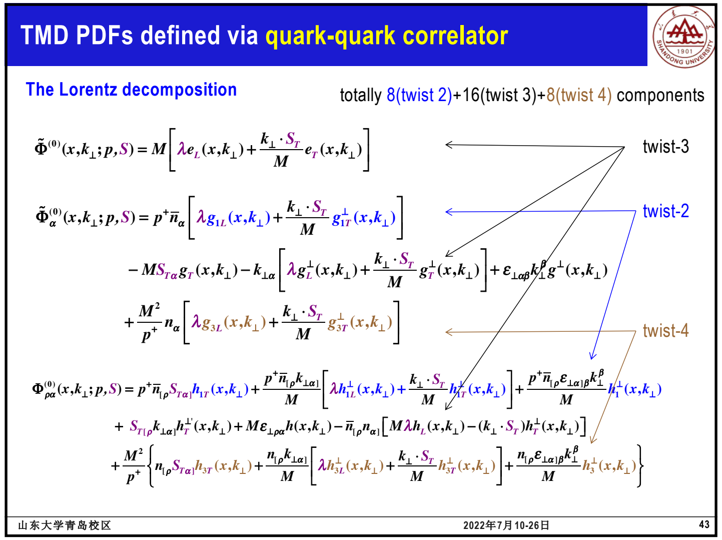

TMD PDFs defined via quark-quark correlator

The Lorentz decomposition totally 8(twist 2)+16(twist 3)+8(twist 4) components

$$\tilde{\Phi}^{(0)}(x,k_\perp;p,S) = M\left[\lambda e_L(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} e_T(x,k_\perp)\right] \leftarrow \text{twist-3}$$

$$\tilde{\Phi}^{(0)}_\alpha(x,k_\perp;p,S) = p^+ \bar{n}_\alpha \left[\lambda g_{1L}(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} g_{1T}^\perp(x,k_\perp)\right] \leftarrow \text{twist-2}$$
$$- M S_{T\alpha} g_T(x,k_\perp) - k_{\perp\alpha}\left[\lambda g_L^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} g_T^\perp(x,k_\perp)\right] + \varepsilon_{\perp\alpha\beta} k_\perp^\beta g^\perp(x,k_\perp)$$
$$+ \frac{M^2}{p^+} n_\alpha \left[\lambda g_{3L}(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} g_{3T}^\perp(x,k_\perp)\right] \leftarrow \text{twist-4}$$

$$\Phi^{(0)}_{\rho\alpha}(x,k_\perp;p,S) = p^+ \bar{n}_{[\rho} S_{T\alpha]} h_{1T}(x,k_\perp) + \frac{p^+ \bar{n}_{[\rho} k_{\perp\alpha]}}{M}\left[\lambda h_{1L}^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} h_{1T}^\perp(x,k_\perp)\right] + \frac{p^+ \bar{n}_{[\rho} \varepsilon_{\perp\alpha]\beta} k_\perp^\beta}{M} h_1^\perp(x,k_\perp)$$
$$+ S_{T[\rho} k_{\perp\alpha]} h_T^{\perp'}(x,k_\perp) + M\varepsilon_{\perp\rho\alpha} h(x,k_\perp) - \bar{n}_{[\rho} n_{\alpha]}\left[M\lambda h_L(x,k_\perp) - (k_\perp \cdot S_T) h_T^\perp(x,k_\perp)\right]$$
$$+ \frac{M^2}{p^+}\left\{n_{[\rho} S_{T\alpha]} h_{3T}(x,k_\perp) + \frac{n_{[\rho} k_{\perp\alpha]}}{M}\left[\lambda h_{3L}^\perp(x,k_\perp) + \frac{k_\perp \cdot S_T}{M} h_{3T}^\perp(x,k_\perp)\right] + \frac{n_{[\rho} \varepsilon_{\perp\alpha]\beta} k_\perp^\beta}{M} h_3^\perp(x,k_\perp)\right\}$$

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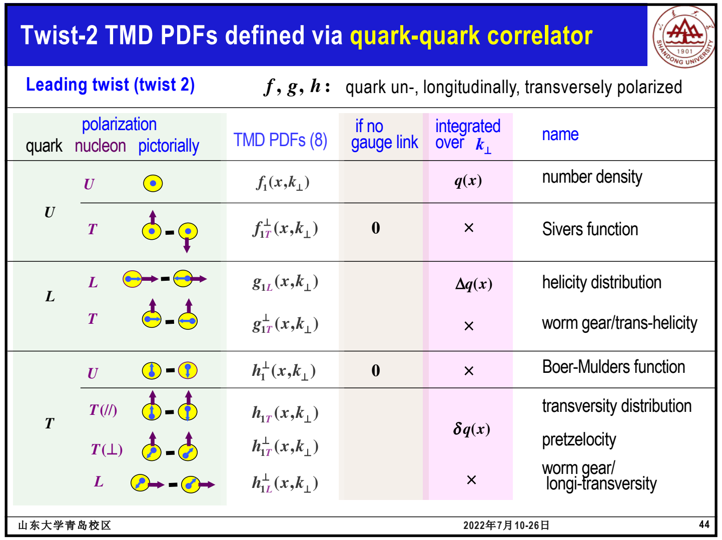

Twist-2 TMD PDFs defined via quark-quark correlator

Leading twist (twist 2) f, g, h: quark un-, longitudinally, transversely polarized

polarization			TMD PDFs (8)	if no gauge link	integrated over $k_\perp$	name
quark	nucleon	pictorially				
U	U		$f_1(x,k_\perp)$		$q(x)$	number density
	T		$f_{1T}^\perp(x,k_\perp)$	0	×	Sivers function
L	L		$g_{1L}(x,k_\perp)$		$\Delta q(x)$	helicity distribution
	T		$g_{1T}^\perp(x,k_\perp)$		×	worm gear/trans-helicity
T	U		$h_1^\perp(x,k_\perp)$	0	×	Boer-Mulders function
	T(//)		$h_{1T}(x,k_\perp)$		$\delta q(x)$	transversity distribution
	$T(\perp)$		$h_{1T}^\perp(x,k_\perp)$			pretzelocity
	L		$h_{1L}^\perp(x,k_\perp)$		×	worm gear/ longi-transversity

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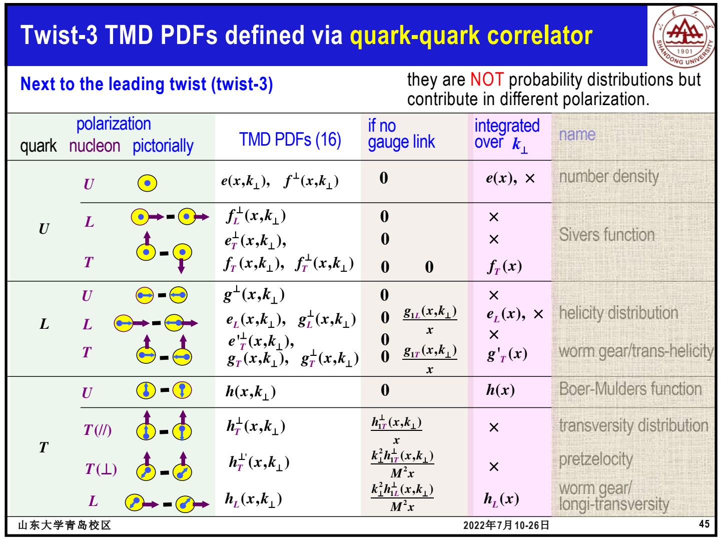
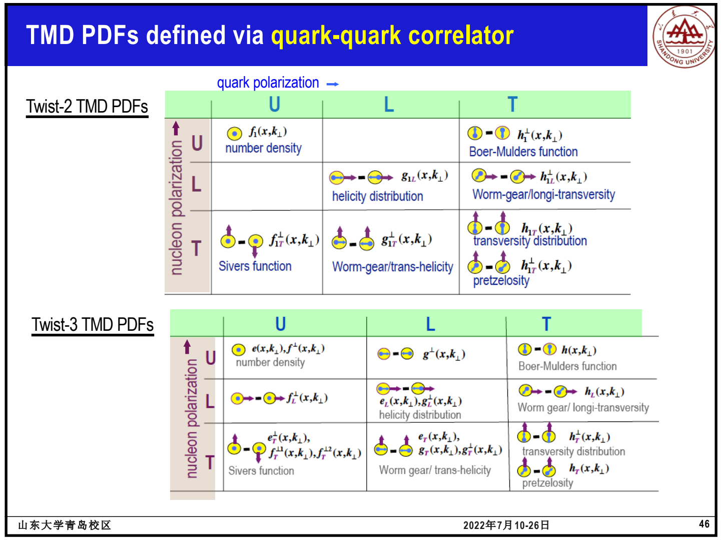


Twist-3 TMD PDFs defined via quark-quark correlator

Next to the leading twist (twist-3)

they are NOT probability distributions but contribute in different polarization.

polarization quark	nucleon	pictorially	TMD PDFs (16)	if no gauge link		integrated over $k_\perp$	name
U	U		$e(x,k_\perp),\ f^\perp(x,k_\perp)$	0		$e(x)$, ×	number density
	L		$f_L^\perp(x,k_\perp)$	0		×	Sivers function
	T		$e_T^\perp(x,k_\perp)$,	0		×	
			$f_T(x,k_\perp),\ f_T^\perp(x,k_\perp)$	0	0	$f_T(x)$	
L	U		$g^\perp(x,k_\perp)$	0		×	
	L		$e_L(x,k_\perp),\ g_L^\perp(x,k_\perp)$	0	$\frac{g_{1L}(x,k_\perp)}{x}$	$e_L(x)$, ×	helicity distribution
	T		$e'^\perp_T(x,k_\perp)$,	0		×	
			$g_T(x,k_\perp),\ g_T^\perp(x,k_\perp)$	0	$\frac{g_{1T}(x,k_\perp)}{x}$	$g'_T(x)$	worm gear/trans-helicity
T	U		$h(x,k_\perp)$	0		$h(x)$	Boer-Mulders function
	$T(//)$		$h_T^\perp(x,k_\perp)$	$\frac{h_{1T}^\perp(x,k_\perp)}{x}$		×	transversity distribution
	$T(\perp)$		$h_T^{\perp'}(x,k_\perp)$	$\frac{k_\perp^2 h_{1T}^\perp(x,k_\perp)}{M^2 x}$		×	pretzelocity
	L		$h_L(x,k_\perp)$	$\frac{k_\perp^2 h_{1L}^\perp(x,k_\perp)}{M^2 x}$		$h_L(x)$	worm gear/ longi-transversity



TMD PDFs defined via quark-quark correlator

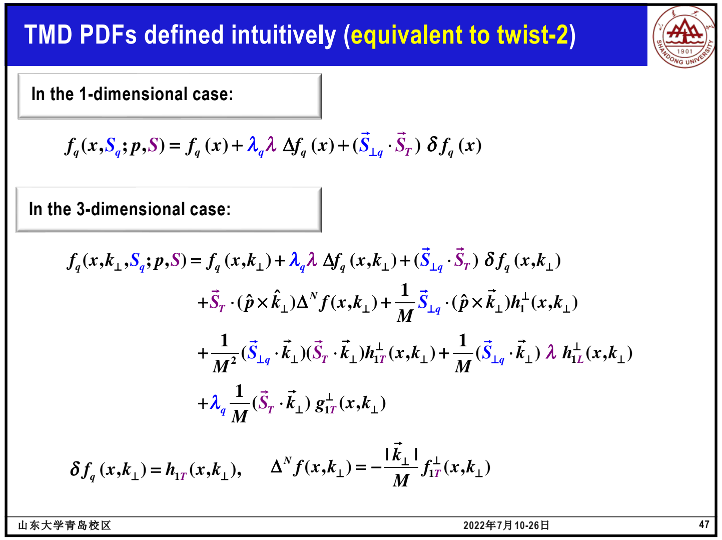
Twist-2 TMD PDFs

quark polarization →

nucleon polarization	U	L	T
U	$f_1(x,k_\perp)$ number density		$h_1^\perp(x,k_\perp)$ Boer-Mulders function
L		$g_{1L}(x,k_\perp)$ helicity distribution	$h_{1L}^\perp(x,k_\perp)$ Worm-gear/longi-transversity
T	$f_{1T}^\perp(x,k_\perp)$ Sivers function	$g_{1T}^\perp(x,k_\perp)$ Worm-gear/trans-helicity	$h_{1T}(x,k_\perp)$ transversity distribution $h_{1T}^\perp(x,k_\perp)$ pretzelosity

Twist-3 TMD PDFs

nucleon polarization	U	L	T
U	$e(x,k_\perp), f^\perp(x,k_\perp)$ number density	$g^\perp(x,k_\perp)$	$h(x,k_\perp)$ Boer-Mulders function
L	$f_L^\perp(x,k_\perp)$	$e_L(x,k_\perp), g_L^\perp(x,k_\perp)$ helicity distribution	$h_L(x,k_\perp)$ Worm gear/ longi-transversity
T	$e_T^\perp(x,k_\perp)$, $f_T^{\perp 1}(x,k_\perp), f_T^{\perp 2}(x,k_\perp)$ Sivers function	$e_T(x,k_\perp)$, $g_T(x,k_\perp), g_T^\perp(x,k_\perp)$ Worm gear/ trans-helicity	$h_T^\perp(x,k_\perp)$ transversity distribution $h_T(x,k_\perp)$ pretzelosity



TMD PDFs defined intuitively (equivalent to twist-2)

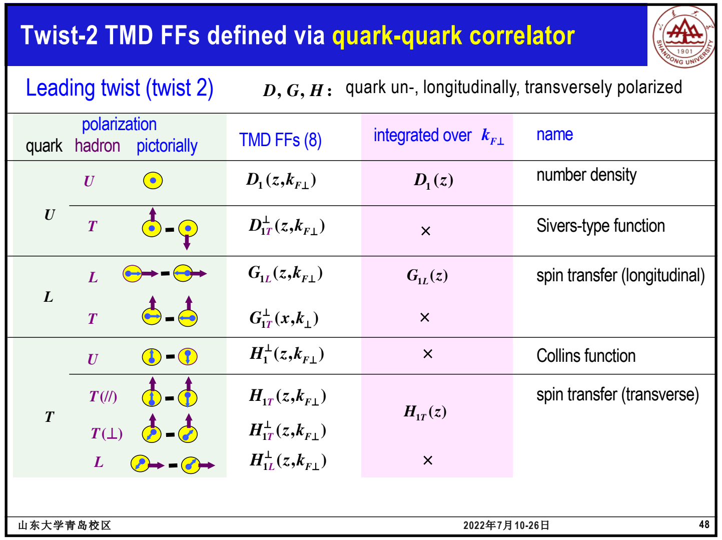
In the 1-dimensional case:

$$f_q(x,S_q;p,S) = f_q(x) + \lambda_q\lambda\ \Delta f_q(x) + (\vec{S}_{\perp q}\cdot\vec{S}_T)\ \delta f_q(x)$$

In the 3-dimensional case:

$$\begin{aligned} f_q(x,k_\perp,S_q;p,S) = {} & f_q(x,k_\perp) + \lambda_q\lambda\ \Delta f_q(x,k_\perp) + (\vec{S}_{\perp q}\cdot\vec{S}_T)\ \delta f_q(x,k_\perp) \\ & + \vec{S}_T\cdot(\hat{p}\times\hat{k}_\perp)\Delta^N f(x,k_\perp) + \frac{1}{M}\vec{S}_{\perp q}\cdot(\hat{p}\times\vec{k}_\perp)h_1^\perp(x,k_\perp) \\ & + \frac{1}{M^2}(\vec{S}_{\perp q}\cdot\vec{k}_\perp)(\vec{S}_T\cdot\vec{k}_\perp)h_{1T}^\perp(x,k_\perp) + \frac{1}{M}(\vec{S}_{\perp q}\cdot\vec{k}_\perp)\ \lambda\ h_{1L}^\perp(x,k_\perp) \\ & + \lambda_q\frac{1}{M}(\vec{S}_T\cdot\vec{k}_\perp)\ g_{1T}^\perp(x,k_\perp) \end{aligned}$$

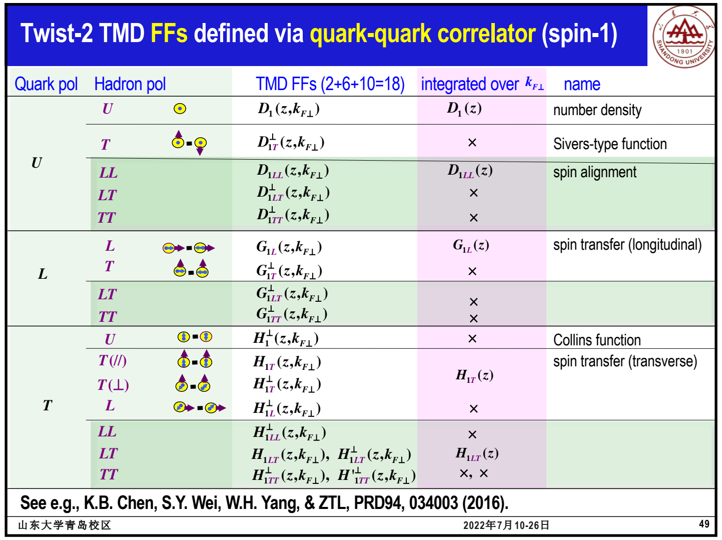
$$\delta f_q(x,k_\perp) = h_{1T}(x,k_\perp), \qquad \Delta^N f(x,k_\perp) = -\frac{|\vec{k}_\perp|}{M} f_{1T}^\perp(x,k_\perp)$$



Twist-2 TMD FFs defined via quark-quark correlator

Leading twist (twist 2) $D, G, H:$ quark un-, longitudinally, transversely polarized

polarization quark	hadron	pictorially	TMD FFs (8)	integrated over $k_{F\perp}$	name
U	U		$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T		$D_{1T}^\perp(z,k_{F\perp})$	×	Sivers-type function
L	L		$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T		$G_{1T}^\perp(x,k_\perp)$	×	
T	U		$H_1^\perp(z,k_{F\perp})$	×	Collins function
	$T(//)$		$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$		$H_{1T}^\perp(z,k_{F\perp})$		
	L		$H_{1L}^\perp(z,k_{F\perp})$	×	



Twist-2 TMD FFs defined via quark-quark correlator (spin-1)

Quark pol	Hadron pol	TMD FFs (2+6+10=18)	integrated over $k_{F\perp}$	name
U	U	$D_1(z,k_{F\perp})$	$D_1(z)$	number density
	T	$D_{1T}^{\perp}(z,k_{F\perp})$	×	Sivers-type function
	LL	$D_{1LL}(z,k_{F\perp})$	$D_{1LL}(z)$	spin alignment
	LT	$D_{1LT}^{\perp}(z,k_{F\perp})$	×	
	TT	$D_{1TT}^{\perp}(z,k_{F\perp})$	×	
L	L	$G_{1L}(z,k_{F\perp})$	$G_{1L}(z)$	spin transfer (longitudinal)
	T	$G_{1T}^{\perp}(z,k_{F\perp})$	×	
	LT	$G_{1LT}^{\perp}(z,k_{F\perp})$	×	
	TT	$G_{1TT}^{\perp}(z,k_{F\perp})$	×	
T	U	$H_1^{\perp}(z,k_{F\perp})$	×	Collins function
	$T(//)$	$H_{1T}(z,k_{F\perp})$	$H_{1T}(z)$	spin transfer (transverse)
	$T(\perp)$	$H_{1T}^{\perp}(z,k_{F\perp})$		
	L	$H_{1L}^{\perp}(z,k_{F\perp})$	×	
	LL	$H_{1LL}^{\perp}(z,k_{F\perp})$	×	
	LT	$H_{1LT}(z,k_{F\perp})$, $H_{1LT}^{\perp}(z,k_{F\perp})$	$H_{1LT}(z)$	
	TT	$H_{1TT}^{\perp}(z,k_{F\perp})$, $H'^{\perp}_{1TT}(z,k_{F\perp})$	×, ×	

See e.g., K.B. Chen, S.Y. Wei, W.H. Yang, & ZTL, PRD94, 034003 (2016).

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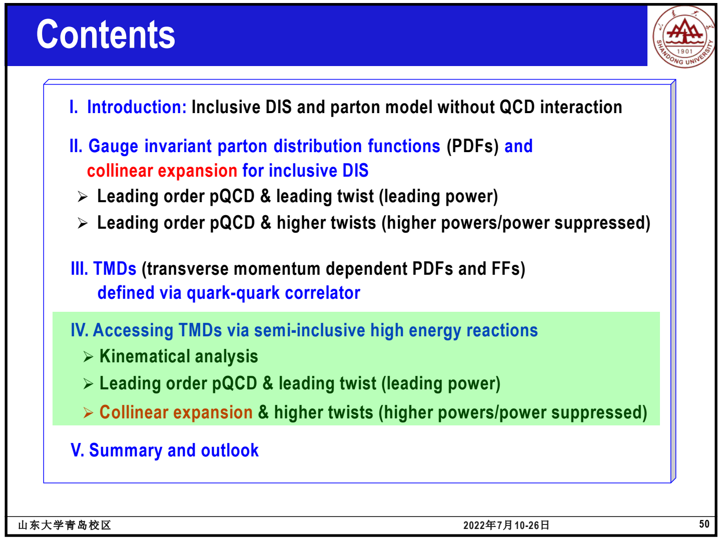

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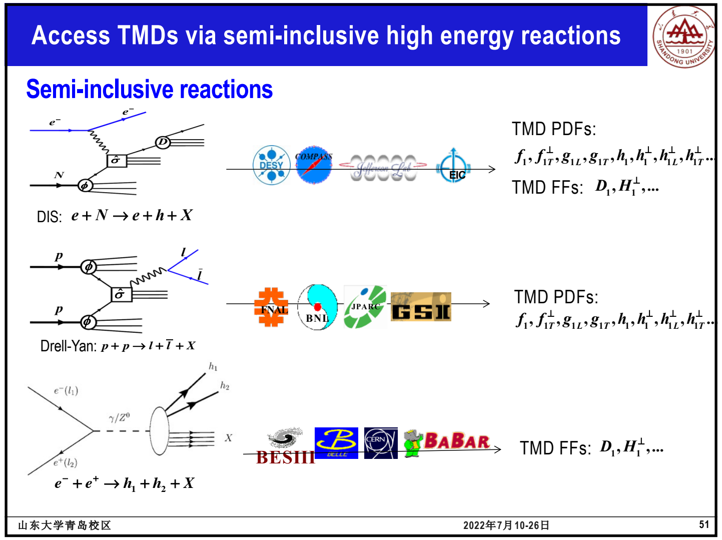

Access TMDs via semi-inclusive high energy reactions

Semi-inclusive reactions

DIS: $e+N\to e+h+X$

TMD PDFs: $f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}, h_1, h_1^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp}$…

TMD FFs: $D_1, H_1^{\perp}$,…

Drell-Yan: $p+p\to l+\bar{l}+X$

TMD PDFs: $f_1, f_{1T}^{\perp}, g_{1L}, g_{1T}, h_1, h_1^{\perp}, h_{1L}^{\perp}, h_{1T}^{\perp}$…

$e^- + e^+ \to h_1 + h_2 + X$

TMD FFs: $D_1, H_1^{\perp}$,…

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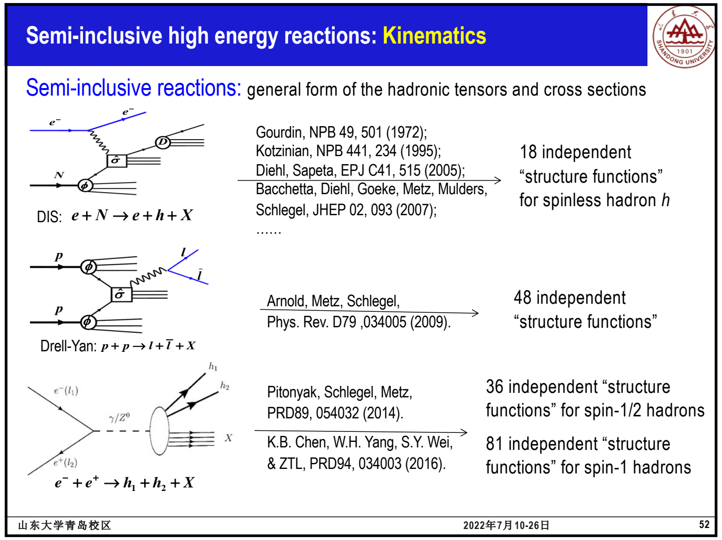

Semi-inclusive high energy reactions: Kinematics

Semi-inclusive reactions: general form of the hadronic tensors and cross sections

DIS: $e+N\to e+h+X$

Gourdin, NPB 49, 501 (1972);
Kotzinian, NPB 441, 234 (1995);
Diehl, Sapeta, EPJ C41, 515 (2005);
Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 02, 093 (2007);
……

18 independent "structure functions" for spinless hadron *h*

Drell-Yan: $p+p\to l+\bar{l}+X$

Arnold, Metz, Schlegel,
Phys. Rev. D79 ,034005 (2009).

48 independent "structure functions"

$e^- + e^+ \to h_1 + h_2 + X$

Pitonyak, Schlegel, Metz,
PRD89, 054032 (2014).

36 independent "structure functions" for spin-1/2 hadrons

K.B. Chen, W.H. Yang, S.Y. Wei,
& ZTL, PRD94, 034003 (2016).

81 independent "structure functions" for spin-1 hadrons

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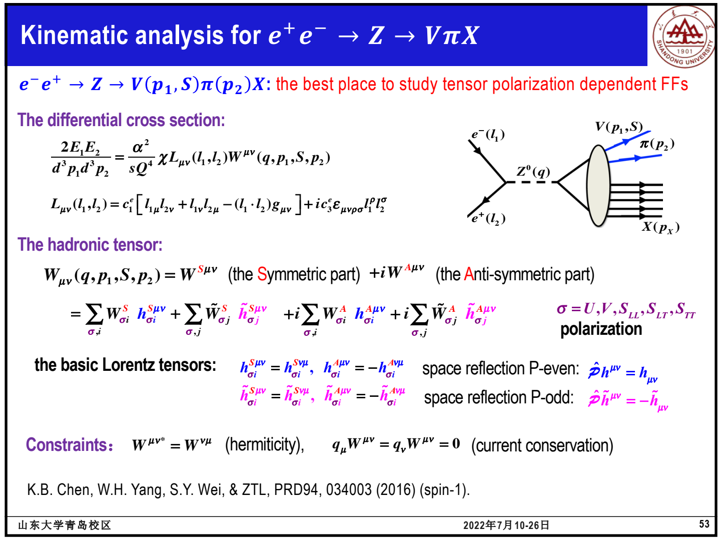


Kinematic analysis for $e^+e^- \to Z \to V\pi X$

$e^-e^+ \to Z \to V(p_1,S)\pi(p_2)X$: the best place to study tensor polarization dependent FFs

The differential cross section:

$$\frac{2E_1E_2}{d^3p_1d^3p_2} = \frac{\alpha^2}{sQ^4}\chi L_{\mu\nu}(l_1,l_2)W^{\mu\nu}(q,p_1,S,p_2)$$

$$L_{\mu\nu}(l_1,l_2) = c_1^e\left[l_{1\mu}l_{2\nu} + l_{1\nu}l_{2\mu} - (l_1\cdot l_2)g_{\mu\nu}\right] + ic_3^e\varepsilon_{\mu\nu\rho\sigma}l_1^\rho l_2^\sigma$$

The hadronic tensor:

$W_{\mu\nu}(q,p_1,S,p_2) = W^{S\mu\nu}$ (the Symmetric part) $+iW^{A\mu\nu}$ (the Anti-symmetric part)

$$= \sum_{\sigma,i} W_{\sigma i}^S\, h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,j} \tilde{W}_{\sigma j}^S\, \tilde{h}_{\sigma j}^{S\mu\nu} + i\sum_{\sigma,i} W_{\sigma i}^A\, h_{\sigma i}^{A\mu\nu} + i\sum_{\sigma,j} \tilde{W}_{\sigma j}^A\, \tilde{h}_{\sigma j}^{A\mu\nu}$$

$\sigma = U,V,S_{LL},S_{LT},S_{TT}$ polarization

the basic Lorentz tensors: $h_{\sigma i}^{S\mu\nu} = h_{\sigma i}^{S\nu\mu}$, $h_{\sigma i}^{A\mu\nu} = -h_{\sigma i}^{A\nu\mu}$ space reflection P-even: $\hat{\mathcal{P}}h^{\mu\nu} = h_{\mu\nu}$

$\tilde{h}_{\sigma i}^{S\mu\nu} = \tilde{h}_{\sigma i}^{S\nu\mu}$, $\tilde{h}_{\sigma i}^{A\mu\nu} = -\tilde{h}_{\sigma i}^{A\nu\mu}$ space reflection P-odd: $\hat{\mathcal{P}}\tilde{h}^{\mu\nu} = -\tilde{h}_{\mu\nu}$

Constraints: $W^{\mu\nu*} = W^{\nu\mu}$ (hermiticity), $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$ (current conservation)

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016) (spin-1).

Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The basic Lorentz tensor sets for the hadronic tensor

unpolarized: 5+4=9

$a^{[\alpha}b^{\beta]} \equiv a^\alpha b^\beta - a^\beta b^\alpha$

$a^{\{\alpha}b^{\beta\}} \equiv a^\alpha b^\beta + a^\beta b^\alpha$

$\varepsilon^{\mu\nu\alpha p} \equiv \varepsilon^{\mu\nu\alpha\beta}p_\beta$, $\varepsilon_\perp^{\mu\nu} \equiv \varepsilon^{\mu\nu\alpha\beta}\bar{n}_\alpha n_\beta$

$p_q \equiv p - \frac{p\cdot q}{q^2}q \quad (p_q\cdot q = 0)$

$h_{Ui}^{S\mu\nu} = \left\{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\ p_{1q}^\mu p_{1q}^\nu,\ p_{2q}^\mu p_{2q}^\nu,\ p_{1q}^{\{\mu}p_{2q}^{\nu\}}\right\}$ symmetric (S), P-even

$\tilde{h}_{Ui}^{S\mu\nu} = \left\{\varepsilon^{\{\mu qp_1p_2}(p_{1q}^{\nu\}},\ p_{2q}^{\nu\}})\right\}$ symmetric (S), P-odd

$h_U^{A\mu\nu} = p_{1q}^{[\mu}p_{2q}^{\nu]}$ anti-symmetric (A), P-even

$\tilde{h}_{Ui}^{A\mu\nu} = \left\{\varepsilon^{\mu\nu qp_1},\ \varepsilon^{\mu\nu qp_2}\right\}$ anti-symmetric (A), P-odd

A regularity: (spin dependent Lorentz tensor set) = (spin dependent Lorentz (pseudo)scalar) × (the unpolarized set)

Unpolarized
5+4=9

Vector polarization S-dependent: 13+14=27

longitudinal polarization $\lambda \sim \vec{p}_1\cdot\vec{S}$

transverse polarization

$$\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix} \qquad \begin{pmatrix} h_{Li}^{S\mu\nu} \\ \tilde{h}_{Li}^{S\mu\nu} \\ h_{Li}^{A\mu\nu} \\ \tilde{h}_{Li}^{A\mu\nu} \end{pmatrix} = \lambda \begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix} \qquad \begin{pmatrix} h_{Ti}^{S\mu\nu} \\ \tilde{h}_{Ti}^{S\mu\nu} \\ h_{Ti}^{A\mu\nu} \\ \tilde{h}_{Ti}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2\cdot S)\begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix},\quad \varepsilon^{Sqp_1p_2}\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix} \right\}$$

Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The basic Lorentz tensor sets for the hadronic tensor (continued)

$S_{TT}^{p\beta} \equiv p_\alpha S_{TT}^{\alpha\beta}$

$\varepsilon^{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta}a_\alpha b_\beta c_\gamma d_\delta$

S_{LL}-dependent part: 5+4=9

$S_{LL}^{\mathcal{P}} = S_{LL}$

$$\begin{pmatrix} h_{LLi}^{S\mu\nu} \\ \tilde{h}_{LLi}^{S\mu\nu} \\ h_{LLi}^{A\mu\nu} \\ \tilde{h}_{LLi}^{A\mu\nu} \end{pmatrix} = S_{LL}\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix}$$

S_{LT}-dependent part: 9+9=18

$S_{LT} = (0,S_{LT}^x,S_{LT}^y,0)$

$p_1\cdot S_{LT} = 0, \quad q\cdot S_{LT} = 0 \qquad S_{LT\mu}^{\mathcal{P}} = S_{LT}^\mu$

$$\begin{pmatrix} h_{LTi}^{S\mu\nu} \\ \tilde{h}_{LTi}^{S\mu\nu} \\ h_{LTi}^{A\mu\nu} \\ \tilde{h}_{LTi}^{A\mu\nu} \end{pmatrix} = \left\{ (p_2\cdot S_{LT})\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix},\quad \varepsilon^{S_{LT}qp_1p_2}\begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix} \right\}$$

S_{TT}-dependent part: 9+9=18

$$S_{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{TT}^{xx} & S_{TT}^{xy} & 0 \\ 0 & S_{TT}^{xy} & -S_{TT}^{xx} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

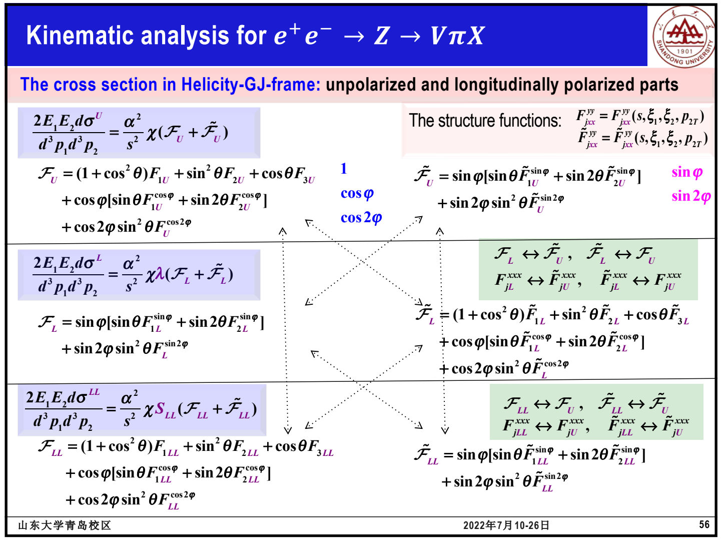
$S_{TT\mu\nu}^{\mathcal{P}} = S_{TT}^{\mu\nu}$

$S_{TT}^{p_1\beta} = S_{TT}^{\alpha p_1} = 0$

$S_{TT}^{q\beta} = S_{TT}^{\alpha q} = 0$

$$\begin{pmatrix} h_{TTi}^{S\mu\nu} \\ \tilde{h}_{TTi}^{S\mu\nu} \\ h_{TTi}^{A\mu\nu} \\ \tilde{h}_{TTi}^{A\mu\nu} \end{pmatrix} = \left\{ S_{TT}^{p_2p_2}\begin{pmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_U^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{pmatrix},\quad \varepsilon^{S_{TT}^{p_2}qp_1p_2}\begin{pmatrix} \tilde{h}_{Ui}^{S\mu\nu} \\ h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \\ h_U^{A\mu\nu} \end{pmatrix} \right\}$$

K.B. Chen, W.H. Yang, S.Y. Wei, & ZTL, PRD94, 034003 (2016).



Kinematic analysis for $e^+e^- \to Z \to V\pi X$

The cross section in Helicity-GJ-frame: unpolarized and longitudinally polarized parts

The structure functions: $F_{jxx}^{yy} = F_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$

$\tilde{F}_{jxx}^{yy} = \tilde{F}_{jxx}^{yy}(s,\xi_1,\xi_2,p_{2T})$

$$\frac{2E_1E_2d\sigma^U}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi(\mathcal{F}_U + \tilde{\mathcal{F}}_U)$$

$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta F_{2U} + \cos\theta F_{3U} + \cos\varphi[\sin\theta F_{1U}^{\cos\varphi} + \sin 2\theta F_{2U}^{\cos\varphi}] + \cos 2\varphi\sin^2\theta F_U^{\cos 2\varphi}$$

1

$\cos\varphi$

$\cos 2\varphi$

$$\tilde{\mathcal{F}}_U = \sin\varphi[\sin\theta\tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta\tilde{F}_{2U}^{\sin\varphi}] + \sin 2\varphi\sin^2\theta\tilde{F}_U^{\sin 2\varphi}$$

$\sin\varphi$

$\sin 2\varphi$

$$\frac{2E_1E_2d\sigma^L}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi\lambda(\mathcal{F}_L + \tilde{\mathcal{F}}_L)$$

$\mathcal{F}_L \leftrightarrow \tilde{\mathcal{F}}_U$, $\tilde{\mathcal{F}}_L \leftrightarrow \mathcal{F}_U$

$F_{jL}^{xxx} \leftrightarrow \tilde{F}_{jU}^{xxx}$, $\tilde{F}_{jL}^{xxx} \leftrightarrow F_{jU}^{xxx}$

$$\mathcal{F}_L = \sin\varphi[\sin\theta F_{1L}^{\sin\varphi} + \sin 2\theta F_{2L}^{\sin\varphi}] + \sin 2\varphi\sin^2\theta F_L^{\sin 2\varphi}$$

$$\tilde{\mathcal{F}}_L = (1+\cos^2\theta)\tilde{F}_{1L} + \sin^2\theta\tilde{F}_{2L} + \cos\theta\tilde{F}_{3L} + \cos\varphi[\sin\theta\tilde{F}_{1L}^{\cos\varphi} + \sin 2\theta\tilde{F}_{2L}^{\cos\varphi}] + \cos 2\varphi\sin^2\theta\tilde{F}_L^{\cos 2\varphi}$$

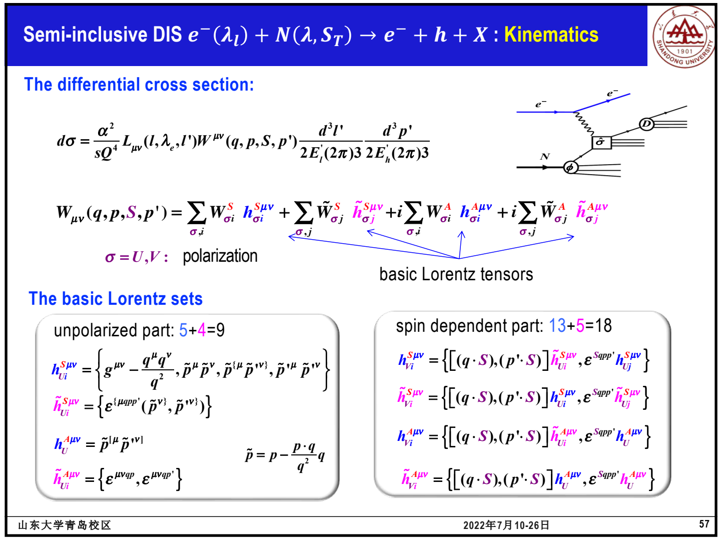
$$\frac{2E_1E_2d\sigma^{LL}}{d^3p_1d^3p_2} = \frac{\alpha^2}{s^2}\chi S_{LL}(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL})$$

$\mathcal{F}_{LL} \leftrightarrow \mathcal{F}_U$, $\tilde{\mathcal{F}}_{LL} \leftrightarrow \tilde{\mathcal{F}}_U$

$F_{jLL}^{xxx} \leftrightarrow F_{jU}^{xxx}$, $\tilde{F}_{jLL}^{xxx} \leftrightarrow \tilde{F}_{jU}^{xxx}$

$$\mathcal{F}_{LL} = (1+\cos^2\theta)F_{1LL} + \sin^2\theta F_{2LL} + \cos\theta F_{3LL} + \cos\varphi[\sin\theta F_{1LL}^{\cos\varphi} + \sin 2\theta F_{2LL}^{\cos\varphi}] + \cos 2\varphi\sin^2\theta F_{LL}^{\cos 2\varphi}$$

$$\tilde{\mathcal{F}}_{LL} = \sin\varphi[\sin\theta\tilde{F}_{1LL}^{\sin\varphi} + \sin 2\theta\tilde{F}_{2LL}^{\sin\varphi}] + \sin 2\varphi\sin^2\theta\tilde{F}_{LL}^{\sin 2\varphi}$$



Semi-inclusive DIS $e^-(\lambda_l) + N(\lambda, S_T) \to e^- + h + X$: Kinematics

The differential cross section:

$$d\sigma = \frac{\alpha^2}{sQ^4} L_{\mu\nu}(l,\lambda_e,l') W^{\mu\nu}(q,p,S,p') \frac{d^3 l'}{2E_l'(2\pi)3} \frac{d^3 p'}{2E_h'(2\pi)3}$$

$$W_{\mu\nu}(q,p,S,p') = \sum_{\sigma,i} W_{\sigma i}^{S}\, h_{\sigma i}^{S\mu\nu} + \sum_{\sigma,j} \tilde{W}_{\sigma j}^{S}\, \tilde{h}_{\sigma j}^{S\mu\nu} + i\sum_{\sigma,i} W_{\sigma i}^{A}\, h_{\sigma i}^{A\mu\nu} + i\sum_{\sigma,j} \tilde{W}_{\sigma j}^{A}\, \tilde{h}_{\sigma j}^{A\mu\nu}$$

$\sigma = U,V$: polarization

basic Lorentz tensors

The basic Lorentz sets

unpolarized part: 5+4=9

$$h_{Ui}^{S\mu\nu} = \left\{ g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \tilde{p}^\mu \tilde{p}^\nu, \tilde{p}^{\{\mu} \tilde{p}'^{\nu\}}, \tilde{p}'^\mu \tilde{p}'^\nu \right\}$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{ \varepsilon^{\{\mu qpp'}(\tilde{p}^{\nu\}}, \tilde{p}'^{\nu\}}) \right\}$$

$$h_{U}^{A\mu\nu} = \tilde{p}^{[\mu} \tilde{p}'^{\nu]}$$

$$\tilde{p} = p - \frac{p \cdot q}{q^2} q$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \left\{ \varepsilon^{\mu\nu qp}, \varepsilon^{\mu\nu qp'} \right\}$$

spin dependent part: 13+5=18

$$h_{Vi}^{S\mu\nu} = \left\{ \left[(q\cdot S),(p'\cdot S)\right] \tilde{h}_{Ui}^{S\mu\nu}, \varepsilon^{Sqpp'} h_{Uj}^{S\mu\nu} \right\}$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \left\{ \left[(q\cdot S),(p'\cdot S)\right] h_{Ui}^{S\mu\nu}, \varepsilon^{Sqpp'} \tilde{h}_{Uj}^{S\mu\nu} \right\}$$

$$h_{Vi}^{A\mu\nu} = \left\{ \left[(q\cdot S),(p'\cdot S)\right] \tilde{h}_{Ui}^{A\mu\nu}, \varepsilon^{Sqpp'} h_{U}^{A\mu\nu} \right\}$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \left\{ \left[(q\cdot S),(p'\cdot S)\right] h_{U}^{A\mu\nu}, \varepsilon^{Sqpp'} h_{U}^{A\mu\nu} \right\}$$

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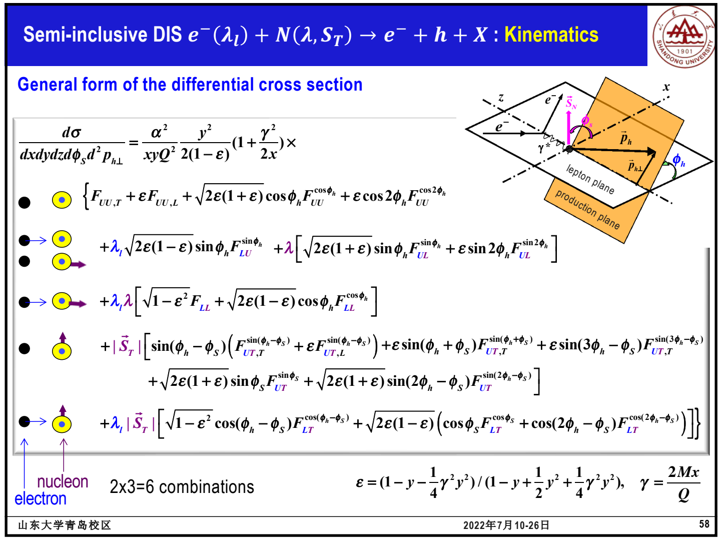

Semi-inclusive DIS $e^-(\lambda_l) + N(\lambda, S_T) \to e^- + h + X$: Kinematics

General form of the differential cross section

$$\frac{d\sigma}{dxdydzd\phi_S d^2 p_{h\perp}} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\varepsilon)} (1+\frac{\gamma^2}{2x}) \times$$

$$\left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right.$$

$$+\lambda_l \sqrt{2\varepsilon(1-\varepsilon)} \sin\phi_h F_{LU}^{\sin\phi_h} \quad +\lambda \left[\sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_h F_{UL}^{\sin\phi_h} + \varepsilon \sin 2\phi_h F_{UL}^{\sin 2\phi_h} \right]$$

$$+\lambda_l \lambda \left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} \cos\phi_h F_{LL}^{\cos\phi_h} \right]$$

$$+|\vec{S}_T| \left[\sin(\phi_h-\phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) + \varepsilon \sin(\phi_h+\phi_S) F_{UT,T}^{\sin(\phi_h+\phi_S)} + \varepsilon \sin(3\phi_h-\phi_S) F_{UT,T}^{\sin(3\phi_h-\phi_S)} \right.$$

$$\left. + \sqrt{2\varepsilon(1+\varepsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)} \sin(2\phi_h-\phi_S) F_{UT}^{\sin(2\phi_h-\phi_S)} \right]$$

$$\left. +\lambda_l |\vec{S}_T| \left[\sqrt{1-\varepsilon^2} \cos(\phi_h-\phi_S) F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)} \left(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi_h-\phi_S) F_{LT}^{\cos(2\phi_h-\phi_S)} \right) \right] \right\}$$

nucleon
electron

2x3=6 combinations

$$\varepsilon = (1 - y - \frac{1}{4}\gamma^2 y^2)/(1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2), \quad \gamma = \frac{2Mx}{Q}$$

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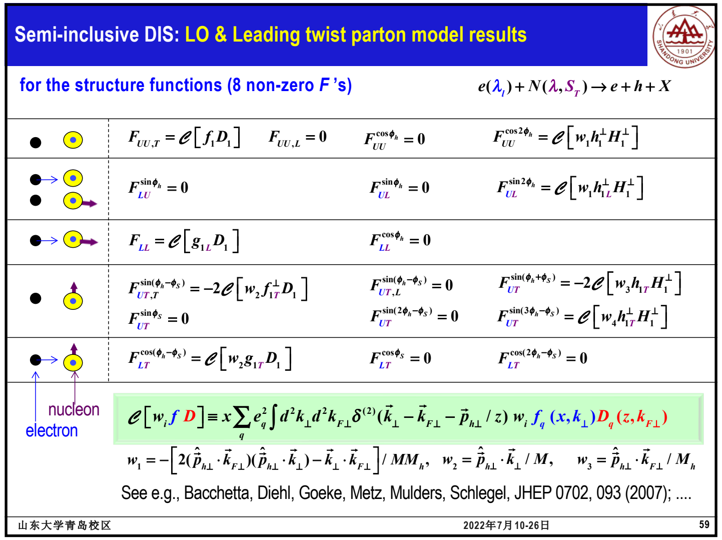

Semi-inclusive DIS: LO & Leading twist parton model results

for the structure functions (8 non-zero *F* 's)

$e(\lambda_l) + N(\lambda, S_T) \to e + h + X$

● ◉	$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right]$ $\quad F_{UU,L} = 0$	$F_{UU}^{\cos\phi_h} = 0$	$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[w_1 h_1^\perp H_1^\perp\right]$
●→ ◉ ● ◉→	$F_{LU}^{\sin\phi_h} = 0$	$F_{UL}^{\sin\phi_h} = 0$	$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[w_1 h_{1L}^\perp H_1^\perp\right]$
●→ ◉→	$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right]$	$F_{LL}^{\cos\phi_h} = 0$	
● ◉↑	$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -2\mathcal{C}\left[w_2 f_{1T}^\perp D_1\right]$ $F_{UT}^{\sin\phi_S} = 0$	$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0$ $F_{UT}^{\sin(2\phi_h-\phi_S)} = 0$	$F_{UT}^{\sin(\phi_h+\phi_S)} = -2\mathcal{C}\left[w_3 h_{1T} H_1^\perp\right]$ $F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[w_4 h_{1T}^\perp H_1^\perp\right]$
●→ ◉↑	$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[w_2 g_{1T} D_1\right]$	$F_{LT}^{\cos\phi_S} = 0$	$F_{LT}^{\cos(2\phi_h-\phi_S)} = 0$

nucleon
electron

$$\mathcal{C}\left[w_i f\, D\right] \equiv x \sum_q e_q^2 \int d^2k_\perp d^2k_{F\perp} \delta^{(2)}(\vec{k}_\perp - \vec{k}_{F\perp} - \vec{p}_{h\perp}/z)\, w_i\, f_q(x,k_\perp) D_q(z,k_{F\perp})$$

$$w_1 = -\left[2(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp})(\hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp) - \vec{k}_\perp\cdot\vec{k}_{F\perp}\right]/MM_h, \quad w_2 = \hat{\vec{p}}_{h\perp}\cdot\vec{k}_\perp/M, \quad w_3 = \hat{\vec{p}}_{h\perp}\cdot\vec{k}_{F\perp}/M_h$$

See e.g., Bacchetta, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP 0702, 093 (2007); ....

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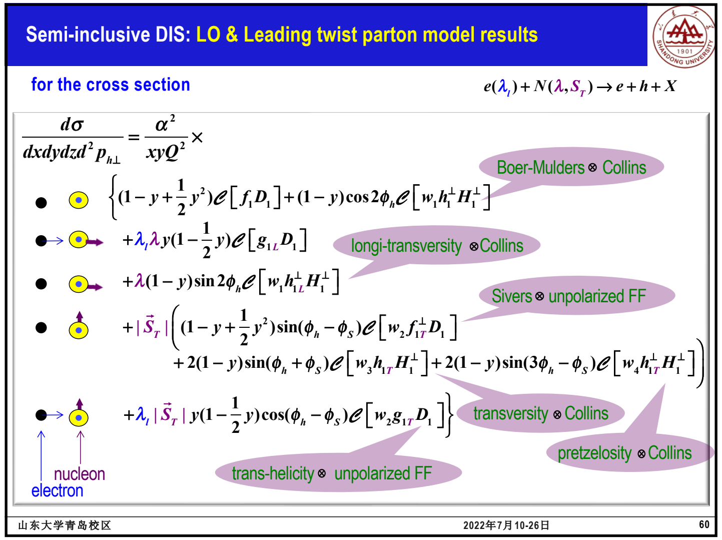

Semi-inclusive DIS: LO & Leading twist parton model results

for the cross section

$e(\lambda_l) + N(\lambda, S_T) \to e + h + X$

$$\frac{d\sigma}{dxdydzd^2 p_{h\perp}} = \frac{\alpha^2}{xyQ^2} \times$$

$$\left\{ (1-y+\frac{1}{2}y^2)\mathcal{C}\left[f_1 D_1\right] + (1-y)\cos 2\phi_h \mathcal{C}\left[w_1 h_1^\perp H_1^\perp\right] \right.$$

$$+\lambda_l \lambda y(1-\frac{1}{2}y)\mathcal{C}\left[g_{1L} D_1\right]$$

$$+\lambda(1-y)\sin 2\phi_h \mathcal{C}\left[w_1 h_{1L}^\perp H_1^\perp\right]$$

$$+|\vec{S}_T| \left((1-y+\frac{1}{2}y^2)\sin(\phi_h-\phi_S)\mathcal{C}\left[w_2 f_{1T}^\perp D_1\right] + 2(1-y)\sin(\phi_h+\phi_S)\mathcal{C}\left[w_3 h_{1T} H_1^\perp\right] + 2(1-y)\sin(3\phi_h-\phi_S)\mathcal{C}\left[w_4 h_{1T}^\perp H_1^\perp\right] \right)$$

$$\left. +\lambda_l |\vec{S}_T| y(1-\frac{1}{2}y)\cos(\phi_h-\phi_S)\mathcal{C}\left[w_2 g_{1T} D_1\right] \right\}$$

Boer-Mulders ⊗ Collins

longi-transversity ⊗Collins

Sivers ⊗ unpolarized FF

transversity ⊗ Collins

pretzelosity ⊗ Collins

trans-helicity ⊗ unpolarized FF

nucleon
electron

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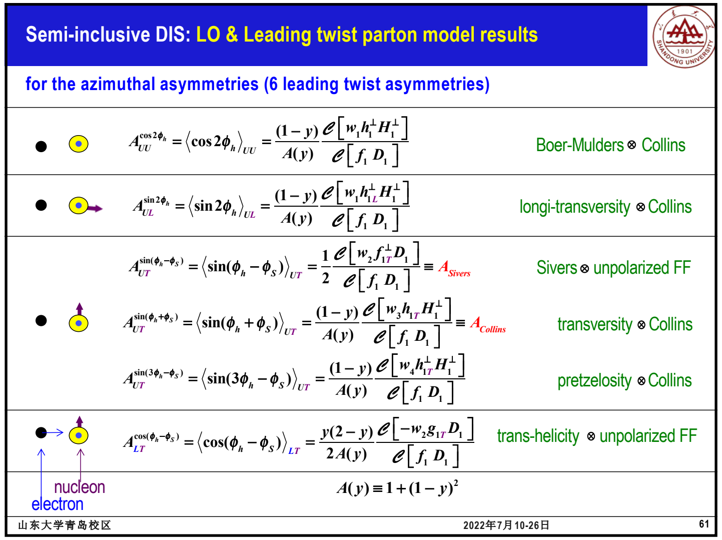

Semi-inclusive DIS: LO & Leading twist parton model results

for the azimuthal asymmetries (6 leading twist asymmetries)

● $\odot$ $$A_{UU}^{\cos 2\phi_h} = \langle \cos 2\phi_h \rangle_{UU} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}[w_1 h_1^\perp H_1^\perp]}{\mathcal{C}[f_1 D_1]}$$ Boer-Mulders ⊗ Collins

● $\odot\rightarrow$ $$A_{UL}^{\sin 2\phi_h} = \langle \sin 2\phi_h \rangle_{UL} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}[w_1 h_{1L}^\perp H_1^\perp]}{\mathcal{C}[f_1 D_1]}$$ longi-transversity ⊗ Collins

$$A_{UT}^{\sin(\phi_h-\phi_S)} = \langle \sin(\phi_h-\phi_S) \rangle_{UT} = \frac{1}{2} \frac{\mathcal{C}[w_2 f_{1T}^\perp D_1]}{\mathcal{C}[f_1 D_1]} \equiv A_{Sivers}$$ Sivers ⊗ unpolarized FF

● $\overset{\uparrow}{\odot}$ $$A_{UT}^{\sin(\phi_h+\phi_S)} = \langle \sin(\phi_h+\phi_S) \rangle_{UT} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}[w_3 h_{1T} H_1^\perp]}{\mathcal{C}[f_1 D_1]} \equiv A_{Collins}$$ transversity ⊗ Collins

$$A_{UT}^{\sin(3\phi_h-\phi_S)} = \langle \sin(3\phi_h-\phi_S) \rangle_{UT} = \frac{(1-y)}{A(y)} \frac{\mathcal{C}[w_4 h_{1T}^\perp H_1^\perp]}{\mathcal{C}[f_1 D_1]}$$ pretzelosity ⊗ Collins

$\bullet\rightarrow$ $\overset{\uparrow}{\odot}$ $$A_{LT}^{\cos(\phi_h-\phi_S)} = \langle \cos(\phi_h-\phi_S) \rangle_{LT} = \frac{y(2-y)}{2A(y)} \frac{\mathcal{C}[-w_2 g_{1T} D_1]}{\mathcal{C}[f_1 D_1]}$$ trans-helicity ⊗ unpolarized FF

nucleon
electron

$$A(y) \equiv 1 + (1-y)^2$$

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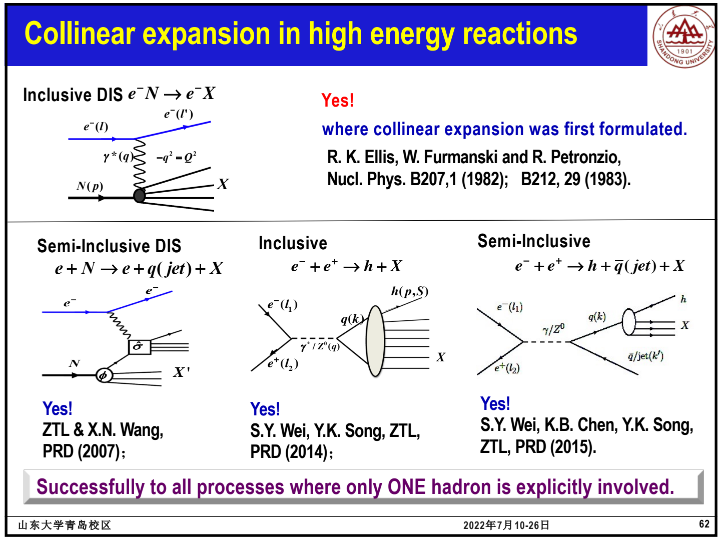

Collinear expansion in high energy reactions

Inclusive DIS $e^- N \to e^- X$

Yes!

where collinear expansion was first formulated.

R. K. Ellis, W. Furmanski and R. Petronzio,
Nucl. Phys. B207,1 (1982); B212, 29 (1983).

Semi-Inclusive DIS
$e + N \to e + q(jet) + X$

Yes!
ZTL & X.N. Wang,
PRD (2007);

Inclusive
$e^- + e^+ \to h + X$

Yes!
S.Y. Wei, Y.K. Song, ZTL,
PRD (2014);

Semi-Inclusive
$e^- + e^+ \to h + \bar{q}(jet) + X$

Yes!
S.Y. Wei, K.B. Chen, Y.K. Song,
ZTL, PRD (2015).

Successfully to all processes where only ONE hadron is explicitly involved.

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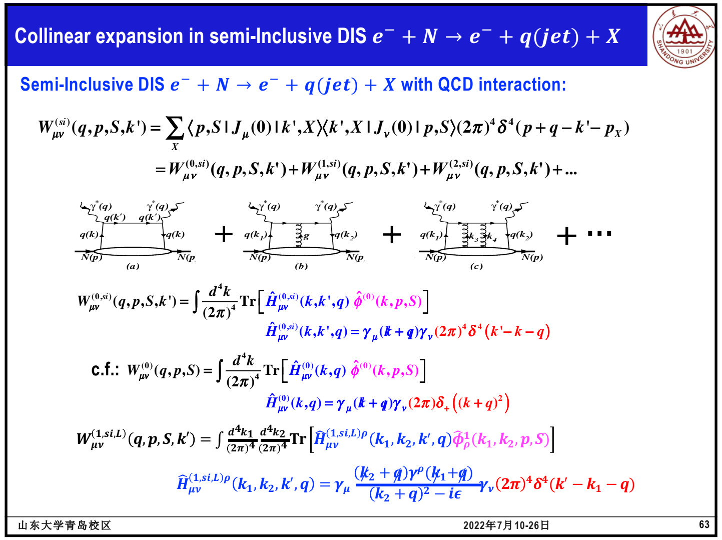

Collinear expansion in semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$ with QCD interaction:

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \sum_X \langle p,S | J_\mu(0) | k',X \rangle \langle k',X | J_\nu(0) | p,S \rangle (2\pi)^4 \delta^4(p+q-k'-p_X)$$

$$= W_{\mu\nu}^{(0,si)}(q,p,S,k') + W_{\mu\nu}^{(1,si)}(q,p,S,k') + W_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0,si)}(k,k',q)\ \hat{\phi}^{(0)}(k,p,S)\right]$$

$$\hat{H}_{\mu\nu}^{(0,si)}(k,k',q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu (2\pi)^4 \delta^4(k'-k-q)$$

c.f.: $$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\ \hat{\phi}^{(0)}(k,p,S)\right]$$

$$\hat{H}_{\mu\nu}^{(0)}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu (2\pi)\delta_+\left((k+q)^2\right)$$

$$W_{\mu\nu}^{(1,si,L)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(1,si,L)\rho}(k_1,k_2,k',q) \hat{\phi}_\rho^1(k_1,k_2,p,S)\right]$$

$$\hat{H}_{\mu\nu}^{(1,si,L)\rho}(k_1,k_2,k',q) = \gamma_\mu \frac{(\not{k}_2+\not{q})\gamma^\rho(\not{k}_1+\not{q})}{(k_2+q)^2 - i\epsilon} \gamma_\nu (2\pi)^4 \delta^4(k'-k_1-q)$$

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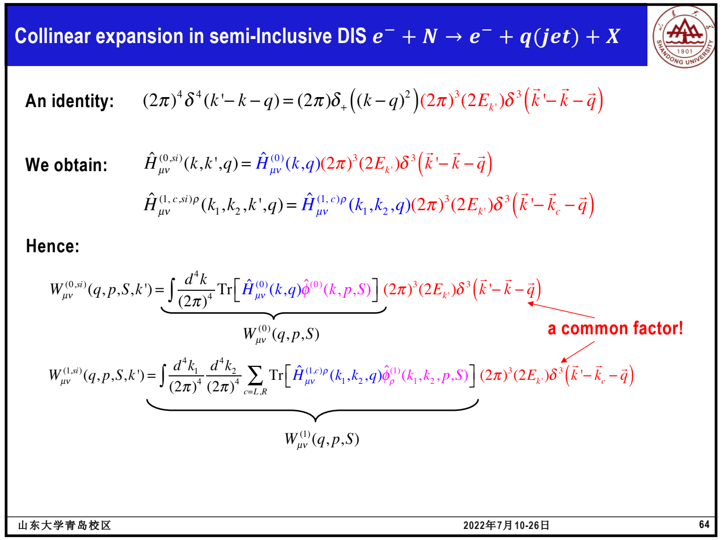

Collinear expansion in semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

An identity: $$(2\pi)^4 \delta^4(k'-k-q) = (2\pi)\delta_+\left((k-q)^2\right)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

We obtain: $$\hat{H}_{\mu\nu}^{(0,si)}(k,k',q) = \hat{H}_{\mu\nu}^{(0)}(k,q)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

$$\hat{H}_{\mu\nu}^{(1,c,si)\rho}(k_1,k_2,k',q) = \hat{H}_{\mu\nu}^{(1,c)\rho}(k_1,k_2,q)(2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}_c-\vec{q}\right)$$

Hence:

$$W_{\mu\nu}^{(0,si)}(q,p,S,k') = \underbrace{\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\hat{\phi}^{(0)}(k,p,S)\right]}_{W_{\mu\nu}^{(0)}(q,p,S)} (2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}-\vec{q}\right)$$

a common factor!

$$W_{\mu\nu}^{(1,si)}(q,p,S,k') = \underbrace{\int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \sum_{c=L,R} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(1,c)\rho}(k_1,k_2,q)\hat{\phi}_\rho^{(1)}(k_1,k_2,p,S)\right]}_{W_{\mu\nu}^{(1)}(q,p,S)} (2\pi)^3(2E_{k'})\delta^3\left(\vec{k}'-\vec{k}_c-\vec{q}\right)$$

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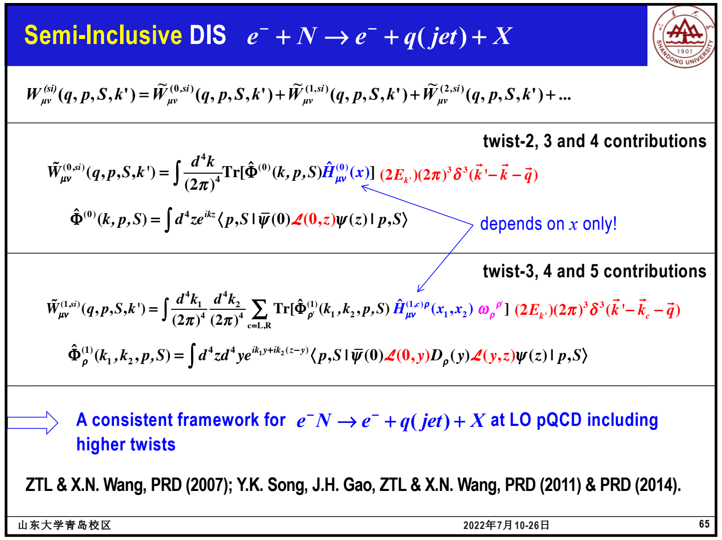

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{\Phi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(x)]\,(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$\hat{\Phi}^{(0)}(k,p,S) = \int d^4z e^{ikz}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|p,S\rangle$$

depends on x only!

twist-3, 4 and 5 contributions

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\sum_{c=L,R}\mathrm{Tr}[\hat{\Phi}_{\rho'}^{(1)}(k_1,k_2,p,S)\,\hat{H}_{\mu\nu}^{(1,c)\rho}(x_1,x_2)\,\omega_\rho^{\ \rho'}]\,(2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$\hat{\Phi}_\rho^{(1)}(k_1,k_2,p,S) = \int d^4z d^4y e^{ik_1y+ik_2(z-y)}\langle p,S|\bar{\psi}(0)\mathcal{L}(0,y)D_\rho(y)\mathcal{L}(y,z)\psi(z)|p,S\rangle$$

⇒ A consistent framework for $e^-N \to e^- + q(jet) + X$ at LO pQCD including higher twists

ZTL & X.N. Wang, PRD (2007); Y.K. Song, J.H. Gao, ZTL & X.N. Wang, PRD (2011) & PRD (2014).

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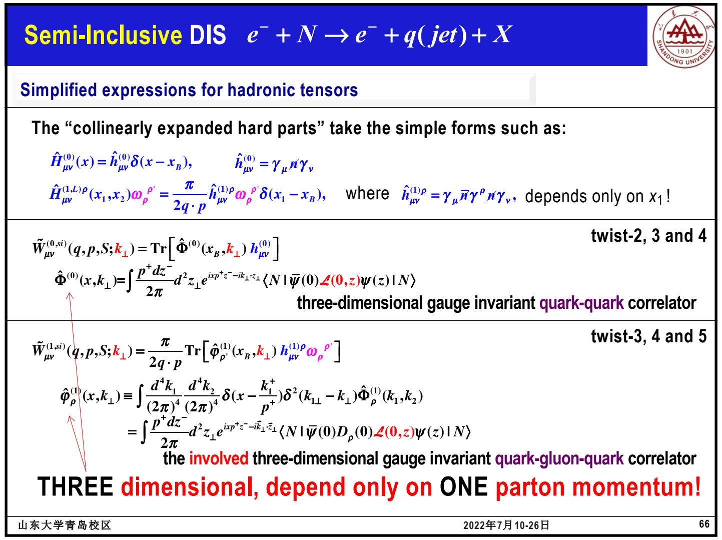

Semi-Inclusive DIS $e^- + N \to e^- + q(jet) + X$

Simplified expressions for hadronic tensors

The "collinearly expanded hard parts" take the simple forms such as:

$$\hat{H}_{\mu\nu}^{(0)}(x) = \hat{h}_{\mu\nu}^{(0)}\delta(x-x_B), \qquad \hat{h}_{\mu\nu}^{(0)} = \gamma_\mu \not{n}\gamma_\nu$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(x_1,x_2)\omega_\rho^{\ \rho'} = \frac{\pi}{2q\cdot p}\hat{h}_{\mu\nu}^{(1)\rho}\omega_\rho^{\ \rho'}\delta(x_1-x_B),$$ where $\hat{h}_{\mu\nu}^{(1)\rho} = \gamma_\mu\bar{\not{n}}\gamma^\rho\not{n}\gamma_\nu$, depends only on x_1!

twist-2, 3 and 4

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S;k_\perp) = \mathrm{Tr}\left[\hat{\Phi}^{(0)}(x_B,k_\perp)\,h_{\mu\nu}^{(0)}\right]$$

$$\hat{\Phi}^{(0)}(x,k_\perp) = \int\frac{p^+dz^-}{2\pi}d^2z_\perp e^{ixp^+z^- - ik_\perp\cdot z_\perp}\langle N|\bar{\psi}(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

three-dimensional gauge invariant quark-quark correlator

twist-3, 4 and 5

$$\tilde{W}_{\mu\nu}^{(1,si)}(q,p,S;k_\perp) = \frac{\pi}{2q\cdot p}\mathrm{Tr}\left[\hat{\varphi}_{\rho'}^{(1)}(x_B,k_\perp)\,h_{\mu\nu}^{(1)\rho}\omega_\rho^{\ \rho'}\right]$$

$$\hat{\varphi}_\rho^{(1)}(x,k_\perp) \equiv \int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(x-\frac{k_1^+}{p^+})\delta^2(k_{1\perp}-k_\perp)\hat{\Phi}_\rho^{(1)}(k_1,k_2)$$
$$= \int\frac{p^+dz^-}{2\pi}d^2z_\perp e^{ixp^+z^- - i\vec{k}_\perp\cdot\vec{z}_\perp}\langle N|\bar{\psi}(0)D_\rho(0)\mathcal{L}(0,z)\psi(z)|N\rangle$$

the involved three-dimensional gauge invariant quark-gluon-quark correlator

THREE dimensional, depend only on ONE parton momentum!

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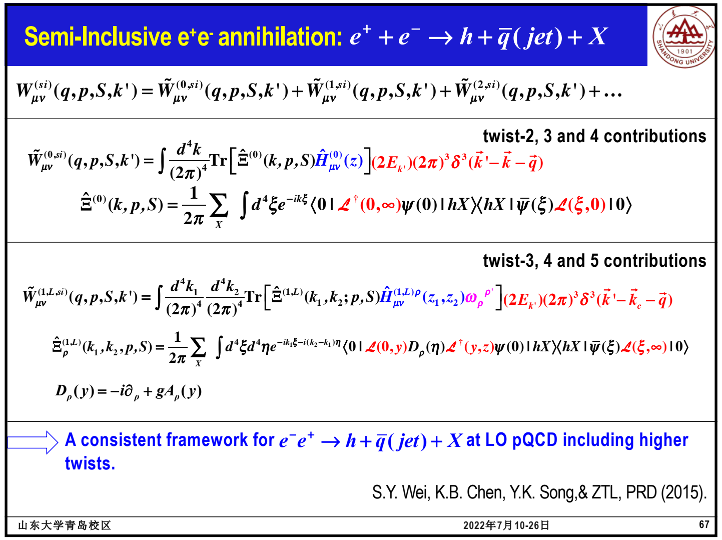

Semi-Inclusive e⁺e⁻ annihilation: $e^+ + e^- \to h + \bar{q}(jet) + X$

$$W_{\mu\nu}^{(si)}(q,p,S,k') = \tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(1,si)}(q,p,S,k') + \tilde{W}_{\mu\nu}^{(2,si)}(q,p,S,k') + \ldots$$

twist-2, 3 and 4 contributions

$$\tilde{W}_{\mu\nu}^{(0,si)}(q,p,S,k') = \int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(0)}(k,p,S)\hat{H}_{\mu\nu}^{(0)}(z)\right](2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$$

$$\hat{\Xi}^{(0)}(k,p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi e^{-ik\xi}\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,0)|0\rangle$$

twist-3, 4 and 5 contributions

$$\tilde{W}_{\mu\nu}^{(1,L,si)}(q,p,S,k') = \int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}\left[\hat{\Xi}^{(1,L)}(k_1,k_2;p,S)\hat{H}_{\mu\nu}^{(1,L)\rho}(z_1,z_2)\omega_\rho^{\ \rho'}\right](2E_{k'})(2\pi)^3\delta^3(\vec{k}'-\vec{k}_c-\vec{q})$$

$$\hat{\Xi}_\rho^{(1,L)}(k_1,k_2,p,S) = \frac{1}{2\pi}\sum_X \int d^4\xi d^4\eta e^{-ik_1\xi - i(k_2-k_1)\eta}\langle 0|\mathcal{L}(0,y)D_\rho(\eta)\mathcal{L}^\dagger(y,z)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$D_\rho(y) = -i\partial_\rho + gA_\rho(y)$$

⇒ A consistent framework for $e^-e^+ \to h + \bar{q}(jet) + X$ at LO pQCD including higher twists.

S.Y. Wei, K.B. Chen, Y.K. Song,& ZTL, PRD (2015).

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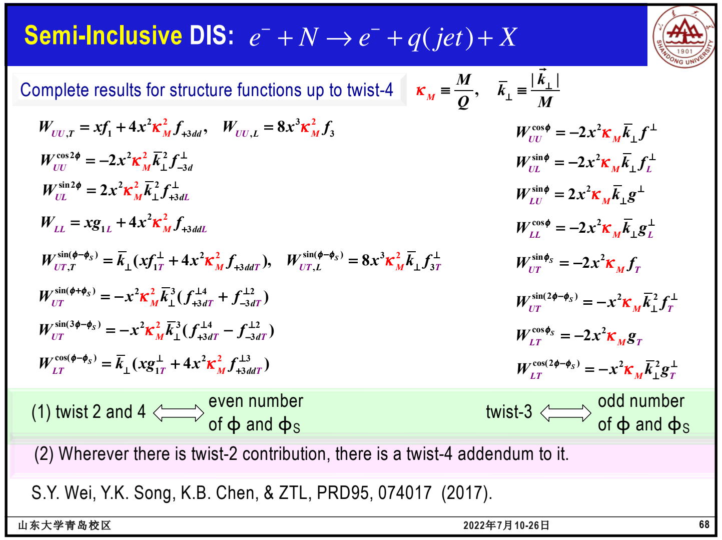

Semi-Inclusive DIS: $e^- + N \to e^- + q(jet) + X$

Complete results for structure functions up to twist-4 $\kappa_M \equiv \frac{M}{Q}$, $\bar{k}_\perp \equiv \frac{|\vec{k}_\perp|}{M}$

$W_{UU,T} = xf_1 + 4x^2\kappa_M^2 f_{+3dd}$, $W_{UU,L} = 8x^3\kappa_M^2 f_3$

$W_{UU}^{\cos 2\phi} = -2x^2\kappa_M^2\bar{k}_\perp^2 f_{-3d}^\perp$

$W_{UL}^{\sin 2\phi} = 2x^2\kappa_M^2\bar{k}_\perp^2 f_{+3dL}^\perp$

$W_{LL} = xg_{1L} + 4x^2\kappa_M^2 f_{+3ddL}$

$W_{UT,T}^{\sin(\phi-\phi_S)} = \bar{k}_\perp(xf_{1T}^\perp + 4x^2\kappa_M^2 f_{+3ddT})$, $W_{UT,L}^{\sin(\phi-\phi_S)} = 8x^3\kappa_M^2\bar{k}_\perp f_{3T}^\perp$

$W_{UT}^{\sin(\phi+\phi_S)} = -x^2\kappa_M^2\bar{k}_\perp^3(f_{+3dT}^{\perp 4} + f_{-3dT}^{\perp 2})$

$W_{UT}^{\sin(3\phi-\phi_S)} = -x^2\kappa_M^2\bar{k}_\perp^3(f_{+3dT}^{\perp 4} - f_{-3dT}^{\perp 2})$

$W_{LT}^{\cos(\phi-\phi_S)} = \bar{k}_\perp(xg_{1T}^\perp + 4x^2\kappa_M^2 f_{+3ddT}^{\perp 3})$

$W_{UU}^{\cos\phi} = -2x^2\kappa_M\bar{k}_\perp f^\perp$

$W_{UL}^{\sin\phi} = -2x^2\kappa_M\bar{k}_\perp f_L^\perp$

$W_{LU}^{\sin\phi} = 2x^2\kappa_M\bar{k}_\perp g^\perp$

$W_{LL}^{\cos\phi} = -2x^2\kappa_M\bar{k}_\perp g_L^\perp$

$W_{UT}^{\sin\phi_S} = -2x^2\kappa_M f_T$

$W_{UT}^{\sin(2\phi-\phi_S)} = -x^2\kappa_M\bar{k}_\perp^2 f_T^\perp$

$W_{LT}^{\cos\phi_S} = -2x^2\kappa_M g_T$

$W_{LT}^{\cos(2\phi-\phi_S)} = -x^2\kappa_M\bar{k}_\perp^2 g_T^\perp$

(1) twist 2 and 4 ⟺ even number of ϕ and ϕS twist-3 ⟺ odd number of ϕ and ϕS

(2) Wherever there is twist-2 contribution, there is a twist-4 addendum to it.

S.Y. Wei, Y.K. Song, K.B. Chen, & ZTL, PRD95, 074017 (2017).

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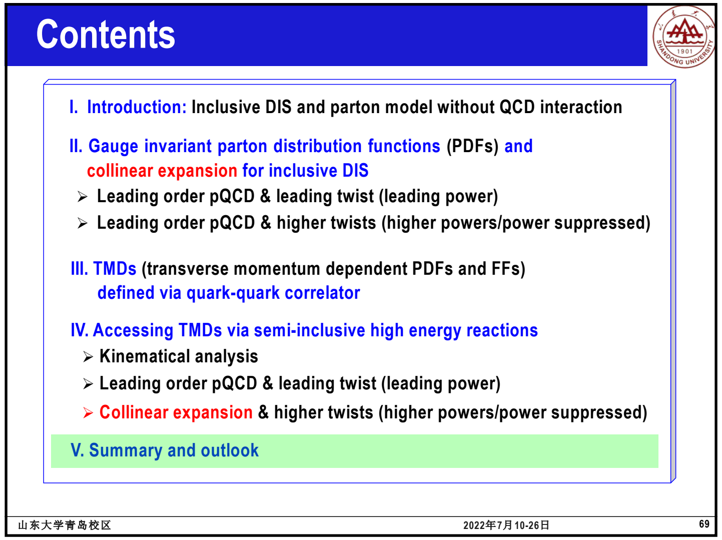

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V. Summary and outlook

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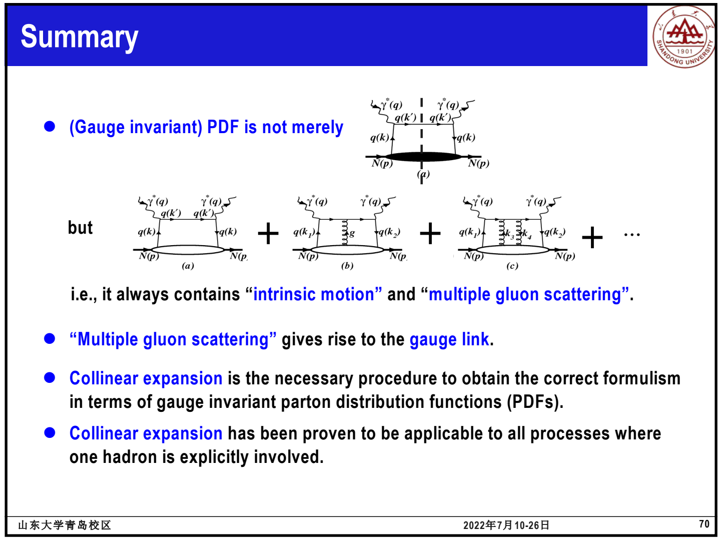

Summary

- (Gauge invariant) PDF is not merely

but

i.e., it always contains "intrinsic motion" and "multiple gluon scattering".

- "Multiple gluon scattering" gives rise to the gauge link.
- Collinear expansion is the necessary procedure to obtain the correct formulism in terms of gauge invariant parton distribution functions (PDFs).
- Collinear expansion has been proven to be applicable to all processes where one hadron is explicitly involved.

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