Introduction to Parton Shower

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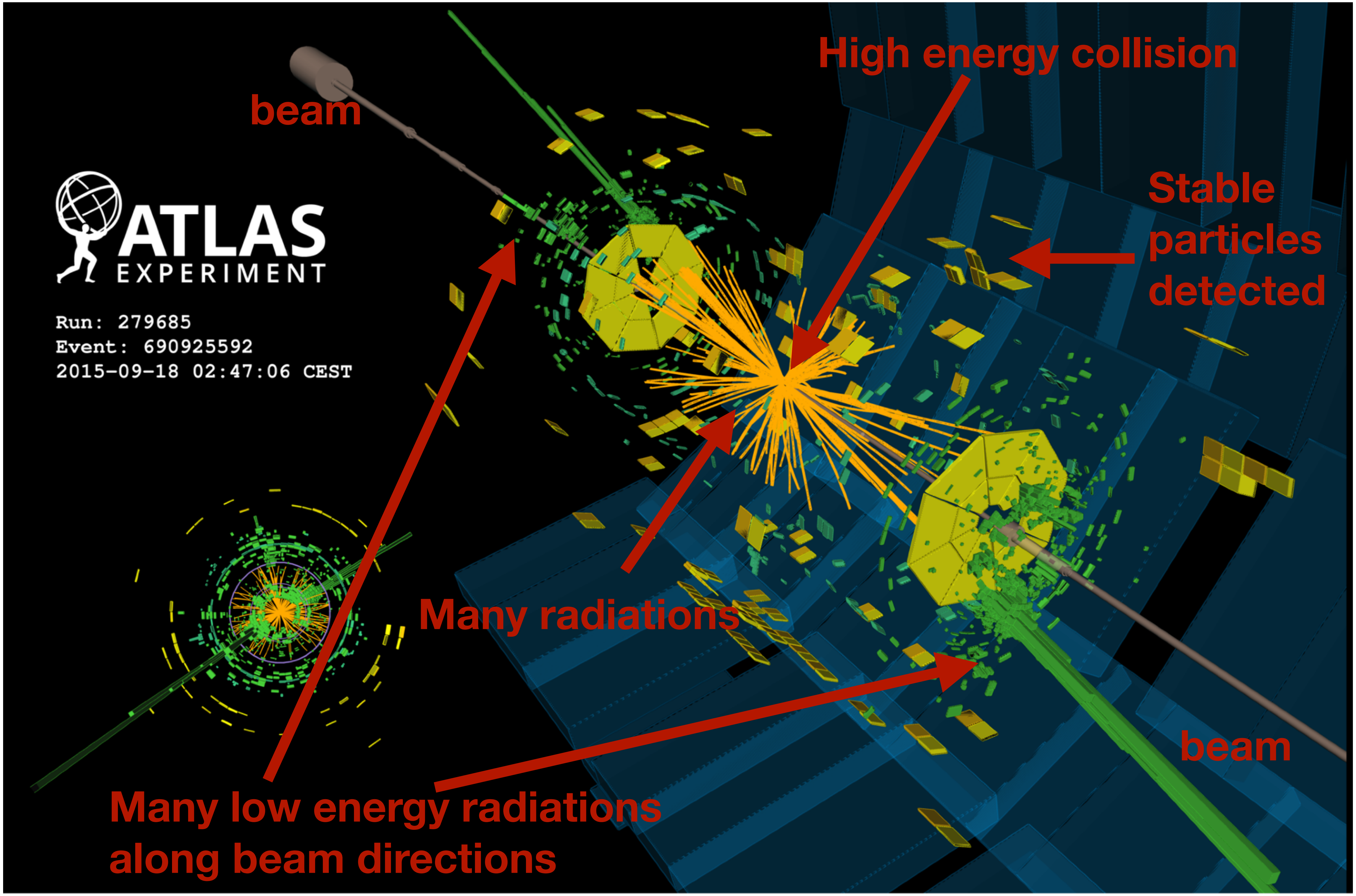
Outline

1. Monte Carlo Event generator
2. Parton Shower
3. Matching and merging
4. Summary

Focus on Theory side of parton showers

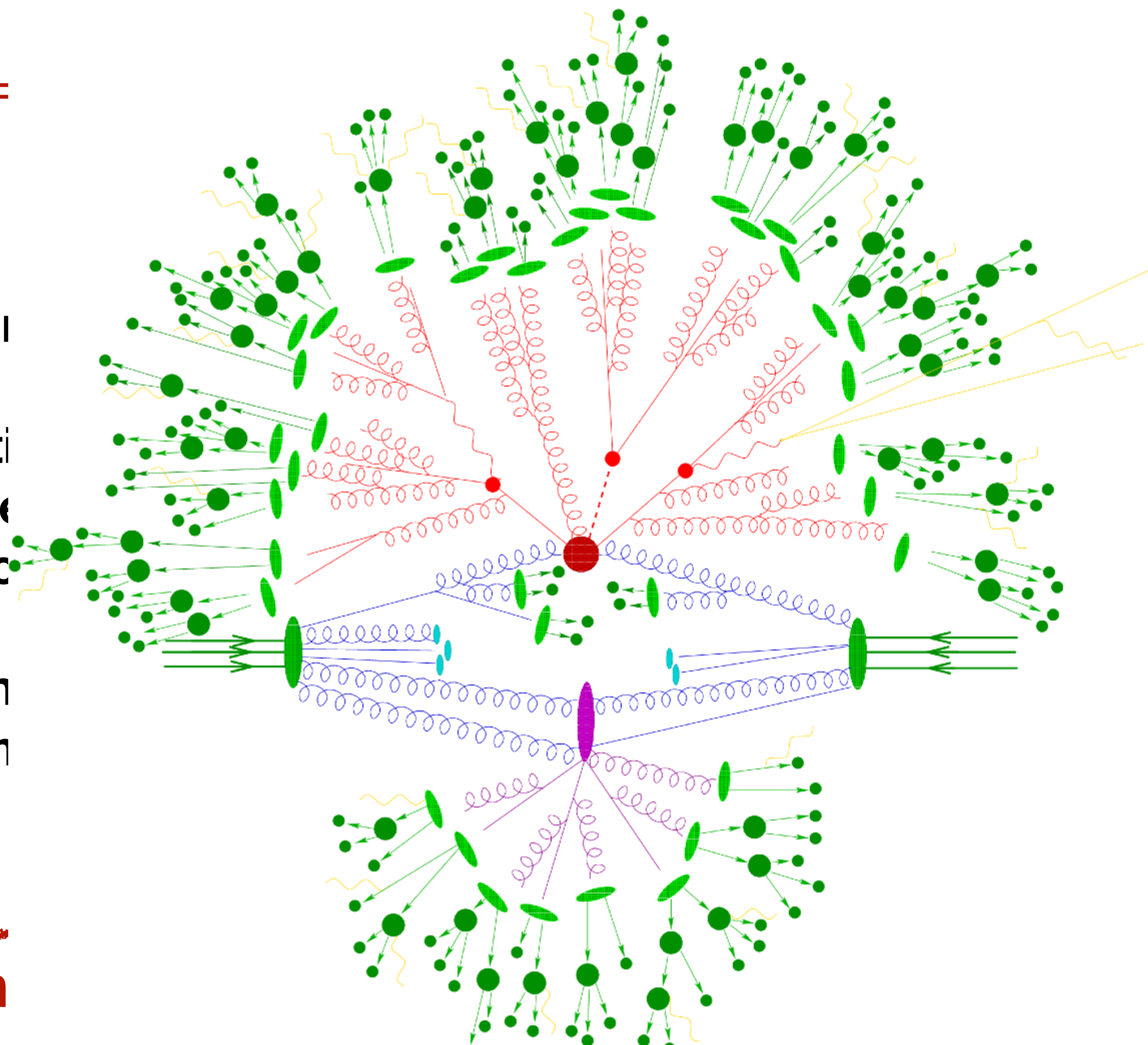
1. Monte Carlo Event generator



1. Monte Carlo Event generator

The purpose of Monte Carlo event generators is to generate events in as much details as nature (generate average and fluctuation right)

$\mathscr{P}_{\text{event}}$ = … lad…



- Hard pr… — hard scale
- Transiti… low ene… —partc… — adronization scale
- Low en… —fragn… — cles

Parton sh… h scale to hadronization scale

1. Monte Carlo Event generator

For multi-scale problem

固定阶计算

$$
\begin{array}{llllll}
 & \textbf{NLO} & \textbf{NNLO} & \textbf{NNNLO} & & \\
z\frac{1}{\sigma_0}\frac{d\sigma}{dz} \propto z + & \frac{\alpha_s}{4\pi} a_{1,2} \ln z + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,4} \ln^3 z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,6} \ln^5 z & + \cdots & \textbf{LL} \\
+ & \frac{\alpha_s}{4\pi} a_{1,1} + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,3} \ln^2 z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,5} \ln^4 z & + \cdots & \textbf{NLL} \\
+ & \frac{\alpha_s}{4\pi} a_{1,0} z + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,2} \ln z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,4} \ln^3 z & + \cdots & \textbf{NNLL} \\
 & + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,1} + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,3} \ln^2 z & + \cdots & \textbf{NNNLL} \\
 & + & \left[\frac{\alpha_s}{4\pi}\right]^2 a_{2,0} z + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} \ln z & + \cdots & \textbf{NNNNLL} \\
 & & + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} & + \cdots & \\
 & & + & \left[\frac{\alpha_s}{4\pi}\right]^3 a_{3,2} z & + \cdots &
\end{array}
$$

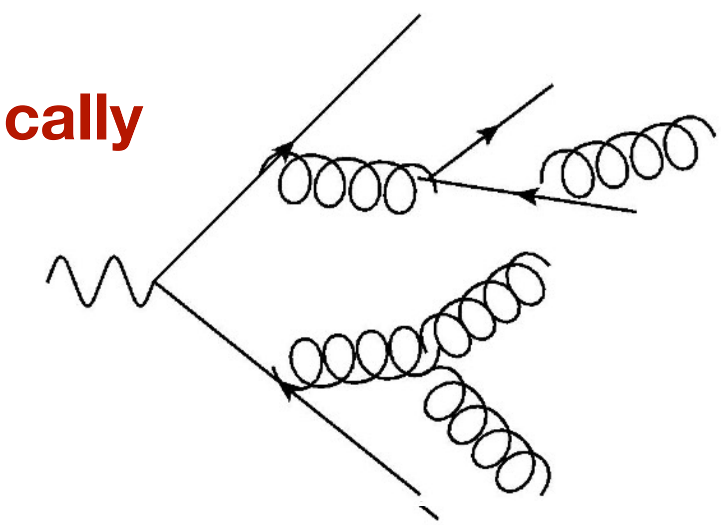
重求和

For observables that involve scale hierarchies resummation is required

1. Monte Carlo Event generator

Parton showers approximate higher-order real-emission corrections to the hard scattering process

- Generate cascades of radiation automatically
- Locally conserved four momentum
- Locally conserved flavor
- Unitarity by construction



Parton showers

- sample infrared configurations
- simulate the evolution of parton (resummation)

Parton shower indispensable tools for particle physics phenomenology

2. Parton Shower

2. Parton Shower

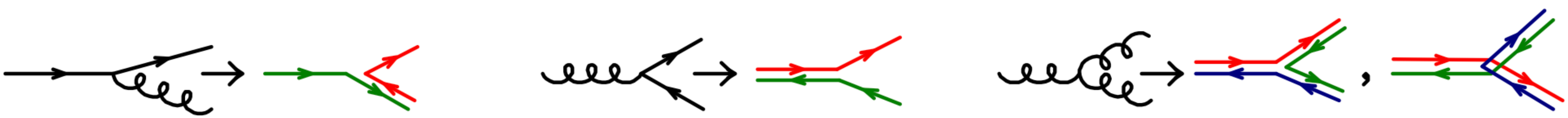
Leading color

Full color coherence

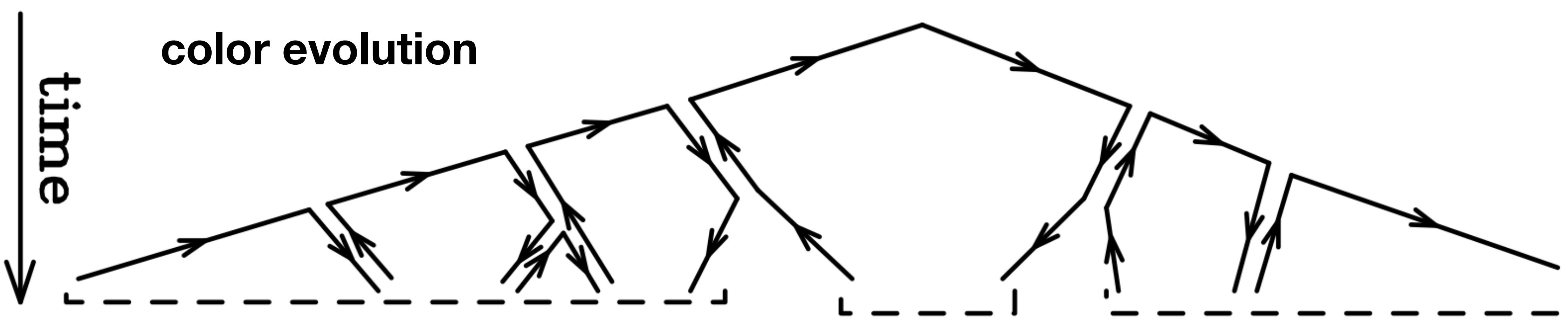
Negative subleading color could lead to non-probabilistic Sudakov factors

More common solution: Leading Color Approximation: Dipole Shower

gluons are replaced by a color triplet-antitriplet pair.



QCD radiation in this approximation is always simulated as the radiation from a single color dipole, rather than a coherent sum from a color multipole.

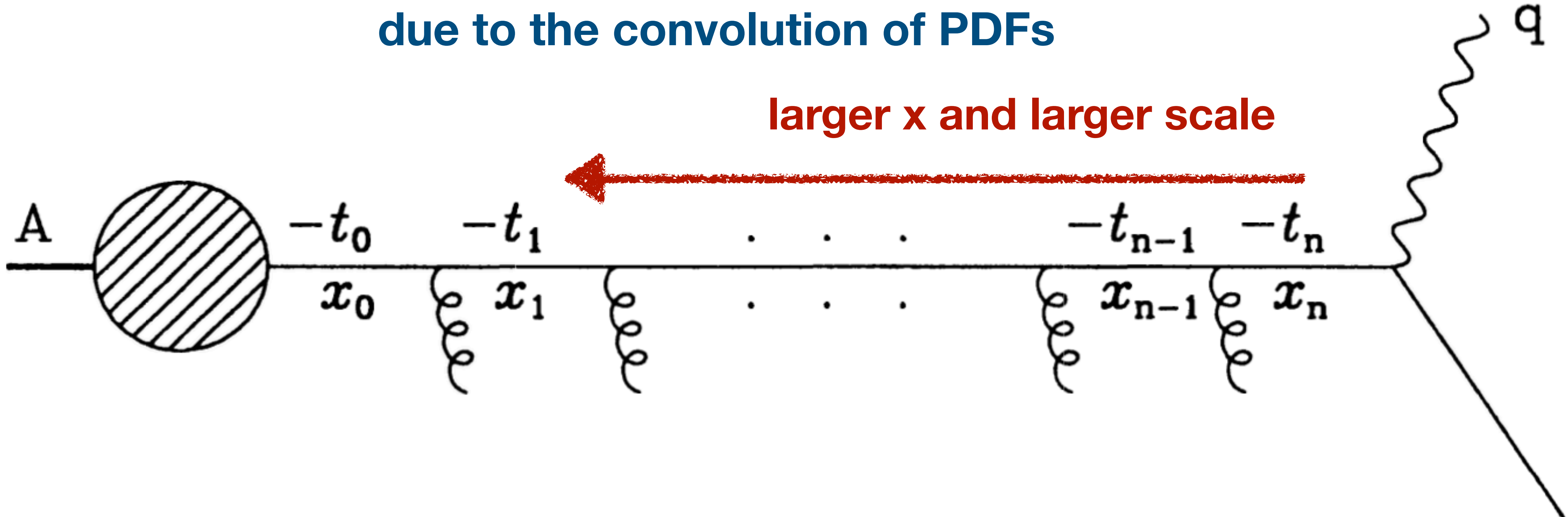


2. Parton Shower

Backward Evolution

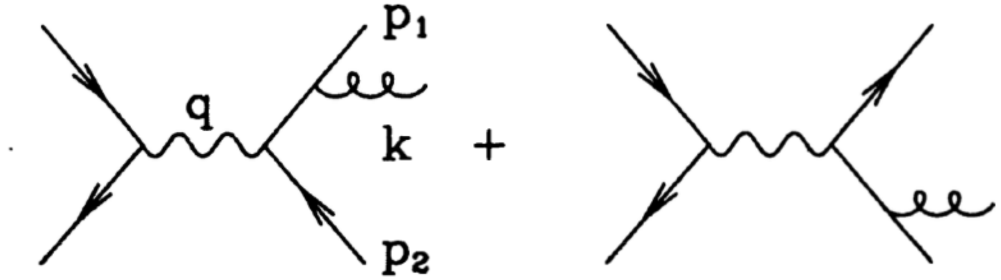
for initial state shower at hadron collider,

a similar construction but more complicated,
due to the convolution of PDFs



The rest of the talk will focus on final state showering

2. Parton Shower



Q is the vitality of the mediated photon

$$|M|^2 \propto \frac{8Q^4}{s_{35}s_{45}} + \frac{8Q^2}{s_{35}} - \frac{4s_{45}}{s_{35}} + \frac{8Q^2}{s_{45}} - \frac{4s_{35}}{s_{45}}$$

Soft **3||5 collinear** **4||5 collinear**

Infrared Divergences come from regions of phase space where massless particles are either soft or collinear to other particles.

Universal properties of the matrix element in collinear and soft limits

$$|M(\cdots, p_i, p_j, \cdots)|^2 \xrightarrow{i||j} g_s^2 \mathcal{C} \frac{P(z)}{s_{ij}} |M(\cdots, p_i + p_j, \cdots)|^2$$

$$|M(\cdots, p_i, p_j, p_k, \cdots)|^2 \xrightarrow{j_g \to 0} g_s^2 \mathcal{C} \frac{2s_{ik}}{s_{ij}s_{jk}} |M(\cdots, p_I, p_K, \cdots)|^2$$

2. Parton Shower

Collinear radiations

Before talking about parton shower, let's take a look at NLO $e^+e^- \to q\bar{q}$

$$\mathrm{d}\sigma_{q\bar{q}g} \approx \sigma_{q\bar{q}} \times \sum_{\mathrm{q,\bar{q}}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\ dz\ \frac{1+(1-z)^2}{z}$$

In the collinear limit, it turns to independent emission distribution for each parton

$$|M(\cdots, p_i, p_j, \cdots)|^2 \overset{i||j}{\to} g_s^2 \mathscr{C} \frac{P(z)}{s_{ij}} |M(\cdots, p_i + p_j, \cdots)|^2$$

From any hard process , the real correction in the collinear limits

$$\mathrm{d}\sigma \approx \sigma_0 \times \sum_{\mathrm{partons,i}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2}\ dz P_{ji}(z, \phi) d\phi$$

The DGLAP splitting function P_{ji} **is universal. After spin averaging, they are**

$$P_{qq}(z) = C_F \tfrac{1+z^2}{1-z}, \qquad P_{gq}(z) = C_F \tfrac{1+(1-z)^2}{z}$$
$$P_{gg}(z) = C_A \tfrac{z^4+1+(1-z)^4}{z(1-z)}, \quad P_{qg}(z) = T_R\left(z^2 + (1-z)^2\right)$$

2. Parton Shower

Soft gluon emission: Coherent branching

- **interference between gluon emission off partons i and j**

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\frac{\alpha_S}{2\pi}\sum_{i,j} C_{ij} W_{ij}$$

$$W_{ij} = \frac{\omega^2 p_i \cdot p_j}{p_i \cdot q\, p_j \cdot q} = \frac{1-\cos\theta_{ij}}{\left(1-\cos\theta_{iq}\right)\left(1-\cos\theta_{jq}\right)}$$

- **partition soft radiations in to i and j collinear sector**

$$W_{ij}^{[i]} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{jq}}\right)$$

- **integrating over the azimuthal angle,**

$$\int_0^{2\pi}\frac{d\phi_{iq}}{2\pi} W_{ij}^{[i]} = \begin{cases} \frac{1}{1-\cos\theta_{iq}} & \text{if } \theta_{iq} < \theta_{ij} \\ 0 & \text{otherwise} \end{cases}$$

leads to angular-ordered parton showers

Soft gluon effects can be correctly taken into account by a collinear patron shower algorithm by angular ordering, dipole showers with transverse momentum ordering.

Parton showers, such as VINCIA, DIRE, are coherent by construction.

2. Parton Shower

Sudakov form factor: Non-branching probability

Probability for generating a branching from parton i between the scale q^2 to $q^2 + dq^2$

$$\mathrm{d}\mathscr{P}_i = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int_{Q_0^2/q^2}^{1-Q_0^2/q^2} \mathrm{d}z P_{ji}(z)$$

The probability that there are no branching from Q to q is $\Delta_i\left(Q^2, q^2\right)$

$$\frac{\mathrm{d}\Delta_i\left(Q^2,q^2\right)}{\mathrm{d}q^2} = \Delta_i\left(Q^2,q^2\right)\frac{\mathrm{d}\mathscr{P}_i}{\mathrm{d}q^2}$$

branching probability at q

no radiation above q

The solution is $\Delta_i\left(Q^2,q^2\right) = \exp\left\{\int_{q^2}^{Q^2}\frac{\mathrm{d}k^2}{k^2}\frac{\alpha_s}{2\pi}\int_{Q_0^2/k^2}^{1-Q_0^2/k^2}\mathrm{d}z P_{ji}(z)\right\}$ **Q_0 is the cutoff scale**

The building block to iterative attach additional partons to a hard process

many choices for the evolution variables

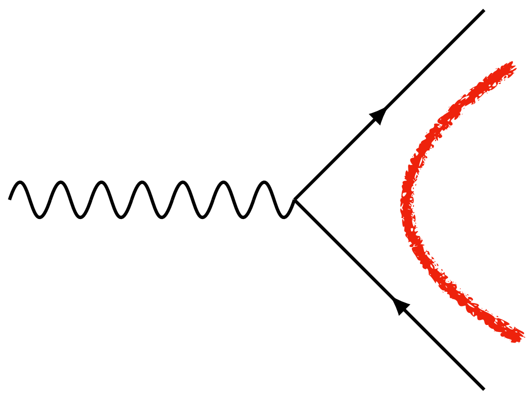
$$\frac{d\theta^2}{\theta^2} = \frac{dq^2}{q^2} = \frac{dk_\perp^2}{k_\perp^2}$$

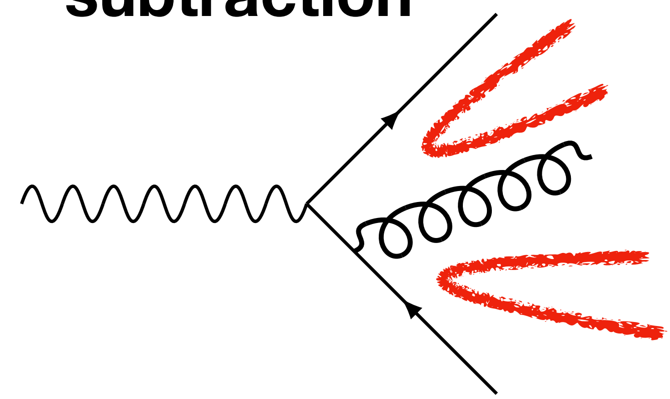
angular **virtuality** **transverse momentum**

2. Parton Shower

NLO cross section $$\sigma_{\rm NLO} = \sigma_0 + \left(\int d\Phi_n V + \int d\Phi_{n+1} S \right) \mathcal{O}_n + \int d\Phi_{n+1} (R \mathcal{O}_{n+1} - S \mathcal{O}_n)$$

virtual **integrated subtraction** **subtracted real**





$1 - \Delta_i(Q^2, Q_0^2)$

From parton shower

$$\sigma^{\rm PS}_{\rm NLO} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$

0-radiation **1-radiation (Sudakov suppressed)**

From the definition of Sudakov factor, we have $\mathcal{P}(\text{unresolved}) + \mathcal{P}(\text{resolved}) = 1$

probability conservation from the definition of Δ

LO parton showers reproduce the NLO singular behavior of the underlying hard process with unitarity assumption $V + \int R = 0$.

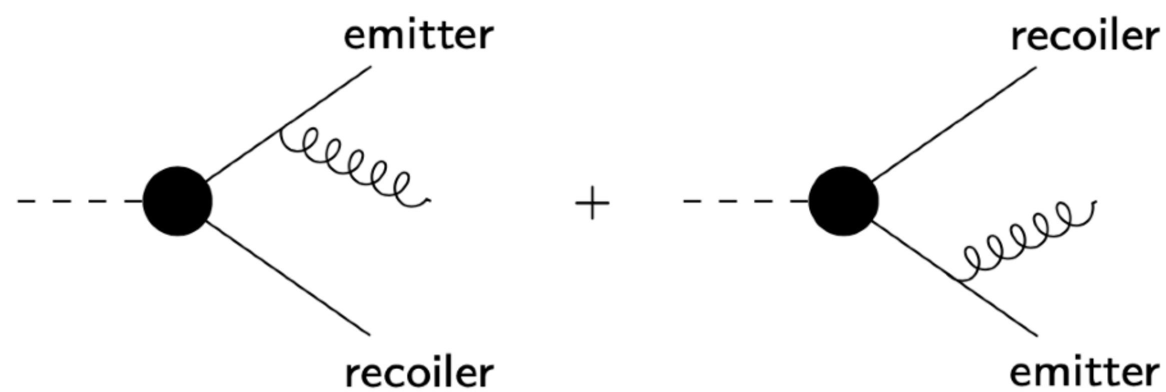
2. Parton Shower

Phase-space mapping

To generate a new radiation, we need to construct three on-shell momenta from two

DGLAP/dipole kinematics distinguish emitter/recoiler:

recoiling by color connected particles



emitter **recoiler**

For branching process $\tilde{ij} + \tilde{k} \to i + j + k$**, with** i **and** j **collinear**

$$p_i^\mu = z\tilde{p}_{ij}^\mu + (1-z)\frac{p_{ij}^2 p_{\tilde{k}}^\mu}{2\tilde{p}_{ij}\cdot\tilde{p}_k} + k_\perp^\mu$$

$$p_j^\mu = (1-z)\tilde{p}_{ij}^\mu + z\frac{p_{ij}^2 p_{\tilde{k}}^\mu}{2\tilde{p}_{ij}\cdot\tilde{p}_k} - k_\perp^\mu$$

$$p_k^\mu = \left(1 - \frac{p_{ij}^2}{2\tilde{p}_{ij}\cdot\tilde{p}_k}\right)p_{\tilde{k}}^\mu$$

Using Monte Carlo method, z **and** $k_\perp$ **can be generated by the Veto Algorithm**

p_{ij}^2 **can be calculated from on-shell conditions**

$$p_{ij}^2 = \frac{k_\perp^2}{z(1-z)}$$

2. Parton Shower

antenna shower: VINCIA

making use of the antenna functions proposed in antenna subtractions

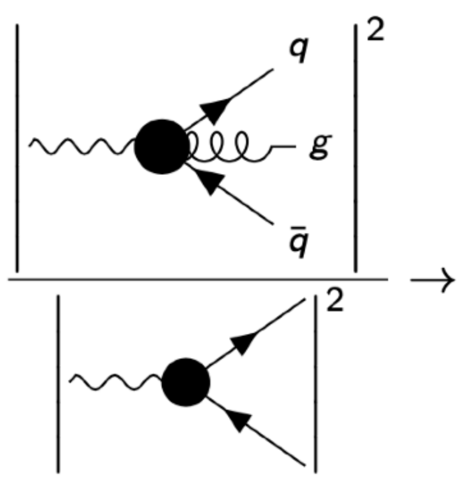
$$\frac{d}{dQ^2}\left(1-\Delta(Q_0^2,Q^2)\right) = -\int \frac{d\Phi_3}{d\Phi_2}\,\delta(Q^2-Q^2(\Phi_3))\;a_3^0\Delta(Q_0^2,Q^2)$$

$$a_3^0 = \frac{|M_{qg\bar{q}}|^2}{|M_{q\bar{q}}|^2}$$

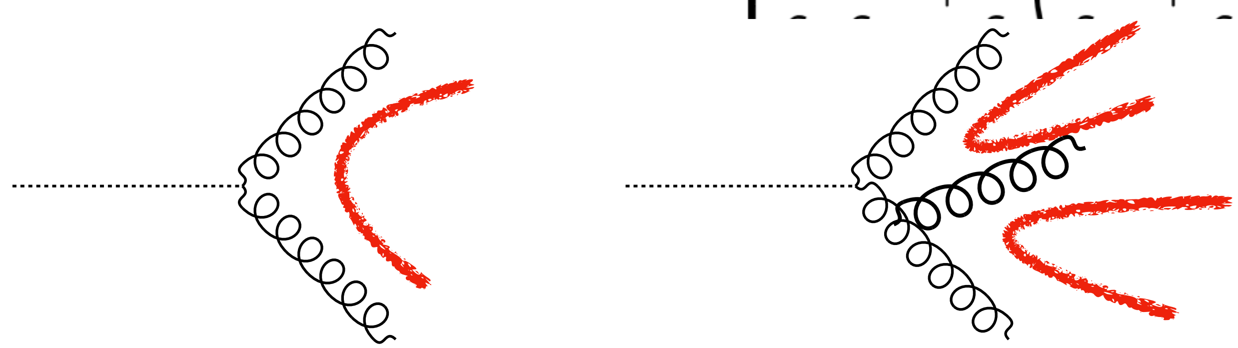
2 to 3 phase space mapping

LO antenna function

include the correct collinear and soft singularities



$$\rightarrow \begin{cases} \underbrace{\frac{P_{q\mapsto qg}(z_q)}{s_{qg}}}_{q\text{-collinear}} + \underbrace{\frac{P_{\bar{q}\mapsto \bar{q}g}(z_{\bar{q}})}{s_{g\bar{q}}}}_{\bar{q}\text{-collinear}} & \text{DGLAP} \\ \underbrace{\frac{K_{q\mapsto qg;\bar{q}}(z_q)}{s_{qg}}}_{q\text{-(soft-)collinear}} + \underbrace{\frac{K_{\bar{q}\mapsto \bar{q}g;q}(z_{\bar{q}})}{s_{g\bar{q}}}}_{\bar{q}\text{-(soft-)collinear}} & \text{CS Dipoles} \\ \frac{2s_{q\bar{q}}}{\ldots} + \frac{1}{\ldots}\left(\frac{s_{g\bar{q}}}{\ldots} + \frac{s_{qg}}{\ldots}\right) & \text{Antennae} \end{cases}$$



gluon antenna functions: singularities are shared with neighboring dipole

2. Parton Shower

antenna sector shower: VINCIA

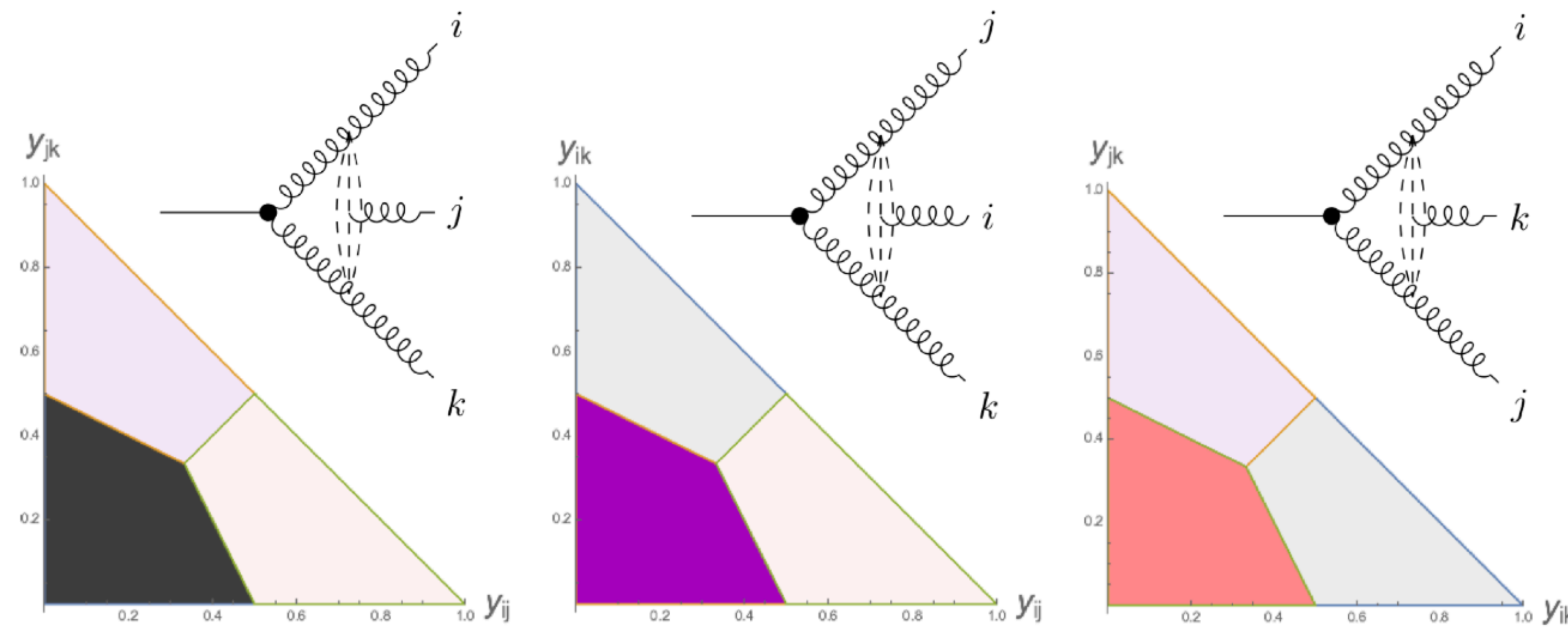
Brooks, Pauss, Skands, 2020

for $g_{\tilde{i}} + g_{\tilde{j}} \to g_i + g_j + g_k$

$$p^2_{T,ijk} = \frac{s_{ij}s_{jk}}{s_{ijk}} = s_{ijk}y_{ij}y_{jk}$$

$$p^2_{T,ikj} = \frac{s_{ik}s_{jk}}{s_{ijk}} = s_{ijk}y_{ik}y_{jk}$$

$$p^2_{T,jik} = \frac{s_{ij}s_{ik}}{s_{ijk}} = s_{ijk}y_{ij}y_{ik}$$



- **Sector defined by the configuration with smallest scale in the event**
  - **Sector Antenna include full soft singularity**
  - **Haft of the collinear singularity for shared sector**
  - **No trivial sector boundary for non-singular contribution**

- **Branchings in the shower are accepted if and only if they correspond**
- **Showering Path Predictable for a given event**

We expect that for sector shower, it is relatively easier to include NLO corrections

2. Parton Shower

NLO DGLAP shower

Prestel, Hoche, 2017

A direct approach: higher-order DGLAP kernels

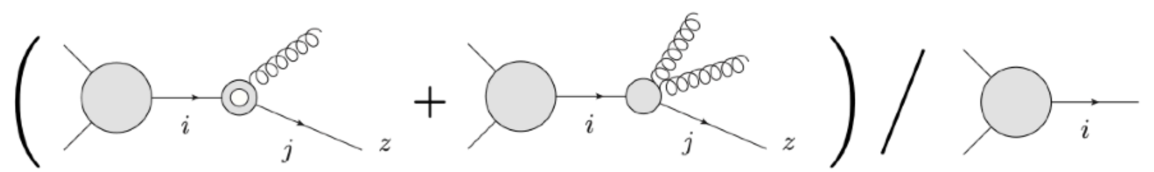
$$D_{ji}^{(0)}(z,\mu) = \delta_{ij}\delta(1-z) \quad \leftrightarrow$$

$$D_{ji}^{(1)}(z,\mu) = -\frac{1}{\varepsilon}P_{ji}^{(0)}(z) \quad \leftrightarrow$$

LO shower, $1 \to 2$

$$D_{ji}^{(2)}(z,\mu) = -\frac{1}{2\varepsilon}P_{ji}^{(1)}(z) + \frac{\beta_0}{4\varepsilon^2}P_{ji}^{(0)}(z) + \frac{1}{2\varepsilon^2}\int_z^1 \frac{\mathrm{d}x}{x}P_{jk}^{(0)}(x)P_{ki}^{(0)}(z/x)$$

$\leftrightarrow$



NLO shower, $1 \to 2$ and $1 \to 3$

Prerequisite for NNLL accuracy in an observable-independent way

Dulat, Prestel, Hoche, 2018

Leading-color fully differential two-loop soft corrections to dipole shower was derived

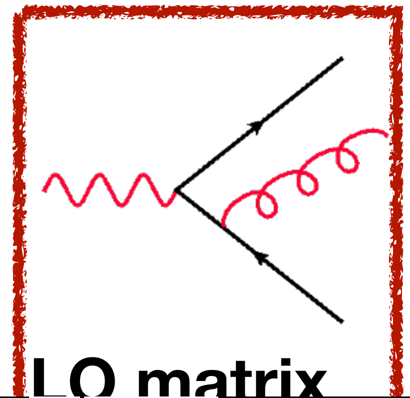
The two-loop cusp anomalous dimension is recovered naturally upon integration over the full phase space.

2. Parton Shower

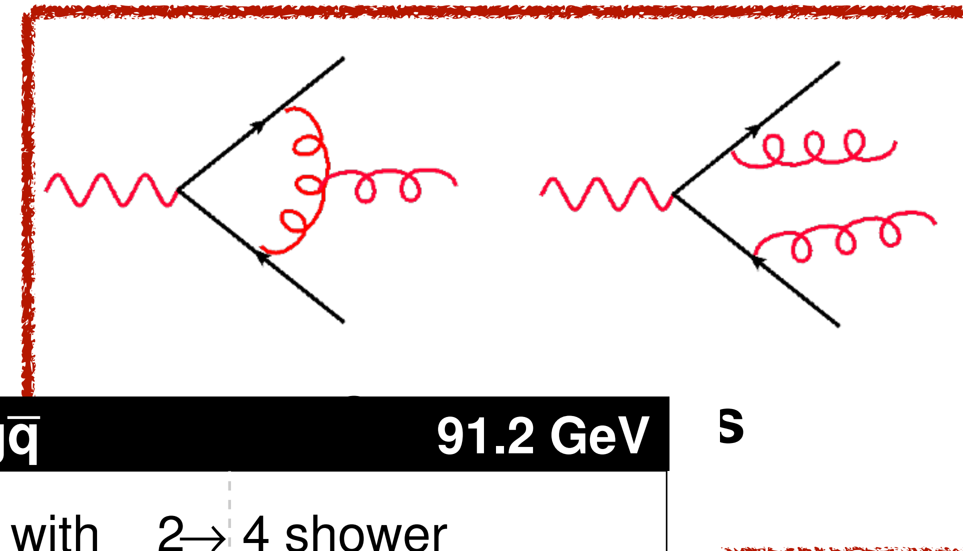
antenna NLO shower: VINCIA

HTL, Skands, 2017

begin with $q + \bar{q}$ dipole



LO matrix



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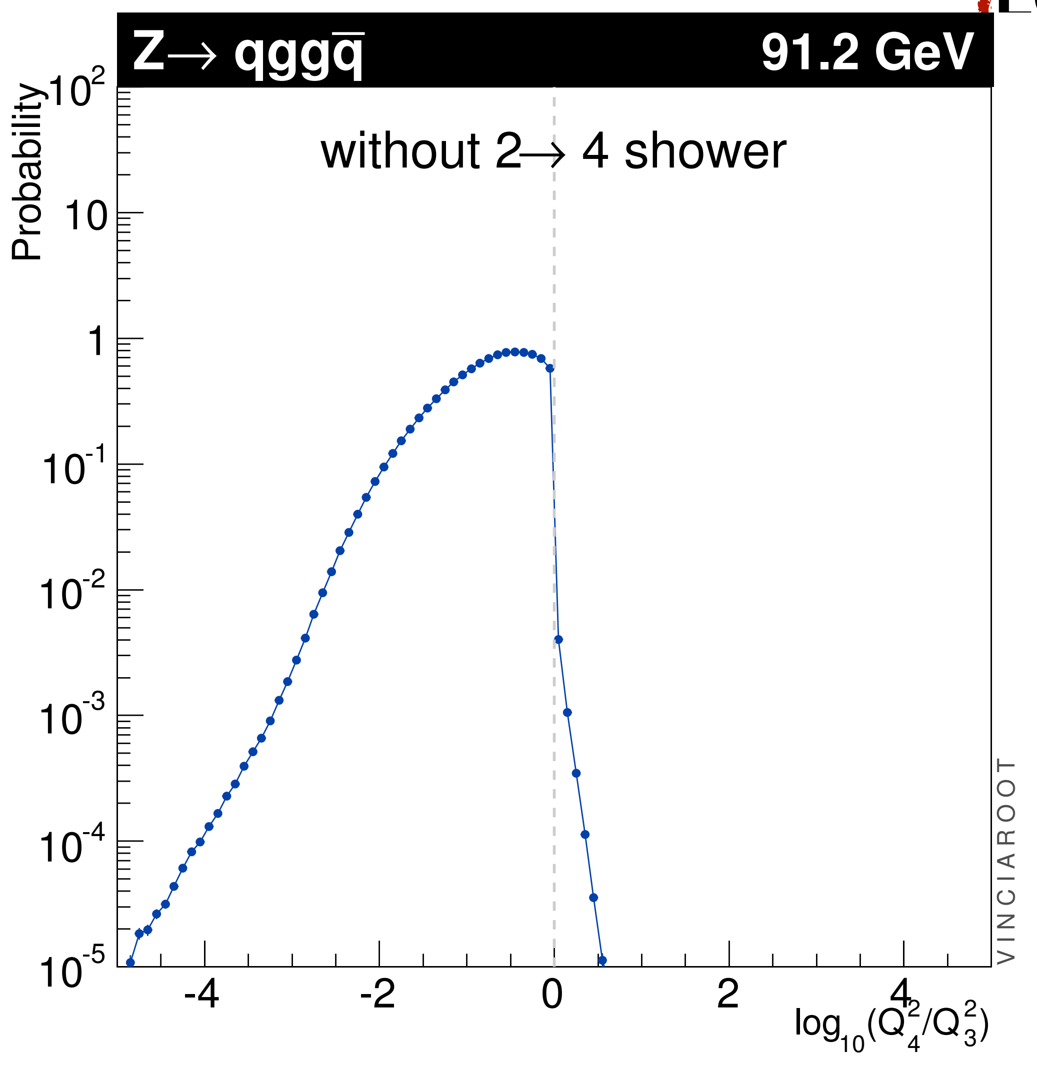
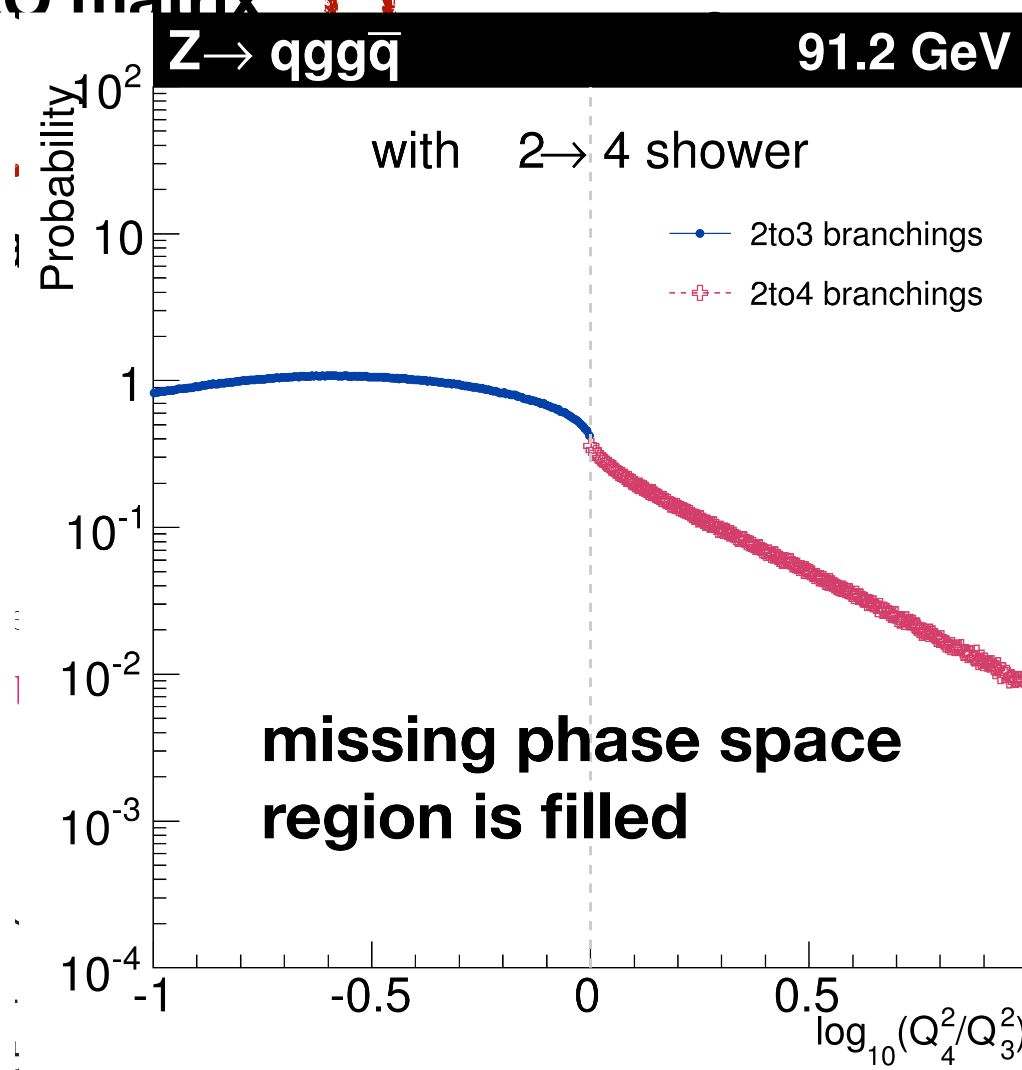




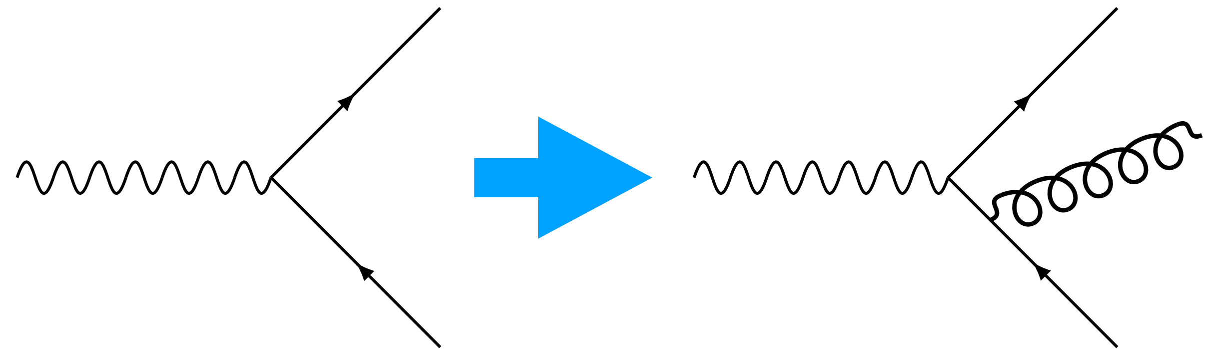
number of branching

2. Parton Shower

Resummation

Proved that parton shower achieves the LL resummation

$$\sigma_{\rm NLO}^{\rm PS} = \sigma_0 \Pi_i \left(\Delta_i(Q^2, Q_0^2) + \int_{Q_0^2}^{Q^2} \frac{dq^2}{q^2} \Delta_i(Q^2, q^2) \int dz P_{ji}(z) \right)$$



LO showers reproduce the IR configuration for ME with one additional radiation
Equivalently LO parton shower includes one-loop anomalous dimensions

To compare with NLL resummation, needs to cover the double soft radiations (double log, governed by cusp anomalous dimensions)

Usually, the two-loop cusp anomalous dimension is included by CMW coupling

$$\alpha_s^{\rm CMW}(\mu) = \alpha_s(\mu)\left(1 + \frac{\alpha_s(\mu)}{2\pi} \times K\right) \qquad K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_f$$

Catani, Webber, Marchesini, 1988

For more meaningful parton showers, in the shower kernel, the scale is set to be the evolution scale

2. Parton Shower

Resummation

LO shower can achieve LL resummation, and include most part of NLL resummation

Why still not NLL?

Hoche, Erichelt, Siegert, 2017

parton shower is momentum conserving (recoil required). (resolved)
parton shower is unitary, NLL is not. (unresolved)

special recoil

special evolution scale

First proved NLL parton shower by PanScale collaboration

Dasgupta et al 2020

$$\frac{d\mathscr{P}_{n\to n+1}}{d\ln v} = \sum_{\text{dipoles}\ \{\tilde{\imath},\tilde{\jmath}\}} \int d\bar{\eta} \frac{d\phi}{2\pi} \frac{\alpha_s(k_t) + K\alpha_s^2(k_t)}{\pi} \times \left[g(\bar{\eta}) a_k P_{\tilde{\imath}\to ik}(a_k) + g(-\bar{\eta}) b_k P_{\tilde{\jmath}\to jk}(b_k) \right]$$

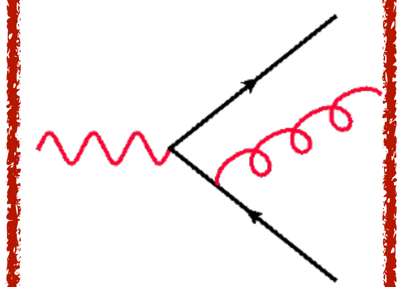
with a careful choice of the momentum mapping schemed and evolution scale

The showers were compared with NLL resummation by runing multiple small values of α_s, extrapolate to $\alpha_s \to 0$ and keeping $\alpha_s L$

2. Parton Shower

Resummation of NLO parton showers

Evolution kernel reproduces the singular of the matrix element at NNLO



LO matrix element

NLO corrections

3 parton final state

3+4 parton final state

Two-loop anomalous dimensions are included correctly at leading color

resummation beyond NLL

NNLL if three loop cusp included

Many efforts in this direction

Dulat, Prestel, Hoche, 2018;
HTL, Skands, 2017
Compbell, Hoche, HTL,Pruss, Skands, 2021

And also parton showers beyond Leading color,
Nagy, Soper, 2019; DeAngels, Forshw, Platzer, 2020; Hamilton etal 2021

2. Parton Shower

Monte-Carlo Methods:

Generating the new scale Q by solving $\Delta(Q_0^2, Q^2) = R$ $\quad \int_{z_{\min}}^{\bar{z}} dz P(z) = R \int_{z_{\min}}^{z_{\max}} dz P(z)$

with a uniform random number $R \in [0,1]$

If $\Delta(Q_0^2, Q^2) = R$ can not be solved, veto algorithm is used in parton shower

To simplify the notation

$$\mathscr{P}_1(t, t') = f(t) \exp\left\{ -\int_t^{t'} d\bar{t} f(\bar{t}) \right\} = \frac{d}{dt} \exp\left\{ -\int_t^{t'} d\bar{t} f(\bar{t}) \right\} \qquad F(t) = \int^t dt f(t)$$

a new scale t determined by $\quad t = F^{-1}\left[F(t') + \log R\right]$

We could find a simple function $g(x) > f(x)$ with a known $G(x)$

Generating a new scale $t = G^{-1}(G(t') + \log R)$

Accept the new scale with probability $\dfrac{f(x)}{g(x)}$

Veto algorithm

$G(x)$ may also overestimate the phase space. Then, phase space veto is required.

2. Parton Shower

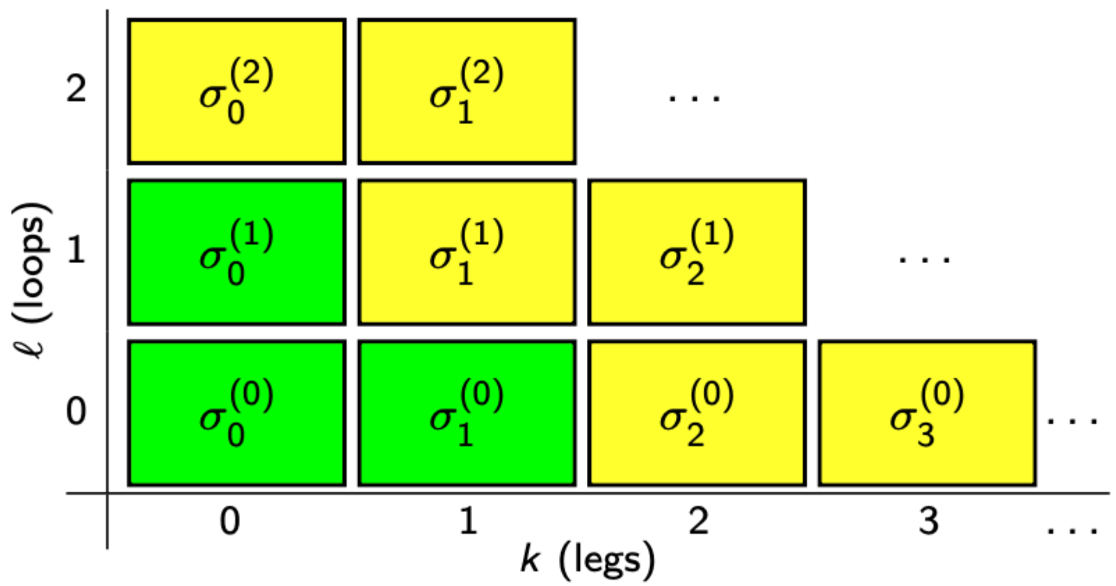
no	id	name	status	mothers		daughters		colours		p_x	p_y	p_z	e	m
0	90	(system)	-11	0	0	0	0	0	0	0.000	0.000	0.000	91.188	91.188
1	11	(e-)	-12	0	0	3	0	0	0	0.000	0.000	45.594	45.594	0.001
2	-11	(e+)	-12	0	0	4	0	0	0	0.000	0.000	-45.594	45.594	0.001
3	11	(e-)	-21	1	0	5	0	0	0	0.000	0.000	45.594	45.594	0.000
4	-11	(e+)	-21	2	0	5	0	0	0	0.000	0.000	-45.594	45.594	0.000
5	23	(Z0)	-22	3	4	6	7	0	0	0.000	0.000	0.000	91.188	91.188
6	3	(s)	-23	5	0	8	9	101	0	-18.850	-40.375	9.661	45.594	0.000
7	-3	(sbar)	-23	5	0	9	10	0	101	18.850	40.375	-9.661	45.594	0.000
8	3	(s)	-51	6	0	11	12	101	0	-18.860	-38.847	10.173	44.365	0.000
9	21	(g)	-51	6	7	12	13	112	101	4.008	1.390	-4.409	6.119	0.000
10	-3	sbar	51	7	0	0	0	0	112	14.852	37.457	-5.764	40.704	0.000
11	3	s	51	8	0	0	0	101	0	-18.216	-34.723	9.239	40.285	0.000
12	21	g	51	8	9	0	0	115	101	0.731	-4.823	0.602	4.915	0.000
13	21	g	51	9	0	0	0	112	115	2.633	2.089	-4.077	5.284	0.000
			Charge sum:		0.000			Momentum sum:		0.000	0.000	-0.000	91.188	91.188

1. **id: particle id.**
2. **Status: negative for intermediate particles, positive for final state particles**
3. **mothers/daughters to track the showering history**
4. **colors store the color information (color, anti-color).**
5. **each step of shower keep the momentum conserved**

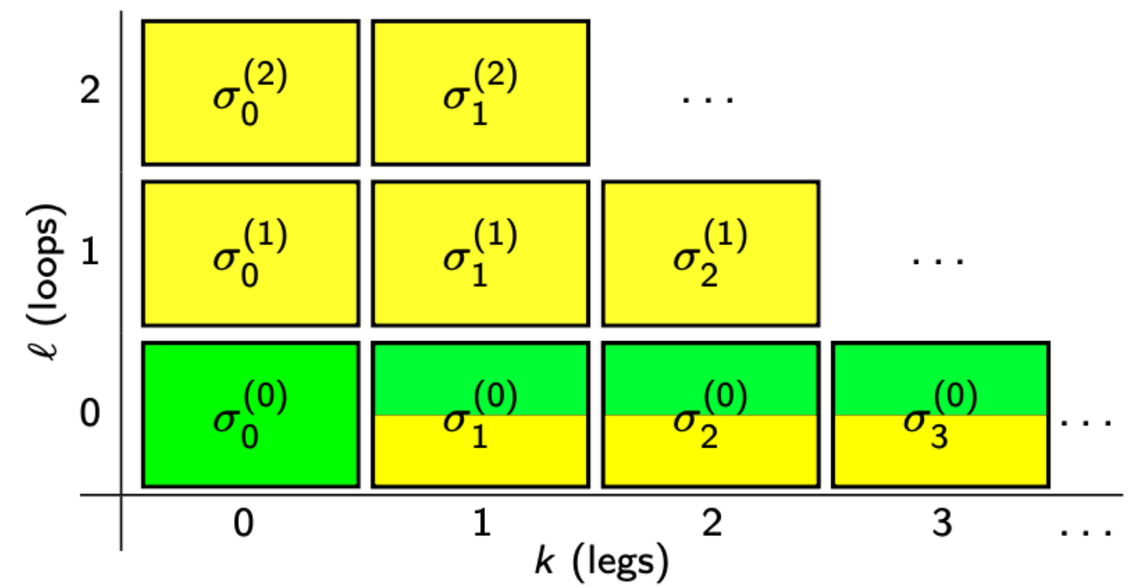
3. Matching and merging

3. Matching and Merging

NLO+PS



LO$_n$+PS (Merging)



Matching:

- combine a fixed-order (typically NLO) calculation with a parton shower, avoiding double-counting in overlap regions

Merging:

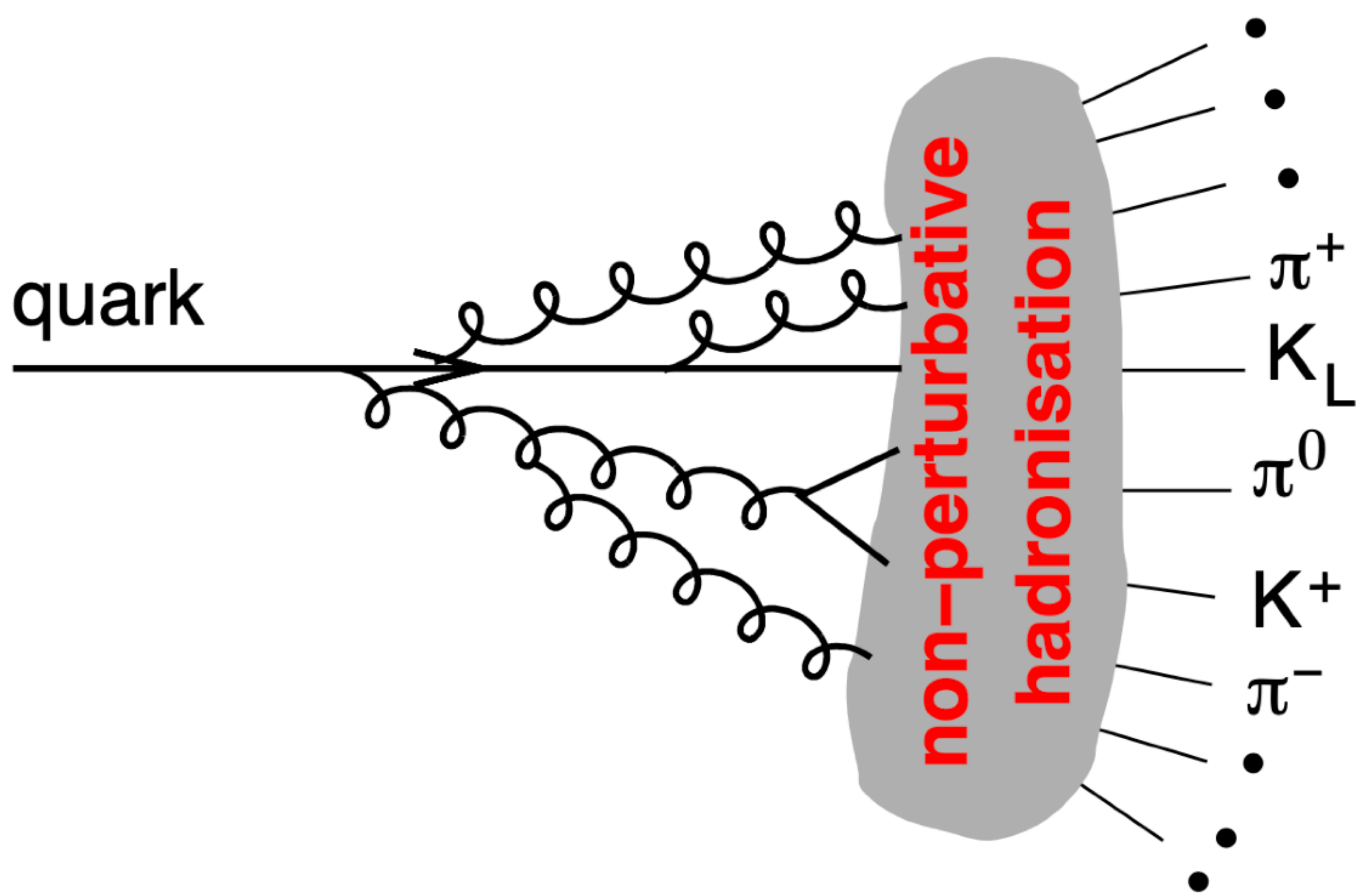
- combine multiple inclusive (N)LO event samples into a single inclusive one with additional shower radiation, accounting for Sudakov suppression and avoiding double-counting in overlap regions (typically via phase-space slicing)

Summary

Indispensable tools for particle physics phenomenology at hadron colliders.

- **Parton showers are built on soft and collinear approximations to the full cross sections**
  - **conserve flavor and four momentum, and**
  - **constructed with the assumption unitarity,**
- **Showers generate singular parts of higher-order matrix elements and evolve events from high scale to hadronization scale.**
- **Recent developments of parton showers**
- **Matrix element corrections improve the prediction for hard radiations; Matching and merging.**

Many components of Monte Carlo Event Generators are not discussed here
Underlying Events, Hadronization, Hadron Decay…..



Thank you!